SAND2019-2842C

New Results in the Fractional Quantum Hall Effect

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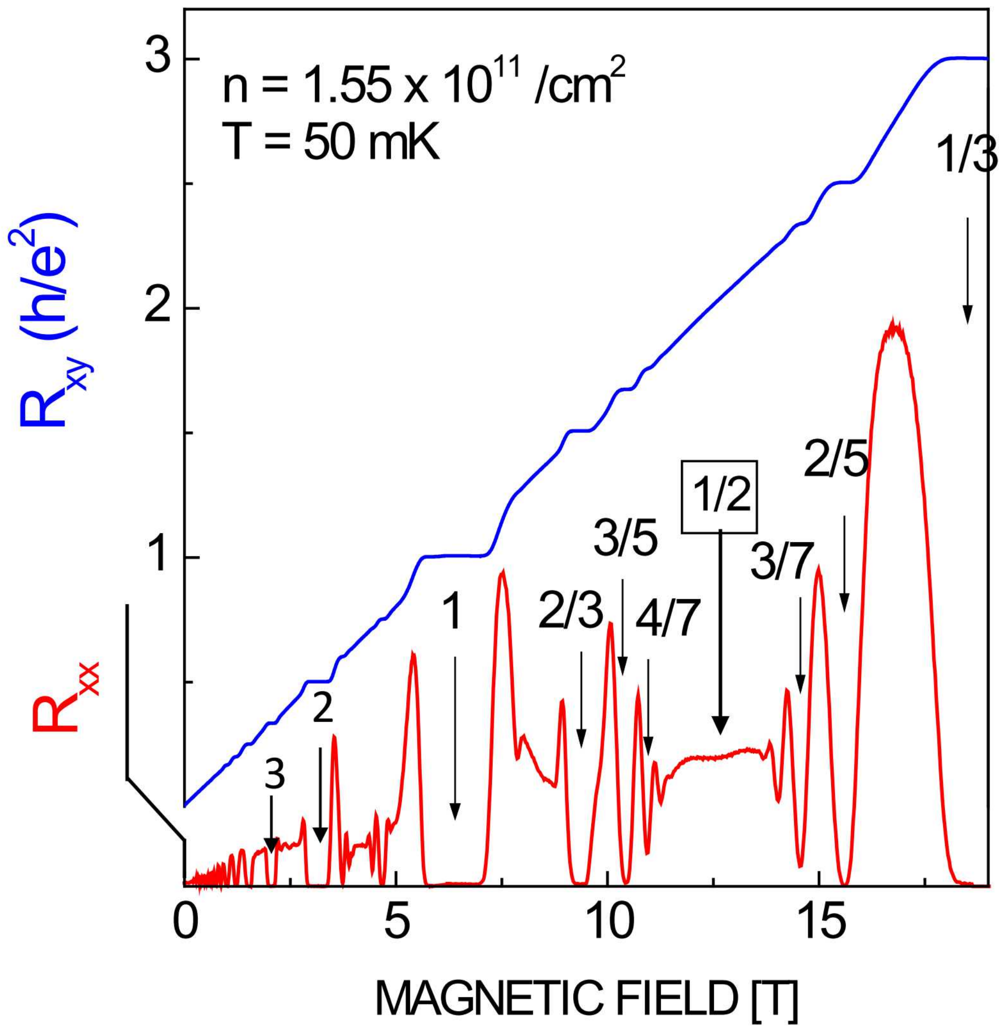
outline

- Quantum Hall effect and composite fermions (CFs)
- Berry phase of CFs
- PH symmetry of the fractional quantum Hall effect

Quantum Hall Effects



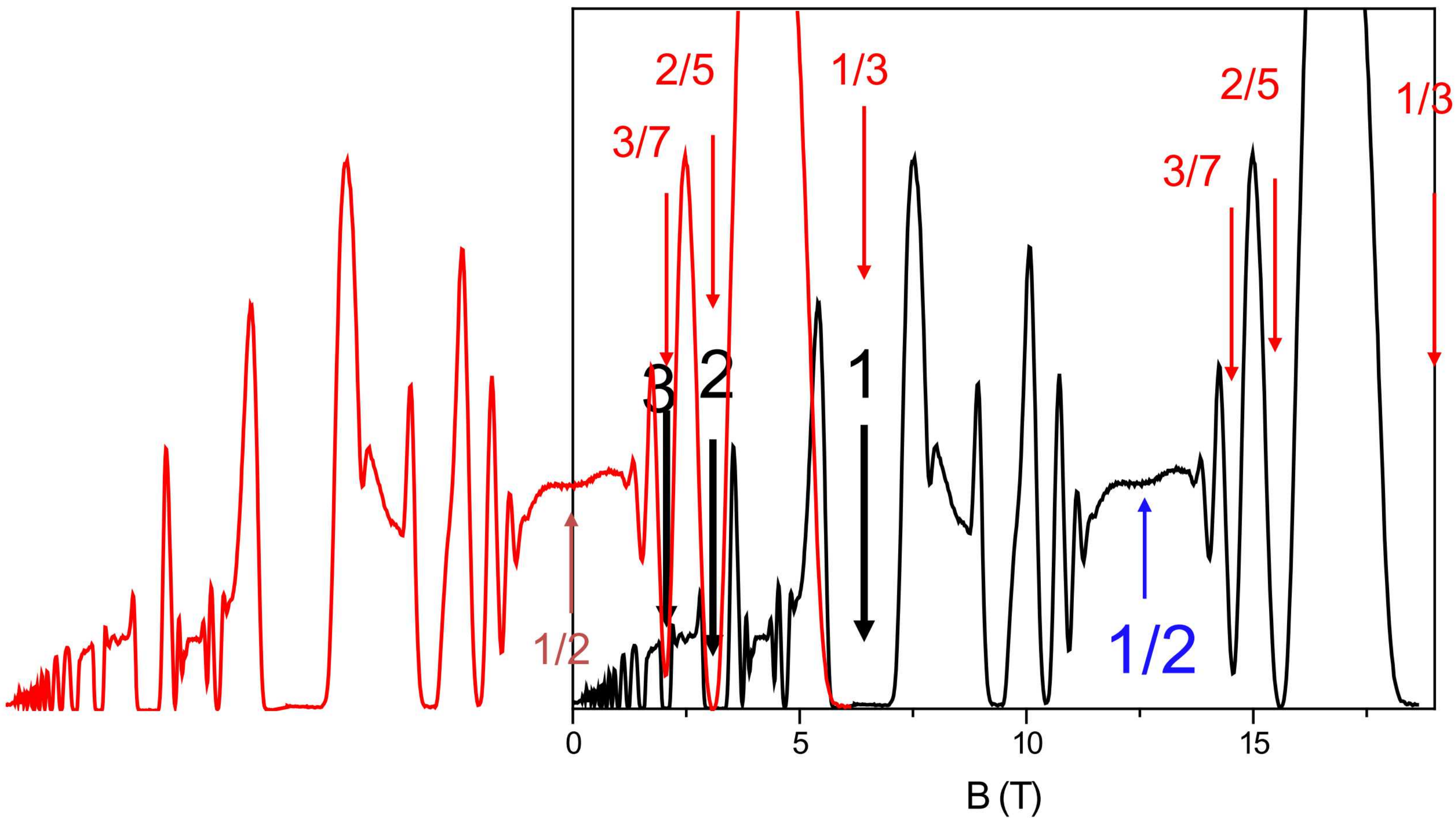
R_{xy} quantized

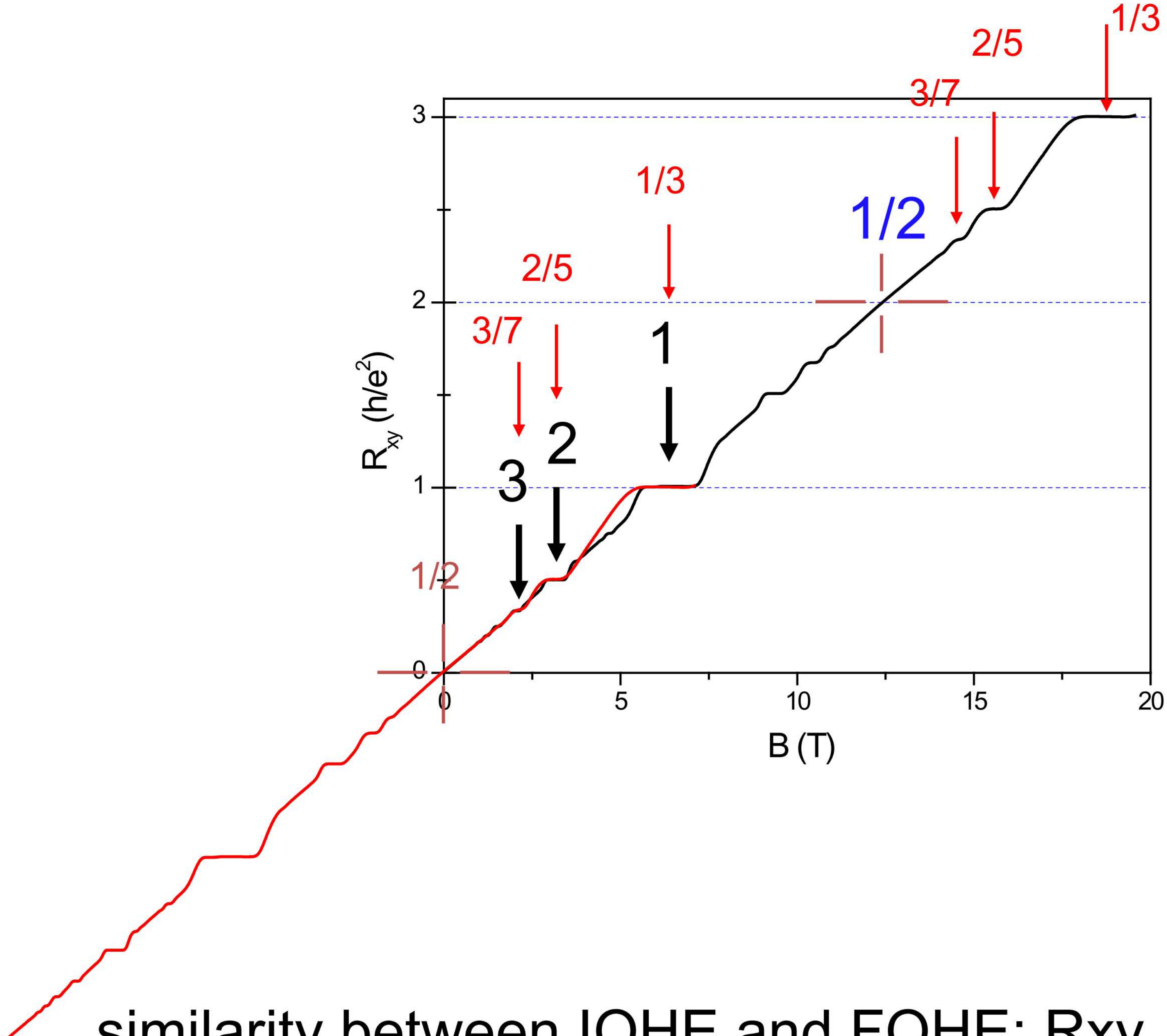
$$R_{xy} = \frac{h}{\nu\, e^2}$$

$\nu =$ 1, 2, 3, ...
1/3, 2/5, 3/7 ...
2/3, 3/5, 4/7 ...

R_{xx} vanishingly small

similarity between IQHE and FQHE: Rxx



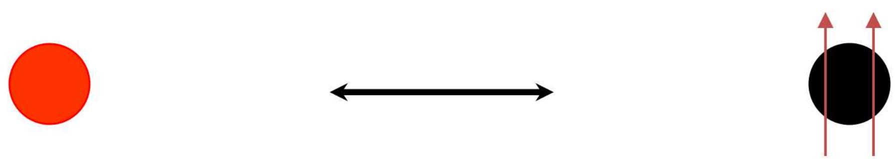


similarity between IQHE and FQHE: Rxy

Halperin-Lee-Read (HLR) Composite Fermion (CF) Model

J.K. Jain, PRL 1989
HLR, PRB, 1993

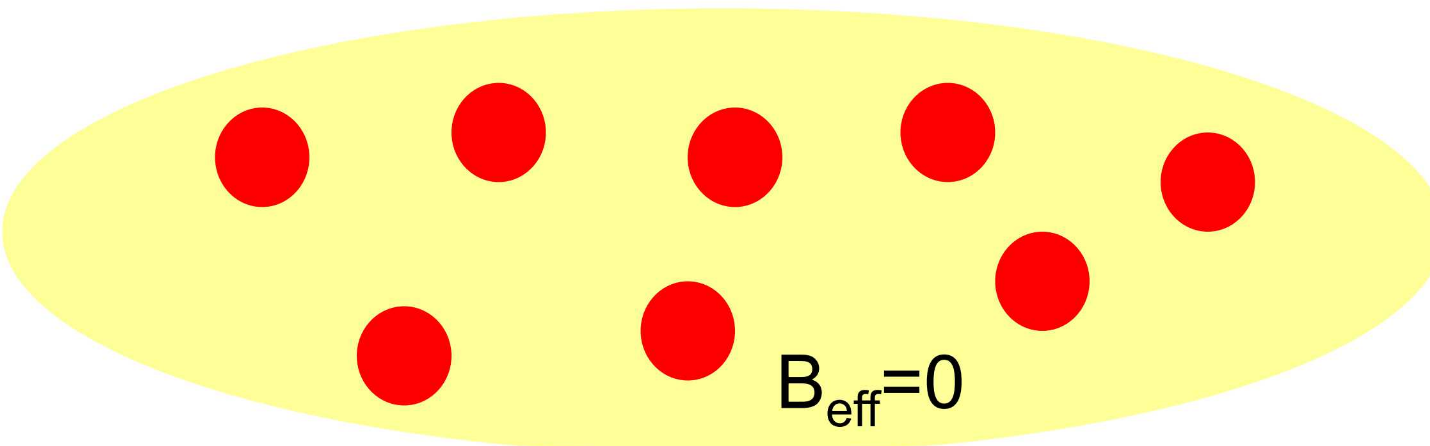
one composite fermion = one electron + **2** flux quanta



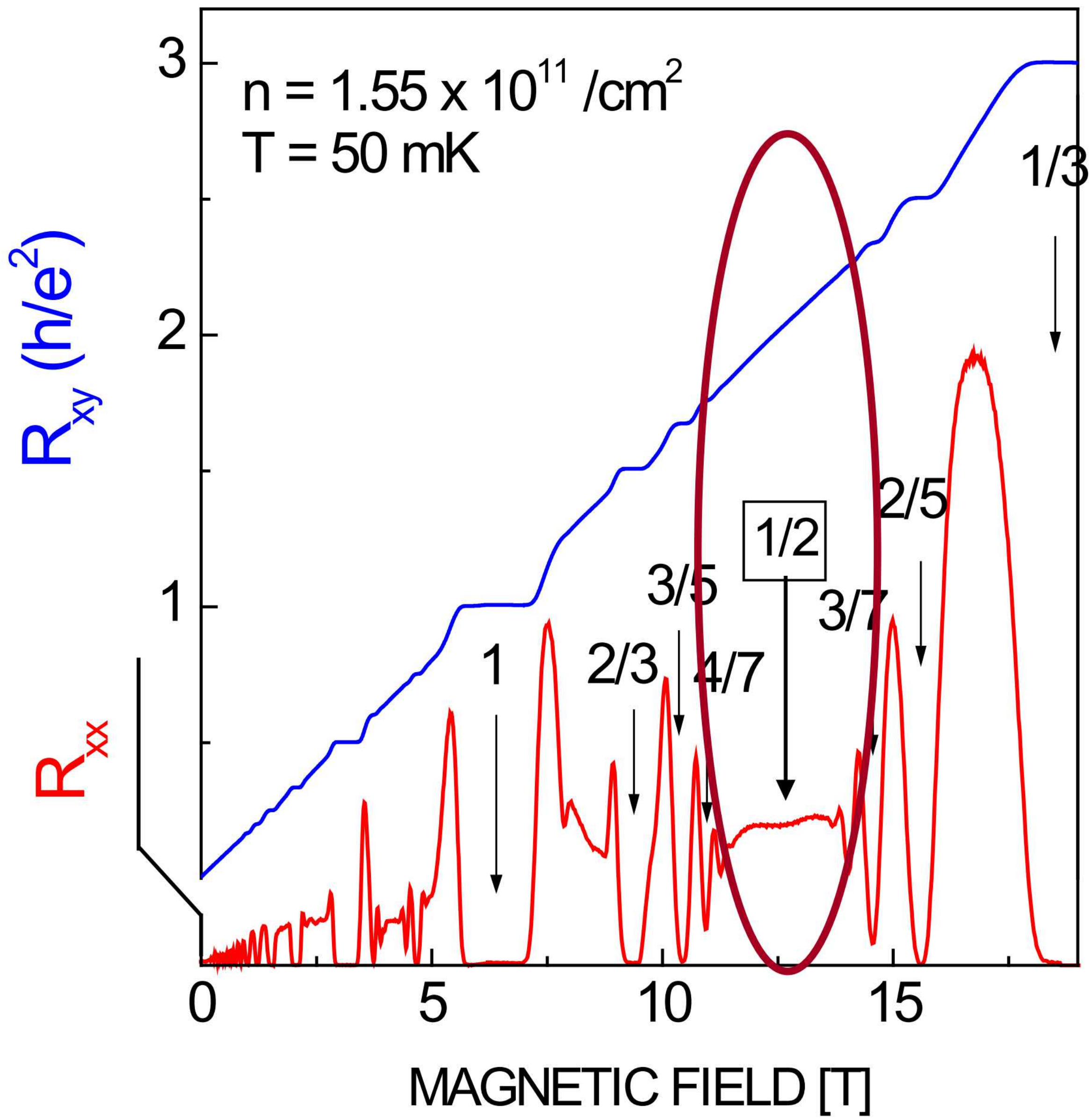
$$\mathbf{B_{eff} = B - 2\Phi_0 \times n = B - 2nh/e = B - B_{\nu=1/2}}$$

At $\nu = 1/2$

- $B_{eff} = B_{\nu=1/2} - B_{\nu=1/2} = 0$
- CF Fermi sea



Featureless transport around ν=1/2 – Fermi sea state

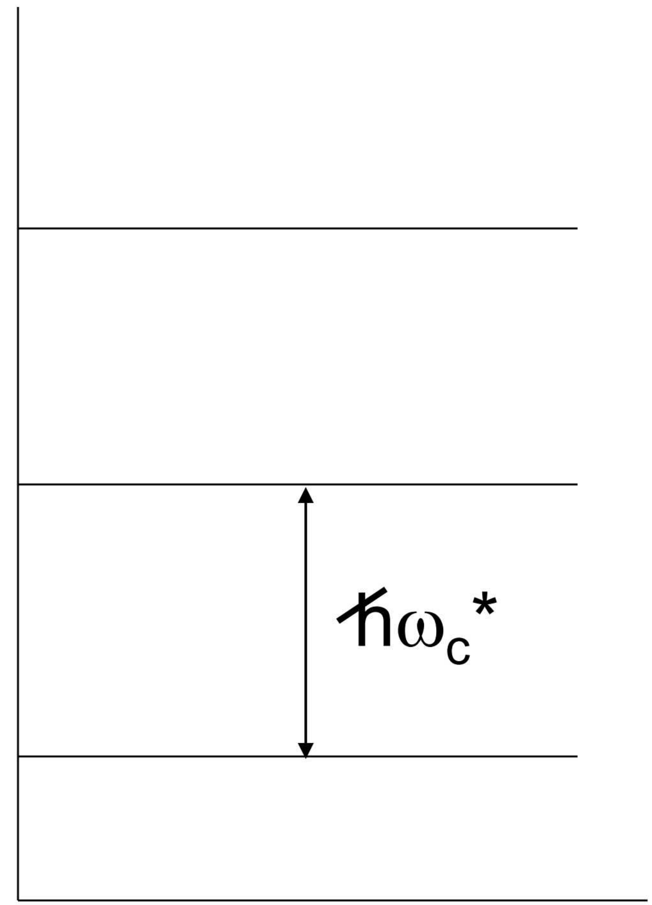


When $\nu \neq 1/2$ and $B_{eff} \neq 0$

- Quantized in Landau levels
- IQHE of CFs

IQHE of CF	FQHE of e^-
p	$\nu = p/(2p+1)$
1	1/3
2	2/5
3	3/7

….



