SAND2019-2597C

A Cross Field Approach to Automatic STL-to-CAD Conversion

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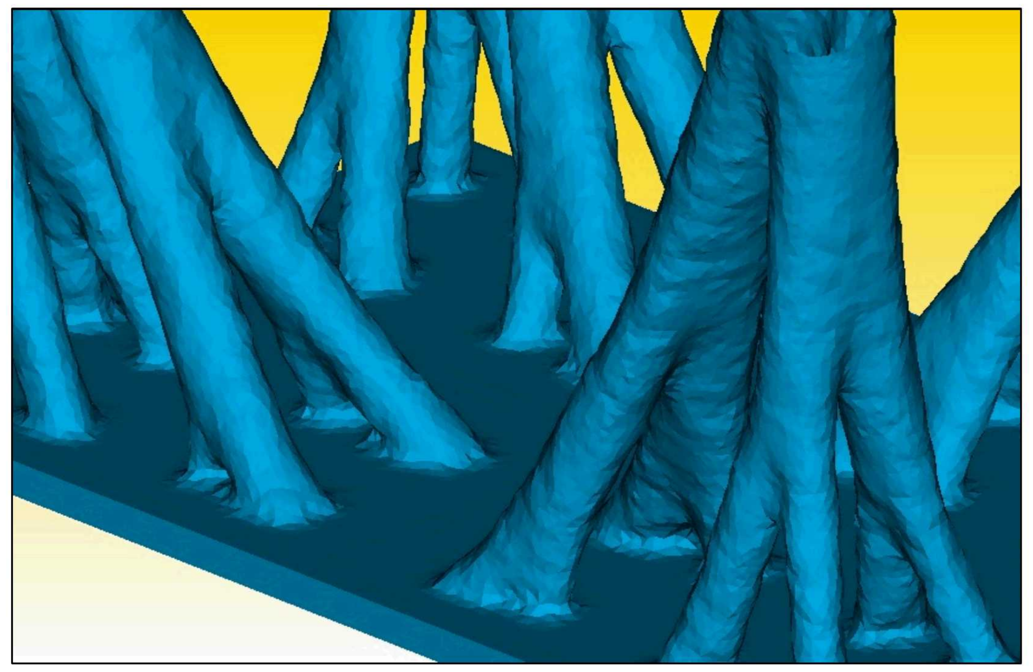
Motivation

While many topology optimization (TO) platforms output discrete geometric representations such as stereolithography (STL) files, many end users require analytic CAD geometry representations for tasks such as analysis, integration with previous designs, manufacturing, and qualification. In practice, generating a CAD model from a TO design is currently a manual process. Automatic STL-to-CAD conversion is a technology gap that is limiting the adoption of TO as a primary design tool.

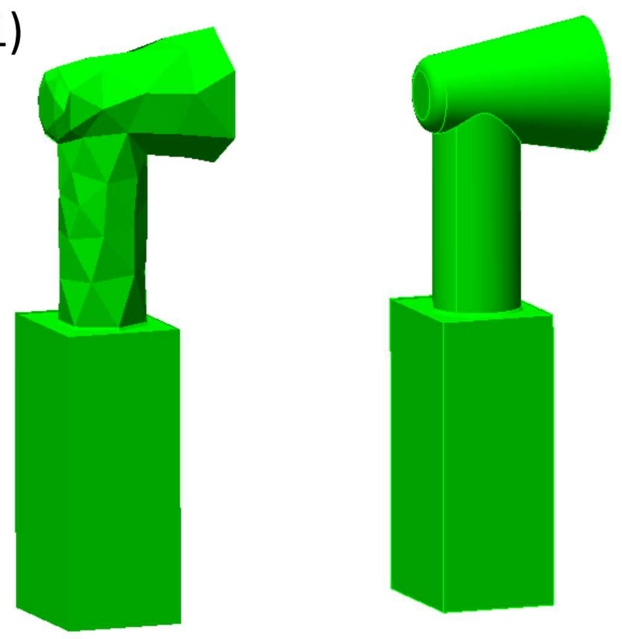


(Courtesy of Paul Stallings)

STL-to-CAD Pipeline

1) Find "special" surfaces
2) Decompose remaining surfaces into patches
3) Compute parameterization
4) Fit a NURBS surface to each patch

1)



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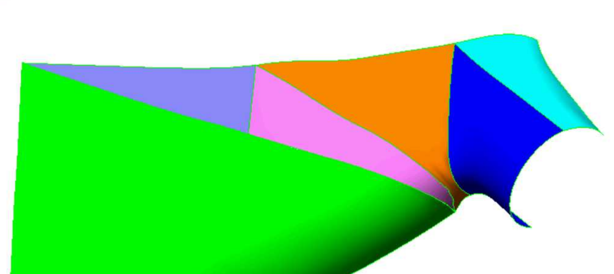

(Courtesy of Paul Stallings)

