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# Finite-Difference Algorithm for 3D Orthorhombic Elastic Wave Propagation

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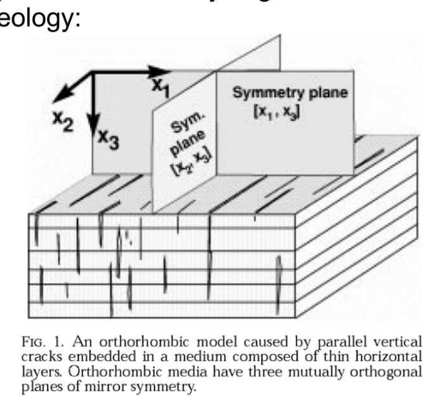
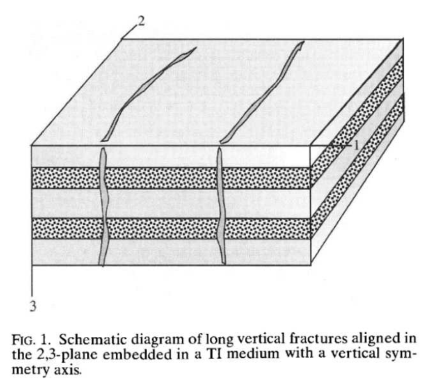
## Summary

Many geophysicists concur that an ***orthorhombic*** elastic medium, characterized by three mutually orthogonal symmetry planes, constitutes a realistic representation of seismic anisotropy in shallow crustal rocks. This symmetry condition typically arises via a dense system of vertically-aligned microfractures superimposed on a finely-layered horizontal geology:

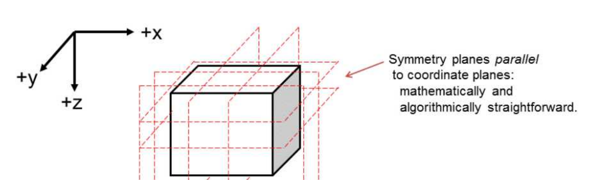


From Tsvankin, 1997, *Geophysics*.

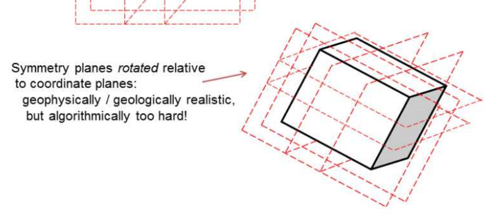


From Schoenberg and Helbig, 1997, *Geophysics*.

However, various geological deformation processes will rotate the symmetry planes away from alignment with the global XYZ coordinate planes:



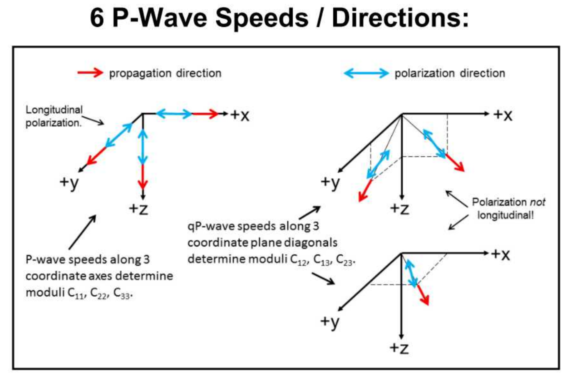

Present algorithmic assumption: 3 principal axes of orthorhombic elastic modulus tensor aligned with global XYZ coordinate axes.



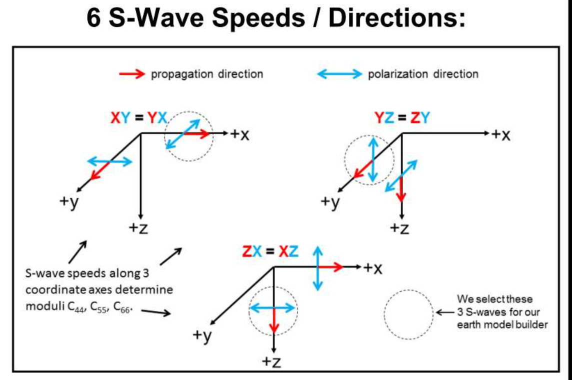

Rotated principal axes lead to significant algorithmic complications! Subject of future R&D.