IQHE of CF FQHE of e^-

p	$\nu = p/(2p+1)$
1	1/3
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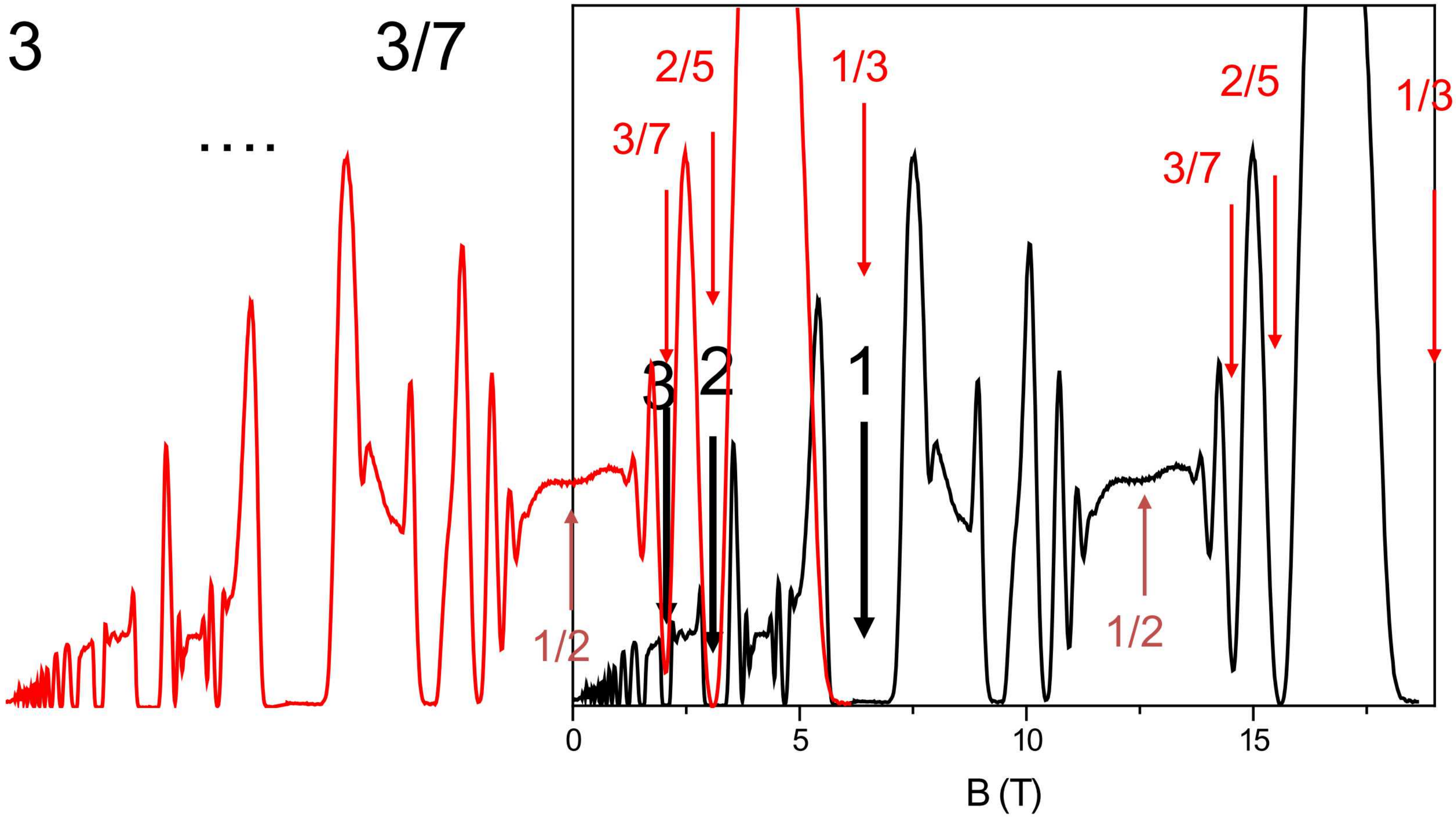
....	



Unclarified aspects of CF theory

apparent lack of particle-hole symmetry

Kivelson et al PRB (1997)
Lee, PRL (1998)
Kamburov et al. PRL (2014)

Under the CF model:

1/3 is the ν=1 IQH state of CFs
2/3 is the ν=2 IQH state of CFs

1/3 and 2/3 states have different topological order!

PHYSICAL REVIEW B VOLUME 29, NUMBER 10 15 MAY 1984

Particle-hole symmetry in the anomalous quantum Hall effect

S. M. Girvin
Surface Science Division, National Bureau of Standards, Washington, D.C. 20234
(Received 27 February 1984)

This paper explores the uses of particle-hole symmetry in the study of the anomalous quantum Hall effect. A rigorous algorithm is presented for generating the particle-hole dual of any state. This is used to derive Laughlin's quasihole state from first principles and to show that this state is exact in the limit $\nu \rightarrow 1$, where ν is the Landau-level filling factor. It is also rigorously demonstrated that the creation of m quasiholes in Laughlin's state with $\nu = 1/m$ is precisely equivalent to creation of one true hole. The charge-conjugation procedure is also generalized to obtain an algorithm for the generation of a hierarchy of states of arbitrary rational filling factors.

state ψ_m defined in (5). Hence this state has the filling factor $\nu = 1 - 1/m$ and (for $N \rightarrow \infty$) is the *exact* particle-hole dual of the state with $\nu = 1/m$. For large but finite M it is

1/3 and 2/3 are the same states

Particle-hole symmetric Dirac theory of composite fermions

Son, *Phys. Rev. X* **5**, 031027 (2015);

Metlitski and Vishwanath, arXiv:1505.05142; Wang and Senthil, arxiv:1507.08290; Geraedts et al, arXiv:1508.04140; …

Composite fermions meet Dirac

Journal Club for Condensed Matter Physics

I. *Is the composite fermion a Dirac particle?*
Dam Thanh Son, Phys. Rev. X **5**, 031027 (2015); arXiv:1502.03446

CFs are treated as Dirac particles

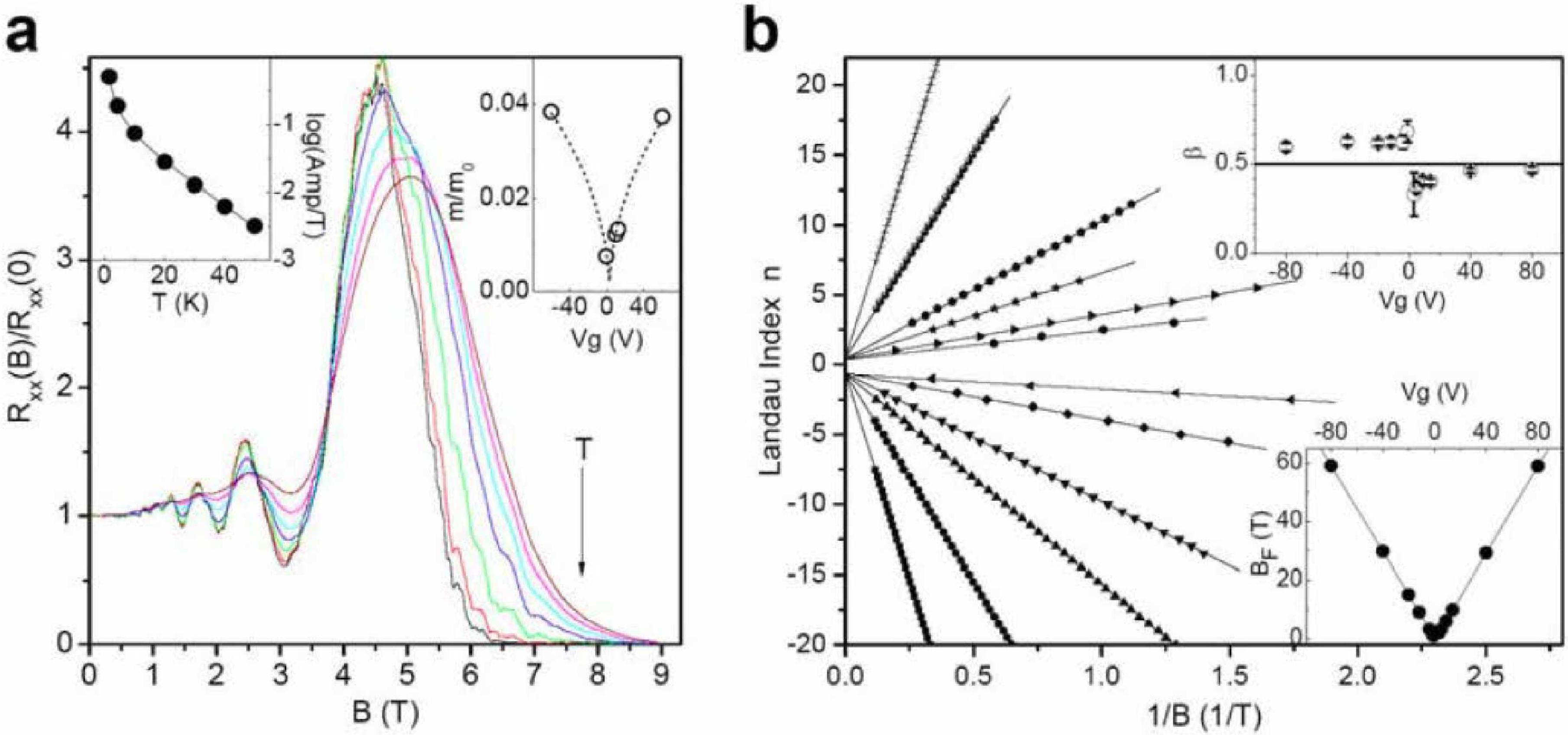
CF filling factor for the 1/3 state is $\nu^* = 3/2$
CF filling factor for the 2/3 state is $\nu^* = -3/2$

Recommended with a commentary by Jason Alicea, Caltech

At first glance, 3D topological insulator surfaces and the half-filled Landau level of a 2D electron gas appear to realize wholly disparate phenomena. Topological insulators typically arise in weakly correlated, time-reversal-invariant systems and support a single Dirac cone at their boundary. The

π Berry phase of composite fermions

Berry’s phase in graphene via SdH oscillations



Y.B. Zhang et al, Nature (2005)

Density of CFs

Son, PRX 5, 031027 (2015);
Wang and Senthil, *PRB* 93, 085110 (2016).

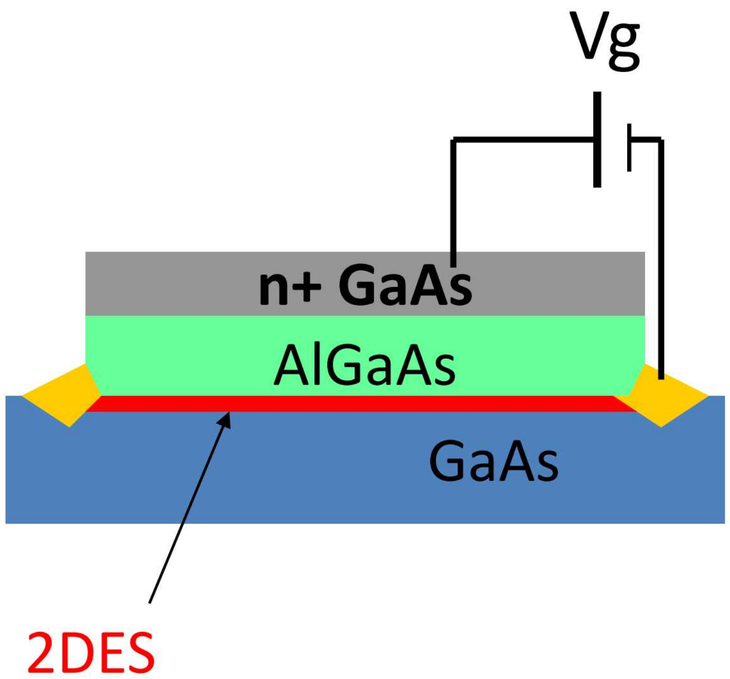
- CF density is determined by magnetic field
- CF density = eB/2h

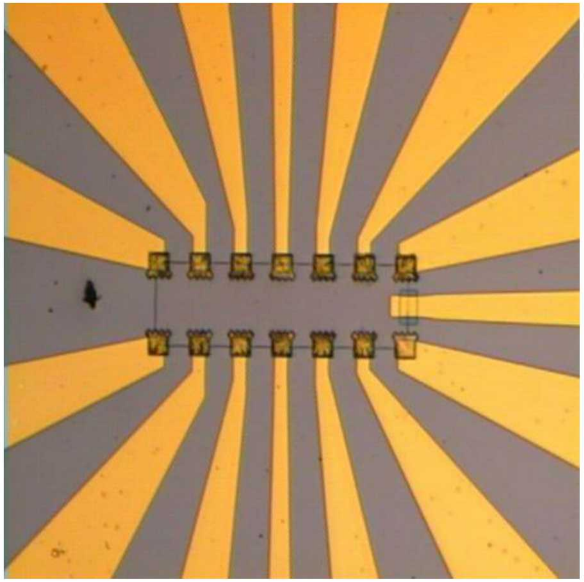
In order to reveal the π Berry phase, one needs to measure SdH oscillations as a function of electron density at a fixed magnetic field.

Heterojunction Insulated-Gate Field-Effect Transistor (HIGFET)

HIGFET

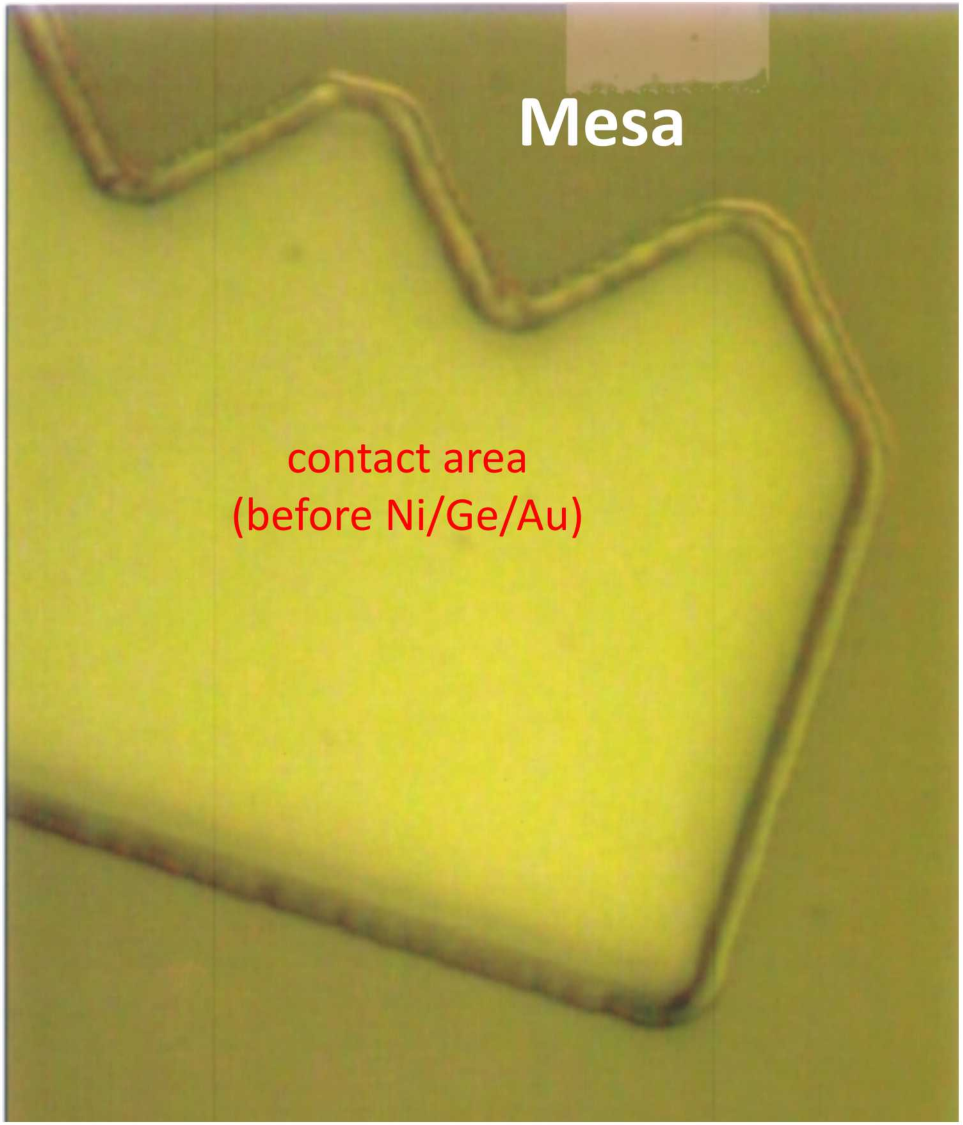
high mobility down to very low densities

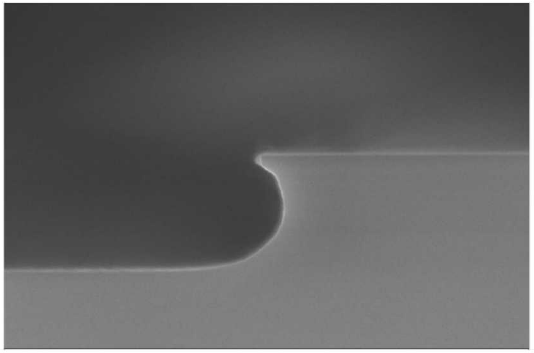


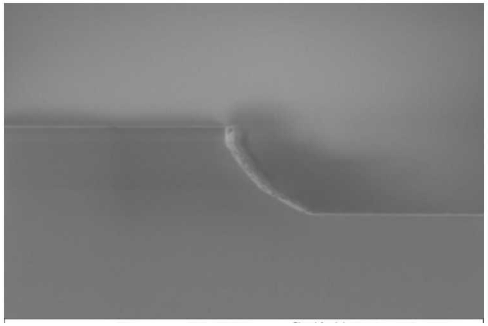


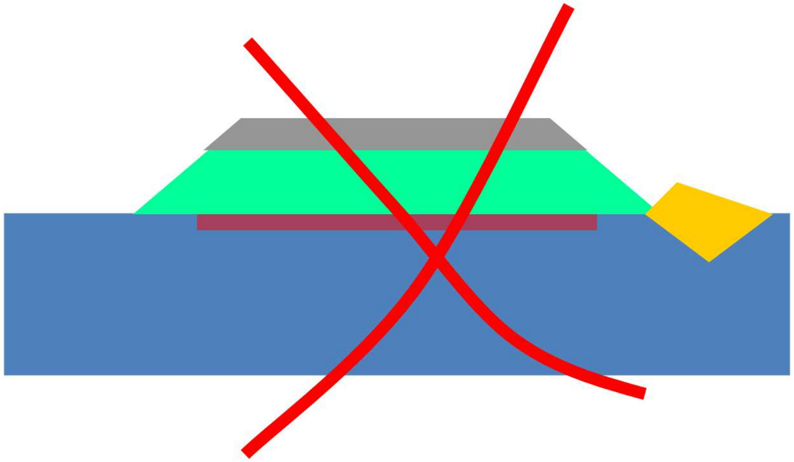
Kane, Pfeiffer, West, and Harnett, APL,1993

