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Chance-constrained Optimization: Approximations, Algorithms, and Applications

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Some Active Collaborators

- Lauren Meyers (Austin, USA)
- David Morton (Northwestern, USA)
- David Pozo (Skoltech, Russia)
- Steffen Rebennack (Karlsruhe, Germany)
- Jean-Paul Watson (Sandia, USA)

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- Chile?

Contents

Chance Constraint Setting

This is a linear Joint Chance Constraint:

$$P(x_t \leq y_t^\omega + w_t^\omega, \forall t \in T) \geq 1 - \varepsilon$$

Background:

- Two-stage stochastic program with recourse
- First stage decision, x_t, second-stage decision, y_t^ω
- Possibly integer restrictions on x and/or y
- i.i.d. samples of uncertainty w_t^ω

Challenges

- CC models are computationally intractable
- A known NP-hard problem
- Existing algorithms not scalable to practical sized problems
- Feasible region is non-convex

Generic CC model

$$\max_{x,y} \quad \sum_{t \in T} (\text{profit}_t - \mathbb{E}[\text{cost}_t] \tag{1a}$$

$$\text{s.t.} \quad \mathbb{P}(\text{reliability}_t, \forall t \in T) \geq 1 - \varepsilon \tag{1b}$$

$$\text{some domain.} \tag{1c}$$

Generic CC model

$$\max_{x,y} \quad \sum_{t\in T} (R_t x_t - \mathbb{E}[B_t y_t^\omega]) \tag{2a}$$

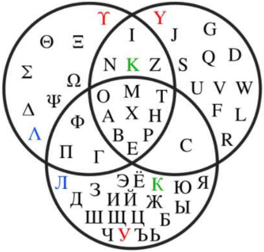
$$\text{s.t.} \quad \mathbb{P}(y_t^\omega + w_t^\omega \geq x_t, \forall t \in T) \geq 1 - \varepsilon \tag{2b}$$

$$(x, y) \in XY. \tag{2c}$$

Contents

Approximations with classical probability bounds

Satisfying a JCC is an intersection of events. Failing a JCC is a union of events.



We can rewrite the JCC as follows:

$$\mathbb{P}(\bigcup_{t \in T} F_t) \leq \varepsilon$$

where $F_t = \{\omega : x_t > y_t^\omega + w_t^\omega\}$.

Approximations with classical probability bounds

$$\mathbb{P}(\bigcup_{t\in T} F_t) \leq \varepsilon$$

Consider an optimization model with a JCC with a maximization objective (such as model (2)).

- Lower Bound (LB): Approximate the LHS using a quantity **larger** than $\mathbb{P}(\bigcup_{t\in T} F_t)$. Feasible region is **restricted**.
- Upper Bound (UB): Approximate the LHS using a quantity **smaller** than $\mathbb{P}(\bigcup_{t\in T} F_t)$. Feasible region is **enlarged**.

Approximations with classical probability bounds

$\mathbb{P}(\bigcup_{t\in T} F_t) = S_1 - S_2 + \cdots (-1)^{|T|-1} S_T$, where $S_k = \mathbb{P}(\sum_{1\le i_1<\cdots<i_k\le|T|} F_{i_1}\cap\cdots\cap F_{i_k})$.

Approximating bounds:

Bonferroni bounds:

$$\mathbb{P}(\bigcup_{t\in T} F_t) \le \quad S_1 \leftarrow LB \tag{3a}$$

$$\mathbb{P}(\bigcup_{t\in T} F_t) \ge \quad S_1 - S_2 \leftarrow UB. \tag{3b}$$

Tighter bounds from Sathe et al. [1980]:

$$\mathbb{P}(\bigcup_{t\in T} F_t) \le \quad S_1 - \tfrac{2}{T} S_2 \leftarrow LB \tag{4a}$$

$$\mathbb{P}(\bigcup_{t\in T} F_t) \ge \quad \frac{S_1 + 2S_2}{T^2} \leftarrow UB. \tag{4b}$$

And more from Dawson and Sankoff [1967]:

$$\mathbb{P}(\bigcup_{t\in T} F_t) \ge \frac{{S_1}^2}{S_1 + 2S_2} \leftarrow \quad UB \tag{5a}$$

$$2\varepsilon S_2 \ge \alpha_n S_1 + \beta_n, n = 0, 1, \ldots |N| - 1 \leftarrow \quad UBlinearized. \tag{5b}$$

