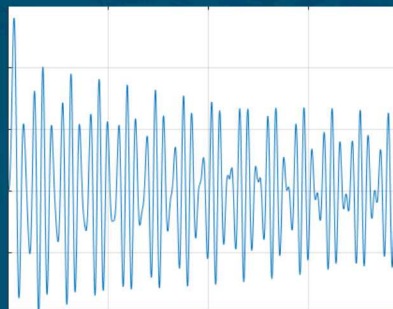
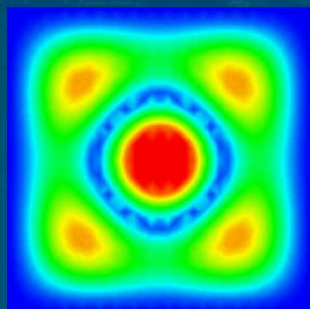
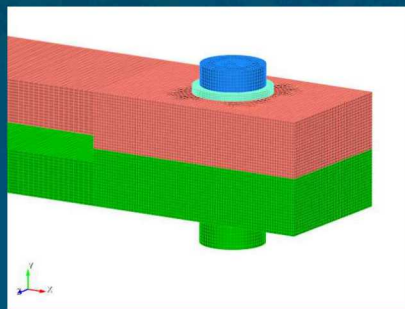


# Structural Dynamics of Mechanical Joints



PRESENTED BY

Robert J. Kuether, Sandia National Laboratories

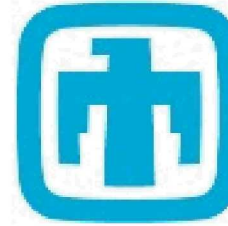
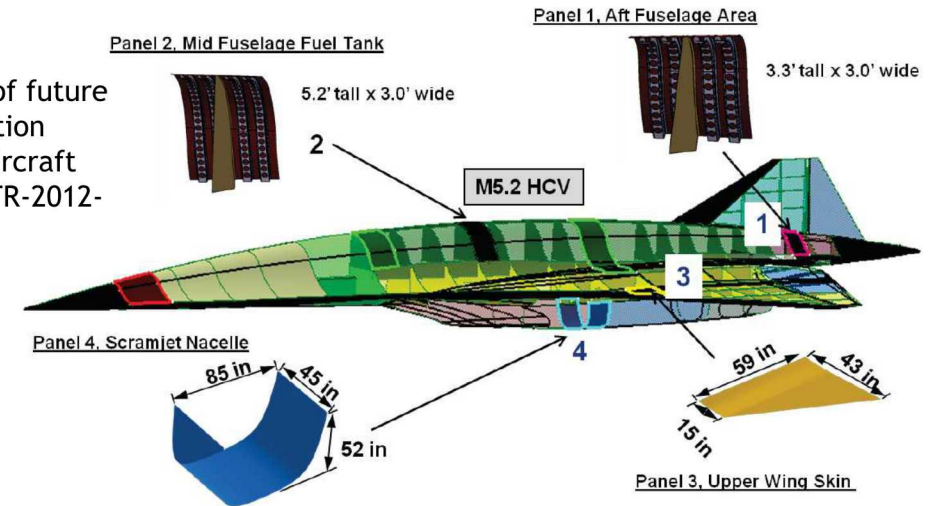
Presented at 2019 CAV Workshop on May 8, 2019

- B.S., M.S. and Ph.D in Engineering Mechanics at University of Wisconsin
  - Focused on computational methods in structural dynamics
  - “Nonlinear Modal Substructuring of Geometrically Nonlinear Finite Element Models”
- Joined Sandia in 2015 as Technical Staff
  - Component Science & Mechanics
  - Research and application work in computational structural dynamics
  - Exploring new nonlinear physics

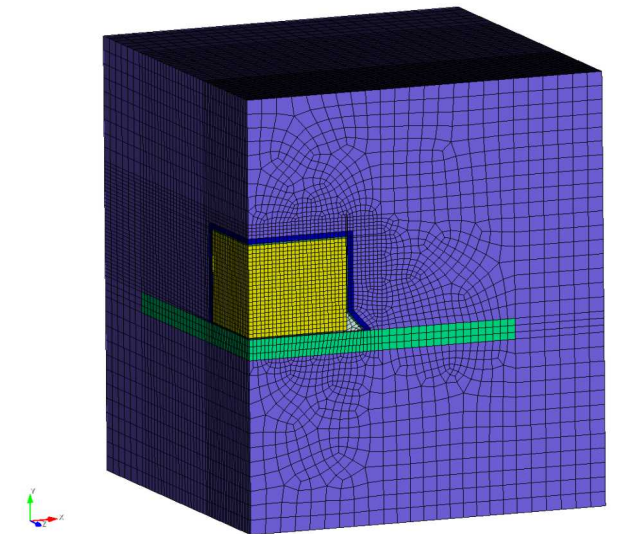
### Keywords:

Structural dynamics; reduced order modeling; nonlinear dynamics and vibrations; test-analysis correlation; interface mechanics

Exploratory design of future reusable, long duration cruise high-speed aircraft from AFRL-RQ-WP-TR-2012-0280

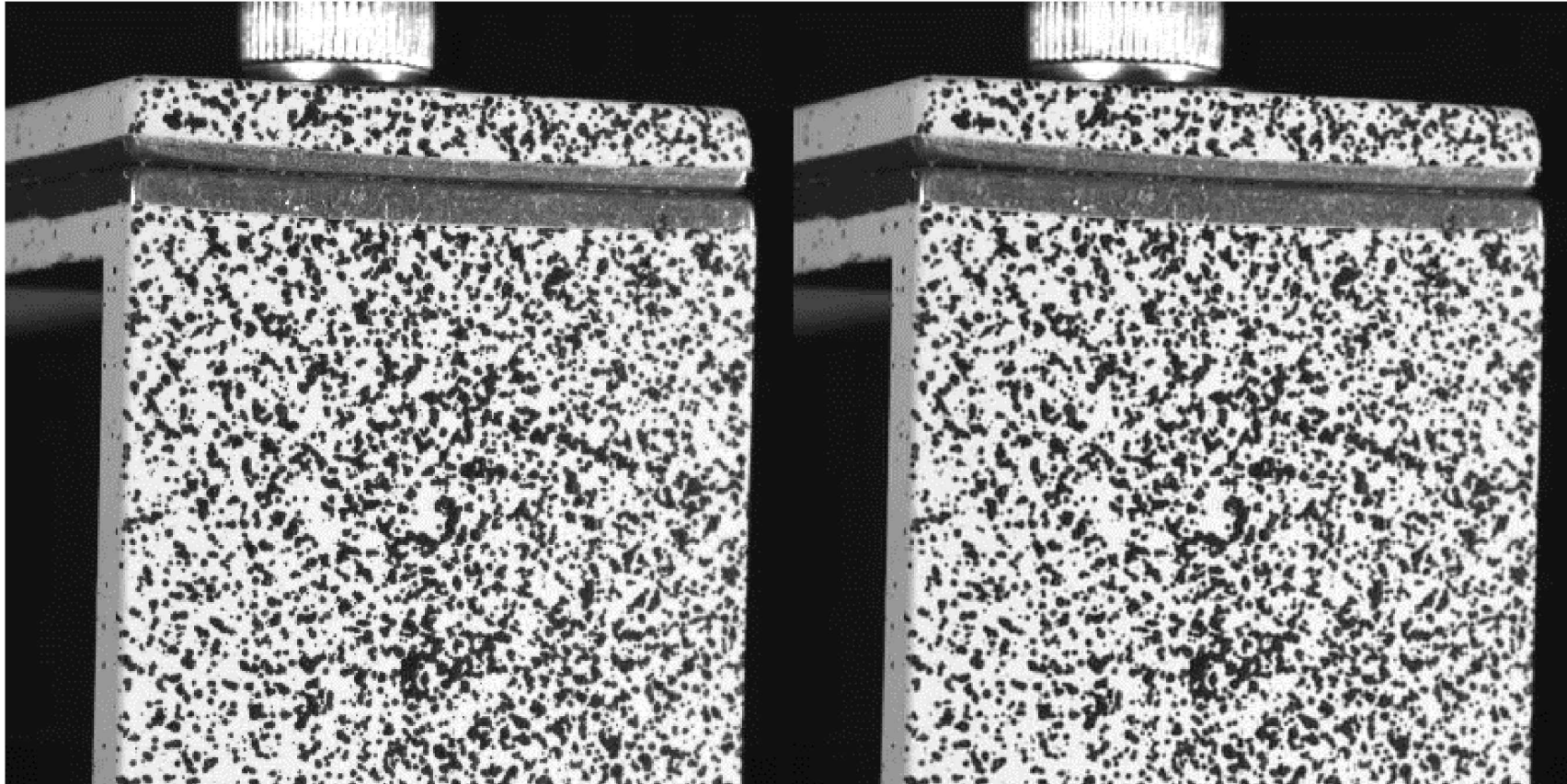


Vibration sensitive electronics potted in foam or polymer to mitigate damaging shock and vibrations





### 3 Structural dynamic considerations with joints



## Motivation and Existing Challenges

Various industries rely on joining technologies to assemble structural systems

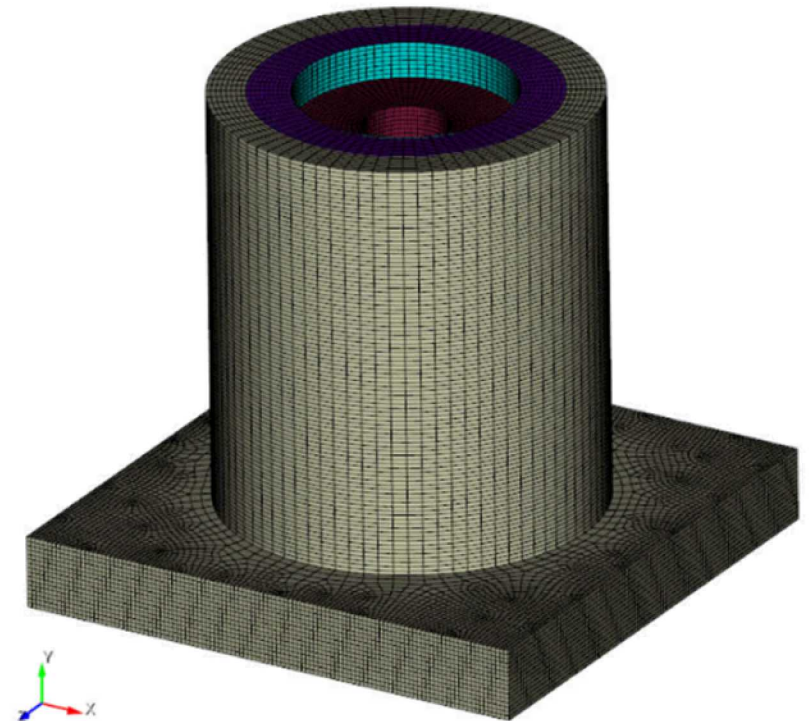
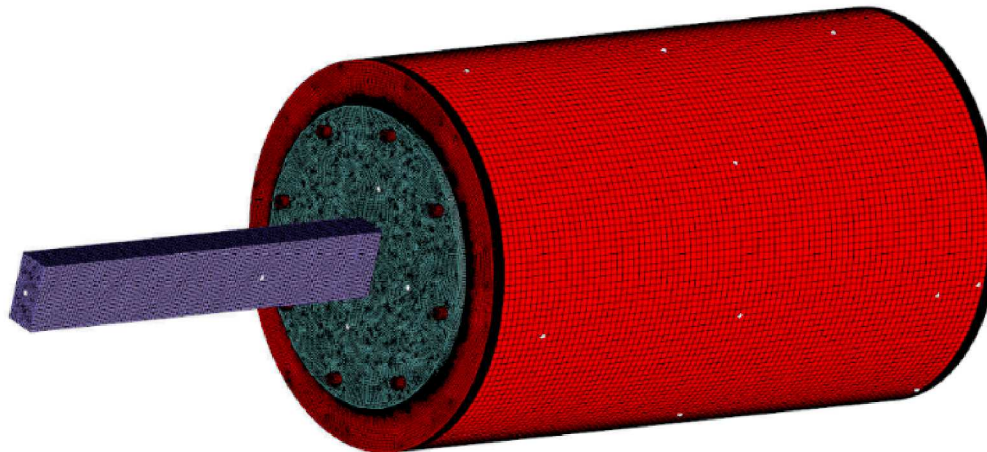
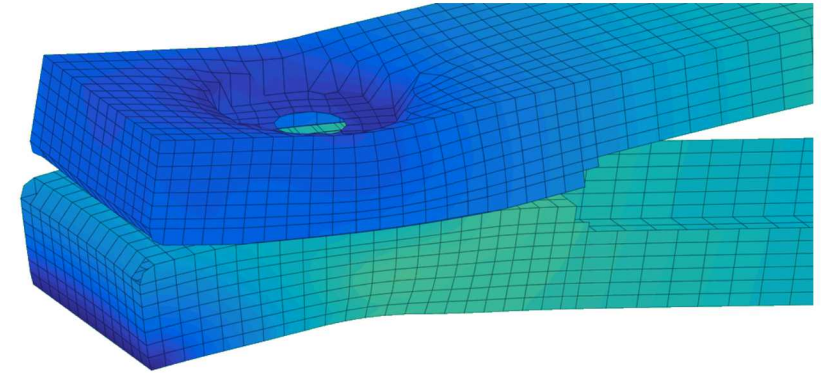
- Automotive, aerospace, civil, etc..

