

Study solid cylinder torsion using 360° DIC

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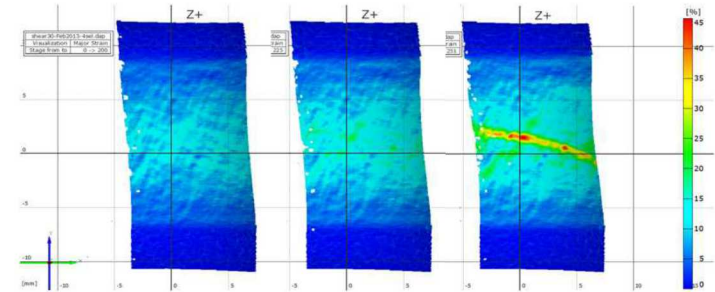
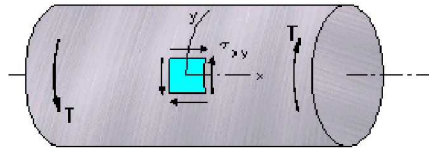
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Ductile Shear Deformation and Failure Experiments

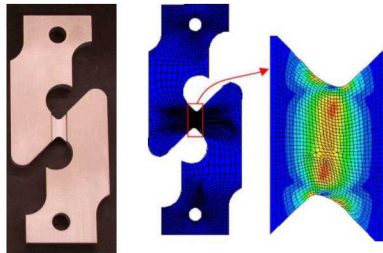
Torsion

- Thin wall tube
- Solid or thick wall cylinder

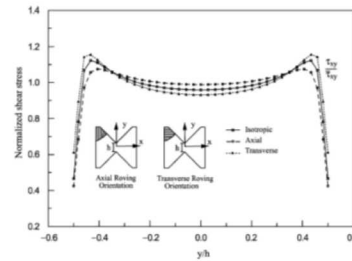
Others (Shear Dominated)



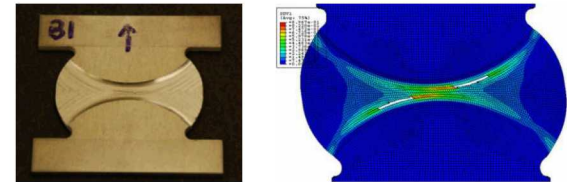
Lu, W. and Jin, H., 2014



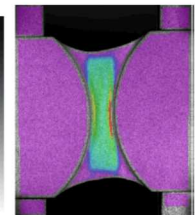
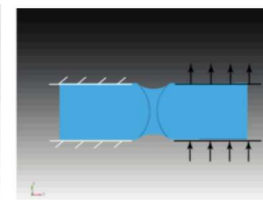
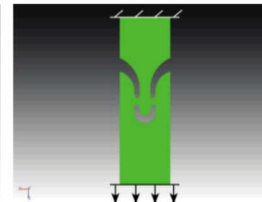
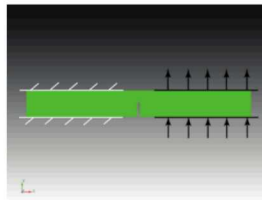
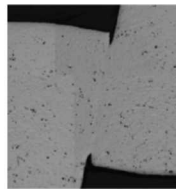
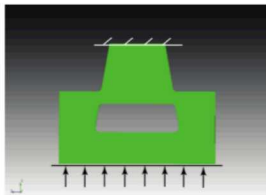
Bao, Y., and Wierzbicki, T., 2004



El-Hajjar, R. and Haj-Ali, R., 2004

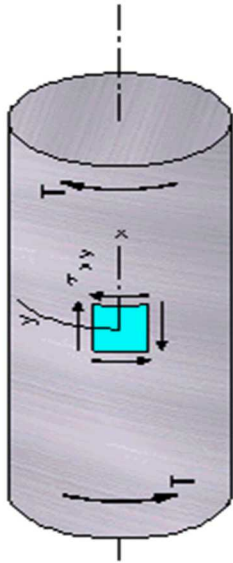


Beese, A. M., Luo, M., Li, Y., Bai, Y., and Wierzbicki, T., 2010



Edmundo Corona et al., SAND2016-9285R, SAND2016-7972C

Solid Cylindrical Specimen Torsion



- Large pure shear deformation
- Stable behavior at large deformation till failure
- Analysis is more complex than thin-walled tube specimens
- Few studies have been done on solid cylinder torsion

(1950) A Nadai, Theory of Flow and Fracture of Solids, McGraw-Hill Book Company

(1991) Richard E Lyon, "Shear Strength of a Ductile Material from Torsion of Solid Cylinders," *Journal of Testing and Evaluation*, JTEVA, Vol. 19, No. 3, pp. 240-243

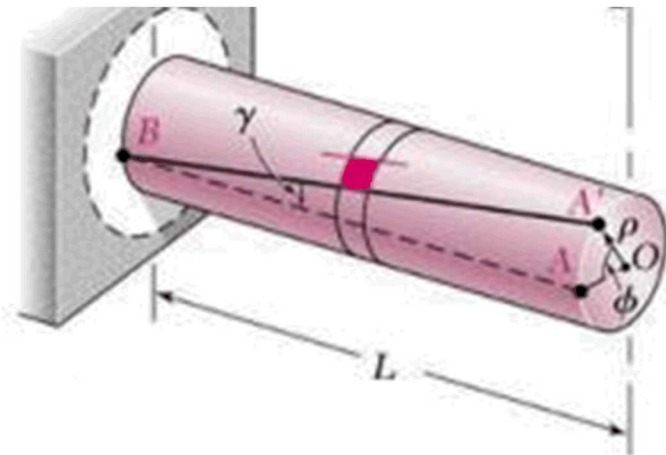
(1992) Han C. Wu, Zhiyou Xu and Paul Wang, "The Shear Stress-Strain Curve Determination from Torsion Test in the Large Strain Range," *Journal of Testing and Evaluation*, JTEVA, Vol. 20, No. 6, pp. 396-402

(2011) Cassandra M. Kingsbury, Preston A. May, Douglas A. Davis, Scott R. White, Jeffrey S. Moore and Nancy R. Sottos, "Shear activation of mechanophore-crosslinked polymers," *Journal Materials Chemistry*, 21, 8381-8388

(2014) Jessica Papasidero, Véronique Doquet, Sebastien Lepeer, "Multiscale investigation of ductile fracture mechanisms and strain localization under shear loading in 2024-T351 aluminum alloy and 36NiCrMo16 steel," *Materials Science & Engineering A*, 610, 203–219

Nadai's Solution

1. Plane sections remain plane
2. All radii remain straight
3. No change in axial length
4. No change in radius
5. Material is isotropic



- Strain $\gamma_o = r_o \omega, \quad \omega = \frac{d\phi}{dz}$

- Stress

- Thin wall tube

$$\tau_o = \frac{T r_o}{I_z}, \quad I_z = \frac{\pi r_o^4}{2} \left(1 - \frac{r_i^4}{r_o^4} \right)$$

- Solid Cylinder

$$\tau_o = \frac{1}{2\pi r_o^3} \left(\omega \frac{dT}{d\omega} + 3T \right)$$

Wu's Method

1. Plane sections remain plane
2. All radii remain straight
3. Change in axial length, second-order strain effect
4. Change in radius, second-order strain effect
5. Material is transversely isotropic

Wu's solution for large torsion adding axial elongation and radius change of cylinder:

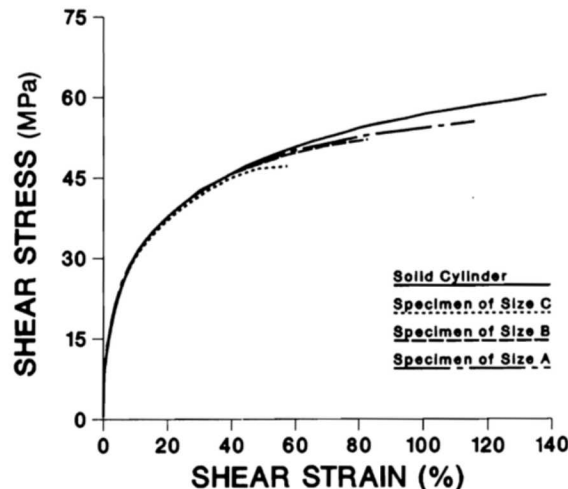
- Strain $\gamma_t = r\omega = r_o(1 + e_x)\omega$

$$e_x = e_z = -\frac{1}{2}e_y$$

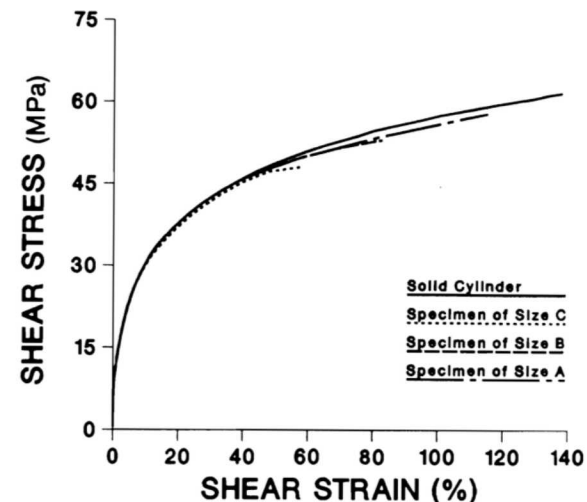
- Stress
 - Solid Cylinder

$$\tau_t = \frac{1}{2\pi r_o^3} \left(\omega \frac{dT}{d\omega} + 3T \right) \left(\frac{1}{1 - \frac{1}{2} \left(3e_y + \omega \frac{de_y}{d\omega} \right)} \right)$$

