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Device-level modeling of hole quantum dots in Ge

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Modeling spin-orbit coupling in Ge

- Goal: understand electric dipole spin resonance (EDSR) experiments for Ge devices
- Approach: work with spin-orbit coupling (SOC) models to understand EDSR details
 - Simplest model:

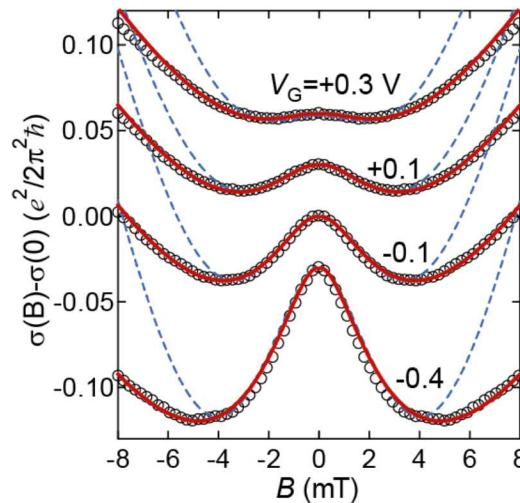
$$\hat{H}_{so1} = i\alpha_1(\hat{\sigma}_- \otimes \hat{k}_+ - \hat{\sigma}_+ \otimes \hat{k}_-)$$

- Expected form from $k \cdot p$ -theory (E. Marcellina et al., Phys. Rev. B, 2017):

$$\begin{aligned}\hat{H}_{so3} = i\alpha_{3,1}(\hat{\sigma}_- \otimes \hat{k}_+^3 - \hat{\sigma}_+ \otimes \hat{k}_-^3) + \\ i\alpha_{3,2}(\hat{\sigma}_- \otimes \hat{k}_- \hat{k}_+ \hat{k}_- - \hat{\sigma}_+ \otimes \hat{k}_+ \hat{k}_- \hat{k}_+)\end{aligned}$$

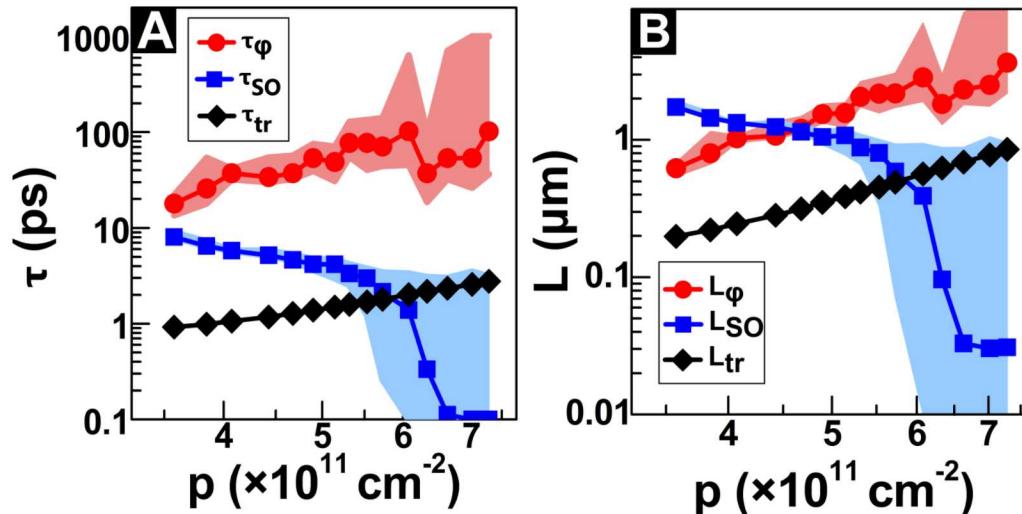
- Developing a starting point for a full device-level model
 - Single-band effective mass theory for now; potentially misses important details (A. Mielnik-Pyszczorski et al., Scientific Reports, 2018)
 - Work toward a multi-band model that includes more parameters of Ge

Background

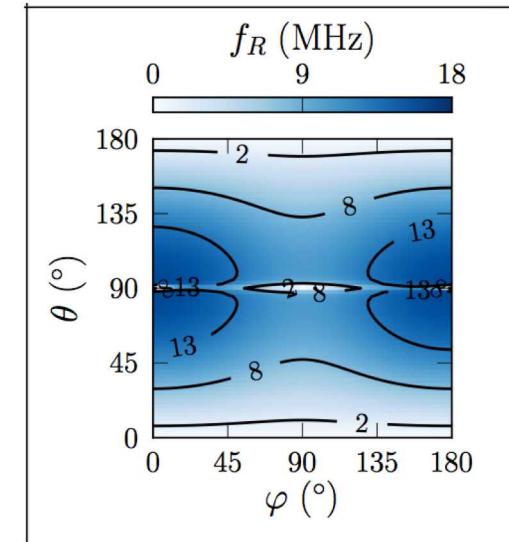


- Magnetoconductivity experiments (2DHGs), suggest SOC model better fits k^3 than k
 - Theory calculations for Ge (and Si) holes based on $k \cdot p$ -theory