Recall...

$$\max_{x,y} \quad \sum_{t \in T} (R_t x_t - \mathbb{E}[B_t y_t^\omega]) \tag{6a}$$

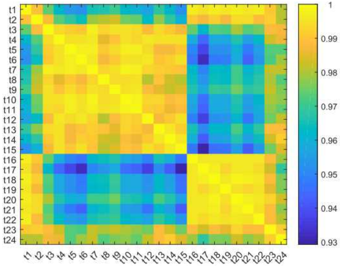
$$\text{s.t.} \quad \mathbb{P}(y_t^\omega + w_t^\omega \geq x_t, \forall t \in T) \geq 1 - \varepsilon \tag{6b}$$

$$0 \leq y_t^\omega \leq \Delta, \forall t \in T, \omega \in \Omega \tag{6c}$$

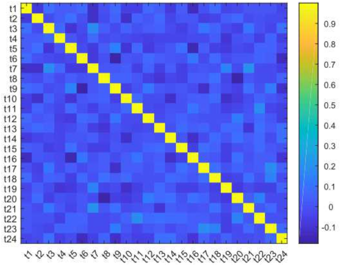
$$x_t \geq 0, \forall t \in T. \tag{6d}$$

Computational results

We compare two sampling procedures: (a) ARMA(2,2) process, and (b) normal random variables. Both samples have the same hourly means and variances.



(a)



(b)

Figure: Correlation structure of w_t

Summary of results

Accepted: *Optimization Letters* (2019)

- Bonferroni lower bound and Dawson & Sankoff linearized bound consistently perform better than others
- Weaker correlation in uncertainty leads to easier-to-solve models
- MIQCP formulation of Dawson & Sankoff bound is challenging

Contents

Stochastic unit commitment

Standard unit commitment (UC) problem: which thermal generators should be scheduled to meet power demand, while ensuring feasible operations?

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Stochastic unit commitment (UC) problem: which thermal generators should be scheduled to meet power demand, while ensuring feasible operations, under uncertainty (of demand, prices, renewables...)?

But...

- Thermal generator operational limits are based on engineering judgments
- Can be exceeded in practice, for short periods
- System operators do run thermal generators beyond these limits

Stochastic unit commitment

Proposed model

- Allow thermal generators to "occasionally" violate operational limits
- Violations should be few (else, increased maintenance costs)
- Violations should not be large (there are absolute ratings of generators)
- 1% savings in energy production is worth $\approx$ \$1 billion per year in the U.S. alone

Stochastic unit commitment

Proposed model

- Let $y_t^{g,\omega}$ denote a "non-nominal" operation in hour t for generator g in scenario ω
- During non-nominal operations, generator's operating region expands from $[\underline{P}^g, \overline{P}^g]$ to $[\underline{\underline{P}}^g, \overline{\overline{P}}^g]$
- Non-nominal mode of generation is more expensive
- Number of non-nominalities is few: $\frac{1}{|\Omega||\mathcal{T}||\mathcal{G}|} \sum_{g \in \mathcal{G}} \sum_{t \in T} \sum_{\omega \in \Omega} y_t^{g,\omega} \leq \varepsilon \leftarrow$ almost a chance-constraint!

Setup

We use:

$$\overline{\overline{P}}^{g} = (1+\beta)\overline{P}^{g}$$
$$\underline{\underline{P}}^{g} = (1-\beta)\underline{P}^{g}$$
$$\overline{C}^{g} = (1+\gamma)C^{L^{g},g}$$
$$\underline{C}^{g} = (1+\gamma)C^{L^{g},g}.$$

WECC240++ system with 85 thermal generators, 50 scenarios and RTS-GMLC system with 73 thermal generators, 16 scenarios

Computational results for the RTS-GMLC 16 scenario case for 10 July 2020.

