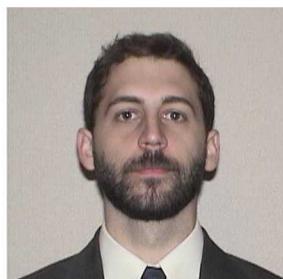
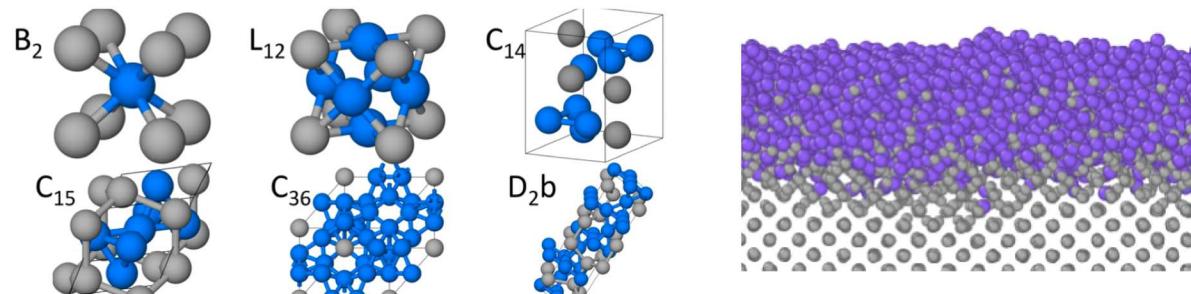


*Exceptional service in the national interest*



**Mitch Wood**



**Mary Alice Cusentino**

## Predictive Atomistic Simulations of Materials using SNAP Machine-Learning Interatomic Potentials

Aidan Thompson  
Sandia National Laboratories

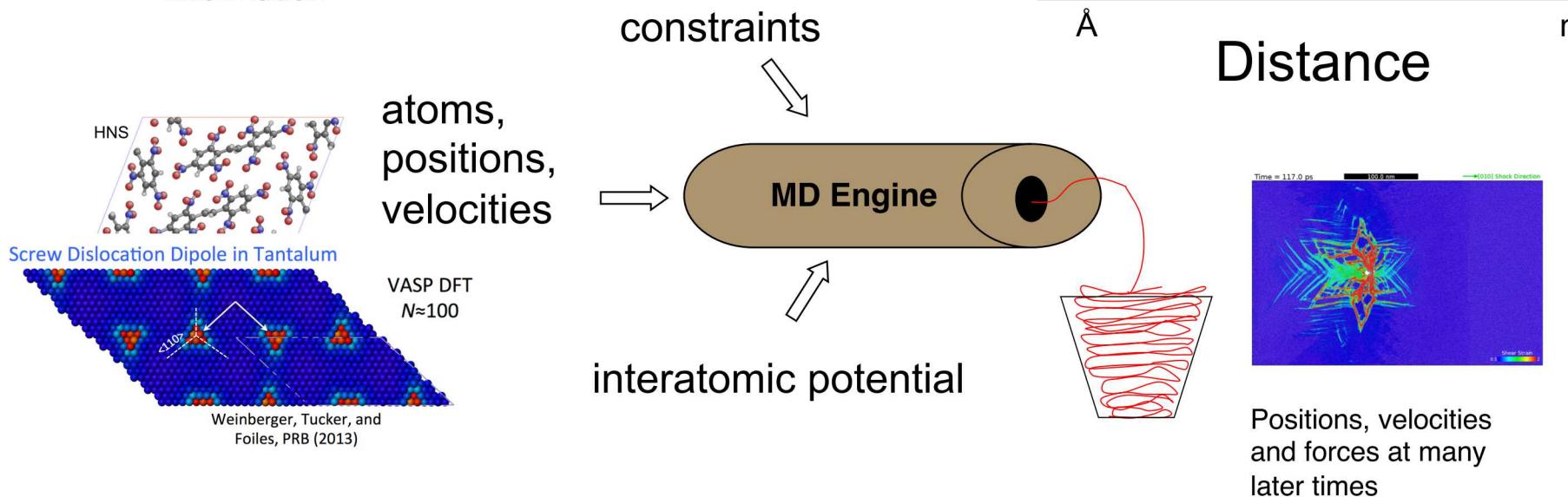


Sandia National Laboratories is a multi-mission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

