

Exceptional service in the national interest



SAND2019-2375C
Sandia
National
Laboratories

Device-level modeling of hole quantum dots in Ge

Mitchell Brickson^{1,3}, Andrew Baczewski¹, Will Hardy², Noah T. Jacobson¹,
Tzu-Ming Lu², Leon Maurer¹, Dwight R. Luhman²

¹ Center for Computing Research, Sandia National Laboratories, Albuquerque, NM 87123, United States

² Sandia National Laboratories, Albuquerque, NM 87123, United States

³ Center for Quantum Information and Control, University of New Mexico, Albuquerque, NM 87131, United States

Modeling spin-orbit coupling in Ge

- Goal: understand electric dipole spin resonance (EDSR) experiments for Ge devices
- Approach: work with spin-orbit coupling (SOC) models to understand EDSR details
 - Simplest model:

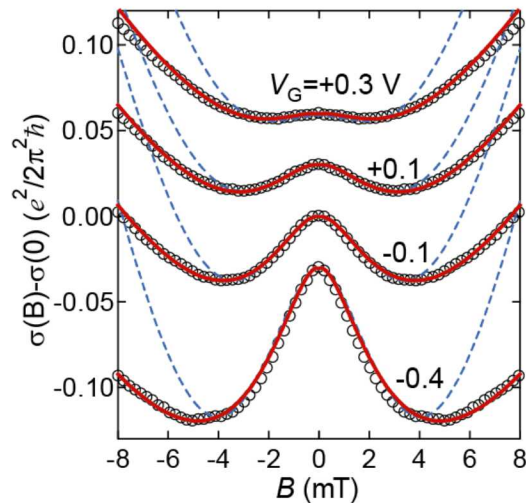
$$\hat{H}_{so1} = i\alpha_1(\hat{\sigma}_- \otimes \hat{k}_+ - \hat{\sigma}_+ \otimes \hat{k}_-)$$

- Expected form from $k \cdot p$ -theory (E. Marcellina et al., Phys. Rev. B, 2017):

$$\begin{aligned}\hat{H}_{so3} = & i\alpha_{3,1}(\hat{\sigma}_- \otimes \hat{k}_+^3 - \hat{\sigma}_+ \otimes \hat{k}_-^3) + \\ & i\alpha_{3,2}(\hat{\sigma}_- \otimes \hat{k}_- \hat{k}_+ \hat{k}_- - \hat{\sigma}_+ \otimes \hat{k}_+ \hat{k}_- \hat{k}_+)\end{aligned}$$

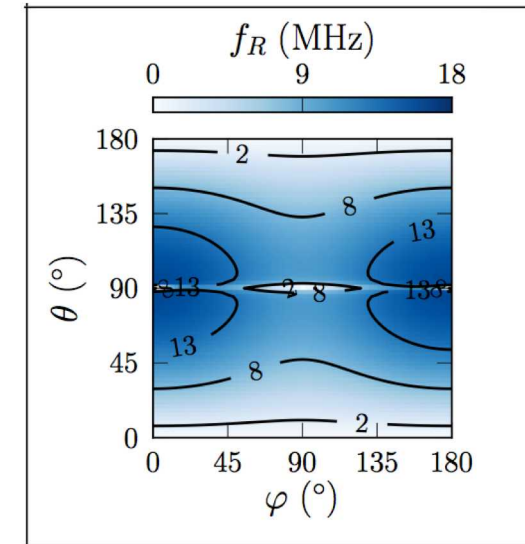
- Developing a starting point for a full device-level model
 - Single-band effective mass theory for now; potentially misses important details (A. Mielnik-Pyszcorski et al., Scientific Reports, 2018)
 - Work toward a multi-band model that includes more parameters of Ge

Background



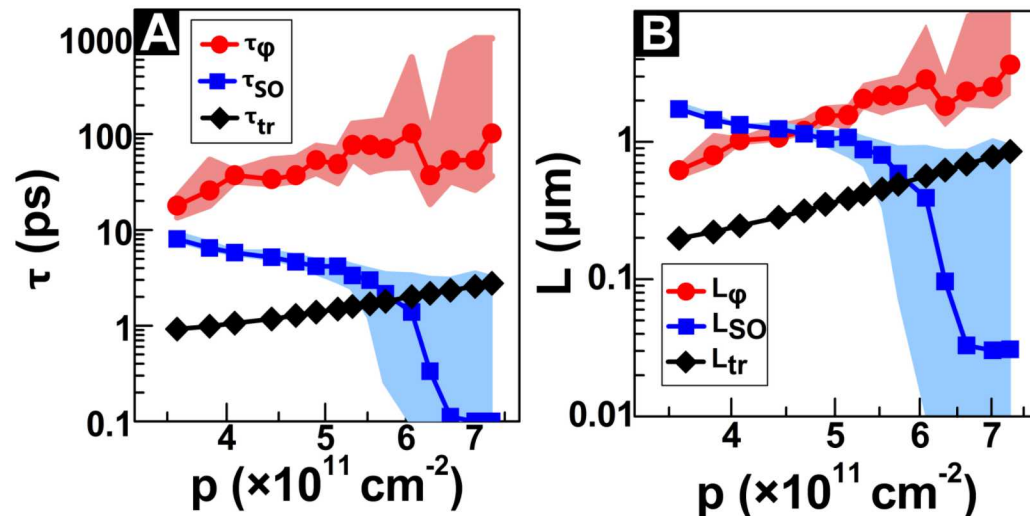
- Magnetoconductivity experiments (2DHGs), suggest SOC model better fits k^3 than k