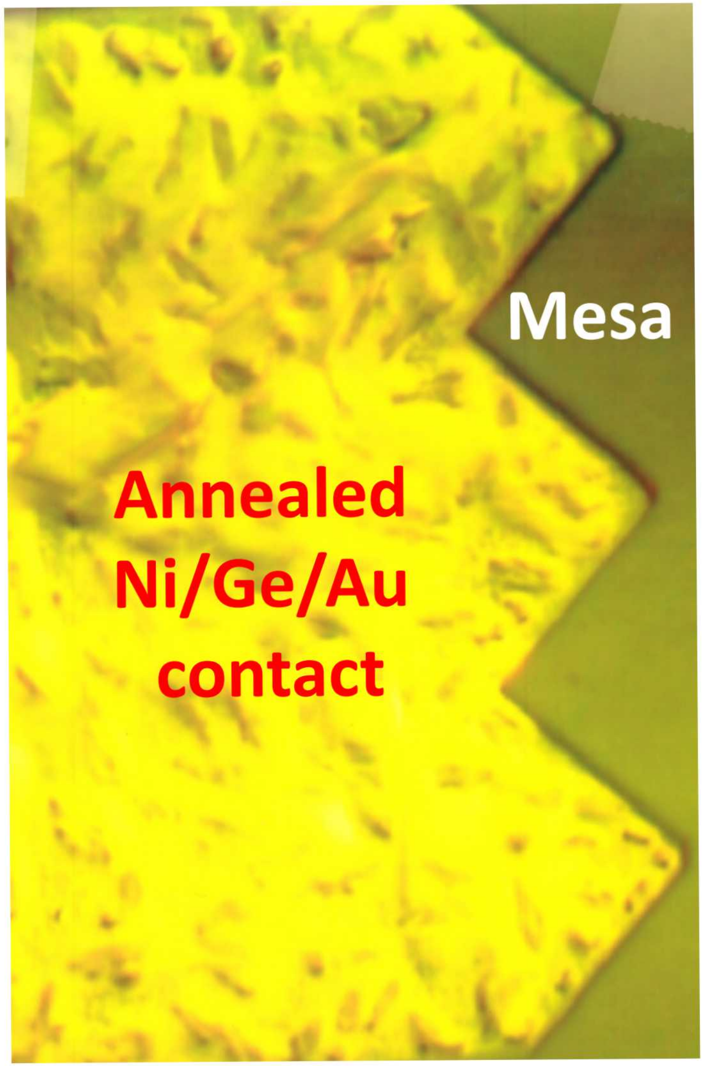
Straight sidewall is important

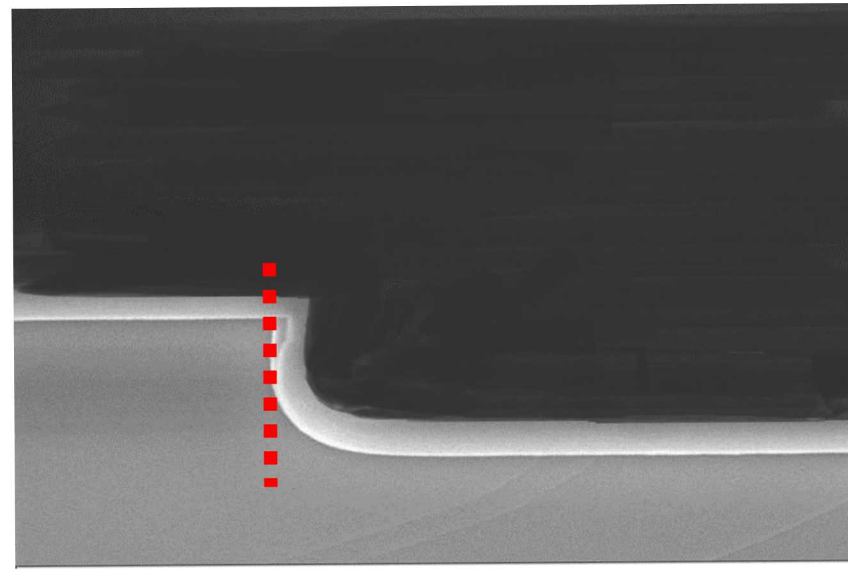




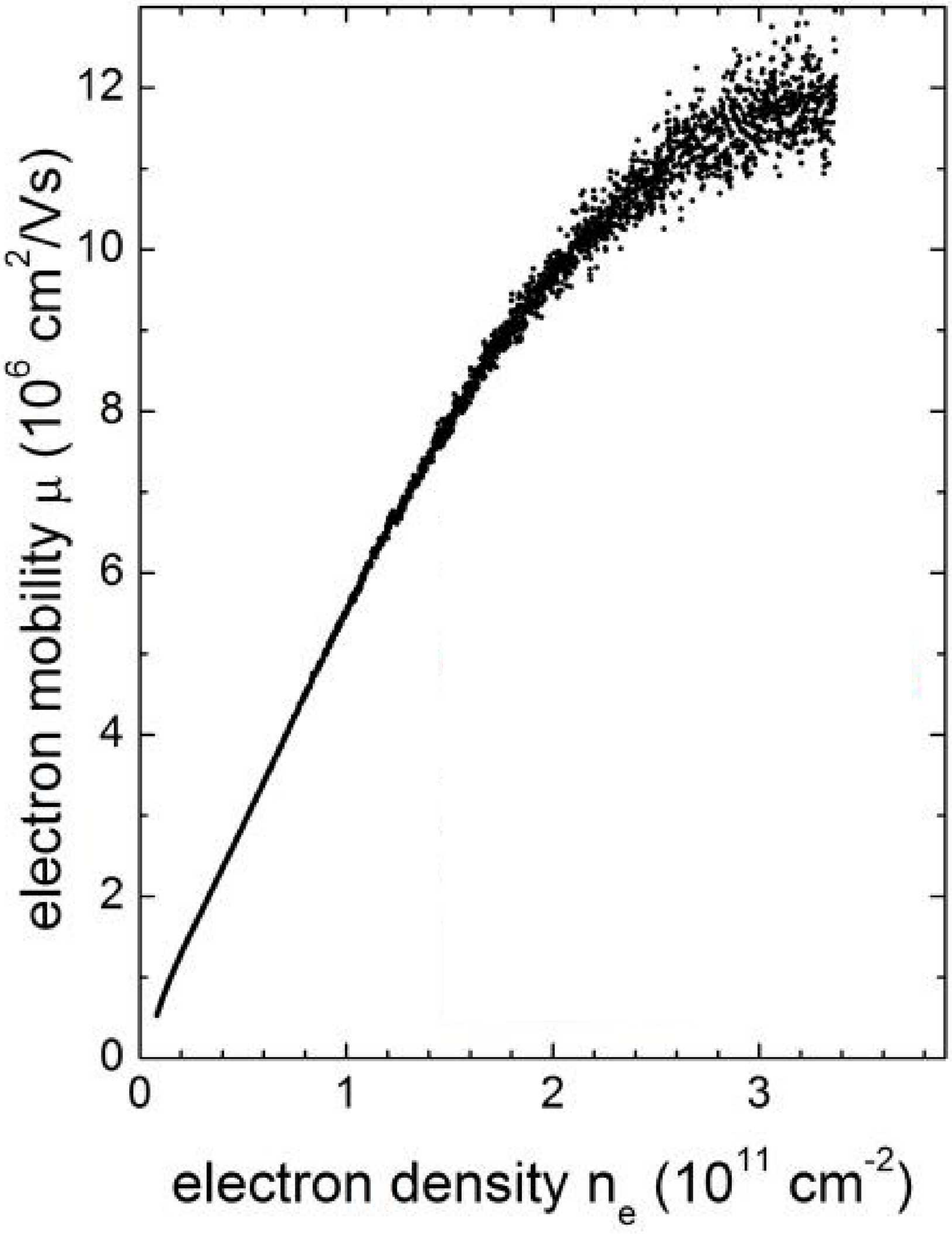








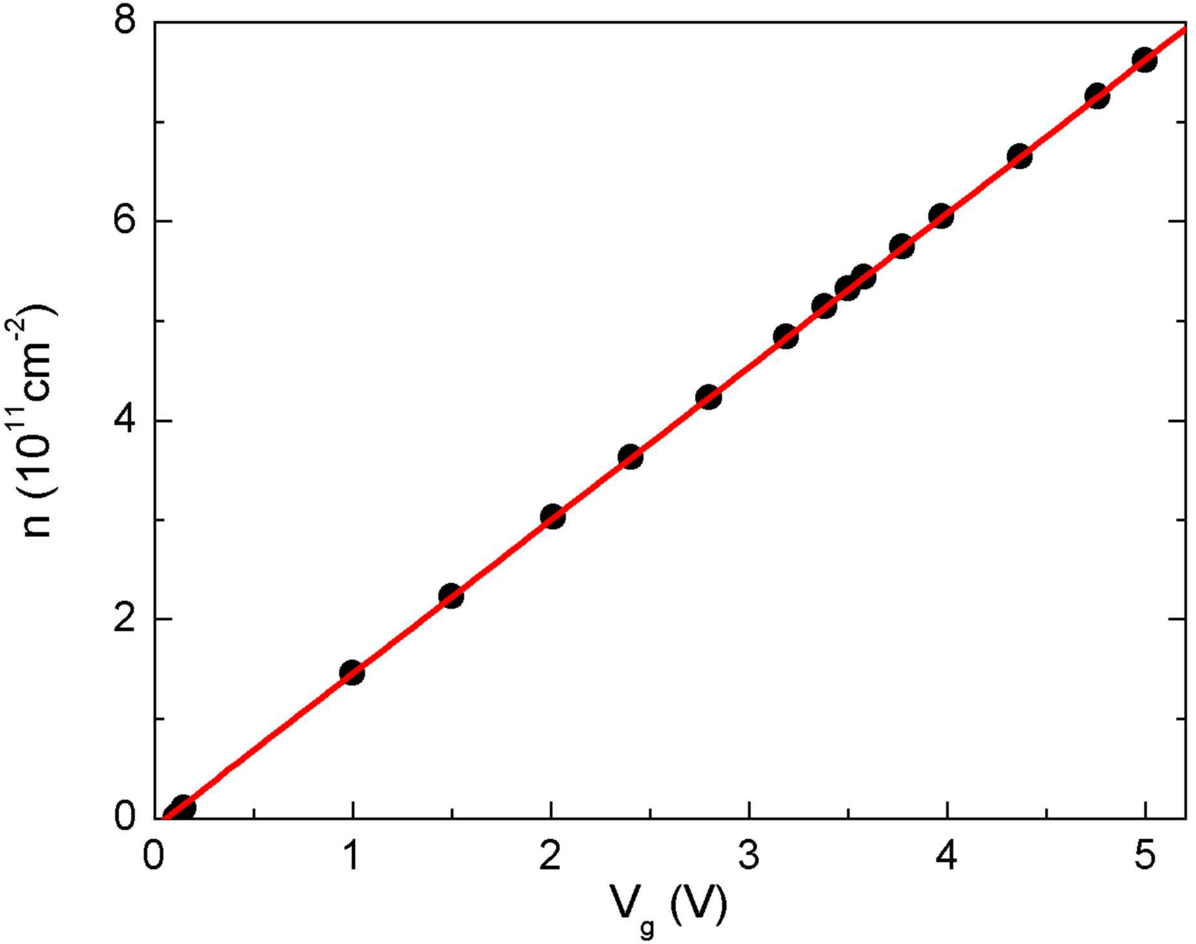
device works!



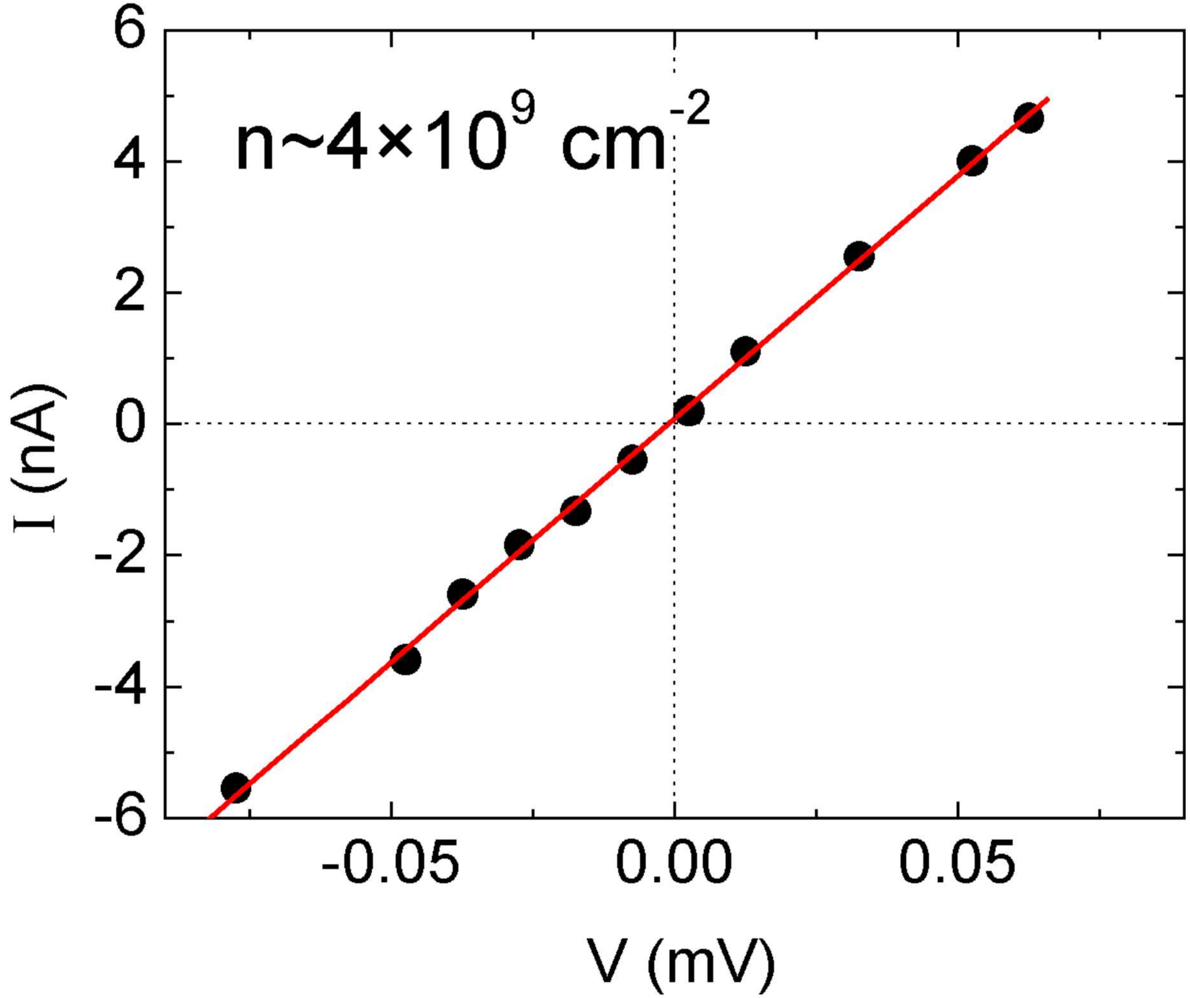
electron mobility μ (10^6 cm^2/Vs)
electron density n_e (10^{11} cm^{-2})
0
2
4
6
8
10
12
0
1
2
3

