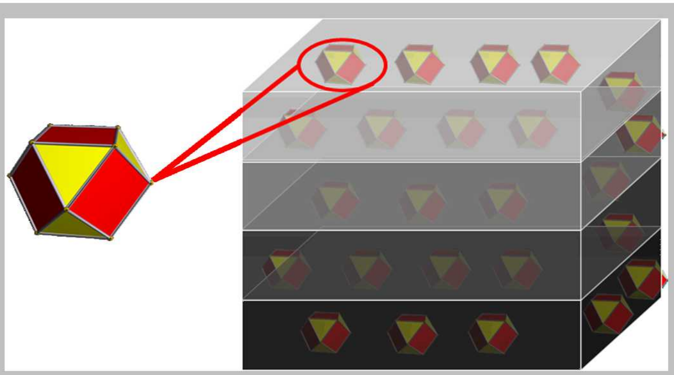
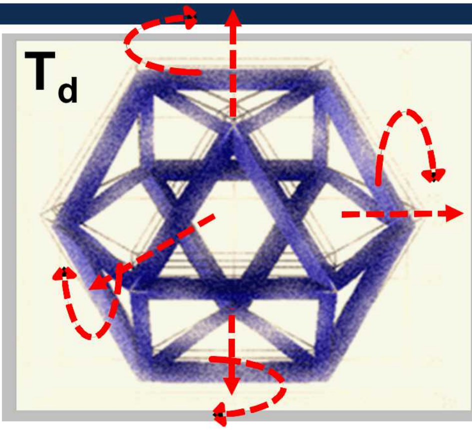
SAND2019-4414C

Group Theory Approach to Pentamode-Like Acoustic Metamaterials for Underwater Cloaking and Impedance Matching to Fluids





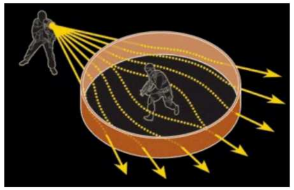
Character table for T_d point group

	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	linear, rotations	quadratic
A_1	1	1	1	1	1		$x^2+y^2+z^2$
A_2	1	1	1	-1	-1		
E	2	-1	2	0	0		$(2z^2-x^2-y^2, x^2-y^2)$
T_1	3	0	-1	1	-1	(R_x, R_y, R_z)	
T_2	3	0	-1	-1	1	(x, y, z)	(xy, xz, yz)

Ihab El-Kady & Charles M. Reinke
Applied Photonic Microsystems, Sandia National Laboratories, Albuquerque, NM, USA

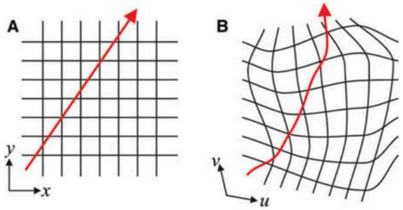
M. Ghasemi Baboly & C.Z. Leseman
Department of Mechanical and Nuclear Engineering, Kansas State University, Manhattan KS, USA

Sandia National Laboratories is a multi-mission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525.

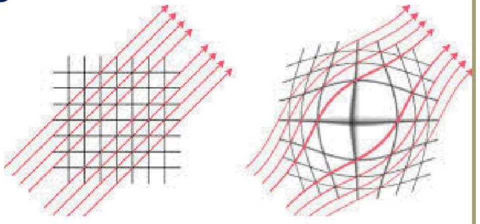


Cloaking in the EM Domain

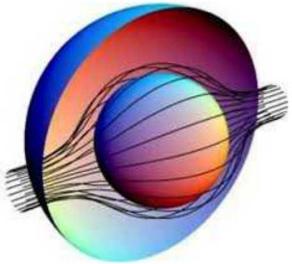
Transformation Optics*[,1]



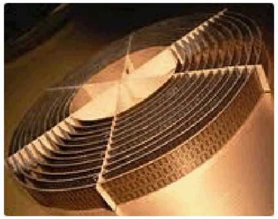
Coordinate Transformation



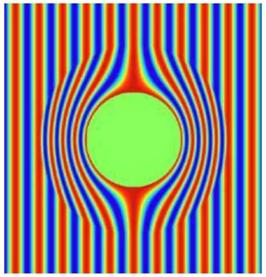
Transformation selected to open up a hole in space