- Theory calculations for Ge (and Si) holes based on $k \cdot p$ -theory



[1] R. Moriya et al., Phys. Rev. Lett. 2014

[2] B. Venitucci and Y. M. Niquet, arXiv preprint 2019

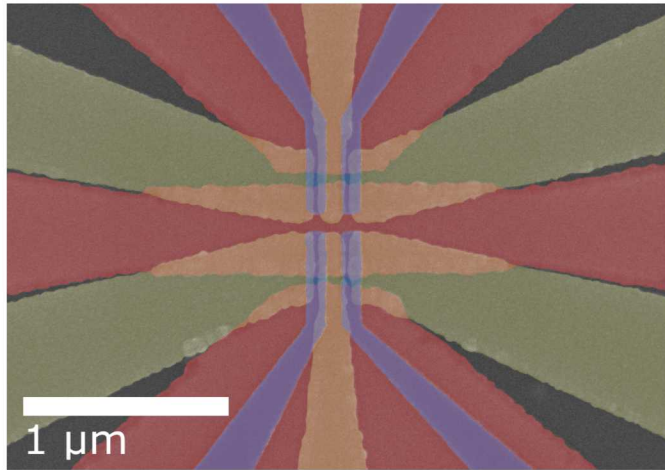


- Magnetoconductivity experiments, fit model assuming SOC is k^3

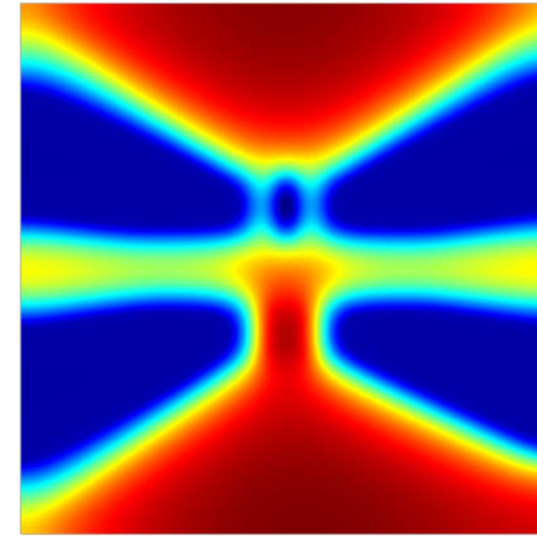
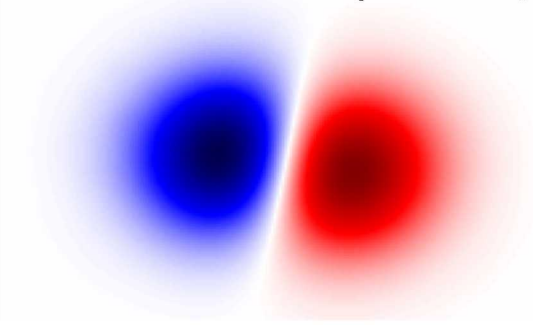
- Experiments from 2DHGs suggest SOC is k^3 in Ge
- Open question for confined QDs

[3] C. T. Chou et al., Nanoscale 2018

Workflow



- Take gate layout and stack dimensions from device [1]
- Use Laconic [2] to calculate wavefunctions (w/o spin)



- Use COMSOL to calculate device potentials
- Use wavefunctions as a basis for calculations with spin

$$k_-^{\alpha\beta} = \langle \psi_\alpha | \hat{k}_- | \psi_\beta \rangle \rightarrow \hat{\sigma}_+ \otimes \hat{k}_-$$

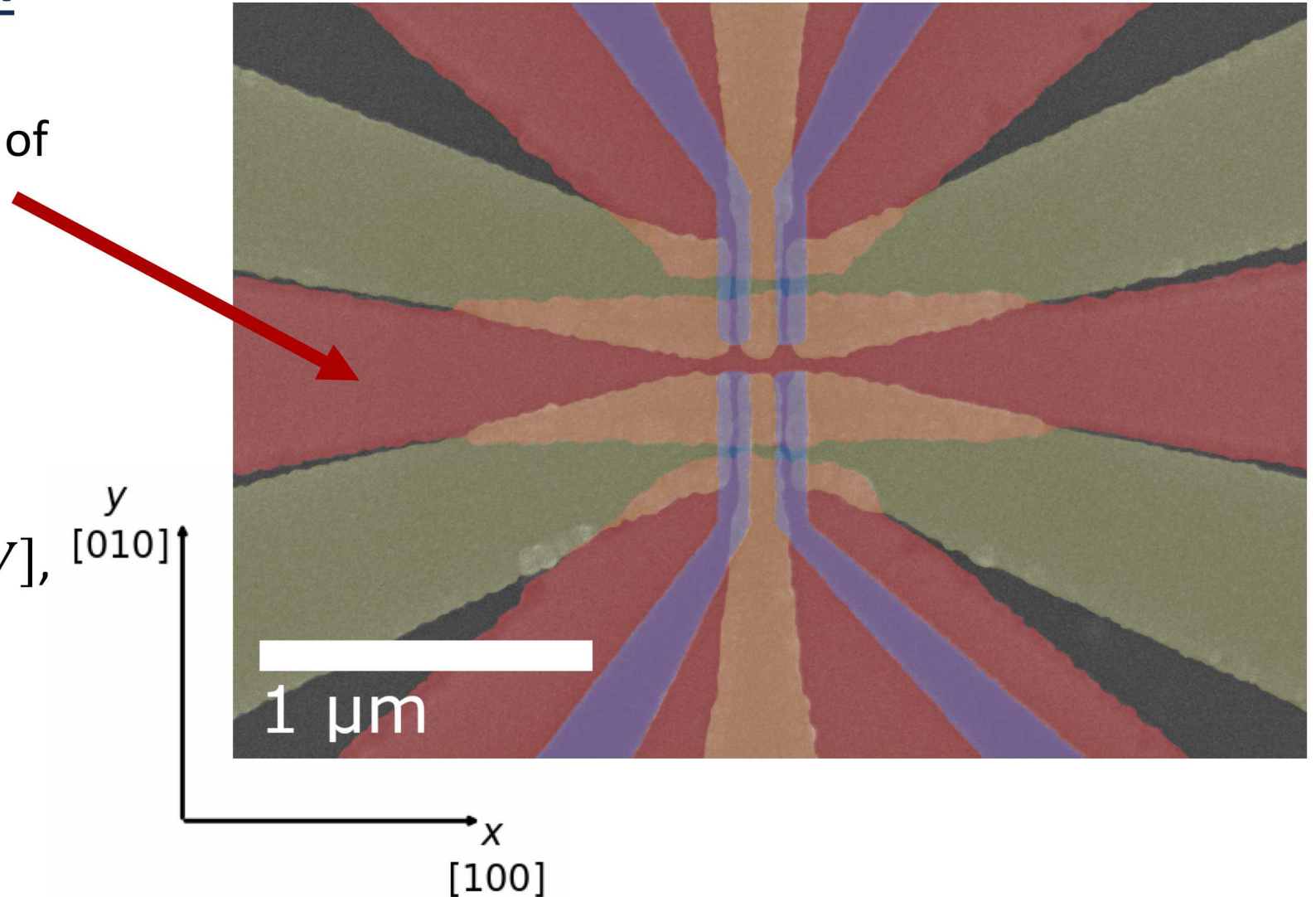
[1] See Will Hardy's talk tomorrow, session L11.00005

