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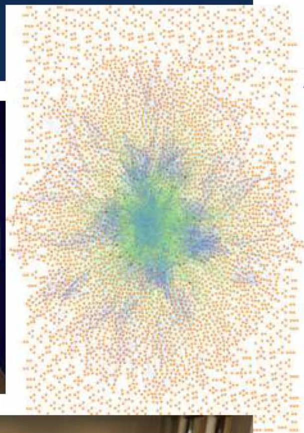

Spatial Error Field Reconstruction using Alpert MultiWavelets

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Outline

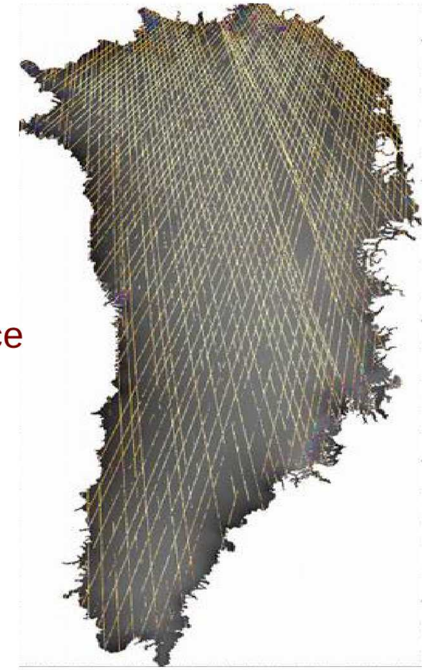
- Background
  - Challenge
  - Hypothesis
- Background on Wavelets
  - Field Data Representation
- Mapping Field Data using wavelets
  - Challenge
- Results
  - Toy Problems
  - *X-specimen Tension Test Data*

Challenge

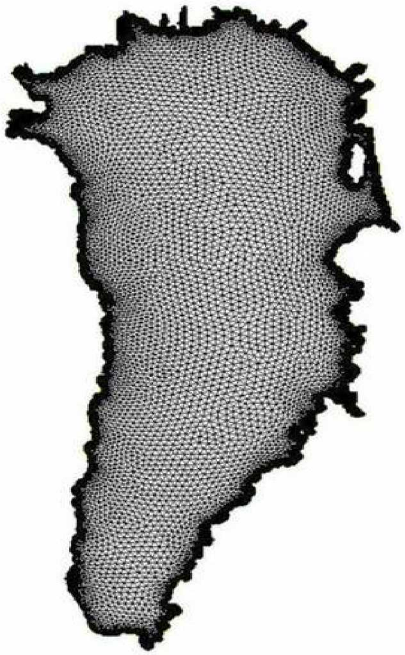
Comparing field data (*e.g.* simulated and measured results) is not always straightforward.

Sensor mesh network to measure surface elevation $\boldsymbol{f}_e$ *

N_e sensors in mesh x



Greenland ice-sheet experiments and models



Computational mesh to simulate surface elevation $\boldsymbol{f}_s$ $

N_s mesh nodes in mesh y

We are seeking global (e.g. $||f_e(x) - f_s(y)||_2$) and local error metrics between the two fields.

- Different number of data points of measured (N_e) and computed (N_s) fields
- Different meshes
- Experimental data is *noisy and incomplete*

* Price S.F. et.al. "An ice sheet model validation framework for the Greenland ice sheet" *Geosci. Model Dev.*, 10, 255–270, 2017.
$ Leng W. et.al. "Finite element three-dimensional Stokes ice sheet dynamics model with enhanced local mass conservation" J. Comp. Phys. 274 (2014) 299–311

Current Methods

Interpolation and Projection

- Applied manually and operate on a case-by-case basis
- *How to filter noise? How to fill incomplete data?*
- *How to treat irregular geometries e.g. non-convex domains?*

Question

Is there a more systematic way to compare data fields involving different meshes, noise, missing points and irregular geometries?

Hypothesis

Comparing data fields of sizes N_e and N_s can be performed in a unified wavelet domain using their large spectral modes.

Global Metric (*e.g.* Objective function for *calibration*)

$$g = \| f_e(x) - f_s(y) \|_2$$

↓ wavelet transform

$$g' = \| \varphi_e - \varphi_s \|_2$$

φ_e and φ_s are of the same size M and encode the information contained in f_e and f_s

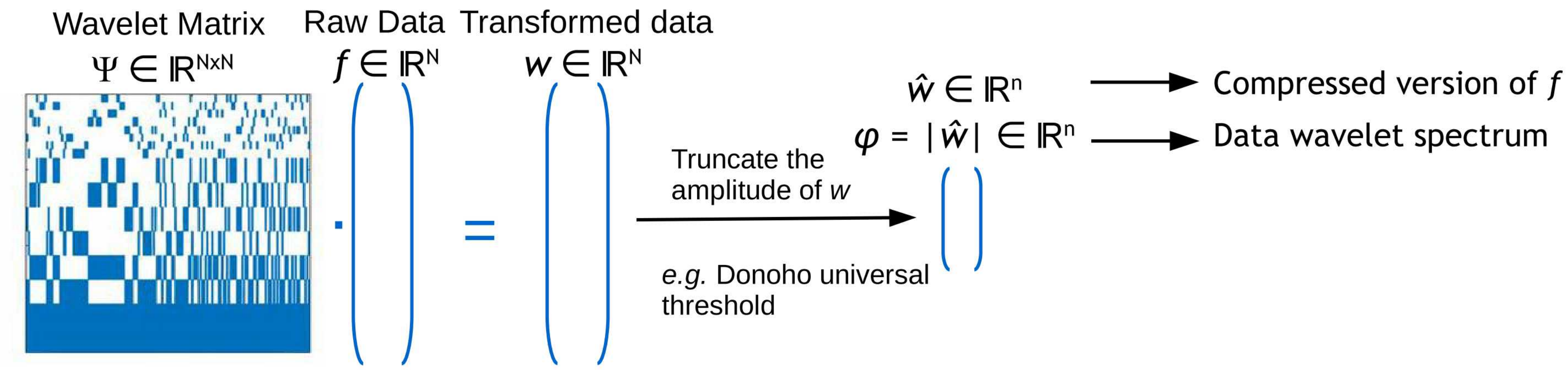
Local Metric (e.g. error field for *validation*)