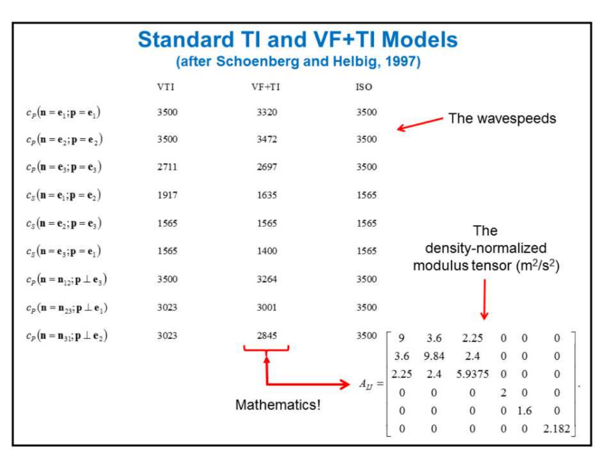
Mathematically, the elastic stress-strain constitutive relations for an orthorhombic body contain nine independent moduli. In turn, these moduli can be determined by observing (or prescribing) nine independent P-wave and S-wave phase speeds along different directions (Brown, 1989):

**6 P-Wave Speeds / Directions:**



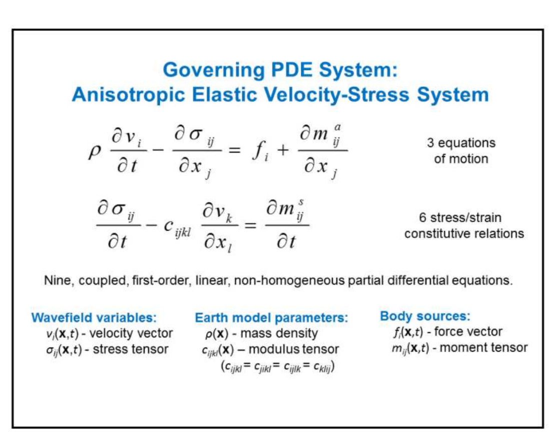
**6 S-Wave Speeds / Directions:**

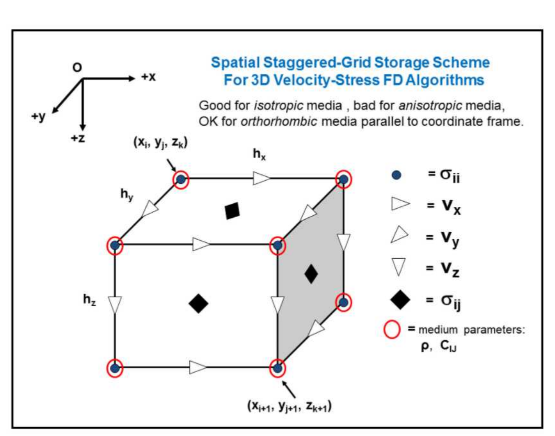





Our initial test modeling utilizes the "standard model" of a VF+TI (vertical fractures + transverse isotropic) elastic model of Schoenberg and Helbig (1997), plus its TI and isotropic counterparts.