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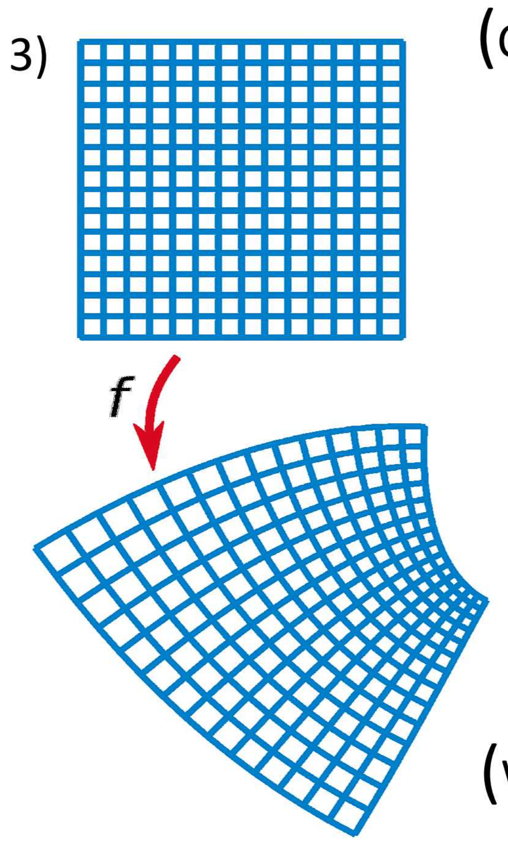

(Wikipedia)

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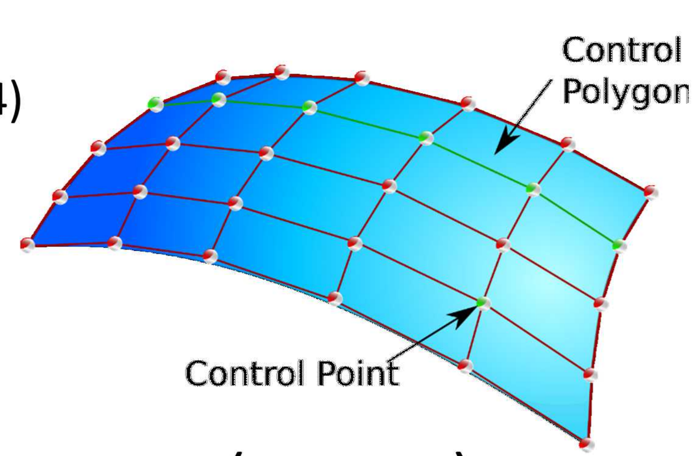
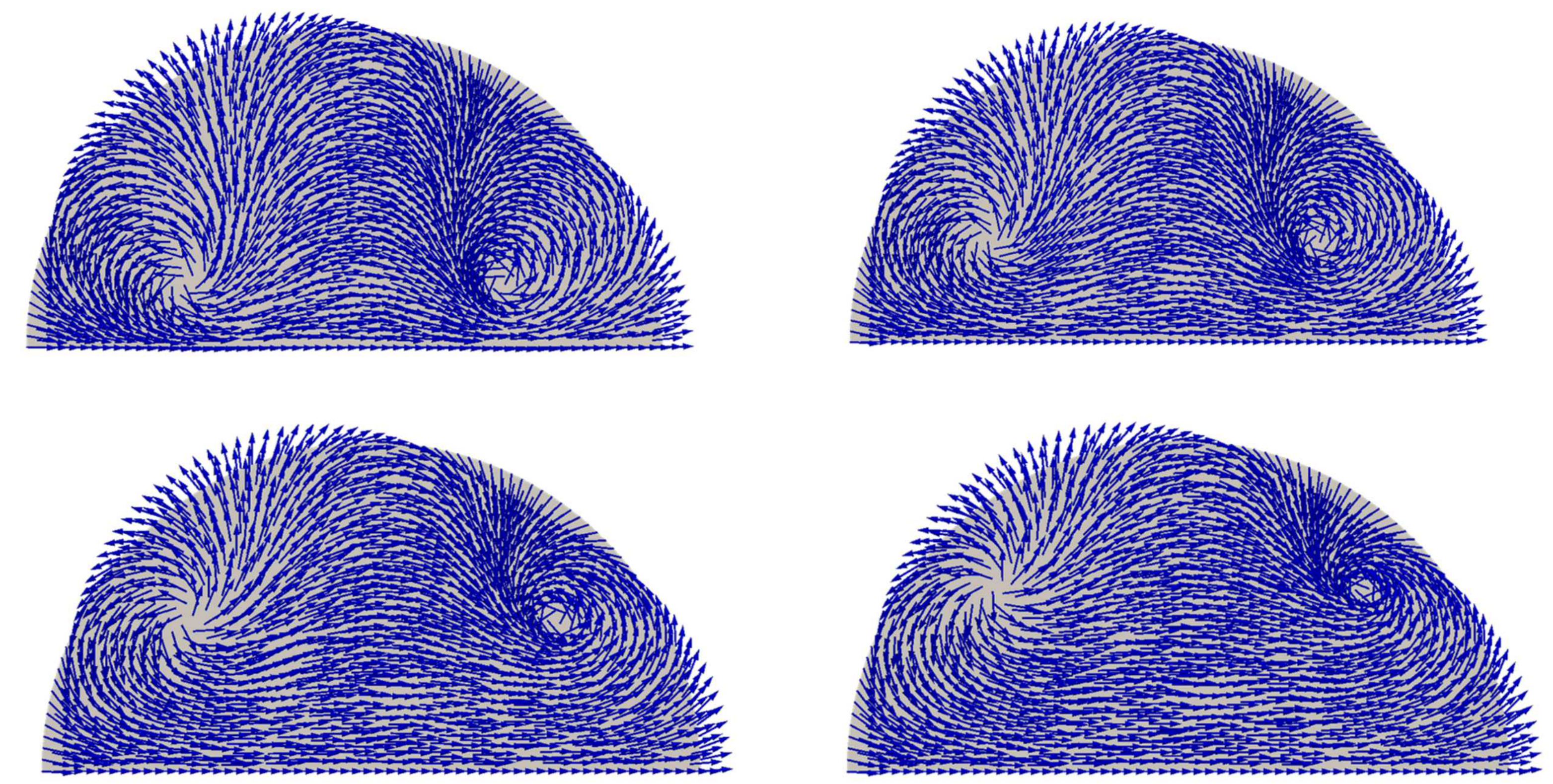


