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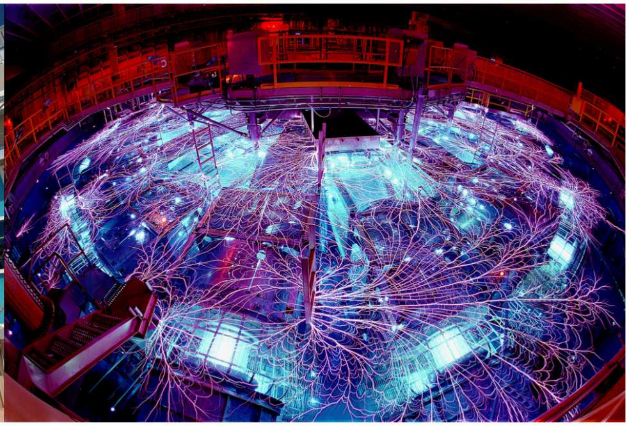

Stabilized continuous finite element schemes for problems in plasma physics

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Outline

- Motivation

- Hydrodynamic and MHD systems
- HD numerical results

- CG scheme for hyperbolic systems
- Low order scalar and tensorial dissipation
- Iterative nodal variation limiters
- High order background dissipation

- Powell term for MHD
- Parabolic cleaning for MHD
- MHD numerical results

Motivation

Solution to strongly coupled multiphysics problems, often involving strong shocks:

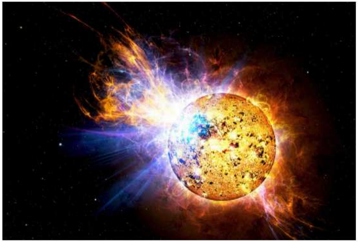
1) be positivity preserving for ρ, pressure etc. (e.g. LED schemes),

2) be applicable to widely varying timescales,

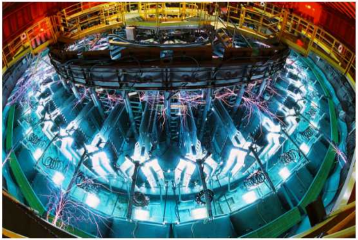
2.1) allow flexible time integrator usage (e.g. RK-IMEX schemes),

3) be on general unstructured meshes, high spatial and temporal order,

4) start with ideal MHD as an effort towards multi-fluid plasma.



Astrophysical plasma (Casey Reed/NASA)



Laboratory plasma (SNL)

Resistive MHD system (Fambri *et al.*, 2017, Comp. Phys. Comm.)

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho \boldsymbol{v} \\ \rho E \\ \boldsymbol{B}\end{bmatrix} + \nabla\cdot\begin{bmatrix}\rho \boldsymbol{v} \\ \rho \boldsymbol{v}\otimes\boldsymbol{v} + pI - \boldsymbol{T}_M \\ \boldsymbol{v}\cdot((\rho E + p)I - \boldsymbol{T}_M) \\ \boldsymbol{B}\otimes\boldsymbol{v} - \boldsymbol{v}\otimes\boldsymbol{B}\end{bmatrix} + \nabla\cdot\begin{bmatrix}\mathbf{0} \\ -\boldsymbol{T} \\ -\boldsymbol{v}\cdot\boldsymbol{T} + \boldsymbol{q} - \frac{\eta}{\mu_0}\boldsymbol{B}\cdot(\nabla\boldsymbol{B} - (\nabla\boldsymbol{B})^T) \\ -\frac{\eta}{\mu_0}(\nabla\boldsymbol{B} - (\nabla\boldsymbol{B})^T)\end{bmatrix} = \mathbf{0}.$$

ρ is the density. $\boldsymbol{v}$ is fluid velocity. ρE is the total energy. $\boldsymbol{B}$ is the magnetic field. μ is the fluid viscosity. η is the resistivity. μ_0 is the fluid permeability. γ is the ratio of specific heats.	Equation of state: $\rho E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho\|\boldsymbol{v}\|^2 + \frac{1}{2\mu_0}\|\boldsymbol{B}\|^2,\ \gamma > 0.$ Stress tensor: $\boldsymbol{T} = \mu\left[\left(\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T\right) - (p + \frac{2}{3}\nabla\cdot\boldsymbol{v}I)\right].$ $\boldsymbol{T}_M = \frac{1}{\mu_0}\left[\boldsymbol{B}\otimes\boldsymbol{B} - \frac{1}{2}\|\boldsymbol{B}\|^2 I\right].$

HD and MHD system

- Set $\eta = \mu = 0$ for now (ideal non-viscous,non-resistive MHD):

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho\boldsymbol{v} \\ \rho E \\ \boldsymbol{B}\end{bmatrix} + \nabla\cdot\begin{bmatrix}\rho\boldsymbol{v} \\ \rho\boldsymbol{v}\otimes\boldsymbol{v} + pI - \boldsymbol{T}_M \\ \boldsymbol{v}\cdot((\rho E + p)I - \boldsymbol{T}_M) \\ \boldsymbol{B}\otimes\boldsymbol{v} - \boldsymbol{v}\otimes\boldsymbol{B}\end{bmatrix} = \boldsymbol{0}.$$