Under second review: *Computational Management Science*

Table: MIP gap = 0.1%

ε	β	γ	Cost (M$)	Savings (%)	Time (sec)	MIP gap (%)
0			3.89	0.00%	33	-
0.01	0.05	0.1	3.84	1.21%	46	-
		0.2	3.84	1.20%	48	-
	0.1	0.1	3.83	1.51%	82	-
		0.2	3.83	1.50%	106	-
0.05	0.05	0.1	3.83	1.53%	65	-
		0.2	3.83	1.45%	100	-
	0.1	0.1	3.81	2.08%	1800	0.22%
		0.2	3.82	1.82%	1800	0.15%

- Increase $\varepsilon \Rightarrow$ increase savings
- Increase $\beta \Rightarrow$ increase savings
- Increase $\gamma \Rightarrow$ decrease savings

Cost savings for the RTS-GMLC 16 scenario case for 10 July 2020.

Under second review: *Computational Management Science*

ε	β	γ	Optimal	Limited	No nuclear
0.01	0.05	0.1	1.21%	0.71%	1.06%
		0.2	1.20%	0.69%	1.04%
	0.1	0.1	1.51%	1.14%	1.15%
		0.2	1.50%	1.10%	1.11%
0.05	0.05	0.1	1.53%	0.70%	1.22%
		0.2	1.45%	0.69%	1.15%
	0.1	0.1	2.08%	1.14%	1.41%
		0.2	1.82%	1.10%	1.28%

Limited = at most one non-nominal operation per generator per day
No nuclear = no non-nominal operation for the nuclear unit in this system

Contents

An algorithm for two-stage CC stochastic program

Comput Manag Sci
https://doi.org/10.1007/s10287-018-0309-x

CrossMark

ORIGINAL PAPER

An adaptive model with joint chance constraints for a hybrid wind-conventional generator system

Bismark Singh[1] · **David P. Morton**[2] · **Surya Santoso**[3]

Received: 8 September 2017 / Accepted: 19 April 2018

Hybrid solar-battery storage system

under review...

A chance-constrained optimization model for day-ahead scheduling of a hybrid solar-battery storage system

Bismark Singh · David Pozo

Received: date / Accepted: date

Abstract We develop a novel chance-constrained optimization model for a hybrid solar-battery storage system. Solar power in excess of the promise is used to charge the battery, while power short of the promise is met by discharging the battery. We ensure reliable operations by using a joint chance constraint. Models with a few hundred scenarios are relatively tractable; for larger models, we demonstrate how a Lagrangian relaxation scheme provides improved results.

Keywords Chance constraints · Stochastic optimization · Solar power · Photovoltaic power station · Battery storage

Contents

Public health: Largely from PhD dissertation

Singh and Meyers *BMC Res Notes* (2017) 10:178
DOI 10.1186/s13104-017-2496-x

BMC Research Notes

RESEARCH NOTE Open Access

Estimation of single-year-of-age counts of live births, fetal losses, abortions, and pregnant women for counties of Texas

Bismark Singh[1*] and Lauren Ancel Meyers[2]

Abstract

Objectives: We provide a methodology for estimating counts of single-year-of-age live-births, fetal-losses, abortions, and pregnant women from aggregated age-group counts. As a case study, we estimate counts for the 254 counties of Texas for the year 2010.

Results: We use interpolation to estimate counts of live-births, fetal-losses, and abortions by women of each single-year-of-age for all Texas counties. We then use these counts to estimate the numbers of pregnant women for each single-year-of-age, which were previously available only in aggregate. To support public health policy and planning, we provide single-year-of-age estimates of live-births, fetal-losses, abortions, and pregnant women for all Texas counties in the year 2010, as well as the estimation method source code.

Keywords: Live birth, Abortion, Fetal loss, Single-year-of-age, Pregnant women

Public health: Largely from PhD dissertation

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RESEARCH NOTE

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Keywords: Live birth, Abortion, Fetal loss, Single-year-of-age, Pregnant women

PLOS | ONE

RESEARCH ARTICLE

Equalizing access to pandemic influenza vaccines through optimal allocation to public health distribution points

Hsin-Chan Huang[1*], Bismark Singh[1], David P. Morton[2], Gregory P. Johnson[3], Bruce Clements[4], Lauren Ancel Meyers[5,6]

1 Operations Research and Industrial Engineering, The University of Texas at Austin, Austin, TX, United States of America, 2 Industrial Engineering and Management Sciences, Northwestern University, Evanston, IL, United States of America, 3 Texas Advanced Computing Center, The University of Texas at Austin, Austin, TX, United States of America, 4 Health Emergency Preparedness and Response Section, Texas Department of State Health Services, Austin, TX, United States of America, 5 Integrative Biology, The University of Texas at Austin, Austin, TX, United States of America, 6 Santa Fe Institute, Santa Fe, New Mexico, United States of America

