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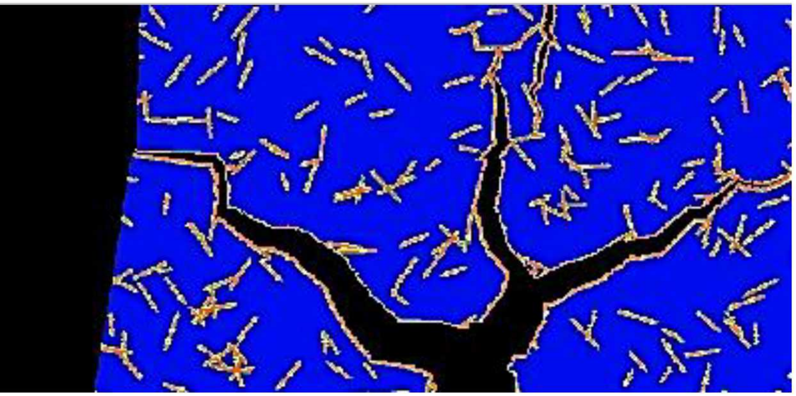
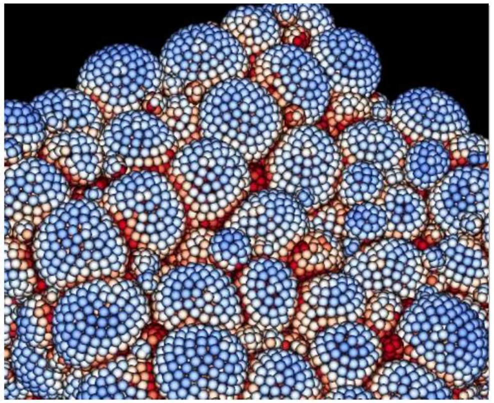
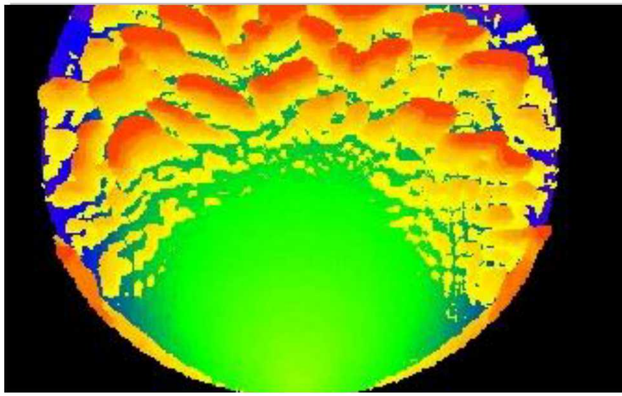

Peridynamic analysis of material failure

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Albuquerque, New Mexico

TMS Conference, San Antonio, TX, March 14, 2019

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Outline

- Peridynamic theory summary
- Some examples of material failure that peridynamics may be good at:
  - Dynamic fracture and complex crack trajectories.
  - Interactions between multiple distributed defects.
  - Direction-dependent failure modes in materials.
  - Accumulated damage.

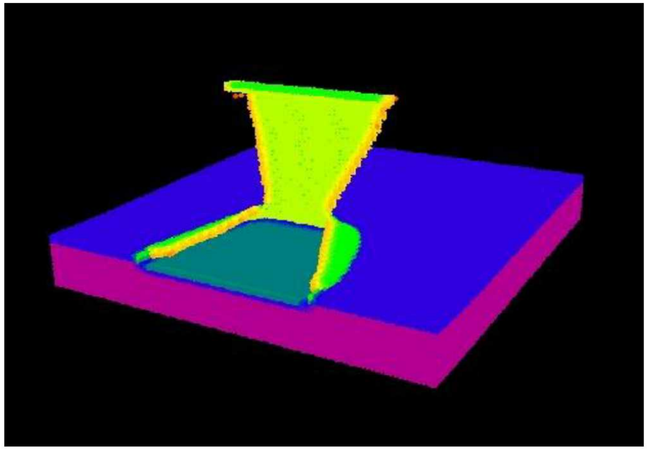
Gaps in classical continuum mechanics

- Local equilibrium equation and material model:

$$\nabla \cdot \sigma + \mathrm{b} = 0, \qquad \sigma = \hat{\sigma}(\mathrm{F}), \qquad \mathrm{F} = \frac{\partial \mathrm{y}}{\partial \mathrm{x}},$$

  $\mathrm{y}(\mathrm{x})$=deformation map, F=deformation gradient tensor, σ=stress tensor, b=external force.

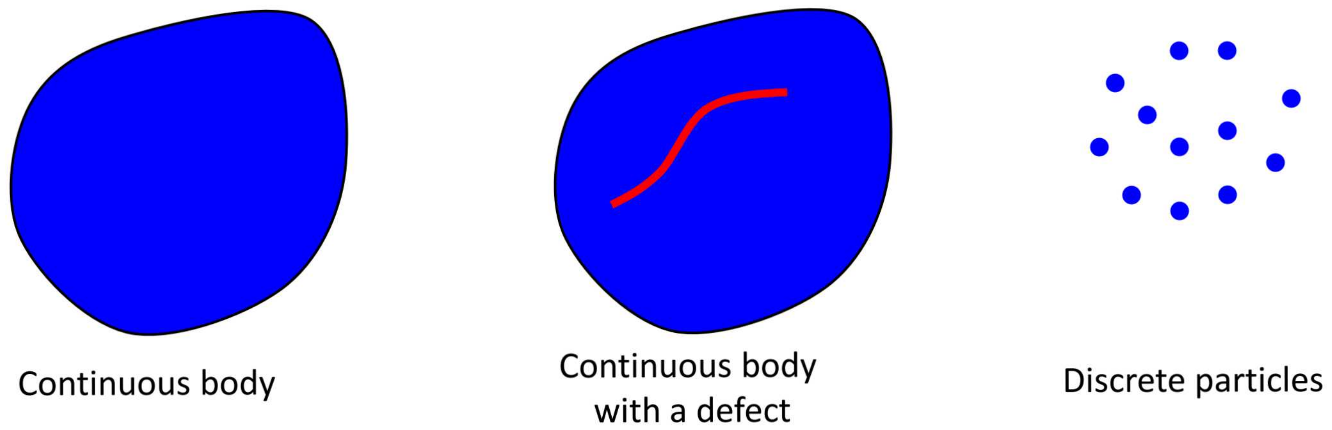
- Requires deformation to be twice differentiable.
- Doesn't apply on growing cracks.
- Can't include nanoscale long-range forces.



We don't have to look far to see the theory's limitations: Humble Scotch® tape

Peridynamics:* What it is

- It's an extension of continuum mechanics to media with cracks and long-range forces.
- It unifies the mechanics of continuous and discontinuous media within a single, consistent set of equations.

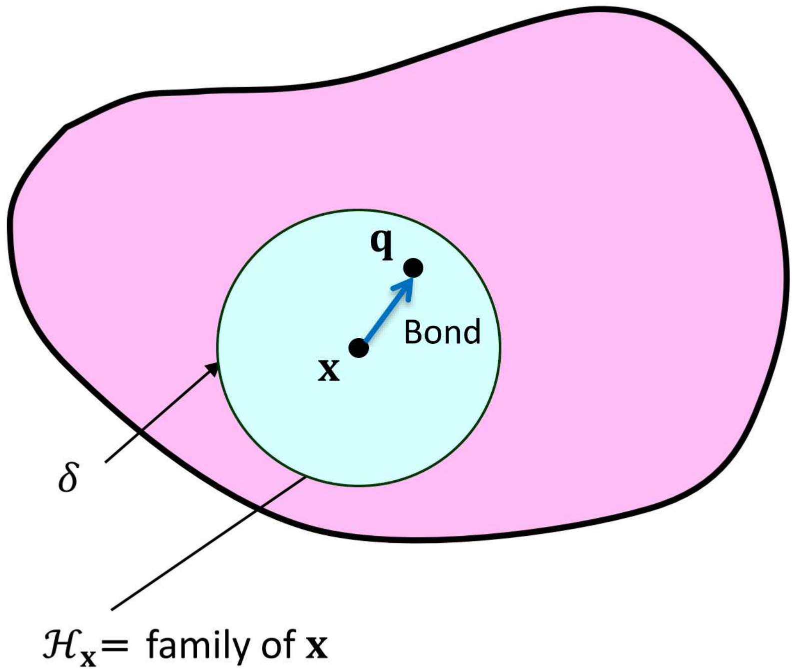


- Our goals
  - Nucleate cracks and seamlessly transition to growth.
  - Model complex fracture patterns.
  - Communicate across length scales.