φ_s can be used to map the field f_s on the mesh x using an inverse wavelet transform then the error field can be computed as:

$$h = | f_e(x) - \hat{f}_s(x) |$$

Background on *Wavelets*

- Wavelets are commonly used as bases to represent multi-resolution functions. (*e.g.* sines and cosines in Fourier analysis)
- We use Alpert multi-wavelets* that adapt to irregular meshes and geometries
- Wavelet transform can be cast as a matrix-vector product:



The wavelet matrix Ψ depends on the mesh representing the data f

Forward wavelet transform: $w = \Psi \cdot f$ ⟶ $\hat{w}$ and $\varphi = |\hat{w}|$ — φ is used is the global comparison metric

Inverse wavelet transform: $\tilde{f} = \hat{\Psi}^T \cdot \hat{w}$

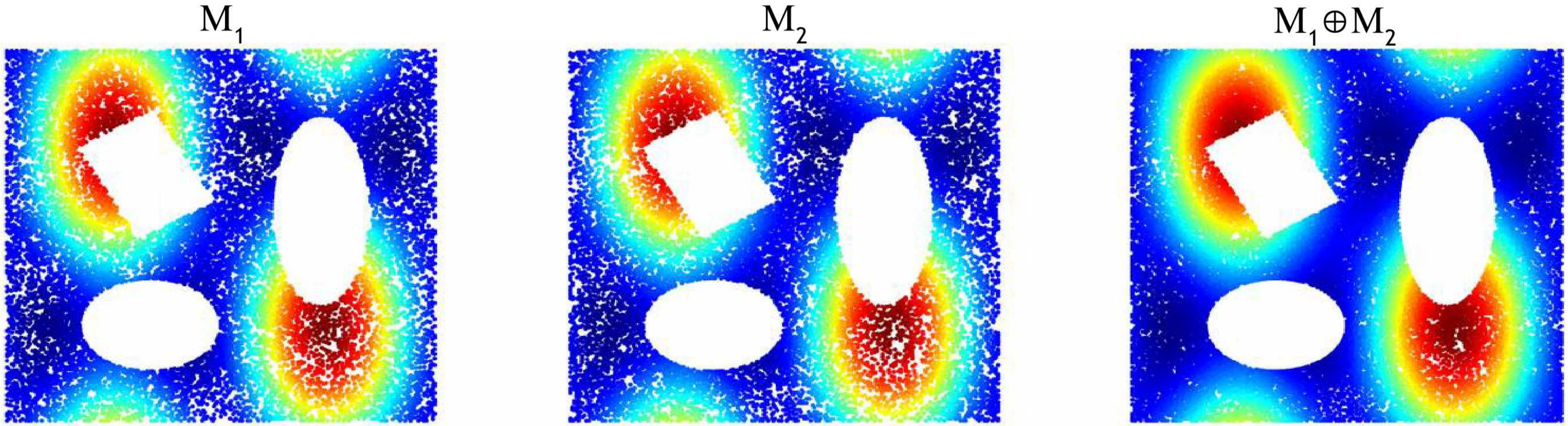
The inverse wavelet transform selectively reconstructs an approximate version of the data f

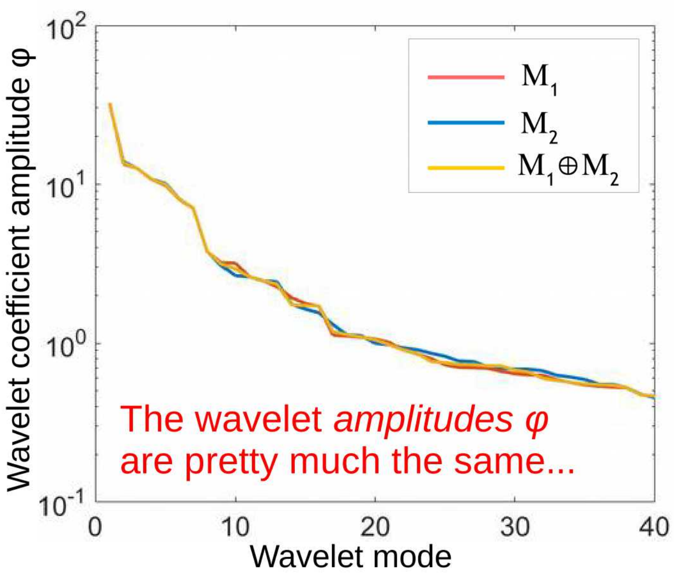
* Alpert B., Beylkin G., Coifman R., and Rokhlin V. Wavelet-like bases for the fast solution of second-kind integral equations. *SIAM Journal on Scientific Computing*, 14(1):159-184, 1993.

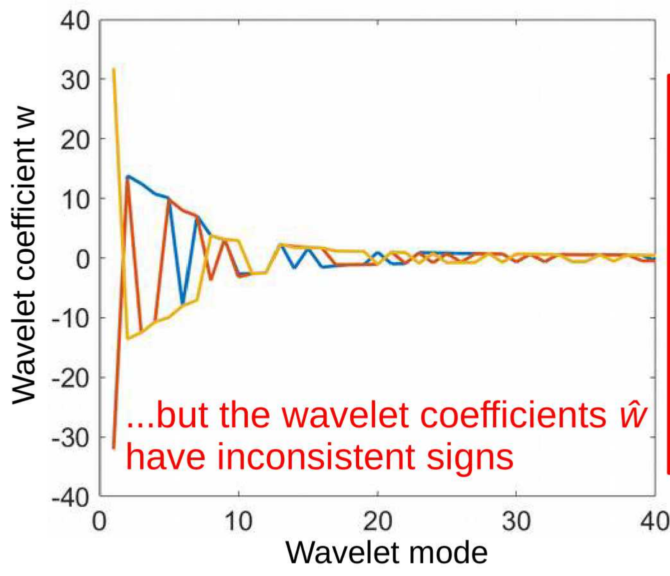
SWinzip v1.0: `http://www.sandia.gov/~mnsallo/SWinzip/swinzip-v1.0.tgz`

Field Data Representation using Wavelets

Consider the function: $f = 48[\sin(2\pi x) - \sin(2\pi y)] \cdot \sin(2\pi x) - 52$ on [0,1]x[0,1] represented on three meshes:







Can we map f_2 on M_1 using:

$$\tilde{f}_2(M_1) = \hat{\Psi}_1^T \cdot \hat{w}_2 \ ?$$

Yes, but the signs of $\hat{w}_2$ have to be adjusted consistently with Ψ_2

Mapping Field Data using Wavelets

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- Consider a function f_1 represented on a mesh M_1
- We would like to estimate the map f_2 of f_1 on another mesh M_2

- We form the combined mesh $M = M_1 \oplus M_2$
- We compute the wavelet matrix Ψ corresponding to the mesh M

The idea is to map f_1 on the combined mesh M
then pull f_2 corresponding to $M_2 \subset M$

- Let f be the map of f_1 on M

$$f = \Psi^T \cdot w$$

- We form the initial guess of w from $w_2 = \Psi \cdot f_2$
- Iterate on the coefficients in w:
  1. Flip the sign of a coefficient in w
  2. Compute $\tilde{f}_i = \Psi^T \cdot w_i$
  3. If $\|\tilde{f}_{1,i} - f_1\|$ decreases, go to 1.
  4. If $\|\tilde{f}_{1,i} - f_1\|$ increases, flip the sign of the coefficient back then go to 1.

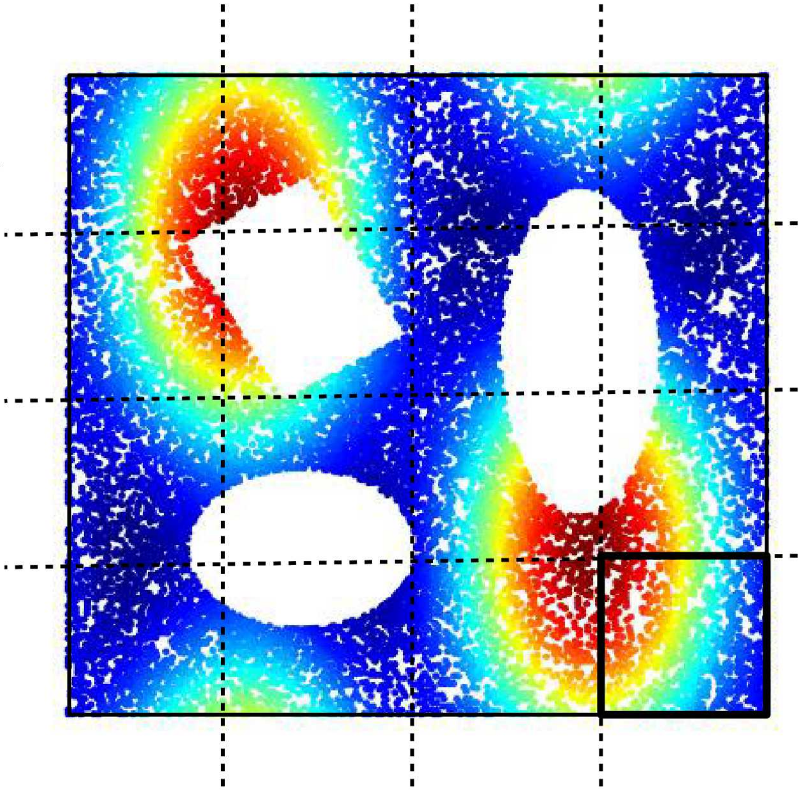
Mapping Field Data using Wavelets

Challenge:

This mapping procedure is expensive for large meshes

- Field data represented on large meshes can be compared by subdividing the mesh into smaller pieces and comparing
  - Divide and conquer
  - Can be run in parallel
  - The full mapping and error fields can be re-assembled like a puzzle

- Data subdivision also results in higher accuracy
  - Fewer features per subdivision block
  - Lower wavelet orders

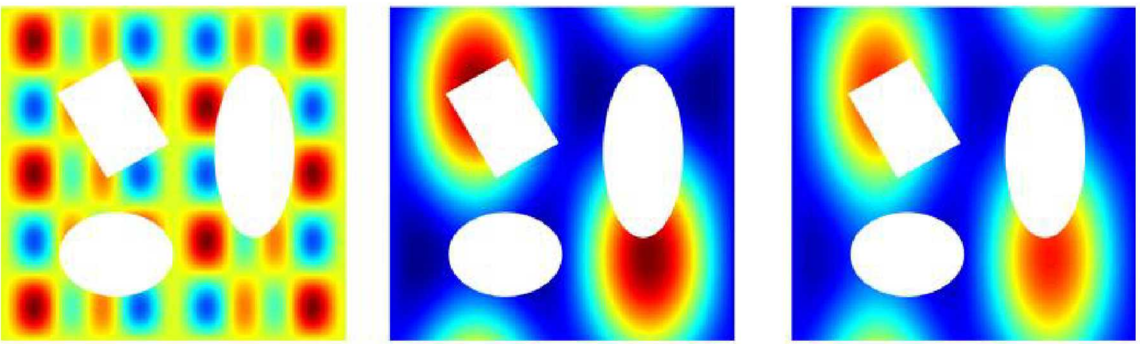


Results: Global Comparison Metric

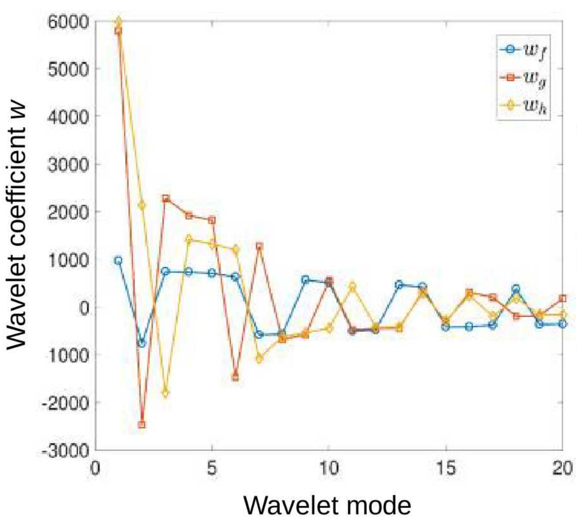
$$f = 48 \cdot \sin(6\pi x) \cdot \sin(5\pi y) \cdot \sin(4\pi x)$$
$$g = 48 \cdot [\sin(2\pi x) - \sin(2\pi y)] \cdot \sin(2\pi x) - 52$$
$$h = 5 \cdot [9 \cdot \sin(2\pi x) - 7 \cdot \sin(2\pi y)] \cdot \sin(2\pi x) - 52$$