(Wikipedia)

Merriman-Bence-Osher (MBO) Method

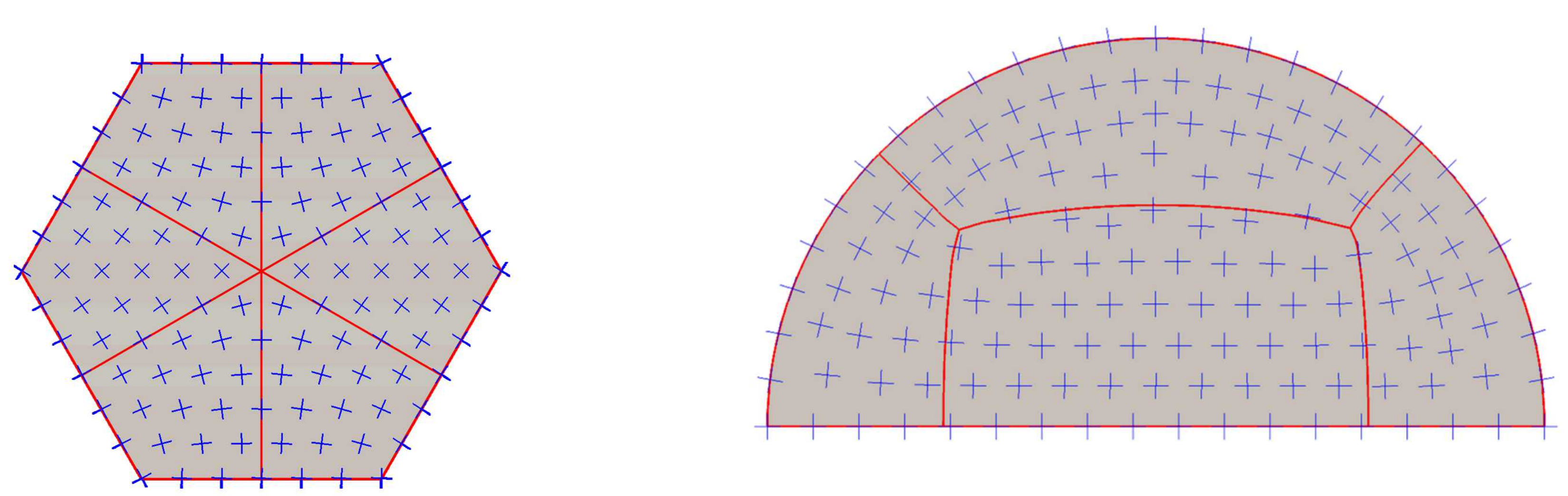
The MBO method was originally introduced as a method for motion by mean curvature. We have adapted it as a method to minimize the cross field energy. The Method is defined by the iterations:

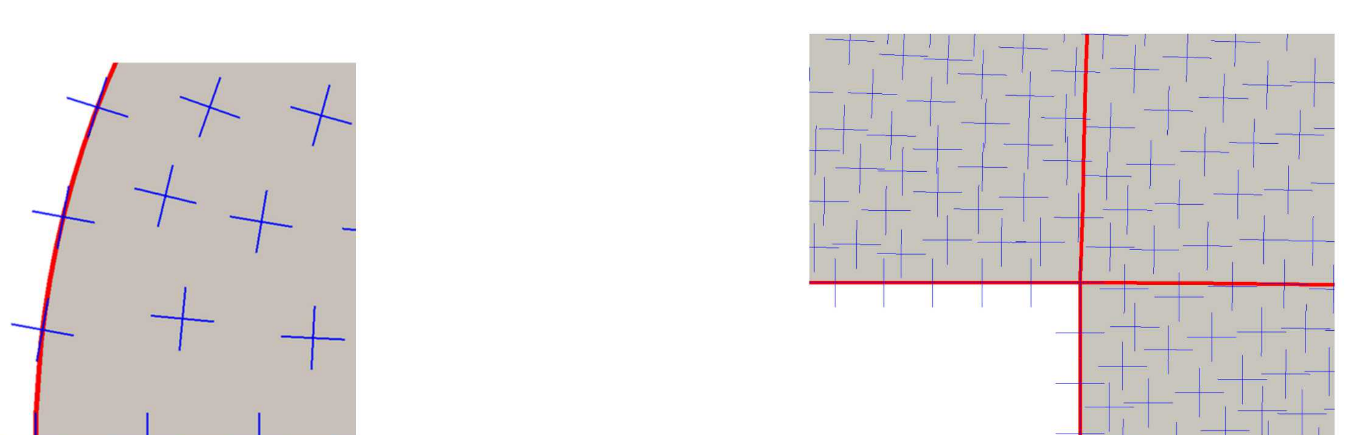
$$u_0 = \frac{\tilde{u}}{|\tilde{u}|} \qquad \text{and} \qquad u_k = \frac{e^{\tau\Delta} u_{k-1}}{|e^{\tau\Delta} u_{k-1}|} \qquad k \geq 1.$$