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Check for updates

OPEN ACCESS

Citation: Huang H-C, Singh B, Morton DP, Johnson GP, Clements B, Meyers LA (2017) Equalizing access to pandemic influenza vaccines through optimal allocation to public health distribution points. PLoS ONE 12(8): e0182720. https://doi.org/10.1371/journal.pone.0182720

Editor: David L. Swerdlow, Pfizer Inc, UNITED STATES

Received: August 14, 2016

Abstract

Vaccines are arguably the most important means of pandemic influenza mitigation. However, as during the 2009 H1N1 pandemic, mass immunization with an effective vaccine may not begin until a pandemic is well underway. In the U.S., state-level public health agencies are responsible for quickly and fairly allocating vaccines as they become available to populations prioritized to receive vaccines. Allocation decisions can be ethically and logistically complex, given several vaccine types in limited and uncertain supply and given competing priority groups with distinct risk profiles and vaccine acceptabilities. We introduce a model for optimizing statewide allocation of multiple vaccine types to multiple priority groups, maxi-

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CDC › EID journal › Volume 21 › Number 2—February 2015

Volume 21, Number 2—February 2015

Research

Optimizing Distribution of Pandemic Influenza Antiviral Drugs

Bismark Singh, Hsin-Chan Huang, David P. Morton, Gregory P. Johnson, Alexander Gutfraind, Alison P. Galvani, Bruce Clements, and Lauren A. Meyers

Author affiliations: The University of Texas at Austin, Austin, Texas, USA (B. Singh, H.-C. Huang, G.P. Johnson, L.A. Meyers); Northwestern University, Evanston, Illinois, USA (D.P. Morton); University of Illinois at Chicago Illinois, USA (A. Gutfraind); Yale University, New Haven, Connecticut, USA (A.P. Galvani); Texas Department of State Health Services, Austin (B. Clements)

Cite This Article

Abstract

We provide a data-driven method for optimizing pharmacy-based distribution of antiviral drugs during an influenza pandemic in terms of overall access for a target population and apply it to the state of Texas, USA. We found that during the 2009 influenza pandemic, the Texas Department of State Health Services achieved an estimated statewide access of 88% (proportion of population willing to travel to the nearest dispensing point). However, access reached only 34.5% of US postal code (ZIP code) areas containing <1,000 underinsured persons. Optimized distribution networks increased expected access to 91% overall and 60% in hard-to-reach regions, and 2 or 3 major pharmacy chains achieved near maximal coverage in well-populated areas. Independent pharmacies were essential for reaching ZIP code areas containing <1,000 underinsured persons. This model was developed during a collaboration between academic researchers and public health officials and is available as a decision support tool for Texas Department of State Health Services at a Web-based interface.

Also under review...

Noname manuscript No.
(will be inserted by the editor)

Evolutionary Stable Strategies in a Queue from *Seinfeld*

Bismark Singh

Received: date / Accepted: date

Abstract Motivated by the episode *The Soup Nazi* of the sitcom *Seinfeld*, we study customers' psychological strategies in a queue with forced abandonments. Using ideas from evolutionary game theory, we investigate whether an aggressive behavior which encourages other customers to leave the queue is an evolutionary stable strategy. We first quantify some of the queue parameters using two queueing routines. Under natural assumptions and using a classical hawk-and-dove model, we show that a few hawk-type customers in a queue full of dove-type customers might not succeed, while a few dove-type customers in a

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TWO-STAGE STOCHASTIC MINIMUM $S-T$ CUT PROBLEMS: FORMULATIONS, COMPLEXITY AND DECOMPOSITION ALGORITHMS

STEFFEN REBENNACK[*], OLEG A. PROKOPYEV[†], AND BISMARK SINGH [*‡]

Abstract. We introduce the two-stage stochastic minimum $s-t$ cut problem. Based on a classical linear 0-1 programming model for the deterministic minimum $s-t$ cut problem, we provide a mathematical programming formulation for the proposed stochastic extension. We show that its constraint matrix loses the total unimodularity property, however, preserves it if the considered graph is a tree. This fact turns out to be not surprising as we prove that the considered problem is $\mathcal{NP}$-hard in general, but admits a linear time solution algorithm when the graph is a tree. We exploit the special structure of the problem and propose a tailored Benders decomposition algorithm. We demonstrate the computational efficiency of this algorithm by solving the Benders dual subproblems as maximum flow problems. For some of the tested instances, we outperform a monolithic formulation by several orders of magnitude.

