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# Modeling Flexible Generator Operating Regions via Chance-constrained Stochastic Unit Commitment

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# Stochastic unit commitment

Standard unit commitment (UC) problem: which thermal generators should be scheduled to meet power demand, while ensuring feasible operations?

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# Stochastic unit commitment

Standard unit commitment (UC) problem: which thermal generators should be scheduled to meet power demand, while ensuring feasible operations?

Stochastic unit commitment (UC) problem: which thermal generators should be scheduled to meet power demand, while ensuring feasible operations, under uncertainty (of demand, prices, renewables...)?

But...

- Thermal generator operational limits are based on engineering judgments
- Can be exceeded in practice, for short periods
- System operators do run thermal generators beyond these limits

## Proposed model

- Allow thermal generators to “occasionally” violate operational limits
- Violations should be few (else, increased maintenance costs)
- Violations should not be large (there are absolute ratings of generators)
- 1% savings in energy production is worth  $\approx \$1$  billion per year in the U.S. alone

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## Proposed model

- Let  $y_t^{g,\omega}$  denote a “non-nominal” operation in hour  $t$  for generator  $g$  in scenario  $\omega$
- During non-nominal operations, generator's operating region expands from  $[\underline{P}^g, \overline{P}^g]$  to  $[\underline{\underline{P}}^g, \overline{\overline{P}}^g]$
- Non-nominal mode of generation is more expensive
- Number of non-nominalities is few:  
$$\frac{1}{|\Omega||\mathcal{T}||\mathcal{G}|} \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega} y_t^{g,\omega} \leq \varepsilon \leftarrow \text{almost a chance-constraint!}$$

## Recall, challenges of chance-constraint (CC) models

- CC models are computationally intractable
- A known NP-hard problem
- Existing algorithms not scalable to practical sized problems
- Feasible region is non-convex

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# Setup

We use:

$$\overline{\overline{P}}^g = (1 + \beta) \overline{P}^g$$

$$\underline{\underline{P}}^g = (1 - \beta) \underline{P}^g$$

$$\overline{C}^g = (1 + \gamma) C^{L^g, g}$$

$$\underline{C}^g = (1 + \gamma) C^{L^g, g}.$$

WECC240++ system with 85 thermal generators, 50 scenarios and  
RTS-GMLC system with 73 thermal generators, 16 scenarios

# Computational results for the RTS-GMLC 16 scenario case for 10 July 2020.

Under second review: *Computational Management Science*

Table: MIP gap = 0.1%

$\varepsilon$	$\beta$	$\gamma$	Cost (M\$)	Savings (%)	Time (sec)	MIP gap (%)
0			3.89	0.00%	33	-
0.01	0.05	0.1	3.84	1.21%	46	-
		0.2	3.84	1.20%	48	-
		0.1	3.83	1.51%	82	-
	0.1	0.2	3.83	1.50%	106	-
		0.1	3.83	1.53%	65	-
		0.2	3.83	1.45%	100	-
0.05	0.05	0.1	3.81	2.08%	1800	0.22%
		0.2	3.82	1.82%	1800	0.15%

- Increase  $\varepsilon \Rightarrow$  increase savings
- Increase  $\beta \Rightarrow$  increase savings
- Increase  $\gamma \Rightarrow$  decrease savings

# Cost savings for the RTS-GMLC 16 scenario case for 10 July 2020.

Under second review: *Computational Management Science*

Table: MIP gap = 0.1%

$\epsilon$	$\beta$	$\gamma$	Optimal	Limited	No nuclear
0.01	0.05	0.1	1.21%	0.71%	1.06%
		0.2	1.20%	0.69%	1.04%
	0.1	0.1	1.51%	1.14%	1.15%
		0.2	1.50%	1.10%	1.11%
	0.05	0.1	1.53%	0.70%	1.22%
		0.2	1.45%	0.69%	1.15%
		0.1	2.08%	1.14%	1.41%
		0.2	1.82%	1.10%	1.28%

Limited = at most one non-nominal operation per generator per day

No nuclear = no non-nominal operation for the nuclear unit in this system

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# Chance Constraint Setting

This is a linear Joint Chance Constraint:

$$P(x_t \leq y_t^\omega + w_t^\omega, \forall t \in T) \geq 1 - \varepsilon$$

Background:

- Two-stage stochastic program with recourse
- First stage decision,  $x_t$ , second-stage decision,  $y_t^\omega$
- Possibly integer restrictions on  $x$  and/or  $y$
- i.i.d. samples of uncertainty  $w_t^\omega$

# Stochastic unit commitment model

## Indices and Sets:

$g \in \mathcal{G}$	Thermal generators.
$t \in \mathcal{T}$	Hourly time steps: $1, \dots, T$ ; i.e., $[a, b] \in \mathcal{T} \times \mathcal{T}$ such that $b \geq a + UT^g$ .
$l \in \mathcal{L}^g$	Piecewise production cost intervals for generator $g$ : $1, \dots, L_g$ .
$s \in \mathcal{S}^g$	Start-up categories for generator $g$ , from hottest (1) to coldest ( $S_g$ ).
$\omega \in \Omega$	Scenarios: $\omega_1, \dots, \omega_N$ .