* Peri (near) + dyn (force)

Peridynamics concepts: Horizon and family

- Any point $\mathbf{x}$ interacts directly with other points within a distance δ called the "horizon."
- The material within a distance δ of $\mathbf{x}$ is called the "family" of $\mathbf{x}$, $\mathcal{H}_{\mathbf{x}}$.

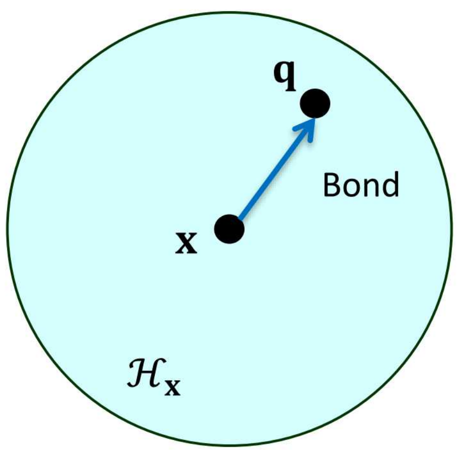


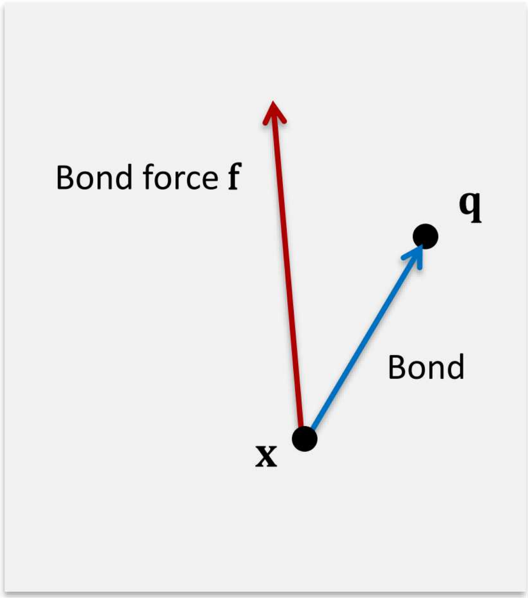
- SS, *J. Mechanics and Physics of Solids* (2000)
- SS & Lehoucq, *Advances in Applied Mechanics* (2010)

Peridynamics concepts: Equilibrium equation

- Momentum balance sums up nonlocal interactions through bond forces $\mathbf{f}$ $(\mathbf{q},\mathbf{x})$.
- Similar to molecular dynamics.
- Bond forces come from the material model.

$$\int_{\mathcal{H}_{\mathbf{x}}} \mathbf{f}(\mathbf{q},\mathbf{x}) + \mathbf{b}(\mathbf{x}) = 0$$

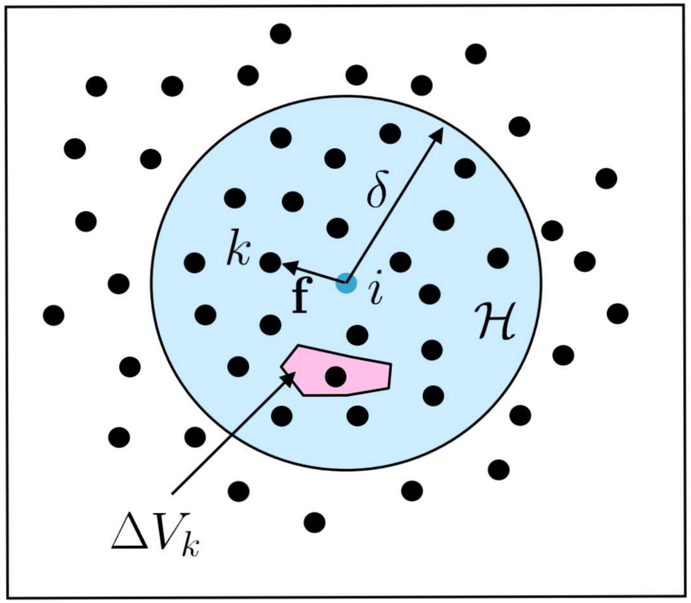




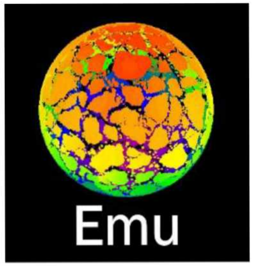
Emu numerical method

- Integral is replaced by a finite sum: resulting method is meshless and Lagrangian.

$$\rho\ddot{\mathbf{y}}(\mathbf{x},t) = \int_{\mathcal{H}} \mathbf{f}(\mathbf{x}',\mathbf{x},t)\, dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t) \longrightarrow \rho\ddot{\mathbf{y}}_i^n = \sum_{k\in\mathcal{H}} \mathbf{f}(\mathbf{x}_k,\mathbf{x}_i,t)\, \Delta V_k + \mathbf{b}_i^n$$

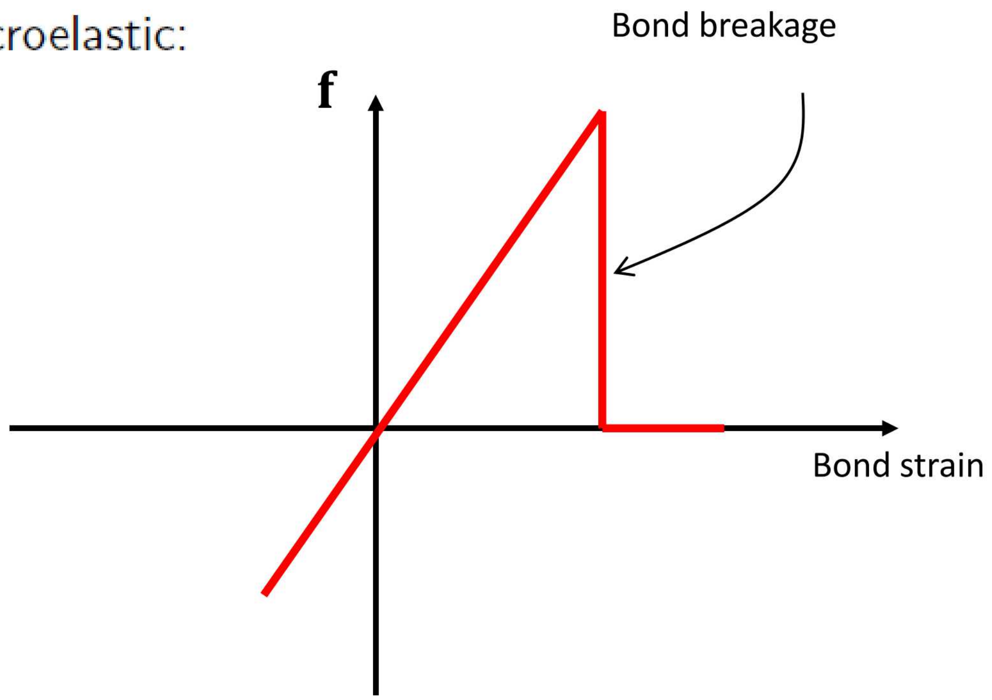


- SS & Askari, *Computers & Structures* (2005)
- *Tian & Du, SIAM Journal on Numerical Analysis* (2014)



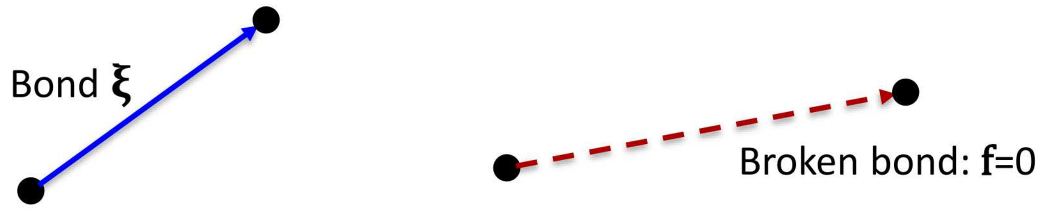
Material model example: Brittle microelastic