Key words. two-stage stochastic programming; minimum $s-t$ cut problem; combinatorial optimization; total unimodularity; complexity; Benders decomposition; maximum flow

1. Introduction. Let $G=(V,E)$ be a directed graph with node set V, arc set $E \subseteq V \times V$, and non-negative costs c_{ij} given for each arc $ij \in E$. The *minimum $s-t$ cut problem* for directed graphs can be defined as follows.

Contents

Optimizing over JCCs

$u_t^\omega = 1$: failure at t in scenario ω
$v_{tt'}^\omega = 1$: failure at t and t' in scenario ω

$$x_t - y_t^\omega - w_t^\omega \leq M_t^\omega u_t^\omega, \forall t \in T, \omega \in \Omega$$

$$\text{McCormick envelope} \begin{cases} v_{t,t'}^\omega \leq u_t^\omega, (t, t') \in T, t < t', \omega \in \Omega \\ v_{t,t'}^\omega \leq u_{t'}^\omega, \forall (t, t') \in T, t < t', \omega \in \Omega \\ v_{t,t'}^\omega \geq u_t^\omega + u_{t'}^\omega - 1, \forall (t, t') \in T, t < t', \omega \in \Omega \end{cases}$$

$$u_t^\omega = \{0, 1\}, \forall t \in T, v_{t,t'}^\omega = \{0, 1\}, \forall (t, t') \in, \omega \in \Omega$$

Computational results: ARMA (large correlation)

ε	Bounding constraint	Optimal objective value Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(3a)	**8,351.3**	8,351.3	0%	2	3.3%
	(3b)	21,282.8	**21,282.8**	0%	12	59.4%
	(4a)	**8,351.3**	8,365 .8	0.1%	2100	3.3%
	(4b)	8,339.6	**10,682.1**	21.9%	2100	19.2%
	(5a)	8,339.7	**8,726.7**	4.5%	2100	1.1%
	(5b)	8,688.9	**8,702.1**	0.2%	2100	0.8%
0.03	(3a)	**8,374.6**	8,374.6	0%	2	8.5%
	(3b)	22,353.2	**22,353.2**	0%	14	59.0%
	(4a)	**8,339.6**	8,755.4	4.7%	2100	8.9%
	(4b)	8,339.6	**13,321.2**	37.4%	2100	31.3%
	(5a)	9,137.3	**9,311.4**	1.9%	2100	1.7 %
	(5b)	9,074.4	**9,252.2**	1.9%	2100	1.1%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 8,351.3 and 8,702.1; true optimal value is 8,634.1
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 8,374.6 and 9,252.2; true optimal value is 9,154.9

Computational results: Gaussian (weak correlation)

ε	Bounding constraint	Optimal objective value Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(3a)	**9,100.8**	9,100.8	0%	1	2.7%
	(3b)	21,606.6	**21,606.6**	0%	18	56.7%
	(4a)	**9,102.0**	9,113.3	0.1%	2100	2.7%
	(4b)	9092.3	**11,365.5**	20%	2100	17.7%
	(5a)	9,434.3	**9,486.3**	0.5%	2100	1.4%
	(5b)	9,421.5	**9,452.3**	0.3%	2100	1.1%
0.03	(3a)	**9,124.3**	9,124.3	0%	2	7.7%
	(3b)	22,762.1	**22,762.1**	0%	21	56.6%
	(4a)	**9,124.8**	9,198.4	0.8%	2100	7.7%
	(4b)	9,092.3	**13,907.6**	34.9%	2100	28.9%
	(5a)	9,092.3	**10,062.6**	9.6%	2100	1.8%
	(5b)	9,092.3	**10,004.8**	9.1%	2100	1.2%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 9,100.8 and 9,449.9; true optimal value is 9,353.2
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 9,124.3 and 10,004.8; true optimal value is 9,884.0