The anisotropic elastic ***velocity-stress system***, a set of 9 coupled, first-order, linear, inhomogeneous PDEs forms the mathematical basis for our explicit time-domain finite-difference (FD) numerical algorithm. All partial derivatives are discretized with centered and staggered FD operators that are 2nd-order in time and 4th-order in space:



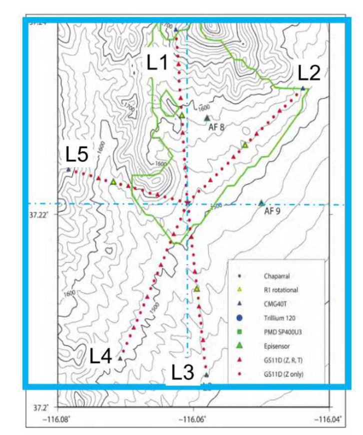




## SPE-1 Model Creation

Model
•1551 x 2001 x 723 grid points at a 2 m grid point spacing (model simulates ~3.1km x 4.0km x 1.4 km).
•Free-surface boundary along X-Y plane at z=0 m.
•40 m thick CPML on all boundaries except free surface.
•Explosion source at varying depths (SPE explosions are all chemical explosions).
•Source is wholly located within a granite inclusion, assumed to be surrounded by alluvium.
•Source is an isotropic point moment tensor source with an error function as the source time function

Test Configurations

| Test | Depth of Burial, (m) | Source Size, (kg) |
|---|---|---|
| SPE1 | 55.0 | 88 |
| SPE2 | 46.0 | 997 |
| SPE3 | 46.0 | 905 |
| SPE4' | 87.0 | 89 |
| SPE5 | 76.5 | 5000 |
| SPE6 | 33.0 | 2200 |

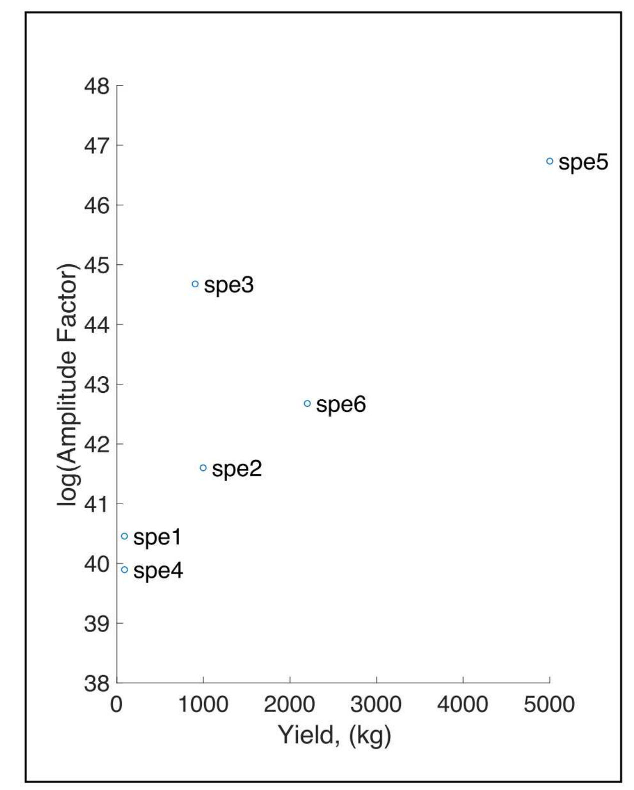



Map showing locations of geophones placed within approximately 2 kilometers of the SPE-1 experiment point. Green line is the approximate outline of the granite body. Equivalent receiver nodes were located in simulation model. Blue line shows approximate extent of the model. Model axes correspond to: +X-axis aligns to East, +Y-axis aligns to South, and +Z-axis is depth from surface. Figure is from: *Data Release Report for Source Physics Experiment 1 (SPE-1), NNSS.*

## Scale Factor

$$observed\ Data = Ar^{B} * (convolved\ Synthetic\ Green's\ Function)$$

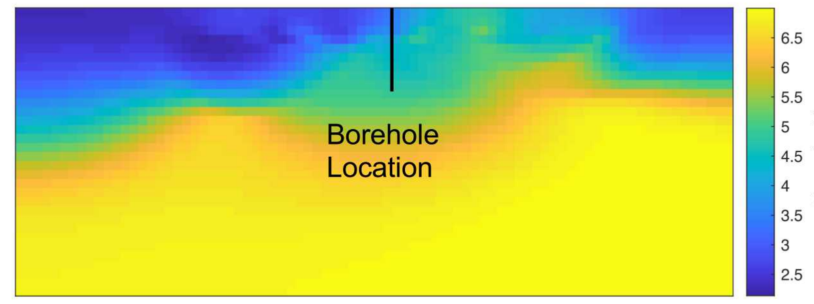
$A$ = amplitude factor
$r$ = distance receiver is from borehole
$B$ = attenuation factor



