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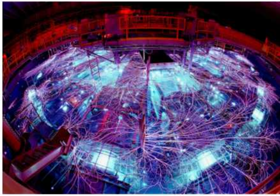

Bayesian model selection for metal yield models in high-velocity impact

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- Many potential models to represent von Mises yield stress in elastoplastic deformation models for hardened steel.
- Need to pick the "best" one for high-velocity impact.
- Ideally, should reproduce data and generalize to other scenarios.
- Bayesian model selection balances these requirements to select the best model, taking uncertainties into account.

Candidate models

- Johnson-Cook

$$\sigma = [A + B\epsilon^n][1 + C \ln \dot{\epsilon}/\dot{\epsilon_0}]\left[1 - \left(\frac{T - T_{room}}{T_{melting} - T_{room}}\right)^m\right]$$

- Zerilli-Armstrong (face-centered model)

$$\sigma = c_0 + (c_1 + c_2\epsilon^{1/2})\exp(-c_3 T + c_4 T \ln \dot{\epsilon}) + c_5\epsilon^N$$

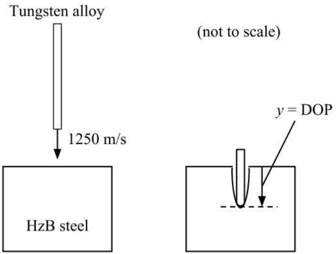
- Steinberg-Guinan-Lund

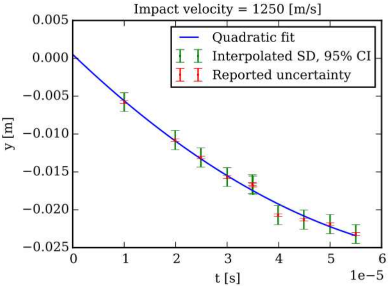
$$\sigma = \sigma_0[1 + \beta\epsilon]^n \left[1 + \frac{AP}{\eta^{1/3}} - B(T - T_{room})\right],$$

$$\sigma_{max} \geq \sigma_0[1 + \beta\epsilon]^n$$

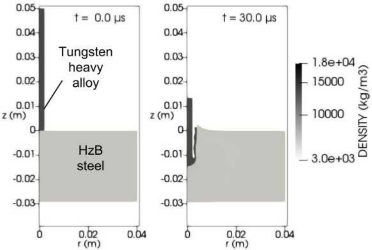
Parameters in red are tunable.

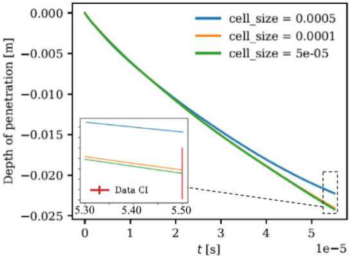
Deformation data for a tungsten rod impacting a HzB (hardened) steel plate at 1250 m/s was used to calibrate and compare the models (Anderson, Hohler, *et al.* 1995).





ALEGRA's CTH elastic-plastic model with JC, ZA, or SGL was used to produce model outputs to compare to data.





Assumed axisymmetry, tracked d.o.p. using Lagrangian tracer at interface between steel plate and penetrator.

Uniform elements, performed mesh convergence study to determine cell size.

Sources of uncertainty

- **Measurement uncertainty**
- Parametric uncertainty
  - **Model parameters**
  - Boundary conditions
  - Initial conditions
- Aleatory uncertainty (e.g. manufacturing variations)

This work focused on measurement and parametric uncertainty.

In the UQ paradigm, uncertainties are represented using probability distributions.

For the Johnson-Cook model, this looks like

- Uncertain parameters $\mathbf{p} = \{A, B, C, n, m\}$. For example,

$$\pi(A) = \mathcal{N}(A_0, (.05A_0)^2)$$

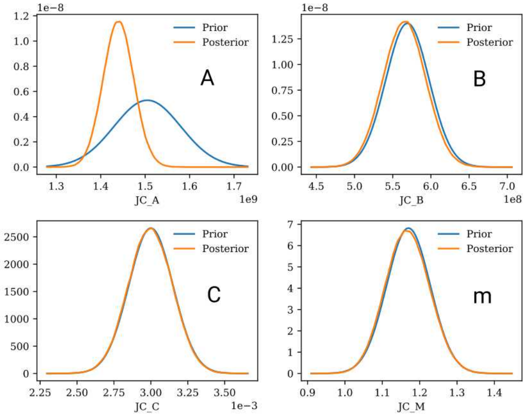
- Uncertain data $\mathbf{d} = [y(t_0), y(t_1), \cdots, y(t_{N_t})]$.

$$\begin{aligned}\pi(d_i|\mathbf{p}) &= \mathcal{N}(y(t_i), (.0006)^2) \\ &\propto \exp\left(-\frac{1}{2(.0006)^2}\left(d_i - m(t_i;\mathbf{p})\right)^2\right).\end{aligned}$$