Joints introduce mechanical interfaces, contributing as sources of nonlinearity in vibratory response

- Frictional slip: micro- and macro-slip
- Variable normal pressure distributions

Frictional contact presents various challenges

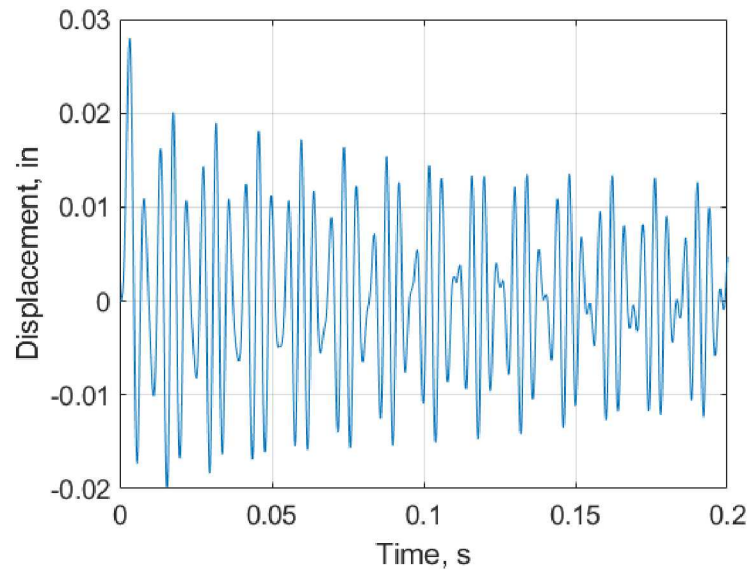
- **Modeling:** simulation cost, stability and convergence, model fidelity and uncertainty
- **Experiments:** measuring kinematics, repeatability, nonlinear dynamics



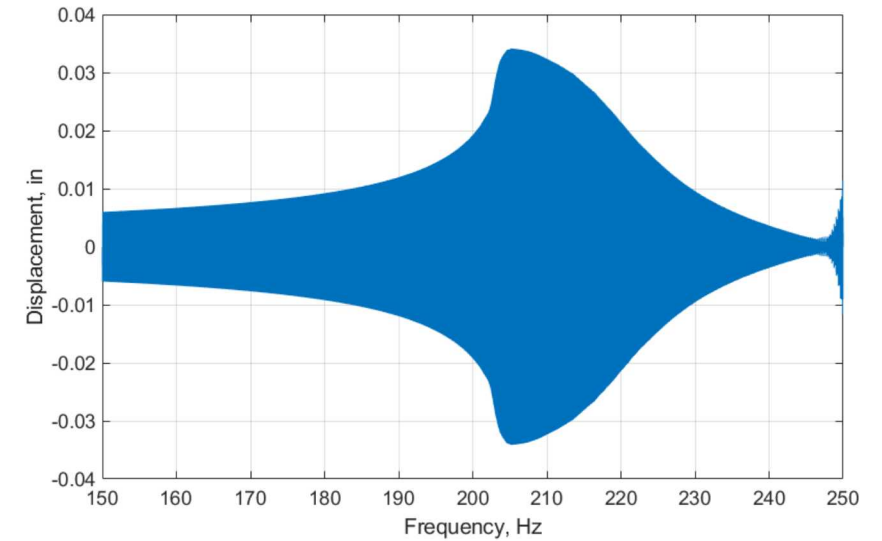


# What global response metrics should be preserved?

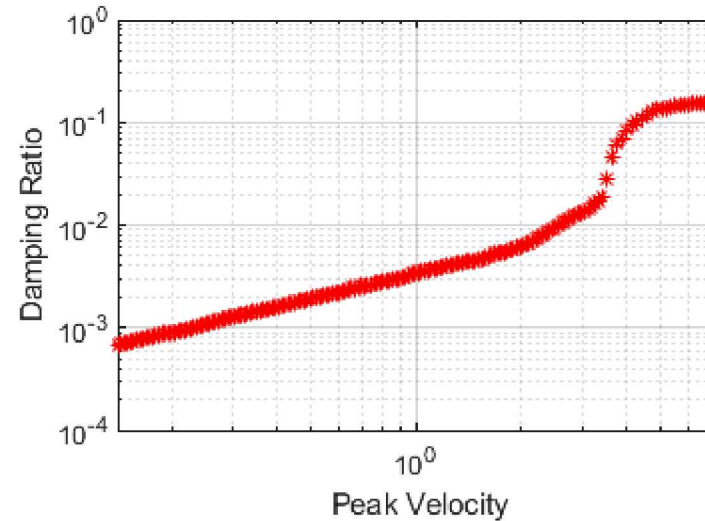
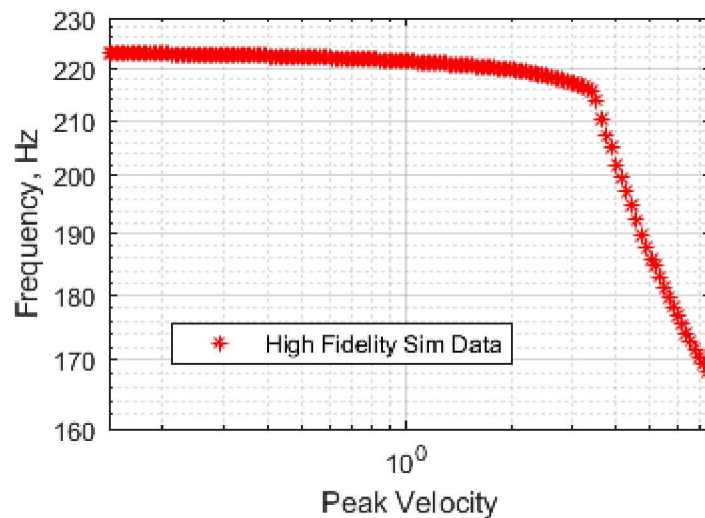
Transient  
Response?



Swept/Stepped  
Sine Response?

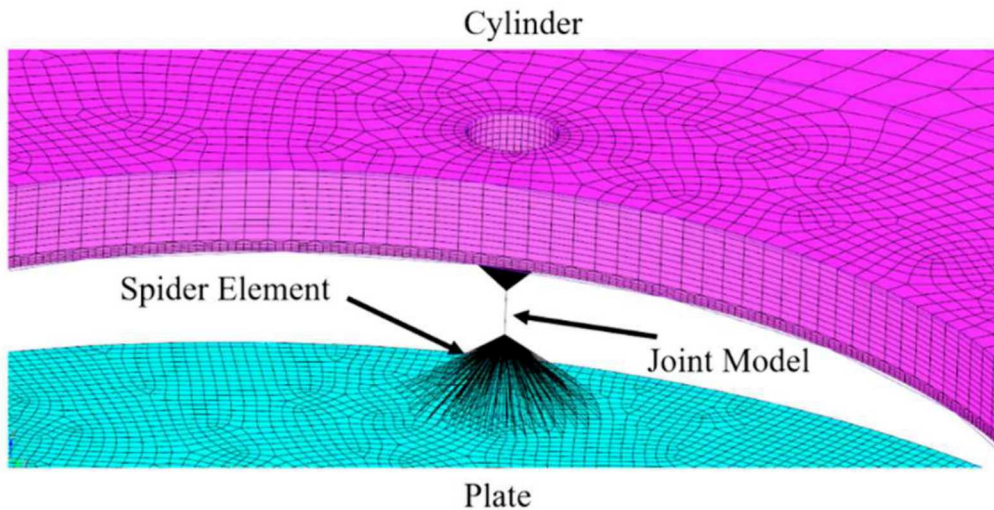


Amplitude dependent natural frequency and damping ratio



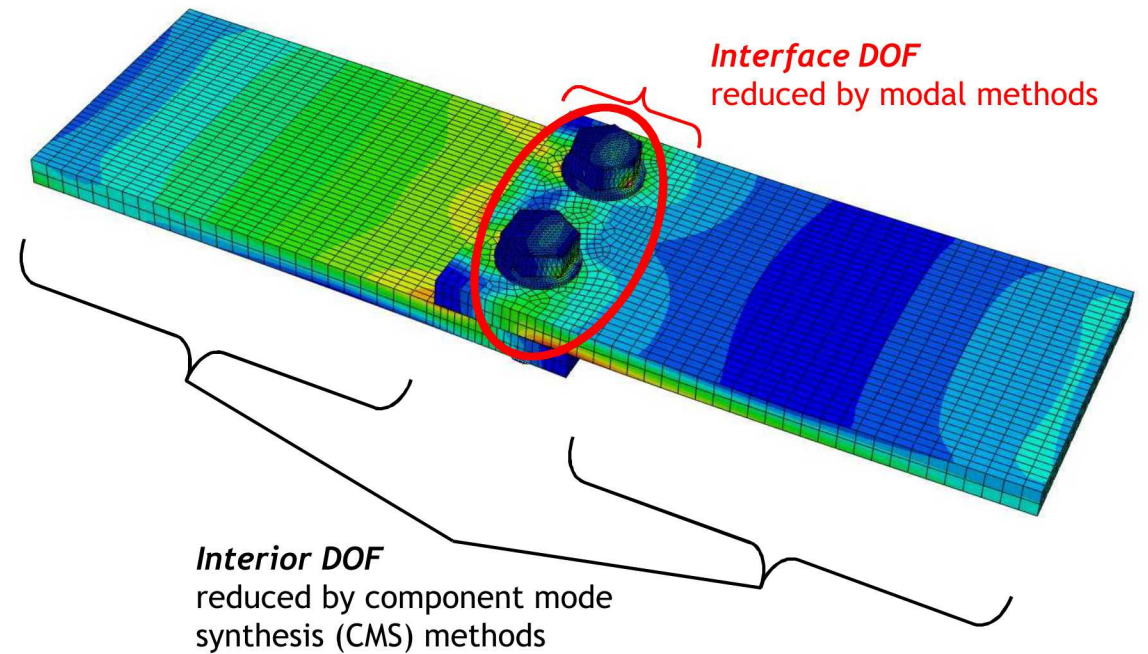
## 6 How to efficiently develop models with mechanical interfaces?

### Whole Joint Modeling (Rigid Interfaces)



- Goal: Estimate/calibrate the joint parameters in the whole joint reduced order models to match response from high-fidelity models and/or experiments

### Interface Reduction (Flexible Interfaces)



- Goal: keep full kinematics and nonlinear elements, and apply interface reduction

# Overview of Structural Dynamics of Mechanical Joints

Nonlinear reduced order models with mechanical interfaces

- Whole joint modeling
- Interface reduction

Experimental methods to measure nonlinear frequency and damping characteristics

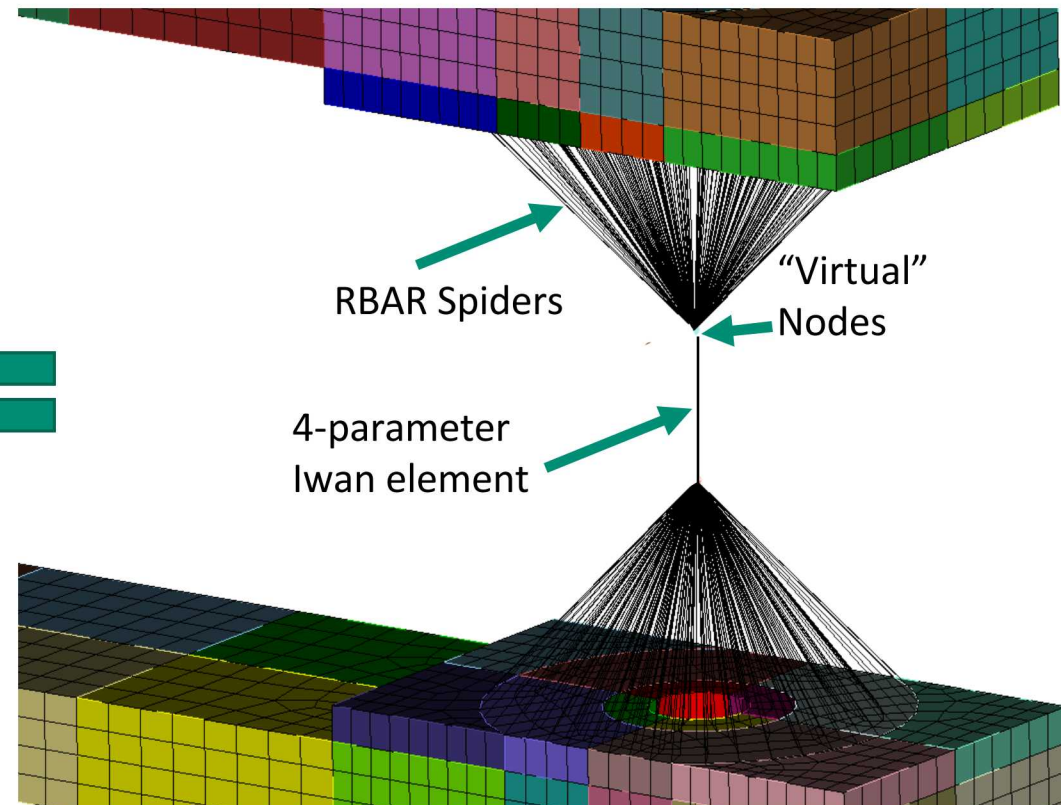
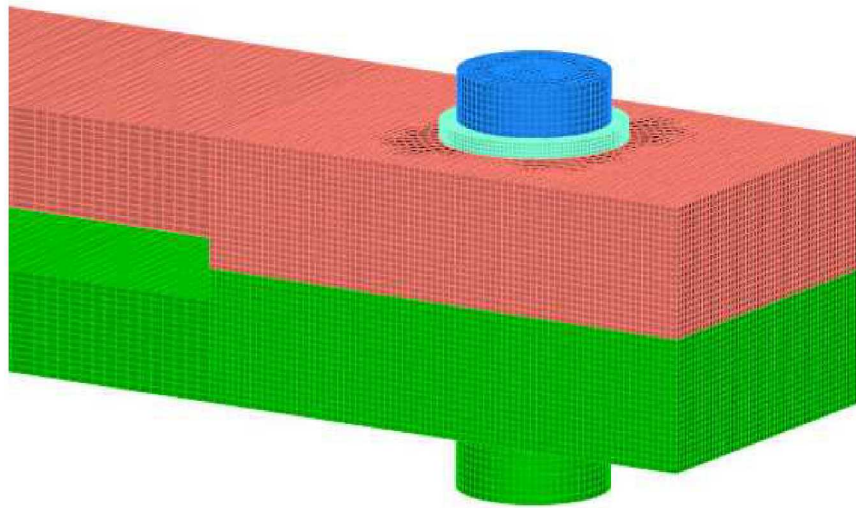
Comparison of various dissipation models to model jointed structures