Computational results: ARMA (large correlation) with 500 scenarios

ε	Bounding constraint	Optimal objective value Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(3a)	**8,453.4**	8,453.4	0%	1	2.9%
	(3b)	21,582.9	**21,582.9**	0%	129	59.7%
	(4a)	**8,701.0**	8,701.0	0%	1717	0%
	(4b)	10,462.7	**11,318.4**	7.5%	2100	23.1%
	(5a)	8,348.9	**40,116.9**	79.2%	2100	78.3%
	(5b)	8,348.9	**8,772.9**	4.8%	2100	0.8%
0.03	(3a)	**8,542.5**	8,542.5	0%	3	7.3%
	(3b)	22,570.6	**22,570.6**	0%	175	59.2%
	(4a)	**8,348.9**	9,396.1	11.1%	2100	9.4%
	(4b)	8,348.9	**15,127.8**	44.8%	2100	39.1%
	(5a)	8,348.9	**41,151.4**	79.8%	2100	77.6 %
	(5b)	8,348.9	**9,352.9**	10.7%	2100	1.5%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 8,701.0 and 8,772.9; true optimal value is 8,701.0
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 8,542.5 and 9,352.9; true optimal value is 9,211.3

Computational results: Gaussian (weak correlation) with 500 scenarios

ε	Bounding constraint	Optimal objective value Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(3a)	**9,005.1**	9,005.1	0%	1	3.7%
	(3b)	21,503.7	**21,503.7**	0%	75	56.5%
	(4a)	**8866.9**	8,889.3	1.3%	2100	5.1%
	(4b)	8,866.9	**11,071.9**	19.9%	2100	15.6%
	(5a)	8,866.9	**40,126.1**	77.9%	2100	76.7%
	(5b)	9,343.6	**9,390.3**	0.5%	2100	0.5%
0.03	(3a)	**9,148.2**	9,148.2	0%	3	7.4%
	(3b)	22,565.4	**22,565.4**	0%	46	56.2%
	(4a)	**8,866.9**	9,315.3	4.8%	2100	10.2%
	(4b)	8,866.9	**13,711.9**	35.3%	2100	27.9%
	(5a)	8,866.9	**41,187.8**	78.5%	2100	76.0 %
	(5b)	8,866.9	**9,990.9**	11.2%	2100	1.2%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 9,005.1 and 9,390.3; true optimal value is 9,346.4
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 9,148.2 and 9,990.9; true optimal value is 9,874.1

Stochastic unit commitment model

Indices and Sets:

$g \in \mathcal{G}$	Thermal generators.
$t \in \mathcal{T}$	Hourly time steps: $1, \ldots, T$; i.e., $[a, b) \in \mathcal{T} \times \mathcal{T}$ such that $b \geq a + UT^g$.
$l \in \mathcal{L}^g$	Piecewise production cost intervals for generator g: $1, \ldots, L_g$.
$s \in \mathcal{S}^g$	Start-up categories for generator g, from hottest (1) to coldest (S_g).
$\omega \in \Omega$	Scenarios: $\omega_1, \ldots, \omega_N$.

Parameters: First Stage

$C^{l,g}$	Marginal cost for piecewise segment l for generator g (\$/MWh).
$\overline{C}^g$	Marginal cost for production above $\overline{P}^g$ (\$/MWh).
$\underline{C}^g$	Marginal cost for production below $\underline{P}^g$ (\$/MWh).
$C^{R,g}$	Cost of generator g running and operating at minimum production $\underline{P}_g$ (\$/h).
$C^{s,g}$	Start-up cost of category s for generator g (\$).
DT^g	Minimum down time for generator g (h).
$\overline{P}^g$	Maximum power output for generator g under normal operations (MW).
$\overline{\overline{P}}^g$	Maximum power output for generator g under non-nominal operations (MW).
$\underline{P}^g$	Minimum power output for generator g under normal operations (MW).
$\underline{P}^g$	Minimum power output for generator g under non-nominal operations (MW).
$\overline{\overline{P}}^{l,g}$	Maximum power available for piecewise segment l for generator g (MW) (with $\overline{P}^{0,g} = \underline{P}^g$).
RD^g	Ramp-down rate for generator g (MW/h).
RU^g	Ramp-up rate for generator g (MW/h).
SD^g	Shutdown ramp rate for generator g (MW/h).
SU^g	Start-up ramp rate for generator g (MW/h).
TC^g	Time down after which generator g goes cold (h).
$\underline{T}^{s,g}$	Time offline after which the start-up category s is available (h) (with $\underline{T}^{1,g} = DT^g$, $\underline{T}^{S^g,g} = TC^g$)
UT^g	Minimum up time for generator g (h).