3D Cloak Concept

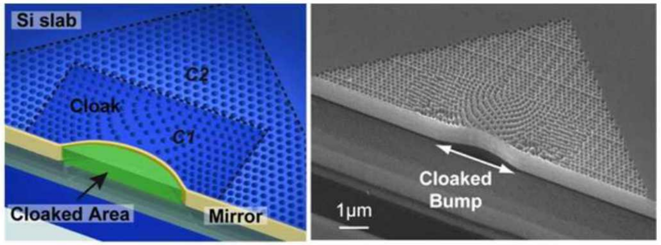


Duke's 2D RF Cloak

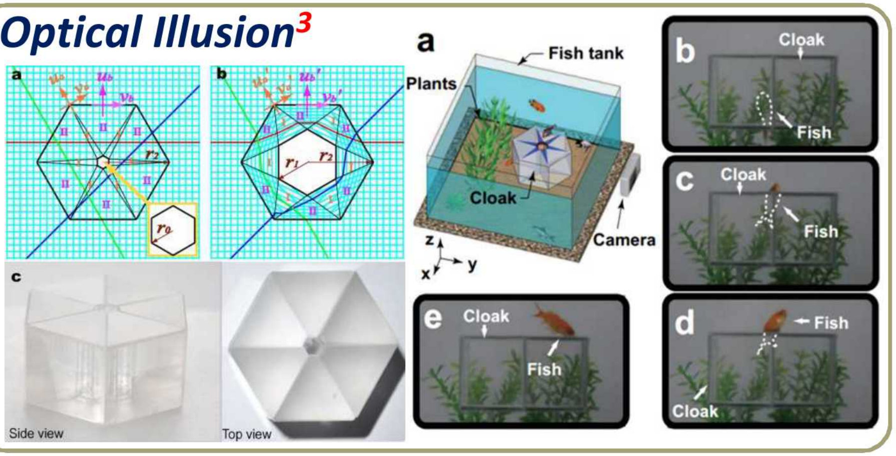


Cloak EM Signature

Diffractive Optics[2]

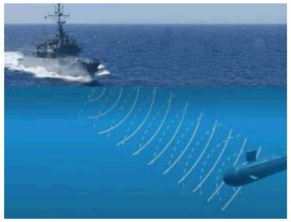


Optical Illusion[3]



Disadvantages:

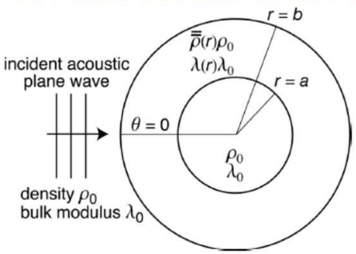
- Cloak's size
- Cloaked area $\sim\lambda^2$
- Blind inside out & outside in!



Cloaking in the Acoustic Domain

Acoustic Cloaking

Transformation Acoustics[*,4]



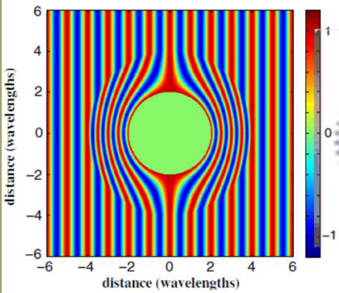
3D Acoustic Cloaking Shell

$$\frac{r^2}{\rho_r} = \frac{(r-a)^2}{k_1}, \qquad \rho_\phi = \rho_\theta = \frac{b-a}{b},$$

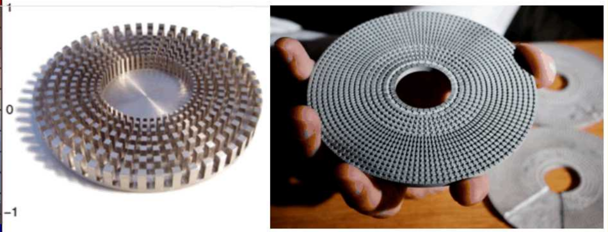
$$\rho_\phi = k_1, \qquad \rho_r = \frac{b-a}{b}\,\frac{r^2}{(r-a)^2},$$

$$\frac{\rho_\phi}{\lambda} k_0^2 r^2 = k_{sh}^2 (r-a)^2, \qquad \lambda = \frac{(b-a)^3}{b^3}\,\frac{r^2}{(r-a)^2}.$$

Coordinate Transformation

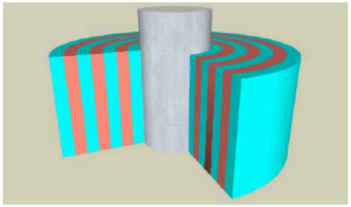


Pressure Field



MIT 2D Ultrasonic Cloak

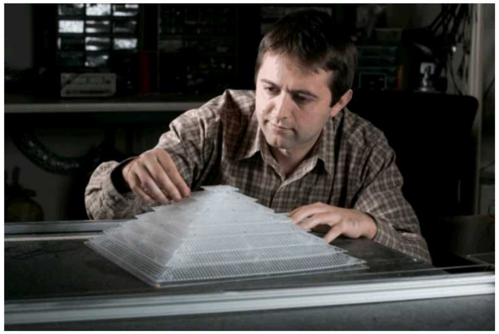
Fluidic Acoustic Cloak[5]

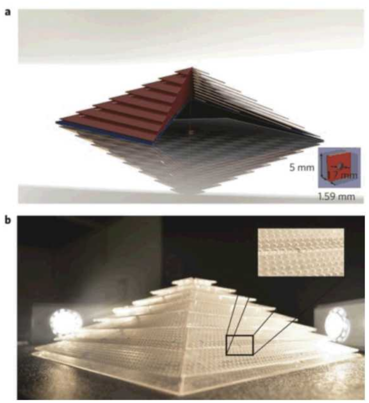


Layers with varying fluid density

3D rendering using fluid filled pipes!

Ground Acoustic Cloak[6]

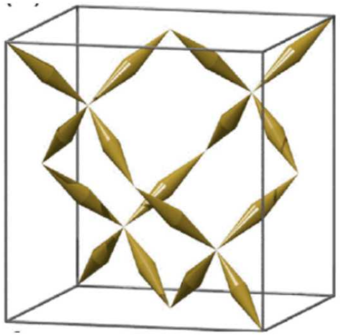




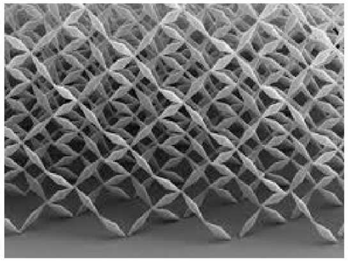
Disadvantages:

- Cloak's size
- Cloaked area $\sim\lambda^2$
- Blind inside out & outside in!