[2] Sandia software, led by Andrew Baczewski

Modeling EDSR

- Compute change in potential for oscillations of middle gate
- Crystal axes determine SOC Hamiltonian
- Driving Hamiltonian:

$$\hat{H}_{ac} \sim V_{ac} \hat{y}$$
- Voltage oscillation: $1[mV]$, $[010]$
 Applied B-field: $1[T]$
- Study Rabi frequency trends in SOC models



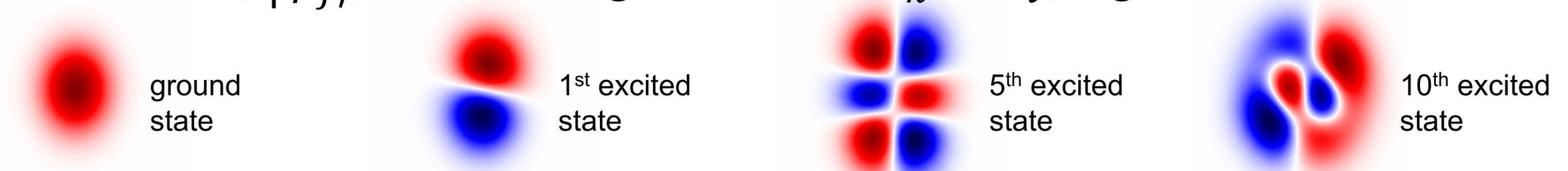
How we compute Rabi frequencies

- Full Hamiltonian:

$$\hat{H}_{full} = \hat{H}_k + \hat{H}_v + \hat{H}_{so} + \hat{H}_{ac}$$

kinetic part (includes B-field) potential part spin-orbit part AC drive part

- Define basis states, $|\psi_j\rangle$, from the eigenstates of $\hat{H}_k + \hat{H}_v$, e.g.



- Using $|\psi_j\rangle$ s, expand and diagonalize $\hat{H}_k + \hat{H}_v + \hat{H}_{so}$ to get a new basis, $|\psi'_j\rangle$
- Calculate the Rabi frequency as

$$f_R = \frac{2\pi}{\hbar} |\langle \psi'_0 | \hat{H}_{ac} | \psi'_1 \rangle|$$

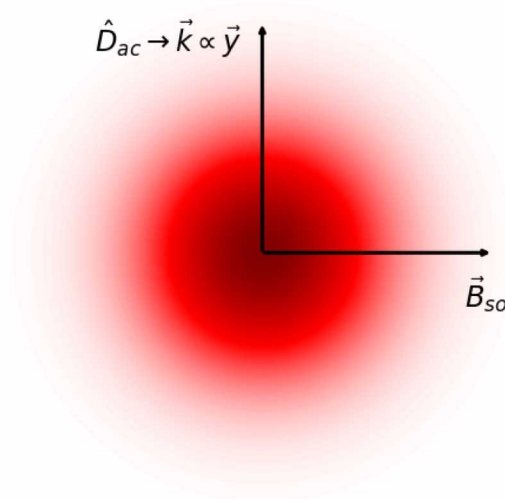
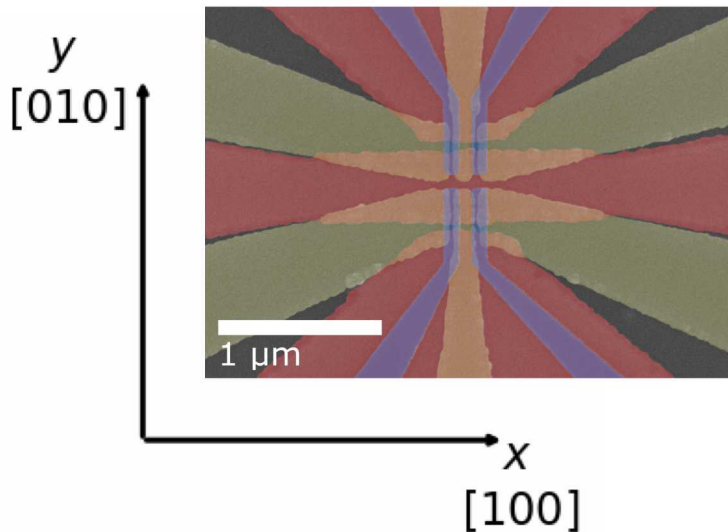
AC E-field along $y \rightarrow$ AC B-field along x