Stochastic unit commitment model

Parameters: Second Stage

D_t^ω	Load (demand) at time t in scenario ω (MW).
$\overline{W}_t^\omega$	Maximum power from renewables at time t in scenario ω (MW).
$\underline{W}_t^\omega$	Minimum power from renewables at time t in scenario ω (MW).

Variables: First Stage

u_t^g	Commitment status of generator g at time t, $\in \{0,1\}$.
v_t^g	Start-up status of generator g at time t, $\in \{0,1\}$.
w_t^g	Shutdown status of generator g at time t, $\in \{0,1\}$.
$x_{[t,t')}^g$	Indicator arc for shutdown at time t, start-up at time t', uncommitted for $i \in [t,t')$, for generator g, $\in \{0,1\}$, $[t,t')$ such that $t + DT^g \leq t' \leq t + TC^g - 1$.

Variables: Second Stage

$p_t^{g,\omega}$	Power above minimum from generator g at time t in scenario ω (MW).
$\overline{P}_t^{g,\omega}$	Power above maximum from generator g at time t in scenario ω (MW).
$\underline{P}_t^{g,\omega}$	Power below minimum from generator g at time t in scenario ω (MW).
$p_t^{l,g,\omega}$	Power from piecewise interval l for generator g at time t in scenario ω (MW).
$r_t^{n,\omega}$	Power from renewables at time t in scenario ω (MW).
$y_t^{g,\omega}$	Non-nominal operation status of generator g at time t in scenario (MW).

Stochastic unit commitment

$$\min \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} \left(\sum_{l\in\mathcal{L}^g} \mathbb{E}[C^{l,g}\, p_t^{l,g,\omega} + \overline{C}^g\, \overline{P}_t^{g,\omega} + \underline{C}^g\, \underline{P}^{g,\omega}] + C^{R,g}\, u_t^g + c_t^{SU,g} \right) \quad (7)$$

subject to:

$$u_t^g - u_{t-1}^g = v_t^g - w_t^g \qquad \forall t\in\mathcal{T},\ \forall g\in\mathcal{G} \quad (8a)$$

$$\sum_{i=t-UT^g+1}^{t} v_i^g \le u_t^g \qquad \forall t\in[UT^g, T],\ \forall g\in\mathcal{G} \quad (8b)$$

$$\sum_{i=t-DT^g+1}^{t} w_i^g \le 1 - u_t^g \qquad \forall t\in[DT^g, T],\ \forall g\in\mathcal{G} \quad (8c)$$

$$\sum_{t'=t-TC^g+1}^{t-DT^g} x_{[t',t)}^g \le v_t^g \qquad \forall t\in\mathcal{T},\ \forall g\in\mathcal{G} \quad (8d)$$

$$\sum_{t'=t+DT^g}^{t+TC^g-1} x_{[t,t')}^g \le w_t^g \qquad \forall t\in\mathcal{T},\ \forall g\in\mathcal{G} \quad (8e)$$

$$c_t^{SU,g} = C^{S,g} v_t^g + \sum_{s=1}^{S^g-1} (C^{s,g} - C^{S,g}) \left(\sum_{t'=t-\underline{T}^{s+1,g}+1}^{t-\underline{T}^{s,g}} x_{[t',t)}^g \right) \qquad \forall t\in\mathcal{T},\ \forall g\in\mathcal{G} \quad (8f)$$