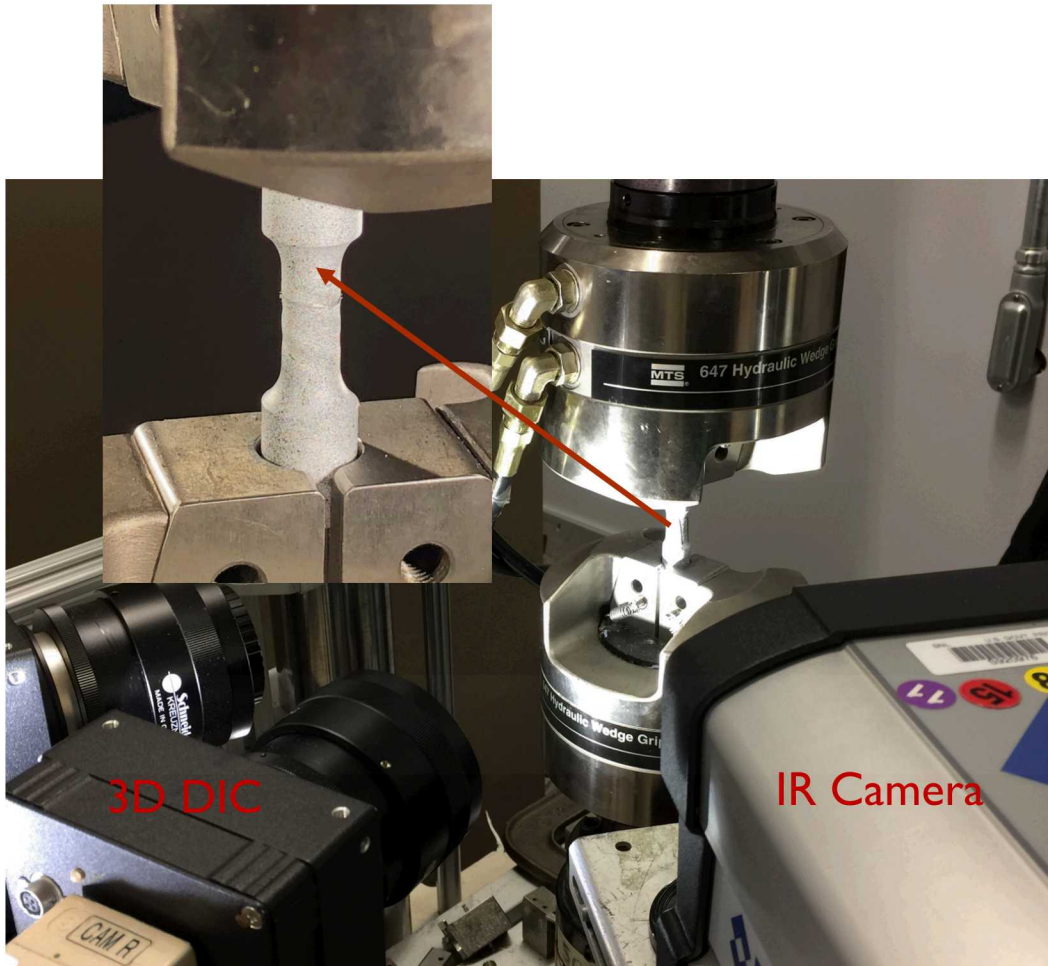
Nadai's method



Wu's method



Torsion of Solid Cylindrical Al Specimens

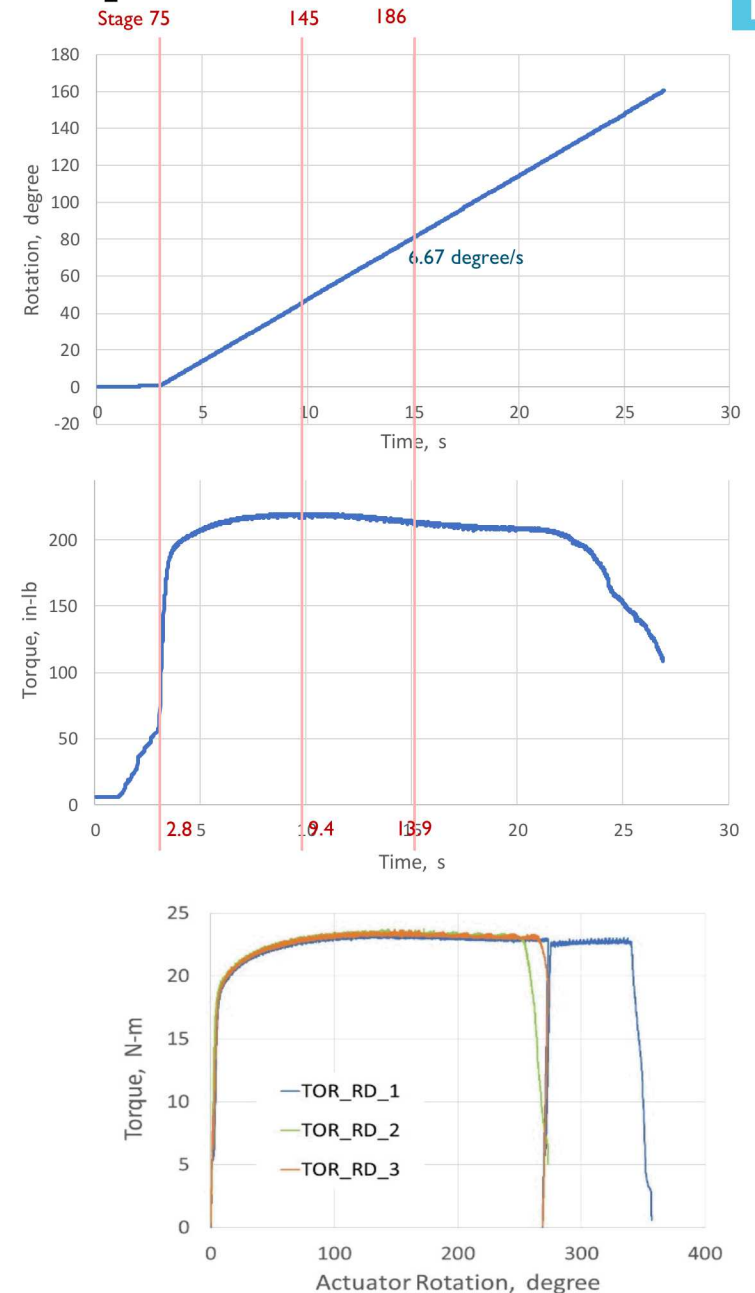
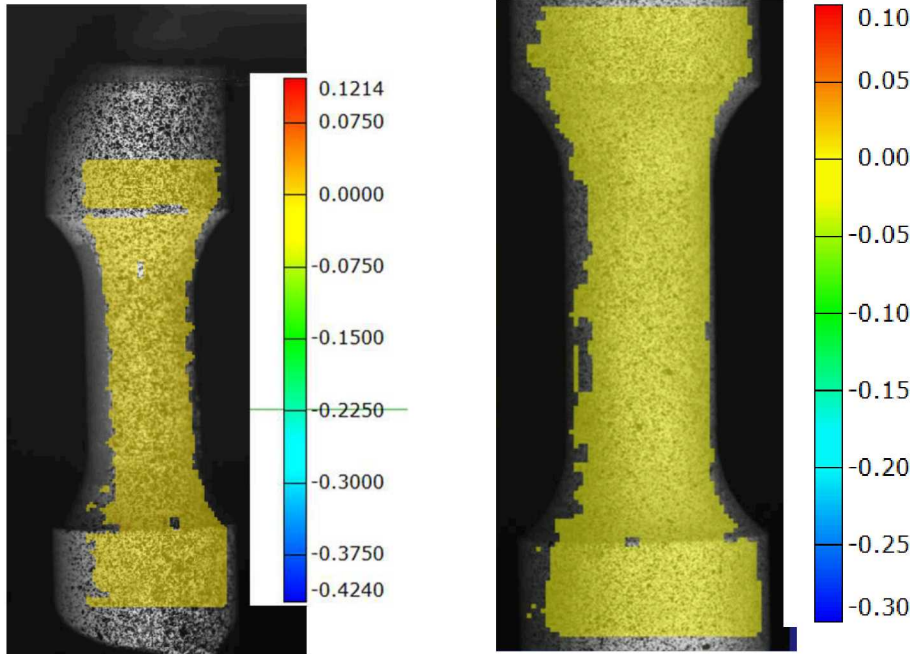