- Omit the $\boldsymbol{B}$ terms (compressible Euler equations):

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho\boldsymbol{v} \\ \rho E\end{bmatrix} + \nabla\cdot\begin{bmatrix}\rho\boldsymbol{v} \\ \rho\boldsymbol{v}\otimes\boldsymbol{v} + pI \\ \boldsymbol{v}\cdot((\rho E + p)I)\end{bmatrix} = \boldsymbol{0}.$$

Solution strategy:

- Do continuous finite element discretization in space (P^1/Q^1).
- Introduce algebraic stabilization (low order diffusion + limiters).
- Do divergence cleaning for MHD (hyperbolic/parabolic/mixed).

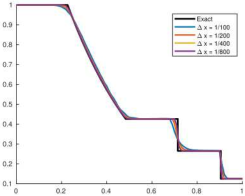
Achievements so far:

- Extensive studies for Euler equations (different limiter designs)
- Applications to steady hydrodynamics problems (Scramjet)
- Applied explicit and implicit time stepping (explicit RK4 & Crank-Nicolson)

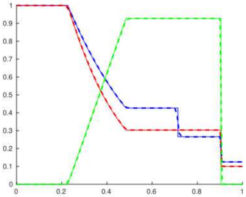
Transient HD results

Problems solved: Sod shocktube, Woodward-Colella blast wave, Sedov point blast etc... (Mabuza, Shadid and Kuzmin, 2018, JCP)

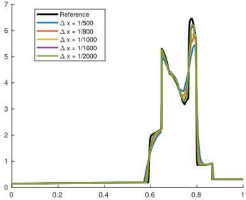
1D results; Top: Sod shock tube, Bottom: Woodward-Colella



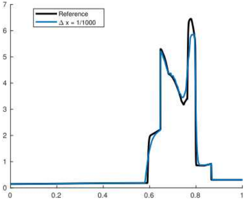
ρ vs x



primitive variables vs x, for $\Delta x = \frac{1}{800}$



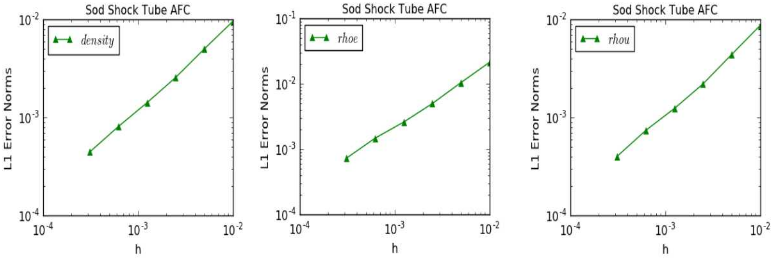
ρ vs x



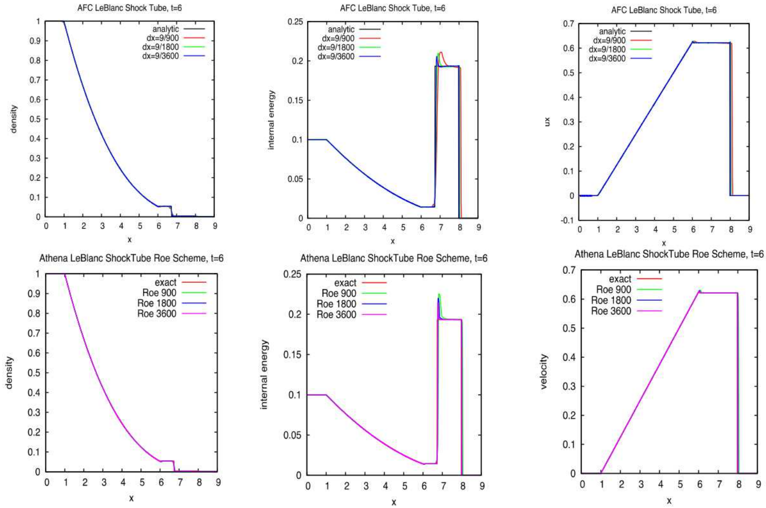
ρ vs x at $\Delta x = \frac{1}{1000}$

Sod ShockTube – Mesh Convergence 100-3200 cells

	L1 Slope (100-200)	L1 Slope (200-400)	L1 Slope (400-800)	L1 Slope (800-1600)	L1 Slope (1600-3200)
rho	9.36689e-01	9.65261e-01	8.52469e-01	8.06609e-01	8.60413e-01
rhoE	1.03158e+00	1.06352e+00	9.39890e-01	8.16929e-01	1.01285e+00
rhou	9.87064e-01	9.94115e-01	8.30446e-01	7.41841e-01	8.76973e-01