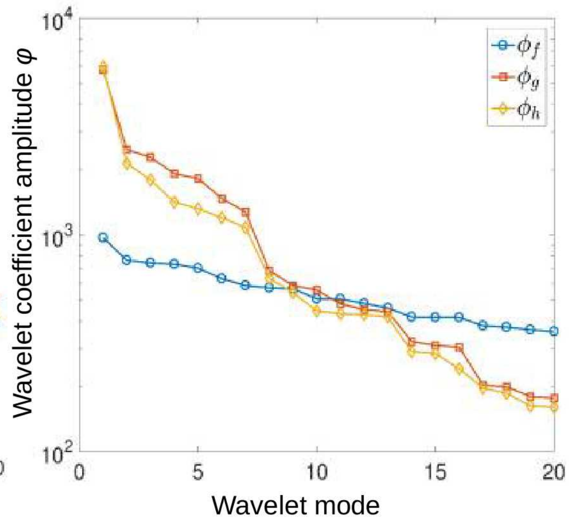
Defined on [0,1]x[0,1] with irregular holes

f g h



$$\epsilon_{12} = \frac{\|\log(\varphi_1/\sqrt{N_1}) - \log(\varphi_2/\sqrt{N_2})\|_2}{\sqrt{n} \cdot \max[\log(\varphi_1/\sqrt{N_1})]}$$