1. MTS Mini Bionix AT Testing System
2. GOM ARAMIS System (3D DIC)
 - Control parameter
 - Axial: Force
 - Torsion: Rotation

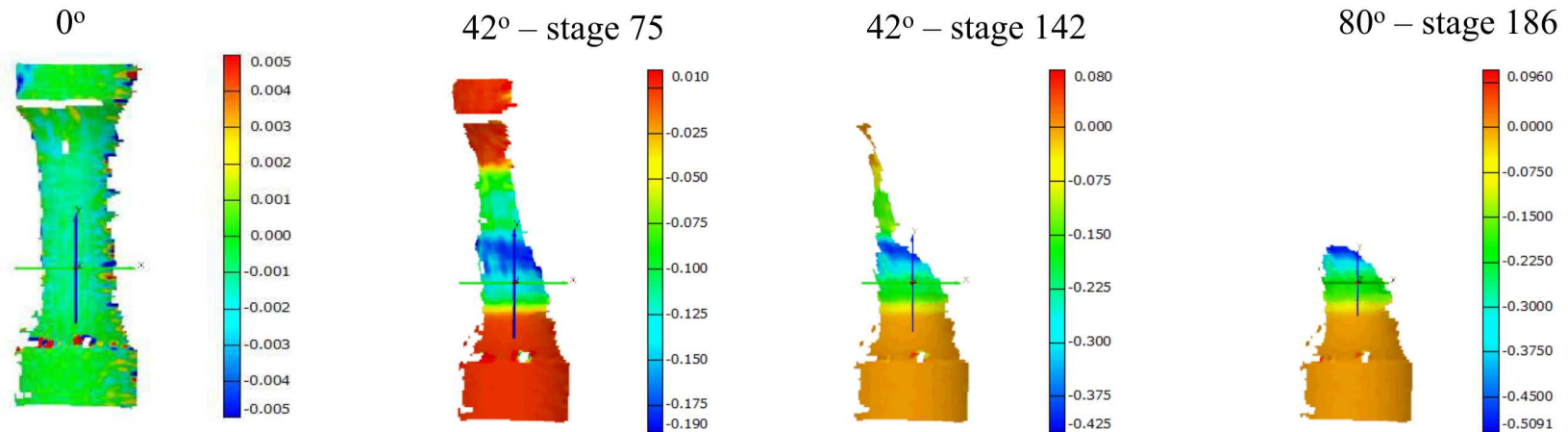
Torsion experiment with 3D-DIC

Shear Strain of Torsion Experiment

Video: Solid_3 ϵ_{xy} 6.67 degree/s, DIC, IR
Actual testing time: ~ 25 s

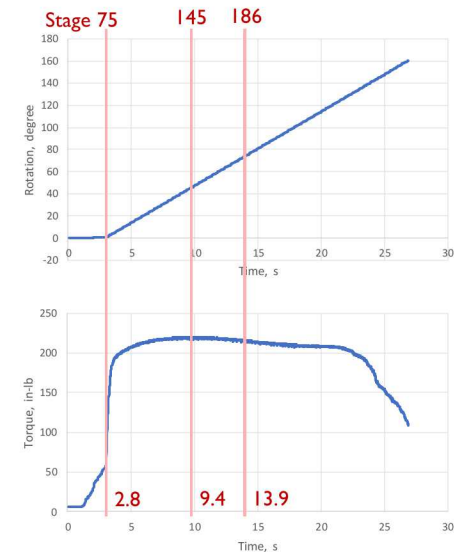
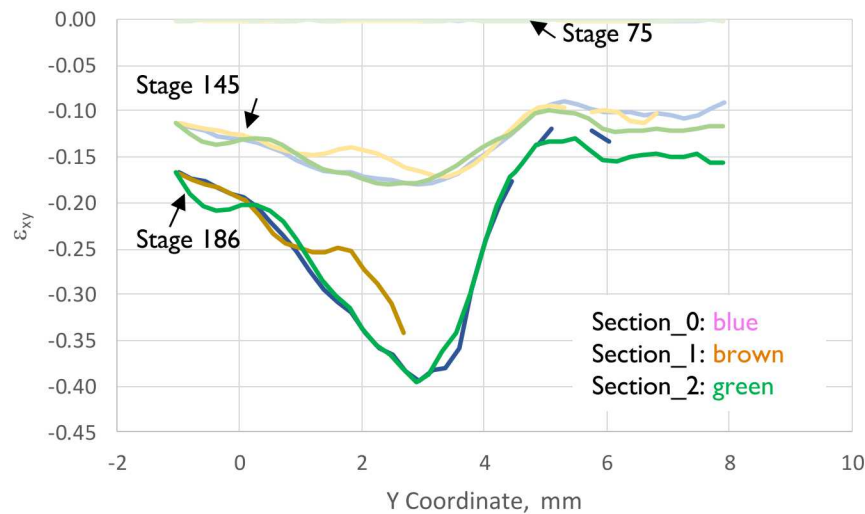
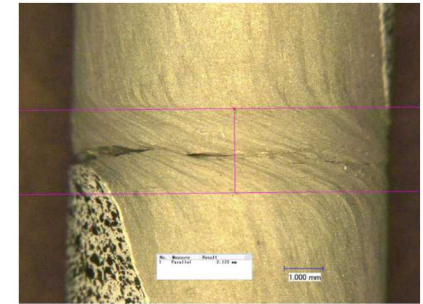
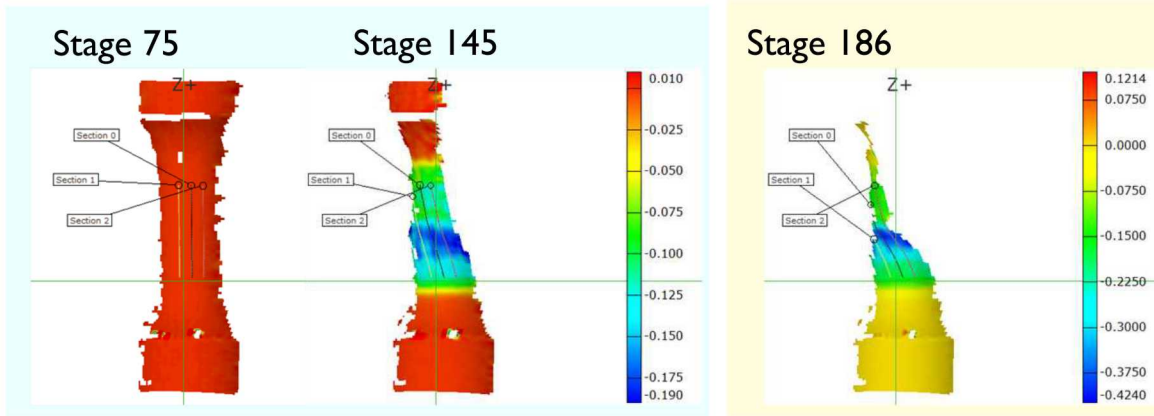


Solid_3 Shear Strain e_{xy} Distribution at Different Stages

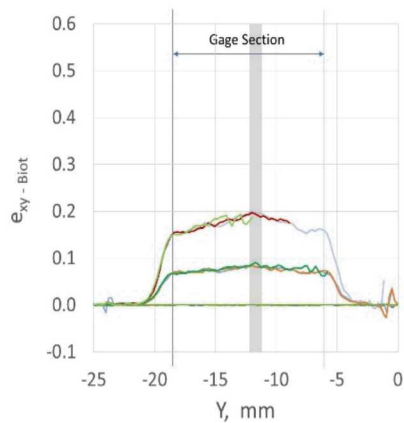


- (1) Full-field shear strain e_{xy} measurement with 3D-DIC system;
- (2) Due to large rotation the area utilized for DIC measurements, or the measurable area, became smaller and smaller. Since a pair of cameras only covered a portion of the cylinder surface, part of the DIC pattern gradually rotated out of the camera view; and those speckles newly rotated into the view did not have a reference state.