Pentamode Approach to Cloaking



Pentamode Milton[7,8].



Why is Acoustic Underwater cloaking difficult?

- Fluids are Shearless
- Solids support both shear and bulk waves
- The impedance mismatch will always create reflections!

Pentamode: The Sufficient but NOT Necessary Condition:

- 3D solid that behaves like a fluid.
- Finite bulk but vanishing shear modulus
- Hard to compress yet easy to deform
- Elasticity tensor with **<u>only one</u>** non-zero eigenvalue and five (penta) vanishing eigenvalues.

$$\tilde{C}_{\text{Pentamode}} = \begin{bmatrix} \begin{matrix} C_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \hat{0} \\ \hat{0} & \hat{0} \end{bmatrix}$$

The Necessary AND Sufficient Condition:

- Modal energy distribution is based on ratio of Bulk (K) to Shear (G) moduli
- Need to minimize G/K [8] or ideally have G/K→0

$$\tilde{C}_{\text{Needed}} = \begin{bmatrix} K_{ij} & KG_{ij} \\ GK_{ij} & G_{ij} \end{bmatrix}$$

$$KG_{ij} \to 0$$

$$GK_{ij} \to 0$$

$$\& \; G_{ij}/K_{ij} \to 0$$

An Elastic Tensor by Design:

- **<u>#1 Sigmund Numerical Search:</u>** Space = boundary of hypercube ~10^9 possibilities[9].
- **<u>#2 Milton logical deduction:</u>** 1D →2D →3D; limited number of starting points →Severely restrict outcome→ Requiring a in infinitely small connection

A Metamaterial Approach

Sandia National Laboratories

❖ ***What is a Metamaterial?***

Engineered artificial materials that exhibit properties different and likely unattainable by constituent components

❖ ***Context:***

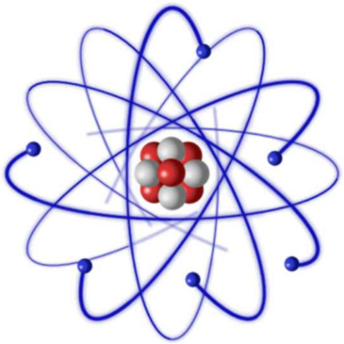
- *Interested in the mechanical properties e.g. stiffness, and elastic modulus*
- *Think of two atoms connected by a spring (often nonlinear)*

❖ ***Stiffness:***

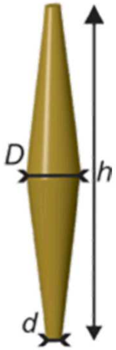
- *Depends on the type of bond*

❖ ***Elastic Modulus:***

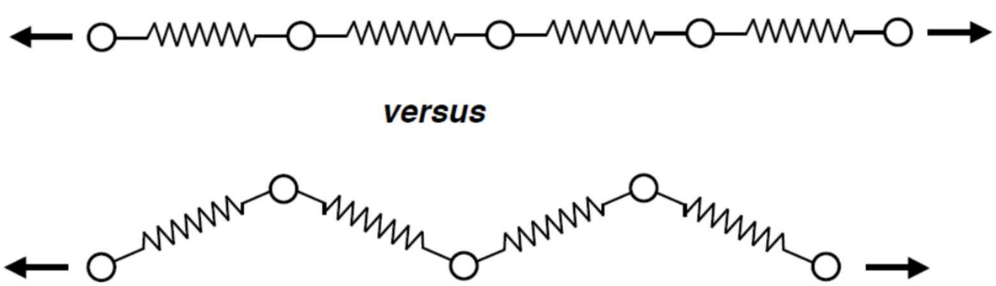
- *Depends on the arrangement/packing of atoms*