$N_1 = N_2 = 33{,}062$

$n = 400$

$\varepsilon_{gf} = 1.033$

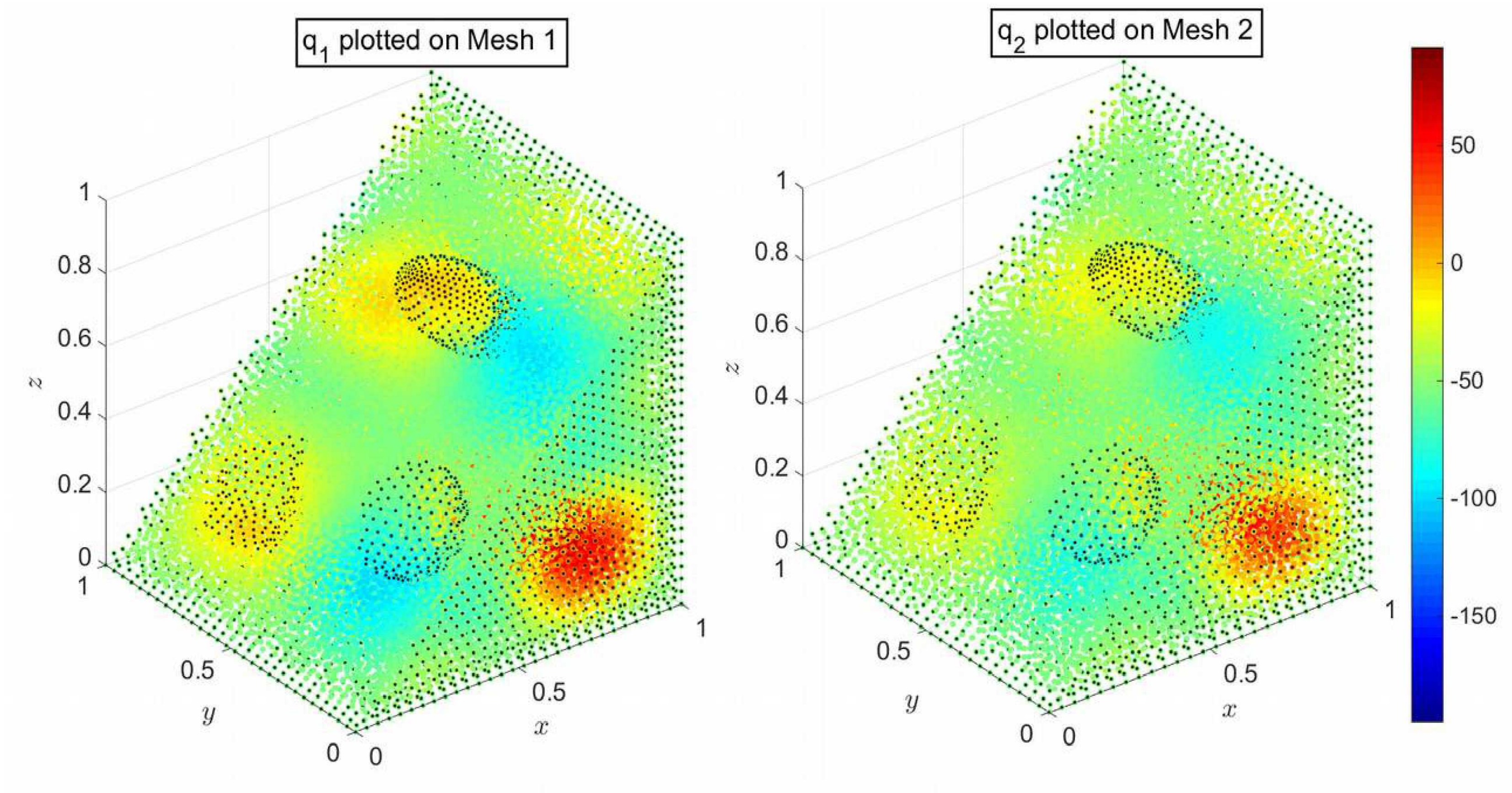
$\varepsilon_{gh} = 0.038$

Results: Error Field

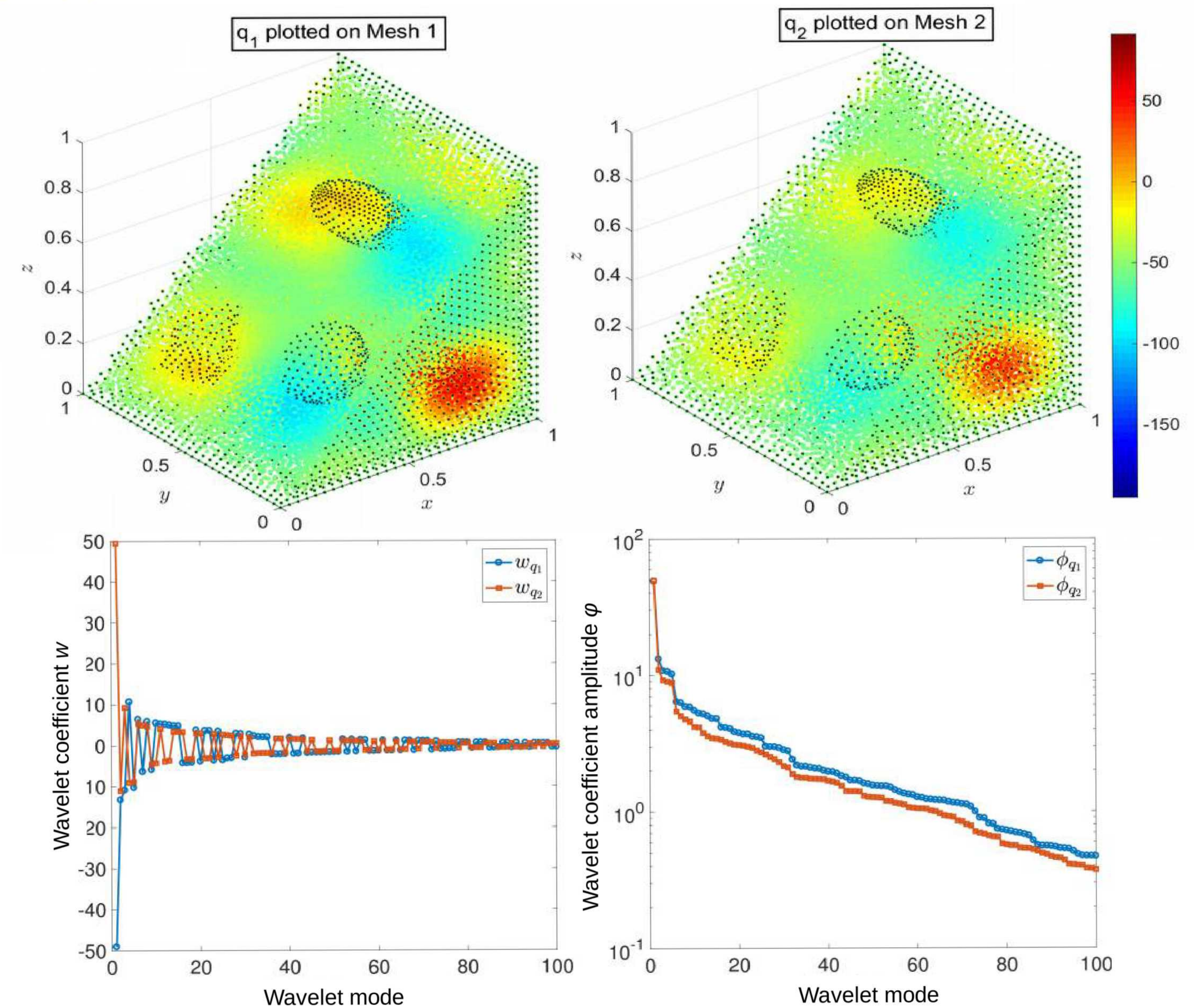
$$q_1 = 48 \cdot [\sin(2\pi x) - \sin(2\pi y) - \sin(2.5\pi z)] \cdot \sin(2\pi x) \cdot \sin(2.5\pi z) - 52$$