Bayesian calibration

The uncertain parameter distributions are updated by comparing to data using Bayes's theorem:

$$\pi(\mathbf{p}|\mathbf{d}) = \frac{\pi(\mathbf{d}|\mathbf{p})\pi(\mathbf{p})}{\int \pi(\mathbf{d}|\mathbf{p})\pi(\mathbf{p})d\mathbf{p}}$$



- Bayesian model selection determines which model is most likely to have produced a piece of observational data.
- Each model is assigned an initial probability. Usually,

$$P(M_i) = \frac{1}{\text{number of models}}$$

- Bayes's theorem is used to update probabilities using data.

$$P(M_i|\mathbf{d}) = \frac{P(\mathbf{d}|M_i)P(M_i)}{\sum_j P(\mathbf{d}|M_j)P(M_j)} = \frac{P(\mathbf{d}|M_i)}{\sum_j P(\mathbf{d}|M_j)}$$

- Results depend on the model evidence:

$$P(\mathbf{d}|M_i) \equiv \int \pi(\mathbf{d}|M_i, \mathbf{p}_i)\pi(\mathbf{p}_i)\mathrm{d}\mathbf{p}_i$$

($\mathbf{p}_i$ are the parameters for model M_i)

Intuition behind model evidence

$$\log\left[P(\mathbf{d}|M_i)\right] = \mathbb{E}_{\pi(\mathbf{p}_i|\mathbf{d})}\left[\log \pi(\mathbf{d}|\mathbf{p}_i)\right] - D_{KL}\left(\pi(\mathbf{p}_i|\mathbf{d}) \,\Big|\Big|\, \pi(\mathbf{p}_i)\right).$$

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How well can the model be calibrated to fit the data?

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How much did the parameter distributions change after comparing to data?

Model evidence can't usually be computed analytically.

There are several approximations, for example:

- Monte Carlo (MC) integration
- Laplace approximation
- Posterior kernel density estimate (KDE) approximation

The posterior KDE approximation is most robust and uses samples of posterior distributions generated during Bayesian calibration.

Bayesian model selection process proceeds as follows:

- Build surrogate models for Johnson-Cook, Zerilli-Armstrong, and Steinberg-Guinan-Lund models.
- Perform a Bayesian calibration for each set of model paremeters with the surrogate.
- Use posterior samples from Bayesian calibration to compute approximations of model evidence for each model.

Bayesian calibration and surrogate construction used Dakota, Sandia's UQ and optimization software.

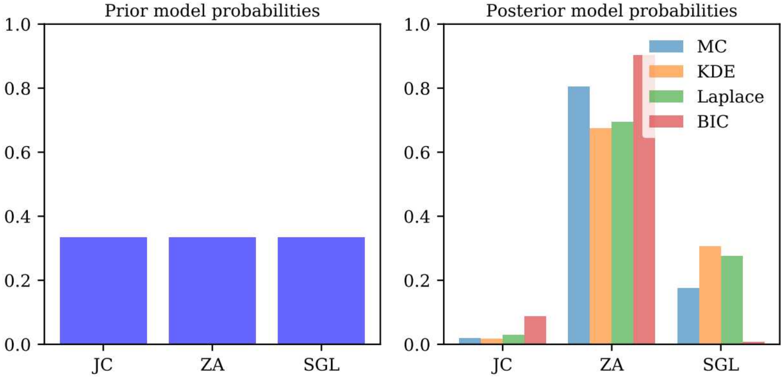
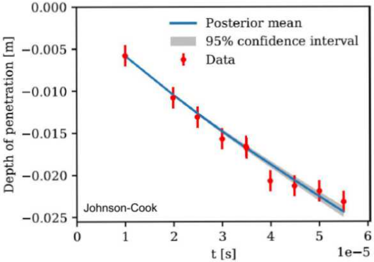


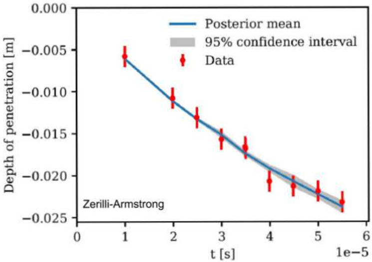
Figure: The probability each model produced the impact data, using MC, Laplace, and KDE approximations of the evidence. Also compared to an information-theory based metric, Bayesian Information Criterion (BIC)

Predicted d.o.p with uncertainty bounds for Johnson-Cook and Zerilli-Armstrong.

Johnson-Cook



Zerilli-Armstrong



ZA best resolves the potentially anomalous data at 40 μs.

- Bayesian calibration automated and more informative calibration method than deterministic approaches.

- Bayesian model selection requires minimal additional computation after Bayesian calibration.

- Zerilli-Armstrong shows promise for applications to high-velocity impact of hardened steel.

- Future work: check predictive ability of models by validating with higher-velocity (1700 m/s) impact data from Anderson, Hohler, *et al.*

Thanks!

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