- A material in which each bond responds independently of all others is called *bond-based*.
- Example: Microelastic:



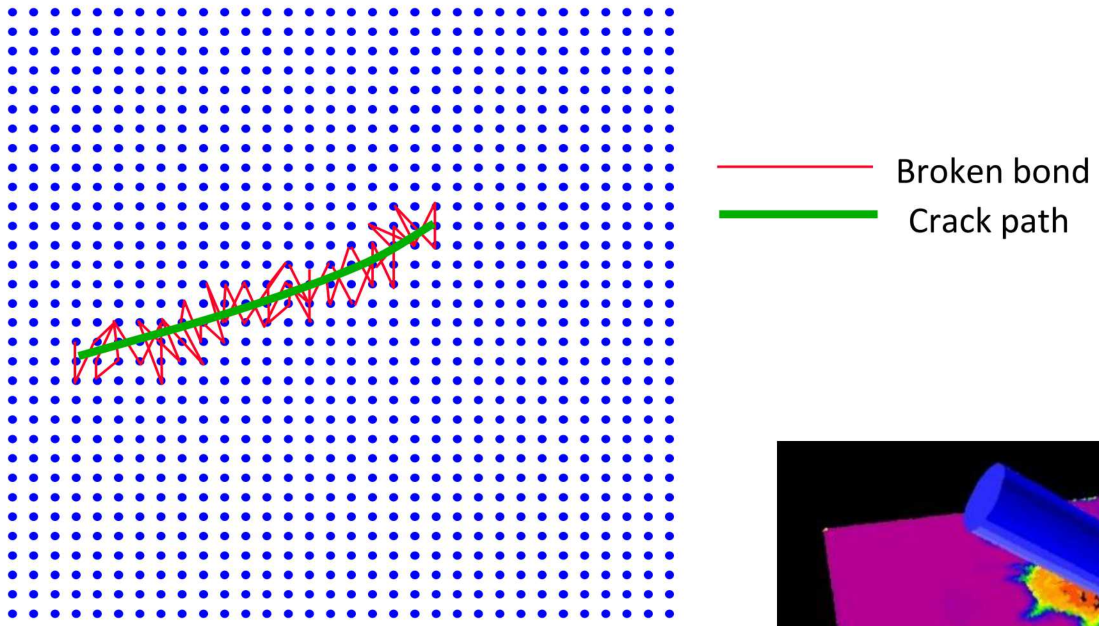
Bond damage criteria

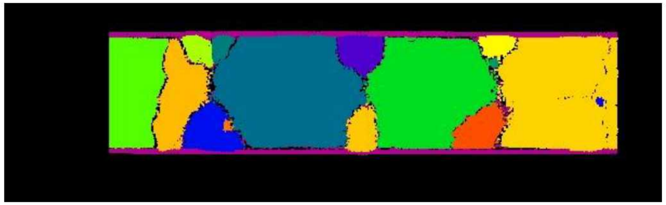
- Damage is usually treated by *bond breakage*.
- When some damage criterion is met, a bond no longer sustains tensile load. Examples:
- $|\mathbf{f}(\mathbf{q}, \mathbf{x})| > f_0$ where f_0 is a critical bond force density.
- $|\mathbf{u}(\mathbf{q}) - \mathbf{u}(\mathbf{x})| > e_0$ where e_0 is a critical bond extension.
- Bond failure due to cyclic extension (fatigue).
- Continuum damage mechanics (nonlocal version).
- Phase field? (conjecture)



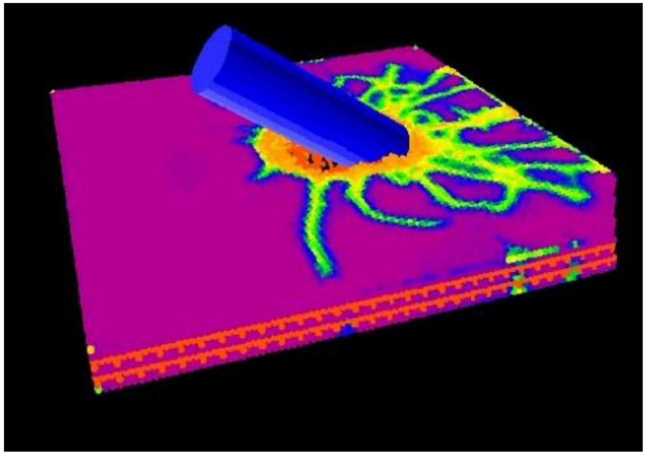
Discontinuities are treated within the basic field equations

- When a bond breaks, its load is shifted to its neighbors, leading to progressive failure.



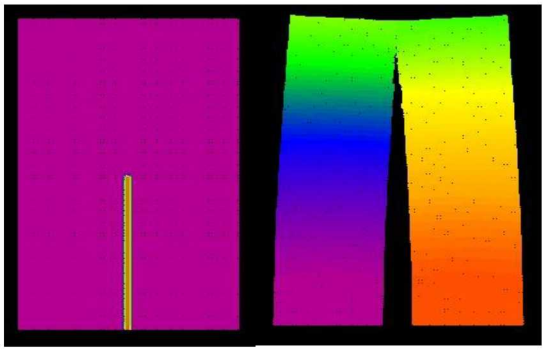


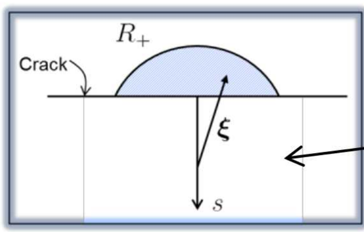
Cracking in a composite lamina



Impact against reinforced concrete

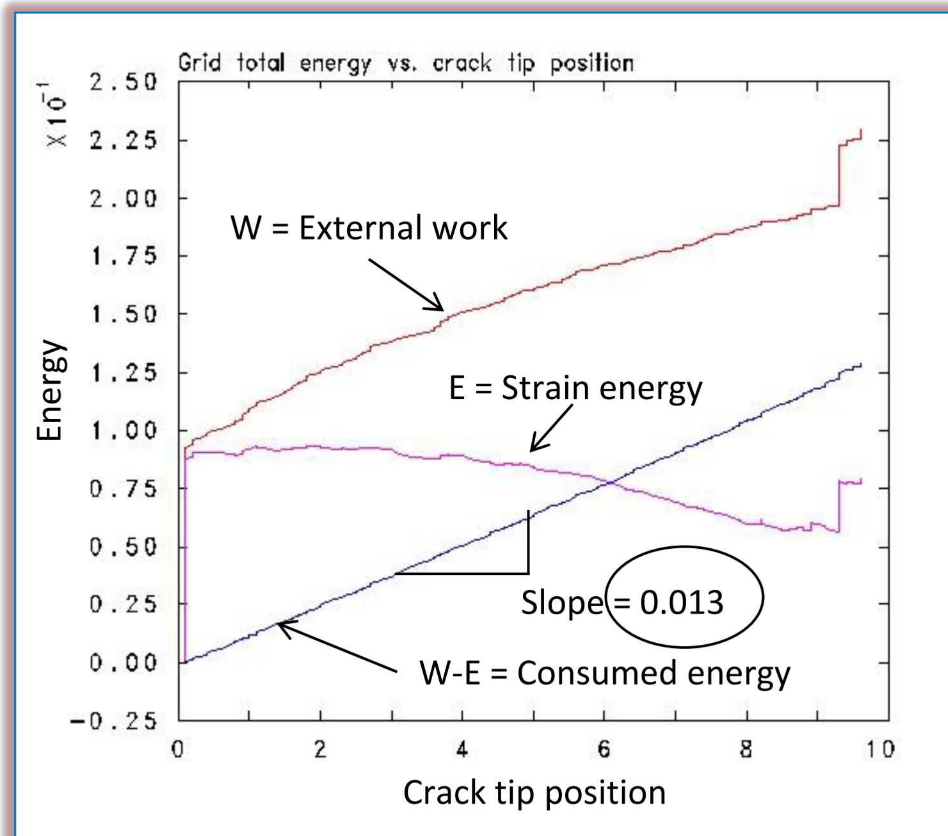
Verification of the energy release rate





From bond properties, energy release rate should be

G = 0.013



- This confirms that the energy consumed per unit crack growth area equals the expected value from bond breakage properties.