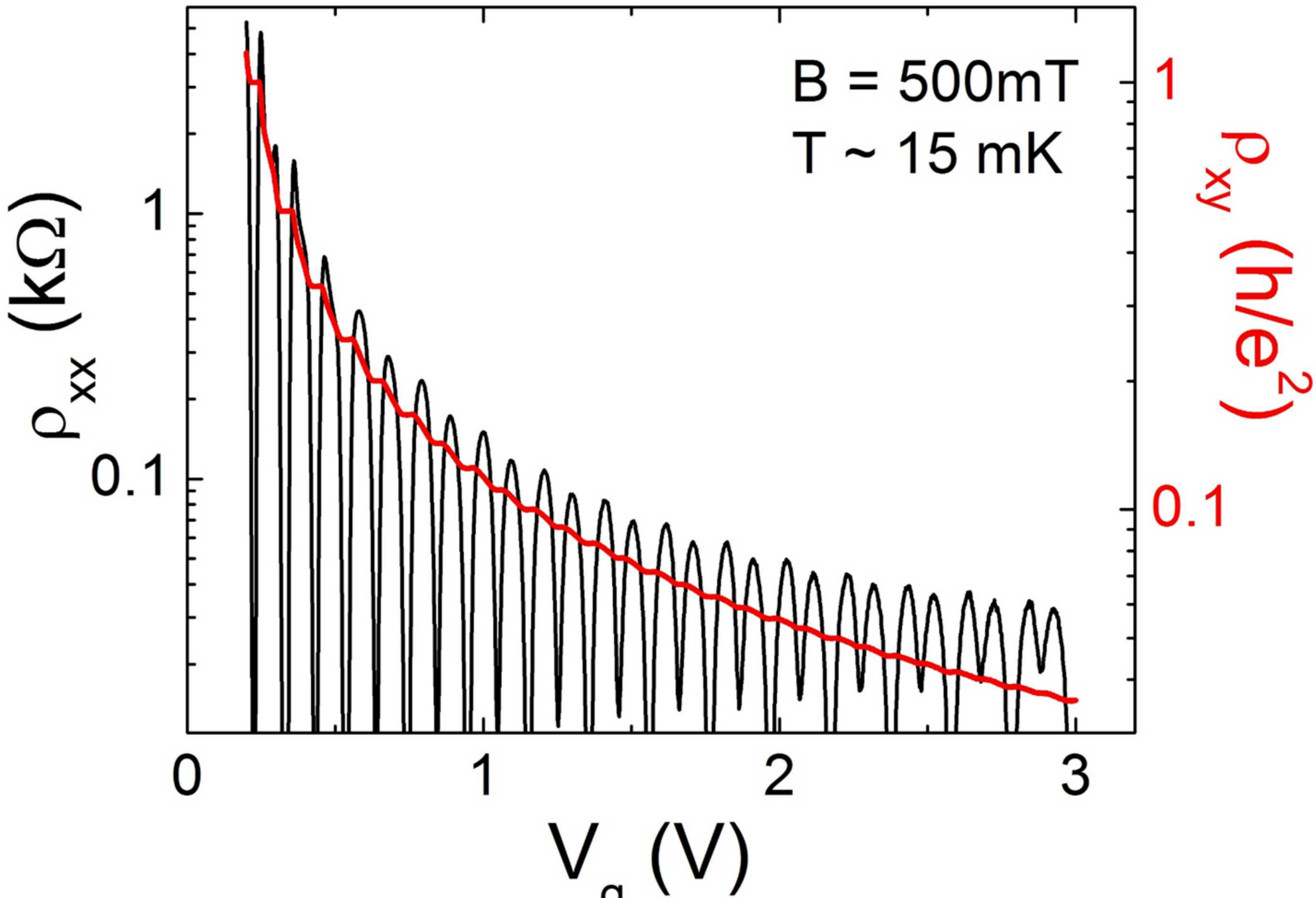
Very large density range

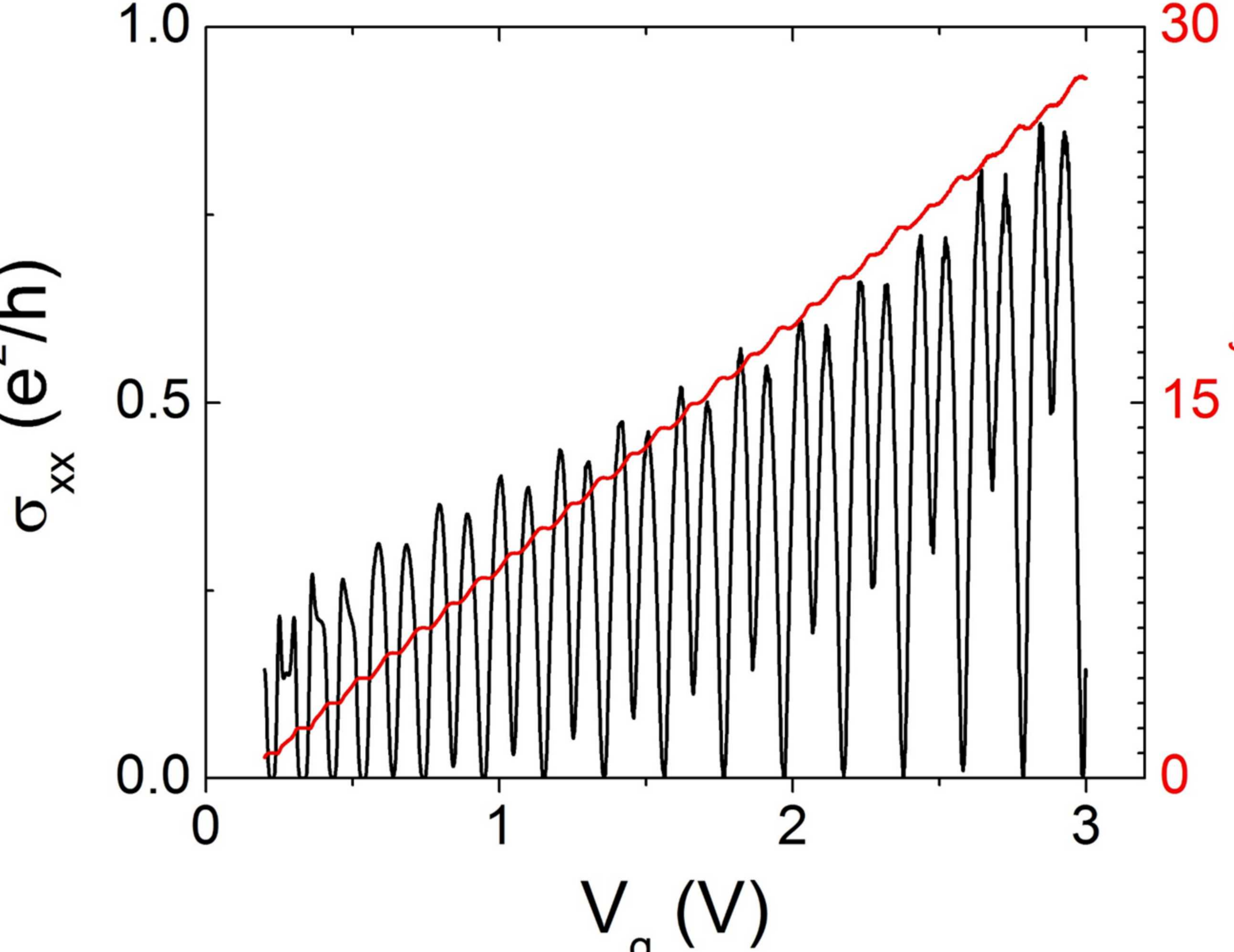
~ 1×10^9 to ~ 7.5×10^{11} cm^{-2}

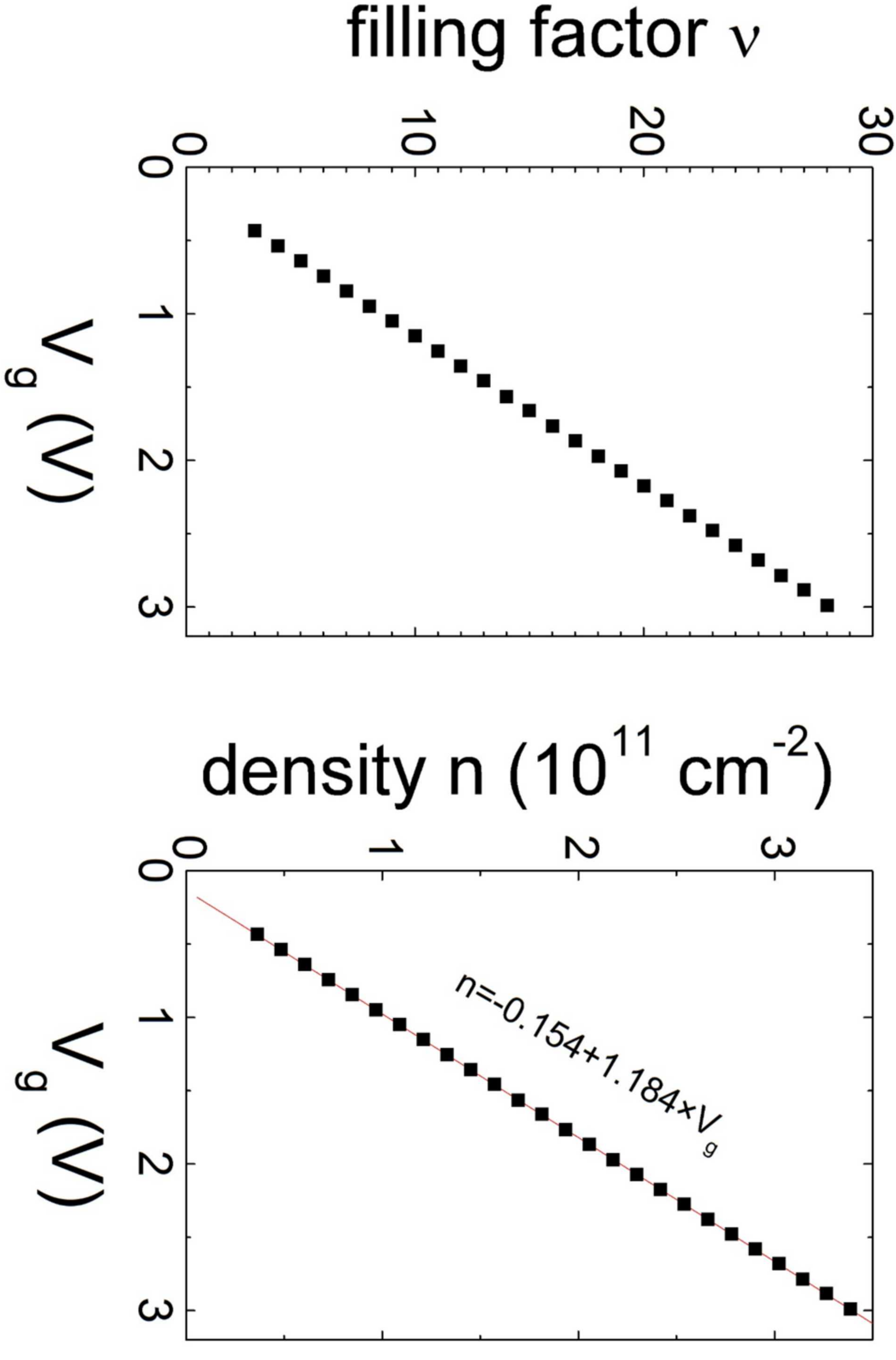


Linear I-V at very low densities



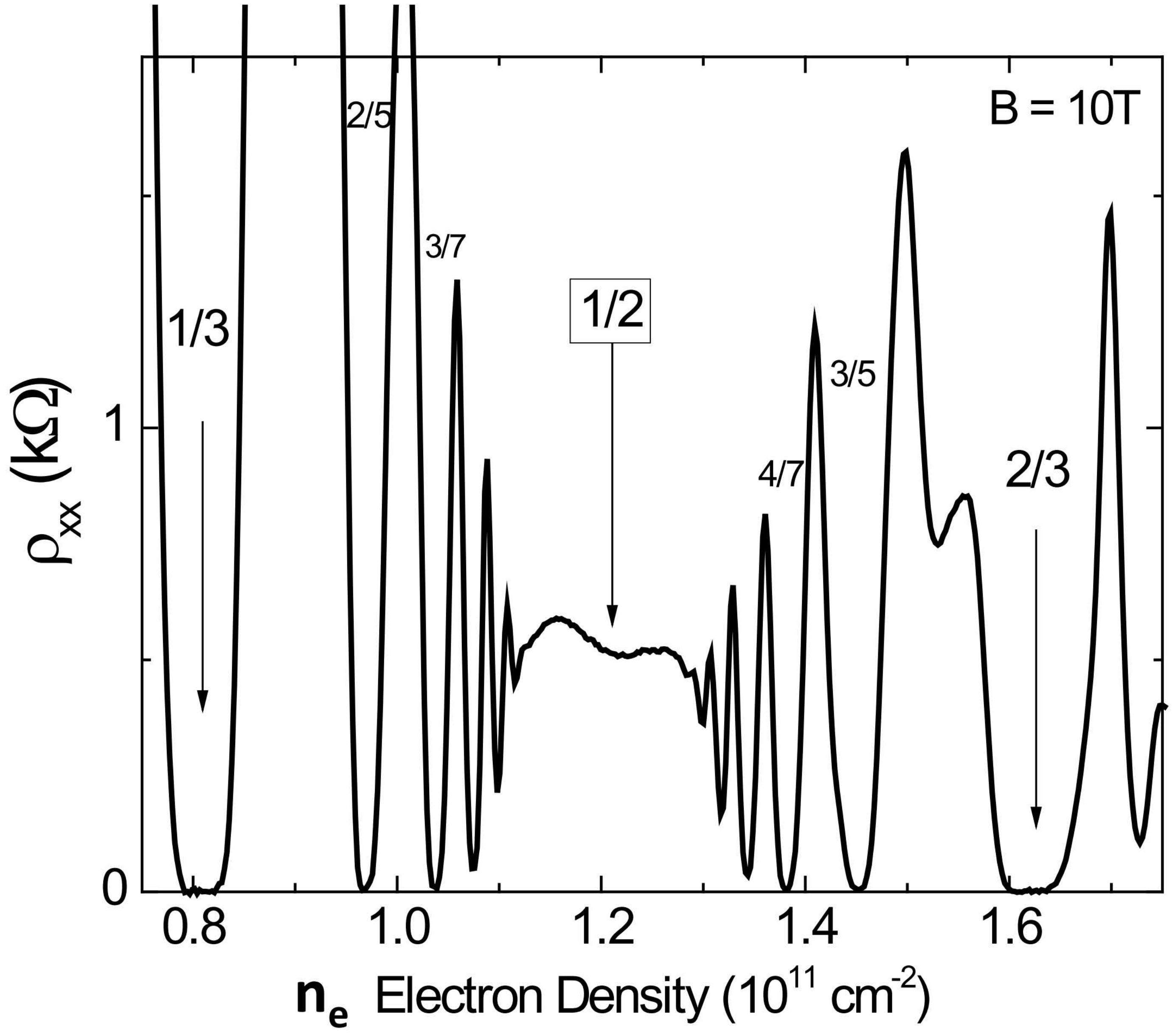

B = 500mT
T ~ 15 mK
ρ_{xx} (kΩ)
ρ_{xy} (h/e^2)
V_g (V)
1
0.1
1
0.1
0
1
2
3


1.0
0.5
0.0
30
15
0
0
1
2
3
σ_{xx} (e^2/h)
σ_{xy}(e^2/h)
V_g (V)

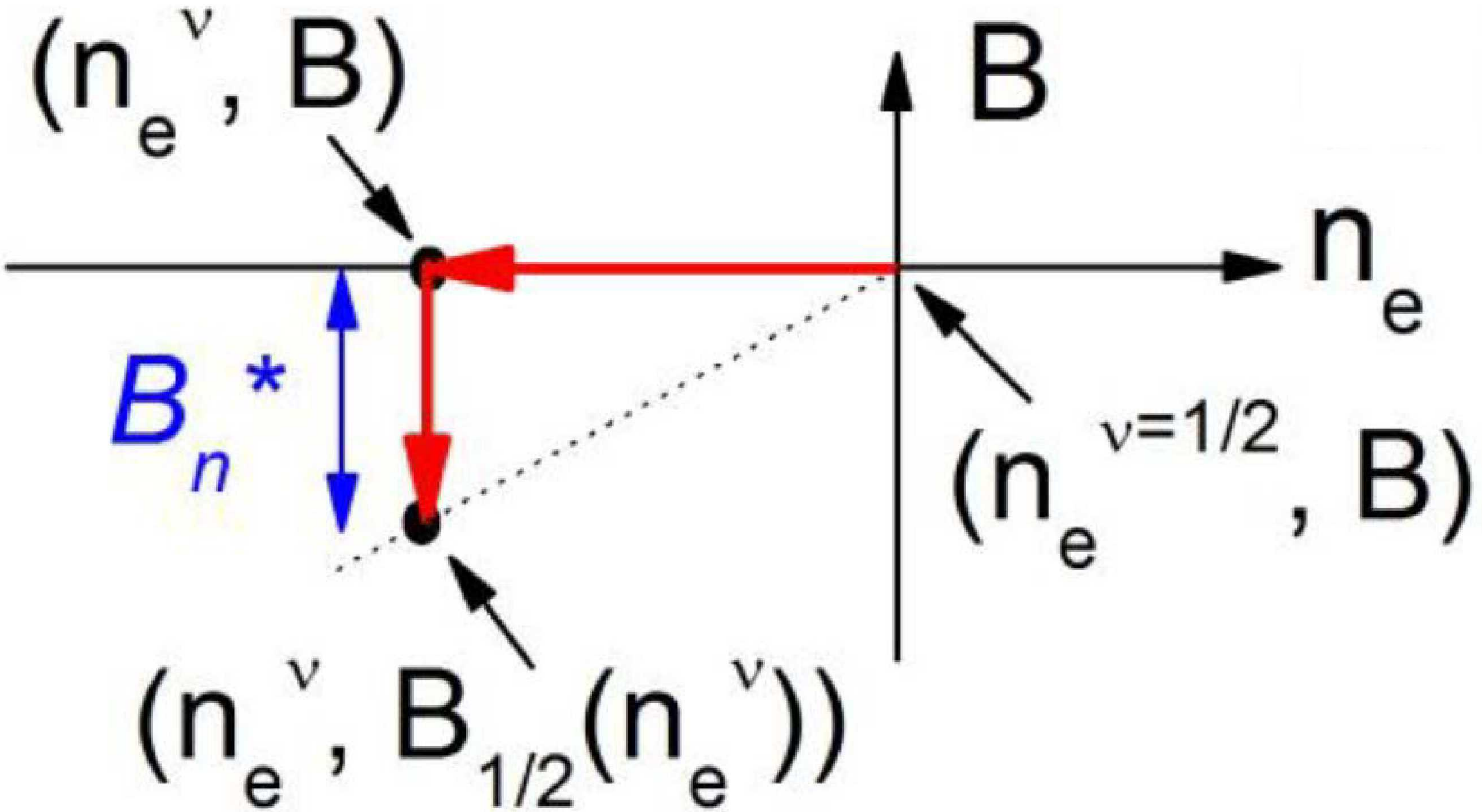


filling factor ν
0
10
20
30
V_g (V)
density n (10^{11} cm^{-2})
n=-0.154+1.184×V_g

SdH of CFs as a function of electron density



Wang and Senthil, arxiv:1507.08290

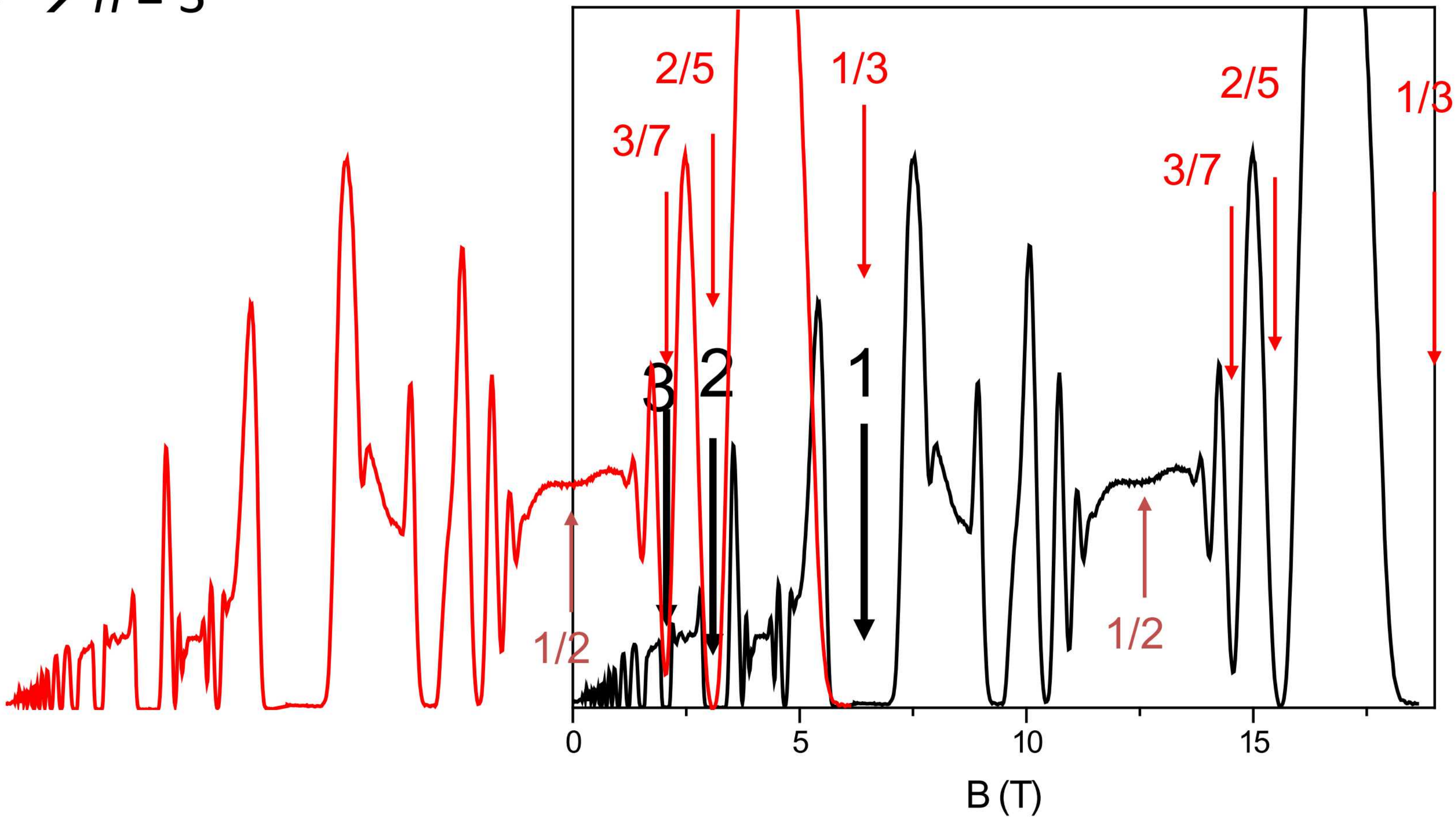


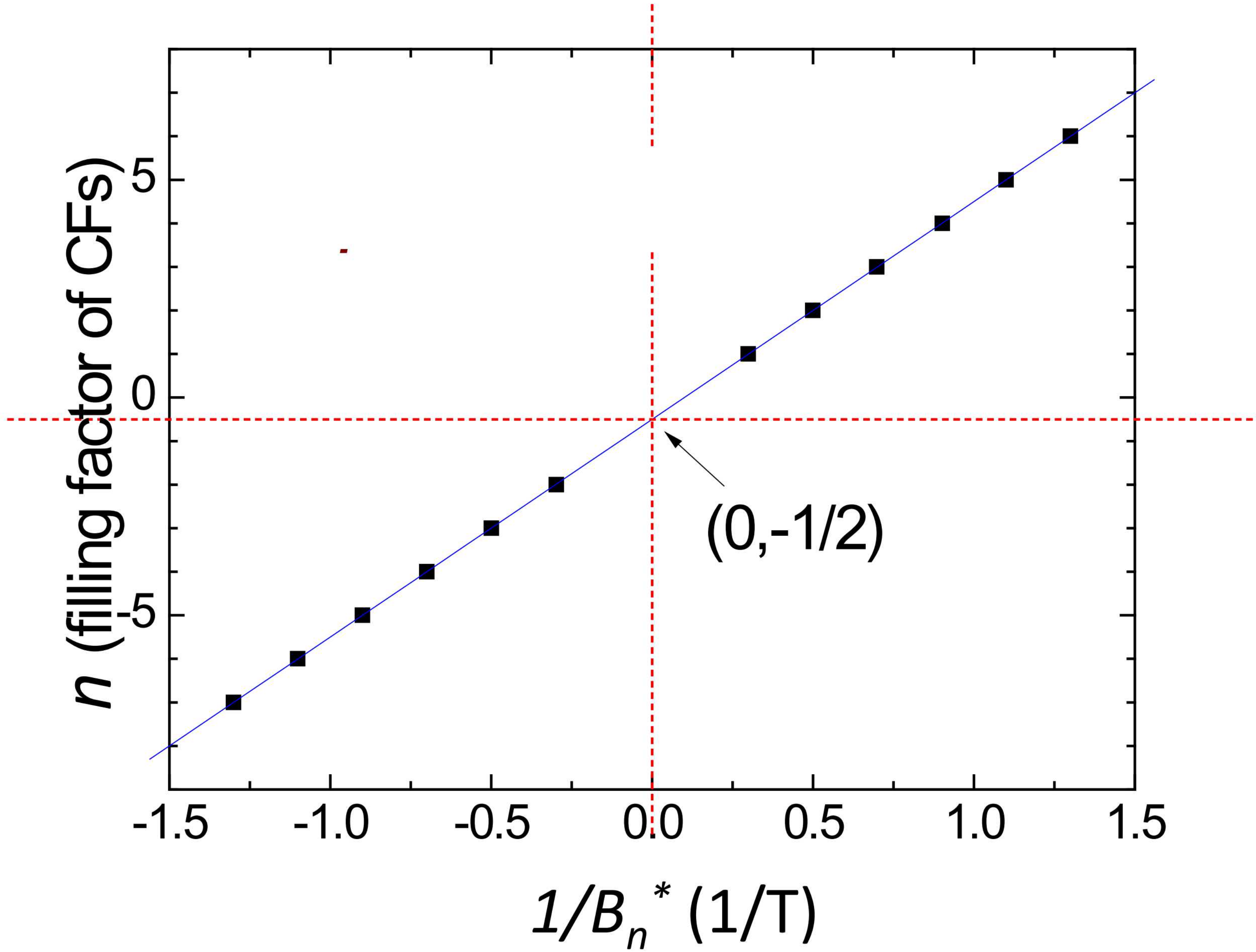
$$B_n^* = B - B_{1/2}(n_e^{\nu}) = B - 2n_e^{\nu}h/e$$

similarity between IQHE and FQHE: R_{xx}

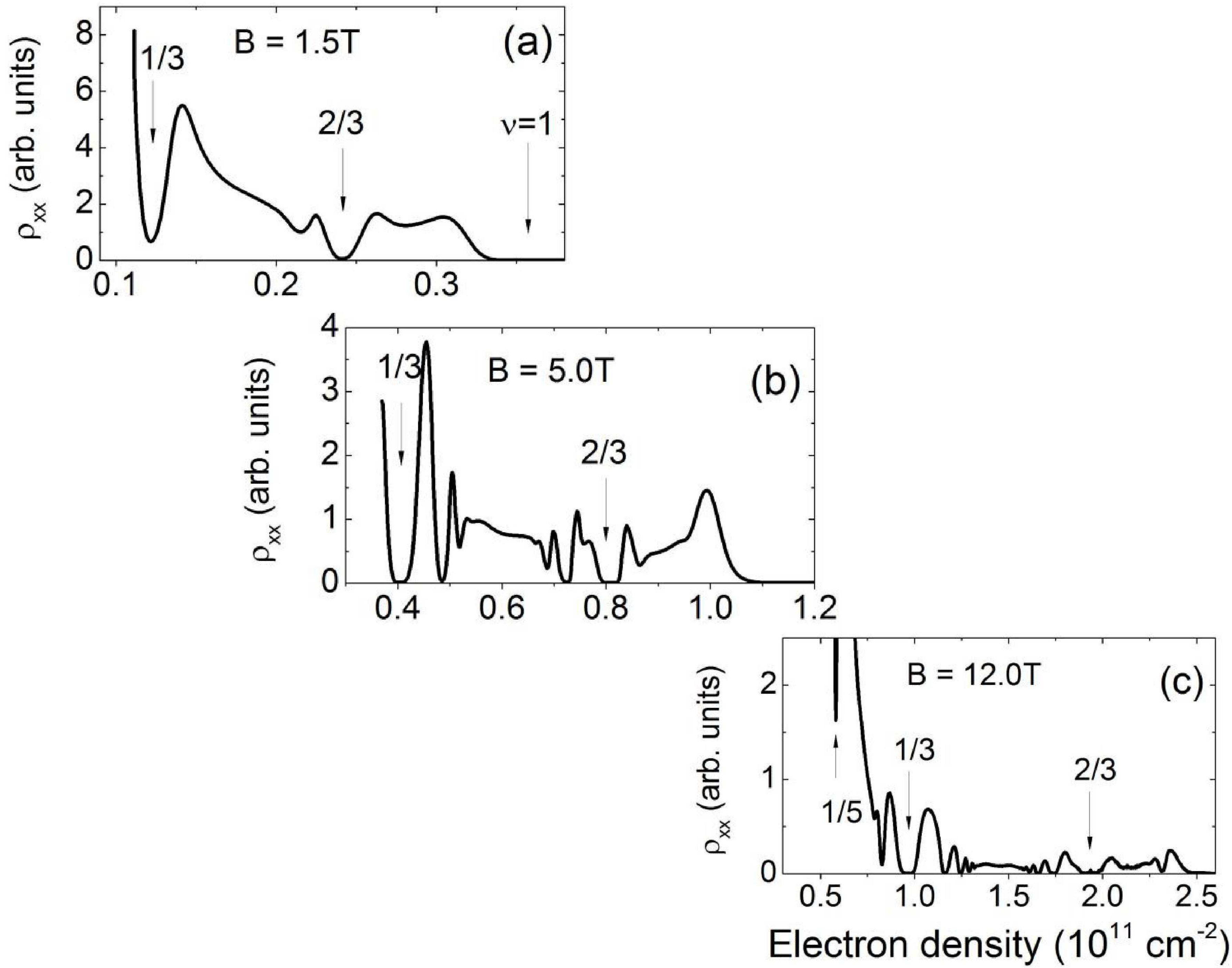
1/3 → n = 1
2/5 → n = 2
3/7 → n = 3

....