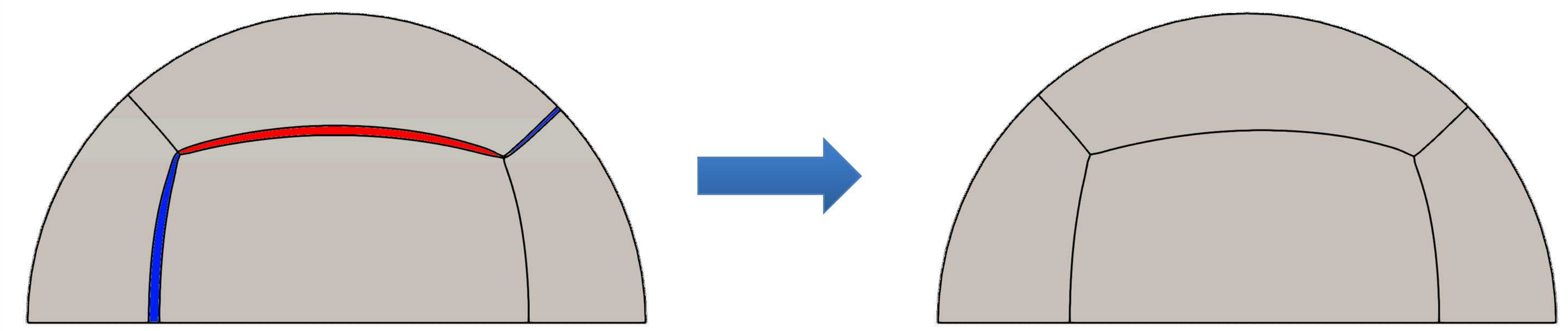
Asymptotic Behavior of Cross Fields Near Singularities

Lemma 5.1. *Let f be a boundary-aligned canonical harmonic cross field on D. Let a be an interior singularity of f of index $d/4$ with $d < 4$. There are exactly $4 - d$ separatrices meeting at a. These separatrices partition a neighborhood of a into $4 - d$ even-angled sectors.*