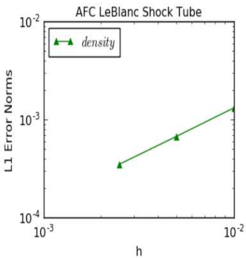


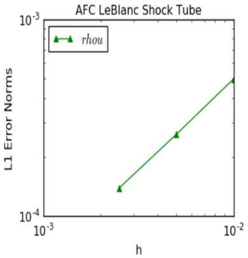
LeBlanc ShockTube Profiles – Conservative CFD Formulation

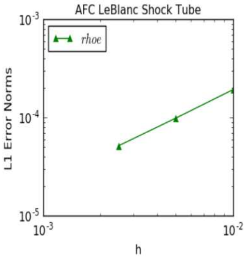


LeBlanc ShockTube – Mesh Convergence

	L1 error (9/900)	L1 error (9/1800)	L1 error (9/3600)	Slope (900-1800)	Slope (1800-3600)
rho	1.31065e-03	6.72218e-04	3.48644e-04	9.63279e-01	9.47172e-01
rhou	4.98398e-04	2.60317e-04	1.38587e-04	9.37030e-01	9.09481e-01
rhoE	1.91591e-04	9.83755e-05	5.14695e-05	9.61657e-01	9.34582e-01

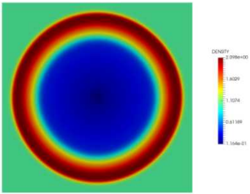


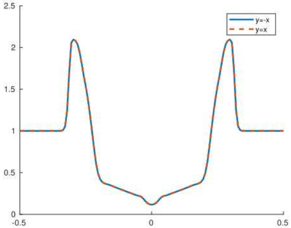




Transient HD results

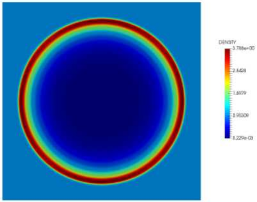
2D results; Top: Radial Riemann, Bottom: Sedov point blast

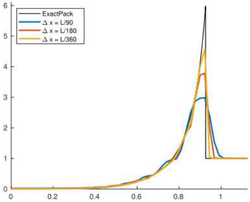




$\Delta x = \Delta y = 1/128$ and $\Delta t = 1 \times 10^{-3}$

ρ vs x trace at $x = y$ and $y = -x$



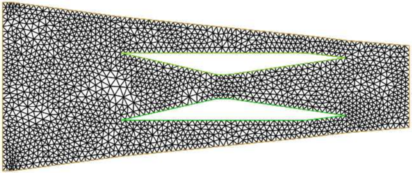


$\Delta x = \Delta y = 0.0125$ and $\Delta t = 1 \times 10^{-5}$.

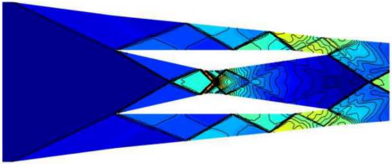
ρ vs x, $L = 2.25$ and $t = 0.24$.

Steady HD result

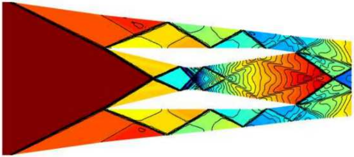
Steady $Ma = 3$ supersonic combustion ramjet engine:



Coarse unstructured mesh



Density



Mach

65256 elements and 33859 degrees of freedom per variable.

CG (high order) scheme

Continuous problem $\Omega \subset \mathbb{R}^d$:

$$\mathbf{u}_t + \nabla \cdot \mathbf{f}(\mathbf{u}) = \mathbf{0},\ (\boldsymbol{x}, t) \in \Omega \times \mathbb{R}_+,\ \mathbf{u}(\boldsymbol{x}, 0) = \mathbf{u}_0(\boldsymbol{x}),$$
$$\mathbf{u} : (\boldsymbol{x}, t) \mapsto \mathbb{R}^m,\ \mathbf{f} : \mathbb{R}^m \to \mathbb{R}^{m \times d}.$$

Semi-discrete problem in $V^h = P^1$ or Q^1:

$$\mathbf{u}_h(\boldsymbol{x}, t) = \{u_h^k(\boldsymbol{x}, t)\}_{k=1}^m,\ u_h^k(\boldsymbol{x}, t) = \sum_j u_j^k(t)\phi_j(\boldsymbol{x}),$$
$$\frac{d}{dt}\int_\Omega \phi \mathbf{u}_h d\boldsymbol{x} - \int_\Omega \nabla\phi \cdot \mathbf{f}(\mathbf{u}_h) d\boldsymbol{x} + \int_\Gamma \phi \mathbf{f}(\mathbf{u}_h) \cdot \boldsymbol{n} d\sigma = \mathbf{0},$$
$$\mathbf{u}_h(\cdot, 0) = \Pi_h \mathbf{u}_0(\cdot) = \sum_j \phi_j \mathbf{u}_0(\boldsymbol{x}_j).$$

$$\mathbf{M}_C \frac{d\boldsymbol{U}}{dt} = \boldsymbol{K}(\boldsymbol{U}) + \boldsymbol{B}(\boldsymbol{U}),\ \boldsymbol{U}(0) = \boldsymbol{U}_0.$$