## Objectives of whole joint modeling R&D

Contact areas in high fidelity finite element models simplified by “spidering” surface to a single node and modeling joint forces as a 1D constitutive law

Global optimization to calibrate whole joint parameters to match global response





## 9 Quasi-static Modal Analysis

### Quasi-static Modal Analysis of Full-order Model

#### Nonlinear Preload Analysis

$$\mathbf{K}\mathbf{x} + \mathbf{f}_{NL}(\mathbf{x}, \boldsymbol{\theta}) = \mathbf{f}_{pre}$$

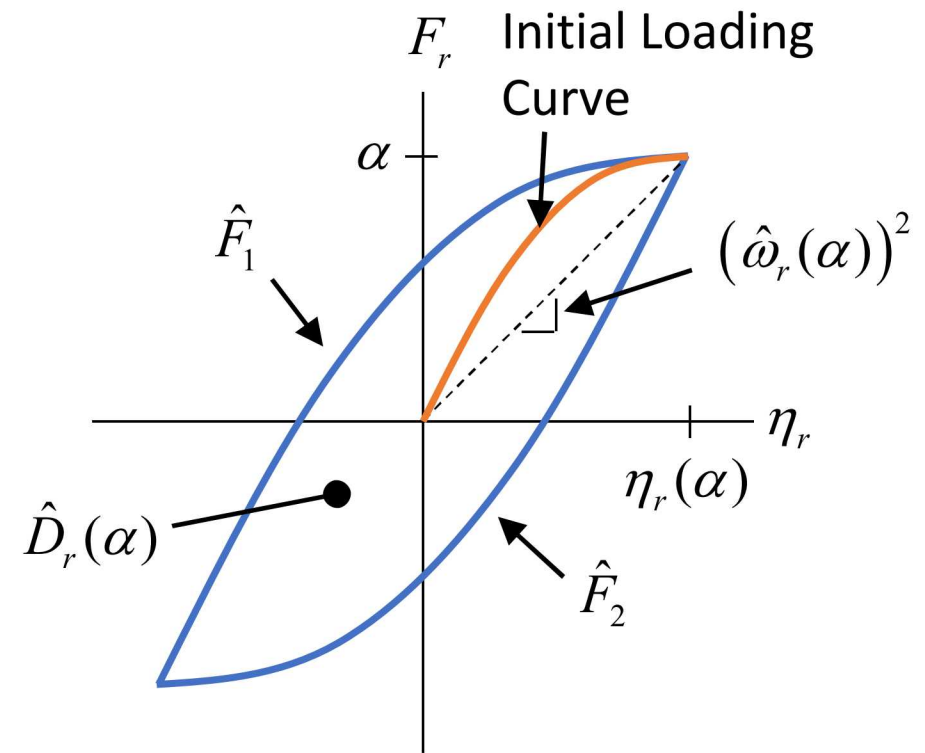
#### Linearized Modal Analysis

$$\left( \mathbf{K} + \left. \frac{d\mathbf{f}_{NL}(\mathbf{x}, \boldsymbol{\theta})}{d\mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_{pre}} - \omega_r^2 \mathbf{M} \right) \boldsymbol{\phi}_r = \mathbf{0}$$

#### Quasi-static Modal Analysis

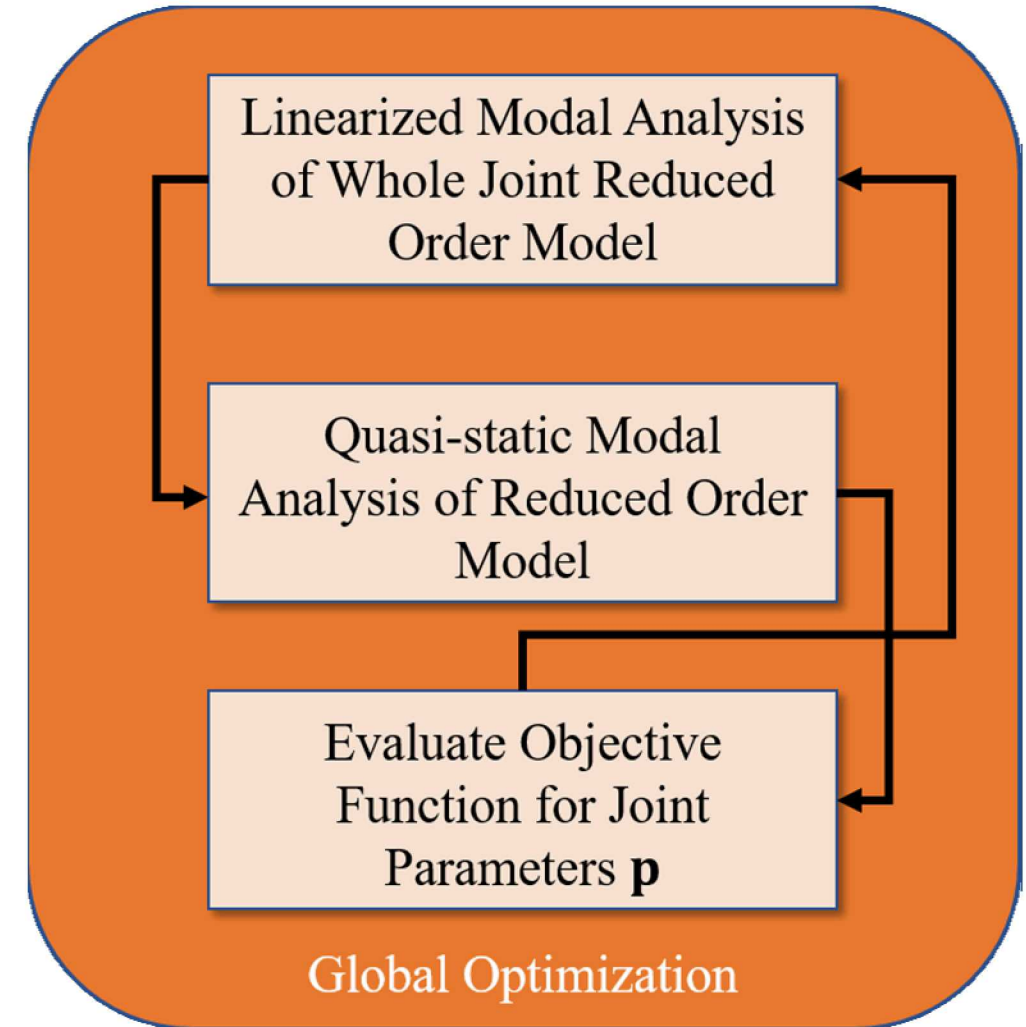
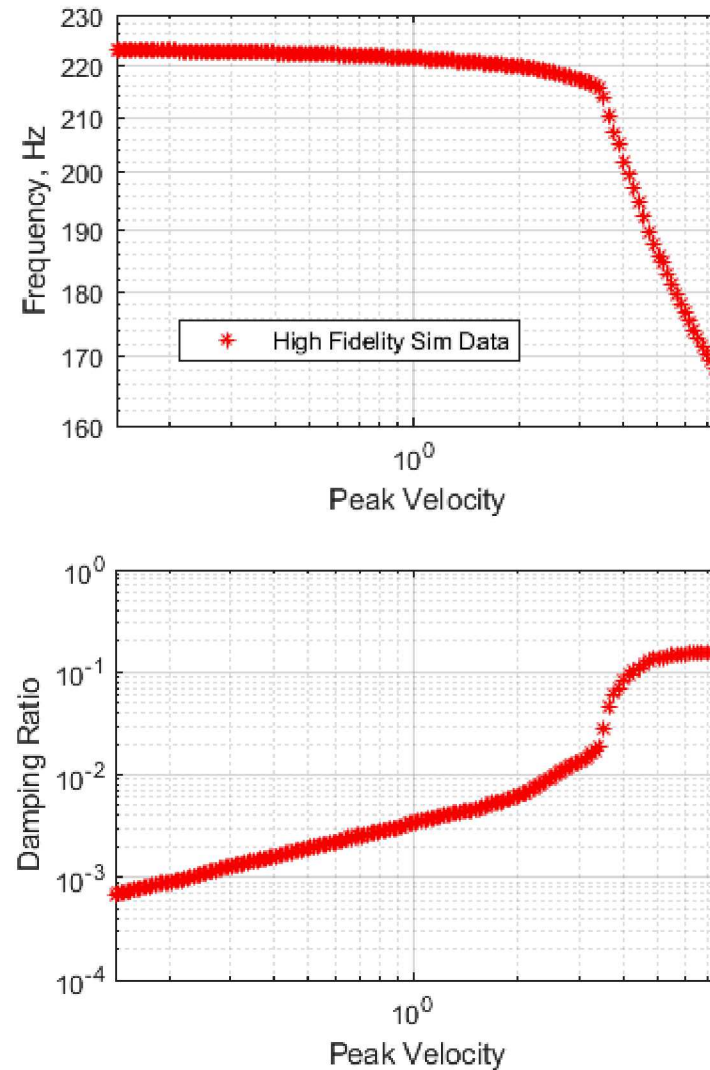
$$\mathbf{K}\mathbf{x} + \mathbf{f}_{NL}(\mathbf{x}, \boldsymbol{\theta}) = \mathbf{f}_{pre} + \mathbf{M}\boldsymbol{\phi}_r\alpha$$

Estimate modal amplitude dependent natural frequencies,  $\omega_r(\alpha)$ , and damping ratios,  $\zeta_r(\alpha)$ , of high-fidelity model and reduced models with whole joints [1]