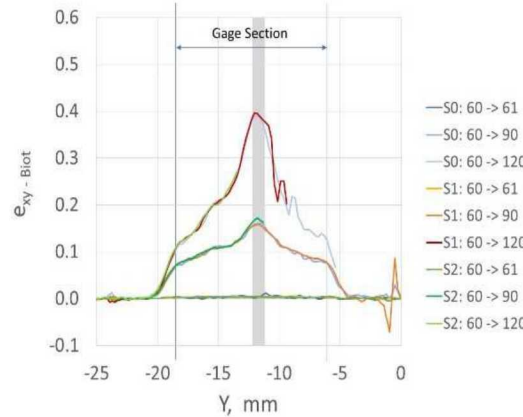
Solid_3 Shear Strain e_{xy} Distribution



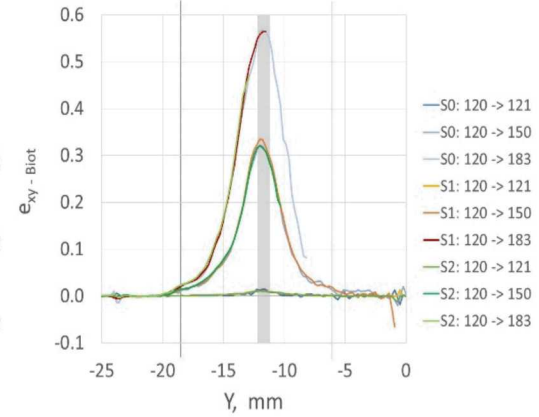
Shear Strain e_{xy} from Incremental DIC Analysis



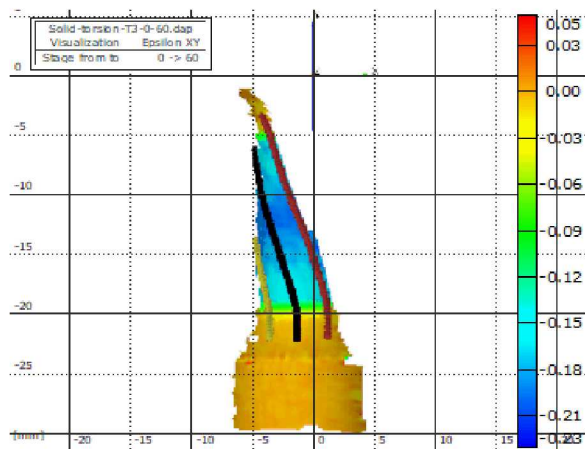
(a) 0 -> 60



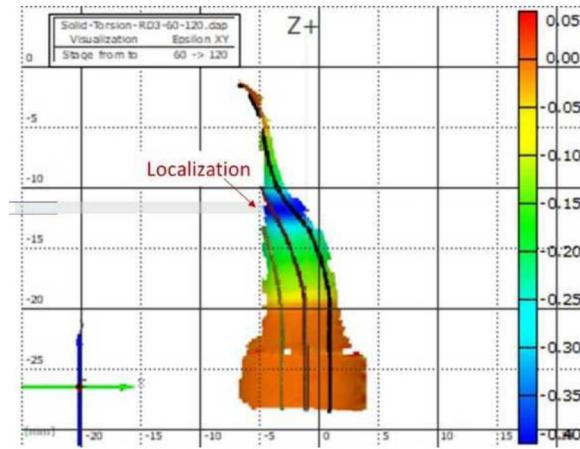
(b) 60 -> 120



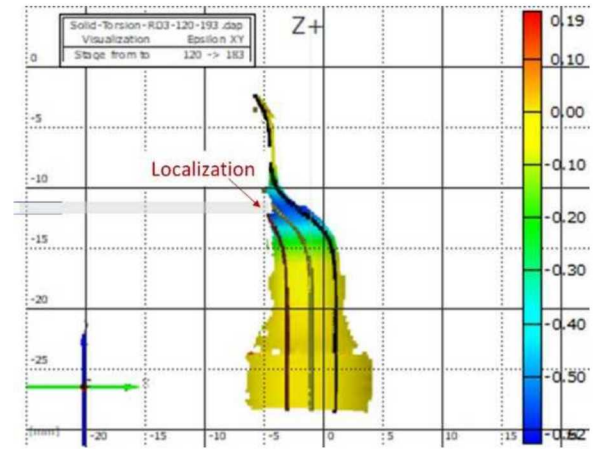
(c) 120 -> 183



(d) Stage 0 -> 60



(e) Stage 60 -> 120

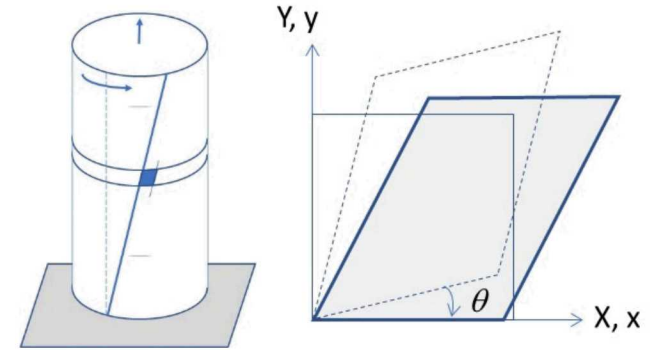


(f) stage120 -> 183

Deformation Gradient and Engineering Shear Strain e_{xy}

Deformation gradient tensor:

$$F = \begin{bmatrix} p & k & 0 \\ 0 & q & 0 \\ 0 & 0 & s \end{bmatrix} = \begin{bmatrix} 1 + e_x & \gamma & 0 \\ 0 & 1 + e_y & 0 \\ 0 & 0 & 1 + e_z \end{bmatrix}$$



$$F = RU$$

Where R is the rotation tensor
U is the right stretch tensor

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} p\cos\theta & p\sin\theta & 0 \\ p\sin\theta & k\sin\theta + q\cos\theta & 0 \\ 0 & 0 & s \end{bmatrix}$$

Biot strain $\bar{U}$ is defined from the right stretch tensor as follows:

$$\bar{U} = \begin{bmatrix} e_{x-B} & e_{xy-B} & 0 \\ e_{xy-B} & e_{y-B} & 0 \\ 0 & 0 & e_{z-B} \end{bmatrix} = U - I$$

- Biot strain is direct output from DIC.
- Deformation gradient can be calculated from Biot strain.

$$p = e_{xy-B} / s \sin\theta = (e_{x-B} + 1) / c \cos\theta,$$

$$q = (1 + e_{y-B} - e_{xy-B} \tan\theta) \cos\theta, \quad \tan\theta = \frac{e_{xy-B}}{e_{x-B} + 1}$$

Mapping Engineering Shear Strain e_{xy} back to the Original Reference

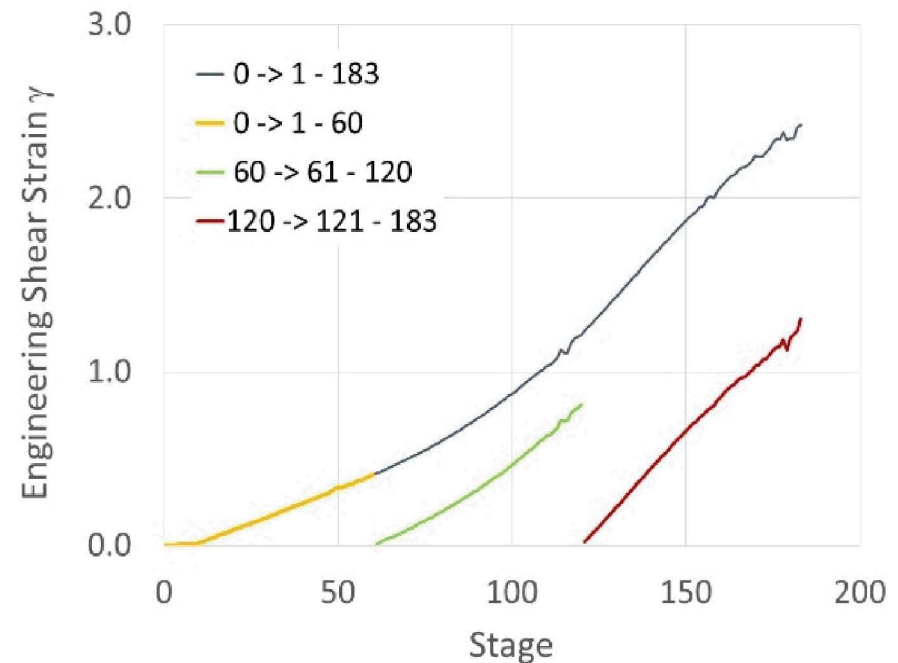
Calculating shear strain from deformation tensor

$$\mathbf{F}_i = \mathbf{F}_i' \mathbf{F}_I = \begin{bmatrix} p_i' & k_i' & 0 \\ 0 & q_i' & 0 \\ 0 & 0 & s_i' \end{bmatrix} \begin{bmatrix} p_I & k_I & 0 \\ 0 & q_I & 0 \\ 0 & 0 & s_I \end{bmatrix}$$

$$\mathbf{F}_i = \mathbf{F}_i' \mathbf{F}_{60}, \text{ for } 60 < i \leq 120$$

$$\mathbf{F}_i = \mathbf{F}_i' \mathbf{F}_{120}, \text{ for } i > 120.$$

Three “incremental” shear strain curves 0 -> 1 - 60, 60 -> 61 - 120 and 120 -> 121 - 183 are based on three distinct references. After transformation and composing, curve 0 -> 1 - 183 is the shear strain history of the specimen.