Consistent mass matrix:

$$\mathbf{M}_C = \{M_{ij}\}_{i,j=1}^{N_h},\ M_{ij} = m_{ij} I_{m\times m},\ m_{ij} = \int_\Omega \phi_i \phi_j d\boldsymbol{x}.$$

Convection term:

$$\boldsymbol{K}(\boldsymbol{U}) = \{\boldsymbol{k}_i\}_{i=1}^{N_h},\ \boldsymbol{k}_i = \sum_{e=1}^{N_K} \int_{K_e} \nabla\phi_i \cdot \mathbf{f}(\mathbf{u}_h) d\boldsymbol{x}.$$

Boundary term:

$$\boldsymbol{B}(\boldsymbol{U}) = \{\boldsymbol{b}_i\}_{i=1}^{N_h},\ \boldsymbol{b}_i = -\sum_{e=1}^{N_K} \int_{\partial K_e \cap \Gamma} \phi_i \mathbf{f}(\mathbf{u}_h) \cdot \boldsymbol{n} d\sigma.$$

Initial condition:

$$\boldsymbol{U}_0 = \{\boldsymbol{U}_{0,i}\}_{i=1}^{N_h},\ \boldsymbol{U}_{0,i} = \mathbf{u}_0(\boldsymbol{x}).$$

Low order scheme and artificial diffusion

$$\mathbf{M}_L \frac{d\boldsymbol{U}}{dt} = \boldsymbol{B}(\boldsymbol{U}) + \mathbf{D}(\boldsymbol{U})\boldsymbol{U} + \boldsymbol{K}(\boldsymbol{U}).$$

Difference between high and low order schemes is given by the anti-diffusion:

$$\boldsymbol{F}(\boldsymbol{U}) = (\mathbf{M}_L - \mathbf{M}_C)\frac{d\boldsymbol{U}}{dt} - \mathbf{D}(\boldsymbol{U})\boldsymbol{U}.$$

The high order scheme can be written as:

$$\mathbf{M}_L \frac{d\boldsymbol{U}}{dt} = \boldsymbol{B}(\boldsymbol{U}) + \mathbf{D}(\boldsymbol{U})\boldsymbol{U} + \boldsymbol{K}(\boldsymbol{U}) + \boldsymbol{F}(\boldsymbol{U}).$$

The artificial diffusion $\mathbf{D}(\boldsymbol{U})$ is defined by

$$\mathbf{D}(\boldsymbol{U}) = \{D_{ij}\},\ D_{ij} = \sum_e D_{ij}^{(e)},$$

where

$$D_{ij}^{(e)} = d_{ij}^{(e)} I_{m\times m},\ \text{for } j \neq i, \&\ D_{ii}^{(e)} = -\sum_{j\neq i} D_{ij}^{(e)}.$$

Rusanov & Roe diffusion for systems (Kuzmin, Möller and Gurris, 2012, FCT book):

- Scalar/Rusanov diffusion:

$$d_{ij}^{(e)}(\boldsymbol{U}) = \max\left(\lambda_{\max}(\boldsymbol{c}_{ij}^{(e)}, \boldsymbol{U}_i, \boldsymbol{U}_j), \lambda_{\max}(\boldsymbol{c}_{ji}^{(e)}, \boldsymbol{U}_j, \boldsymbol{U}_i)\right), \; j \neq i,$$

$$D_{ij}^{(e)} = d_{ij}^{(e)} I_{m\times m}, \text{ for } j \neq i, \& \; D_{ii}^{(e)} = -\sum_{j\neq i} D_{ij}^{(e)}.$$

- Tensorial/Roe diffusion:

$$\boldsymbol{c}_{ij}^{(e)} \cdot \mathbf{f}'(\mathbf{u}) = R(\mathbf{u}; \boldsymbol{c}_{ij}^{(e)})\Lambda(\mathbf{u}; \boldsymbol{c}_{ij}^{(e)})R^{-1}(\mathbf{u}; \boldsymbol{c}_{ij}^{(e)}),$$

$$D_{ij}^{(e)} = \left[R(\boldsymbol{U}_{ij}; \boldsymbol{c}_{ij}^{(e)})|\Lambda(\boldsymbol{U}_{ij}; \boldsymbol{c}_{ij}^{(e)})|R^{-1}(\boldsymbol{U}_{ij}; \boldsymbol{c}_{ij}^{(e)})\right], \; j \neq i, \& \; D_{ii}^{(e)} = \sum_{j\neq i} -D_{ij}^{(e)}.$$