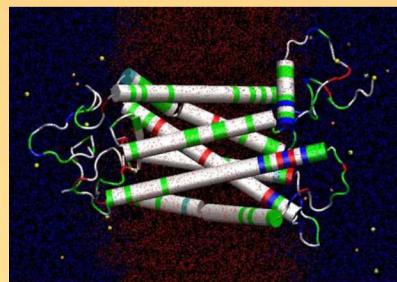
# What is Molecular Dynamics Simulation?



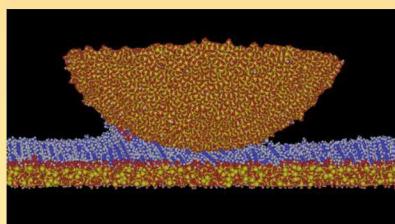
- Continuum models require underlying models of the materials behavior
- Quantum methods can provide very complete description for 100s of atoms
- Molecular Dynamics acts as the “missing link”
  - Bridges between quantum and continuum models
  - Moreover, extends quantum accuracy to continuum length scales; retaining atomistic information



# What is LAMMPS?



Initial positions  
and velocities



Interatomic  
potential



- Biomolecules
- Polymers (soft materials)
- Materials science (hard materials)
- Mesoscale to continuum

Large-scale Atomic/Molecular  
Massively Parallel Simulator



Positions and  
velocities at  
later times

Thanks to Aidan Thompson

Mike  
Chandross

# MD Approximations Change Over Time

## Twobody (B.C.)

Lennard-Jones, Hard Sphere, Coulomb, Bonded

## Manybody (1980s)

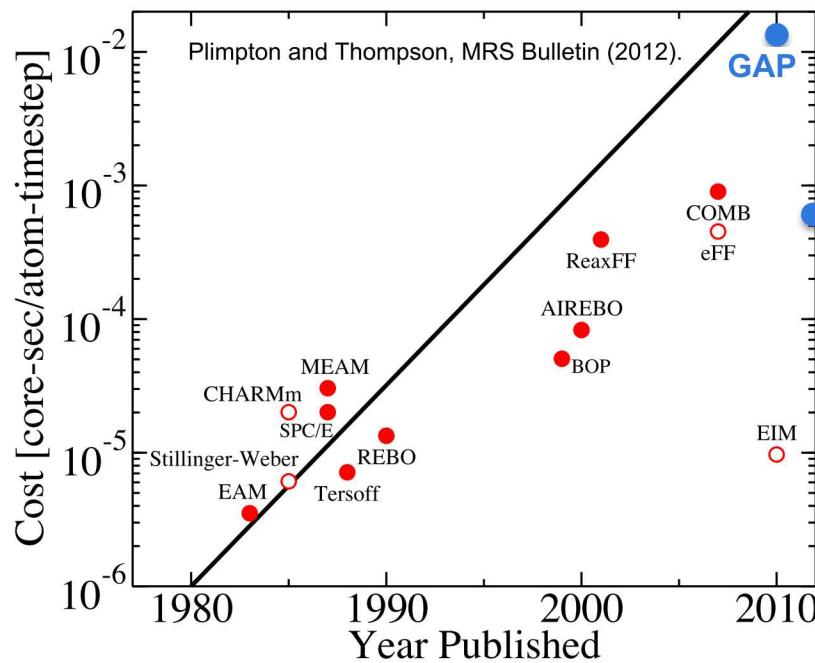
Stillinger-Weber, Tersoff, Embedded Atom Method