[1] R. Moriya et al., Phys. Rev. Lett. 2014



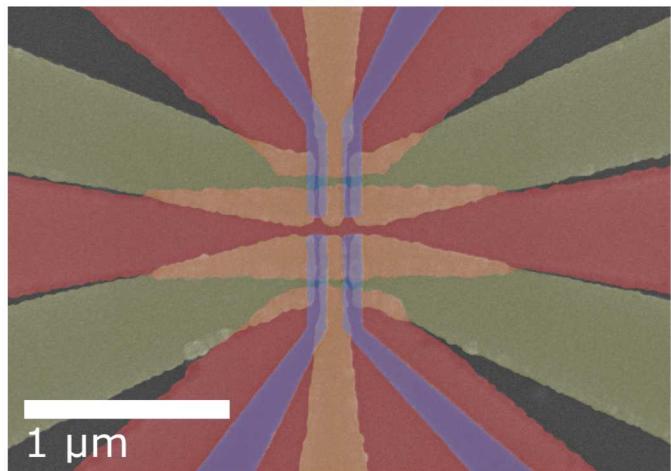
[3] C. T. Chou et al., Nanoscale 2018



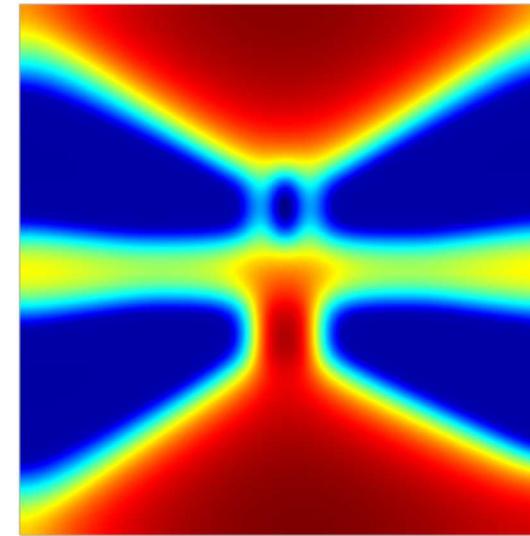
[2] B. Venitucci and Y. M. Niquet, arXiv preprint 2019

- Magnetoconductivity experiments, fit model assuming SOC is k^3
 - Experiments from 2DHGs suggest SOC is k^3 in Ge
 - Open question for confined QDs

Workflow



- Take gate layout and stack dimensions from device [1]
- Use Laconic [2] to calculate wavefunctions (w/o spin)



- Use COMSOL to calculate device potentials
- Use wavefunctions as a basis for calculations with spin

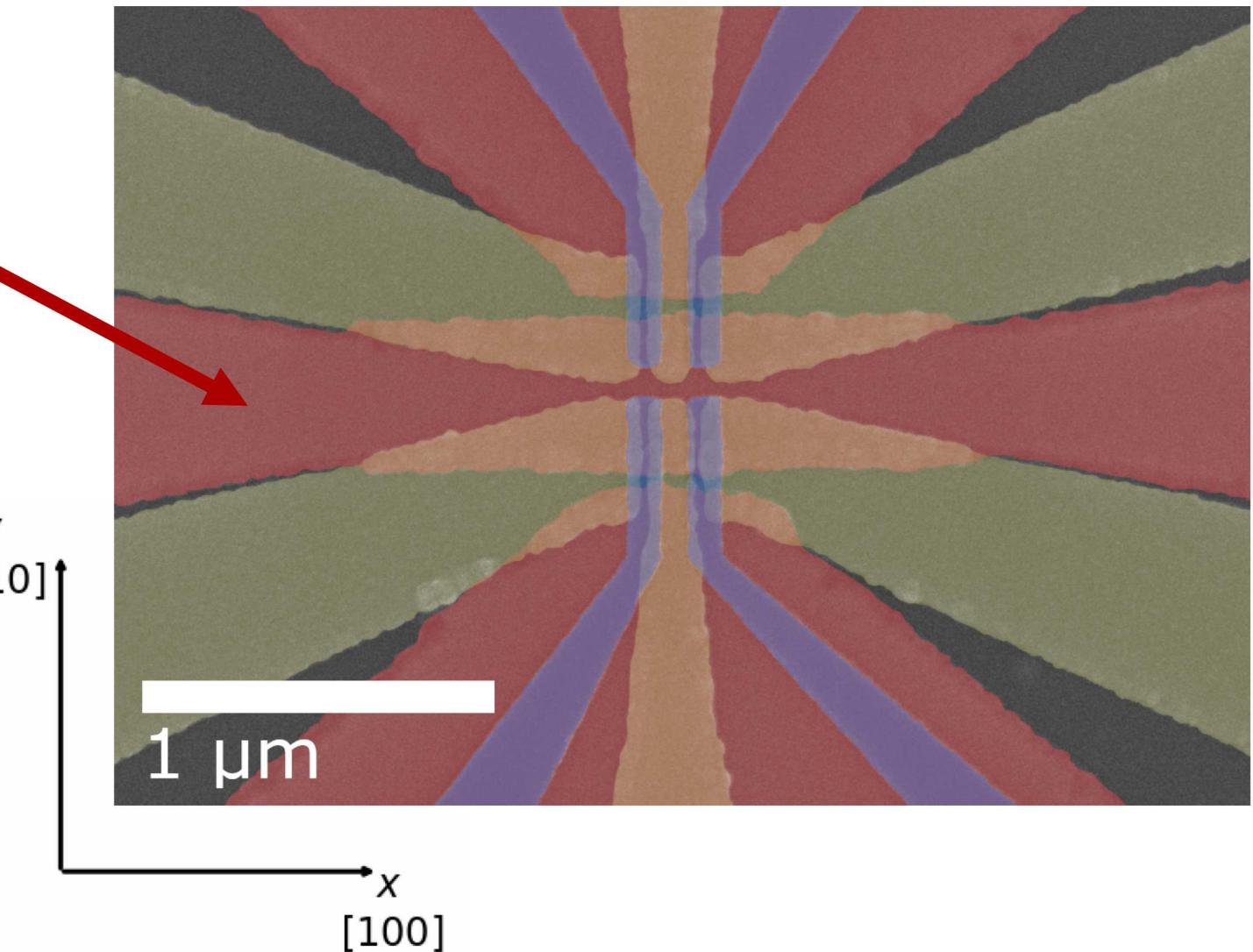
$$k_-^{\alpha\beta} = \langle \psi_\alpha | \hat{k}_- | \psi_\beta \rangle \rightarrow \hat{\sigma}_+ \otimes \hat{k}_-$$