- Hamiltonian:

$$\hat{H}_{3,2} \approx \alpha_{3,2}(\hat{\sigma}_y \otimes (\hat{k}_x^2 + \hat{k}_y^2)\hat{k}_x - \hat{\sigma}_x \otimes (\hat{k}_x^2 + \hat{k}_y^2)\hat{k}_y)$$

- AC drive reduces Hamiltonian to effective magnetic field along x :

$$\hat{H}_{ac} \sim V_{ac}\hat{y} \rightarrow k_x \sim 0$$



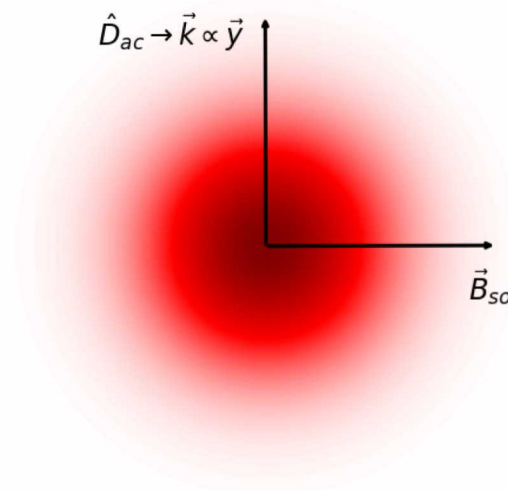
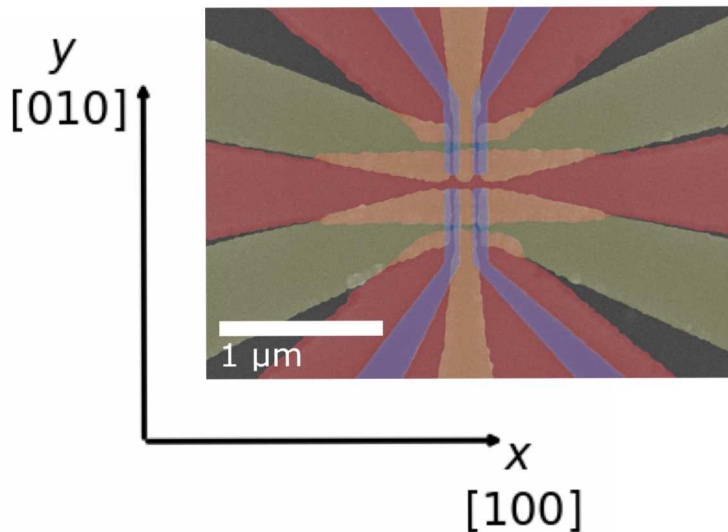
AC E-field along $y \rightarrow$ AC B-field along x

- Hamiltonian:

$$\hat{H}_{3,2} \approx \alpha_{3,2} (\hat{\sigma}_y \otimes (\hat{k}_x^2 + \hat{k}_y^2) \hat{k}_x - \hat{\sigma}_x \otimes (\hat{k}_x^2 + \hat{k}_y^2) \hat{k}_y)$$

- AC drive reduces Hamiltonian to effective magnetic field along x :

$$\hat{H}_{ac} \sim V_{ac} \hat{y} \rightarrow k_x \sim 0 \rightarrow \hat{H}_{3,2} \sim \hat{\sigma}_x \otimes (\hat{k}_x^2 + \hat{k}_y^2) \hat{k}_y \rightarrow \mathbf{B}_{so} \propto \vec{x}$$



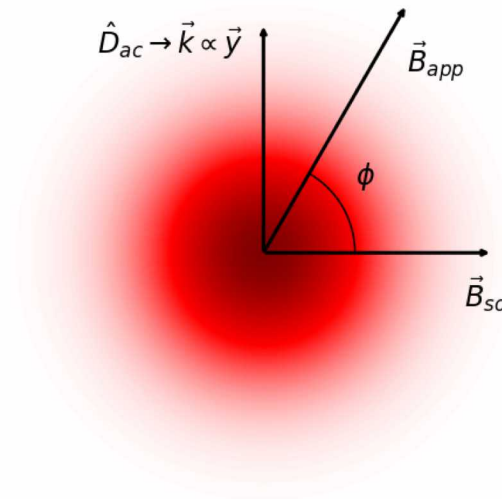
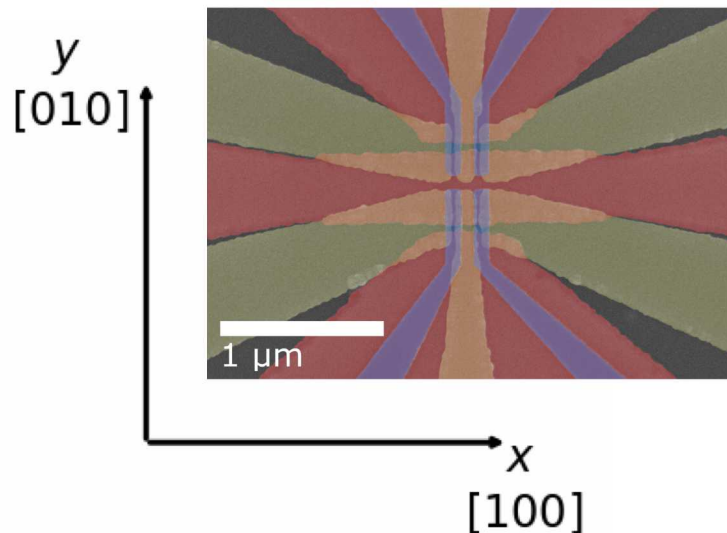
Strong rotations require orthogonal B-fields