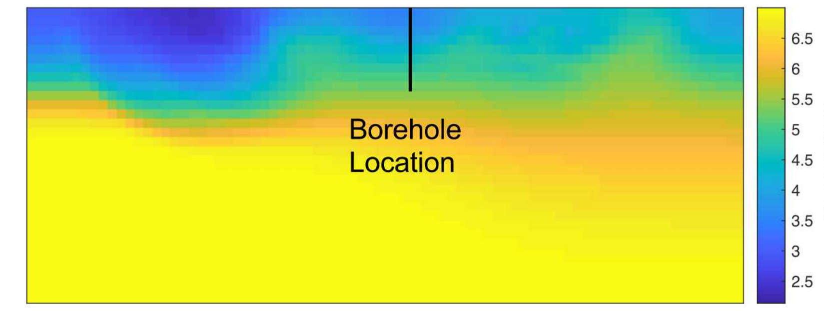

## Anisotropic Wave Speed Used in Model

**•No published anisotropic wave speeds for site geology**
**•Reconstructed model from 2D wave speed profiles from observed data**
**•Assumed wave speeds along lines 1 and 3 represented Vpyy**
**•Assumed wave speeds along lines 2 and 4 represented Vpxy**
**•Assumed wave speed along line 5 represented Vpxx**
**•Performed isotropic tomographic inversion from all five receiver lines to obtained average isotropic value for Vp**
**•Interpolation relationships between Vpxx, Vpxy, Vpyy and Vp based on ratio between the wave speeds at a common point. This location of the borehole was the common point used.**
**•If a point was outside of a specified distance from one of the receiver lines, it was assumed to have the isotropic wave speed velocity. If it was within the specified distance to the receiver line, the wave speed of the point was linearly interpolated between the isotropic wave speed and the wave speed associated to the nearest receiver line (Vpxx → Line #5, Vpyy → Line #1 and #3, Vpxy → Line #2 and #4). All other wave speeds at the point are assigned base on interpolation table.**
**•Assumed Vpxx correlates to Vpxz and Vsxz, Vpyy correlates to Vpyz and Vsyz, and Vpyx correlates to Vpzz and Vsxy.**

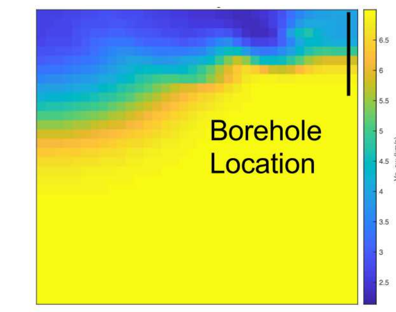
Wave Velocities along Lines #1 and #3 → Vpyy