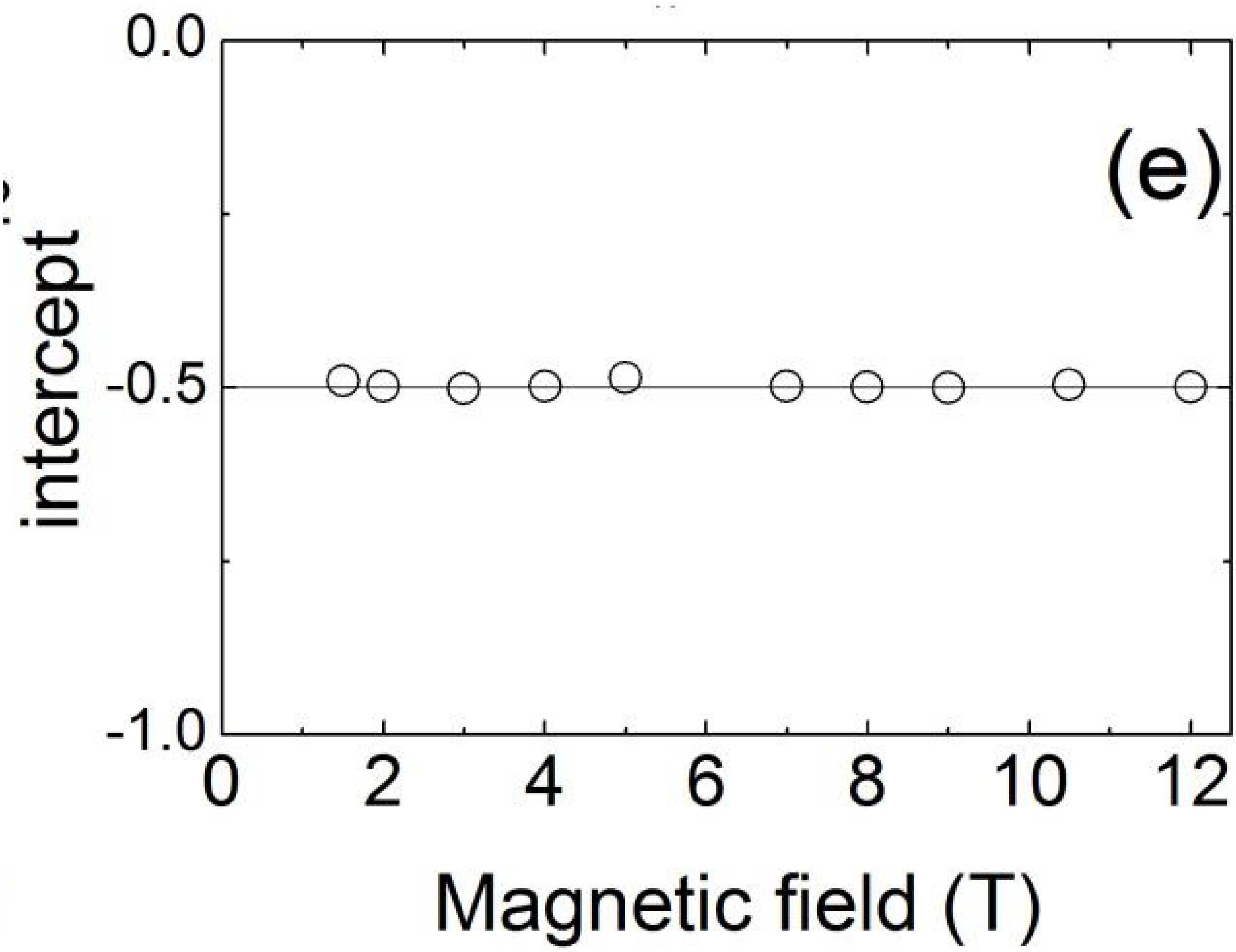




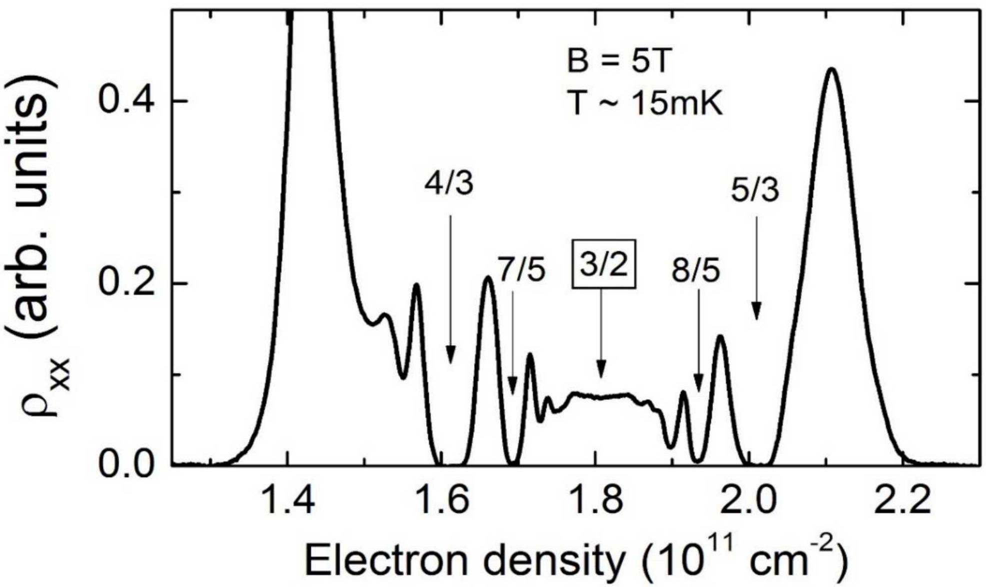
Nature Physics (2017)

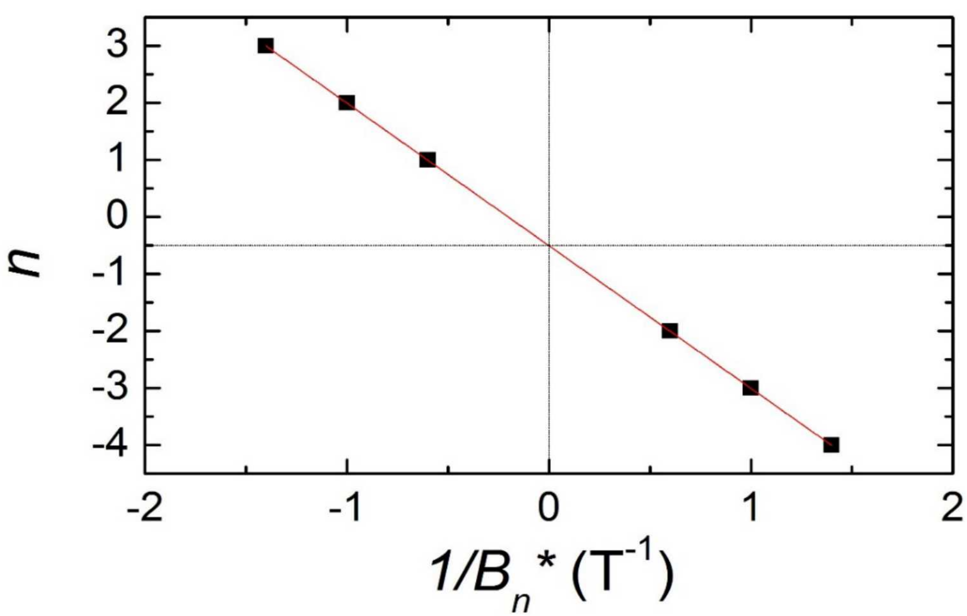


B = 1.5T
(a)
1/3
2/3
ν=1
ρxx (arb. units)
0
2
4
6
8
0.1
0.2
0.3
B = 5.0T
(b)
1/3
2/3
ρxx (arb. units)
0
1
2
3
4
0.4
0.6
0.8
1.0
1.2
B = 12.0T
(c)
1/5
1/3
2/3
ρxx (arb. units)
0
1
2
0.5
1.0
1.5
2.0
2.5
Electron density (10^11 cm^-2)



(e)
intercept
0.0
-0.5
-1.0
0
2
4
6
8
10
12
Magnetic field (T)





CFs around 3/2

FQHE occurs at ν=2-n/(2n+1)
(n integer)

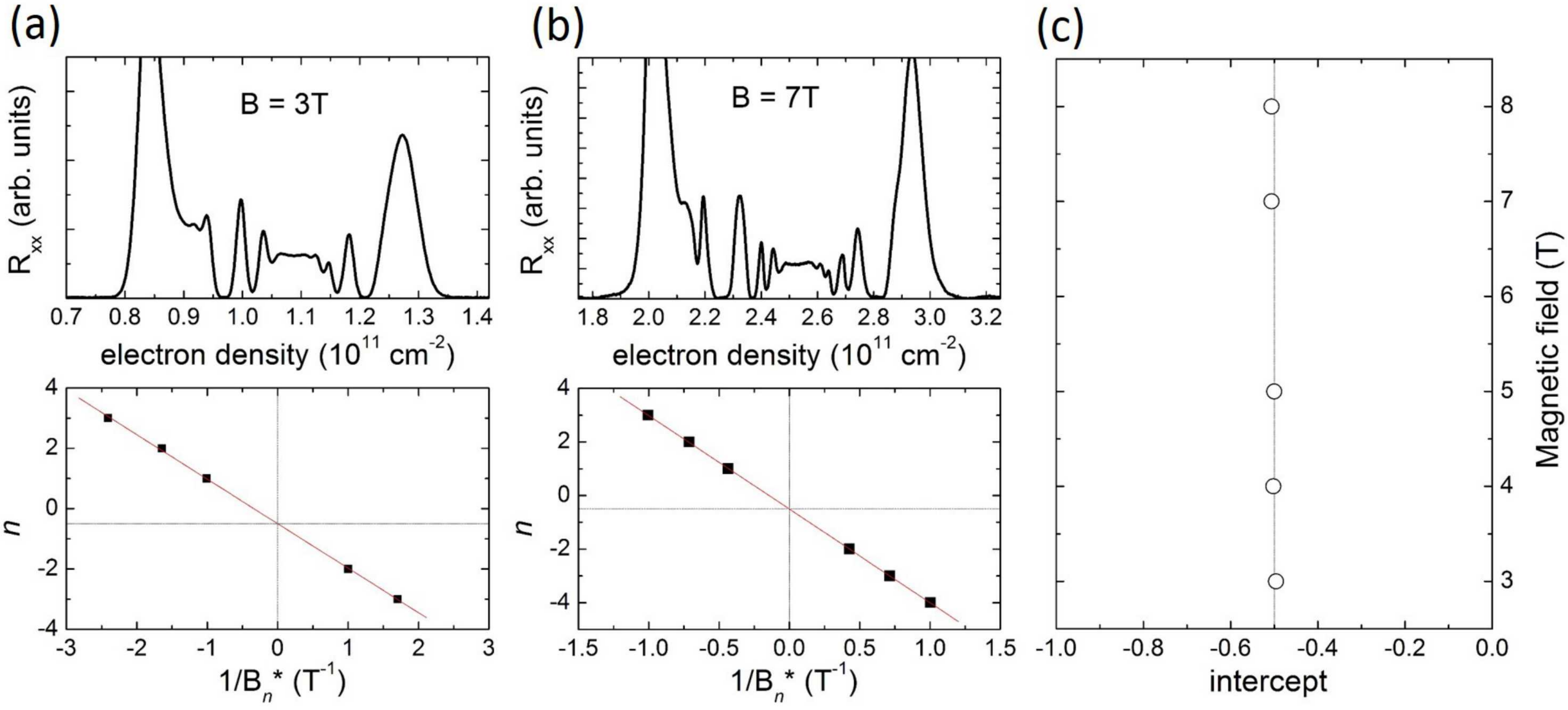
5/3 $\leftrightarrow$ n = 1
4/3 $\leftrightarrow$ n = -2

For SdH oscillations at a constant B, under the DCF model

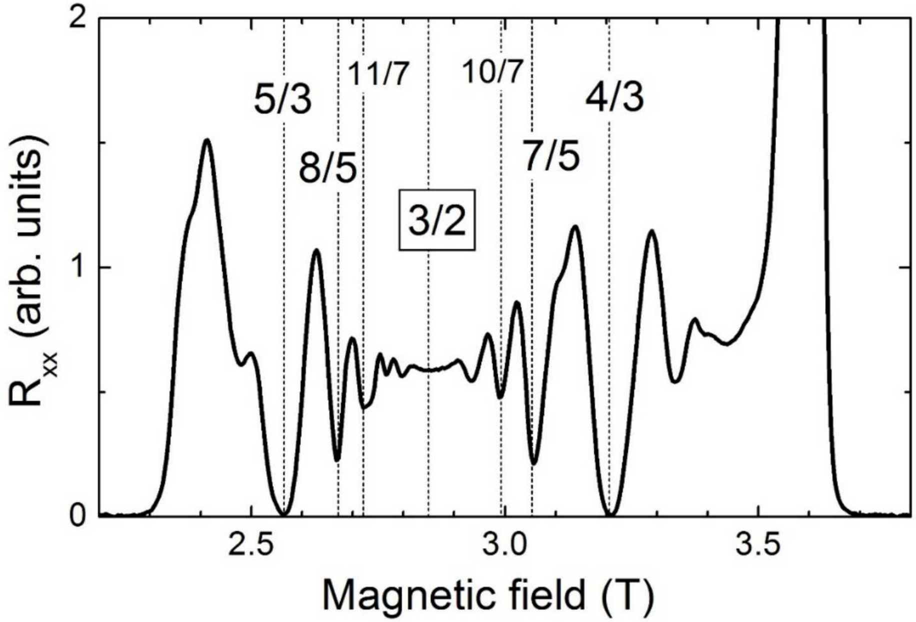
$$B_n^* = 3\times(B - 2n_e h/3e)$$

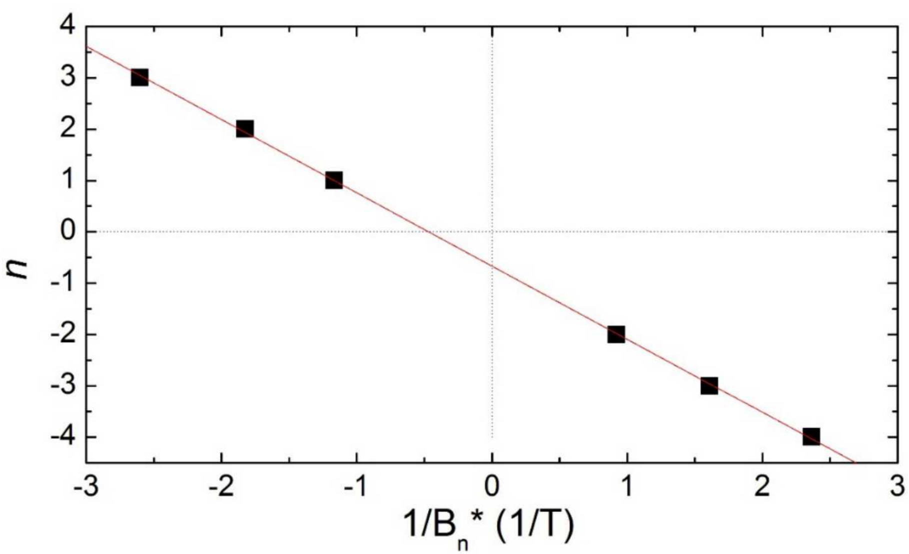
$$n = -(B/2) \times 1/B_n^* - 1/2$$

$\rightarrow$Experiment ~ -0.5



(a)
(b)
(c)
B = 3T
B = 7T
R_xx (arb. units)
electron density (10^11 cm^-2)
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
n
1/B_n* (T^-1)
4
2
0
-2
-4
-3
-1
1
3
-1.5
-1.0
-0.5
0.0
0.5
1.5
intercept
-0.8
-0.6
-0.4
-0.2
Magnetic field (T)
5
6
7
8





FQHE occurs at ν=2-n/(2n+1)
(n integer)