$$q_2 = 38 \cdot [1.1\sin(2\pi x) - 0.9\sin(2\pi y) - 1.05\sin(2.5\pi z)] \cdot \sin(2\pi x) \cdot \sin(2.5\pi z) - 52$$

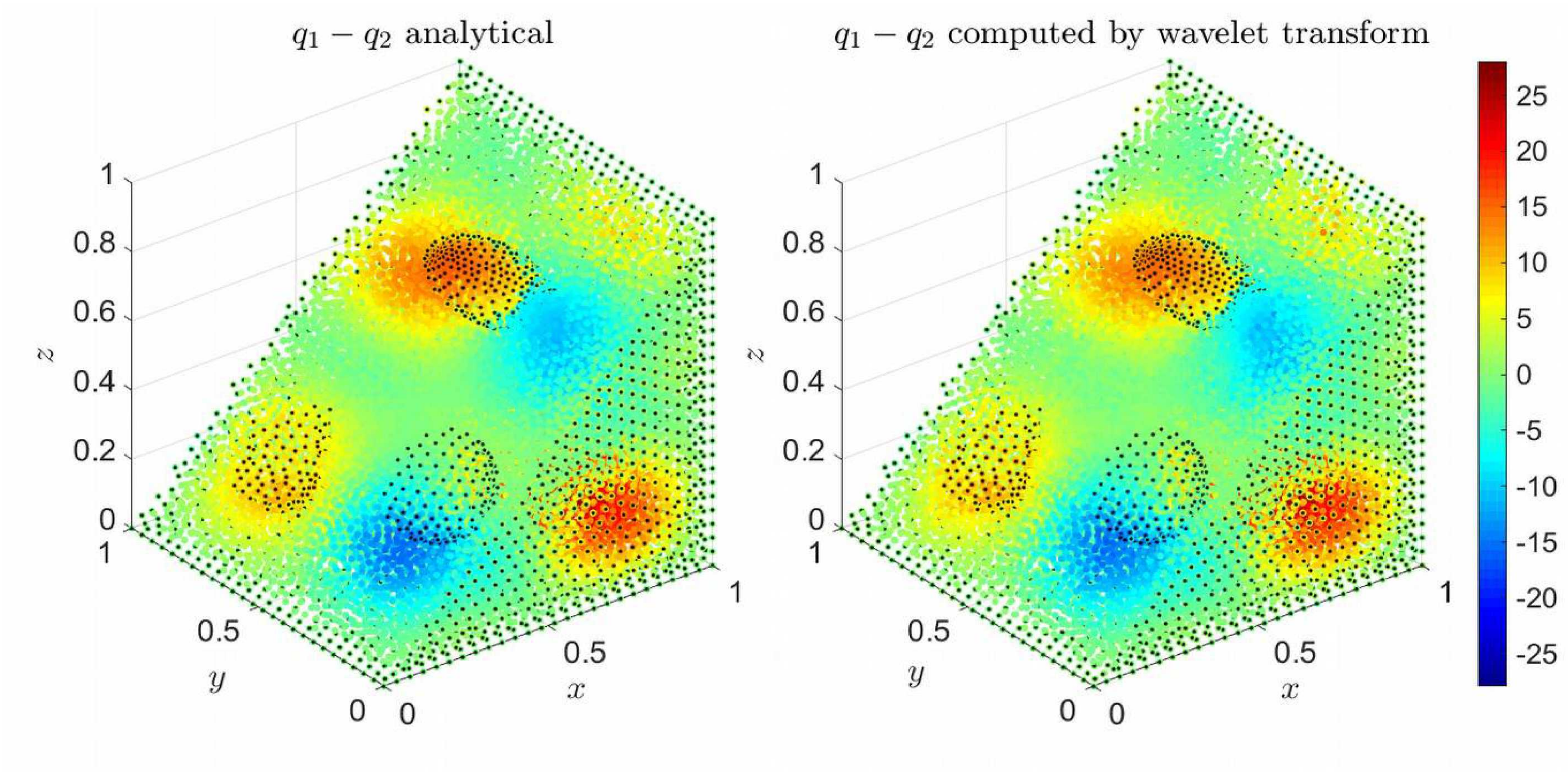
Defined on [0,1]x[0,1]x[0,1] with irregular holes, using two different meshes