Wave Velocities along Lines #2 and #4 → Vpxy




Wave Velocities along Lines #5→ Vpxx




## Acknowledgements

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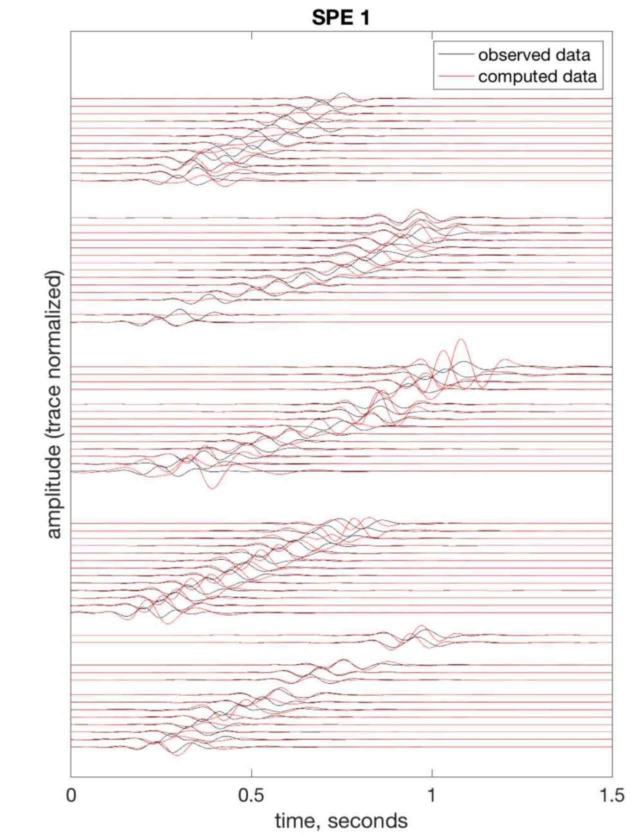
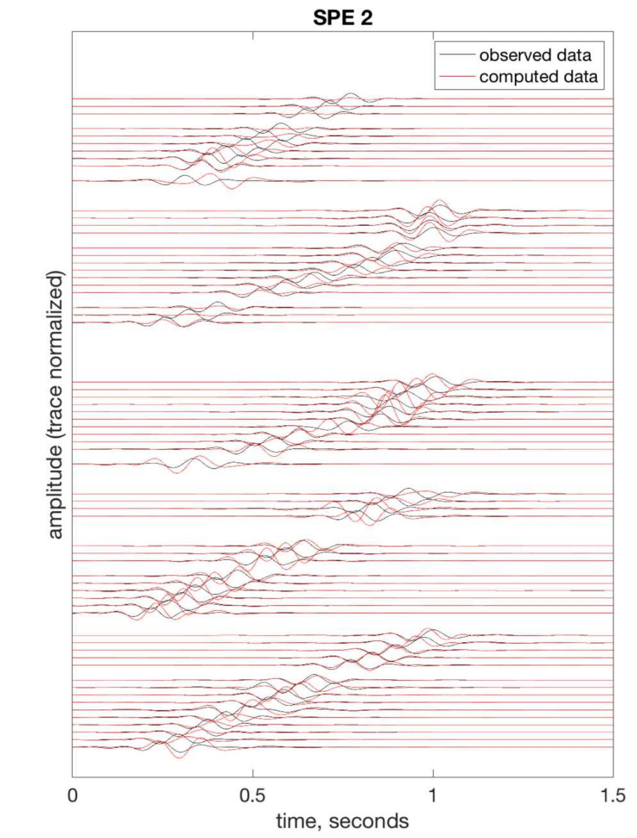
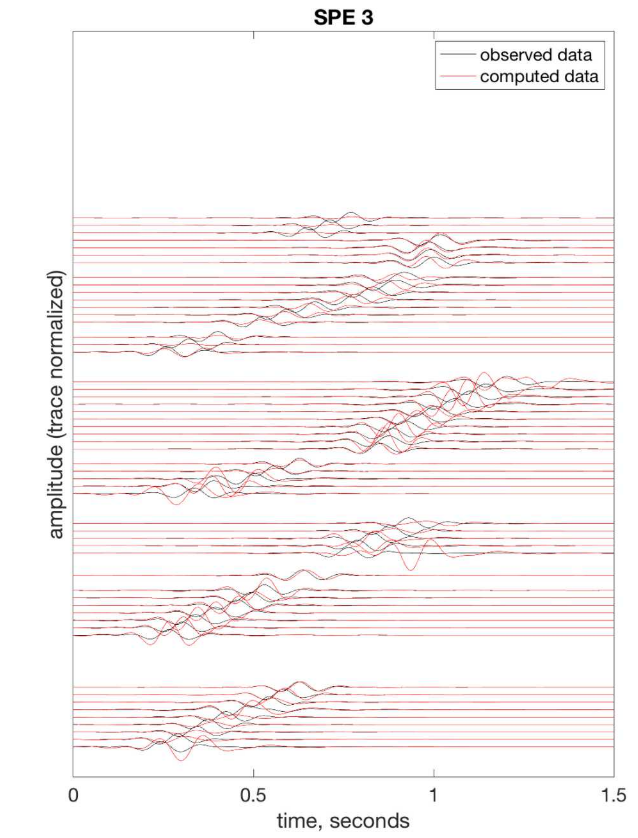
Brown, R.J., 1989, Relationships between the velocities and the elastic constants of an anisotropic solid possessing orthorhombic symmetry: Research Report 1989-17, Consortium for Elastic Wave Exploration Seismology (CREWES), University of Calgary.
Preston, L.A., 2018, *Pararhombi: Parallel Implementation of 3-D Seismic Wave Propagation in Orthorhombic Media*, SAND2018-9477, Technical Report Sandia National Laboratories, Albuquerque, NM.
Schoenberg, M., and Helbig, K., 1997, Orthorhombic media: modeling of elastic wave behavior in a vertically fractured earth: Geophysics, **62**, 1954-1974.
Tsvankin, I., 1997, Anisotropic parameters and P-wave velocity for orthorhombic media: Geophysics, **62**, 1292-1309.
National Security Technologies, LLC, 2014. *Data Release Report for Source Physics Experiment 1 (SPE-1), Nevada National Security Site.* Technical Report DOE/NV/25946--2018, 197 pages. Las Vegas, NV.