Rusanov diffusion for MHD:

$$d_{ij}^{(e)}(\boldsymbol{U}) = \max\left(\lambda_{\max}(\boldsymbol{c}_{ij}^{(e)}, \boldsymbol{U}_i, \boldsymbol{U}_j), \lambda_{\max}(\boldsymbol{c}_{ji}^{(e)}, \boldsymbol{U}_j, \boldsymbol{U}_i)\right),\ j \neq i,$$

$$D_{ij}^{(e)} = d_{ij}^{(e)} I_{m\times m},\ \text{for } j \neq i, \&\ D_{ii}^{(e)} = -\sum_{j\neq i} D_{ij}^{(e)}.$$

Let $\boldsymbol{n}_{ij}^{(e)} = \boldsymbol{c}_{ij}^{(e)}/\|\boldsymbol{c}_{ij}^{(e)}\|$. The maximum eigenvalue is:

$$\lambda_{\max}(\boldsymbol{c}_{ij}^{(e)}, \boldsymbol{U}_i, \boldsymbol{U}_j) = \|\boldsymbol{c}_{ij}^{(e)}\|\left[|\boldsymbol{v}\cdot\boldsymbol{n}_{ij}^{(e)}| + c_f(\boldsymbol{U}_j; \boldsymbol{n}_{ij}^{(e)})\right],$$

where the fast magneto-sonic wave speed is

$$c_f(\mathbf{u}; \boldsymbol{n}) = \frac{1}{2}(a^2 + |\boldsymbol{b}|^2) + \frac{1}{2}\sqrt{(a^2 + |\boldsymbol{b}|^2)^2 - 4a^2c_a^2}.$$

$\boldsymbol{n} \in \mathbb{R}^d$ is a unit vector, $c_a = \boldsymbol{b}\cdot\boldsymbol{n}$, $\boldsymbol{b} = |\boldsymbol{B}|^2/\sqrt{\rho}$, $a = \sqrt{\frac{\gamma p}{\rho}}$.

Limiting strategy

High order scheme:

$$\mathbf{M}_L \frac{d\boldsymbol{U}}{dt} = \boldsymbol{K}(\boldsymbol{U}) + \mathbf{D}(\boldsymbol{U})\boldsymbol{U} + \boldsymbol{B}(\boldsymbol{U}) + \boldsymbol{F}(\boldsymbol{U}).$$

Element-wise limiting:

$$\boldsymbol{F}^{(e)} = (\mathbf{M}_L^{(e)} - \mathbf{M}_C^{(e)})\dot{\boldsymbol{U}}^{(e)} - \mathbf{D}^{(e)}(\boldsymbol{U}^{(e)})\boldsymbol{U}^{(e)}, \ \bar{\boldsymbol{F}}^{(e)} = \alpha_e \boldsymbol{F}^{(e)} \implies \bar{\boldsymbol{F}} = \sum_e \bar{\boldsymbol{F}}^{(e)}.$$

Stabilized semi-discrete scheme:

$$\mathbf{M}_L \frac{d\boldsymbol{U}}{dt} = \boldsymbol{K}(\boldsymbol{U}) + \mathbf{D}(\boldsymbol{U})\boldsymbol{U} + \boldsymbol{B}(\boldsymbol{U}) + \bar{\boldsymbol{F}}.$$

Desired properties of the element limiter:

- $\alpha_e \in [0, 1]$.
- $\alpha_e = 1$ in smooth regions.
- $\alpha_e = 0$ in the vicinity of steep-fronts/shocks.
- $\alpha_e = 1$ if the solution is linear.

Element-wise limiter α_e calculation (Kuzmin, Basting and Shadid, 2017, CMAME):

- Construct a linearity preserving nodal limiter :
  - $\Phi_i(u_i - u_j)$ depends continuously on u_j, $j \in \mathcal{N}(i)$.
  - $\Phi_i(u_i - u_j) = 0$ at a local maximum and local minimum.
  - $\Phi_i(u_i - u_j) = (u_i - u_j)$ if u_h is linear on $\Omega_i = \mathrm{supp}\{\phi_i\}$.

$$\Phi_i^u = 1 - \left[\frac{\left| \sum_{j \neq i} \beta_{ij}(u_j - u_i) \right| + \epsilon}{\sum_{j \neq i} \beta_{ij} |u_j - u_i| + \epsilon} \right]^q ,$$

where $\beta_{ij} \geq 0$ such that $\sum_{j \neq i} \beta_{ij} \boldsymbol{g}_i \cdot (\boldsymbol{x}_j - \boldsymbol{x}_i) = 0$, $q = 10$, and $\epsilon = 1 \times 10^{-16}$.