Stochastic unit commitment

$$p_t^{g,\omega} \leq (\overline{P}^g - \underline{P}^g)u_t^g - (\overline{P}^g - SU^g)v_t^g - (\overline{P}^g - SD^g)w_{t+1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{>1}, \forall \omega \in \Omega \tag{9a}$$

$$p_t^{g,\omega} \leq (\overline{P}^g - \underline{P}^g)u_t^g - (\overline{P}^g - SU^g)v_t^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \tag{9b}$$

$$p_t^{g,\omega} \leq (\overline{P}^g - \underline{P}^g)u_t^g - (\overline{P}^g - SD^g)w_{t+1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \tag{9c}$$

$$p_t^{g,\omega} - p_{t-1}^{g,\omega} \leq (SU^g - RU^g - \underline{P}^g)v_t^g + RU^g u_t^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \tag{9d}$$

$$p_{t-1}^{g,\omega} - p_t^{g,\omega} \leq (SD^g - RD^g - \underline{P}^g)w_t^g + RD^g u_{t-1}^g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \tag{9e}$$

$$p_t^{g,\omega} = \sum_{l \in \mathcal{L}^g} p_t^{l,g,\omega} \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \tag{9f}$$

$$p_t^{l,g,\omega} \leq (\overline{P}^{l,g} - \overline{P}^{l-1,g})u_t^g \quad \forall t \in \mathcal{T}, \forall l \in \mathcal{L}^g, \forall g \in \mathcal{G}, \forall \omega \in \Omega \tag{9g}$$

Stochastic unit commitment

$$y_t^{g,\omega} \leq u_t^g - v_t^g - w_{t+1}^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{>1}, \forall \omega \in \Omega \tag{10a}$$

$$y_t^{g,\omega} \leq u_t^g - v_t^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \tag{10b}$$

$$y_t^{g,\omega} \leq u_t^g - w_{t+1}^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \tag{10c}$$

$$\overline{P}_t^{g,\omega} \leq \left(\overline{\overline{P}} - \overline{P}\right) y_t^{g,\omega} \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{10d}$$

$$\underline{P}_t^{g,\omega} \leq \left(\underline{P} - \underline{\underline{P}}\right) y_t^{g,\omega} \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{10e}$$

$$\sum_{g \in \mathcal{G}} \left(p_t^{g,\omega} + \overline{P}_t^{g,\omega} - \underline{P}_t^{g,\omega} + \underline{P}^g u_t^g\right) + r_t^\omega = D_t^\omega \forall t \in \mathcal{T},\ \forall \omega \in \Omega \tag{11}$$

$$\frac{1}{|\mathcal{G}||\mathcal{T}||\Omega|} \sum_{\omega \in \Omega} \sum_{g \in \mathcal{G}} \sum_{t \in T} y_t^{g,\omega} \leq \varepsilon \tag{12}$$

$$p_t^{l,g,\omega} \in \mathbb{R}_+ \qquad \forall t \in \mathcal{T},\ \forall l \in \mathcal{L}^g,\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{13a}$$

$$p_t^{g,\omega}, \overline{P}_t^{g,\omega}, \underline{P}_t^{g,\omega} \in \mathbb{R}_+ \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{13b}$$

$$r_t^{n,\omega} \in \left[\underline{W}_t^{n,\omega}, \overline{W}_t^{n,\omega}\right] \qquad \forall t \in \mathcal{T}, \forall n \in \mathcal{N}, \forall \omega \in \Omega \tag{13c}$$

$$u_t^g, v_t^g, w_t^g \in \{0,1\} \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{13d}$$

$$x_{[t,t')}^g \in \{0,1\} \qquad \forall [t,t') \in \mathcal{X}^g,\ \forall g \in \mathcal{G} \tag{13e}$$

$$y_t^{g,\omega} \in \{0,1\} \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega. \tag{13f}$$

Computational results for the WECC240++ 50 scenario test case for 11 May 2013.

Table: MIP gap = 0.1%

ε	β	γ	Cost (K$)	Savings (%)	Time (sec)	MIP gap (%)
0			64.41	0.00%	183	-
0.01	0.05	0.1	64.20	0.33%	275	-
		0.2	64.21	0.31%	242	-
	0.1	0.1	64.03	0.59%	258	-
		0.2	64.04	0.58%	317	-
0.05	0.05	0.1	63.86	0.85%	275	-
		0.2	63.90	0.80%	343	-
	0.1	0.1	63.35	1.64%	378	-
		0.2	63.42	1.55%	371	-

- Increase $\varepsilon \Rightarrow$ increase savings
- Increase $\beta \Rightarrow$ increase savings
- Increase $\gamma \Rightarrow$ decrease savings

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