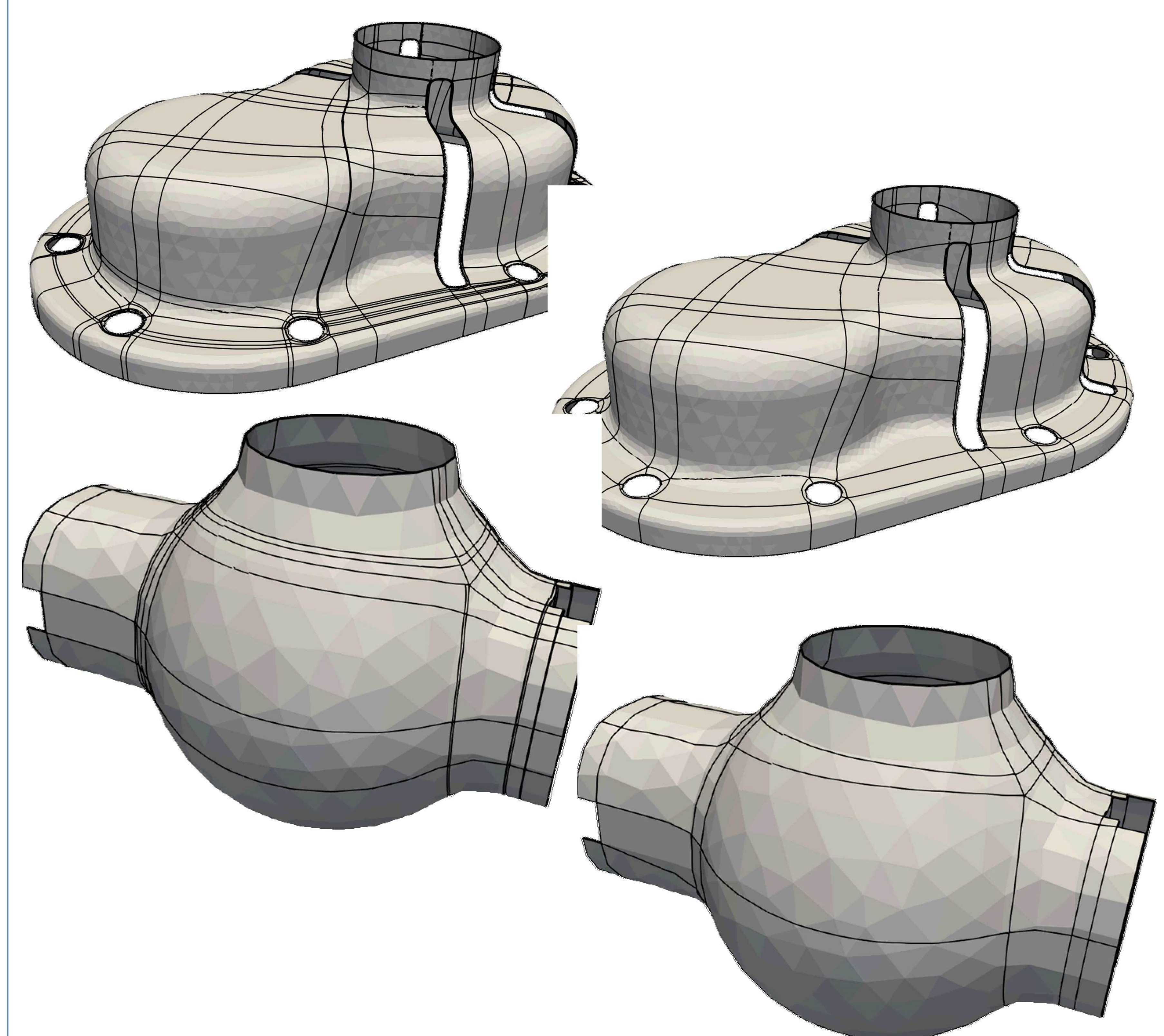


Partition Simplification



Algorithm 3 Partition Simplification

Let Γ be the set of collapsible chords of the partition
while $|\Gamma| > 0$ **do**
  if No chords meet the conditions for collapse **then**
    Stop.
  else
    Collapse the chord with the smallest minimum width
    Determine new set of collapsible chords Γ
  end if
end while



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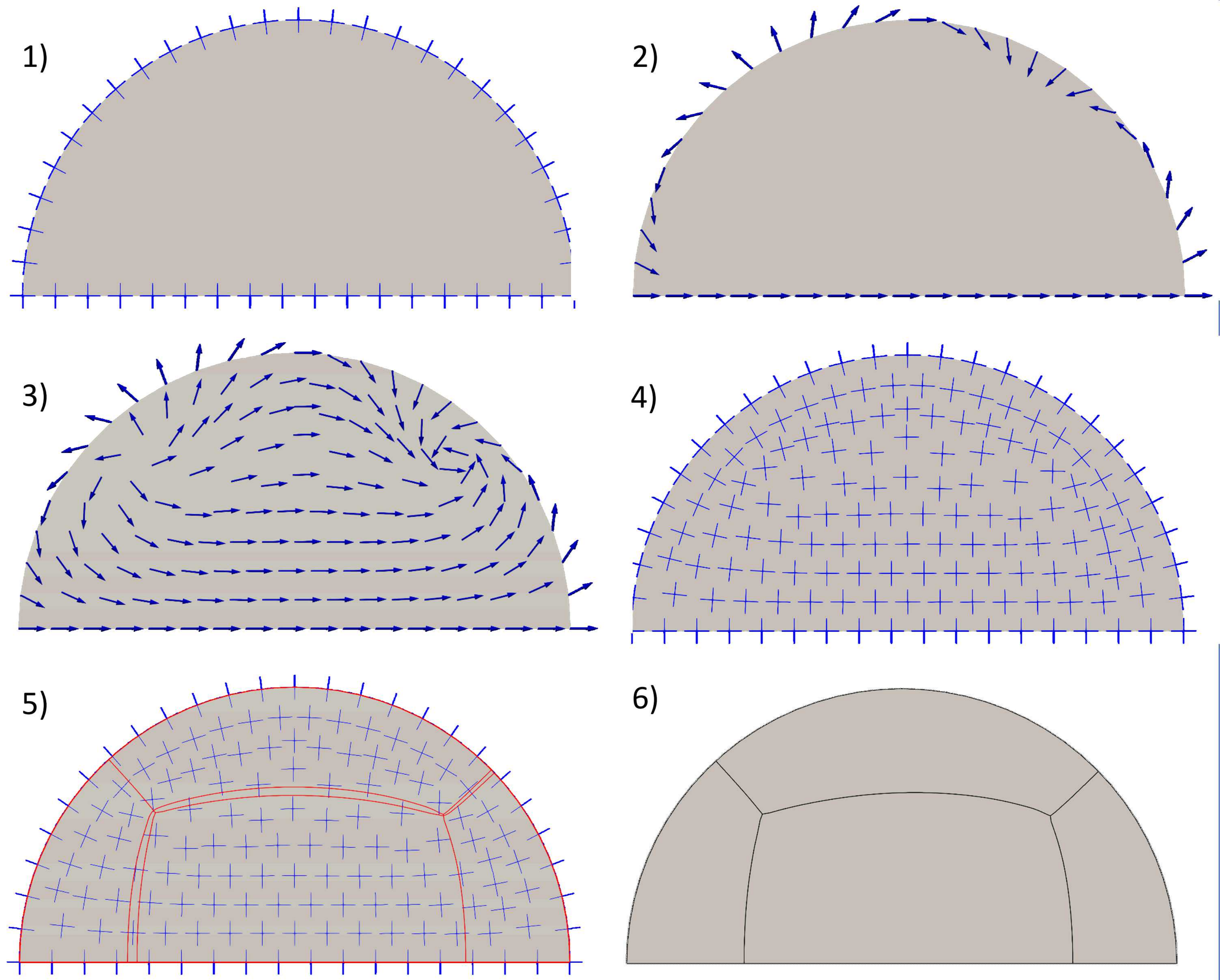
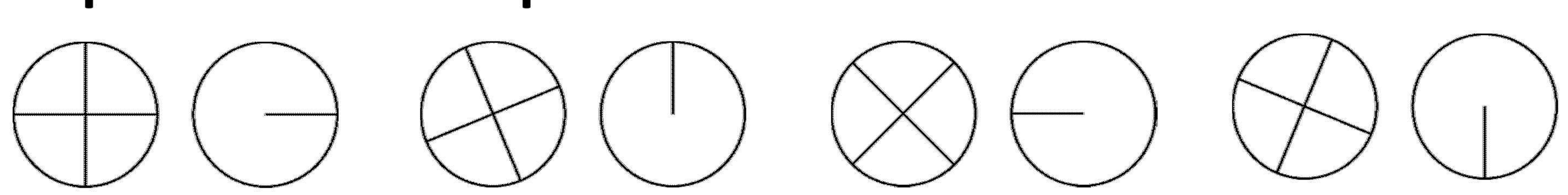
Generating a Coarse Quad Decomposition