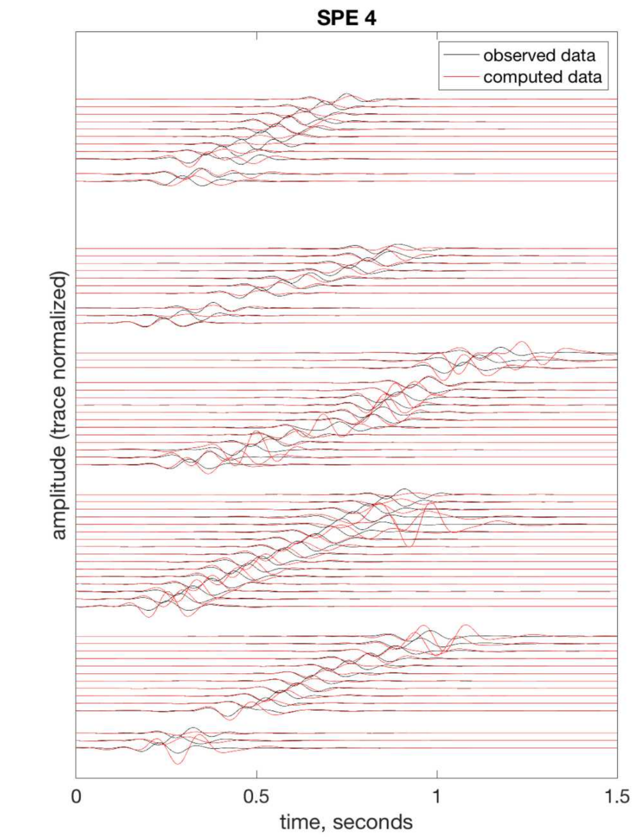
## Comparison of Observed Data to Synthetic Data