Cracks nucleate due to a material instability

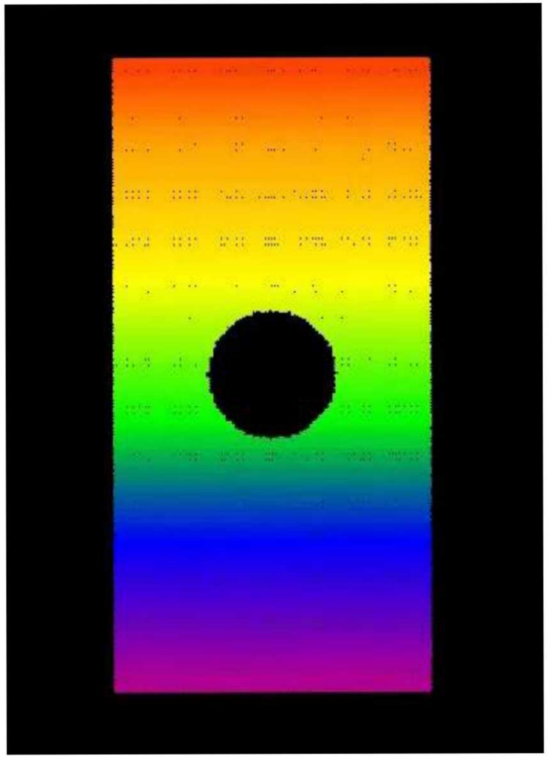
- Suppose the bond force density is given by

$$\mathbf{f}(\mathbf{q},\mathbf{x}) = \mathbf{F}(\boldsymbol{\eta},\boldsymbol{\xi})$$

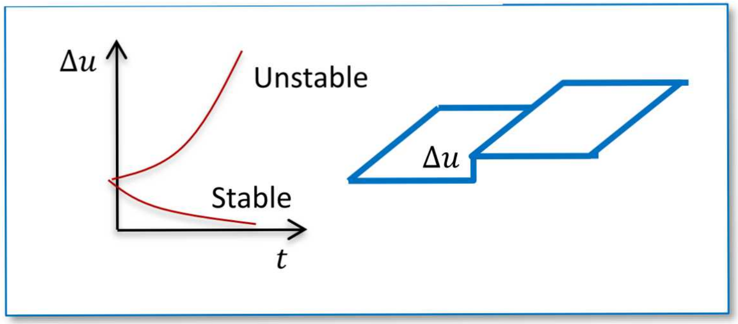
where $\boldsymbol{\xi} = \mathbf{q} - \mathbf{x}$, $\boldsymbol{\eta} = \mathbf{u}(\mathbf{q}) - \mathbf{u}(\mathbf{x})$.

- Condition for growth of a jump perturbation:

$$\det \int_{\mathcal{H}_{\mathbf{x}}} \frac{\partial \mathbf{F}}{\partial \boldsymbol{\eta}} \, dV_{\xi} < 0$$

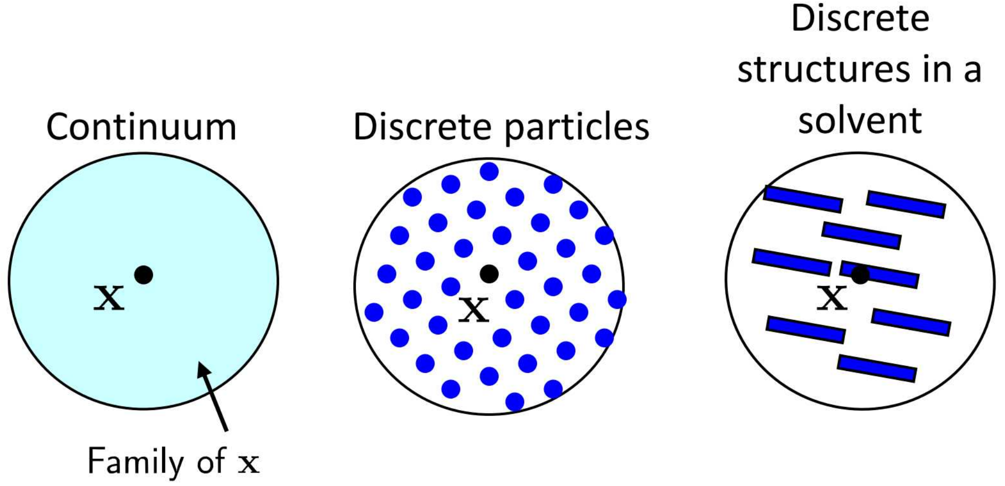


- Related to the convexity of the elastic energy density function for bonds.

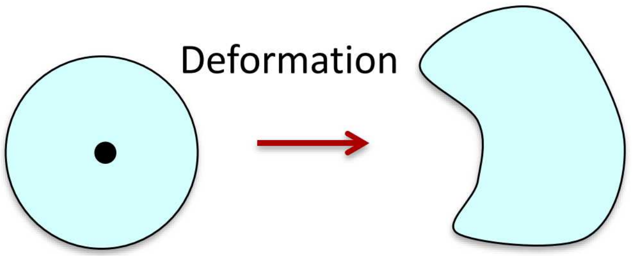


- SS, Weckner, Askari, & Bobaru, *Int. J. Fracture* (2010)
- Lipton, *J. Elast.* (2014)
- Lipton, *J. Elast.* (2015)

State-based concept of strain energy density at a point

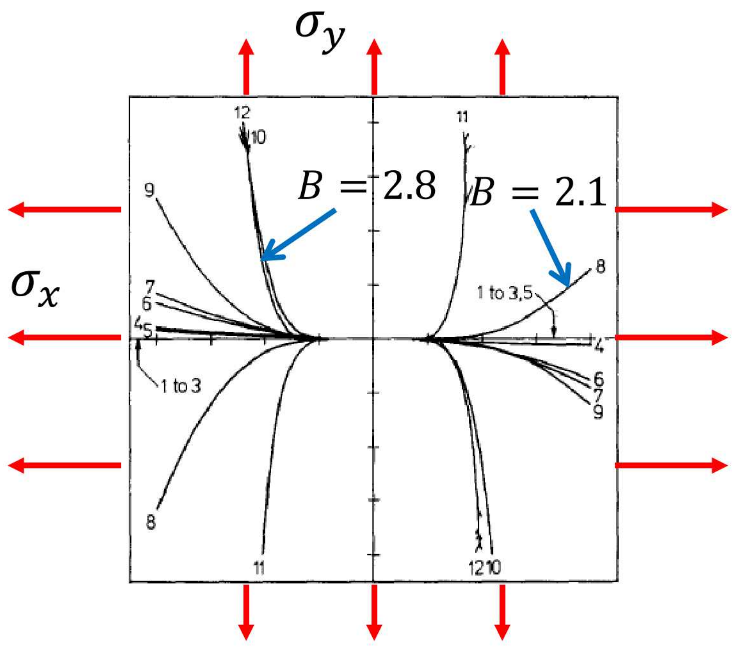


- The strain energy density $W(\mathbf{x})$ is determined by the deformation of the entire family of $\mathbf{x}$.
- How to describe this dependence? **States:** Nonlocal operators similar to second order tensors.