- Element-wise limiter:

$$\alpha_e^u = \min\{\Phi_i^u\}.$$

- Synchronized limiter:

$$\alpha_e = \min\{\alpha_e^\rho, \alpha_e^{\rho p}\}, \text{ OR } \alpha_e = \min\{\alpha_e^\rho, \alpha_e^s\}, \text{ where } s = \log p - \gamma \log \rho.$$

Add a high order diffusion term:

$$\tilde{\boldsymbol{F}}_i^{(e)}(\boldsymbol{U}) = \bar{\boldsymbol{F}}_i^{(e)}(\boldsymbol{U}) + \omega\alpha_e \sum_{j\neq i} D_{ij}^{(e)}(\boldsymbol{U}) \left[\frac{\boldsymbol{G}_i + \boldsymbol{G}_j}{2} \cdot (\boldsymbol{x}_i - \boldsymbol{x}_j) - (\boldsymbol{U}_i - \boldsymbol{U}_j) \right],$$

where

$$\boldsymbol{G}_i = \frac{1}{m_i} \int_\Omega \phi_i \nabla \mathbf{u}_h d\boldsymbol{x} = \frac{1}{m_i} \sum_j \boldsymbol{c}_{ji} \boldsymbol{U}_j, \ \boldsymbol{c}_{ij} = \sum_e \boldsymbol{c}_{ij}^{(e)}.$$

The final stabilized scheme is assembled from

$$\boldsymbol{R}^{(e)}(\boldsymbol{U}) = -\mathbf{M}_L^{(e)} \frac{d\boldsymbol{U}^{(e)}}{dt} + \boldsymbol{K}^{(e)}(\boldsymbol{U}) + \boldsymbol{B}^{(e)}(\boldsymbol{U}) + \mathbf{D}^{(e)}(\boldsymbol{U})\boldsymbol{U}^{(e)} + \tilde{\boldsymbol{F}}^{(e)}(\boldsymbol{U}).$$

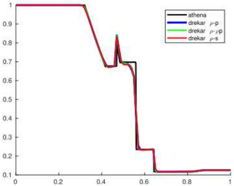
In summary:

- Semi-discrete *stabilized scheme*.
- We can use various time-steppers to discretize it.

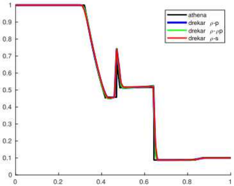
Brio-Wu test

Comparison of various synchronized limiting procedures.

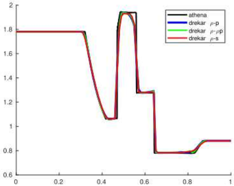
$$\Delta x = \frac{1}{800}, \ \Delta t = 2.5 \times 10^{-4}.$$



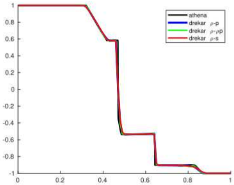
ρ



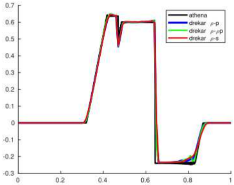
p



ρE



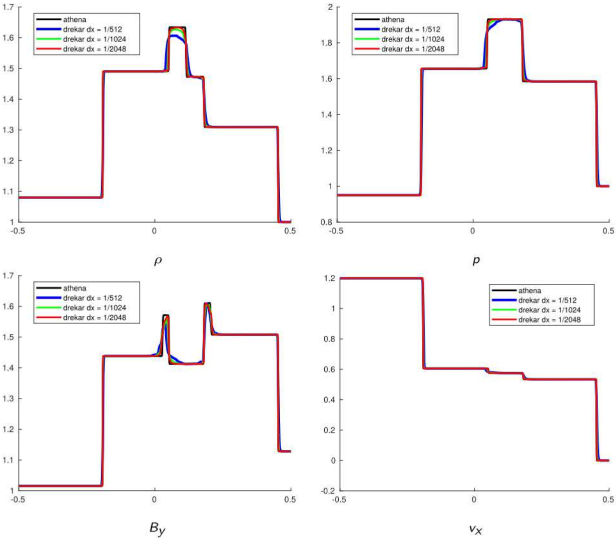
B_y



v_x

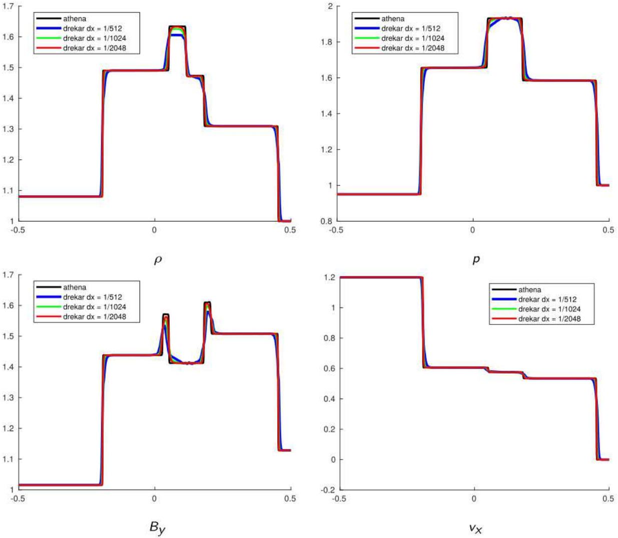
Ryu-Jones problem

Mesh levels: $\Delta x = 1/512, 1/1024$ and $1/2048$. Limiter $\alpha_e = \min\{\alpha_e^{\rho}, \alpha_e^{\rho p}\}$.