## Advanced (90s-2000s)

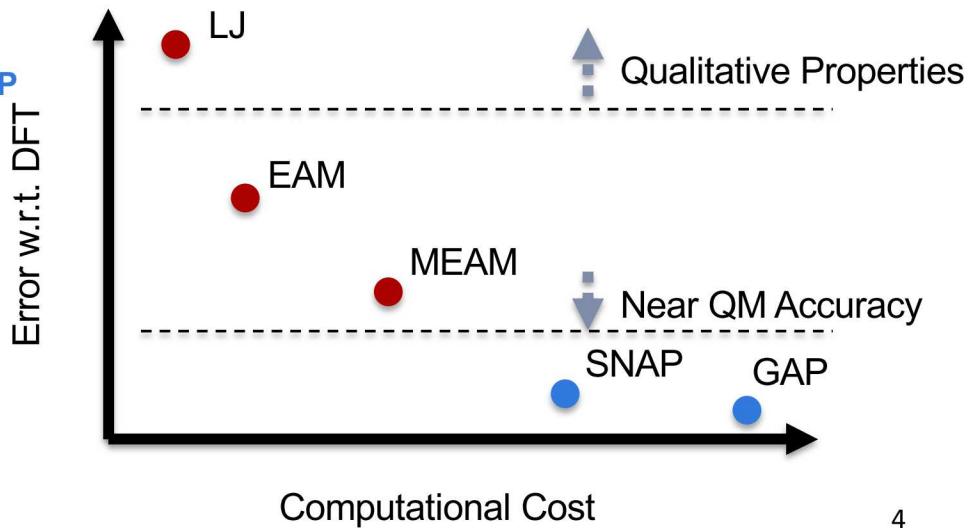
REBO, BOP, COMB, ReaxFF

## Big Data / Deep / Machine Learning (2010s)

GAP, SNAP, NN, ...



- Resources are limited, which is your best choice?



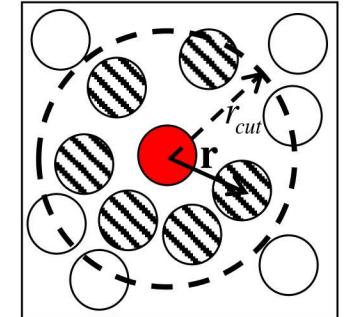
# Bispectrum Components as Descriptor

- Neighbors of each atom are mapped onto unit sphere in 4D

$$(\theta_0, \theta, \phi) = \left( \theta_0^{\max} r/r_{cut}, \cos^{-1}(z/r), \tan^{-1}(y/x) \right)$$

- Expand density around each atom in a basis of **4D hyperspherical harmonics**,
- Bispectrum components of the 4D hyperspherical harmonic expansion are used as the geometric descriptors of the local environment
  - Preserves universal physical symmetries
  - Rotation, translation, permutation
  - Size-consistent

$$u_{m,m'}^j = U_{m,m'}^j(0,0,0) + \sum_{r_{ii'} < R_{cut}} f_c(r_{ii'}) w_i U_{m,m'}^j(\theta_0, \theta, \phi)$$