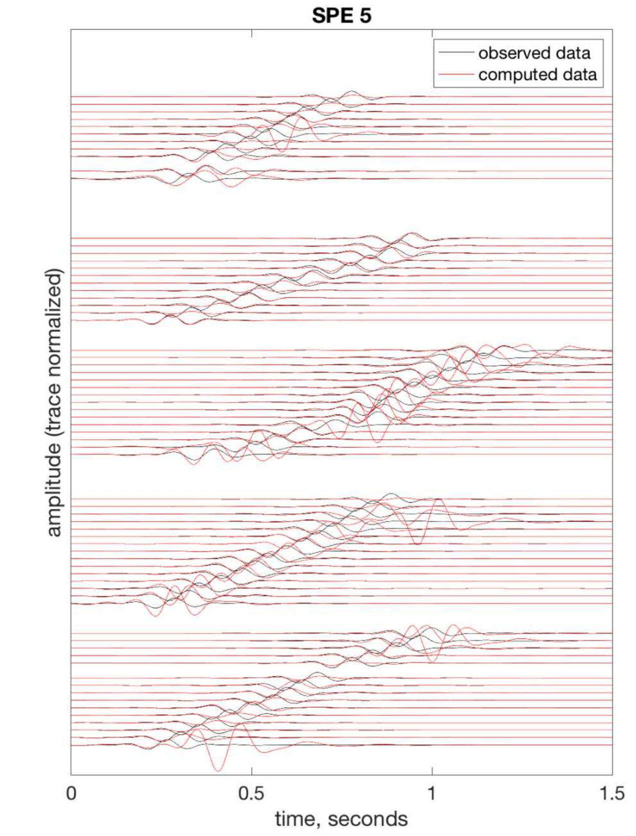


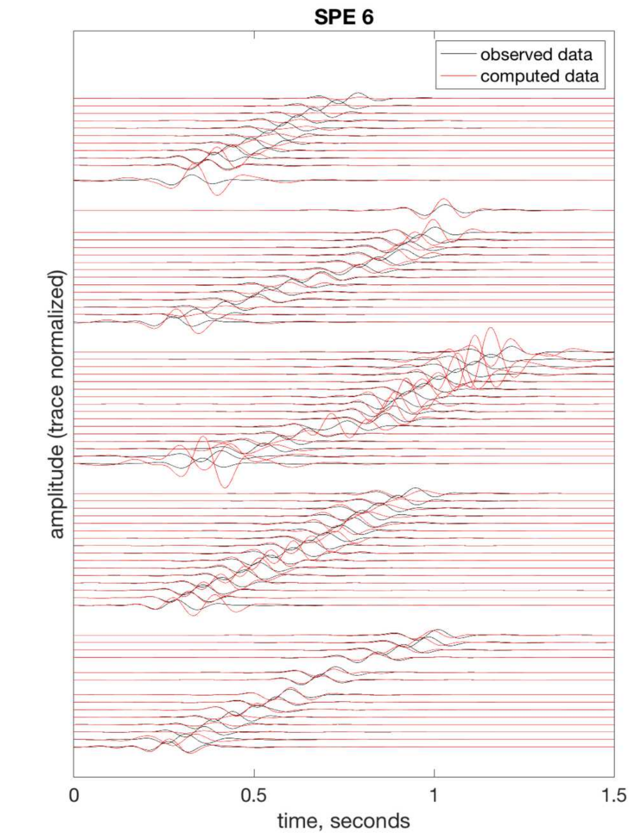




Observed and Synthetic data has been processed with a band pass filter (10-25 Hz).
"Box car" windowing was used to reduce data and process first arrival wave.




Green's Functions are simulated with the linear orthorhombic algorithm using a point moment tensor source at the source depth. All receivers are used simultaneously in the inversion. A massively parallel implementation of a 3-D seismic waveform simulation algorithm, with the capability to handle fully anisotropic media, was used to generate the Green's functions. This code, *Pararhombi*, was developed at Sandia National Laboratories (Preston, 2018).

## Conclusions

**1) Completed synthetic predictions for SPE-1 through SPE-6, azimuthally anisotropic and orthorhombic model of site, 2) Derived scale factors relating synthetics to observed, and 3) Scale factor needed to account for attenuation, which is not implemented in the waveform simulation code *Pararhombi***

Limitations

**1) No published anisotropy models of site, 2) Model wave velocities derived from interpolating values from tomographic inversion of observed data.**

Future work:

**Source time function estimated from DAG data and experiments will be modeled**