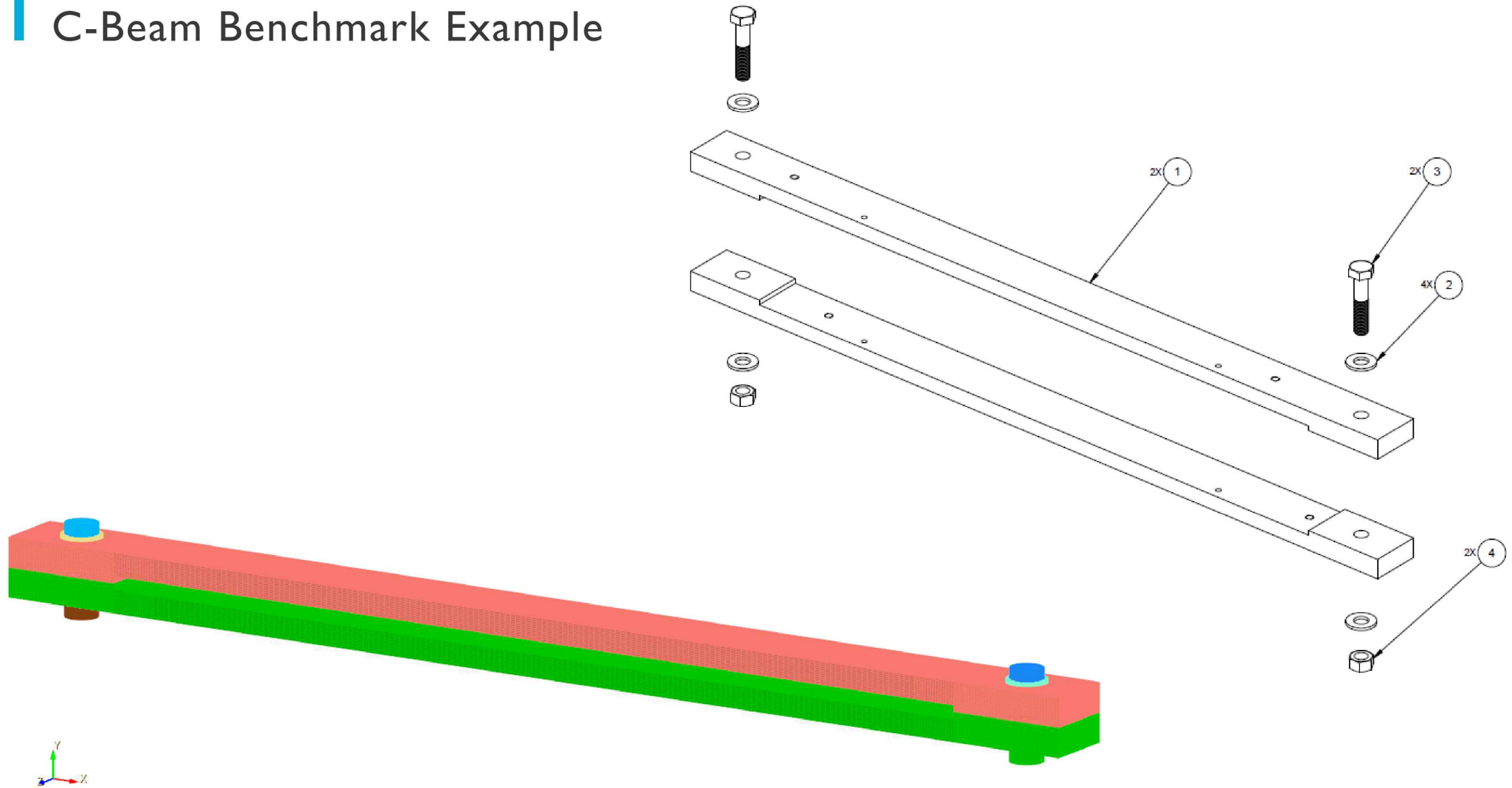
[1] M. S. Allen, R. M. Lacayo, and M. R. W. Brake, "Quasi-static Modal Analysis based on Implicit Condensation for Structures with Nonlinear Joints," presented at the ISMA2016 - International Conference on Noise and Vibration Engineering, Leuven, Belgium, 2016.

# Whole joint calibration via multi-objective optimization [1]

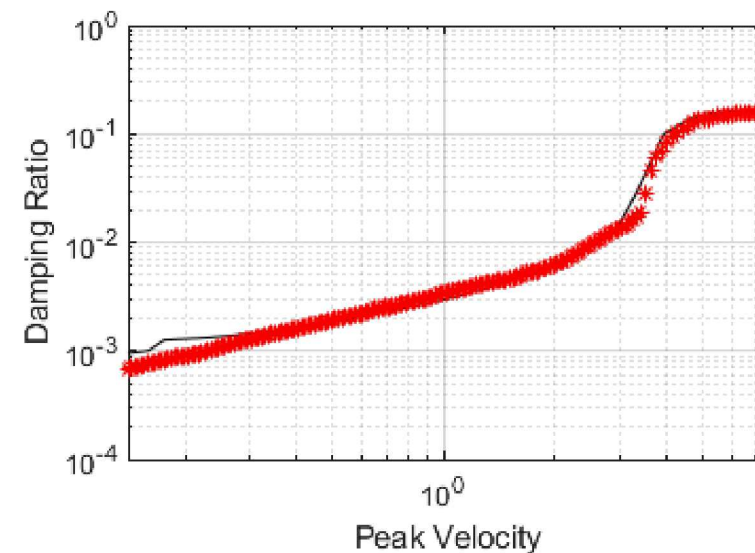
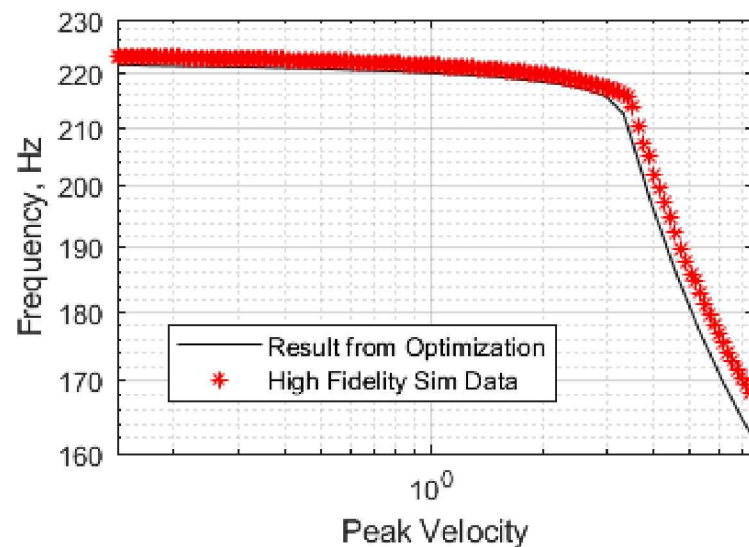
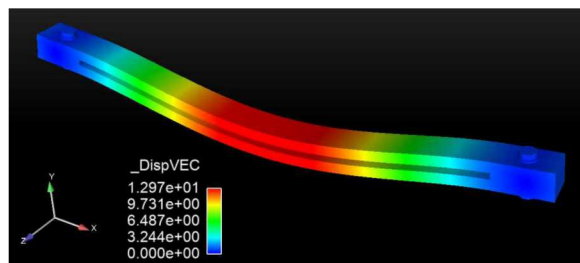




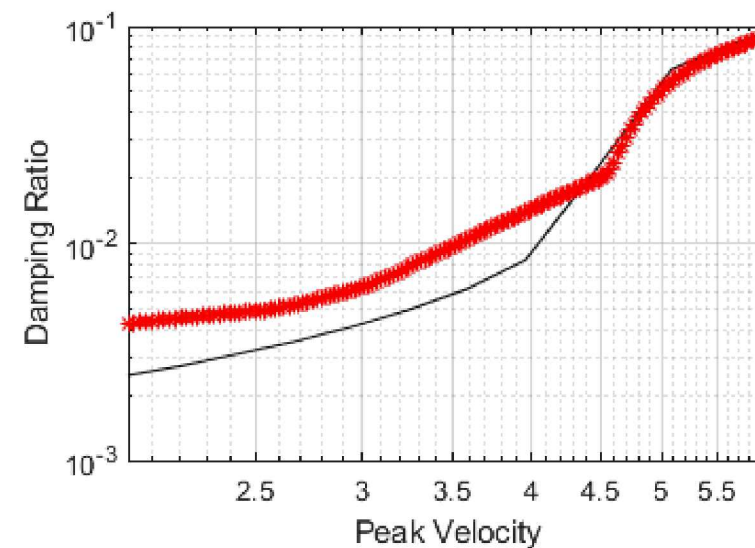
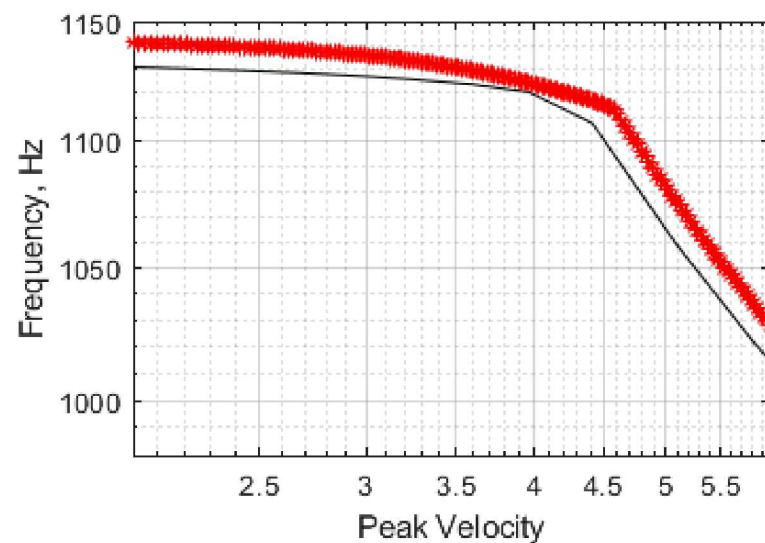
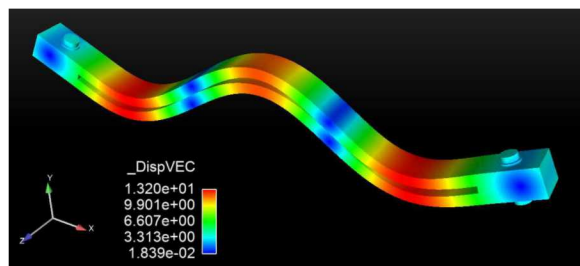
# C-Beam Benchmark Example



Mode 1 223 Hz



Mode 8 1142 Hz





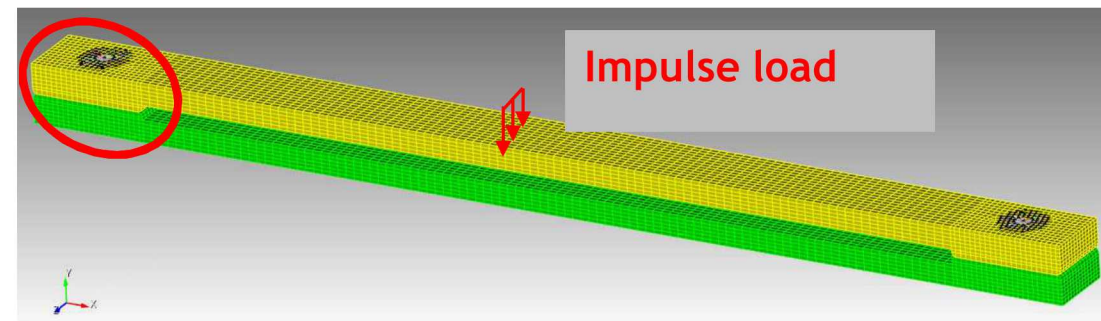
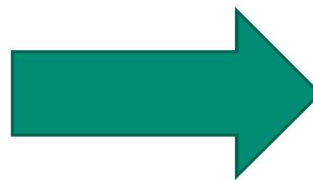
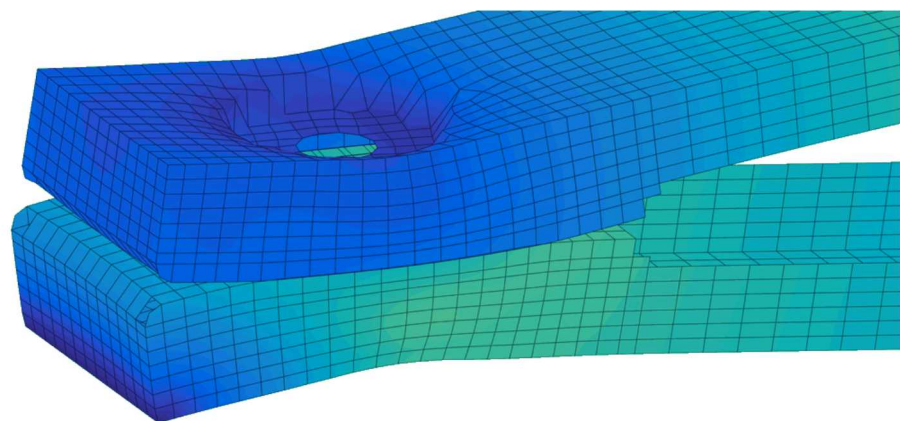
# Overview of Structural Dynamics of Mechanical Joints

Nonlinear reduced order models with mechanical interfaces

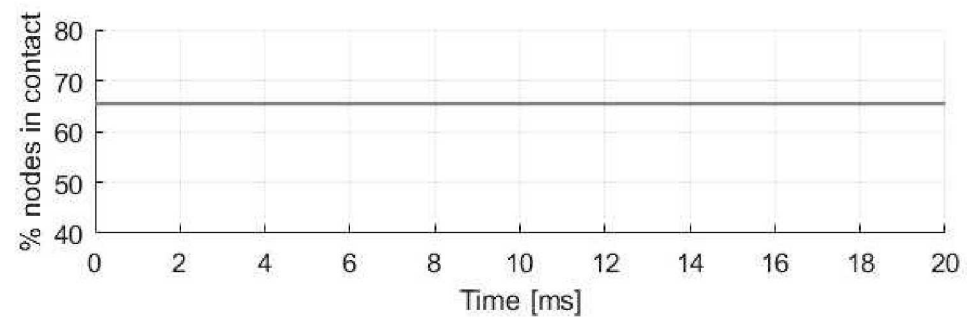
- Whole joint modeling
- Interface reduction