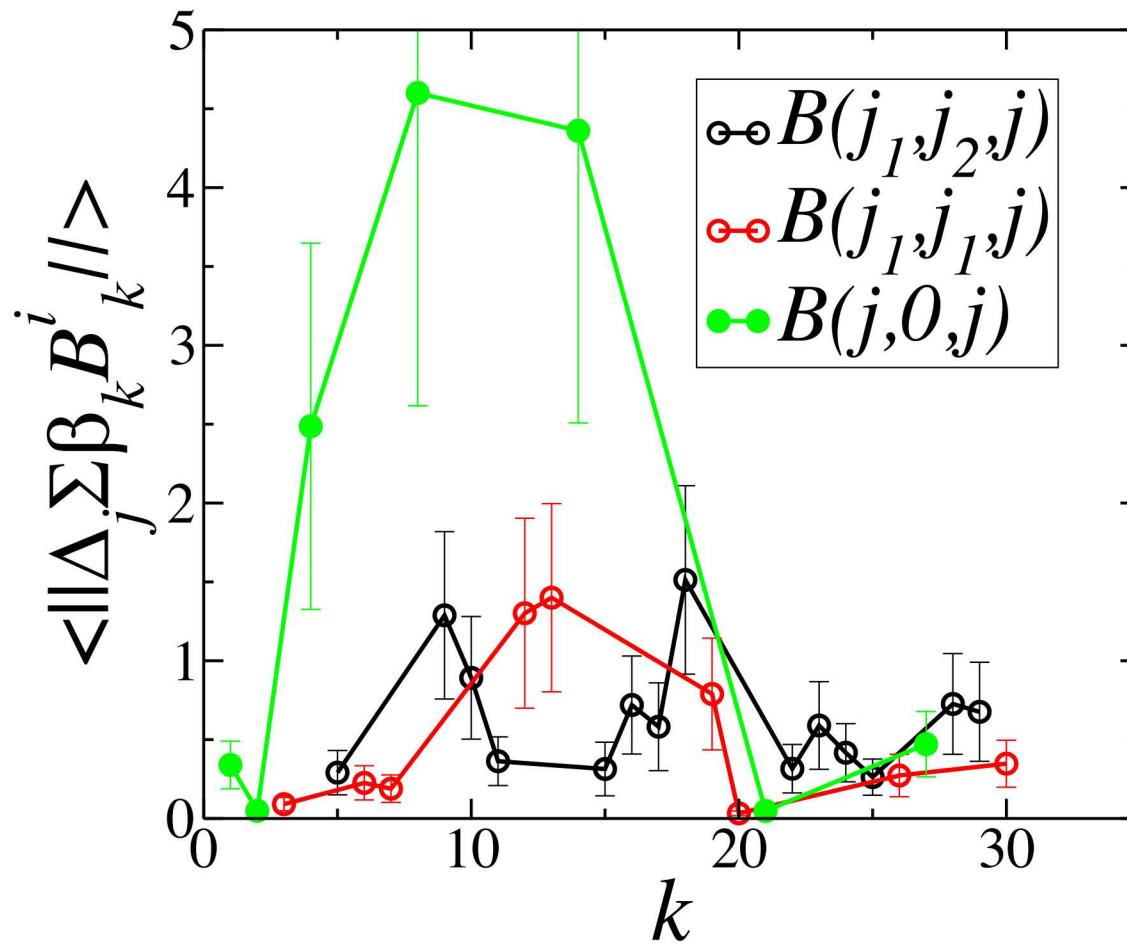
$$B_{j_1, j_2, j} = \sum_{m_1, m'_1 = -j_1}^{j_1} \sum_{m_2, m'_2 = -j_2}^{j_2} \sum_{m, m' = -j}^j (u_{m,m'}^j)^* H_{j_1 m_1 m'_1 \atop j_2 m_2 m'_2}^{j m m'} u_{m_1, m'_1}^{j_1} u_{m_2, m'_2}^{j_2}$$

Symmetry relation:  $\frac{B_{j_1, j_2, j}}{2j + 1} = \frac{B_{j, j_2, j_1}}{2j_1 + 1} = \frac{B_{j_1, j, j_2}}{2j_2 + 1}$

Drautz showed that Steinhardt, SOAP, SNAP and many other descriptors are all special forms of the atomic cluster expansion (*Phys. Rev. B* 2019) 5