- Should have weak rotations for

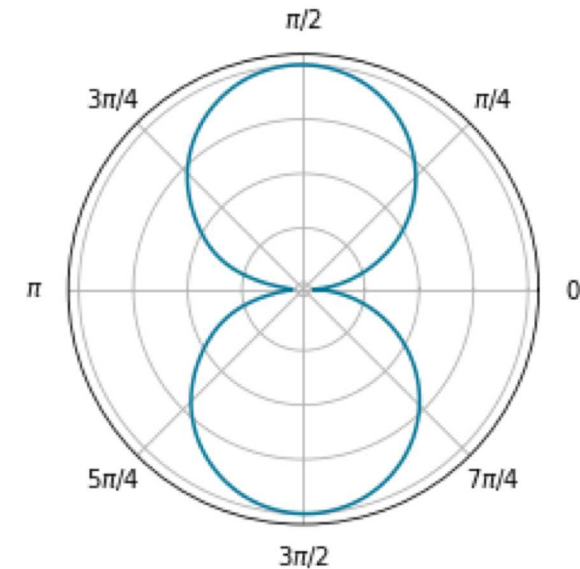
$$\mathbf{B}_{app} \parallel \mathbf{B}_{so}$$

- This fits the simulations:

$$f_R \propto |\mathbf{B}_{app} \times \mathbf{B}_{so}|$$

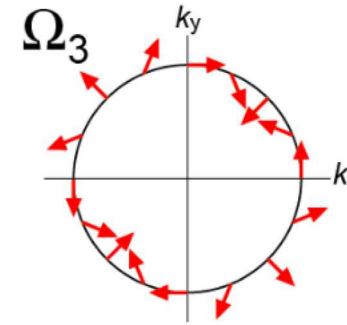


Rabi frequency as a function of magnetic field rotation (normalized units)



Importance of crystal axes to device design

- Coordinate system determines form of \hat{H}_{so3}
 - \hat{k}_x and \hat{k}_y set by crystal axes
 - Rotation about the z-axis changes \hat{H}_{so3}
- B_{so} is then a function of the crystal axes as well as the dipole perturbation
 - This adjusts the previous structure

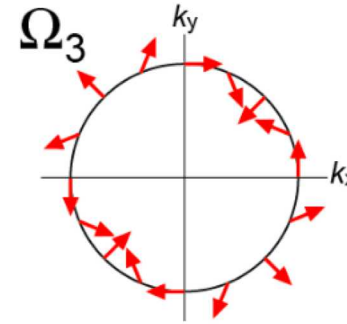


- Rotation of the spin-orbit field as a function of charge-carrier direction of motion for
$$\hat{H}_{3,1} = i\alpha_{3,1}(\hat{\sigma}_- \otimes \hat{k}_+^3 - \hat{\sigma}_+ \otimes \hat{k}_-^3)$$

R. Moriya et al., PRL 2014

Importance of crystal axes to device design

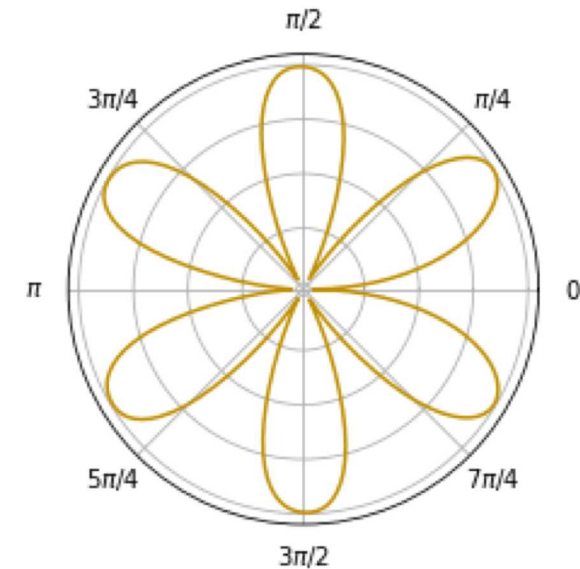
- Coordinate system determines form of \hat{H}_{so3}
 - \hat{k}_x and \hat{k}_y set by crystal axes
 - Rotation about the z-axis changes \hat{H}_{so3}
- B_{so} is then a function of the crystal axes as well as the dipole perturbation
 - This adjusts the previous structure
- We can explore the relative alignment of the crystal, dipole, and applied B-field
 - Fix dipole to original y-axis
 - Fix B-field to original x-axis
 - Rotate crystal axes



- Rotation of the spin-orbit field as a function of charge-carrier direction of motion for
$$\hat{H}_{3,1} = i\alpha_{3,1}(\hat{\sigma}_- \otimes \hat{k}_+^3 - \hat{\sigma}_+ \otimes \hat{k}_-^3)$$

R. Moriya et al., PRL 2014

Rabi frequency as a function of crystal axis rotation (normalized units)



Future work

- Incorporate more microscopic details
 - QW band structure as a function of material conditions (effective masses)
 - Multi-band effective mass theory (light and heavy hole)
 - Static and dynamic noise sources
- Understand and optimize single-qubit gate fidelities
- Model two-qubit gate

Acknowledgements

Project PI: Dwight Luhman

Mentor: Andrew Baczewski

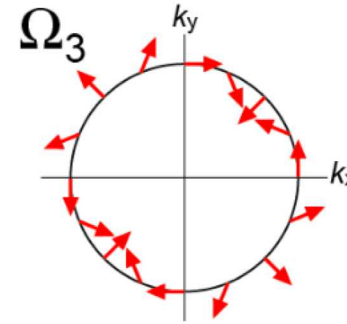
Experimentalists: Will Hardy, Tzu-Ming Lu

Theorists: Toby Jacobson, Leon Maurer

Sandia National Laboratories is a multi-missions laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for DOE's National Nuclear Security Administration under contract DE-NA0003525.