Ryu-Jones problem

Mesh levels: $\Delta x = 1/512, 1/1024$ and $1/2048$. Limiter $\alpha_e = \min\{\alpha_e^{\rho}, \alpha_e^{s}\}$.



In 2D and 3D we need $\nabla \cdot \boldsymbol{B} = 0$. Try adding the Powell term to the MHD system:

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho \boldsymbol{v} \\ \rho E \\ \boldsymbol{B} \end{bmatrix} + \nabla \cdot \begin{bmatrix} \rho \boldsymbol{v} \\ \rho \boldsymbol{v} \otimes \boldsymbol{v} + pI - \boldsymbol{T}_M \\ \boldsymbol{v} \cdot ((\rho E + p)I - \boldsymbol{T}_M) \\ \boldsymbol{B} \otimes \boldsymbol{v} - \boldsymbol{v} \otimes \boldsymbol{B} \end{bmatrix} + \begin{bmatrix} 0 \\ \boldsymbol{B} \\ \boldsymbol{v} \cdot \boldsymbol{B} \\ \boldsymbol{v} \end{bmatrix} (\nabla \cdot \boldsymbol{B}) = \boldsymbol{0}.$$

Notable things with the Godunov-Powell formulation

- Aids symmetrization for the MHD system
- Helps to guarantee Galilean invariance.
- May result in incorrect Rankine-Hugoniot conditions (for discontinuous solutions)
- Provides some form of divergence cleaning.

Multi-dim MHD + parabolic divergence cleaning

We add a penalty term to the induction equation to get:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho\boldsymbol{v} \\ \rho E \\ \boldsymbol{B}\end{bmatrix} + \nabla\cdot\begin{bmatrix}\rho\boldsymbol{v} \\ \rho\boldsymbol{v}\otimes\boldsymbol{v} + pI - \boldsymbol{T}_M \\ \boldsymbol{v}\cdot((\rho E + p)I - \boldsymbol{T}_M) \\ \boldsymbol{B}\otimes\boldsymbol{v} - \boldsymbol{v}\otimes\boldsymbol{B} - c_p^2(\nabla\cdot\boldsymbol{B})I\end{bmatrix} = \mathbf{0}.$$

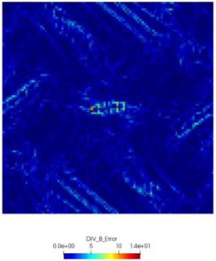
- Diffuses out the $\nabla\cdot\boldsymbol{B}$ error.
- Controls the global value $\|\nabla\cdot\boldsymbol{B}\|_{L^1(\Omega)}$.
- Requires an implicit time stepping scheme to be effective.

Comparison between cleaning strategies (Orszag-Tang)

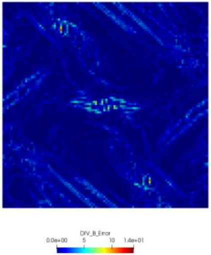
$\|\nabla \cdot \boldsymbol{B}\|_{L^1(K)}$ range for *no cleaning*, *Powell term* and *parabolic cleaning*:

No cleaning	8.7×10^{-3} to 13.955
Powell term	4.802×10^{-3} to 11.0264
Parabolic cleaning	2.6318×10^{-3} to 0.6333

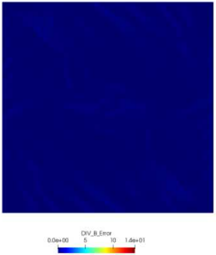
Plot of $\|\nabla \cdot \boldsymbol{B}\|_{L^1(K)}$ *color-scheme* between 0 and 13.955:



No cleaning



Powell term

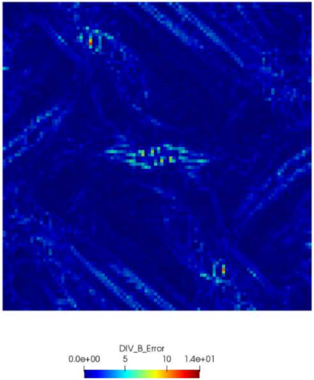


Parabolic cleaning

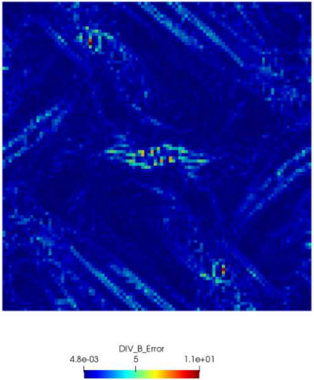
Powell term (Orszag-Tang)