## Parameters: First Stage

$C^{l,g}$	Marginal cost for piecewise segment $l$ for generator $g$ (\$/MWh).
$\bar{C}^g$	Marginal cost for production above $\bar{P}^g$ (\$/MWh).
$\underline{C}^g$	Marginal cost for production below $\underline{P}^g$ (\$/MWh).
$C^{R,g}$	Cost of generator $g$ running and operating at minimum production $P_g$ (\$/h).
$C^{s,g}$	Start-up cost of category $s$ for generator $g$ (\$).
$DT^g$	Minimum down time for generator $g$ (h).
$\bar{P}^g$	Maximum power output for generator $g$ under normal operations (MW).
$\underline{\bar{P}}^g$	Maximum power output for generator $g$ under non-nominal operations (MW).
$\bar{P}^g$	Minimum power output for generator $g$ under normal operations (MW).
$\underline{\bar{P}}^g$	Minimum power output for generator $g$ under non-nominal operations (MW).
$\bar{P}^{l,g}$	Maximum power available for piecewise segment $l$ for generator $g$ (MW) (with $\bar{P}^{0,g} = \bar{P}^g$ ).
$RD^g$	Ramp-down rate for generator $g$ (MW/h).
$RU^g$	Ramp-up rate for generator $g$ (MW/h).
$SD^g$	Shutdown ramp rate for generator $g$ (MW/h).
$SU^g$	Start-up ramp rate for generator $g$ (MW/h).
$TC^g$	Time down after which generator $g$ goes cold (h).
$T^{s,g}$	Time offline after which the start-up category $s$ is available (h) (with $T^{1,g} = DT^g$ , $T^{S_g,g} = TC^g$ )
$UT^g$	Minimum up time for generator $g$ (h).

# Stochastic unit commitment model

## Parameters: Second Stage

$D_t^\omega$	Load (demand) at time $t$ in scenario $\omega$ (MW).
$W_t^\omega$	Maximum power from renewables at time $t$ in scenario $\omega$ (MW).
$\underline{W}_t^\omega$	Minimum power from renewables at time $t$ in scenario $\omega$ (MW).

## Variables: First Stage

$u_t^g$	Commitment status of generator $g$ at time $t$ , $\in \{0, 1\}$ .
$v_t^g$	Start-up status of generator $g$ at time $t$ , $\in \{0, 1\}$ .
$w_t^g$	Shutdown status of generator $g$ at time $t$ , $\in \{0, 1\}$ .
$x_{[t, t']}$	Indicator arc for shutdown at time $t$ , start-up at time $t'$ , uncommitted for $i \in [t, t')$ , for generator $g$ , $\in \{0, 1\}$ , $[t, t')$ such that $t + DT^g \leq t' \leq t + TC^g - 1$ .

## Variables: Second Stage

$p_t^{g, \omega}$	Power above minimum from generator $g$ at time $t$ in scenario $\omega$ (MW).
$\bar{P}_t^{g, \omega}$	Power above maximum from generator $g$ at time $t$ in scenario $\omega$ (MW).
$\underline{P}_t^{g, \omega}$	Power below minimum from generator $g$ at time $t$ in scenario $\omega$ (MW).
$p_{t, l, g, \omega}$	Power from piecewise interval $l$ for generator $g$ at time $t$ in scenario $\omega$ (MW).
$r_t^{h, \omega}$	Power from renewables at time $t$ in scenario $\omega$ (MW).
$y_t^{g, \omega}$	Non-nominal operation status of generator $g$ at time $t$ in scenario $\omega$ (MW).

# Stochastic unit commitment

$$\min \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \left( \sum_{l \in \mathcal{L}^g} \mathbb{E}[C^{l,g} p_t^{l,g,\omega} + \bar{C}^g \bar{P}_t^{g,\omega} + \underline{C}^g \underline{P}_t^{g,\omega}] + C^{R,g} u_t^g + c_t^{SU,g} \right) \quad (1)$$

subject to:

$$u_t^g - u_{t-1}^g = v_t^g - w_t^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (2a)$$

$$\sum_{i=t-UT^g+1}^t v_i^g \leq u_t^g \quad \forall t \in [UT^g, T], \forall g \in \mathcal{G} \quad (2b)$$

$$\sum_{i=t-DT^g+1}^t w_i^g \leq 1 - u_t^g \quad \forall t \in [DT^g, T], \forall g \in \mathcal{G} \quad (2c)$$

$$\sum_{t'=t-TC^g+1}^{t-DT^g} x_{[t',t)}^g \leq v_t^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (2d)$$

$$\sum_{t'=t+DT^g}^{t+TC^g-1} x_{[t,t')}^g \leq w_t^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (2e)$$

$$c_t^{SU,g} = C^{S,g} v_t^g + \sum_{s=1}^{S^g-1} (C^{s,g} - C^{S,g}) \left( \sum_{t'=t-\underline{T}^{s+1,g}+1}^{t-\bar{T}^{s,g}} x_{[t',t)}^g \right) \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (2f)$$