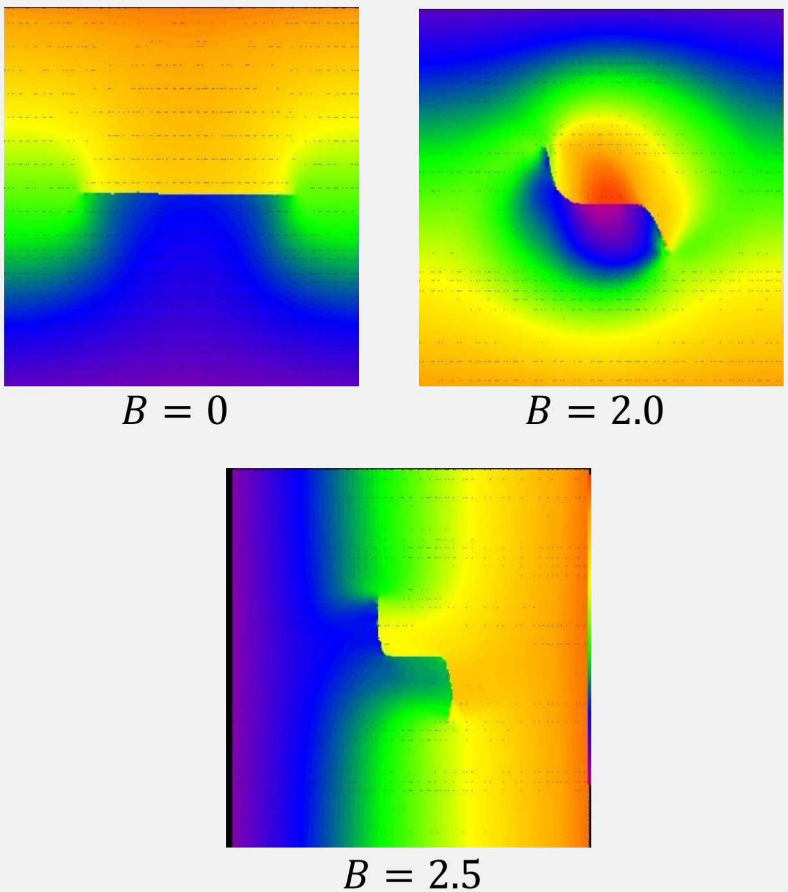


Crack stability and mode transition

- Biaxial loading makes a crack turn.
- Center defect can grow in an S-shape.
- Biaxiality: $B = \sigma_x / \sigma_y$.



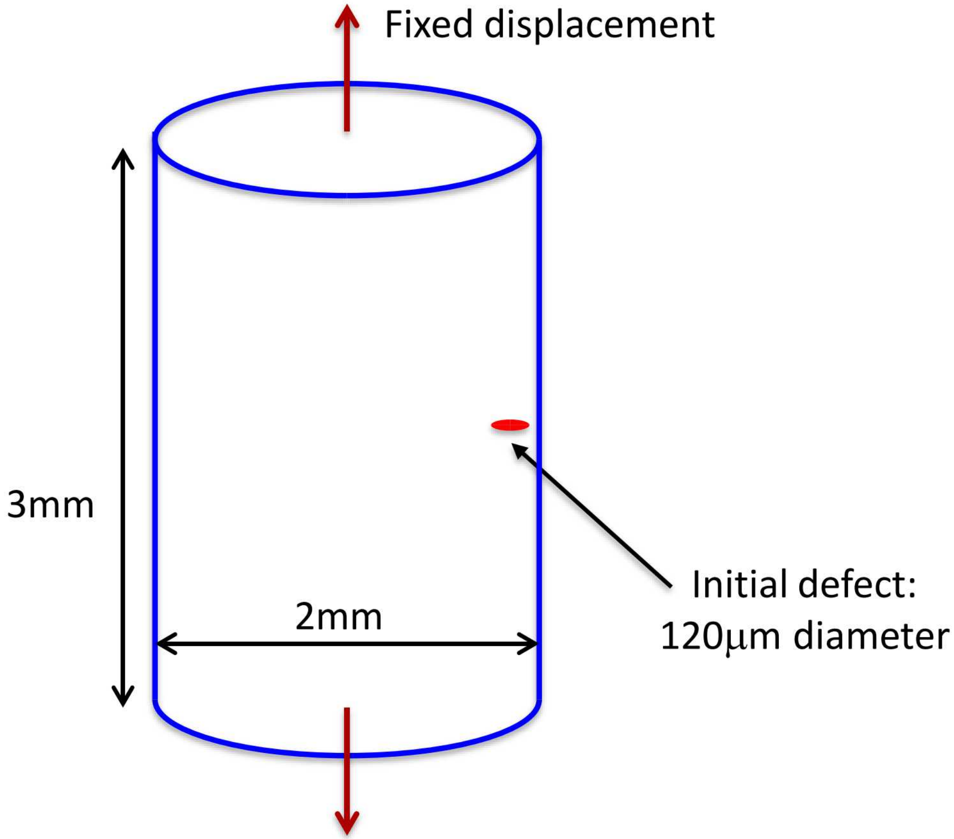
Observed crack paths in PMMA



Leevers, Radon, & Culver JMPS (1976)

Failure of a glass rod in tension

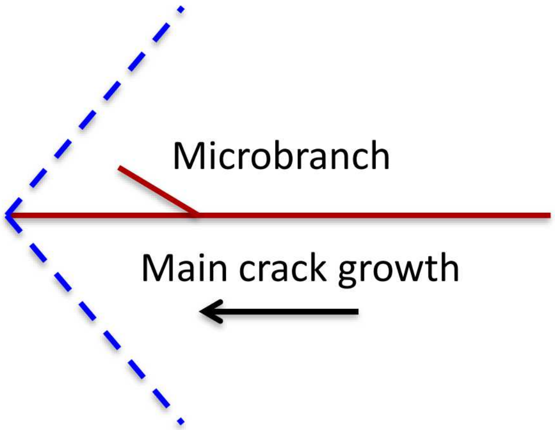
- A classical test problem for fractography.
- We will try to reproduce key fractographic features.
- Multiscale approach allows us to make the horizon << geometric length scales.



ρ = 3000 kg/m^3
E = 70.5 GPa
ν= 0.25
G_{Ic} = 7.0 J/m^2
δ = 25μm

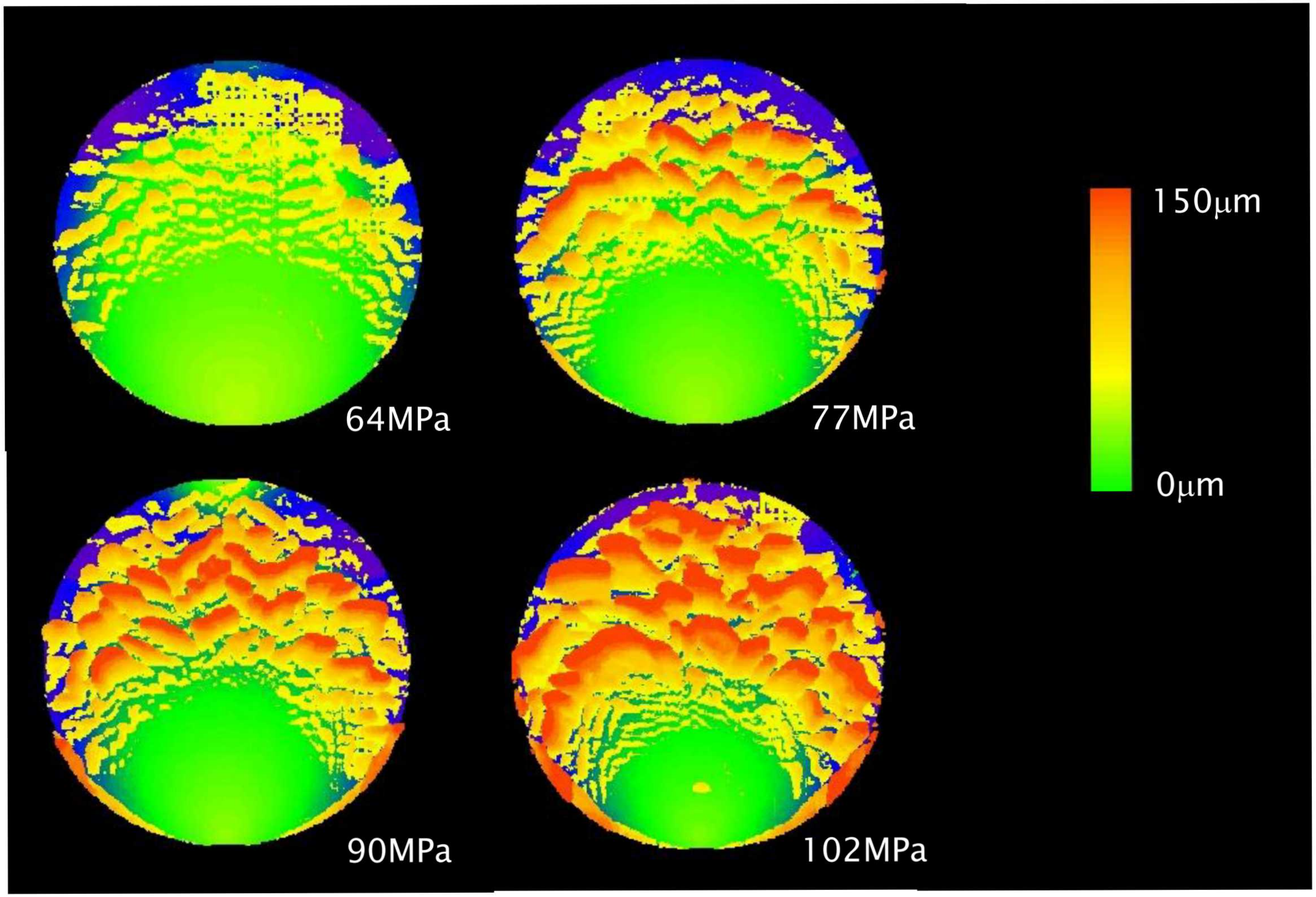
Failure of a glass rod in tension (movie)