Shear Stress ~ Shear Strain

Calculating Shear Stress

$$\tau = \frac{1}{2\pi r_o^2} \left(\omega \frac{dT}{d\omega} + 3T \right)$$

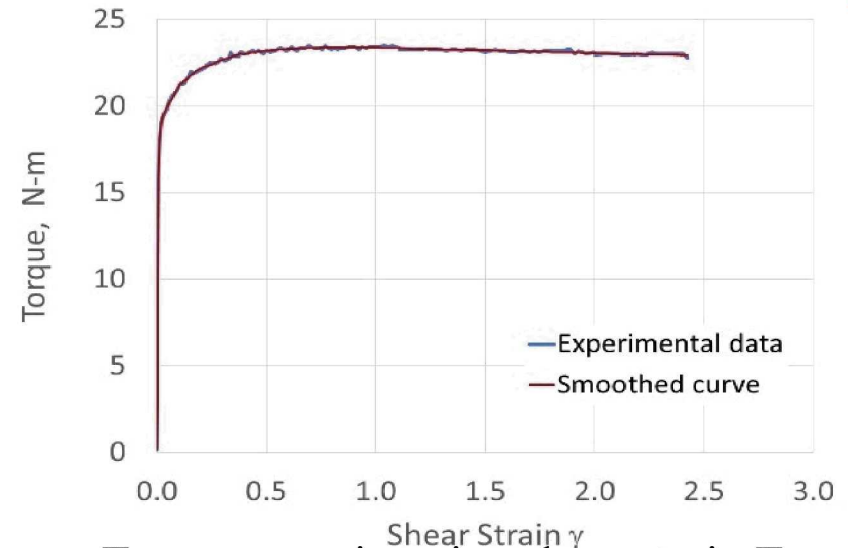
The radius r is constant for circular cylinder during torsion, so $\gamma = r_o \omega$

$\omega \frac{dT}{d\omega}$ is replaced by $\gamma \frac{dT}{d\gamma}$

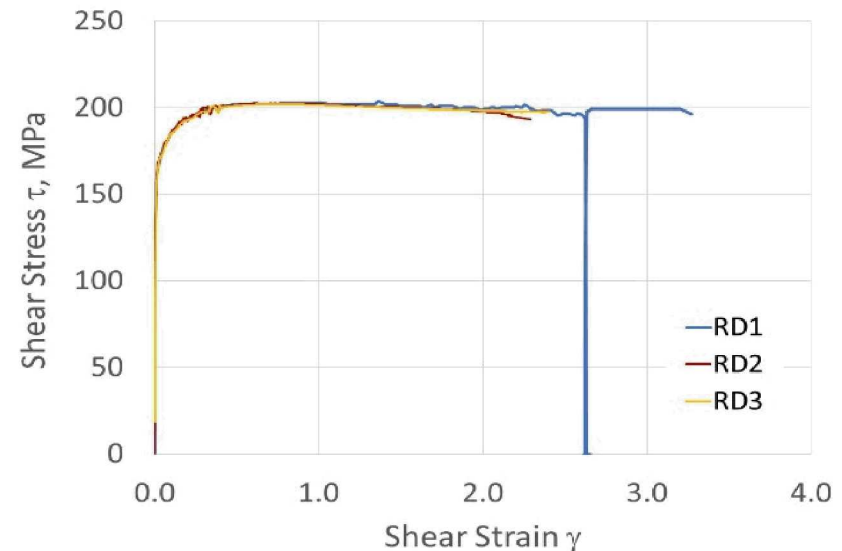
$$\tau = \frac{1}{2\pi r_o^2} \left(\gamma \frac{dT}{d\gamma} + 3T \right)$$

$\frac{dT}{d\gamma}$ This derivative can be calculated by differentiating the polynomial fitting function of T- γ curve.

The engineering shear stress-strain curve of the specimen is determined, shown on the right. Three specimens exhibit consistent stress-strain behavior.