1) Align crosses with the normal and tangent directions of the boundary
2) Obtain a vector-valued boundary condition via the representation map
3) Find a field of unit vectors that minimizes the Dirichlet energy subject to the boundary condition
4) Convert the representation field into a boundary-aligned, smooth cross field
5) Trace separatrices of the cross field to partition the domain into four-sided regions
6) Simplify the partition using a "chord collapse" operation



Representation Map



Problem Formulation

Original Problem

$$\begin{cases} \min_u E(u) \\ E(u) = \frac{1}{2}\int_D |\nabla u|^2 dA \\ u(x) = g(x) \qquad \forall x \in \partial D \\ |u(x)| = 1 \quad a.e. x \in D \end{cases}$$

Relaxed Problem

$$\min_{u \in H^1_g(D,\mathbb{C})} E_\varepsilon(u)$$

$$E_\epsilon(\mathbf{u}) = \frac{1}{2}\int_G |\nabla \mathbf{u}|^2 + \frac{1}{4\epsilon^2}\int_G (|\mathbf{u}|^2 - 1)^2$$

Algorithm 1 Partitioning D into a quad layout with T-junctions.

Input: A domain D satisfying Assumption 3.1, and a boundary-aligned canonical harmonic cross field f with singularities of index $\leq 1/4$.

Output: A set $\mathcal{B}$ containing limit cycles and separatrices that define a quad layout with T junctions.

Let $\mathcal{S}$ be the set of separatrices that do not converge to a limit cycle. Let $\mathcal{P}$ be the set of separatrices that do. Let $\mathcal{L}$ be the set of limit cycles.

Initialize the set $\mathcal{B} = \mathcal{S}$.

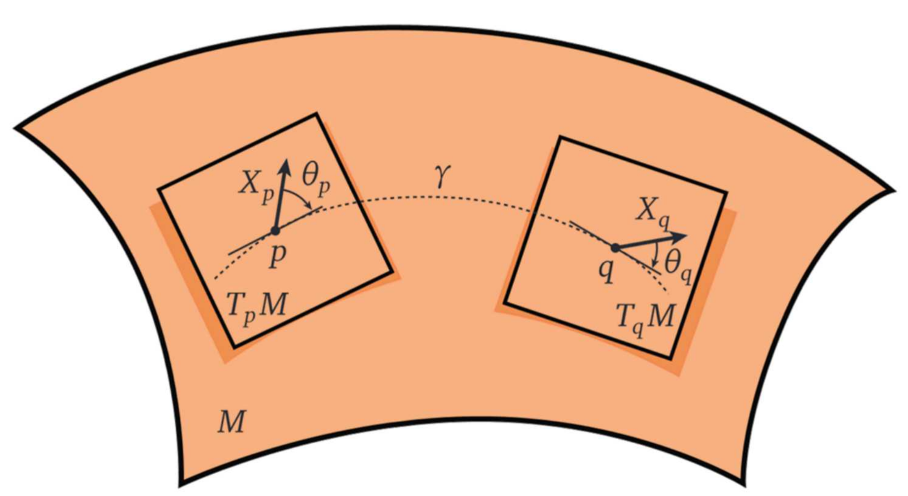
for $l \in \mathcal{L}$ **do**
  if no element of $\mathcal{B}$ intersects l **then**
    (i) Add l to $\mathcal{B}$.
    (ii) By Corollary 5.8, there is an element of $\mathcal{P}$ that intersects l. Let ρ' be the portion of that separatrix beginning at the singularity and ending in a T-junction with l.
    (iii) Add ρ' to $\mathcal{B}$.
    (iv) remove ρ from $\mathcal{P}$.
  end if
end for
for $\rho \in \mathcal{P}$ **do**
  Let ρ' be the curve segment of ρ beginning at the singularity and continuing until it intersects an element of $\mathcal{B}$. Add ρ' to $\mathcal{B}$.
end for