Evolution of surface roughness (movie)



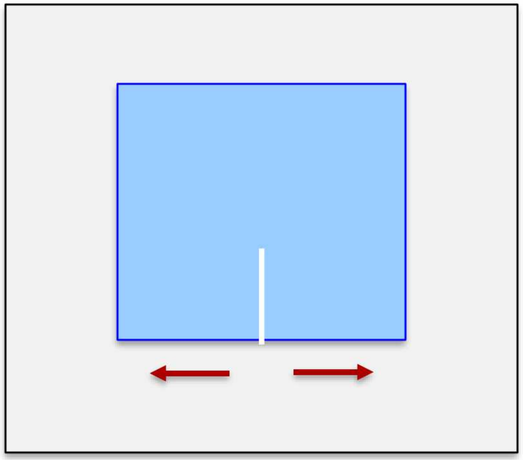
- Rough features branch off from the main crack.
- Each one grows slower than the main crack and eventually dies.

Crack surface for four values of initial stress: mirror-mist-hackle

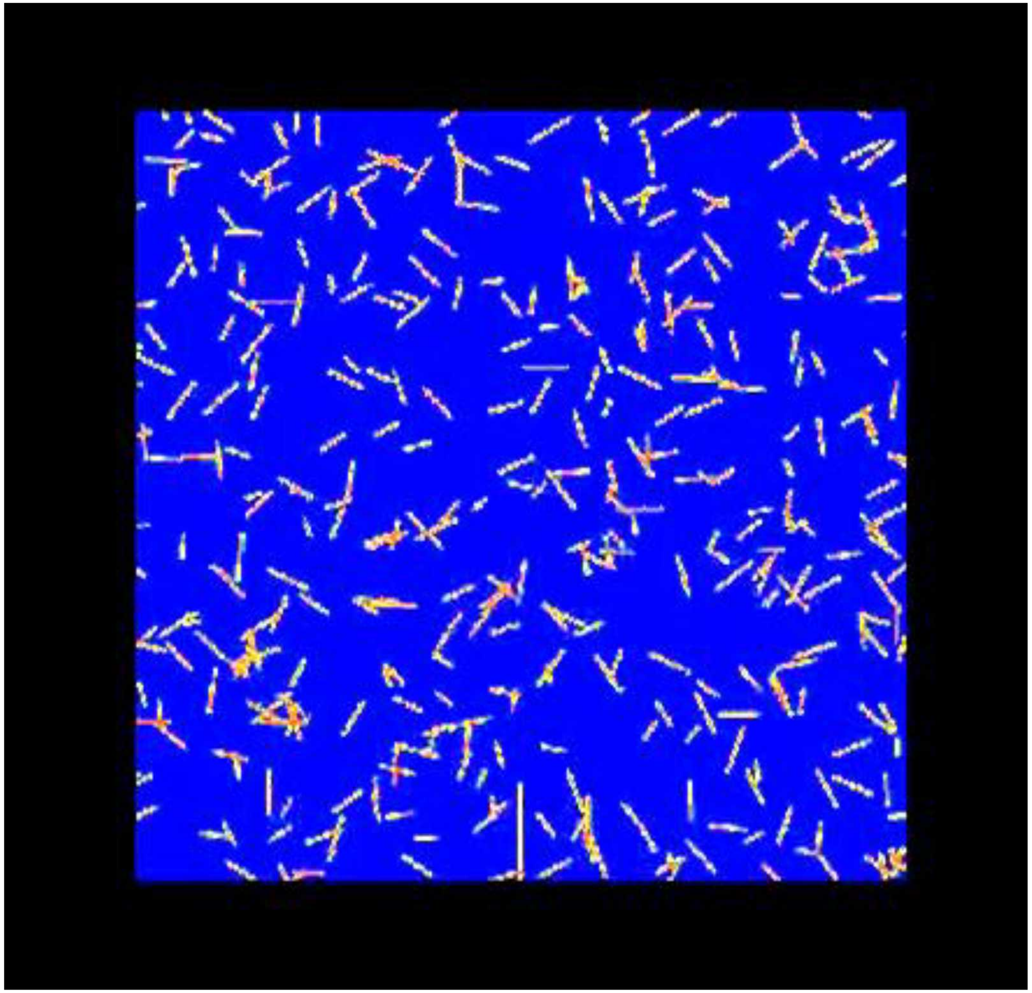


Colors show elevation of the fracture surface above the initial defect position.

Fracture in a brittle plate with a lot of defects

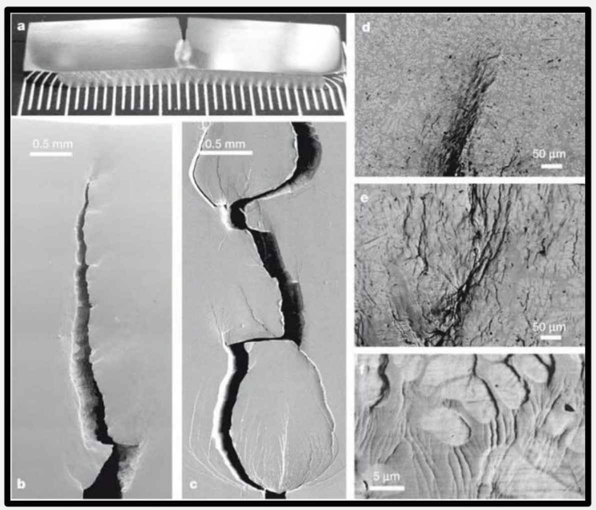


VIDEO

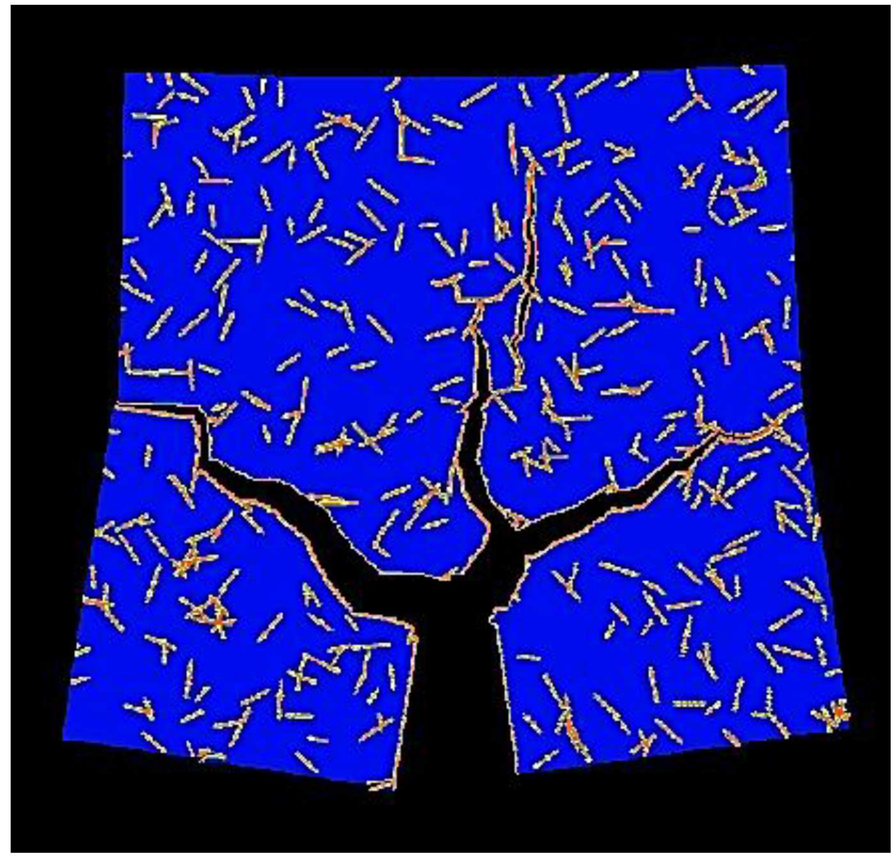


Fracture in a brittle plate with a lot of defects

- How do defects join up to form a macroscopic crack?