Experimental methods to measure nonlinear frequency and damping characteristics

Comparison of various dissipation models to model jointed structures



Nodes in contact: 66%



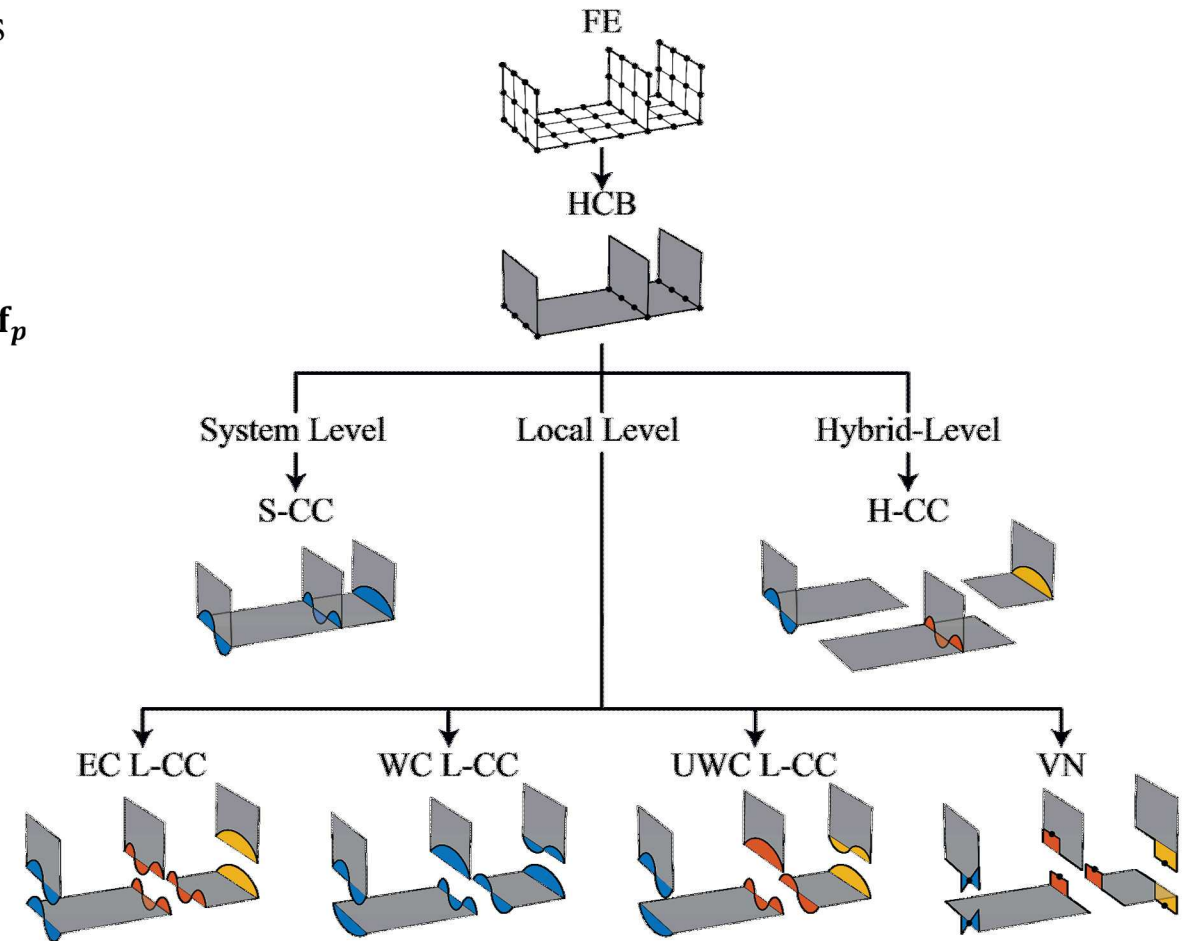


HCB reduced model dominated by potentially thousands of  $r$ -set DOF

$$\begin{bmatrix} \mathbf{I}_{ii} & \mathbf{M}_{ir}^{\text{HCB}} & \mathbf{M}_{ip}^{\text{HCB}} \\ \mathbf{M}_{ri}^{\text{HCB}} & \mathbf{M}_{rr}^{\text{HCB}} & \mathbf{M}_{rp}^{\text{HCB}} \\ \mathbf{M}_{pi}^{\text{HCB}} & \mathbf{M}_{pr}^{\text{HCB}} & \mathbf{M}_{pp}^{\text{HCB}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_i \\ \ddot{\mathbf{u}}_r \\ \ddot{\mathbf{u}}_p \end{Bmatrix} + \begin{bmatrix} \mathbf{\Lambda}_{ii}^{\text{FI}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{rr}^{\text{HCB}} & \mathbf{K}_{rp}^{\text{HCB}} \\ \mathbf{0} & \mathbf{K}_{pr}^{\text{HCB}} & \mathbf{K}_{pp}^{\text{HCB}} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_i \\ \mathbf{u}_r \\ \mathbf{u}_p \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_r(\mathbf{u}_r) \\ \mathbf{0} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_p \end{Bmatrix}$$

Research challenge: how can we further reduce these equations?

Explored the extension of interface reduction techniques [1] to problems involving nonlinear contact [2,3]



[1] Krattiger, D. et al. "Interface reduction for Hurty/Craig-Bampton substructured models: Review and improvements," *Mechanical Systems and Signal Processing*, 114, pp 579-603, 2019.

[2] Kuether RJ, Coffin PB, Brink AR "On Hurty/Craig-Bampton Substructuring With Interface Reduction on Contacting Surfaces," *ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Volume 8: 29th Conference on Mechanical Vibration and Noise*.

[3] Hughes, P.J. et al. "Interface Reduction on Hurty/Craig-Bampton Substructures with Frictionless Contact," *2018 International Modal Analysis Conference (IMAC) XXXVI*, Orlando, FL, 2018.

Solve the quasi-static version of the HCB model for preloaded equilibrium

$$\begin{bmatrix} \Lambda_{ii}^{FI} & 0 & 0 \\ 0 & \mathbf{K}_{rr}^{HCB} & \mathbf{K}_{rp}^{HCB} \\ 0 & \mathbf{K}_{pr}^{HCB} & \mathbf{K}_{pp}^{HCB} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_i \\ \mathbf{u}_r \\ \mathbf{u}_p \end{Bmatrix} + \begin{Bmatrix} 0 \\ \mathbf{f}_r(\mathbf{u}_r) \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \mathbf{f}_{pre} \end{Bmatrix}$$

Apply a secondary reduction about the preloaded equilibrium such that

$$\mathbf{v} = \begin{Bmatrix} \mathbf{q}_i \\ \mathbf{u}_r \\ \mathbf{u}_p \end{Bmatrix} = \mathbf{v}_{pre} + \begin{bmatrix} \mathbf{I} & 0 & 0 \\ 0 & \boldsymbol{\Phi}^{SCC} & \boldsymbol{\Psi}^{SCCe} \\ 0 & 0 & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_i \\ \mathbf{q}_r \\ \mathbf{u}_p \end{Bmatrix} = \mathbf{v}_{pre} + \mathbf{T}^{SCCe} \mathbf{w}$$

where the tangent S-CC modes and static constraint modes computed about preloaded state

$$\left[ \mathbf{K}_{rr}^{HCB} + \underbrace{\frac{\partial \mathbf{f}_r(\mathbf{u}_r)}{\partial \mathbf{u}_r}}_{\text{Tangent stiffness contributions about deformed state}} \bigg|_{\mathbf{v}_{pre}} - (\omega^{SCC})^2 \mathbf{M}_{rr}^{HCB} \right] \boldsymbol{\Phi}_s^{SCC} = 0 \quad \boldsymbol{\Psi}^{SCCe} = - \left( \mathbf{K}_{rr}^{HCB} + \underbrace{\frac{\partial \mathbf{f}_r(\mathbf{u}_r)}{\partial \mathbf{u}_r}}_{\text{Tangent stiffness contributions about deformed state}} \bigg|_{\mathbf{v}_{pre}} \right)^{-1} \mathbf{K}_{rp}^{HCB}$$

Tangent stiffness contributions  
about deformed state

Using the S-CC modes from the initial reduction on the interface

$$\mathbf{v} = \begin{Bmatrix} \mathbf{q}_i \\ \mathbf{u}_r \\ \mathbf{u}_p \end{Bmatrix} = \mathbf{v}_{\text{pre}} + \begin{bmatrix} \mathbf{I} & 0 & 0 \\ 0 & \boldsymbol{\Phi}^{\text{SCC}} & \boldsymbol{\Psi}^{\text{SCCe}} \\ 0 & 0 & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_i \\ \mathbf{q}_r \\ \mathbf{u}_p \end{Bmatrix} = \mathbf{v}_{\text{pre}} + \mathbf{T}^{\text{SCCe}} \mathbf{w}$$

Take Taylor series expansion around preloaded configuration to get modal derivatives

$$\mathbf{T}_{i(\mathbf{w})} = \mathbf{T}_i|_c + \underbrace{\sum_{j=1}^{n_w} \frac{\partial \mathbf{T}_i}{\partial w_j} \bigg|_c}_{\text{Describe how modes change for a given modal amplitude of response}} (w_j - w_j(\text{PL})) + \text{H. O. T.}$$

Describe how modes change for a given modal amplitude of response

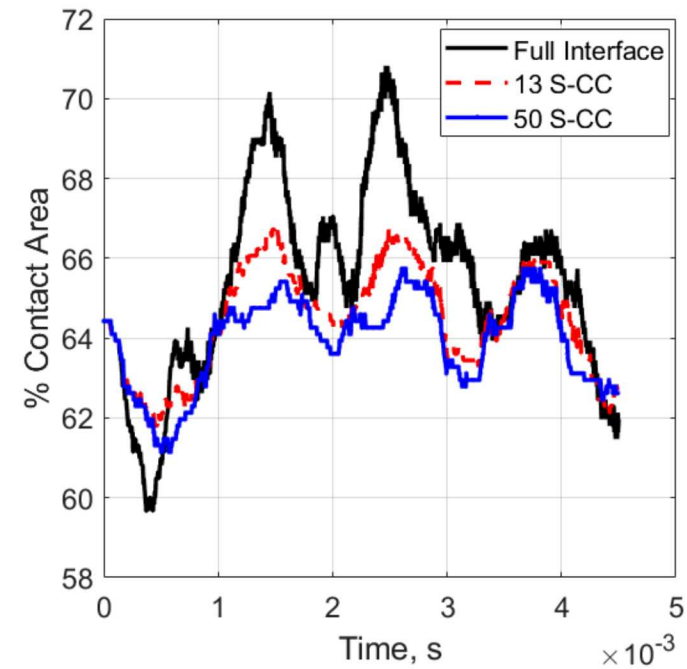
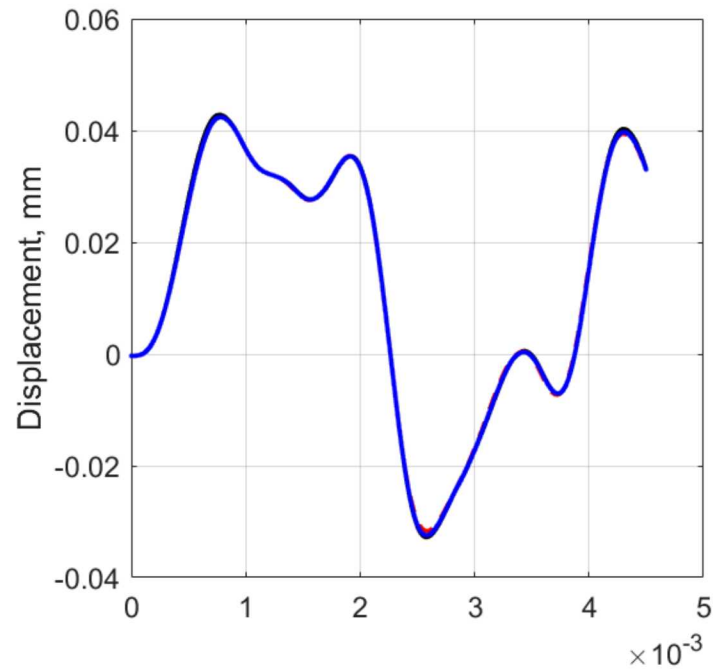
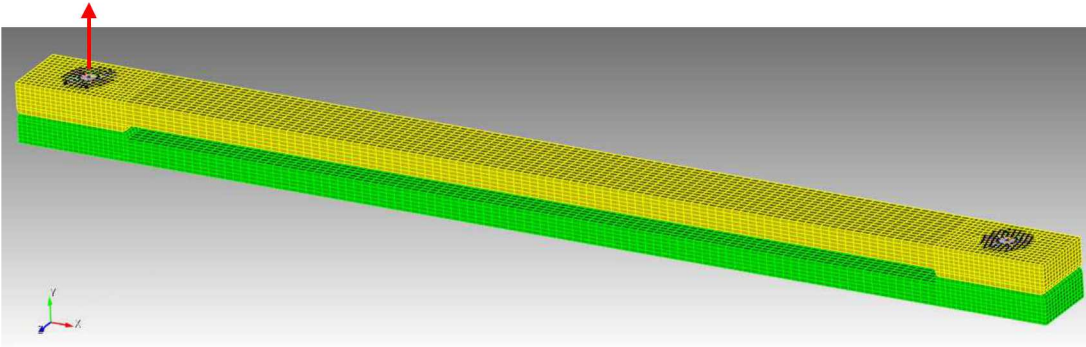
Take Taylor series expansion around preloaded configuration to get modal derivatives

$$\mathbf{T}^{\text{TVD}} = \begin{bmatrix} \mathbf{T}^{\text{SCCe}} & \frac{\partial \mathbf{T}^{\text{SCCe}}}{\partial \mathbf{w}} \end{bmatrix}$$