5/3 $\leftrightarrow$ n = 1
4/3 $\leftrightarrow$ n = -2

For standard SdH oscillations of constant electron density

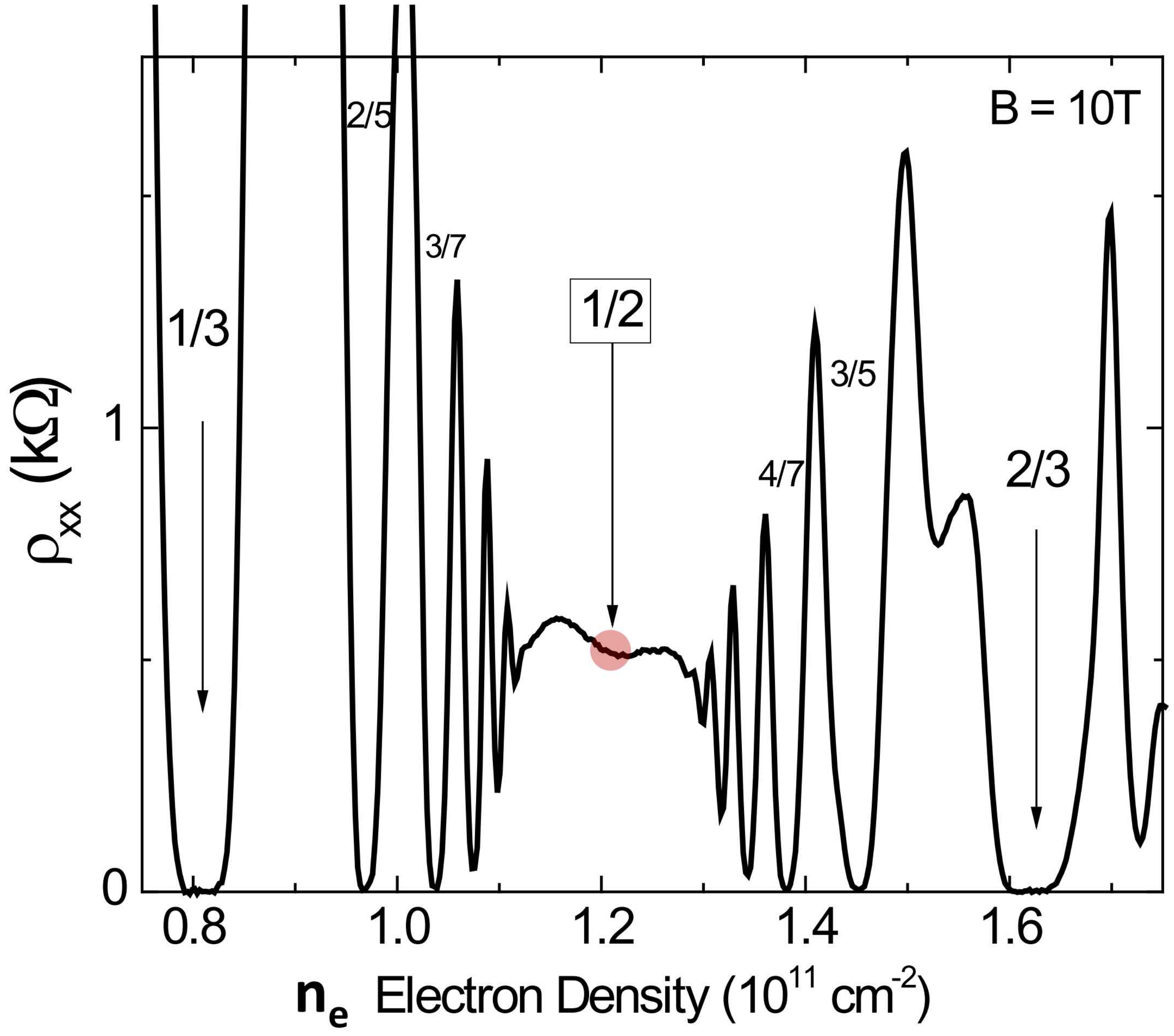
Under HLR picture

$B_n^* = 3\text{x}(B_\nu - B_{3/2}) = -(3n+2)e/n_e h$

→Intercept at $1/B_n^* = 0$ is -2/3

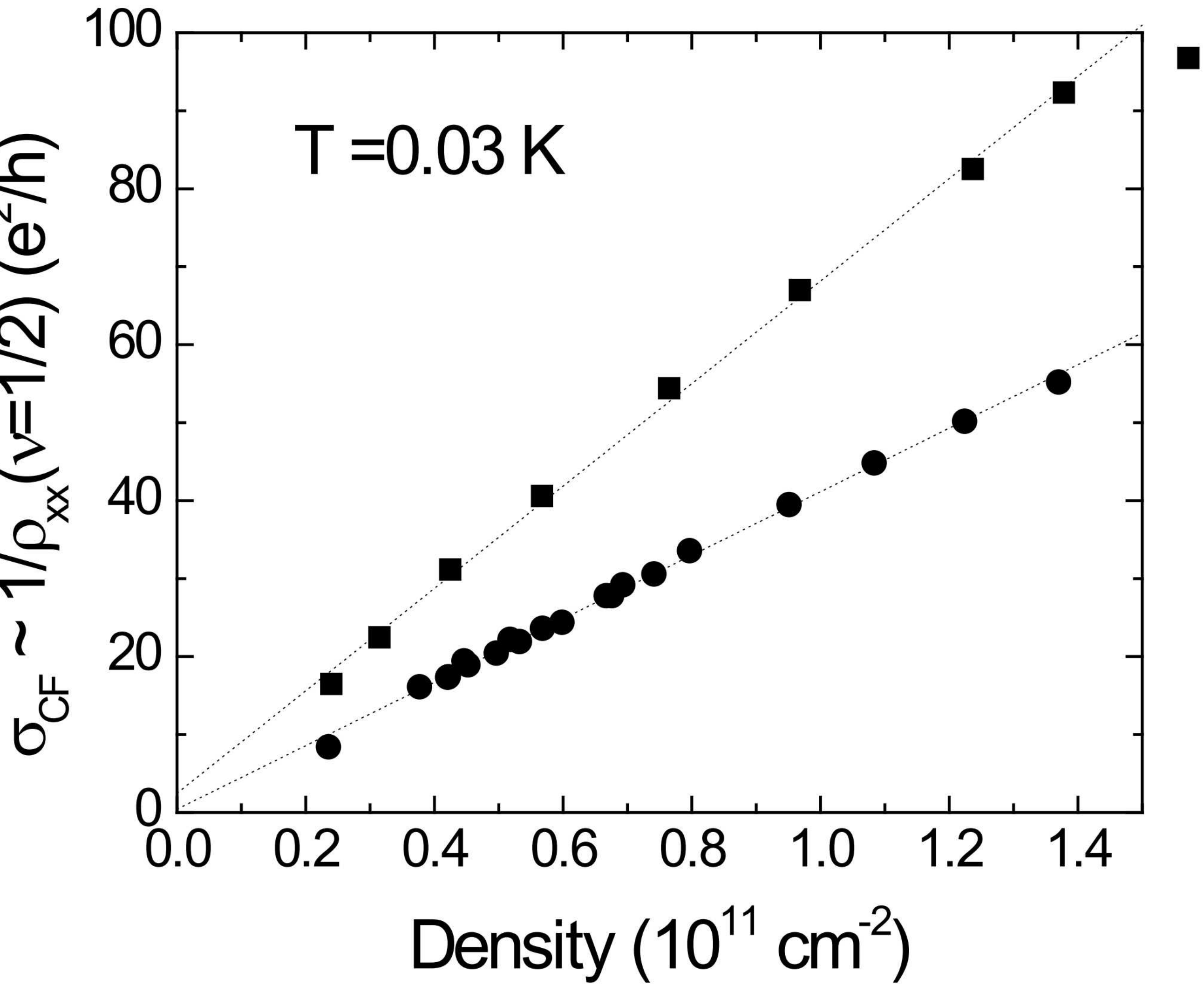
→Experimentally ~ -0.665

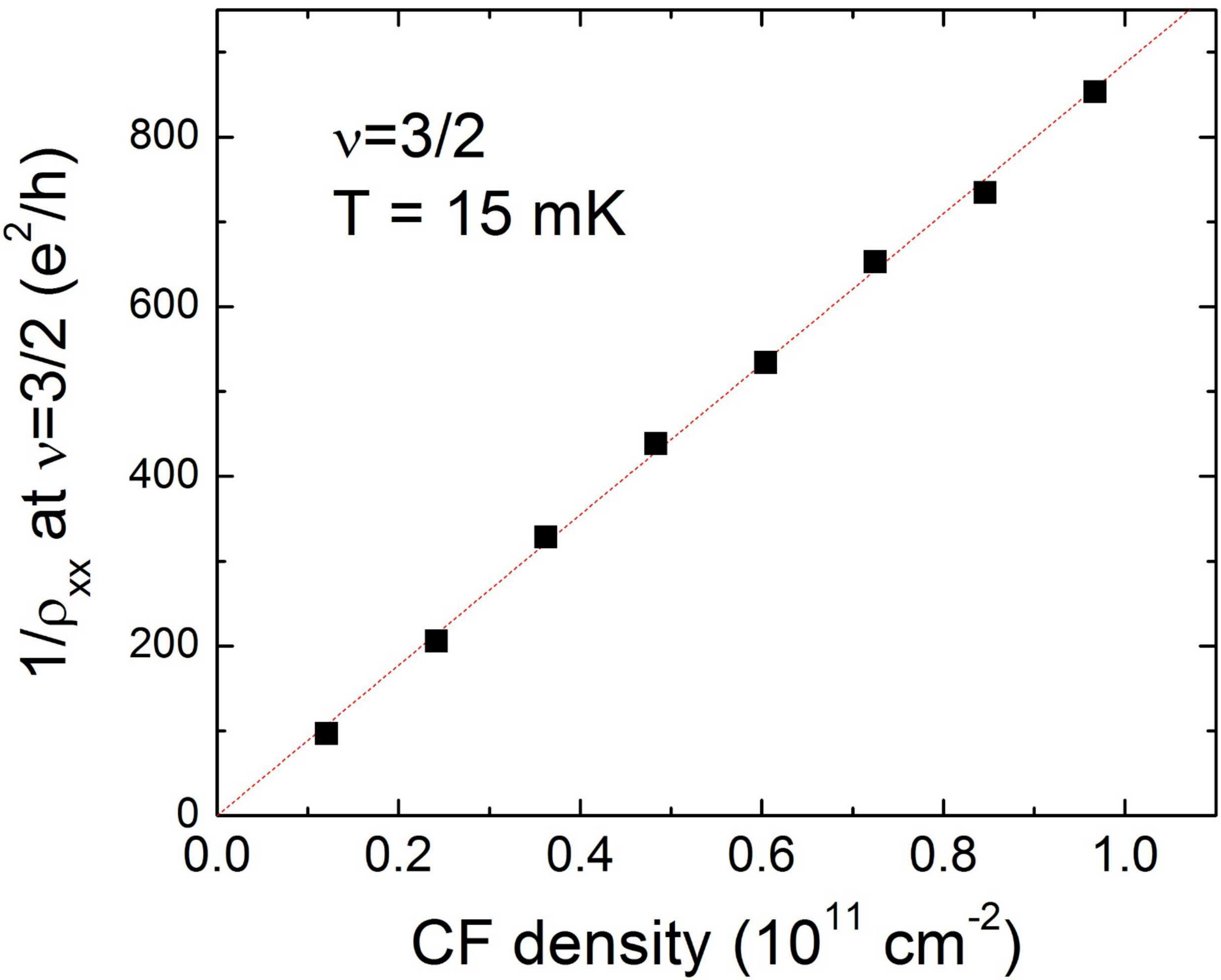
linear density dependence of CF conductivity



B = 10T
1/3
2/5
3/7
1/2
4/7
3/5
2/3
ρxx (kΩ)
1
0
0.8
1.0
1.2
1.4
1.6
ne Electron Density (10^11 cm^-2)

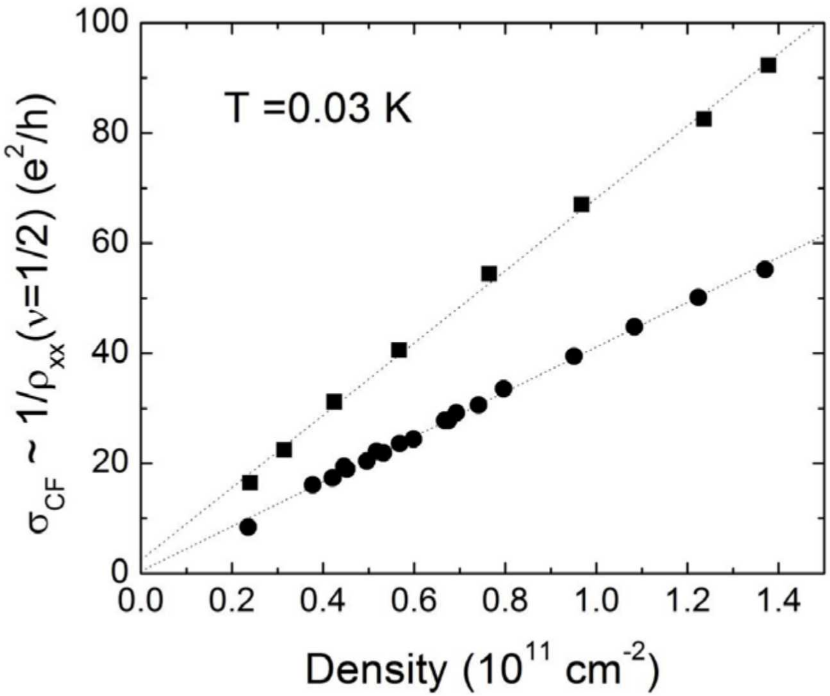
CF conductivity *versus* density



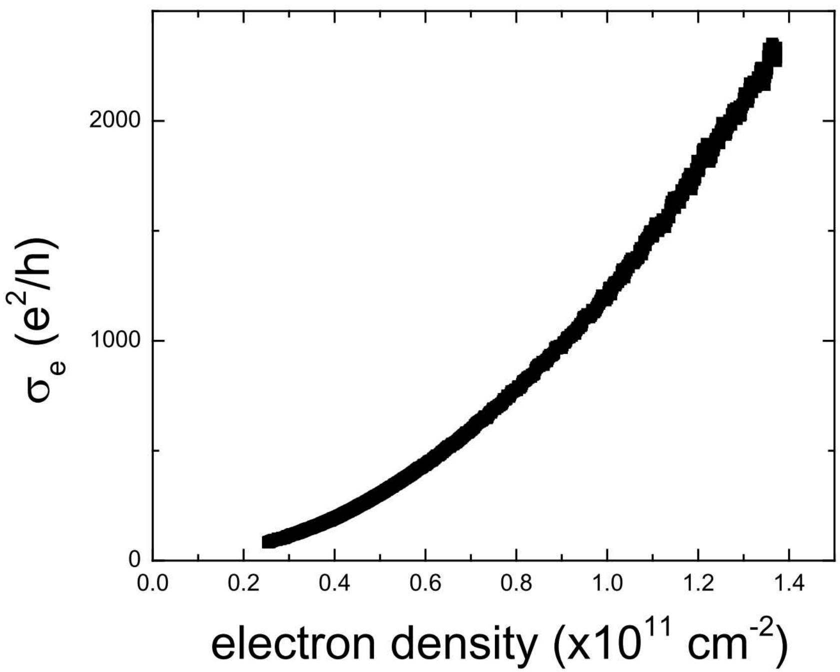


ν=3/2
T = 15 mK
1/ρ_xx at ν=3/2 (e^2/h)
0
200
400
600
800
0.0
0.2
0.4
0.6
0.8
1.0
CF density (10^11 cm^-2)

CF conductivity



Electron conductivity



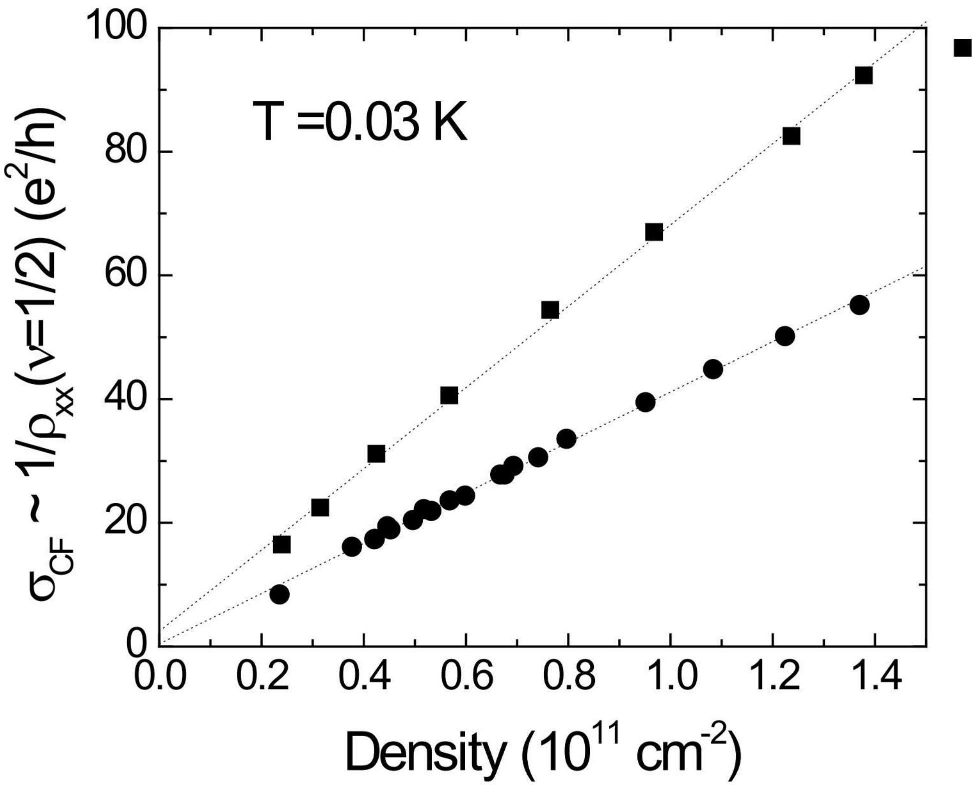
Halperin-Lee-Read theory of CFs:

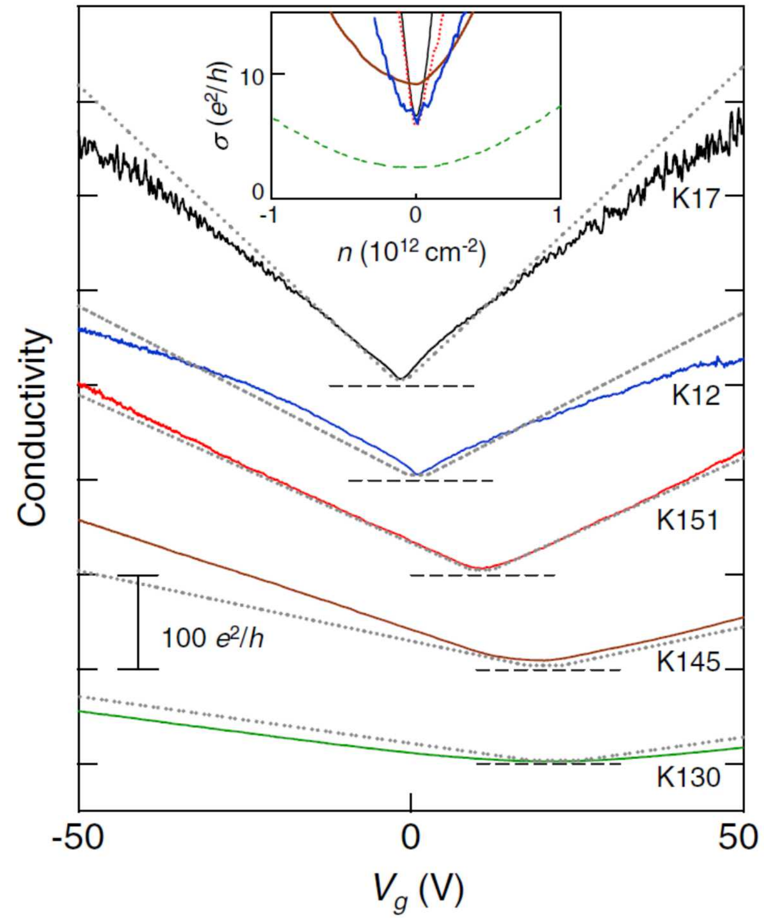
$$\rho_{xx}^{CF} \approx (n_{imp}/n) \times (\pi\phi^2/k_f d_s e^2)$$

$\rho_{xx}^{CF} \propto n^{-3/2}$

$\sigma_{xx}^{CF} \sim 1/\rho_{xx}^{CF} \propto n^{+3/2}$

Linear dependence in graphene





Tan *et al.* PRL (2007).

Particle-hole symmetry (PHS) in the FQHE

PHYSICAL REVIEW B VOLUME 29, NUMBER 10 15 MAY 1984

Particle-hole symmetry in the anomalous quantum Hall effect

S. M. Girvin
Surface Science Division, National Bureau of Standards, Washington, D.C. 20234
(Received 27 February 1984)

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state ψ_m defined in (5). Hence this state has the filling factor $\nu = 1 - 1/m$ and (for $N \to \infty$) is the *exact* particle-hole dual of the state with $\nu = 1/m$. For large but finite M it is

$$\nu \longleftrightarrow 1-\nu$$

Examining energy gap of FQHE

$E_g(\nu) = g^{\nu} \times e^2/(\varepsilon l_B)$

l_B magnetic length

$E_g(1\text{-}\nu) = g^{1\text{-}\nu} \times e^2/(\varepsilon l_B)$

$l_B = (\hbar/eB)^{1/2}$

PHS → $g^{\nu} = g^{1-\nu} = g$

$E_g(\nu) = E_g(1\text{-}\nu)$ at fixed B

Disorder broadening

$E_g(\nu) = g \times e^2/(\varepsilon l_B) - \Gamma$

$E_g(1\text{-}\nu) = g \times e^2/(\varepsilon l_B) - \Gamma$

PH symmetric disorder → $\Gamma(\nu) = \Gamma(1-\nu) = \Gamma$

$E_g(\nu) = E_g(1\text{-}\nu)$ at fixed B

PHYSICAL REVIEW B VOLUME 36, NUMBER 15 15 NOVEMBER 1987-II

Activation energies and localization in the fractional quantum Hall effect

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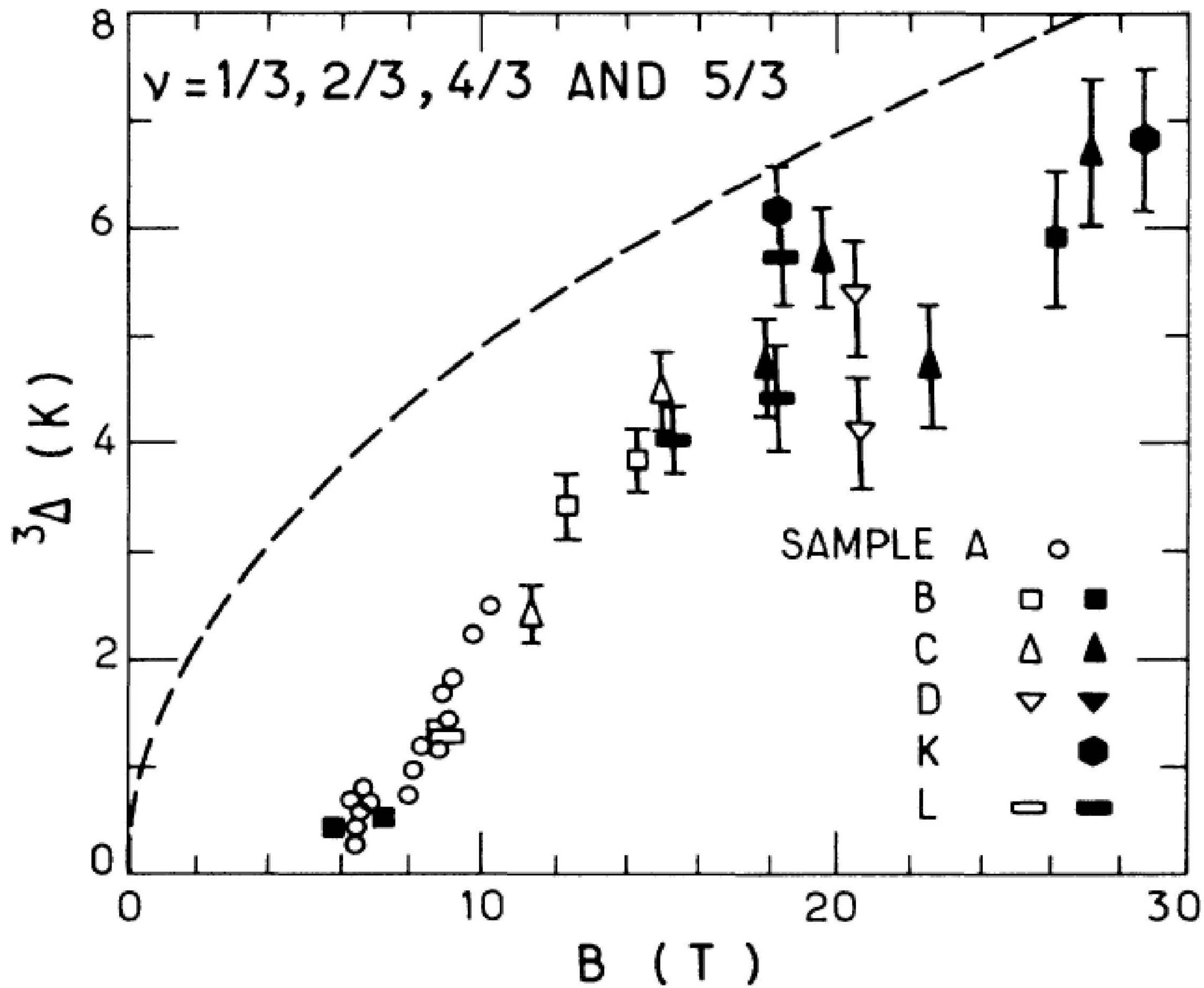
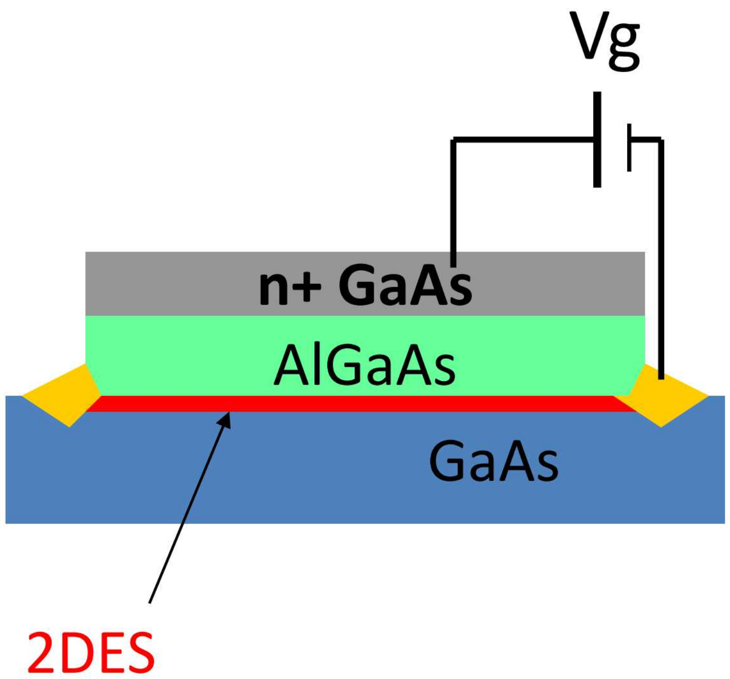
(Received 29 April 1987)