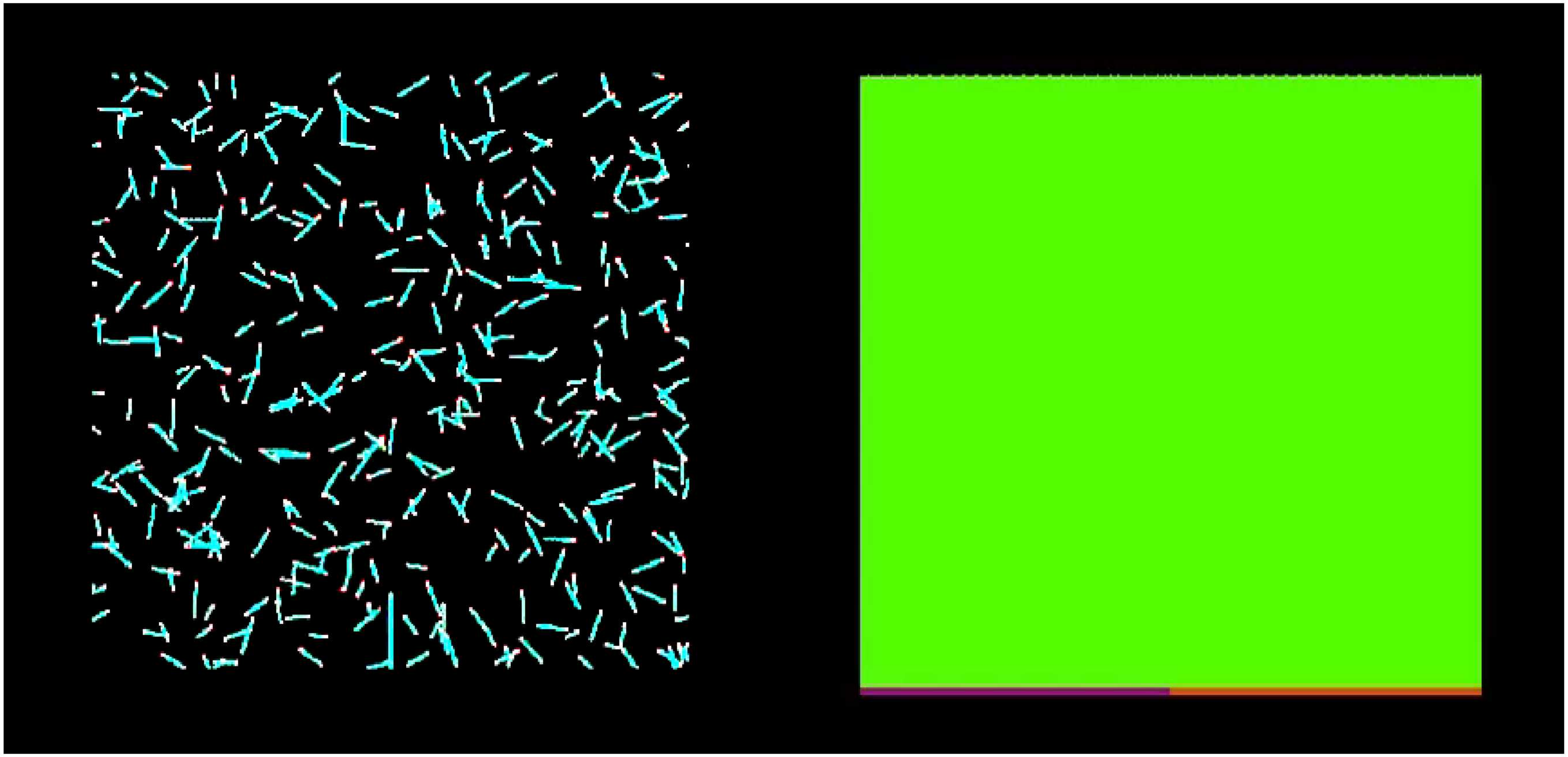
Metallic glass fracture (Hofmann et al, Nature 2008)



Fracture in an elastic-plastic plate with a lot of defects

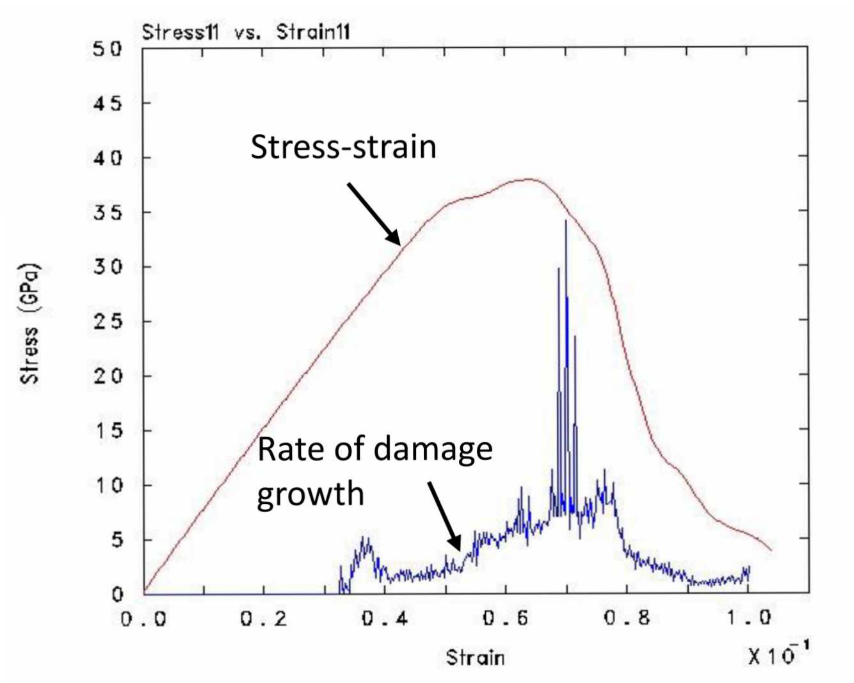
VIDEOS

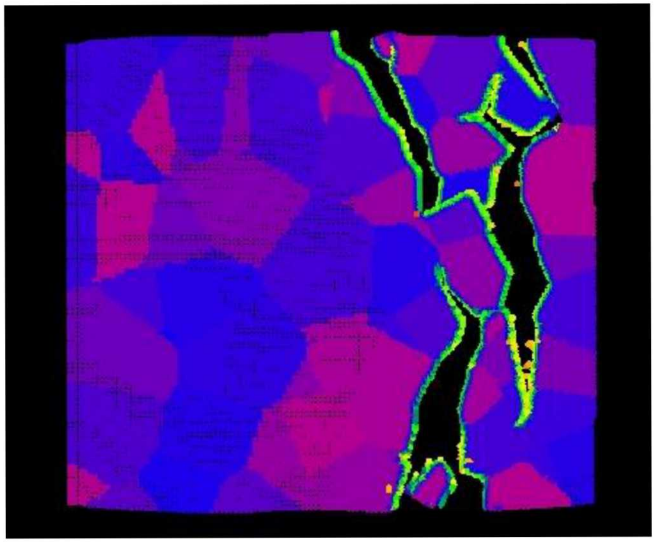
Defects　　　　　　　　　　　　Displacement



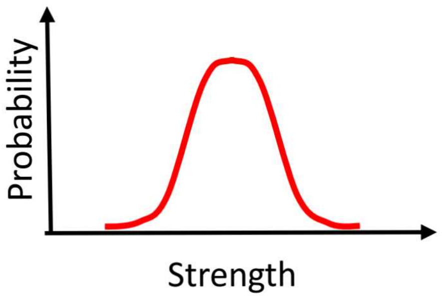
Failure of a graphene sheet

- Assign strength randomly to grain boundaries in each of many realizations.
- This one realization fails at some stress under uniaxial tension.
- Repeating with more realizations leads to statistical distribution of strength of the polycrystal.