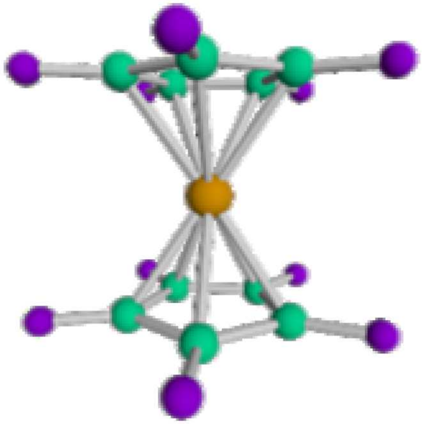
Actual Natural Atom



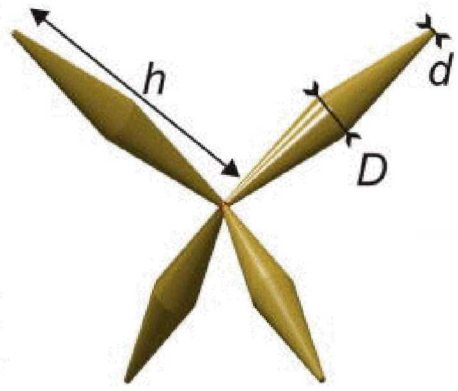
Artificial Metamaterial Atom

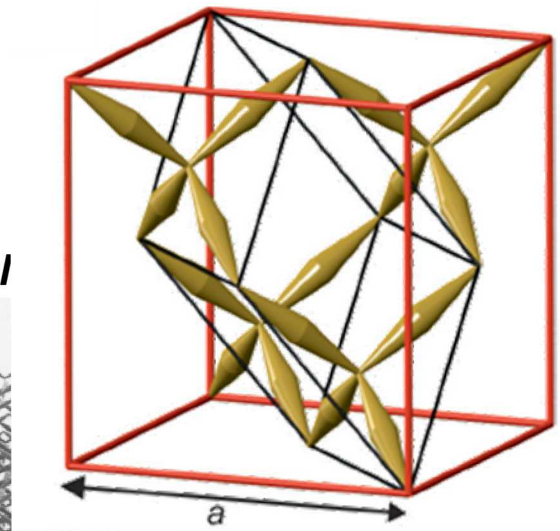


What is a Metamaterial?

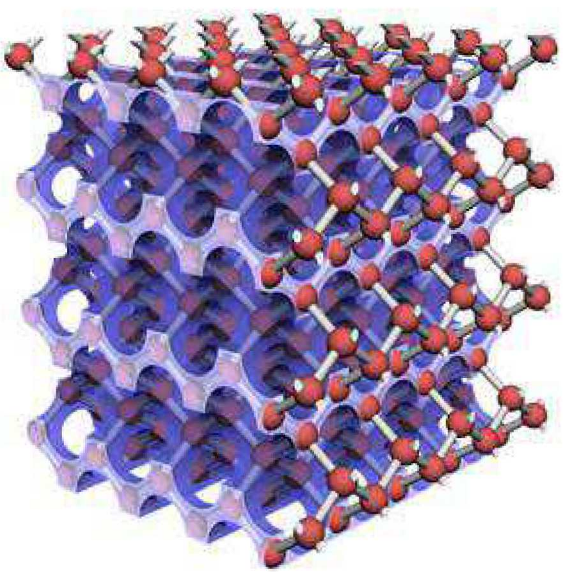


Natural Molecule

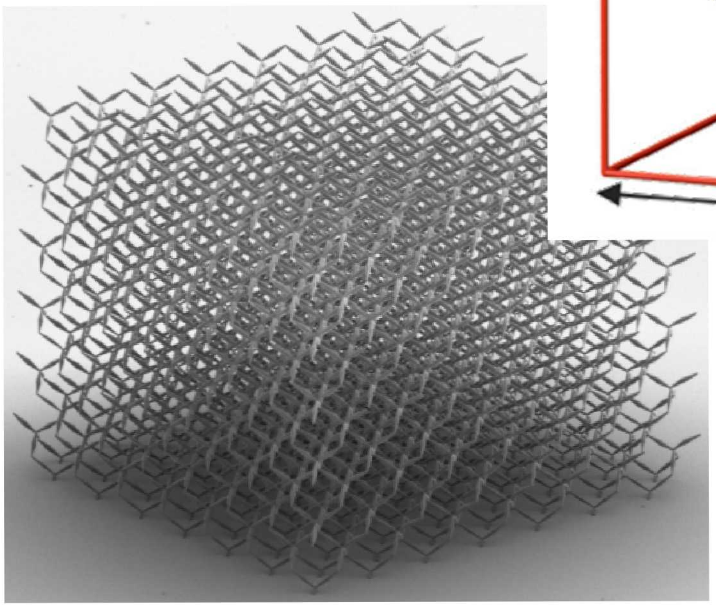




Metamaterial Molecul



Molecular Lattice

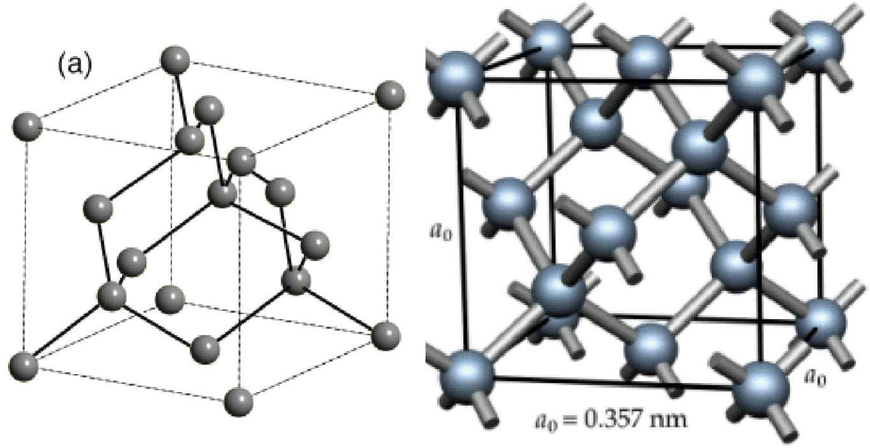


Metamaterial Lattice

Arrangement Matters!