THEOREM 5.9. *Given a domain D satisfying Assumption 3.1, and a boundary-aligned canonical harmonic cross field f with singularities of index $\leq 1/4$, Algorithm 1 is well defined, terminates in finite time, and partitions D into a quad layout with exactly $|\mathcal{P}|$ T-junctions.*

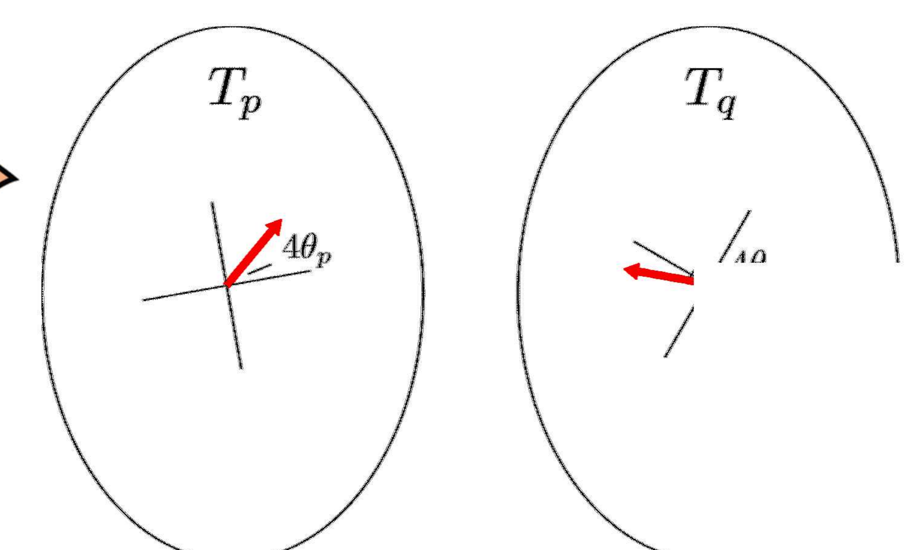
Cross Fields on Surfaces

On a surface there is no global coordinate system so we need the concept of a "connection"

Levi-Civita Connection



Discrete Cross Field Connection



$$Q_{pq}(e^{4i\theta_p}) = e^{4i(\theta_p + \phi_{pq})}$$
$$\implies Q_{pq}(v) = e^{4i\phi_{pq}} v$$

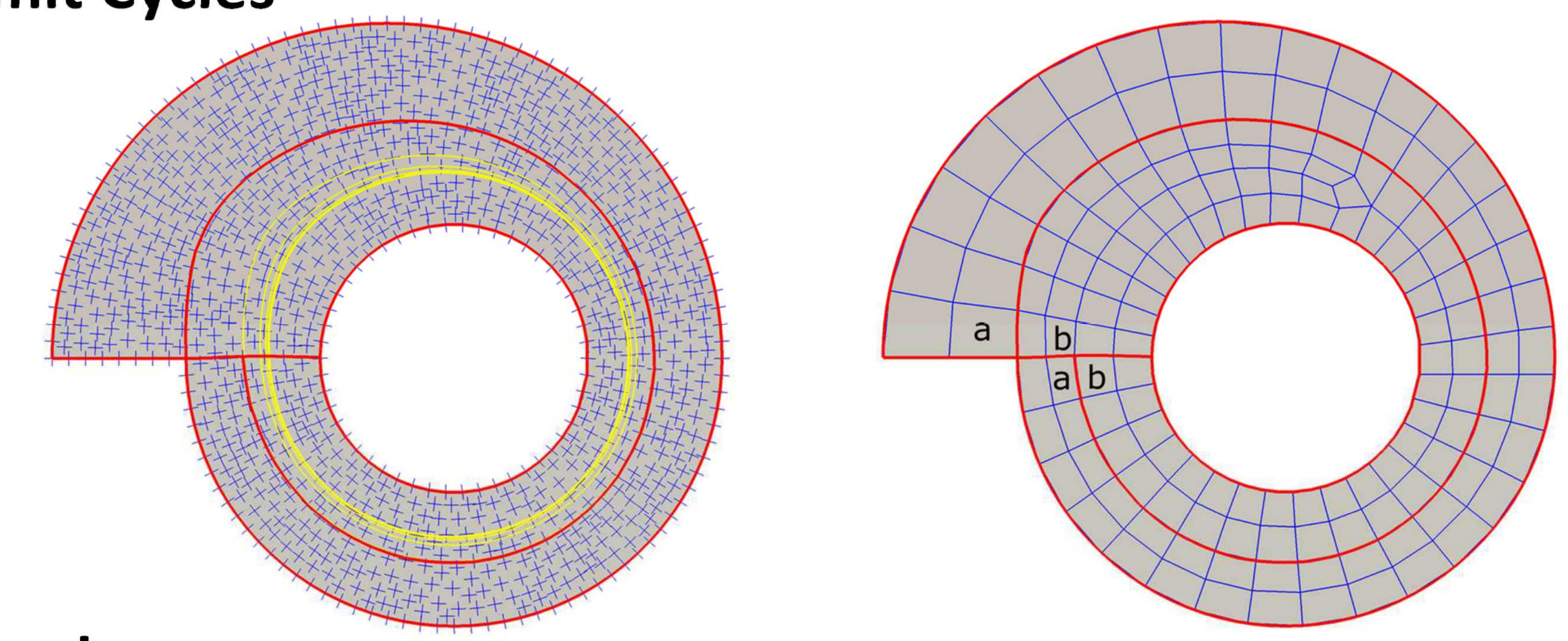
Discrete Laplacian for Cross Field Design