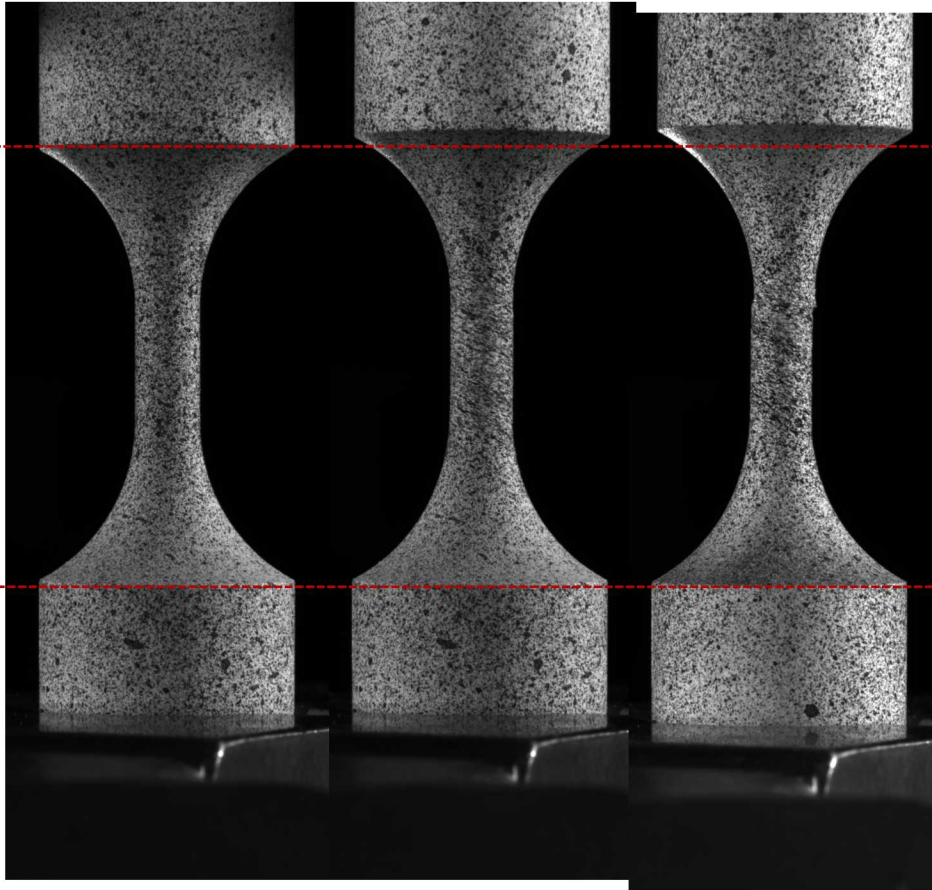


Torque – engineering shear strain T- γ curve of TOR_RD_3



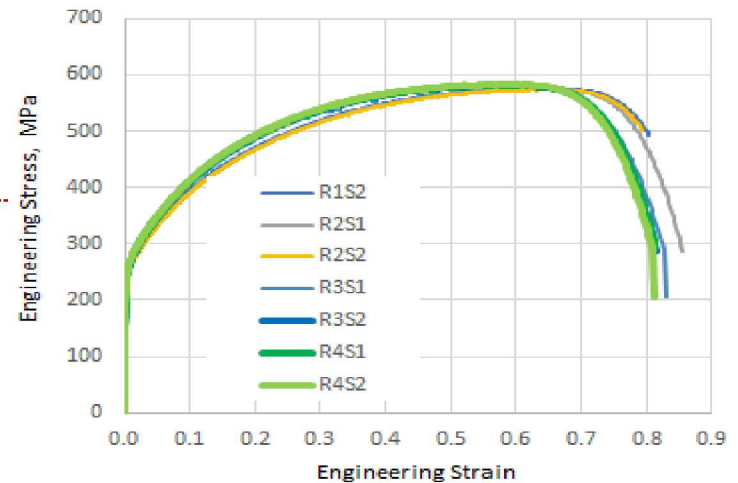
Shear stress vs shear strain for Al

Torsion of Solid Cylindrical 304L Specimens

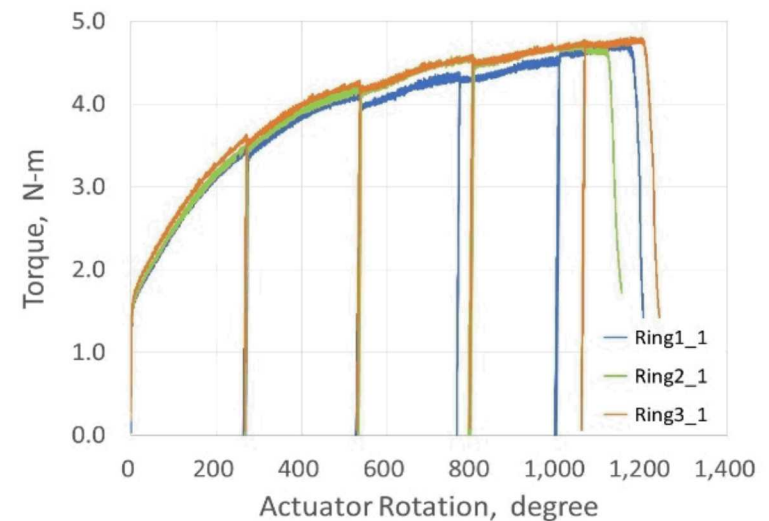


304L VAR specimens have higher strengths and larger ductility;

Specimen dimension: diameter = 3.175mm (0.125"),
length = 5.766 mm (0.227")

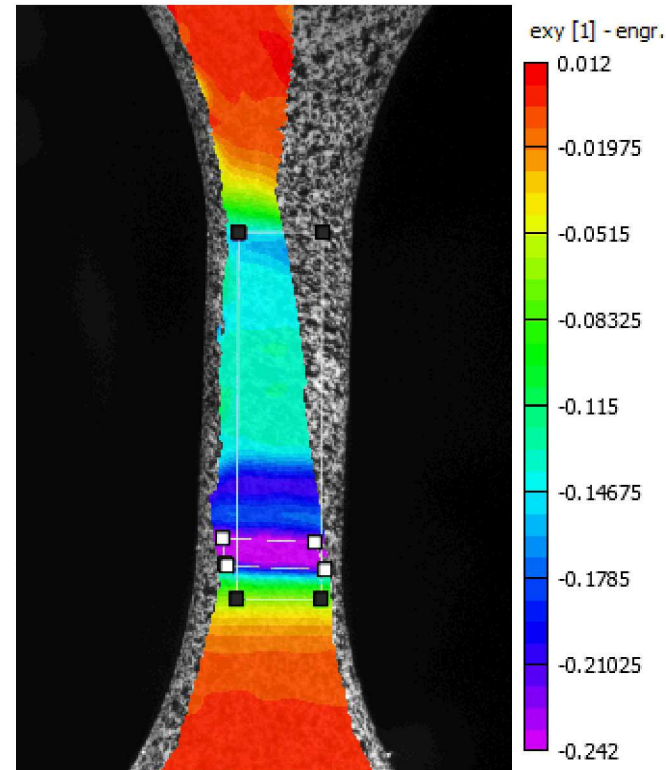
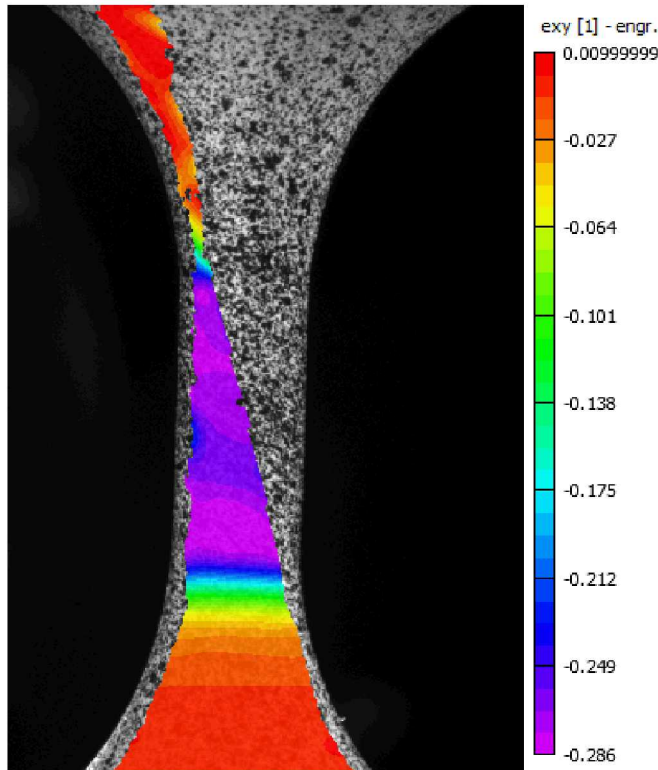


Axial direction tensile stress-strain curves of SS304-VAR



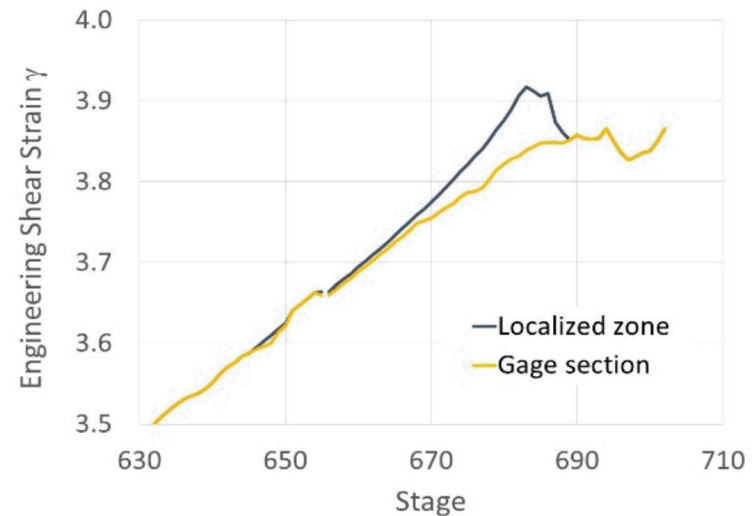
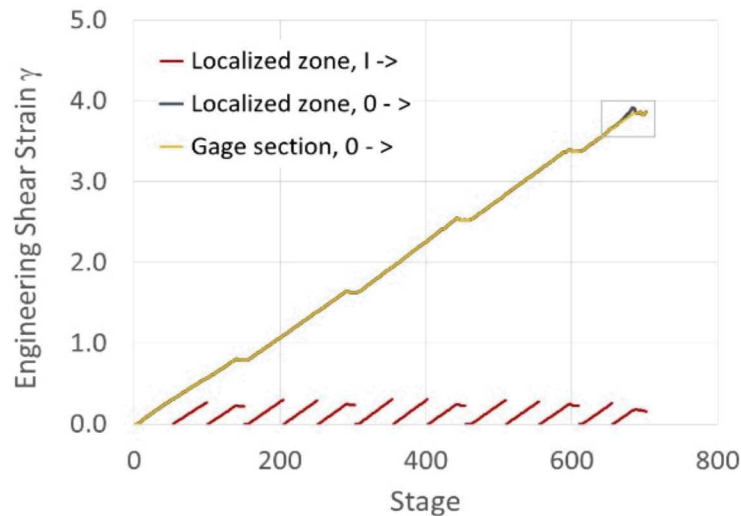
Torque-rotation curves of SS304L-VAR specimens.

Shear Strain e_{xy} Distribution of 304L VAR Specimens using Different Reference

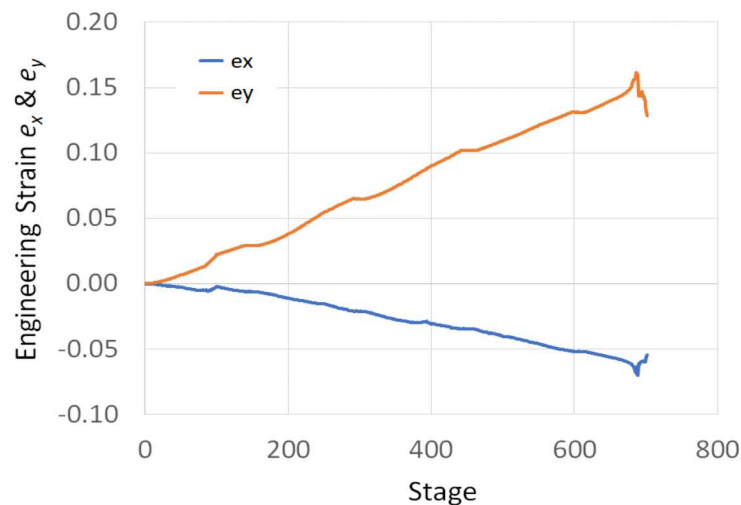


304VAR Ring3-1 native shear strain e_{xy-B} . (a) 510 -> 555 showing uniform deformation and (b) 656 -> 682 showing shear strain localization.