Take Carbon atom as an example

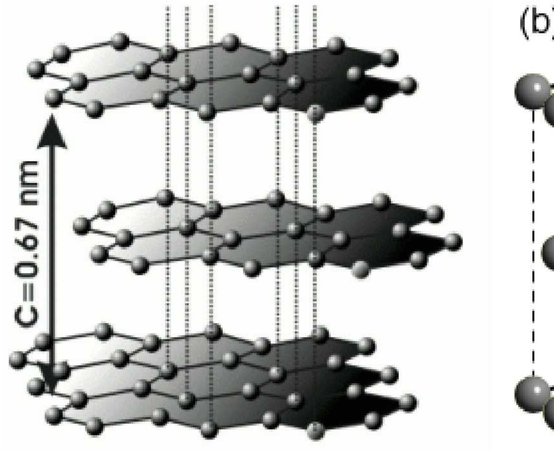
Diamond



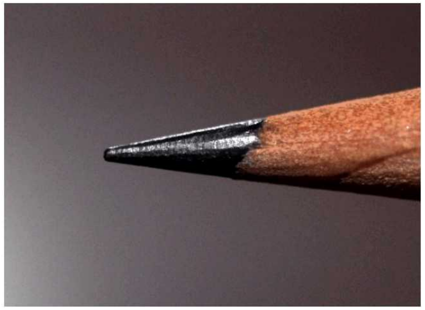
Diamond Structure

One of the **hardest** yet optically **transparent** materials known to man!

Graphite



Graphite Structure



One of the **softest** yet optically **opaque** materials known to man!

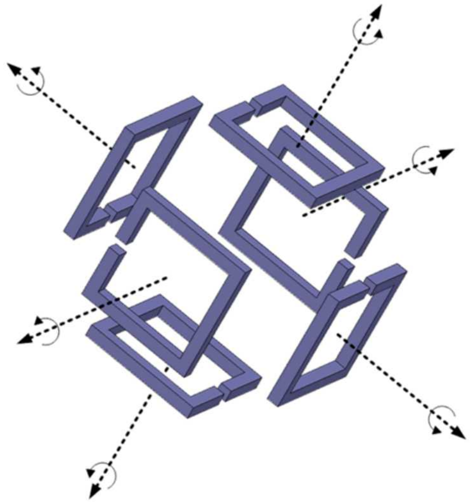
In the language of metamaterials: "Not only does the local resonance (atom) matter, but also the global resonance and the interactions of the adjacent cells!"

How do we arrange Metamaterial "atoms" to achieve a desired functionality?

❖ ***Why is this such a hard problem?***

➢ ***Take the simplest MM element; Split Ring Resonator:***

- ***Trial and Error?***
  - ***For a simple SRR on the 6 faces of a cube:***
    - — ***In generally: 4^6 = 4096 ways!***
    - — ***Invoking reciprocity: 4^3 = 256 ways!***
    - — ***Invoking the Quasi-static limit: $4^3/2$ = 128 ways!***



➢ ***No Design rules of thumb! Physical intuition can only get you so far!***

- ***Element-Element Interactions:***
  - ***Brute force EM simulations can only tell us the net result with very limited insight in to element-element interactions.***
- ***Extrapolation:***
  - ***Starting with a well studied MM element (e.g. SRR) can we predict the outcome of a chosen topological arrangement?***

❖ ***Turn back to Natural atoms, and ask the same question!***

Question: How are natural atoms arranged to achieve a given property?

Sandia National Laboratories

- ***Bare in mind a few issues:***
  - ***Natural atoms assemble to give the minimum energy configuration, NOT necessarily our objective here!***
  - ***NOT all properties are awarded by naturally occurring materials! Entire field of MM arose in an attempt to achieve unusual functionalities!***
  - ***Manybody Problem:***
    - ***Equations of motion are almost impossible to solve.***
  - ***We are only primarily interested in Mechanical properties***
    - ***Basically vibrations and rotations in our artificial atoms***
  - ***Quantum Mechanics tricks → come to selection rules, "solutionless approach":***
    - ***Symmetry of Wavefunction***
    - ***Symmetry of the interaction Hamiltonian/potential***
    - ***Symmetry of the boundary value problem***

Code Word → Symmetry

Molecular Spectroscopy: Determining the properties of Natural Molecules

- ***Symmetry elements of a molecule constitute a complete group, "Point Group"***

- ***Catalogue molecules into " 58 Character" tables:***
  - *Despite sharing **same symmetry**, molecules may have vastly **different** motional resonances!*

- ***Generalized rotational-vibrational motion of a molecule:***
  - *Can be viewed as a superposition of elemental/fundamental motions called "**normal modes**".*
  - *Use normal modes as basis ➔ Symmetry elements are diagonal Matrices*

- ***Good news:***
  - ***1-to-1 analogy** with a molecule's spectroscopic activity*
    - *Active **Vibrational** modes ➔ **Raman** active*
    - *Active **Rotational** modes ➔ **IR** active*

Nonaxial groups	C_1	C_s	C_i	-	-	-	-
C_n groups	C_2	C_3	C_4	C_5	C_6	C_7	C_8
D_n groups	D_2	D_3	D_4	D_5	D_6	D_7	D_8
C_{nv} groups	C_{2v}	C_{3v}	C_{4v}	C_{5v}	C_{6v}	C_{7v}	C_{8v}
C_{nh} groups	C_{2h}	C_{3h}	C_{4h}	C_{5h}	C_{6h}	-	-
D_{nh} groups	D_{2h}	D_{3h}	D_{4h}	D_{5h}	D_{6h}	D_{7h}	D_{8h}
D_{nd} groups	D_{2d}	D_{3d}	D_{4d}	D_{5d}	D_{6d}	D_{7d}	D_{8d}
S_n groups	S_2	S_4	S_6	S_8	S_{10}	S_{12}	-
Cubic groups	T	T_h	T_d	O	O_h	I	I_h
Linear groups	$C_{\infty v}$	$D_{\infty h}$	-	-	-	-	-