# Stochastic unit commitment

$$p_t^{g,\omega} \leq (\bar{P}^g - \underline{P}^g)u_t^g - (\bar{P}^g - SU^g)v_t^g - (\bar{P}^g - SD^g)w_{t+1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{>1}, \forall \omega \in \Omega \quad (3a)$$

$$p_t^{g,\omega} \leq (\bar{P}^g - \underline{P}^g)u_t^g - (\bar{P}^g - SU^g)v_t^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \quad (3b)$$

$$p_t^{g,\omega} \leq (\bar{P}^g - \underline{P}^g)u_t^g - (\bar{P}^g - SD^g)w_{t+1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \quad (3c)$$

$$p_t^{g,\omega} - p_{t-1}^{g,\omega} \leq (SU^g - RU^g - \underline{P}^g)v_t^g + RU^g u_t^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \quad (3d)$$

$$p_{t-1}^{g,\omega} - p_t^{g,\omega} \leq (SD^g - RD^g - \underline{P}^g)w_t^g + RD^g u_{t-1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \quad (3e)$$

$$p_t^{g,\omega} = \sum_{l \in \mathcal{L}^g} p_t^{l,g,\omega} \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \quad (3f)$$

$$p_t^{l,g,\omega} \leq (\bar{P}^{l,g} - \bar{P}^{l-1,g})u_t^g \quad \forall t \in \mathcal{T}, \forall l \in \mathcal{L}^g, \forall g \in \mathcal{G}, \forall \omega \in \Omega \quad (3g)$$

# Stochastic unit commitment

$$y_t^{g,\omega} \leq u_t^g - v_t^g - w_{t+1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{>1}, \forall \omega \in \Omega \quad (4a)$$

$$y_t^{g,\omega} \leq u_t^g - v_t^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \quad (4b)$$

$$y_t^{g,\omega} \leq u_t^g - w_{t+1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \quad (4c)$$

$$\bar{P}_t^{g,\omega} \leq (\bar{\bar{P}} - \bar{P}) y_t^{g,\omega} \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \quad (4d)$$

$$\underline{P}_t^{g,\omega} \leq (\underline{\underline{P}} - \underline{P}) y_t^{g,\omega} \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \quad (4e)$$

$$\sum_{g \in \mathcal{G}} (p_t^{g,\omega} + \bar{P}_t^{g,\omega} - \underline{P}_t^{g,\omega} + \underline{P}_t^{g,\omega} u_t^g) + r_t^\omega = D_t^\omega \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega \quad (5)$$

$$\frac{1}{|\mathcal{G}| |\mathcal{T}| |\Omega|} \sum_{\omega \in \Omega} \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} y_t^{g,\omega} \leq \varepsilon \quad (6)$$

$$p_t^{I,g,\omega} \in \mathbb{R}_+ \quad \forall t \in \mathcal{T}, \forall I \in \mathcal{L}^g, \forall g \in \mathcal{G}, \forall \omega \in \Omega \quad (7a)$$

$$p_t^{g,\omega}, \bar{P}_t^{g,\omega}, \underline{P}_t^{g,\omega} \in \mathbb{R}_+ \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \quad (7b)$$

$$r_t^{n,\omega} \in [\underline{W}_t^{n,\omega}, \bar{W}_t^{n,\omega}] \quad \forall t \in \mathcal{T}, \forall n \in \mathcal{N}, \forall \omega \in \Omega \quad (7c)$$

$$u_t^g, v_t^g, w_t^g \in \{0, 1\} \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (7d)$$

$$x_{[t, t')}^g \in \{0, 1\} \quad \forall [t, t') \in \mathcal{X}^g, \forall g \in \mathcal{G} \quad (7e)$$

$$y_t^{g,\omega} \in \{0, 1\} \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega. \quad (7f)$$

# Computational results for the WECC240++ 50 scenario test case for 11 May 2013.

Table: MIP gap = 0.1%

$\varepsilon$	$\beta$	$\gamma$	Cost (K\$)	Savings (%)	Time (sec)	MIP gap (%)
0			64.41	0.00%	183	-
0.01	-0.05	0.1	64.20	0.33%	275	-
		0.2	64.21	0.31%	242	-
	0.05	0.1	64.03	0.59%	258	-
		0.2	64.04	0.58%	317	-
0.05	0.05	0.1	63.86	0.85%	275	-
		0.2	63.90	0.80%	343	-
	0.1	0.1	63.35	1.64%	378	-
		0.2	63.42	1.55%	371	-

- Increase  $\varepsilon \Rightarrow$  increase savings
- Increase  $\beta \Rightarrow$  increase savings
- Increase  $\gamma \Rightarrow$  decrease savings

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