$\|\nabla \cdot \boldsymbol{B}\|_{L^1(K)}$ range for *Powell term*:

Powell term 4.802×10^{-3} to 11.0264



$\|\nabla \cdot \boldsymbol{B}\|_{L^1(K)}$ between 0 and 13.955

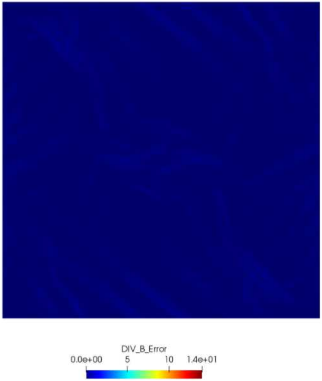


$\|\nabla \cdot \boldsymbol{B}\|_{L^1(K)}$ between 4.802×10^{-3} to 11.0264

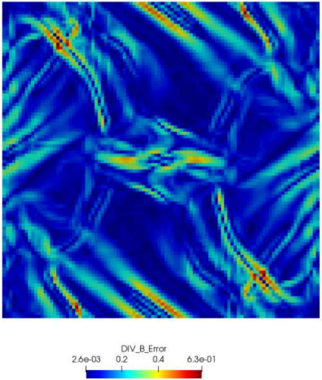
Parabolic cleaning (Orszag-Tang)

$\|\nabla \cdot \boldsymbol{B}\|_{L^1(K)}$ range for *Powell term*:

Parabolic cleaning $\quad 2.6318 \times 10^{-3}$ to 0.6333



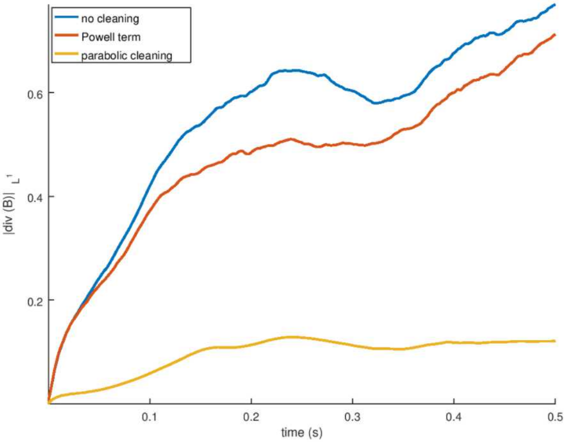
$\|\nabla \cdot \boldsymbol{B}\|_{L^1(K)}$ between 0 and 13.955



$\|\nabla \cdot \boldsymbol{B}\|_{L^1}$ between 2.6318×10^{-3} to 0.6333

Comparison between cleaning strategies (Orszag-Tang)

$\|\nabla \cdot \boldsymbol{B}\|_{L^1(\Omega)}$ plot versus time:



MHD + Powell term (Orszag-Tang)

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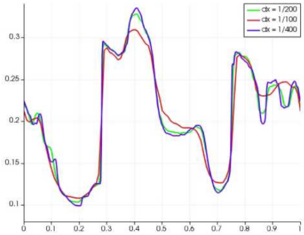
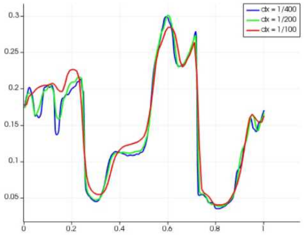
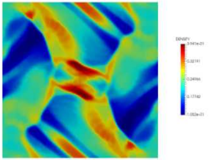
Orszag-Tang profile at 3 mesh levels.



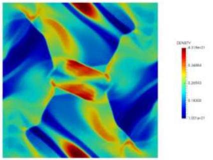
ρ at $y = 0.75$



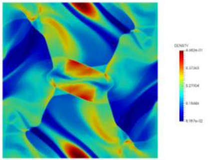
p at $y = 0.25$



ρ for $\Delta x = \Delta y = 1/100$



ρ for $\Delta x = \Delta y = 1/200$



ρ for $\Delta x = \Delta y = 1/400$

Summary

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Aims:

- To have a flexible CG scheme for shock MHD.
- To have stabilization that works for MHD systems.
- To have a divergence cleaning scheme.

What has been achieved:

- Some fairly good shock HD results in multi-D
- Some preliminary results for MHD in 1D and 2D
- Use of the Powell term for MHD that slightly reduces $\|\nabla \cdot \boldsymbol{B}\|_{L^1}$.
- Use of the parabolic cleaning that keeps $\|\nabla \cdot \boldsymbol{B}\|_{L^1}$ under control.

What's next:

- To do divergence (hyperbolic/mixed) cleaning for MHD
- To extend the current formulation to multi-fluid equations.

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