Application of Group Theory to Molecular Spectroscopy:

❖ **Group Theory Terminology:**

- *Reducible Representation Γ : Most general form of vibration (current, mechanical, ..etc).*
- *Irreducible Representations (IrredRep) A_1, A_2, B_1, B_2, ..etc. : The normal modes. classified and named based upon their symmetry*
- *Characters χ: The trace of the matrix representation of a symmetry op in the normal mode basis*
- *L&Q Functions (X, Y, Z) and (R_x, R_y, R_z) Represent the behavior of the Irred. Rep. in a Cartesian basis.*

Character Table:

Most compact form of symmetry Representation

Point Group

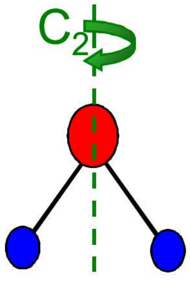
Symmetry Elements/Operations

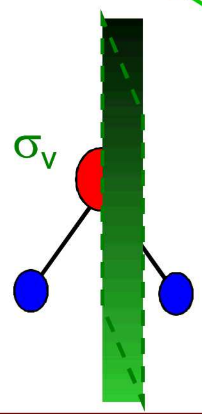
functions that transform as the various irreps of the group.

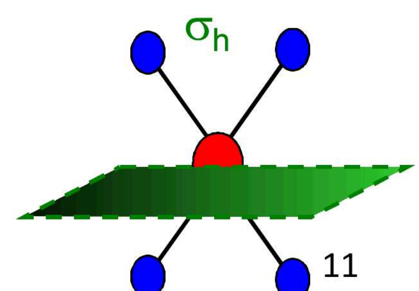
C_{2v}	E	C_2	$\sigma_v(xz)$	$\sigma_v(yz)$	Linear	Quadratic
A_1	1	1	1	1	z	x^2, y^2, z^2
A_2	1	1	-1	-1	R_z	xy
B_1	1	-1	1	-1	x, R_y	xz
B_2	1	-1	-1	1	y, R_x	yz

Irreducible Representations

Characters (trace of the matrix representation) of the group elements







Group Theory Mapping

❖ Mapping:

Bulk modes ↔ Generalized vibrations about structural joints
⇒ Transform like $\vec{r} = (x, y, z)$

Shear modes ↔ Generalized rotations about structural joints
⇒ Transform like axial vector $\vec{R} = (R_x, R_y, R_z)$

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46}\\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56}\\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{bmatrix}$$

Bulk

Shear

Stress

Strain

❖ Quadrant Access:

Coupled Linear Oscillations (r_i, r_j)

Coupled Linear/axial Motions r_i, R_j

$$\overline{\overline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46}\\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56}\\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}\end{bmatrix}$$