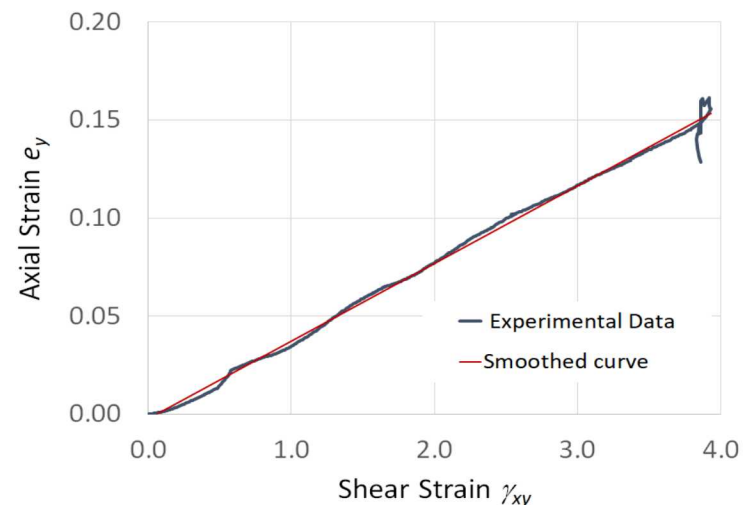
Shear Strain e_{xy} and Normal Strain e_{xx} & e_{yy}



304VAR Ring3-1 native shear strain e_{xy-B} . (a) 510 -> 555 showing uniform deformation and (b) 656 -> 682 showing shear strain localization.

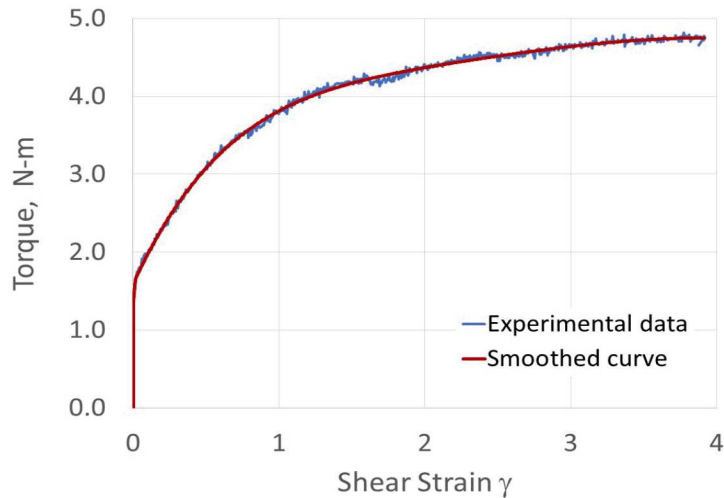


Engineering normal strain histories



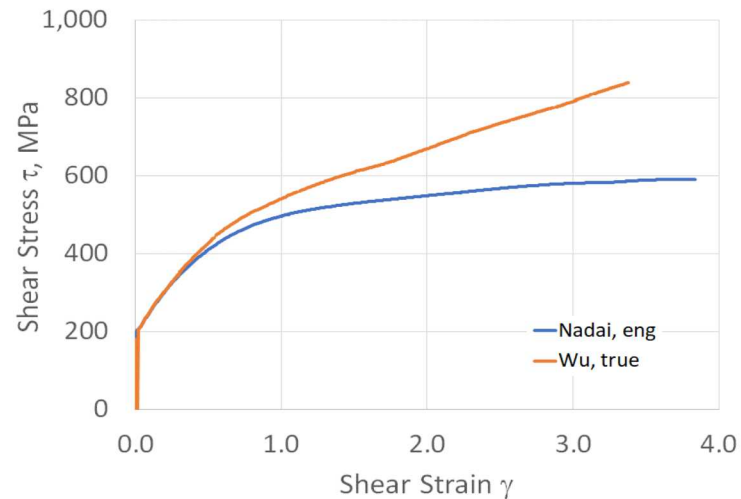
axial strain vs shear strain

Shear Stress



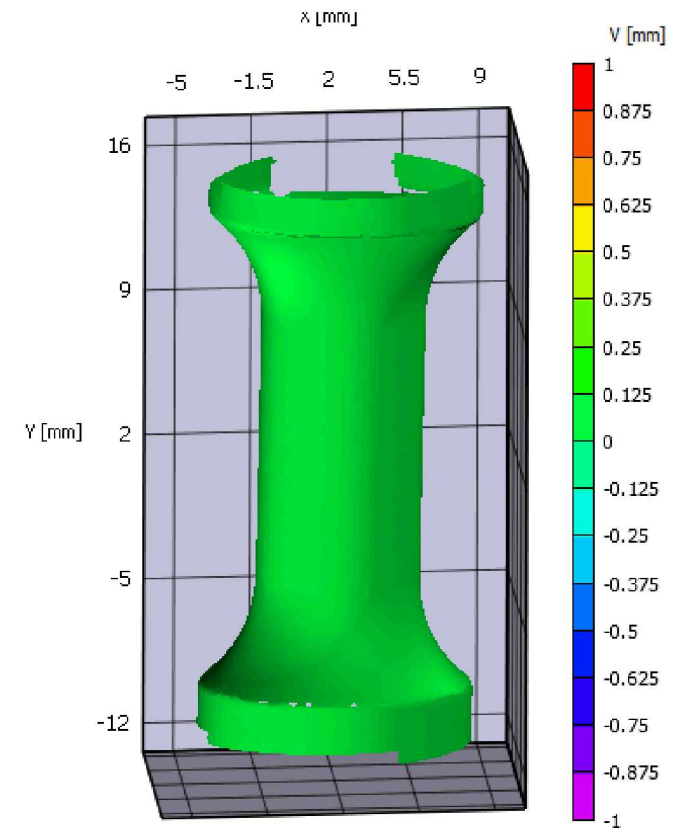
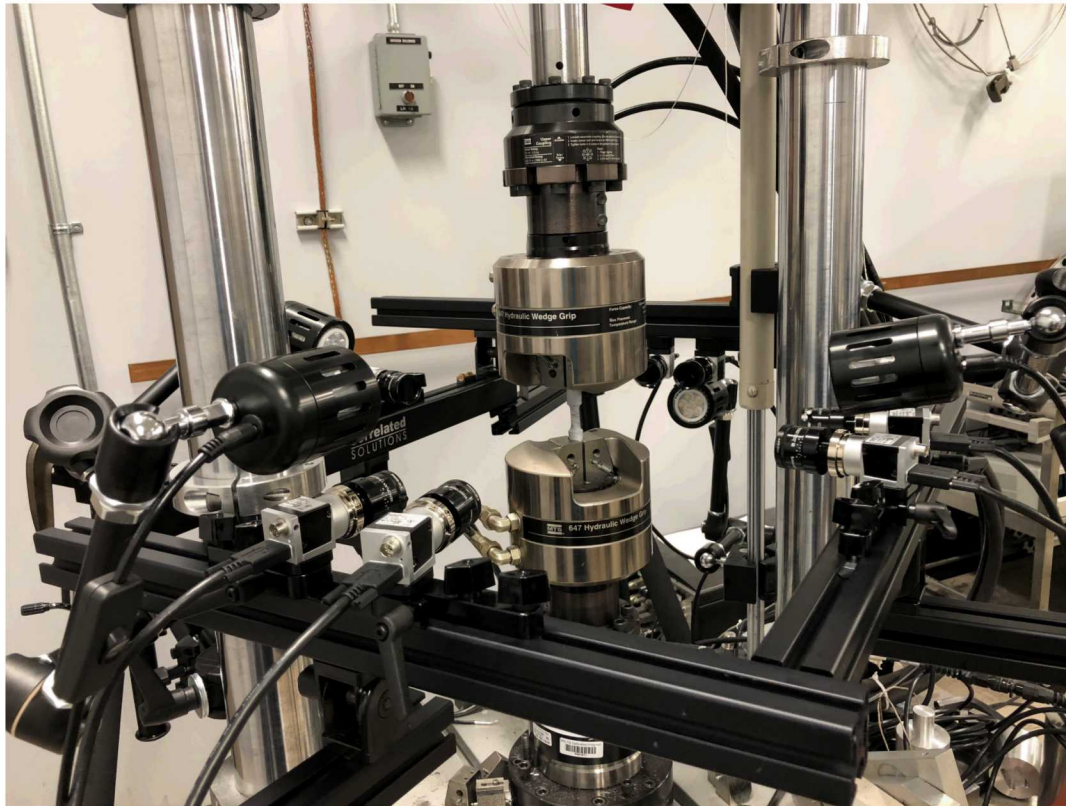
Data and fitted curves of torque and shear strain