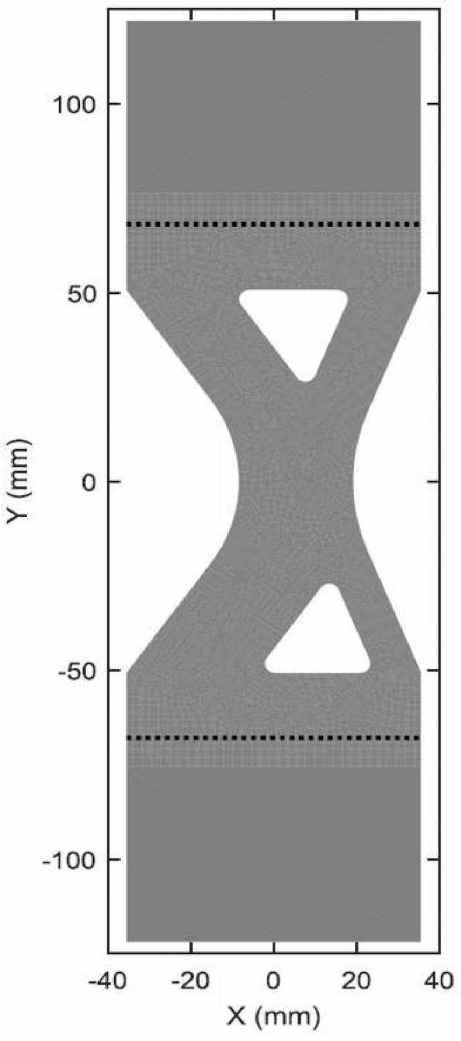
Results: Error Field

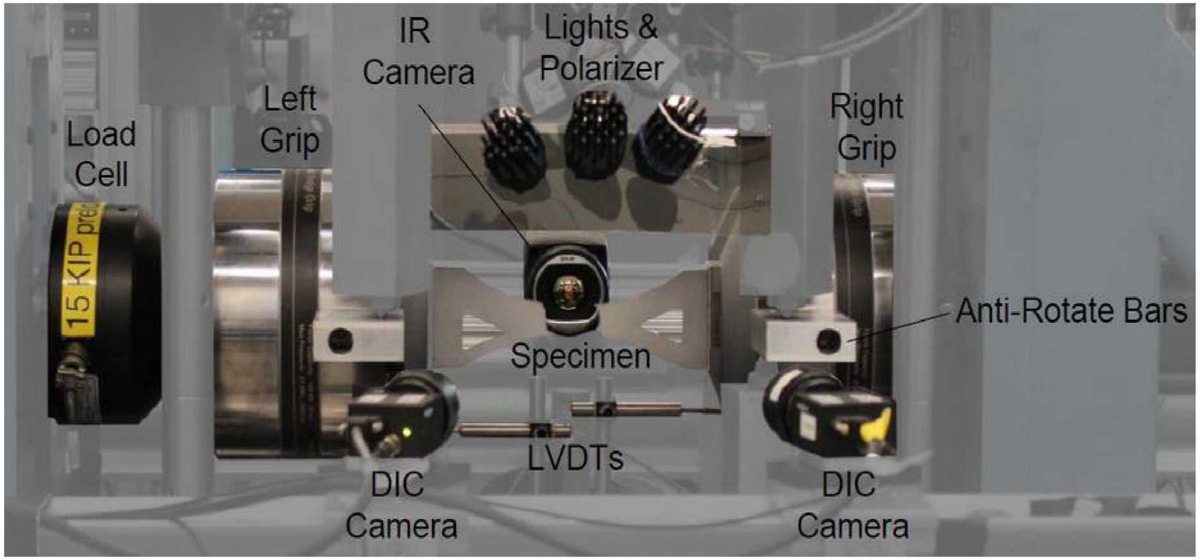


Results: Error Field



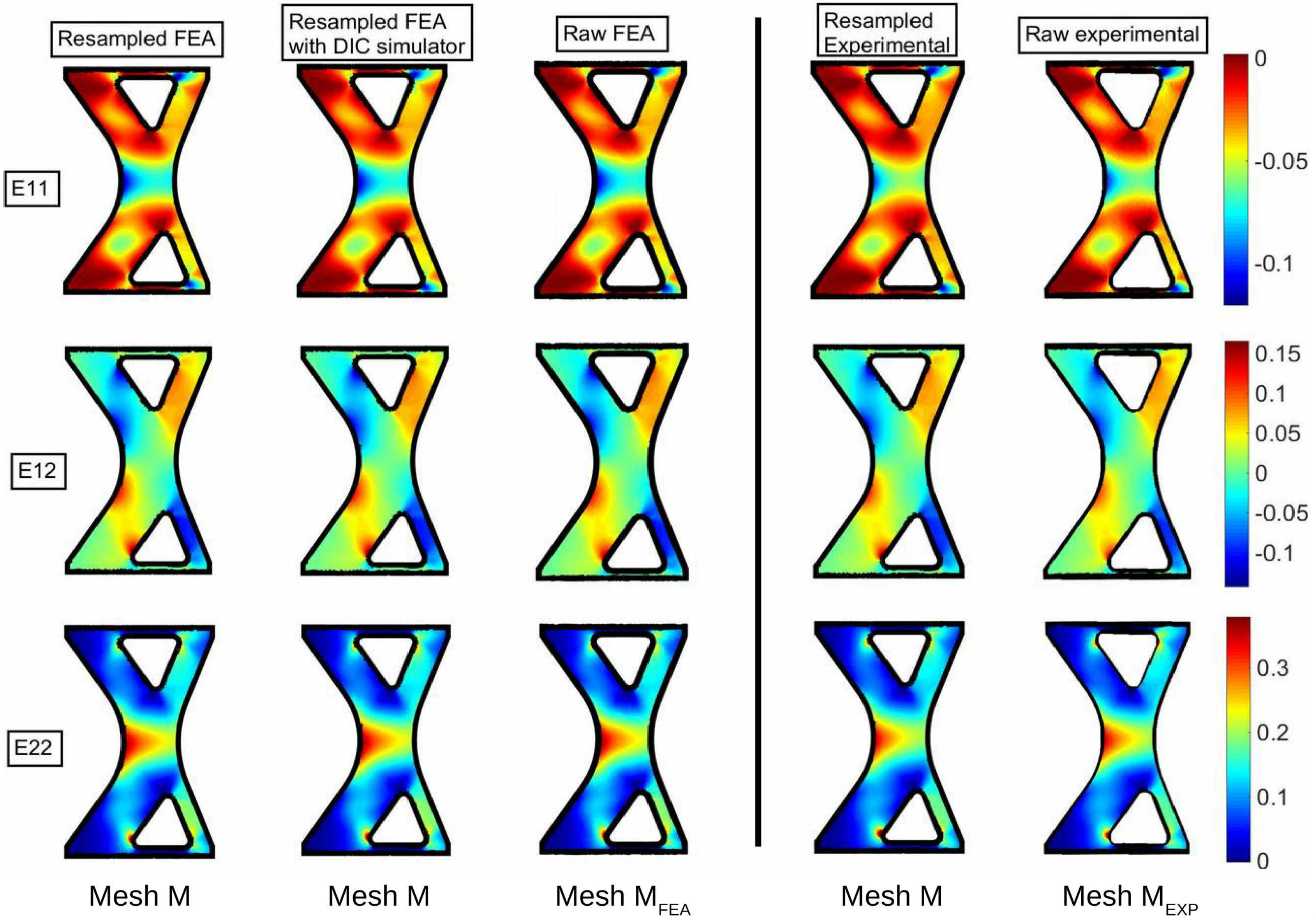
Application:
Full-field Validation of X-specimen tension test



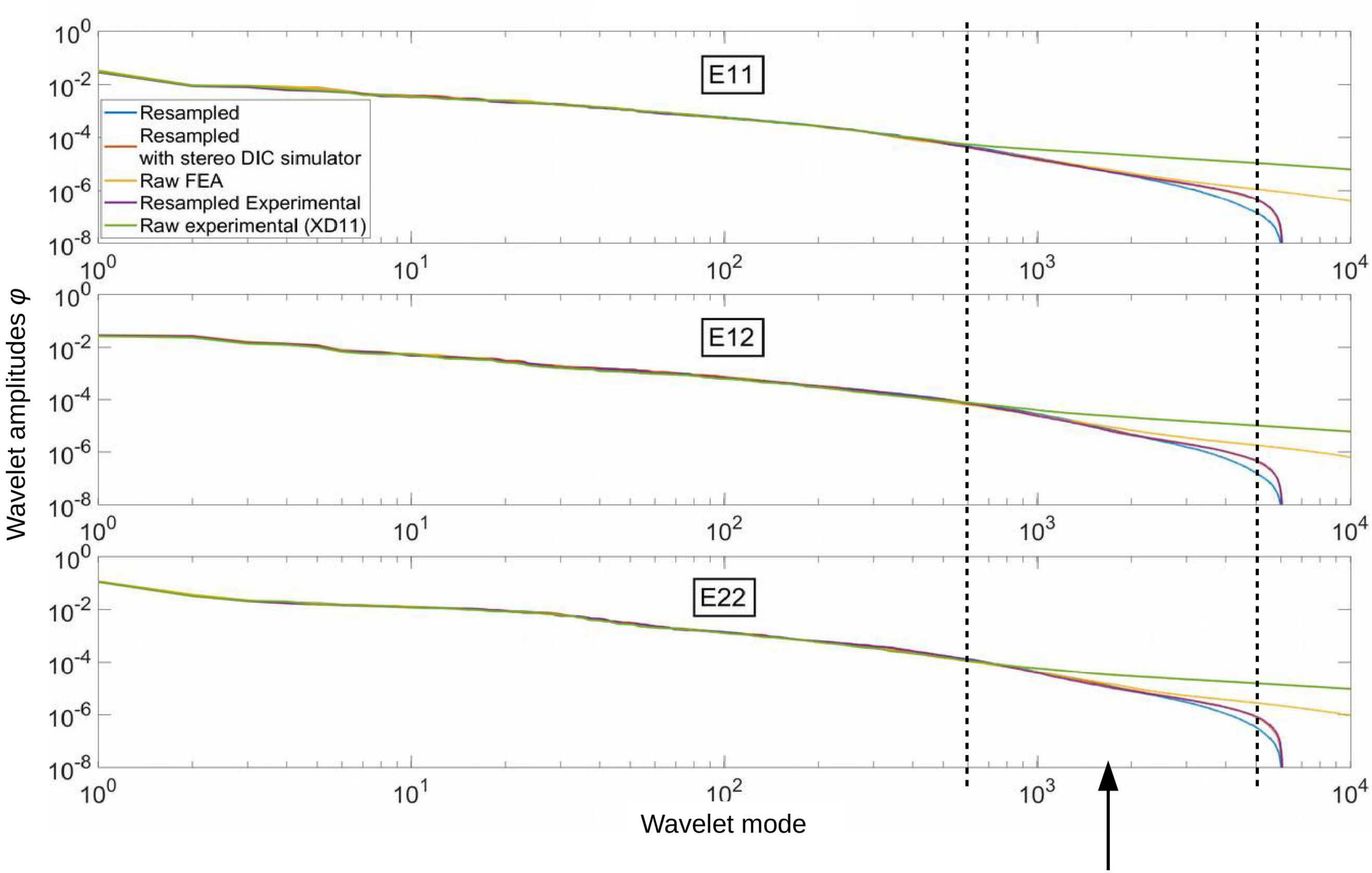


Experimental setup used for the X-specimen tension test. Note that in the load frame, the specimen was oriented horizontally.