[1] See Will Hardy's talk tomorrow, session L11.00005
[2] Sandia software, led by Andrew Baczewski

Modeling EDSR

- Compute change in potential for oscillations of middle gate
- Crystal axes determine SOC Hamiltonian
- Driving Hamiltonian:
$$\hat{H}_{ac} \sim V_{ac} \hat{y}$$
- Voltage oscillation: 1[mV], $[010]$
Applied B-field: 1[T]
- Study Rabi frequency trends in SOC models



How we compute Rabi frequencies

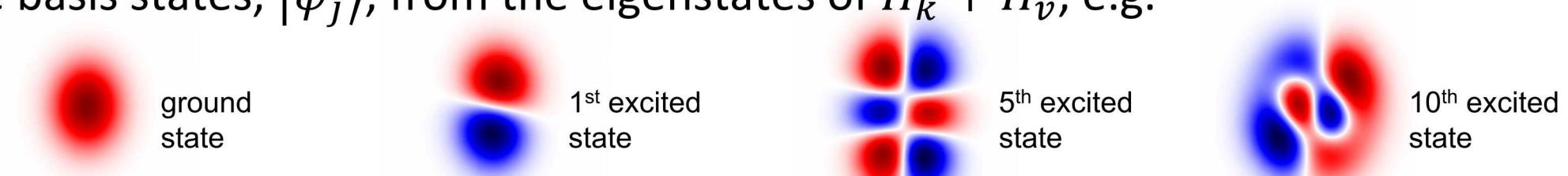
- Full Hamiltonian:

$$\hat{H}_{full} = \hat{H}_k + \hat{H}_v + \hat{H}_{so} + \hat{H}_{ac}$$

kinetic part potential part spin-orbit part AC drive part

(includes B-field)

- Define basis states, $|\psi_j\rangle$, from the eigenstates of $\hat{H}_k + \hat{H}_v$, e.g.



- Using $|\psi_j\rangle$ s, expand and diagonalize $\hat{H}_k + \hat{H}_v + \hat{H}_{so}$ to get a new basis, $|\psi'_j\rangle$
- Calculate the Rabi frequency as

$$f_R = \frac{2\pi}{\hbar} |\langle \psi'_0 | \hat{H}_{ac} | \psi'_1 \rangle|$$

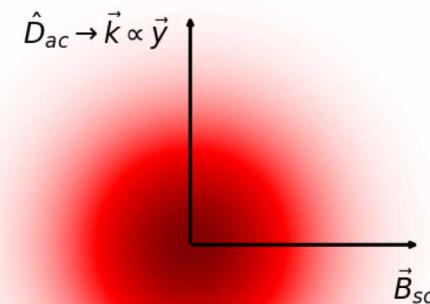
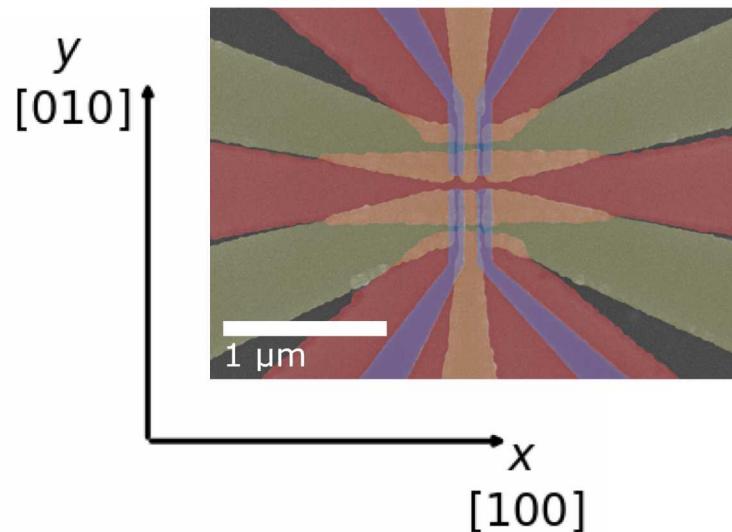
AC E-field along y → AC B-field along x