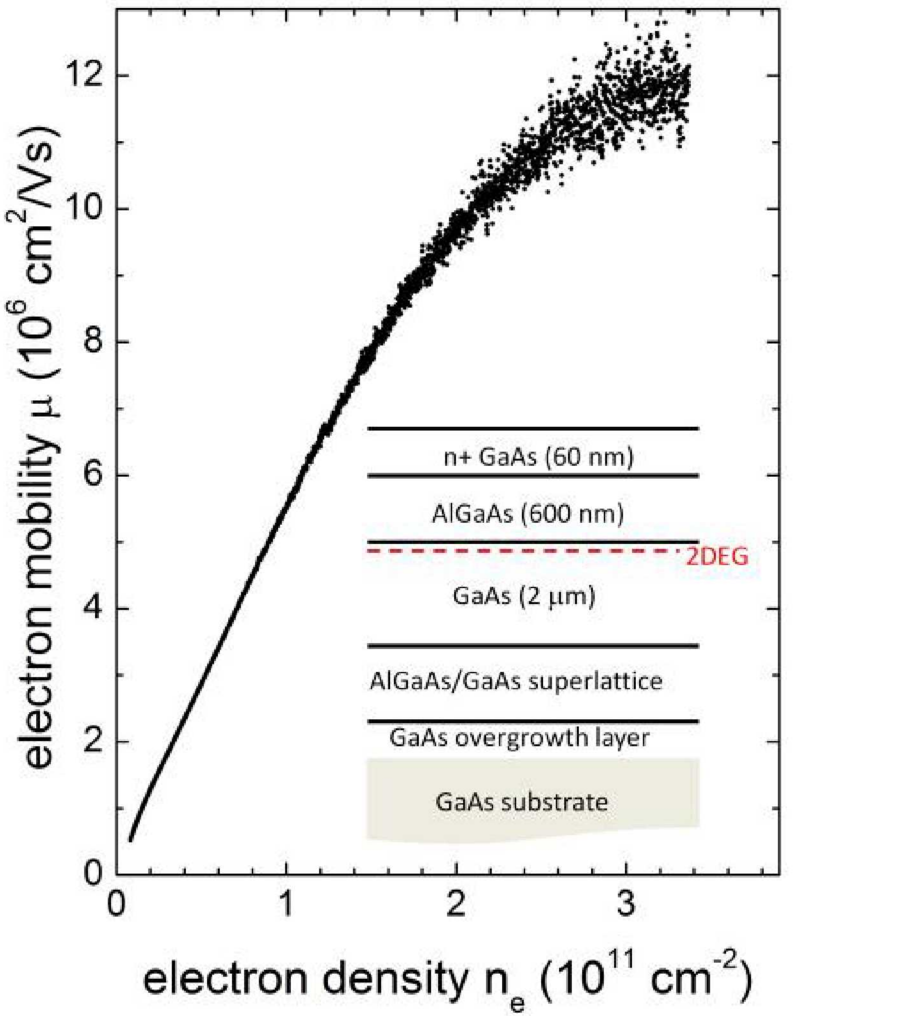
ν = 1/3, 2/3, 4/3 AND 5/3
$^3\Delta$ (K)
B (T)
SAMPLE A
B
C
D
K
L
0
2
4
6
8
0
10
20
30

HIGFET

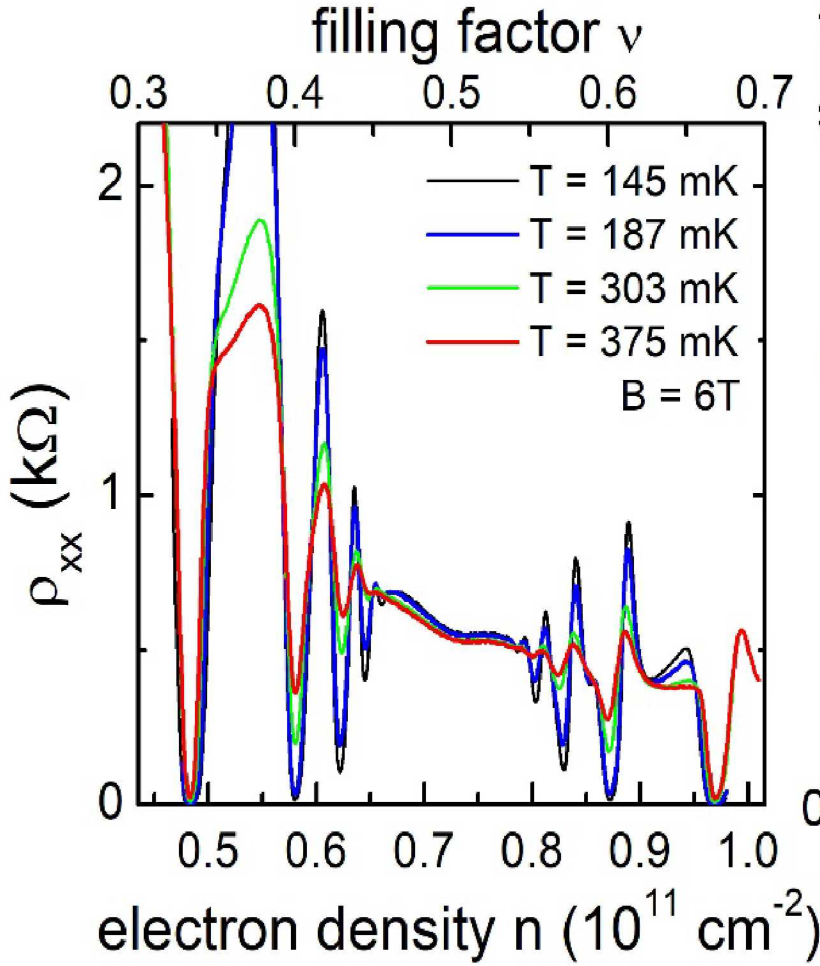
high mobility down to very low densities

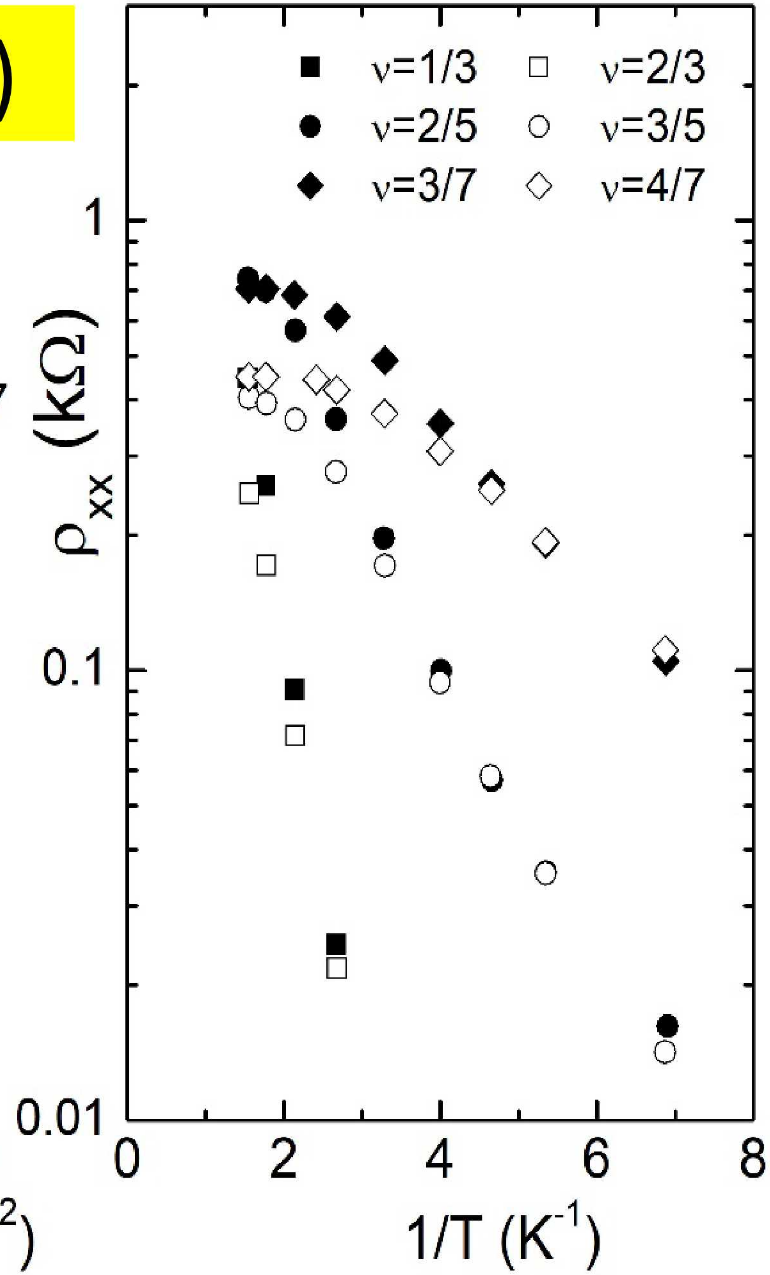


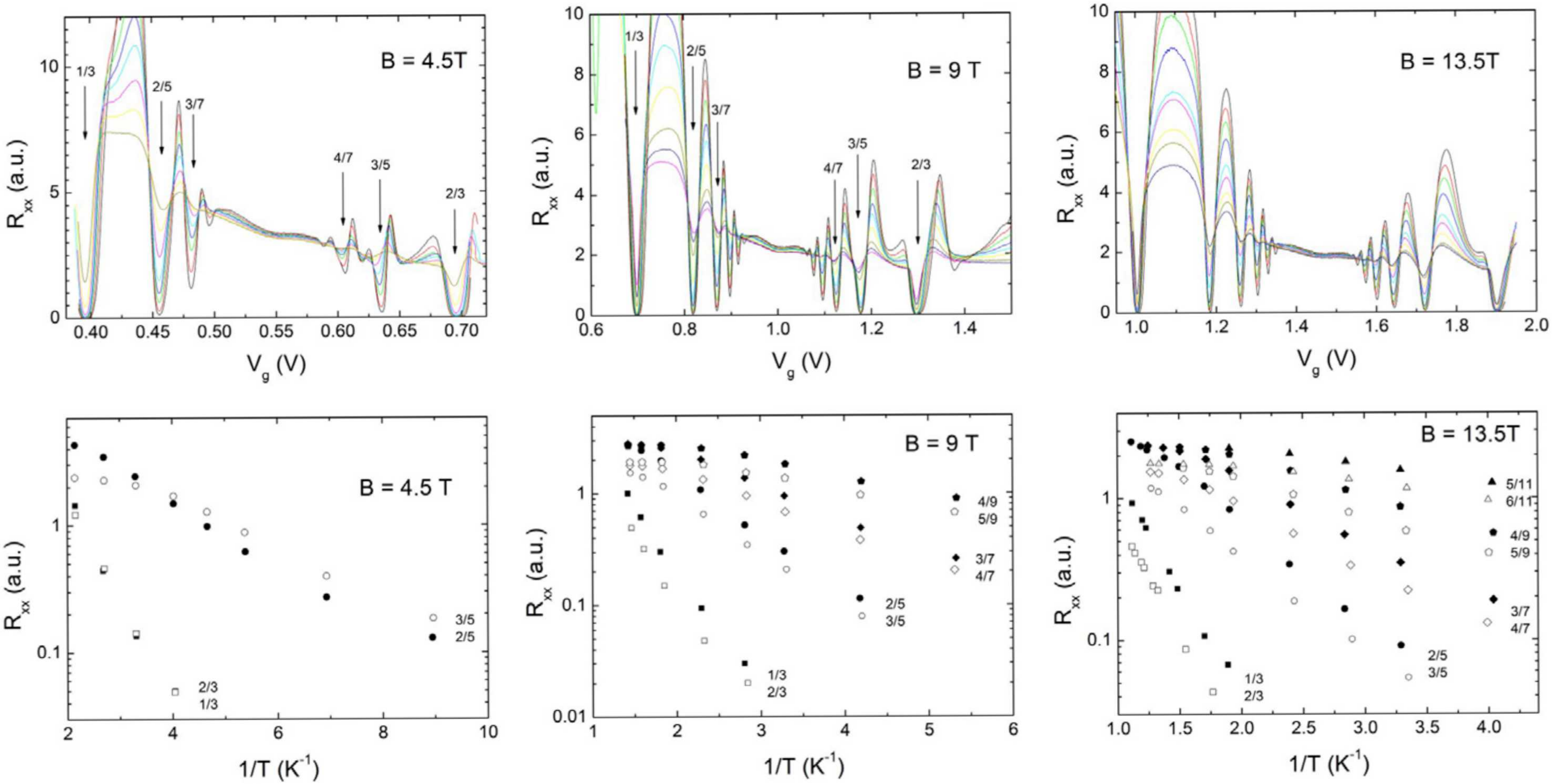
Kane, Pfeiffer, West, and Harnett, APL,1993



$$\rho_{xx} = \rho_{xx}^0 \exp(-E_g/2k_BT)$$







B = 4.5T
1/3
2/5
3/7
4/7
3/5
2/3
R_{xx} (a.u.)
V_g (V)
B = 9 T
1/3
2/5
3/7
4/7
3/5
2/3
R_{xx} (a.u.)
V_g (V)
B = 13.5T
R_{xx} (a.u.)
V_g (V)
B = 4.5 T
3/5
2/5
2/3
1/3
R_{xx} (a.u.)
1/T (K^{-1})
B = 9 T
4/9
5/9
3/7
4/7
2/5
3/5
1/3
2/3
R_{xx} (a.u.)
1/T (K^{-1})
B = 13.5T
5/11
6/11
4/9
5/9
3/7
4/7
2/5
3/5
1/3
2/3
R_{xx} (a.u.)
1/T (K^{-1})

E_g measured at various magnetic fields

ν	B = 4.5T	B = 6T	B = 7.5T	B = 9T	B = 13.5T
1/3	4.01	5.15	5.96	6.13	7.54
2/3	3.85	4.68	5.61	6.15	7.73
2/5	1.19	1.75	2.20	2.60	3.55
3/5	1.09	1.65	1.99	2.45	3.36
3/7		0.86	1.22	1.50	2.18
4/7		0.77	1.12	1.40	2.18
4/9			0.55	0.75	1.24
5/9			0.50	0.69	1.28
5/11					0.57
6/11					0.56

Since energy gap of FQHE of electrons can be viewed as cyclotron gap of IQHE of CFs

$E_g = \hbar e B_{eff}/m^*$

$B_{eff} = B/(2p+1)$ (B fixed)

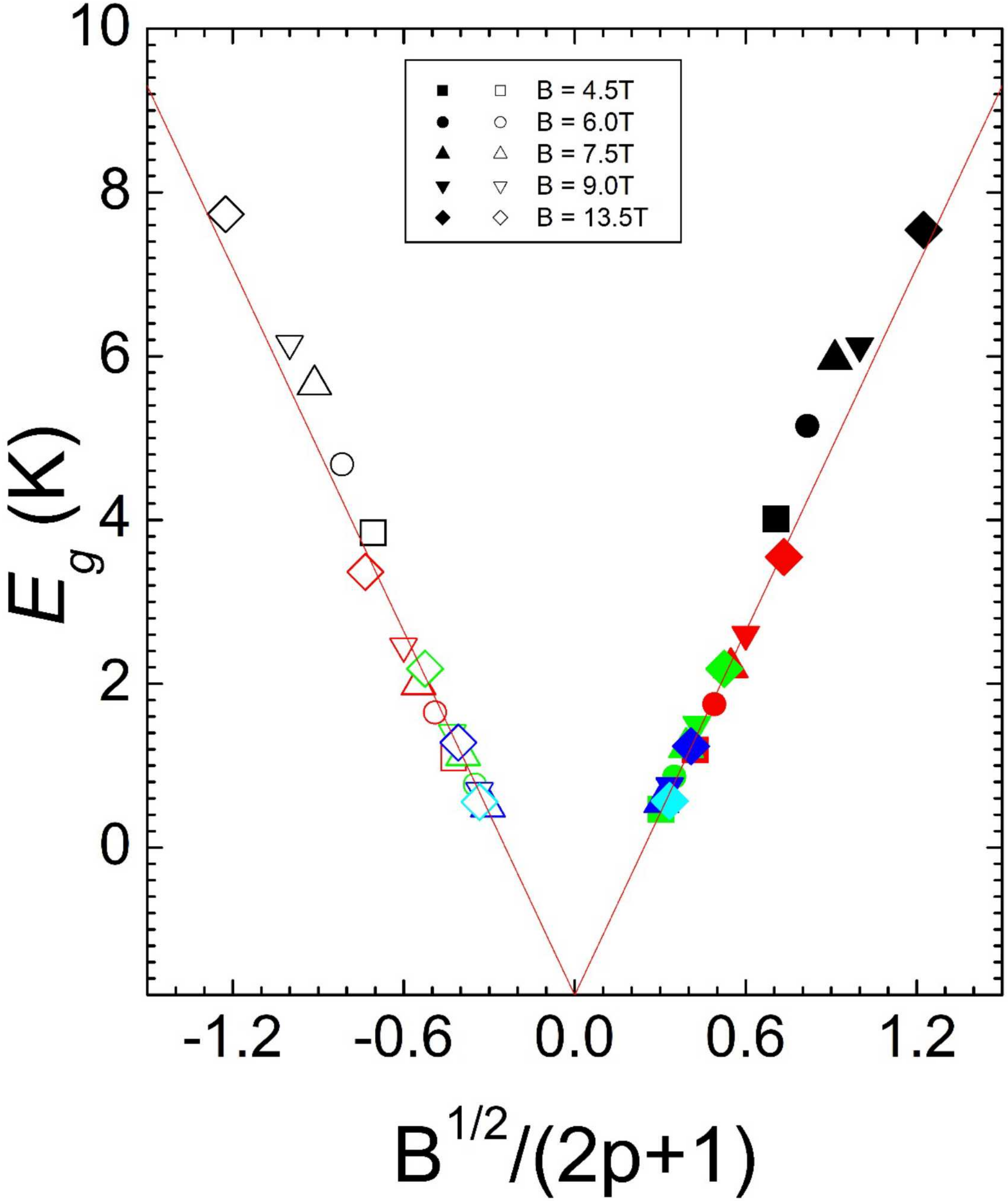
$m^* \propto e^2/\varepsilon l_B$

$$E_g \approx \frac{C}{|2p+1|} \frac{e^2}{\varepsilon l_B}$$

$\propto B^{1/2}/(2p+1)$

$\nu = p/(2p+1)$

(this scaling does not need to assume PHS – Kun Yang)