$$\tau = \frac{1}{2\pi r_o^2} \left(\gamma \frac{dT}{d\gamma} + 3T \right)$$



Engineering and true stress-strain curves of SS304L-VAR Ring3-1

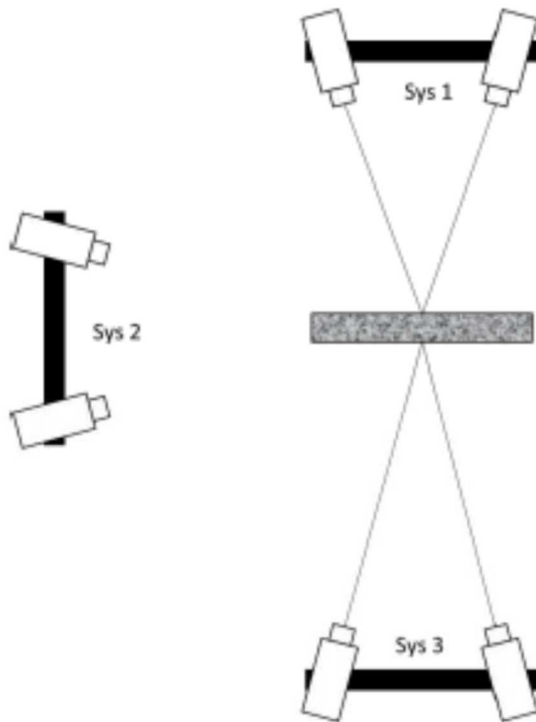
Torsion Experiment with 360° DIC



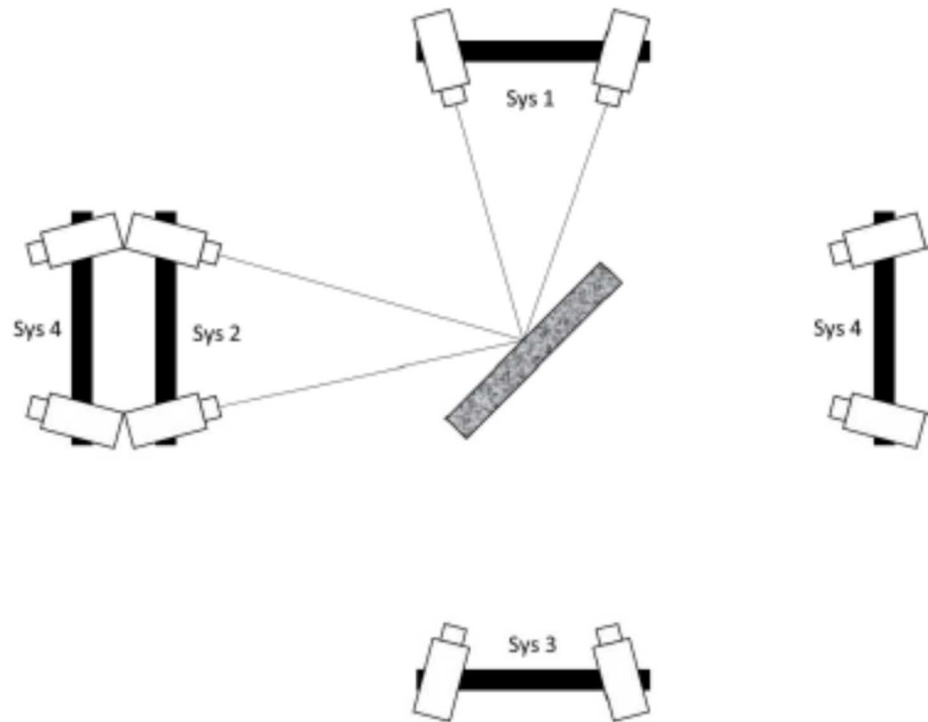
360° DIC:

- Four pairs of cameras
- Full surface view of the specimen

Multi-view Registration for 360° DIC



Dual sided rigid plate needed

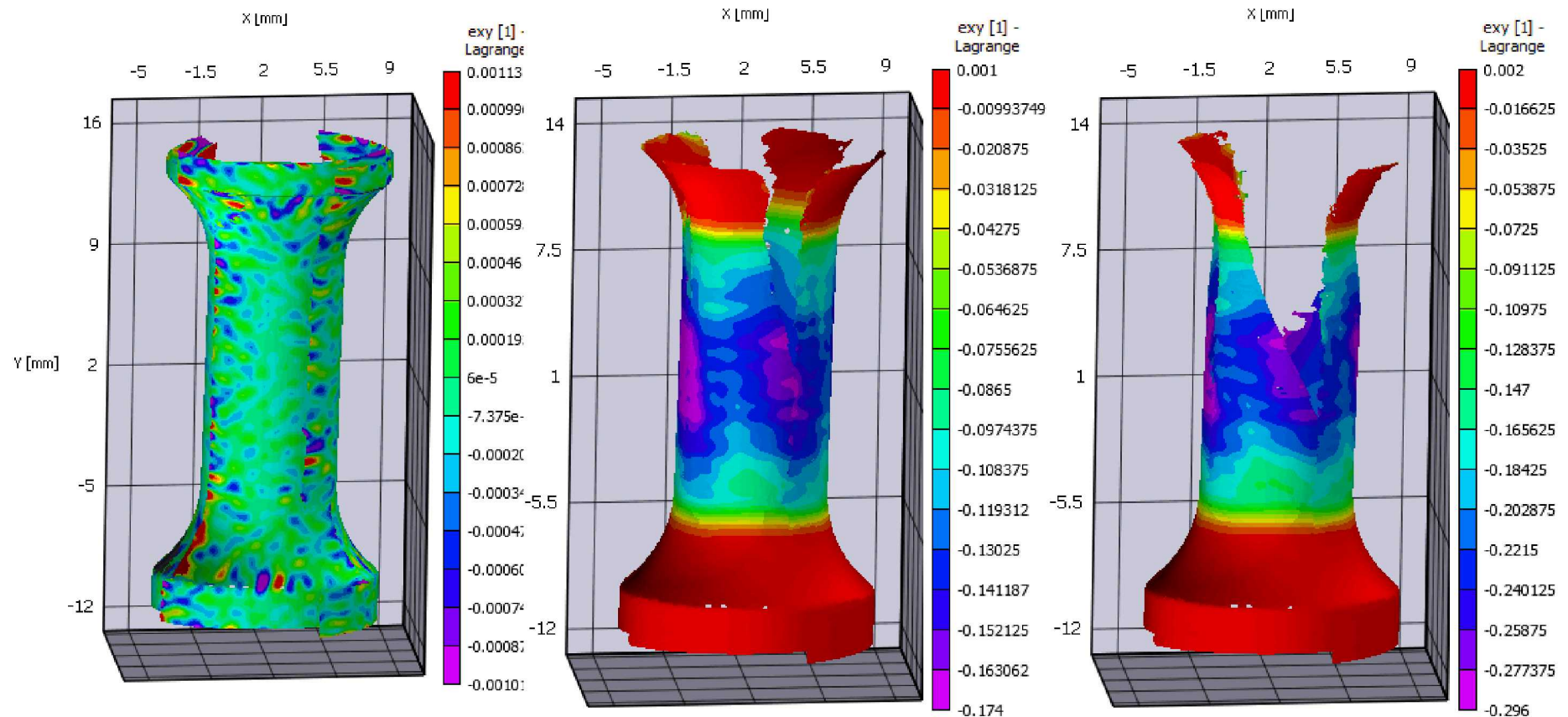


Same side of rigid speckle can be used

360° DIC:

- Use rigid speckle pattern specimen to calibrate and register the relative position of the multiple camera systems
- Select one camera system as reference, transfer the rest of systems to the reference system
- Combine the transferred data

Challenges in 360° DIC



360° DIC:

- Common area rotate out of the view of same pair of camera for torsion experiment

Summary

- 360° 3D-DIC system is developed for torsion tests of solid bar of 304L VAR specimens.
- Large shear strain of 304L VAR is experimentally measured from each individual pair of DIC cameras and added up from the incremental strain.
- Strain is localized at very large strain which is close to failure.
- 360° 3D-DIC system is able to put the whole specimen together and generate the 360° view of the specimen, however, the extraction of the data can be only worked with each individual camera set.
- Surface stress evaluation is based on the results of Nadai and Wu. Pure shear stress-strain curves of engineering materials 304L VAR are obtained.
- Setup of the DIC system needs to be precise to avoid the addition of the systematic error throughout all the steps.