- Hamiltonian:

$$\hat{H}_{3,2} \approx \alpha_{3,2} (\hat{\sigma}_y \otimes (\hat{k}_x^2 + \hat{k}_y^2) \hat{k}_x - \hat{\sigma}_x \otimes (\hat{k}_x^2 + \hat{k}_y^2) \hat{k}_y)$$

- AC drive reduces Hamiltonian to effective magnetic field along x :

$$\hat{H}_{ac} \sim V_{ac} \hat{y} \rightarrow k_x \sim 0$$



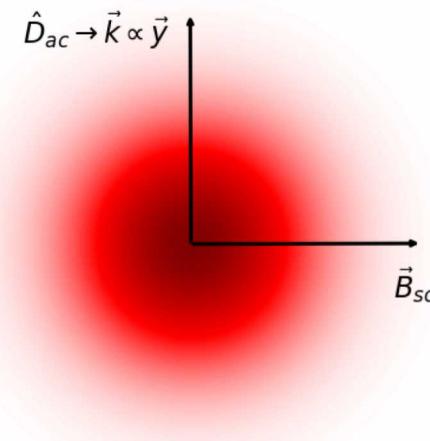
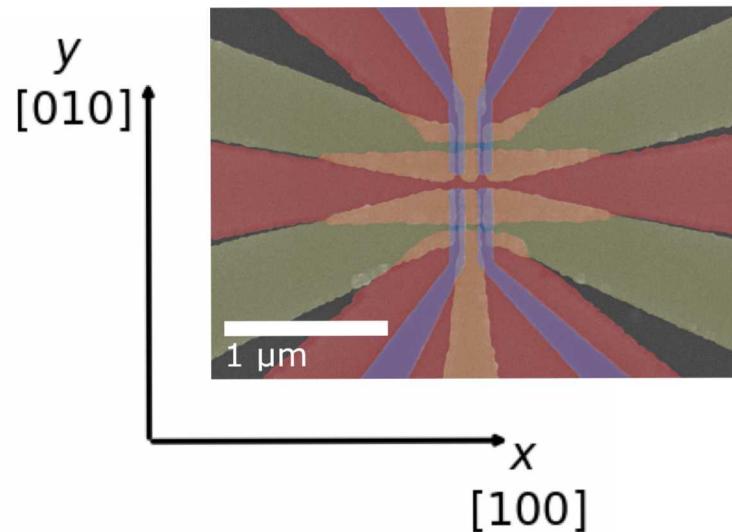
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- AC drive reduces Hamiltonian to effective magnetic field along x :

$$\hat{H}_{ac} \sim V_{ac} \hat{y} \rightarrow k_x \sim 0 \rightarrow \hat{H}_{3,2} \sim \hat{\sigma}_x \otimes (\hat{k}_x^2 + \hat{k}_y^2) \hat{k}_y \rightarrow B_{so} \propto \vec{x}$$



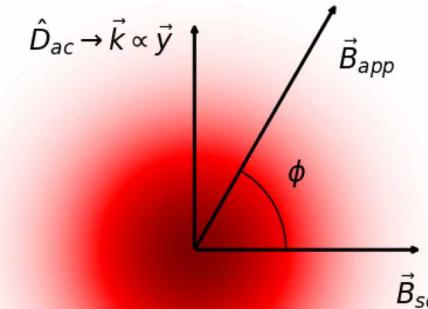
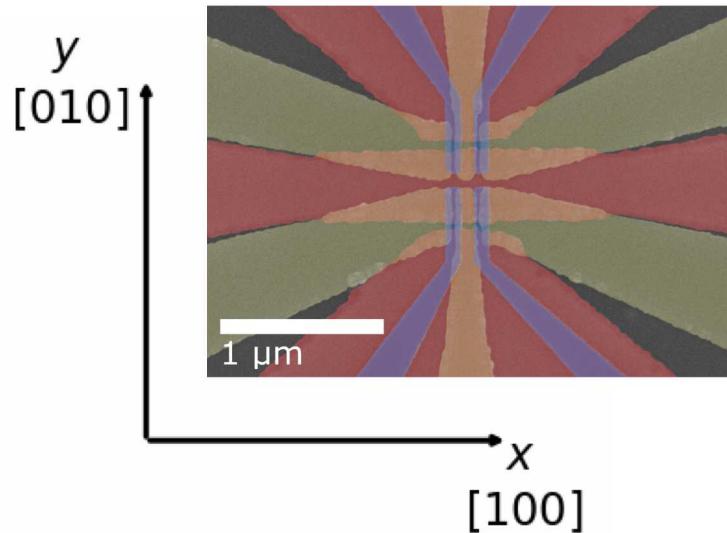
Strong rotations require orthogonal B-fields

- Should have weak rotations for

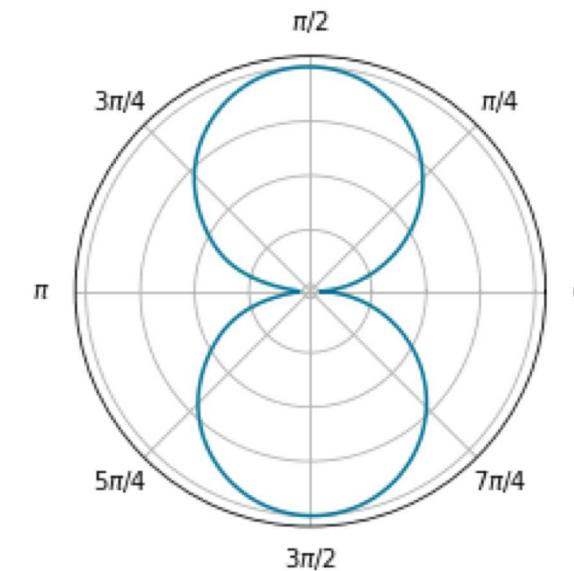
$$\mathbf{B}_{app} \parallel \mathbf{B}_{so}$$