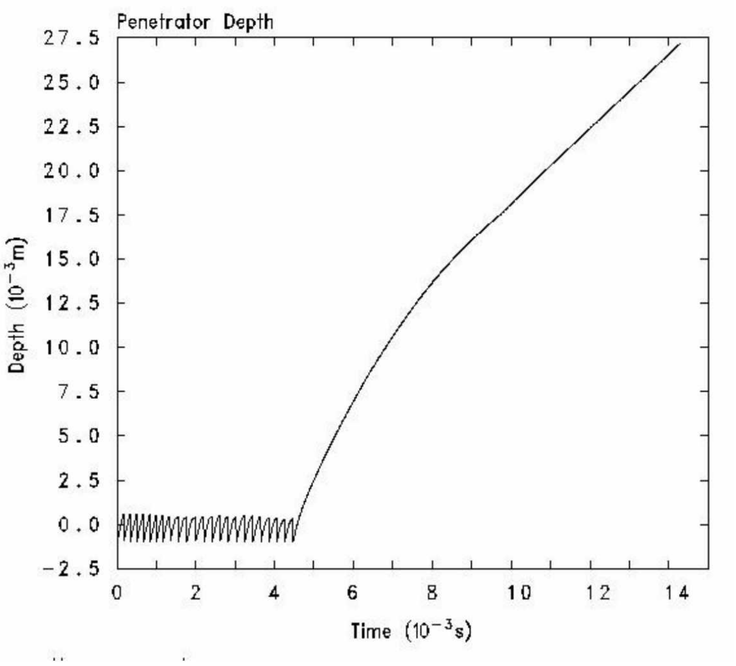




Cracks are mostly along grain boundaries



Accumulation of damage: Hammering on a block

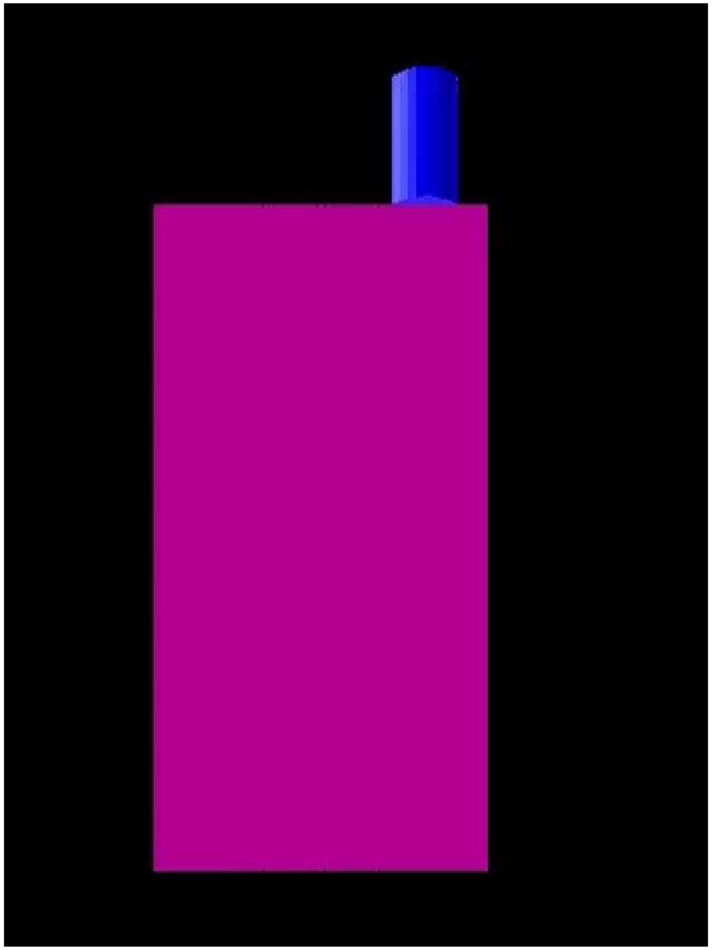


Penetration depth vs. time

Each peak is a strike.

Break through at 24 strikes.

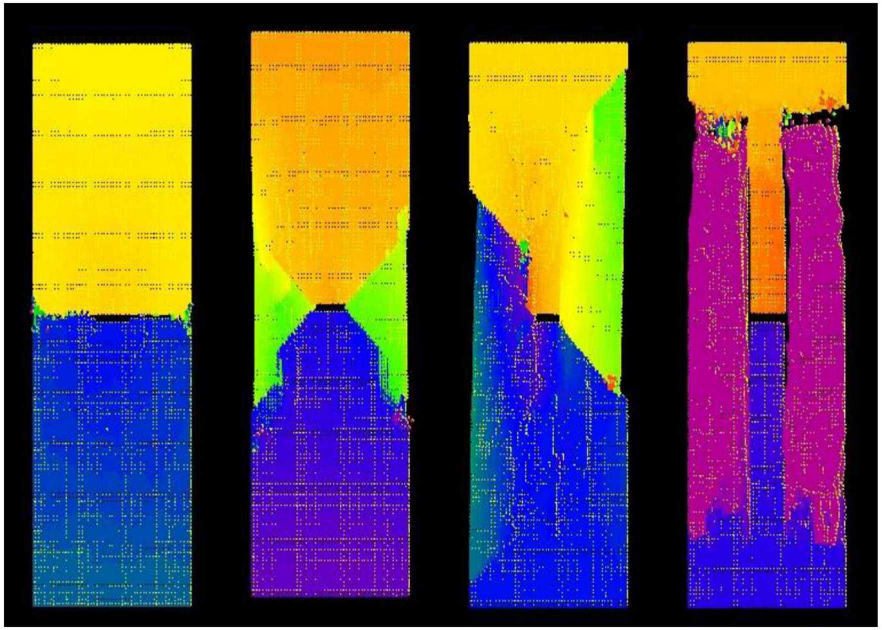
VIDEO



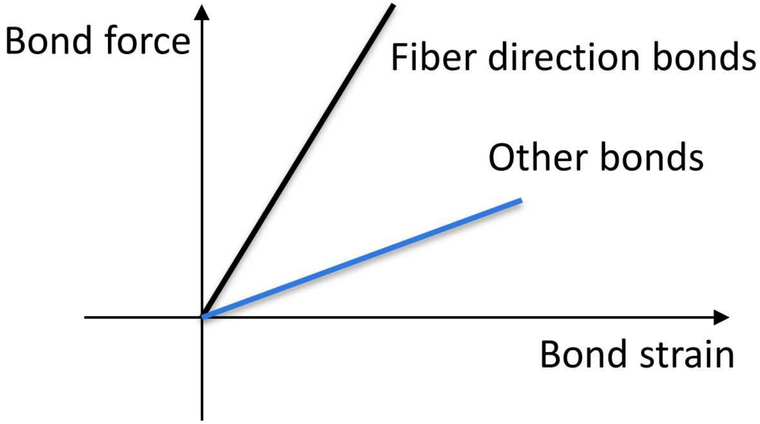
Colors show damage

Composites: Anisotropy in both elastic and damage response

- Bond response depends on bond direction.



Stretching of a panel with a center crack:
Crack paths depend on stacking sequence



Summary

- By treating discontinuous and continuous deformation within the same field equations we gain a lot in modeling some aspects of materials science.
  - Autonomous nucleation and growth of defects.
  - Phase boundaries evolve according to driving force.
  - We avoid the need for supplemental equations that govern defect evolution.

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