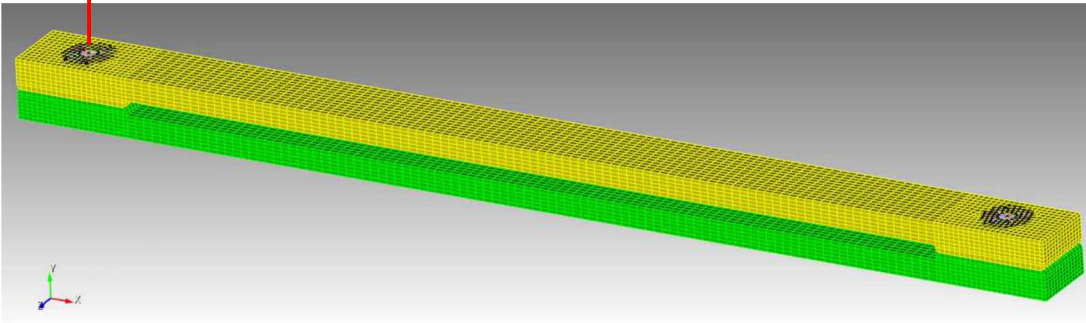
## Time-domain simulations due to impulse load

Impulse load  $A = 2000 \text{ N}$



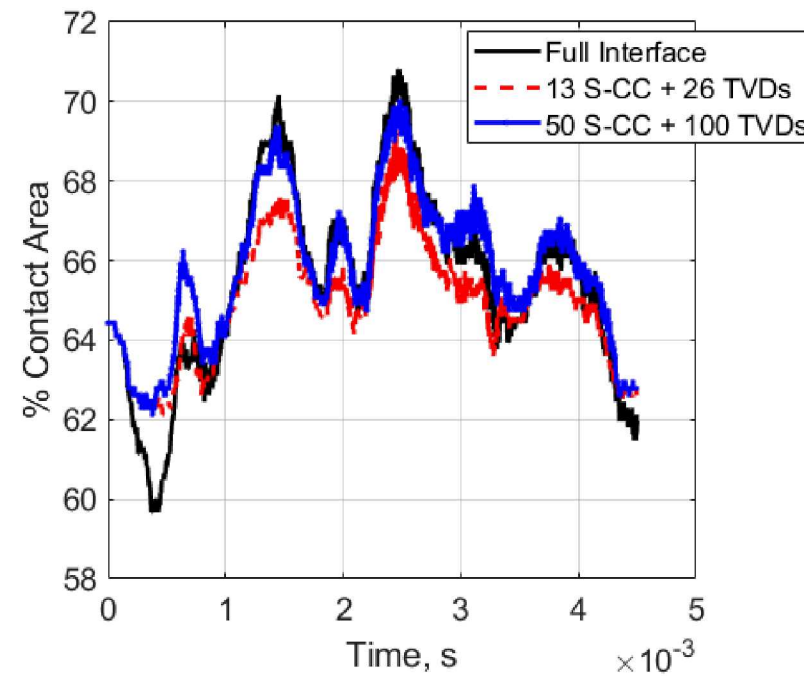
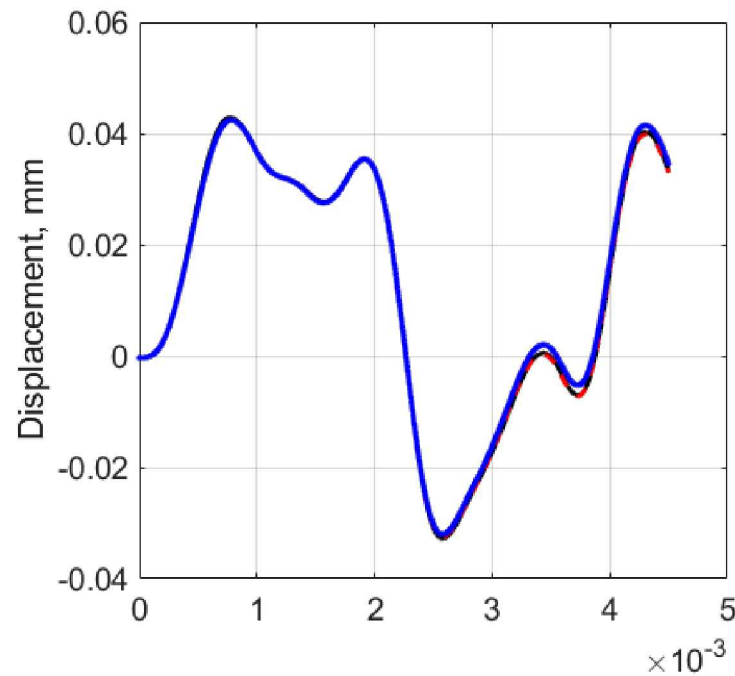
## Time-domain simulations due to impulse load

Impulse load  $A = 2000 \text{ N}$



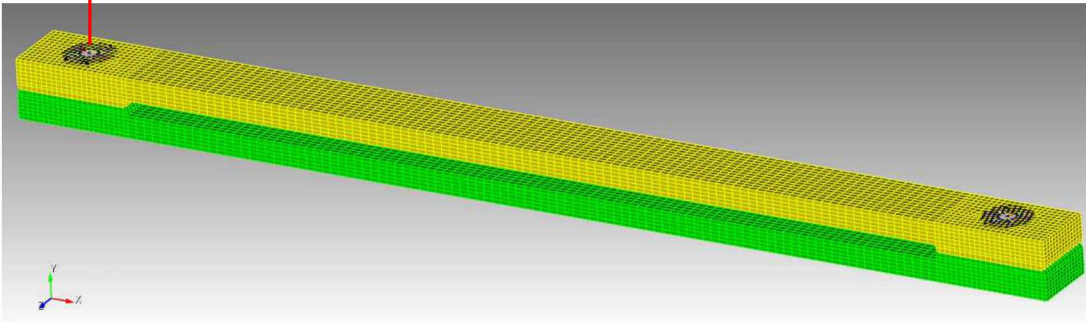
\*Full interface ~ 90 minutes

\*\* IR ROMs - 2 minutes



# Time-domain simulations due to impulse load

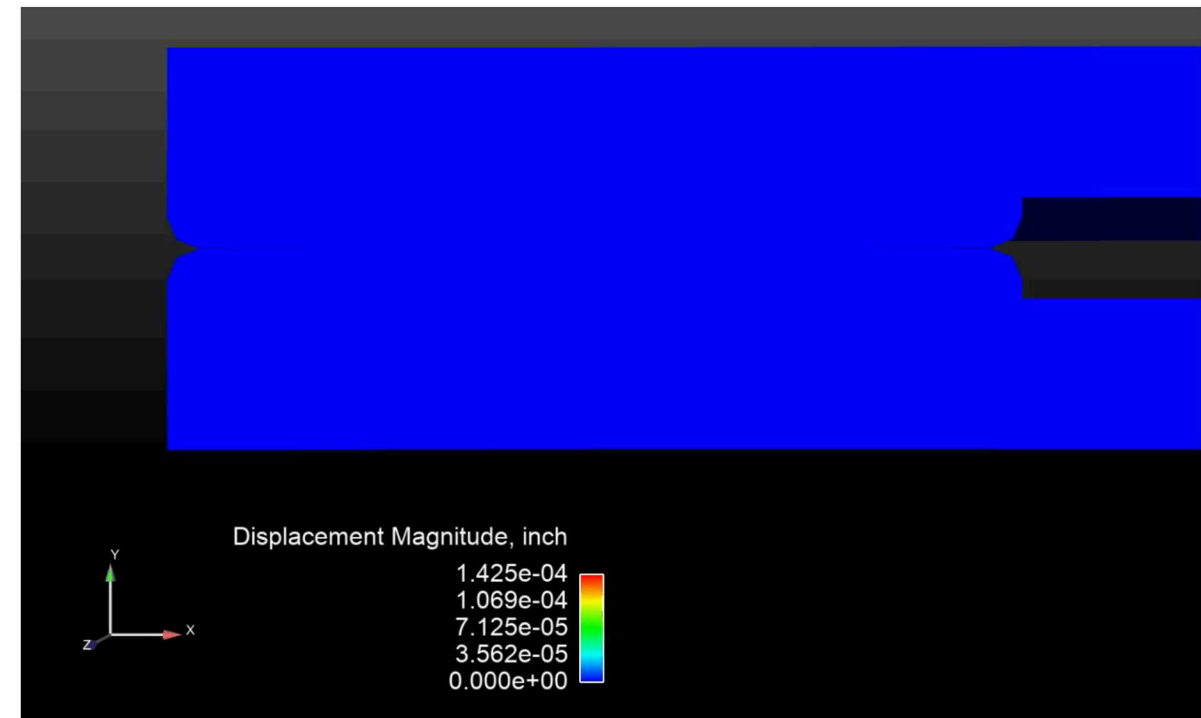
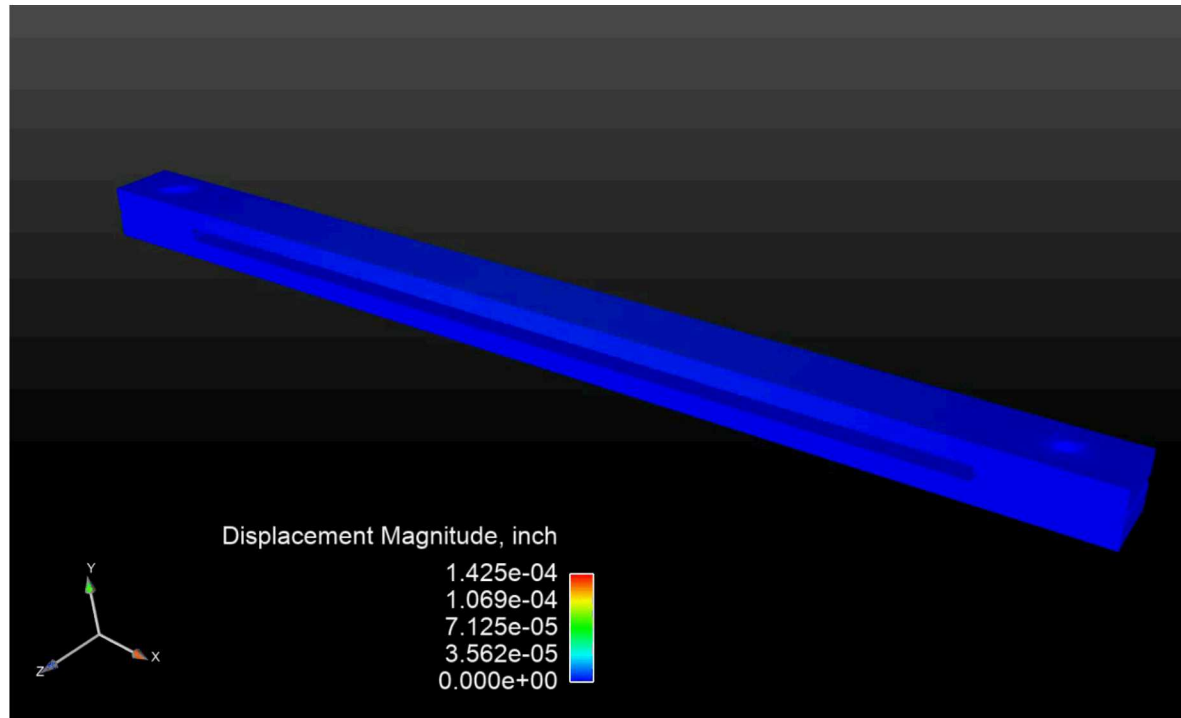
Impulse load A = 2000 N