r_i, R_j Coupled Linear/axial Motions

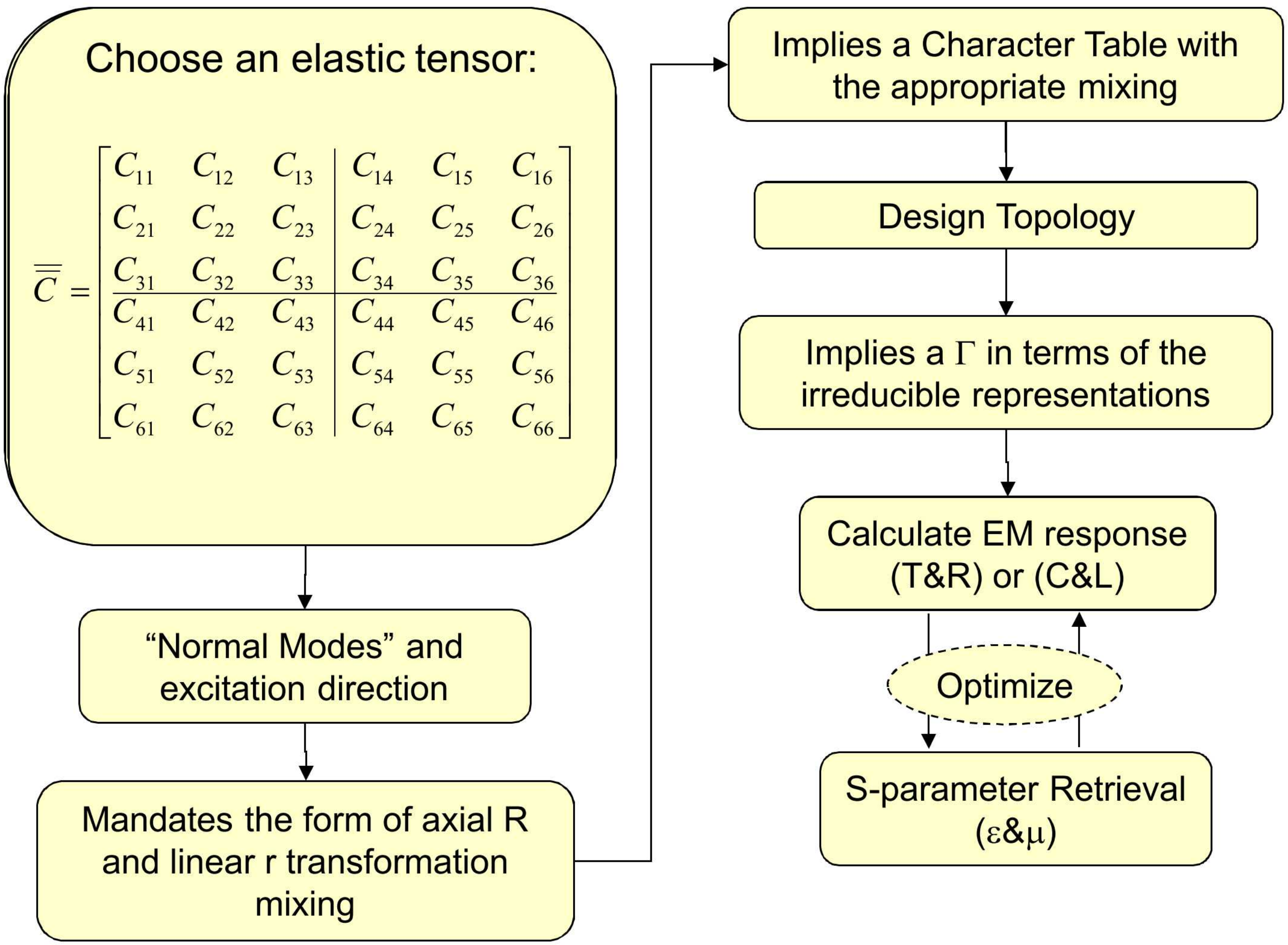
(R_i, R_j) Coupled Axial motion

Independent Linear Oscillations r_i's

$$\overline{\overline{C}} = \begin{bmatrix} C_{11} & 0 & 0 & 0 & 0 & 0\\ 0 & C_{22} & 0 & 0 & 0 & 0\\ 0 & 0 & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}$$

Independent Axial motion R_i's

Group Theory Inverse Problem Approach
Engineering the MM Constitutive Tensor



Group Theory Approach Pentamode

Practically what we require is:

a. Diagonal elasticity tensor
b. Optimize such that all diagonal elements are small relative to C_{11}

Linear and axial mixing:

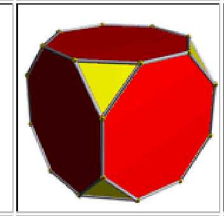
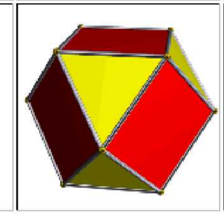
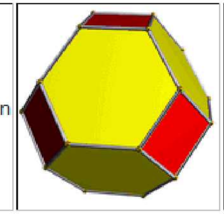
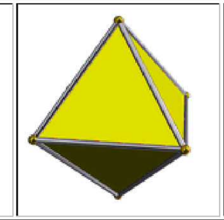
1. Need a block diagonal tensor with no shear/bulk coupling
2. (x, y, z) & (R_x, R_y, R_z)

Cavite:

GpTh only tells us about the symmetry but <u>nothing</u> about the <u>structural</u> or <u>material</u> parameters!

Character Table for O_h point group

O_h	E	$8C_3$	$6C_2$	$6C_4$	$3C_2=(C_4)^2$	i	$6S_4$	$8S_6$	$3\sigma_h$	$6\sigma_d$	linear functions, rotations
A_{1g}	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	-
A_{2g}	+1	+1	-1	-1	+1	+1	-1	+1	+1	-1	-
E_g	+2	-1	0	0	+2	+2	0	-1	+2	0	-
T_{1g}	+3	0	-1	+1	-1	+3	+1	0	-1	-1	(R_x, R_y, R_z)
T_{2g}	+3	0	+1	-1	-1	+3	-1	0	-1	+1	-
A_{1u}	+1	+1	+1	+1	+1	-1	-1	-1	-1	-1	-
A_{2u}	+1	+1	-1	-1	+1	-1	+1	-1	-1	+1	-
E_u	+2	-1	0	0	+2	-2	0	+1	-2	0	-
T_{1u}	+3	0	-1	+1	-1	-3	-1	0	+1	+1	(x, y, z)
T_{2u}	+3	0	+1	-1	-1	-3	+1	0	+1	-1	-

truncated cube	
cuboctahedron	
truncated octahedron	
octahedron	

Character table for T_d point group

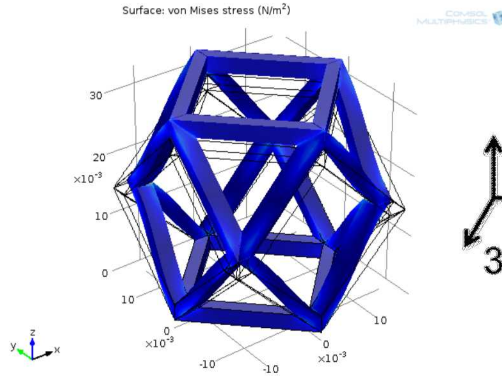
	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	linear, rotations	quadratic
A_1	1	1	1	1	1		$x^2+y^2+z^2$
A_2	1	1	1	-1	-1		
E	2	-1	2	0	0		($2z^2-x^2-y^2$, x^2-y^2)
T_1	3	0	-1	1	-1	(R_x, R_y, R_z)	
T_2	3	0	-1	-1	1	(x, y, z)	(xy, xz, yz)