$E_g \propto B^{1/2}/(2p+1)$

PHS → m* is the same for particle and hole

Or slope is the same for positive and negative p's

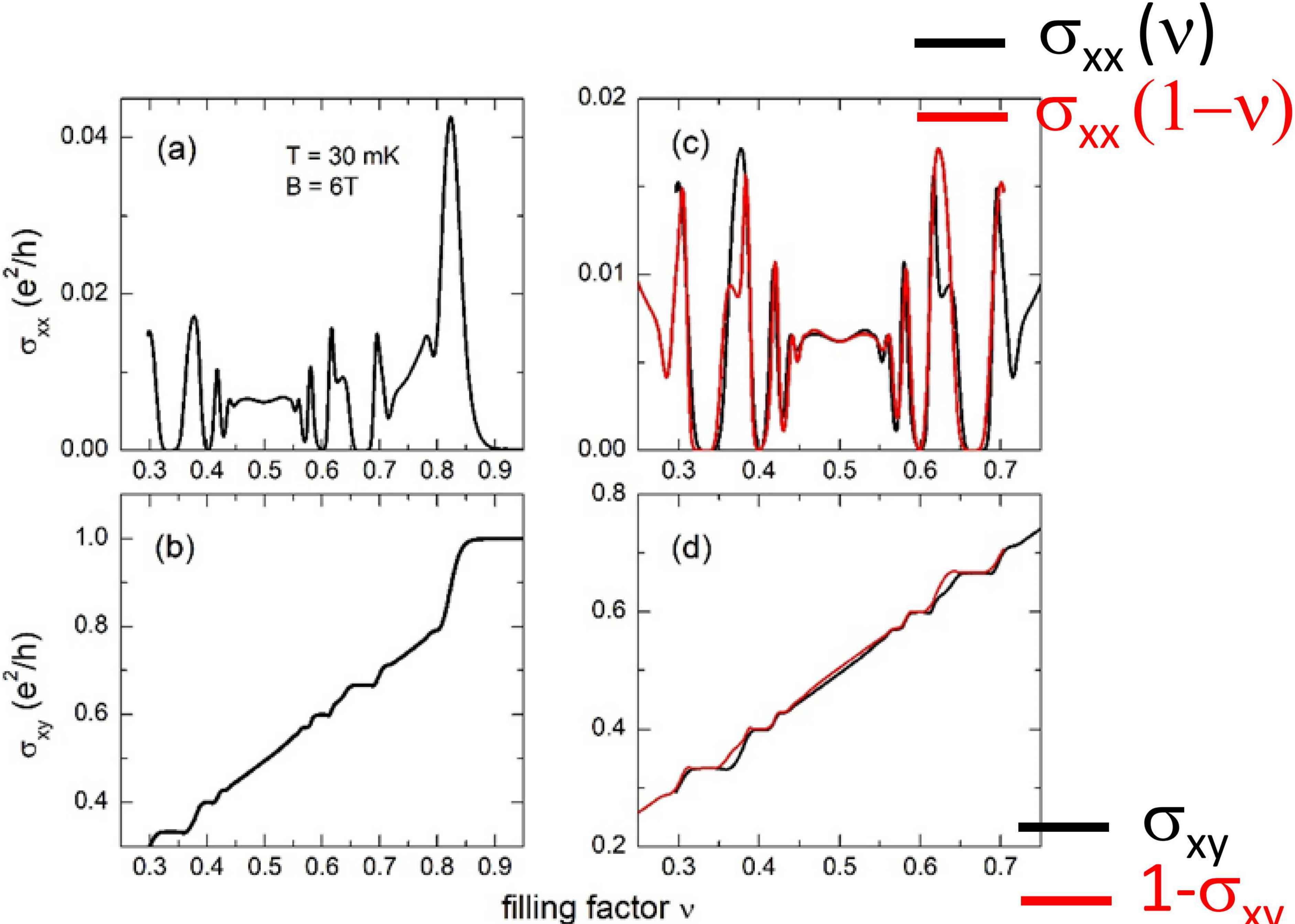
$m^* \approx 0.2\ m_e$

Examining reflection symmetry of FQHE

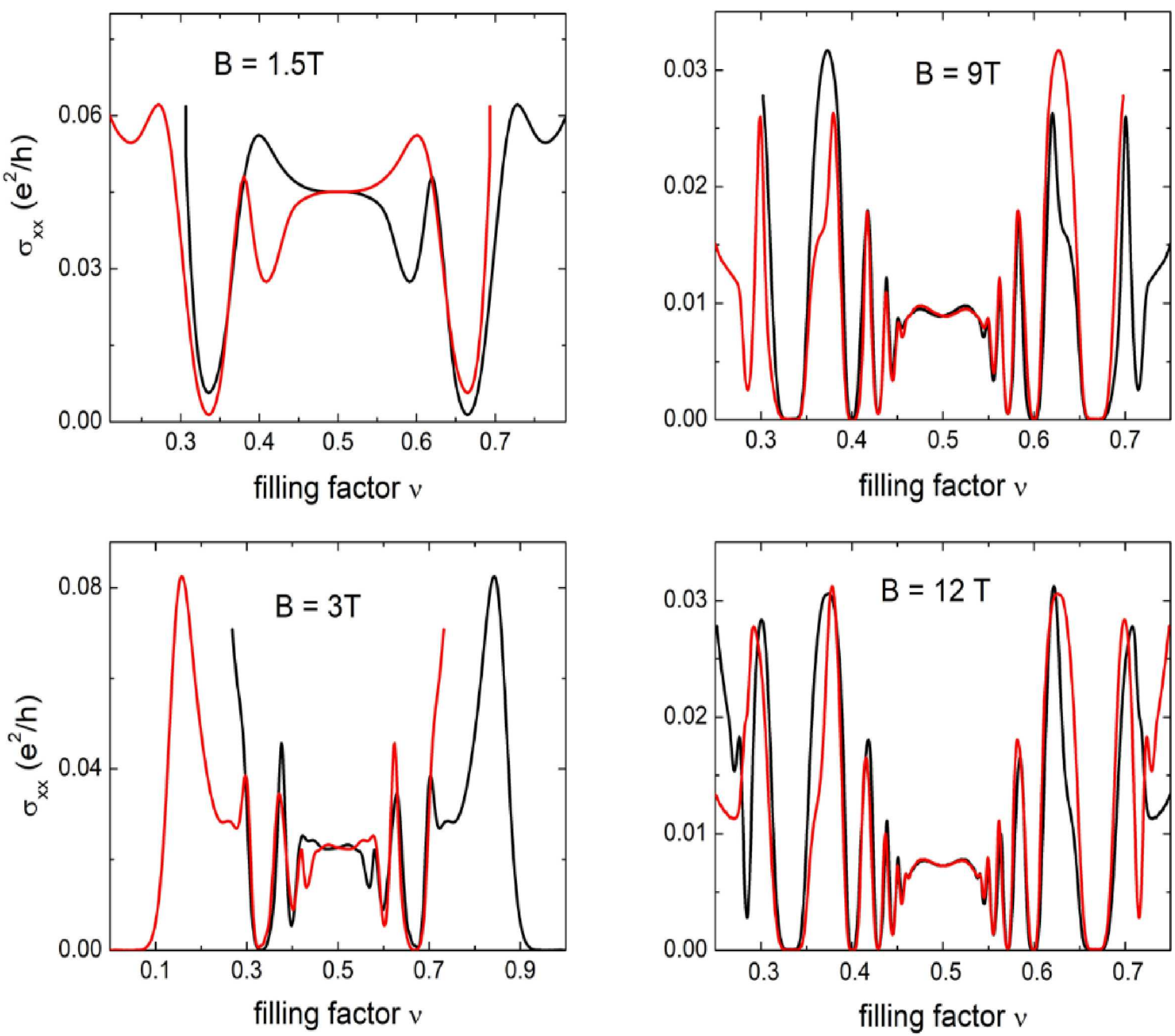
electron conductance

$$\sigma_{xx}(\nu) = \sigma_{xx}\,(1\text{-}\nu)$$

$$\sigma_{xy}(\nu) = 1\text{-}\ \sigma_{xy}\,(1\text{-}\nu)$$



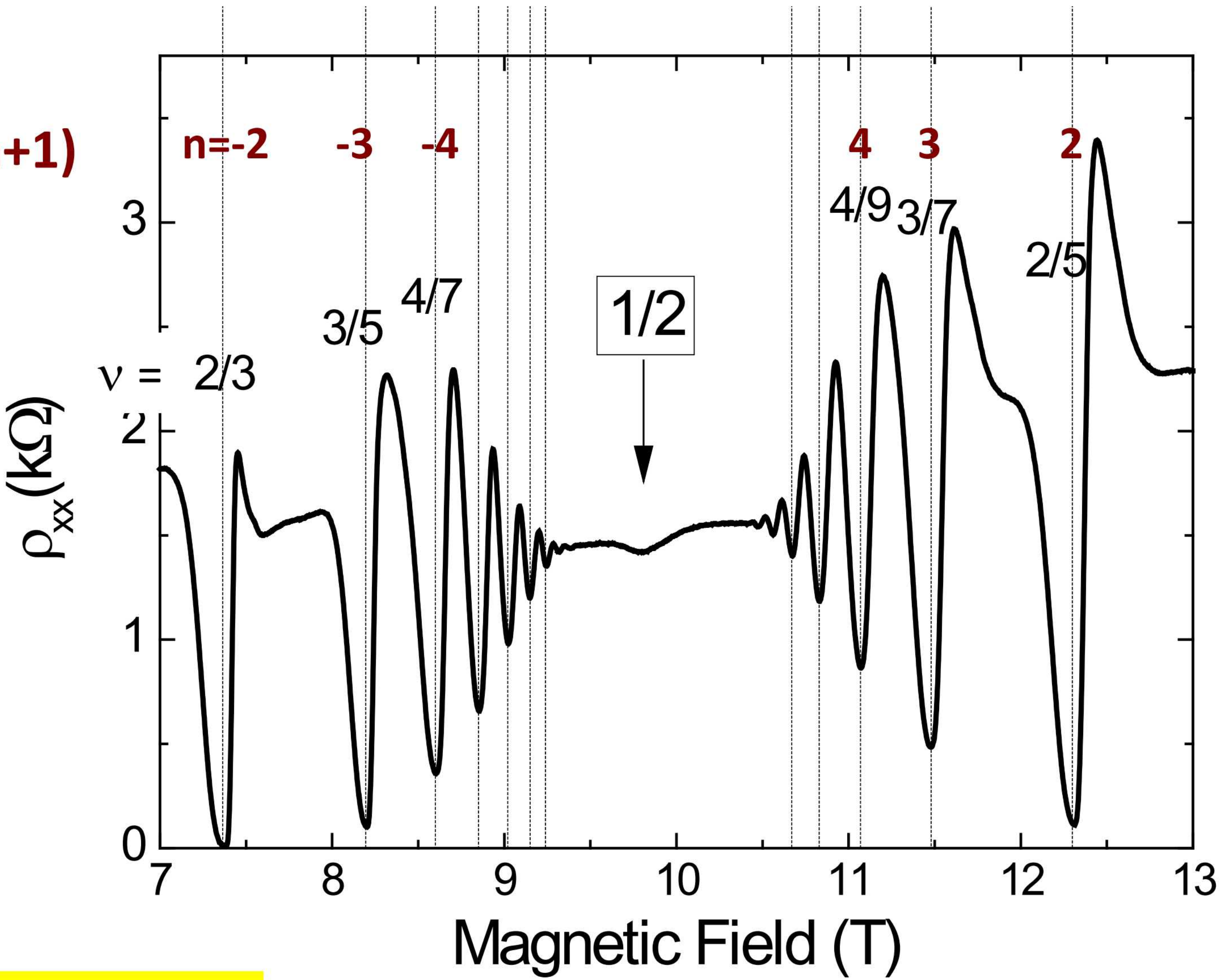
σxx (ν)
σxx (1−ν)
σxy
1-σxy
(a)
T = 30 mK
B = 6T
(b)
(c)
(d)
σxx (e²/h)
σxy (e²/h)
filling factor ν



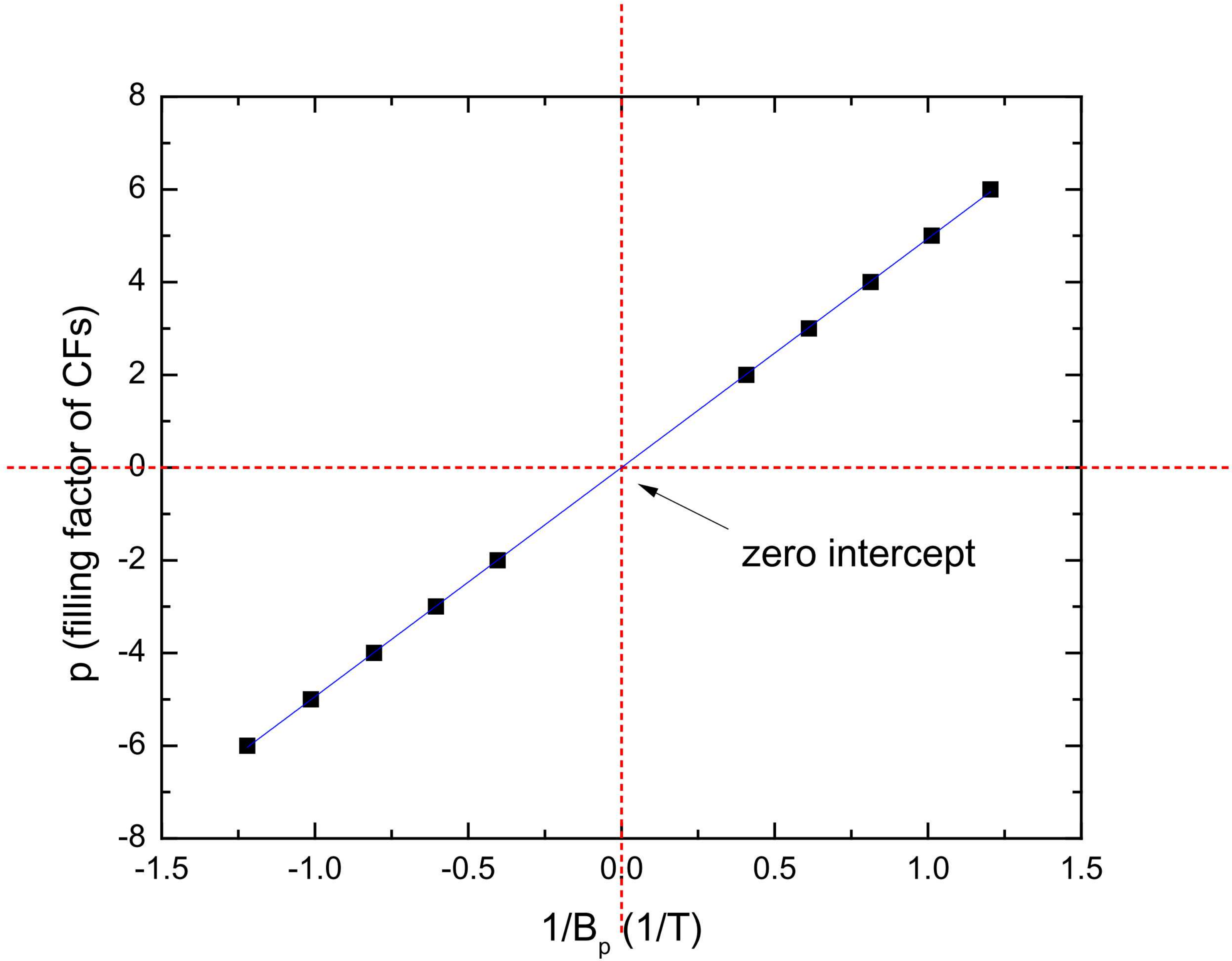
B = 1.5T
σxx (e²/h)
0.00
0.03
0.06
0.3
0.4
0.5
0.6
0.7
filling factor ν
B = 9T
0.00
0.01
0.02
0.03
0.3
0.4
0.5
0.6
0.7
filling factor ν
B = 3T
σxx (e²/h)
0.00
0.04
0.08
0.1
0.3
0.5
0.7
0.9
filling factor ν
B = 12 T
0.00
0.01
0.02
0.03
0.3
0.4
0.5
0.6
0.7
filling factor ν

Thank you

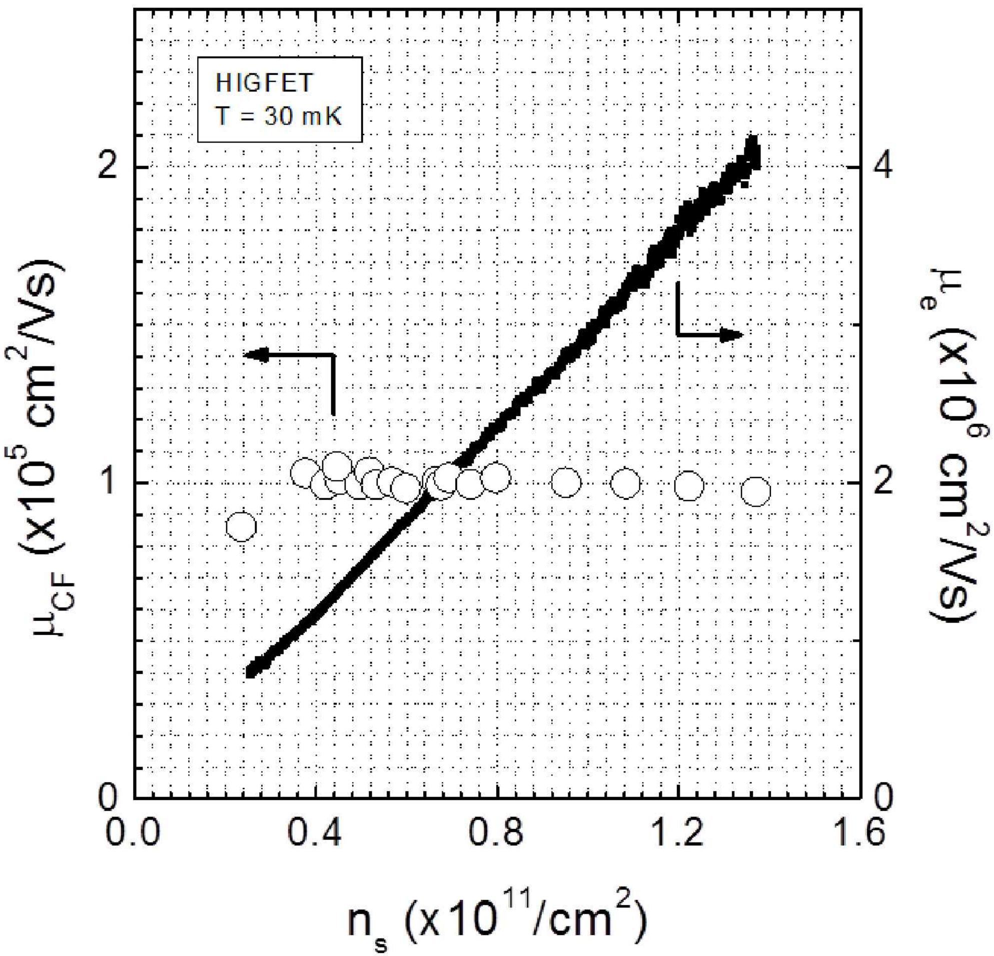
$\nu = n/(2n+1)$

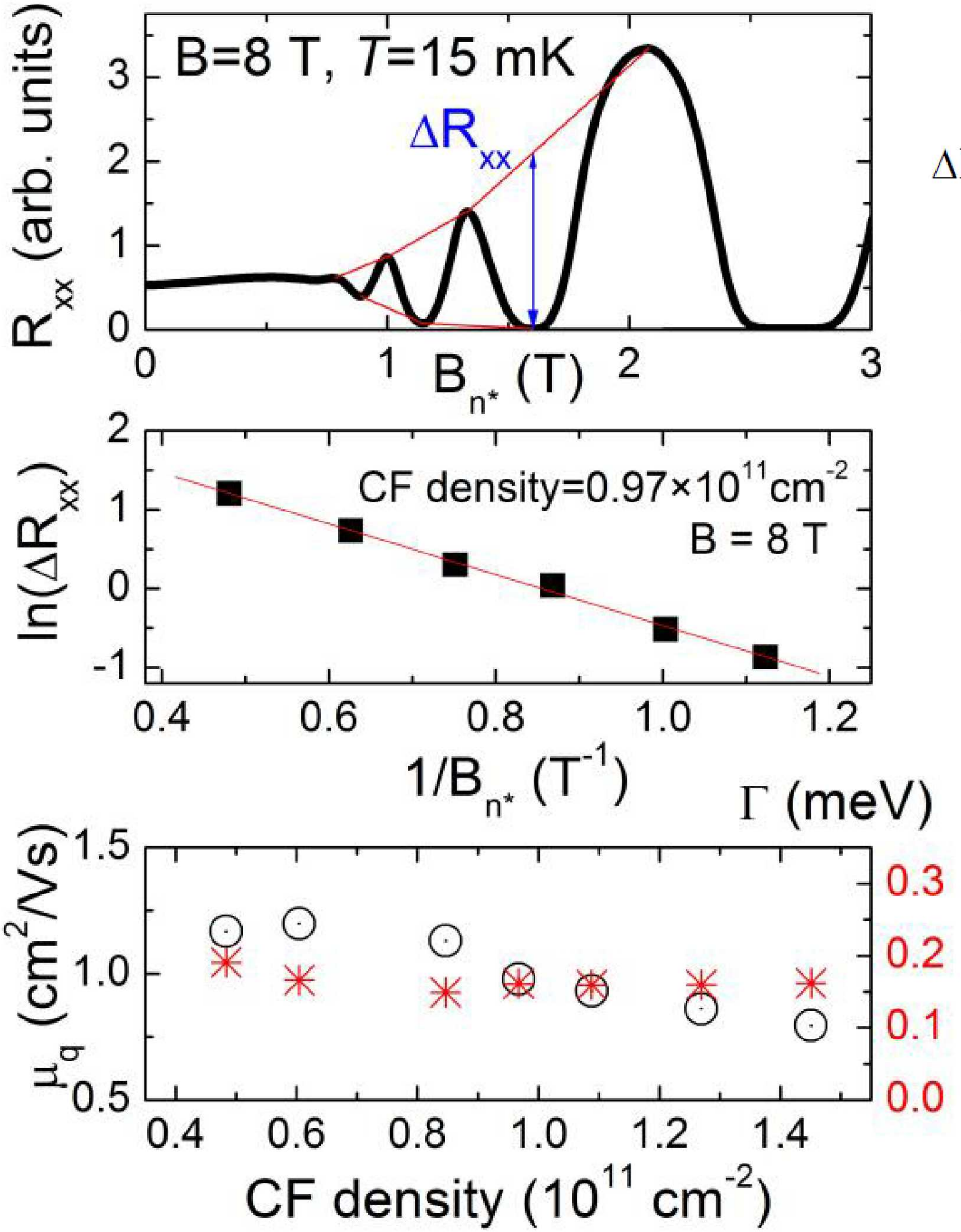


$B_n^* = B_\nu - B_{1/2}$



p (filling factor of CFs)
1/B_p (1/T)
zero intercept
8
6
4
2
0
-2
-4
-6
-8
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5


HIGFET
T = 30 mK
μ_{CF} ($x10^5$ cm^2/Vs)
μ_e ($x10^6$ cm^2/Vs)
n_s ($x10^{11}/cm^2$)
0
1
2
0
2
4
0.0
0.4
0.8
1.2
1.6



$$\Delta R_{xx} = 4R_0 \times X/\sinh(X) \times \exp(-\pi/\mu_q B_n^*)$$

$$X = 2\pi^2 k_B T/\hbar\omega_c, \ \omega_c = eB_n^*/m^*$$

Since X < 0.2

$$X/\sinh(X) \approx 1 - X^2/6 \approx 1$$

$$\Delta R_{xx} = 4R_0 \times \exp(-\pi/\mu_q B_n^*).$$

$$m^*/m_0 = 0.26 \times \sqrt{B_{\nu=1/2}}$$