- This fits the simulations:

$$f_R \propto |\mathbf{B}_{app} \times \mathbf{B}_{so}|$$

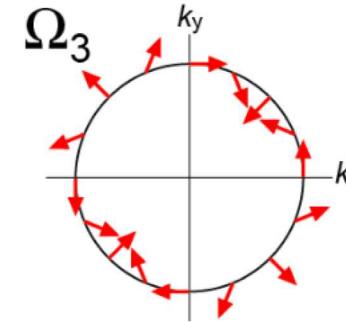


Rabi frequency as a function of magnetic field rotation
(normalized units)



Importance of crystal axes to device design

- Coordinate system determines form of \hat{H}_{so3}
 - \hat{k}_x and \hat{k}_y set by crystal axes
 - Rotation about the z-axis changes \hat{H}_{so3}
- \mathbf{B}_{so} is then a function of the crystal axes as well as the dipole perturbation
 - This adjusts the previous structure



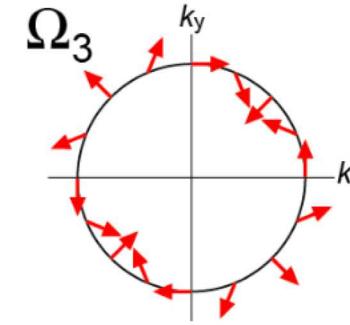
- Rotation of the spin-orbit field as a function of charge-carrier direction of motion for

$$\hat{H}_{3,1} = i\alpha_{3,1}(\hat{\sigma}_- \otimes \hat{k}_+^3 - \hat{\sigma}_+ \otimes \hat{k}_-^3)$$

R. Moriya et al., PRL 2014

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- We can explore the relative alignment of the crystal, dipole, and applied B-field
 - Fix dipole to original y -axis
 - Fix B-field to original x -axis
 - Rotate crystal axes

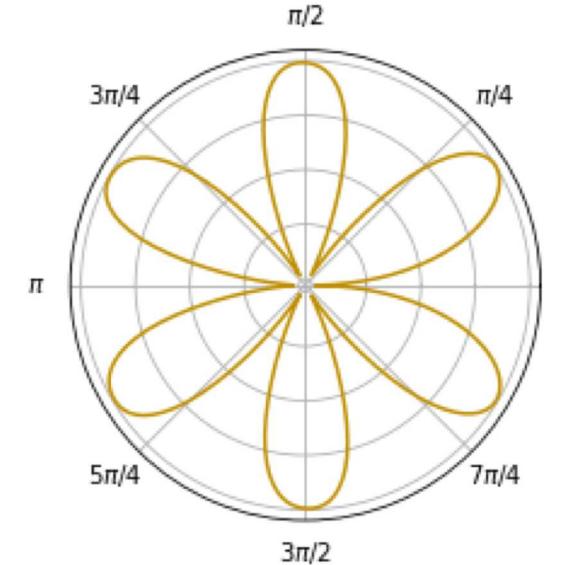


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R. Moriya et al., PRL 2014

Rabi frequency as a function of crystal axis rotation (normalized units)



Future work

- Incorporate more microscopic details
 - QW band structure as a function of material conditions (effective masses)
 - Multi-band effective mass theory (light and heavy hole)
 - Static and dynamic noise sources
- Understand and optimize single-qubit gate fidelities
- Model two-qubit gate

Acknowledgements

Project PI: Dwight Luhman

Mentor: Andrew Baczewski

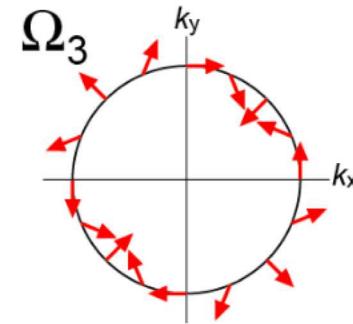
Experimentalists: Will Hardy, Tzu-Ming Lu

Theorists: Toby Jacobson, Leon Maurer

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Crystallographic axes relative to device design