HCB-SCCe-TVD ROM

272 DOF

2.0 min





# Overview of Structural Dynamics of Mechanical Joints

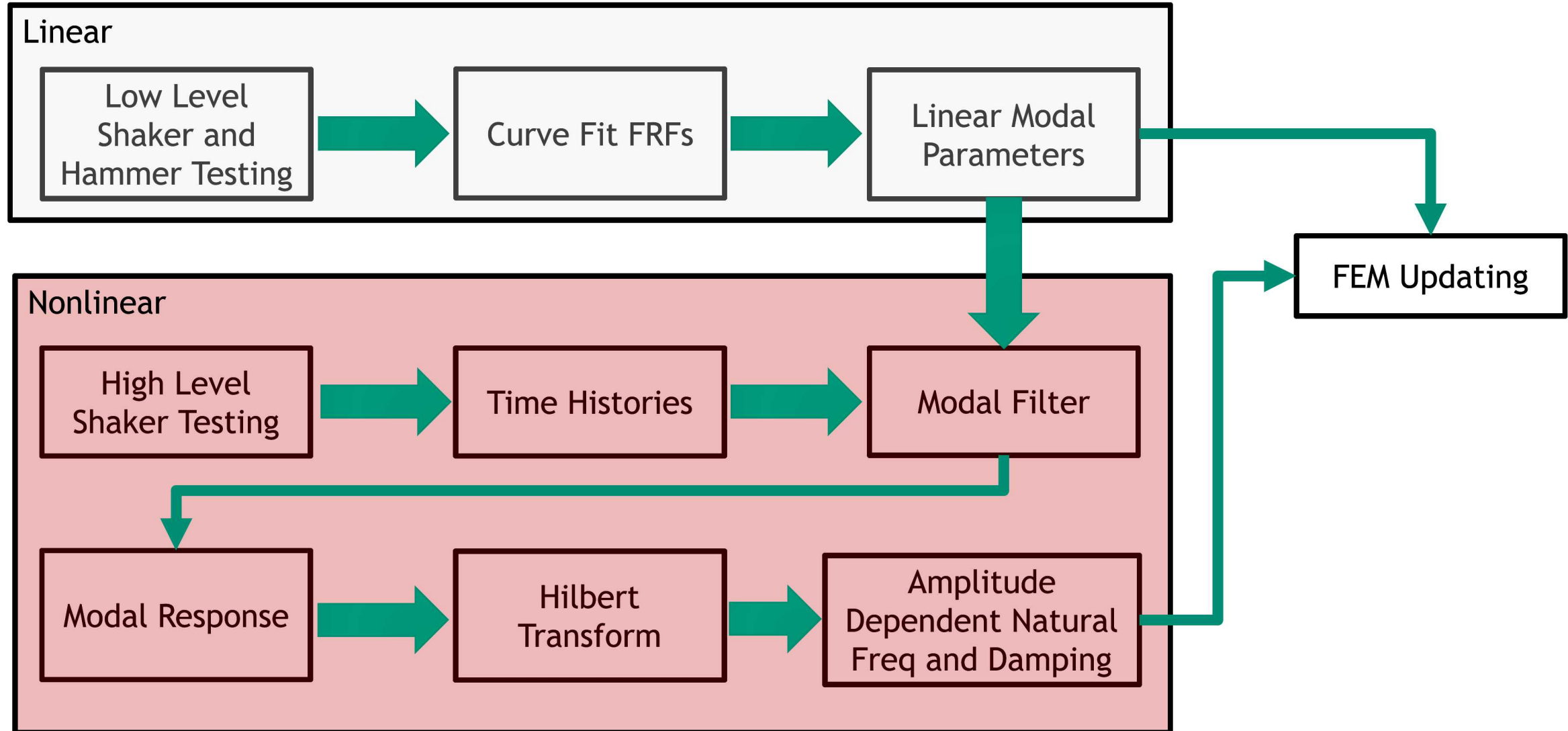
Nonlinear reduced order models with mechanical interfaces

- Whole joint modeling
- Interface reduction

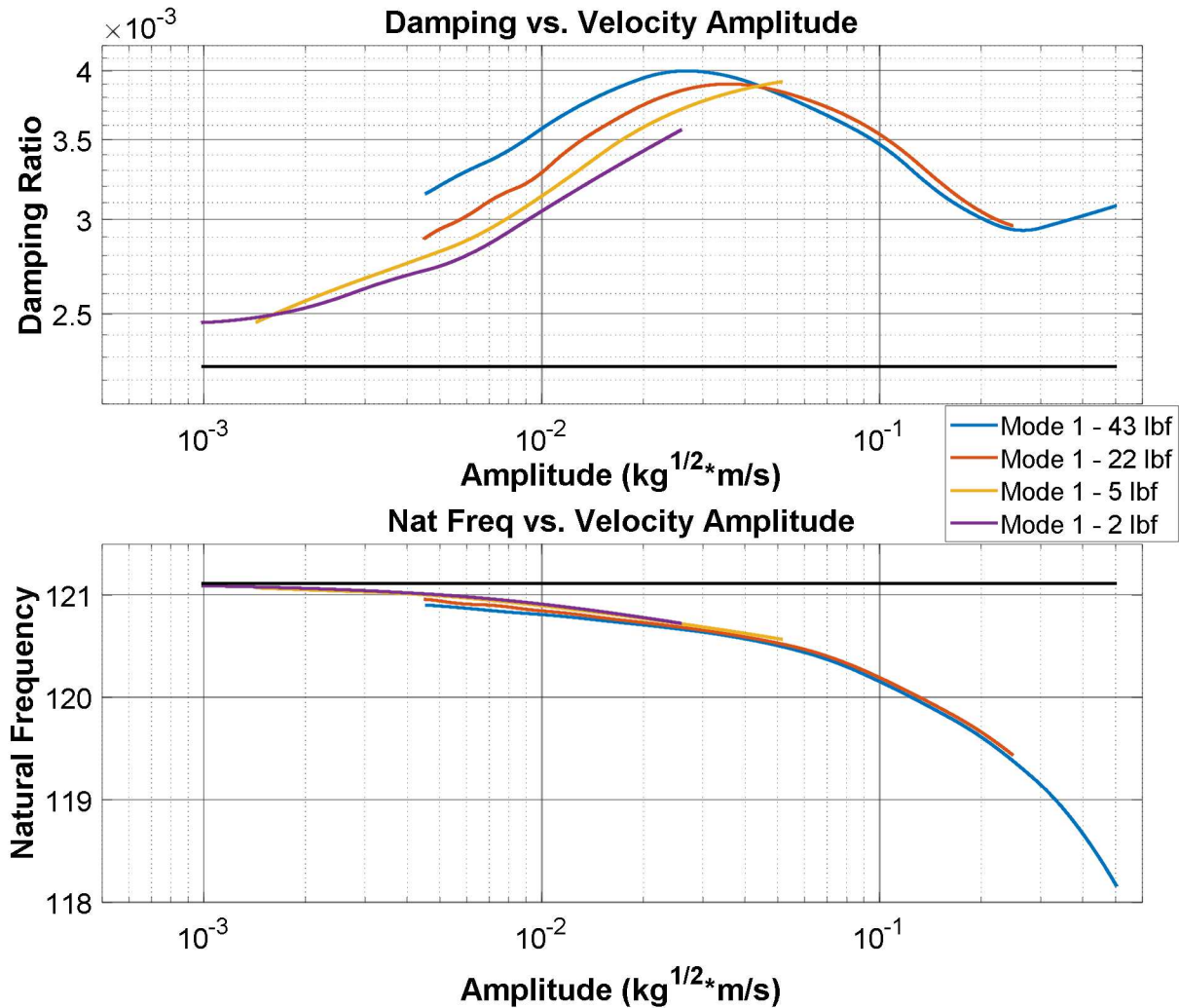
Experimental methods to measure nonlinear frequency and damping characteristics

Comparison of various dissipation models to model jointed structures

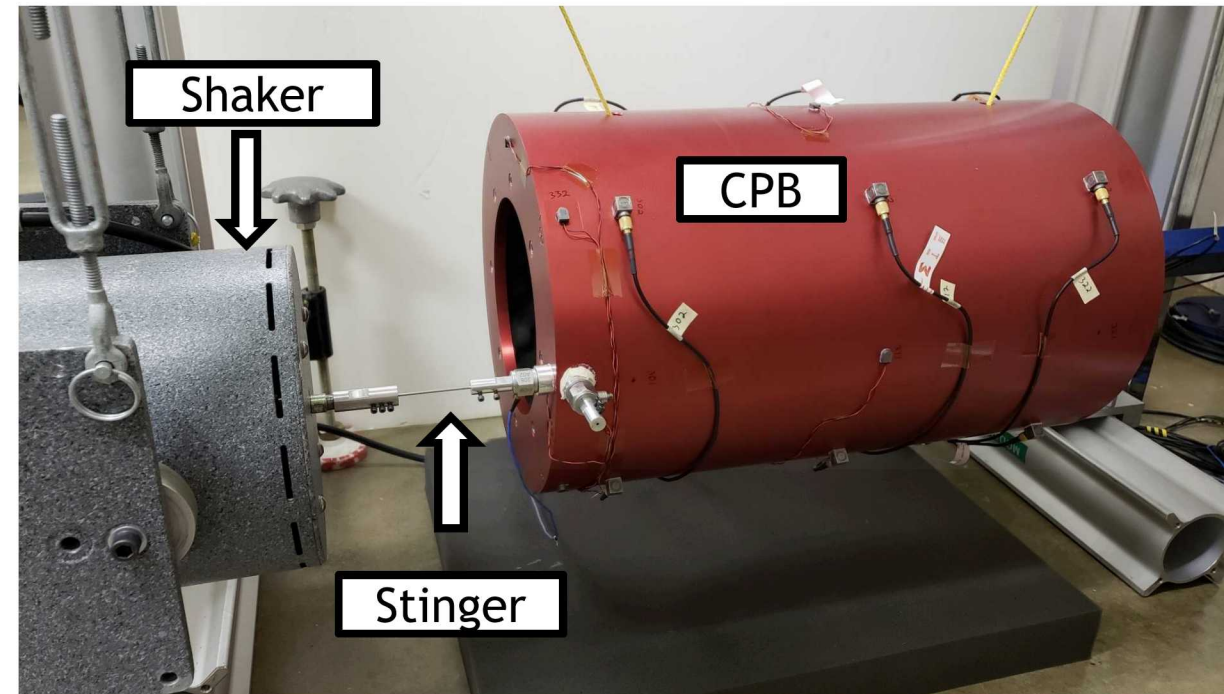
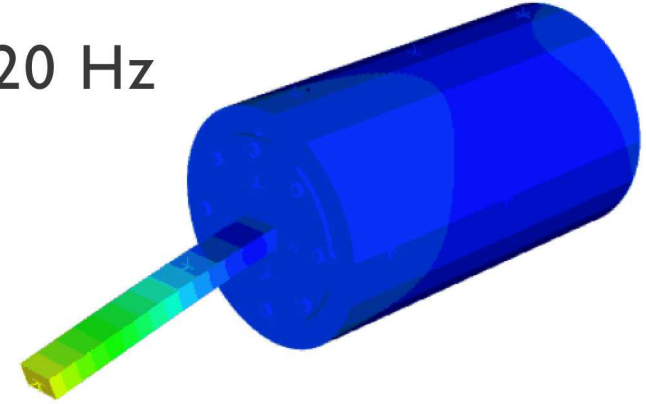
# Experimental procedure to nonlinear system identification



# Measured frequency and damping data on cylinder-plate-beam (NOMAD)



Mode I @ 120 Hz





# Overview of Structural Dynamics of Mechanical Joints

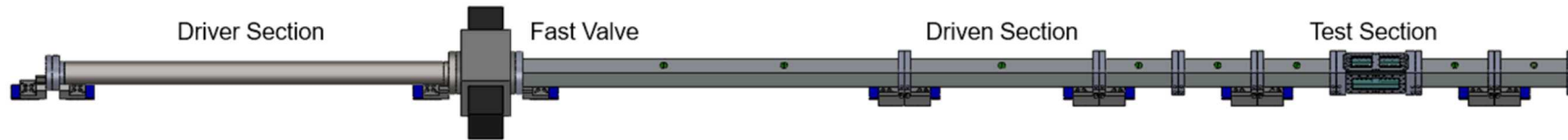
Nonlinear reduced order models with mechanical interfaces

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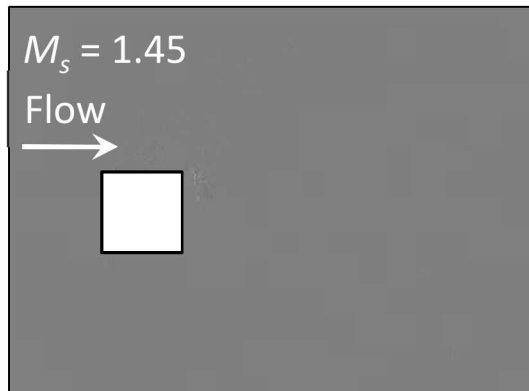
Experimental methods to measure nonlinear frequency and damping characteristics

Comparison of various dissipation models to model jointed structures

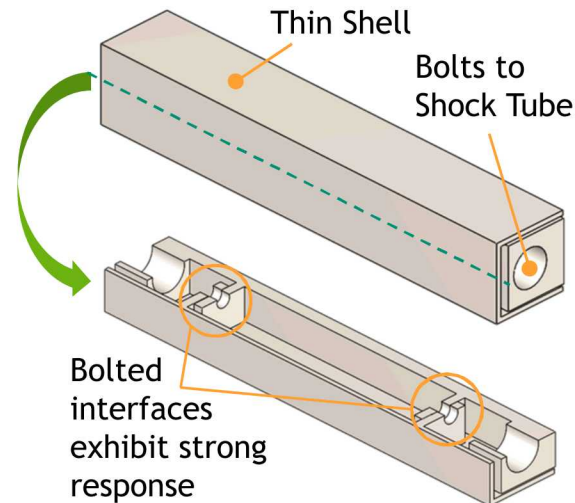
Predicting transfer of energy from a flowfield to structures is challenging, and is often based on semi-empirical models and simple modal testing, with large uncertainties



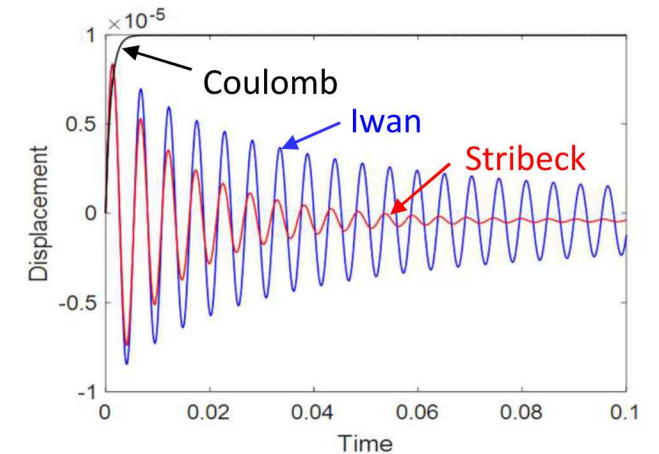
Shock tube creates impulsive start and periodic vortex shedding loads



Loads a susceptible jointed structure



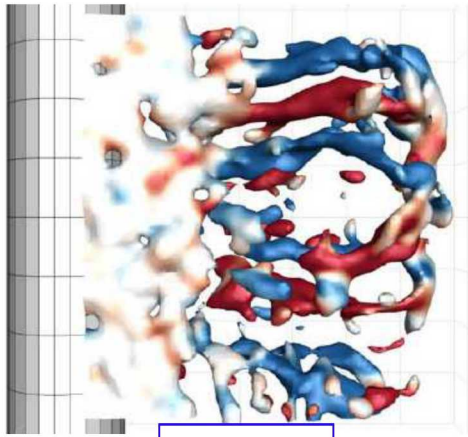
Evaluate response of structural joint models



Primary goal: measure the input loading and output structural response to test the predictivity of constitutive models on jointed structures under real fluid dynamic loading.