Application: Green strains from X-specimen tension test

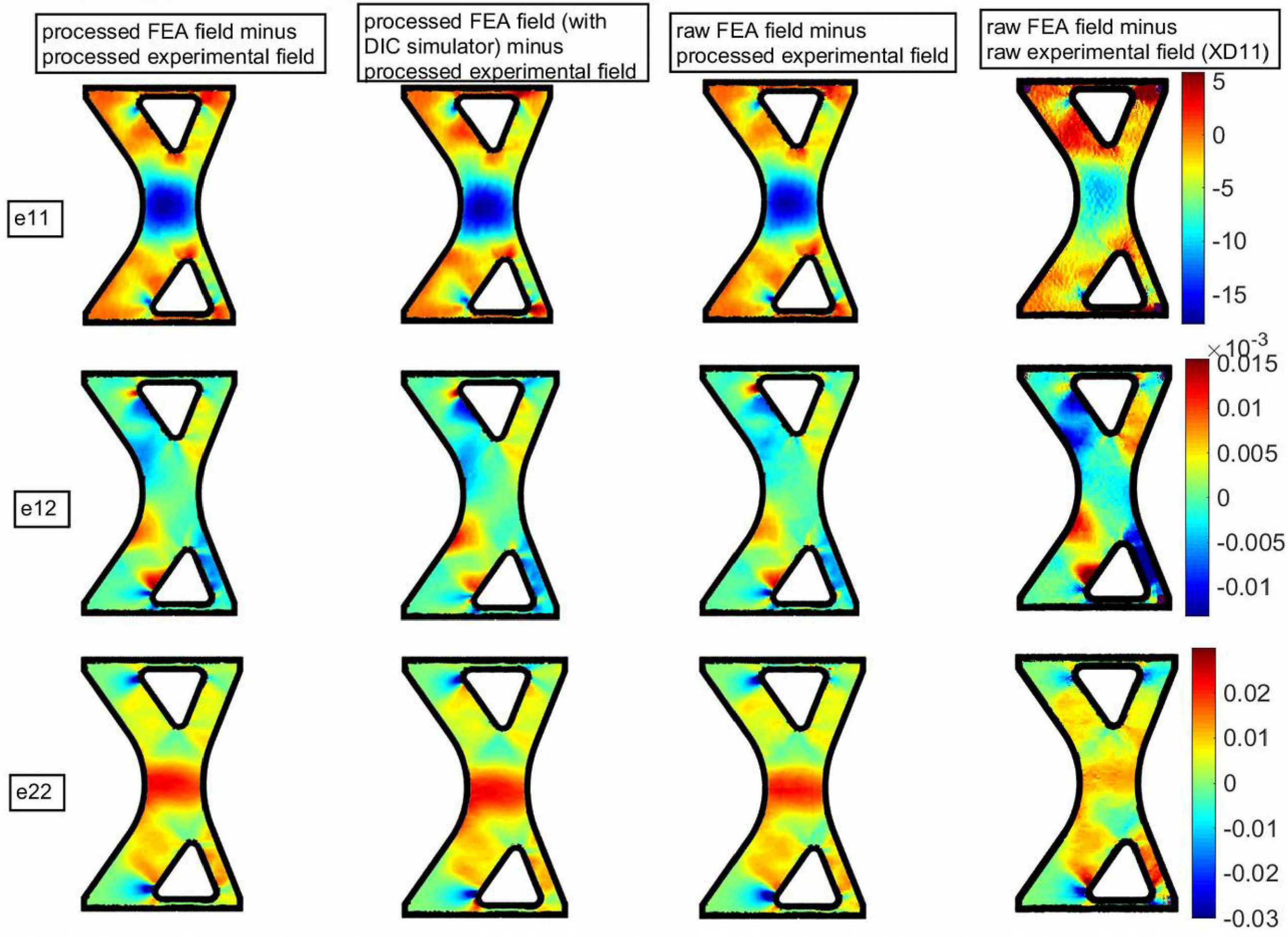


Application: Green Strains Wavelet Spectra



Resampling the raw FEA and experimental data induces attenuation of some high-frequency modes i.e. smoothing in the fields.

Application: Green Strains Error Fields



Summary

- We implemented Alpert wavelets to systematically compare measured and computed data fields.
- Alpert wavelets map field data to a given mesh and produce error fields bypassing collateral effects of common interpolation methods.
- Implementing wavelet mapping using mesh subdivision in enhances its performance and accuracy.
- We implemented Alpert wavelets to compare field data in toy problems and Green strains obtained from X-specimen tension test.
  - Good agreement between analytical and estimated error fields.
  - Alpert wavelets allow a more systematic estimation of the error field

- SWinzip v1.0: `http://www.sandia.gov/~mnsallo/SWinzip/swinzip-v1.0.tgz` *(SWinzip v2.0 coming up soon!)*
- Salloum, M., Fabian, N.D., Hensinger, D.M., Lee, J., Allendorf, E.M., Bhagatwala, A., Blaylock, M.L., Chen, J.H., Templeton, J.A., Tezaur, I. "Optimal Compressed Sensing and Reconstruction of Unstructured Mesh Datasets", *Data Science and Engineering,* 3(1), pp. 1-23 (2018)
- Salloum, M., Johnson, K.L., Bishop, J.E., Aytac, J.M., Dagel, D. and van Bloemen Waander, B.G. "Adaptive Wavelet Compression of Large Additive Manufacturing Experimental and Simulation Data", *Computational Mechanics,* 63(3), pp. 491-510 (2019)

THANK YOU!!
Questions?