$$\Delta_Q(\vec{u})|_i = \frac{1}{|\mathcal{N}(n_i)|}\sum_{n_j \in \mathcal{N}(n_i)} (u_j - Q_{ij}(u_i))$$

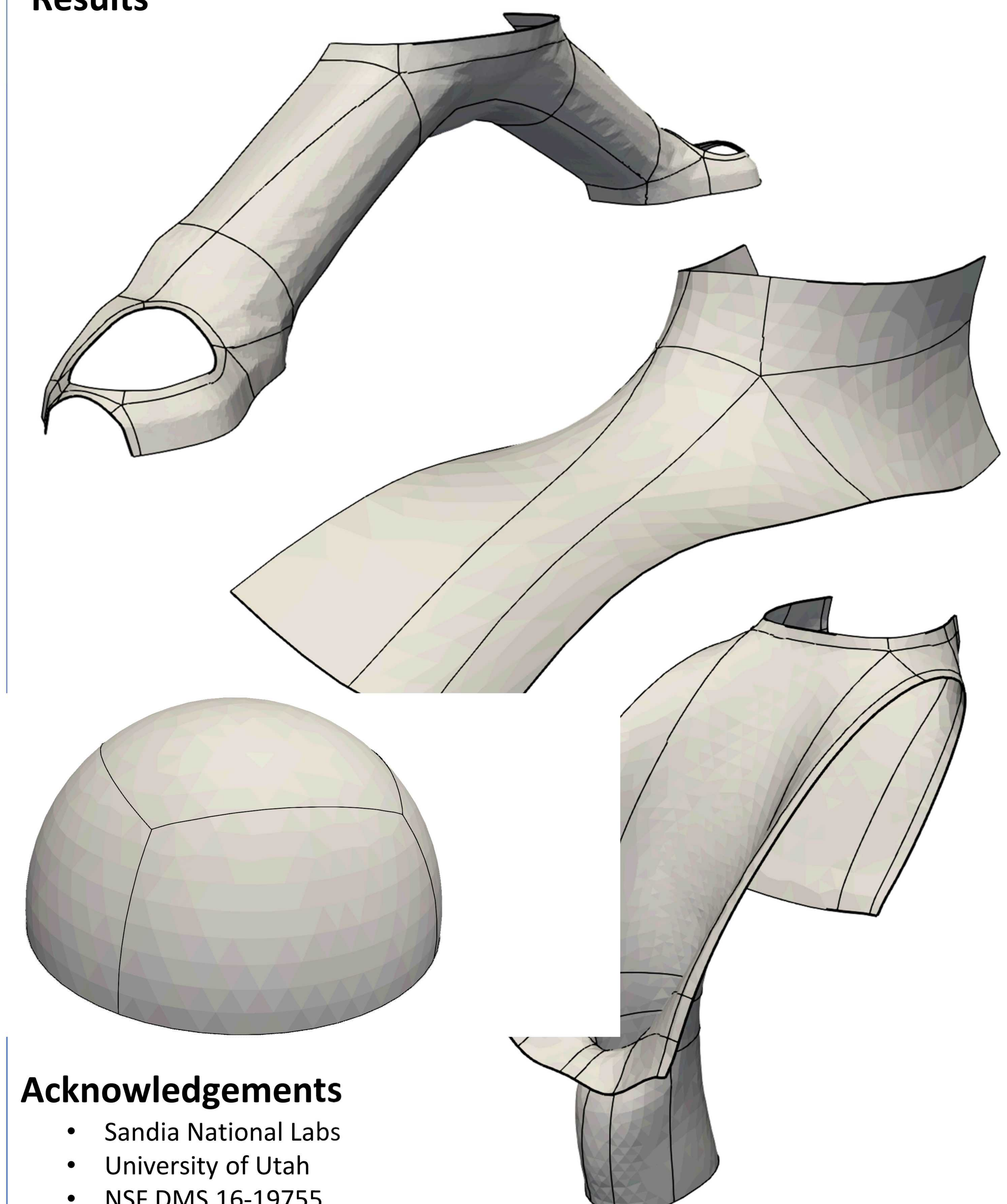
Future Work

1) Implement the full STL-to-CAD pipeline
2) Extend partitioning algorithm to work on assemblies and closed surfaces
3) Research new discrete cross field representation to improve robustness on rough surfaces
4) Find a solution to the limit cycle problem

Limit Cycles



Results



Acknowledgements

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