# Diagnostic Approach

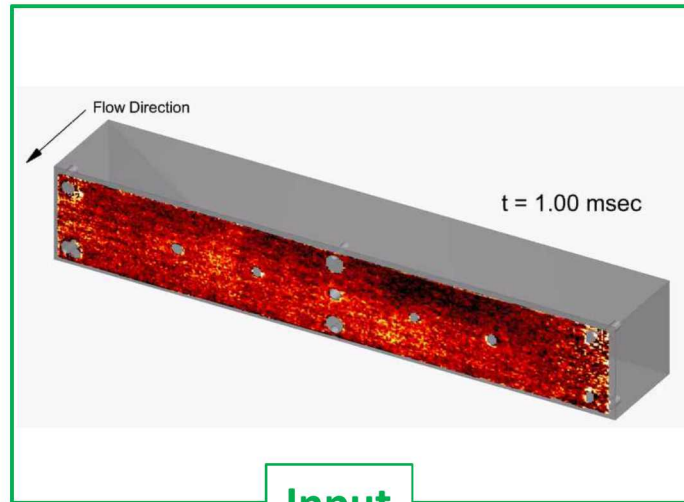
## High-Speed Tomographic-PIV<sup>1</sup>



Source

- Measure the flow-field responsible for pressure loading in a time-resolved fashion over a volume

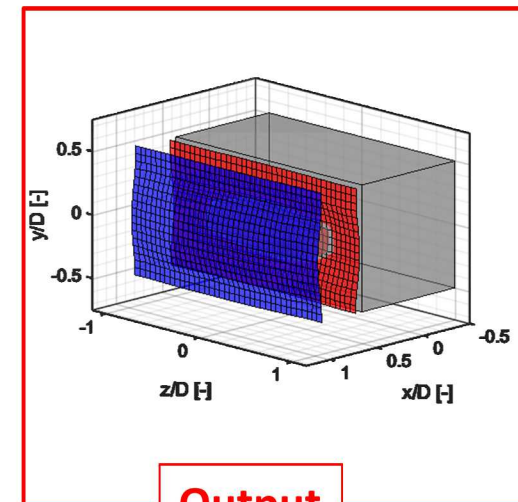
## High-Speed PSP



Input

- Obtain the pressure field on the body surface responsible for the loading. This is used directly as the input in simulations.

## High-Speed DIC



Output

- Measure the output using DIC. Directly compare simulations to these measurements.

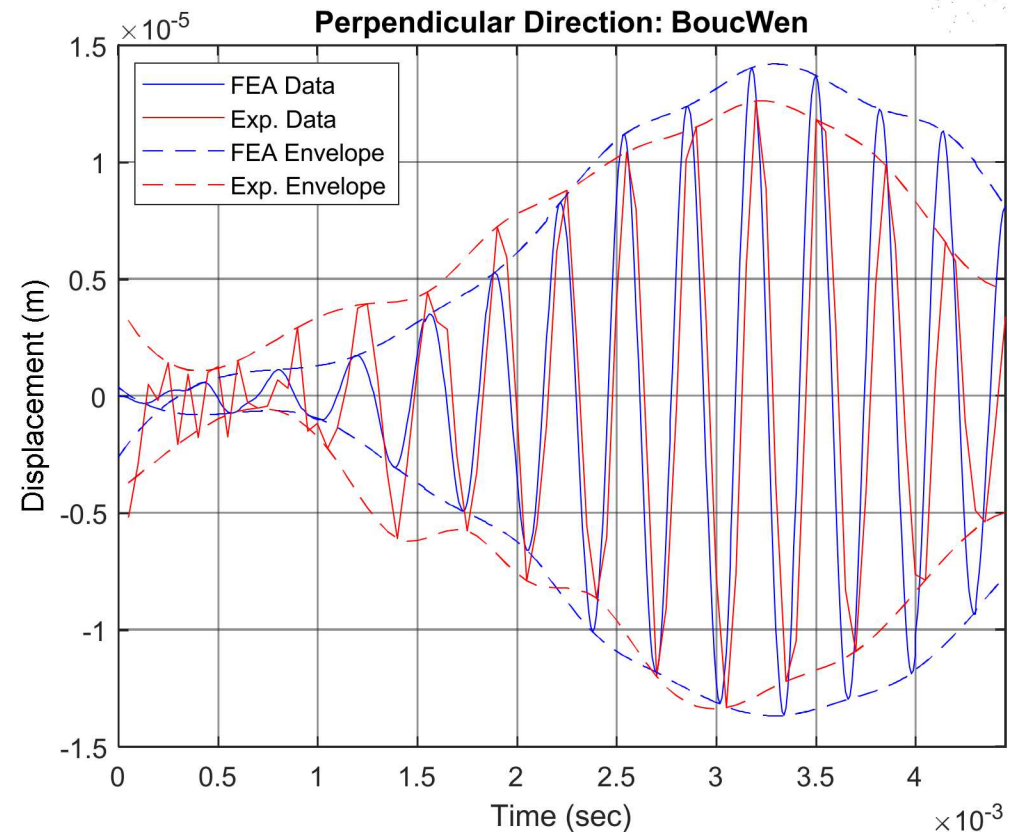
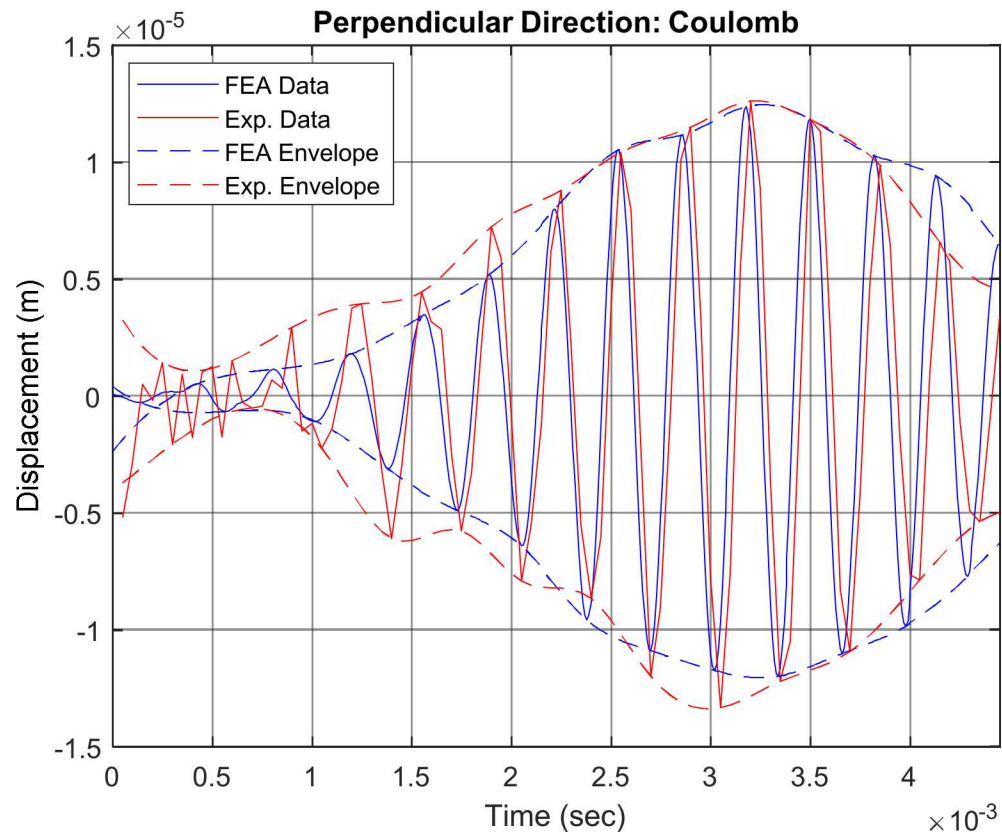
This work: perform high-speed PSP and DIC *simultaneously*



# Comparison of friction models for jointed structure in shock tube

Coulomb friction and Bouc-Wen element appear to match test data well

- Rocking mode is main contribution to loading the joint



Joints in structural dynamics realize new challenges that make modeling and experiments challenging

- Linear theory no longer valid
- Excessive simulation times for models
- Issues with convergence, repeatability, etc..

Reduced order models provide a unique opportunity for model calibration

- System identification based on nonlinear frequency and damping
- Ability to quickly sample the model and understand joint behavior

Many options available for modeling friction: which one is correct?

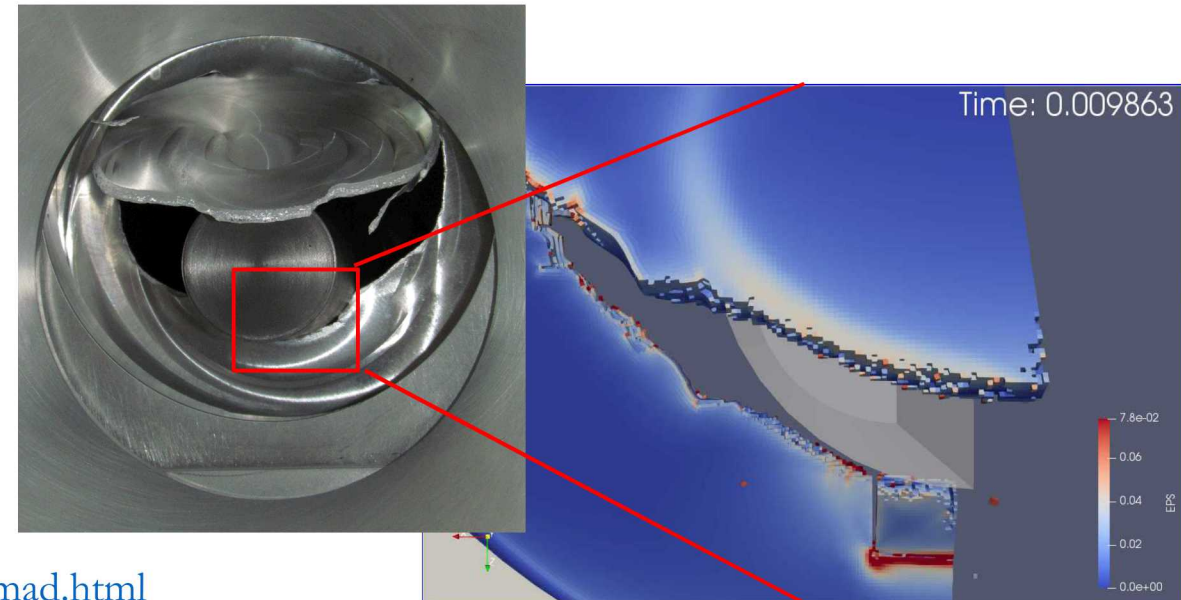
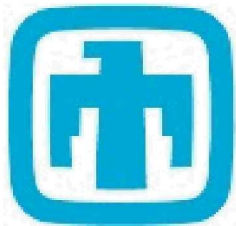
- Small length scales beyond our macro-scale models
- Coulomb most widely available/used

## 29 Research collaboration opportunities for students and professors

- Hosted by Sandia National Laboratories and University of New Mexico
- Collaborative opportunity to work on research in topic areas across nonlinear mechanics and dynamics
- 7 week program held in Albuquerque, New Mexico; open to graduate and highly qualified undergraduate level students



[nomad@sandia.gov](mailto:nomad@sandia.gov)



For more information, please visit:

[http://www.sandia.gov/careers/students\\_postdocs/internships/institutes/nomad.html](http://www.sandia.gov/careers/students_postdocs/internships/institutes/nomad.html)



## Any questions?

Special thanks to my collaborators:

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Contact information: [rjkueth@sandia.gov](mailto:rjkueth@sandia.gov)