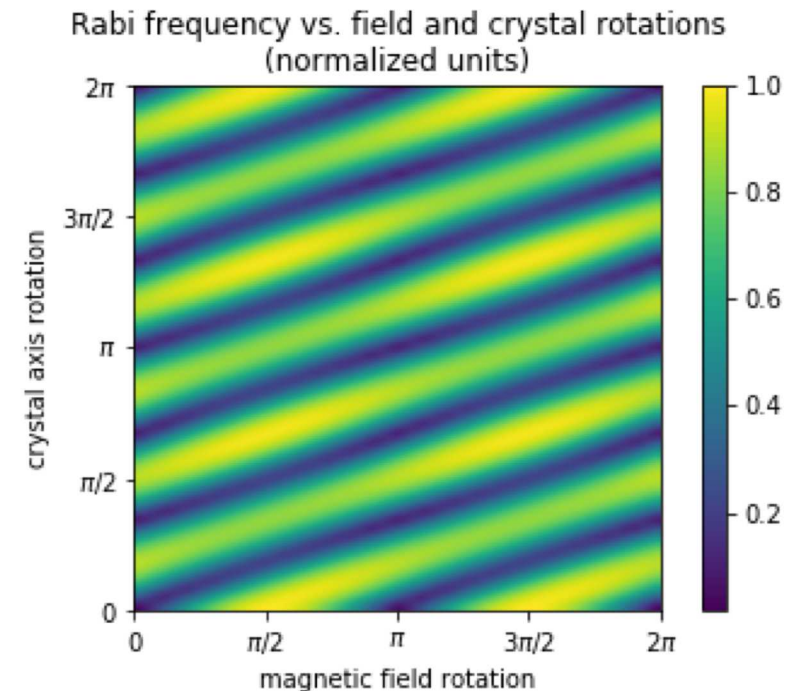
Crystallographic axes relative to device design

- Form of \hat{H}_{so3} imposes coordinate system
 - Crystal axes set by \hat{k}_x and \hat{k}_y
 - Rotation about the z-axis changes \hat{H}_{so3}
- B_{so} is then a function of the crystal axes as well as the dipole perturbation
 - This adjusts the previous structure
- We can explore the relative alignment of the crystal, dipole, and applied B-field
 - Fix dipole to original y-axis
 - Rotate B-field
 - Rotate crystal axes



- Rotation of the spin-orbit field as a function of charge-carrier direction of motion for
$$\hat{H}_{3,1} = i\alpha_{3,1}(\hat{\sigma}_- \otimes \hat{k}_+^3 - \hat{\sigma}_+ \otimes \hat{k}_-^3)$$

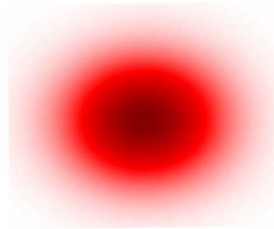
R. Moriya et al., PRL 2014



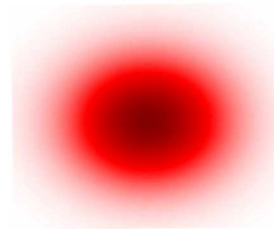
Qubit basis

- Without SOC:

$$|0\rangle = |\uparrow\rangle \otimes$$

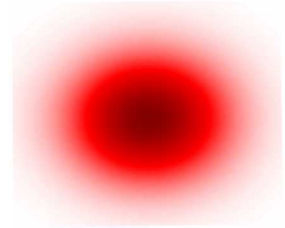


$$|1\rangle = |\downarrow\rangle \otimes$$

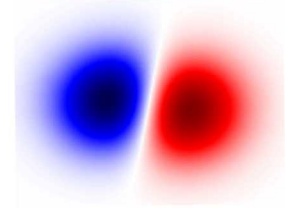


- With SOC:

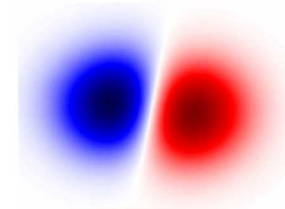
$$|0\rangle = \alpha|\uparrow\rangle \otimes$$



$$+\beta|\downarrow\rangle \otimes$$



$$|1\rangle = \gamma|\uparrow\rangle \otimes$$



$$+\delta|\downarrow\rangle \otimes$$



Linear SOC

- Hamiltonian:

$$\hat{H}_{so1} = i\alpha_1(\hat{\sigma}_- \otimes \hat{k}_+ - \hat{\sigma}_+ \otimes \hat{k}_-)$$

- Gate oscillation coupling:

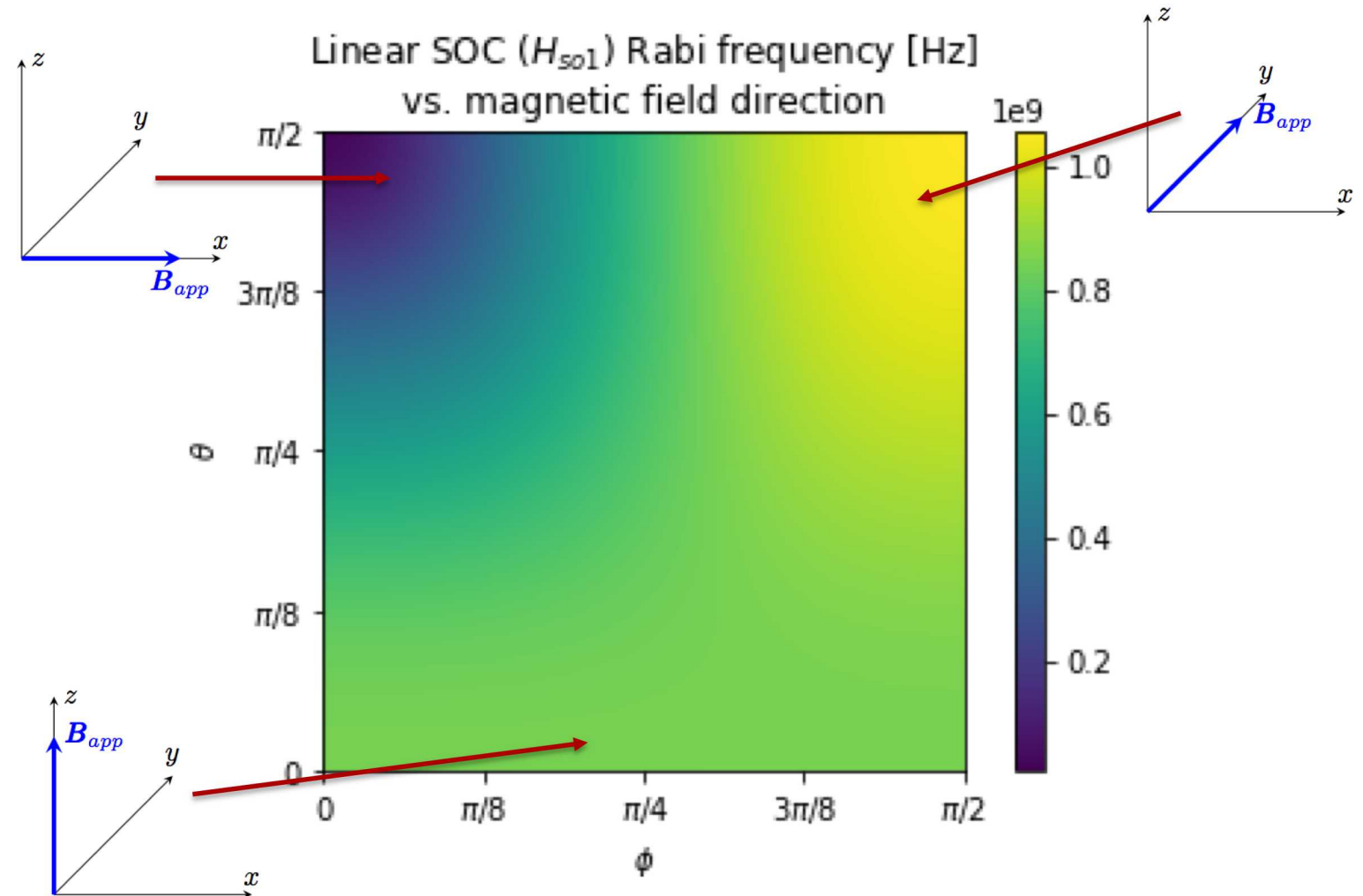
$$\hat{D}_{ac} \sim V_{ac}\hat{y}$$

- Rabi frequency low when

$$\mathbf{B}_{app} \perp \hat{y}$$

in the xy -plane, high when

$$\mathbf{B}_{app} \parallel \hat{y}$$



Cubic SOC

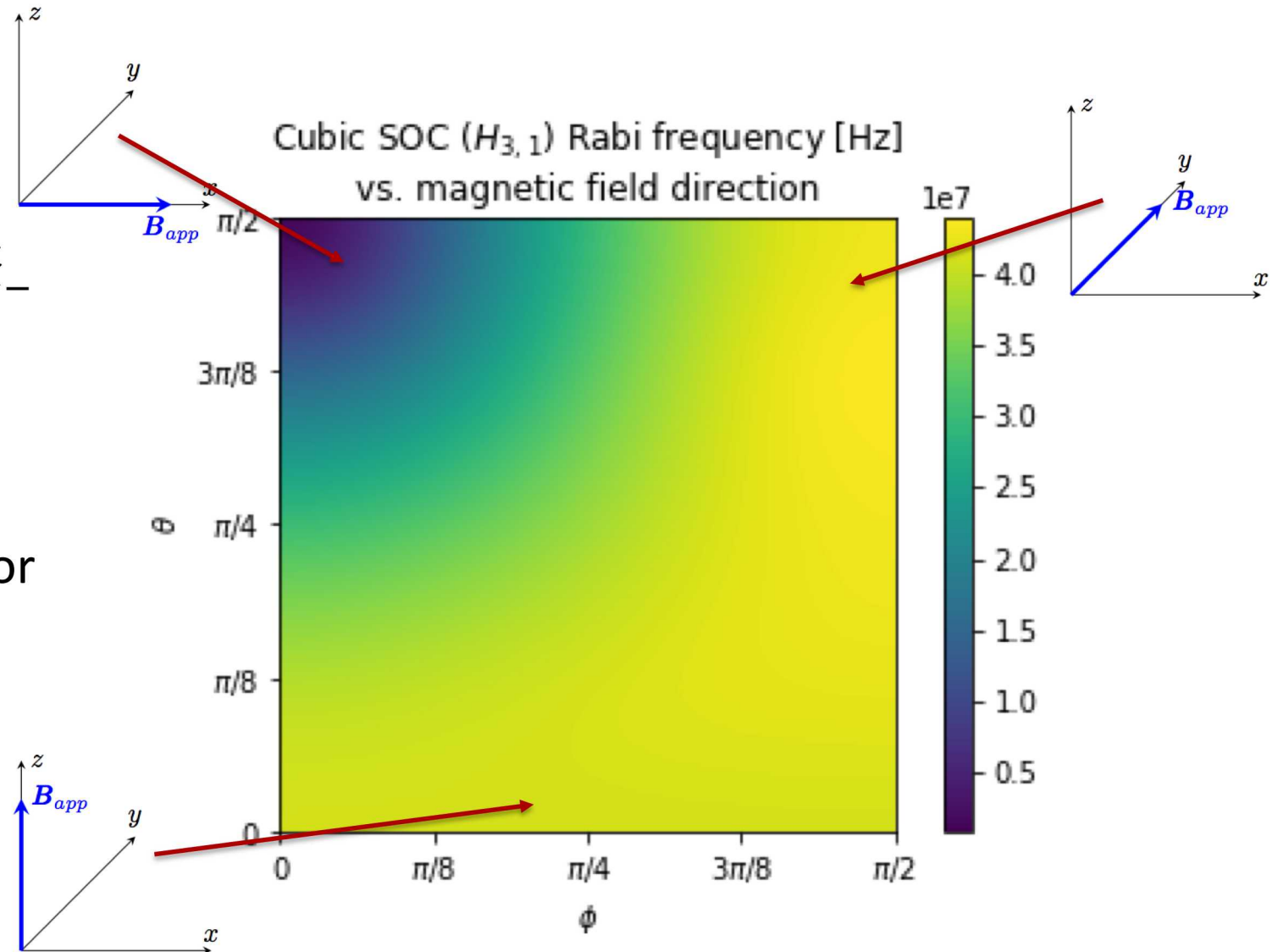
- Hamiltonian:

$$\hat{H}_{3,2} = i\alpha_{3,2}(\hat{\sigma}_- \otimes \hat{k}_- \hat{k}_+ \hat{k}_- - \hat{\sigma}_+ \otimes \hat{k}_+ \hat{k}_- \hat{k}_+)$$

- Gate oscillation coupling:

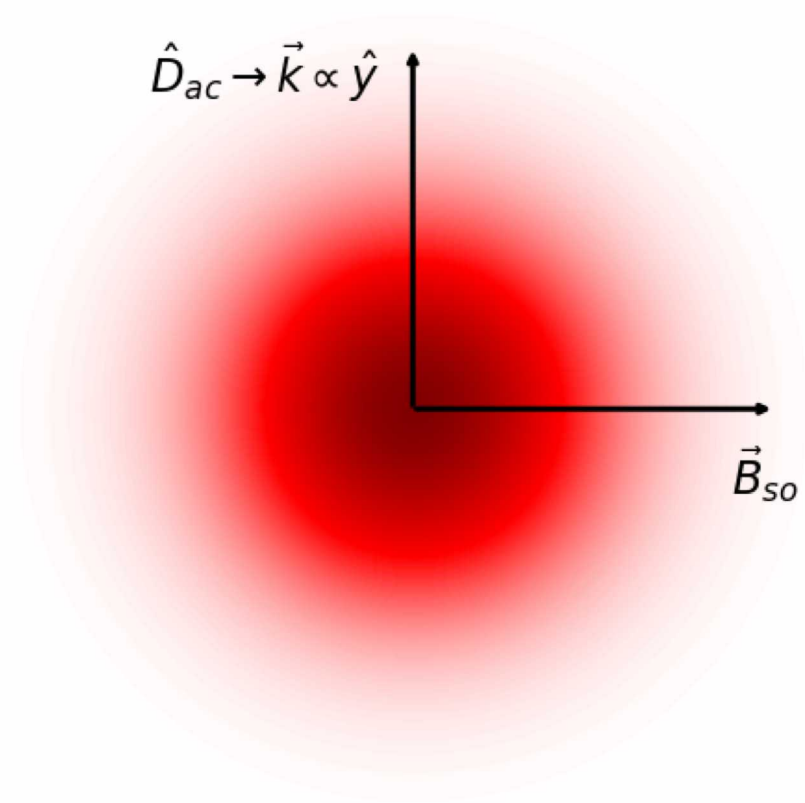
$$\hat{D}_{ac} \sim V_{ac} \hat{y}$$

- Similar qualities to linear for $\mathbf{B}_{app} \perp \hat{y}$ and $\mathbf{B}_{app} \parallel \hat{y}$ situations



Understanding SOC trends

- $\hat{H}_{so1} = \alpha_1(\hat{\sigma}_y \otimes \hat{k}_x - \hat{\sigma}_x \otimes \hat{k}_y)$
- $\hat{H}_{3,1} \approx \alpha_{3,1}(\hat{\sigma}_y \otimes (\hat{k}_x^2 - 3\hat{k}_y^2)\hat{k}_x - \hat{\sigma}_x \otimes (\hat{k}_y^2 - 3\hat{k}_x^2)\hat{k}_y)$
- $\hat{H}_{3,2} \approx \alpha_{3,2}(\hat{\sigma}_y \otimes (\hat{k}_x^2 + \hat{k}_y^2)\hat{k}_x - \hat{\sigma}_x \otimes (\hat{k}_x^2 + \hat{k}_y^2)\hat{k}_y)$
- $\hat{D}_{ac} \sim V_{ac}\hat{y} \rightarrow k_x \sim 0 \rightarrow B_{so} \propto \hat{x}$



Understanding SOC trends

- $\hat{H}_{so1} \sim \hat{\sigma}_x \otimes \hat{k}_y$
 - $\hat{H}_{3,1} \sim \hat{\sigma}_x \otimes \hat{k}_y^3$
 - $\hat{H}_{3,2} \sim \hat{\sigma}_x \otimes \hat{k}_y^3$
-
- $\hat{D}_{ac} \sim V_{ac} \hat{y} \rightarrow k_x \sim 0 \rightarrow B_{so} \propto \hat{x}$