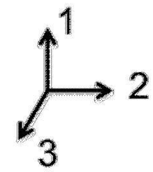
Modeling & Results

Sandia National Laboratories

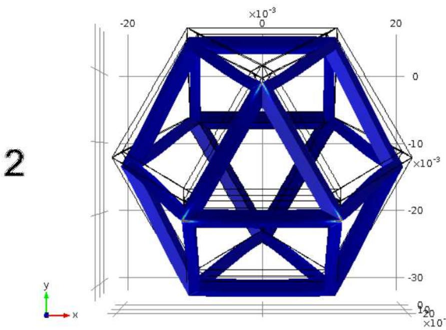
Results:

$$\widetilde{C} = \begin{bmatrix} 1 & 0.0821 & 0.003 & \sim 0 & \sim 0 & \sim 0 \\ 0.01 & 0.1 & 0.007 & \sim 0 & \sim 0 & \sim 0 \\ 0.067 & 0.006 & 0.008 & \sim 0 & \sim 0 & \sim 0 \\ \sim 0 & \sim 0 & \sim 0 & 0.009 & \sim 0 & \sim 0 \\ \sim 0 & \sim 0 & \sim 0 & \sim 0 & 0.0098 & \sim 0 \\ \sim 0 & \sim 0 & \sim 0 & \sim 0 & \sim 0 & 0.002 \end{bmatrix}$$





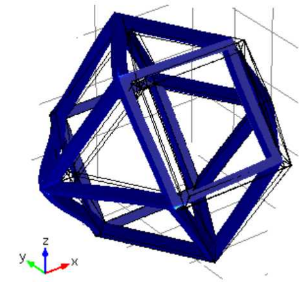
L=2cm; w=16mm; t=20mm



Calculating G/K:

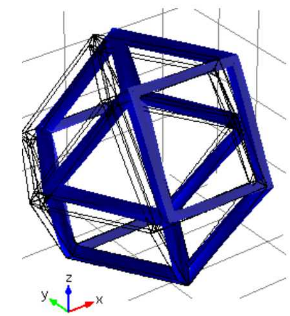
1. *Calculated the volumetric strain on each side.(ΔV/V)*
2. *Fixed the same boundary on the opposite side and calculated the force and displacement on those boundaries.*

$$K = \frac{\frac{1}{3}(f_{11}+f_{22}+f_{33})}{\frac{\Delta V}{V}}$$

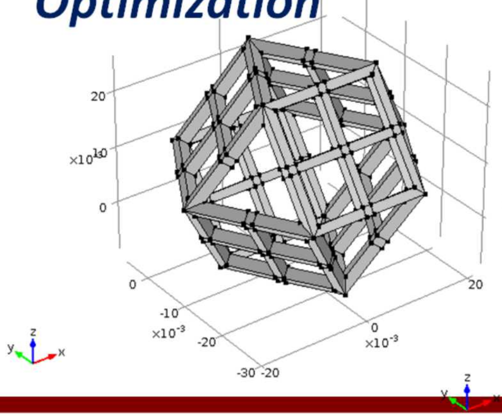


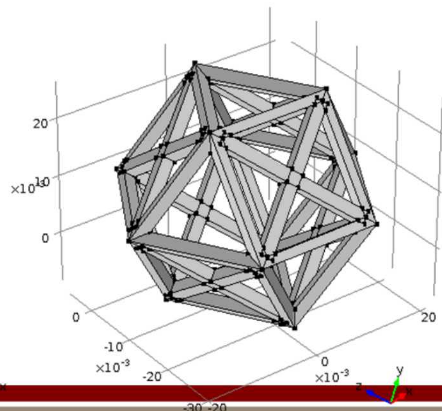
G/K~7.14x10⁻⁴

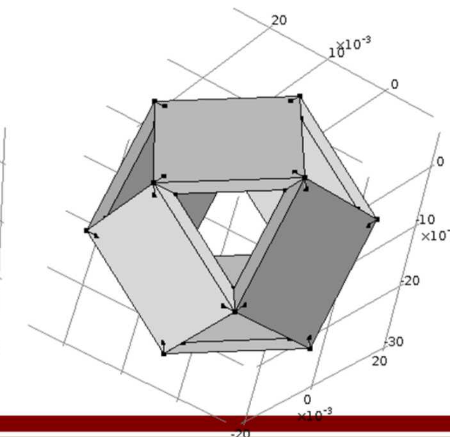
$$\varepsilon_{13} = \frac{dispacement}{L} = \frac{\Delta l}{L};\quad G = \frac{f_{13}}{\varepsilon_{13}}$$

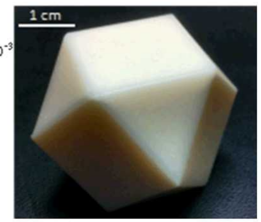


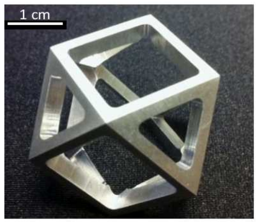
Optimization











Summary and Conclusions

a. Introduced the Group theory as an inverse design methodology to engineered metamaterials[10]
b. Used symmetry arguments to engineer an elastic tensor with the desired format
c. Used modeling to optimize the bulk to shear modulus ratio
d. Were able to achieve Pentamode-like behavior with $G/K \sim 10^{-4}$