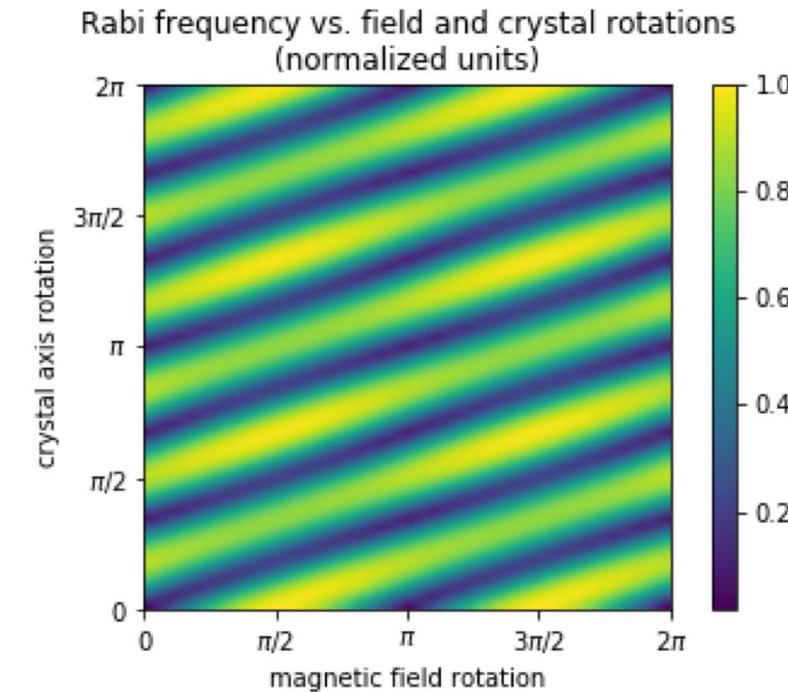
- Form of \hat{H}_{so3} imposes coordinate system
 - Crystal axes set by \hat{k}_x and \hat{k}_y
 - Rotation about the z-axis changes \hat{H}_{so3}
- \mathbf{B}_{so} is then a function of the crystal axes as well as the dipole perturbation
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 - Rotate B-field
 - Rotate crystal axes



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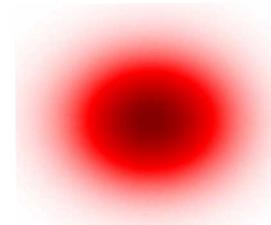
R. Moriya et al., PRL 2014



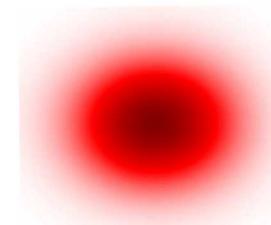
Qubit basis

- Without SOC:

$$|0\rangle = |\uparrow\rangle \otimes$$



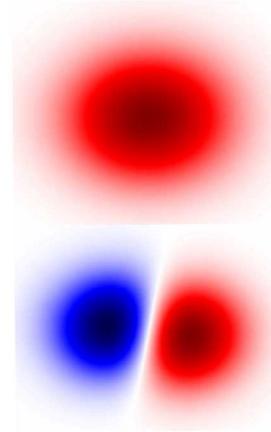
$$|1\rangle = |\downarrow\rangle \otimes$$



- With SOC:

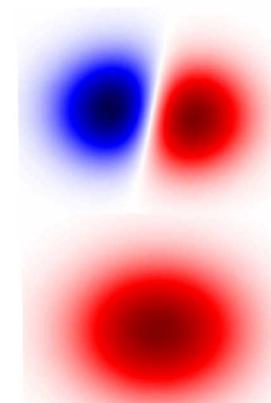
$$|0\rangle = \alpha|\uparrow\rangle \otimes$$

$$+ \beta|\downarrow\rangle \otimes$$



$$|1\rangle = \gamma|\uparrow\rangle \otimes$$

$$+ \delta|\downarrow\rangle \otimes$$



Linear SOC

- Hamiltonian:

$$\hat{H}_{so1} = i\alpha_1(\hat{\sigma}_- \otimes \hat{k}_+ - \hat{\sigma}_+ \otimes \hat{k}_-)$$

- Gate oscillation coupling:

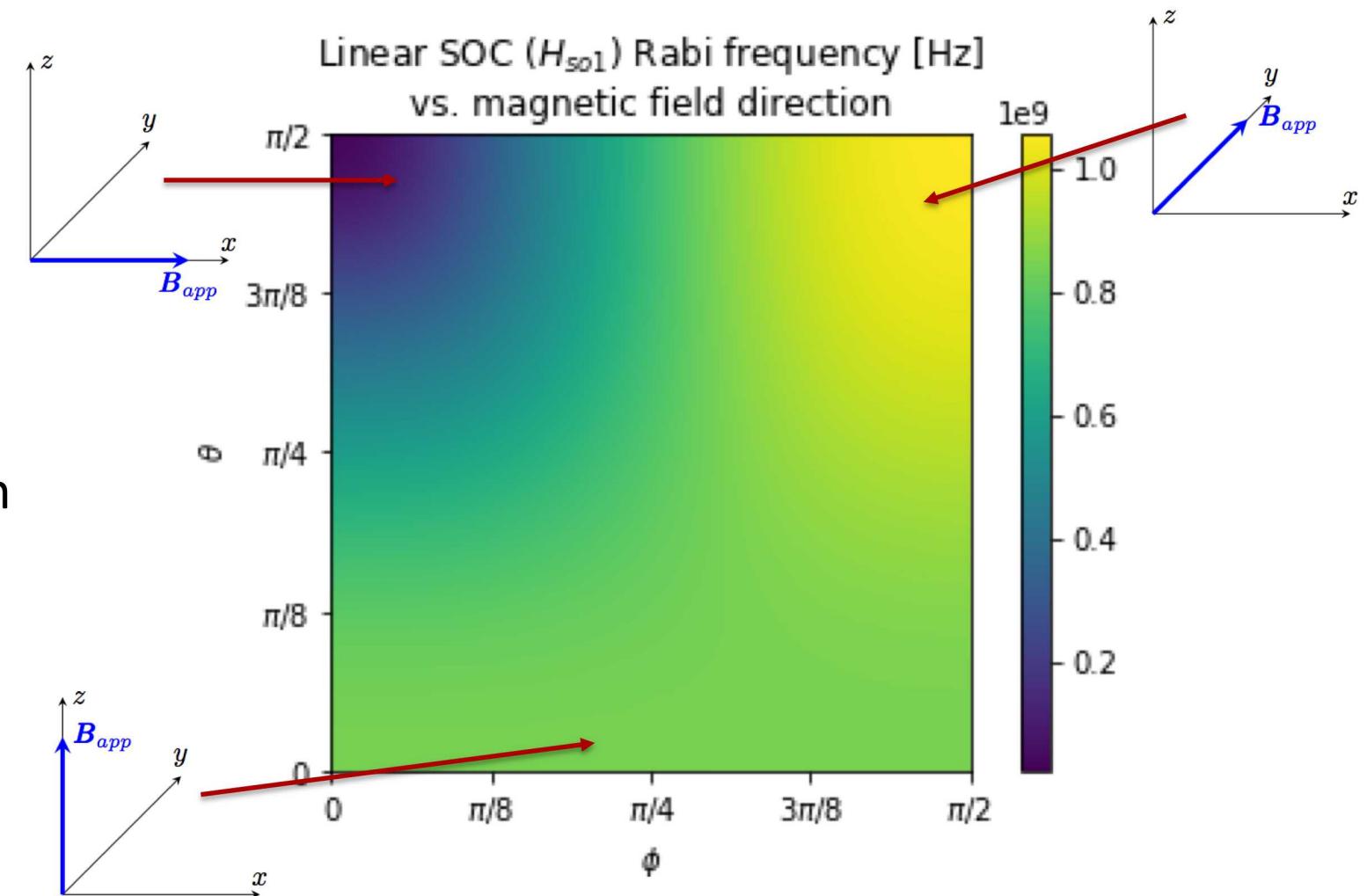
$$\hat{D}_{ac} \sim V_{ac} \hat{y}$$

- Rabi frequency low when

$$\mathbf{B}_{app} \perp \hat{y}$$

in the xy -plane, high when

$$\mathbf{B}_{app} \parallel \hat{y}$$



Cubic SOC

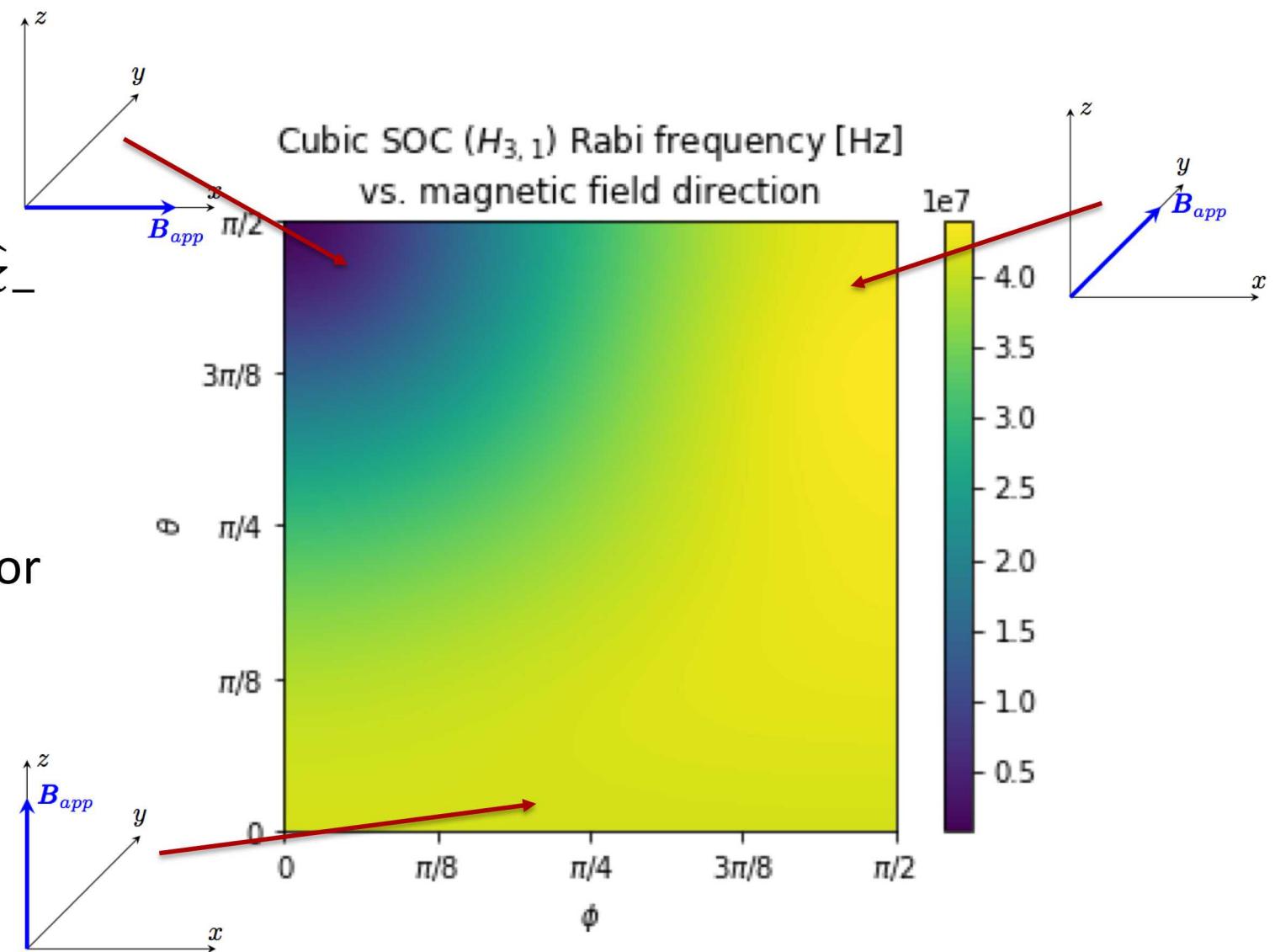
- Hamiltonian:

$$\hat{H}_{3,2} = i\alpha_{3,2}(\hat{\sigma}_- \otimes \hat{k}_- \hat{k}_+ \hat{k}_- \\ - \hat{\sigma}_+ \otimes \hat{k}_+ \hat{k}_- \hat{k}_+)$$

- Gate oscillation coupling:

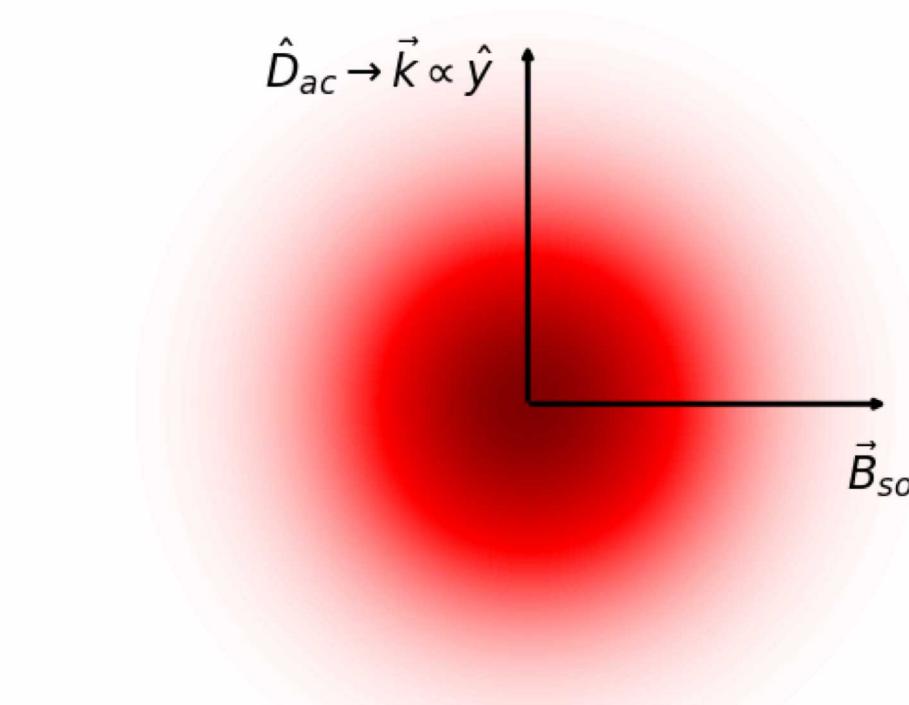
$$\hat{D}_{ac} \sim V_{ac} \hat{y}$$

- Similar qualities to linear for $B_{app} \perp \hat{y}$ and $B_{app} \parallel \hat{y}$ situations



Understanding SOC trends

- $\hat{H}_{so1} = \alpha_1(\hat{\sigma}_y \otimes \hat{k}_x - \hat{\sigma}_x \otimes \hat{k}_y)$
- $\hat{H}_{3,1} \approx \alpha_{3,1}(\hat{\sigma}_y \otimes (\hat{k}_x^2 - 3\hat{k}_y^2)\hat{k}_x - \hat{\sigma}_x \otimes (\hat{k}_y^2 - 3\hat{k}_x^2)\hat{k}_y)$
- $\hat{H}_{3,2} \approx \alpha_{3,2}(\hat{\sigma}_y \otimes (\hat{k}_x^2 + \hat{k}_y^2)\hat{k}_x - \hat{\sigma}_x \otimes (\hat{k}_x^2 + \hat{k}_y^2)\hat{k}_y)$
- $\hat{D}_{ac} \sim V_{ac}\hat{y} \rightarrow k_x \sim 0 \rightarrow B_{so} \propto \hat{x}$



Understanding SOC trends

- $\hat{H}_{so1} \sim \hat{\sigma}_x \otimes \hat{k}_y$
- $\hat{H}_{3,1} \sim \hat{\sigma}_x \otimes \hat{k}_y^3$
- $\hat{H}_{3,2} \sim \hat{\sigma}_x \otimes \hat{k}_y^3$
- $\hat{D}_{ac} \sim V_{ac} \hat{y} \rightarrow k_x \sim 0 \rightarrow B_{so} \propto \hat{x}$