# Effect of High-Order Bispectrum Components

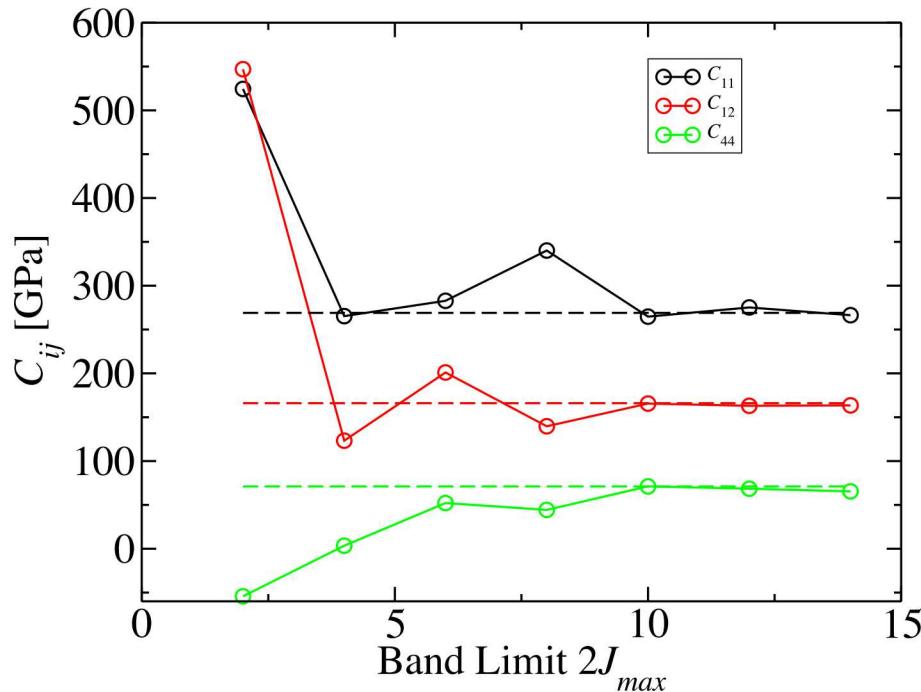
- MD simulation of molten tantalum using SNAP Ta06A potential
- Magnitude of average force contributed by each bispectrum component



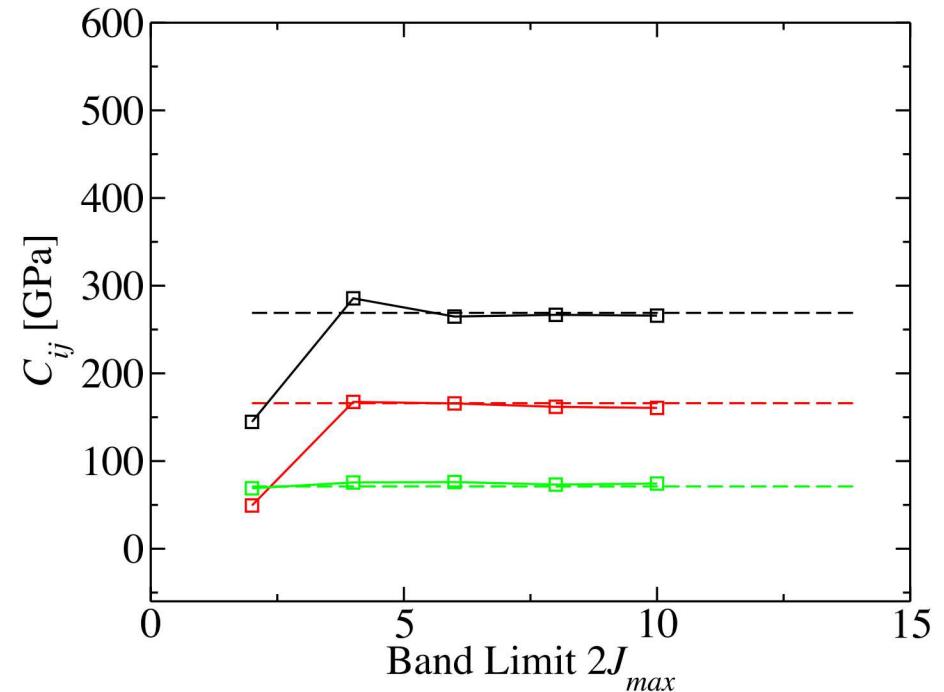
# Effect of High-Order Bispectrum Components

- Elastic constants for tantalum versus band limit

Linear SNAP



Quadratic SNAP



# ML-IAP Definition

## Model Form

- Energy of atom  $i$  expressed as a basis expansion over  $K$  components of the bispectrum ( $B_k^i$ )

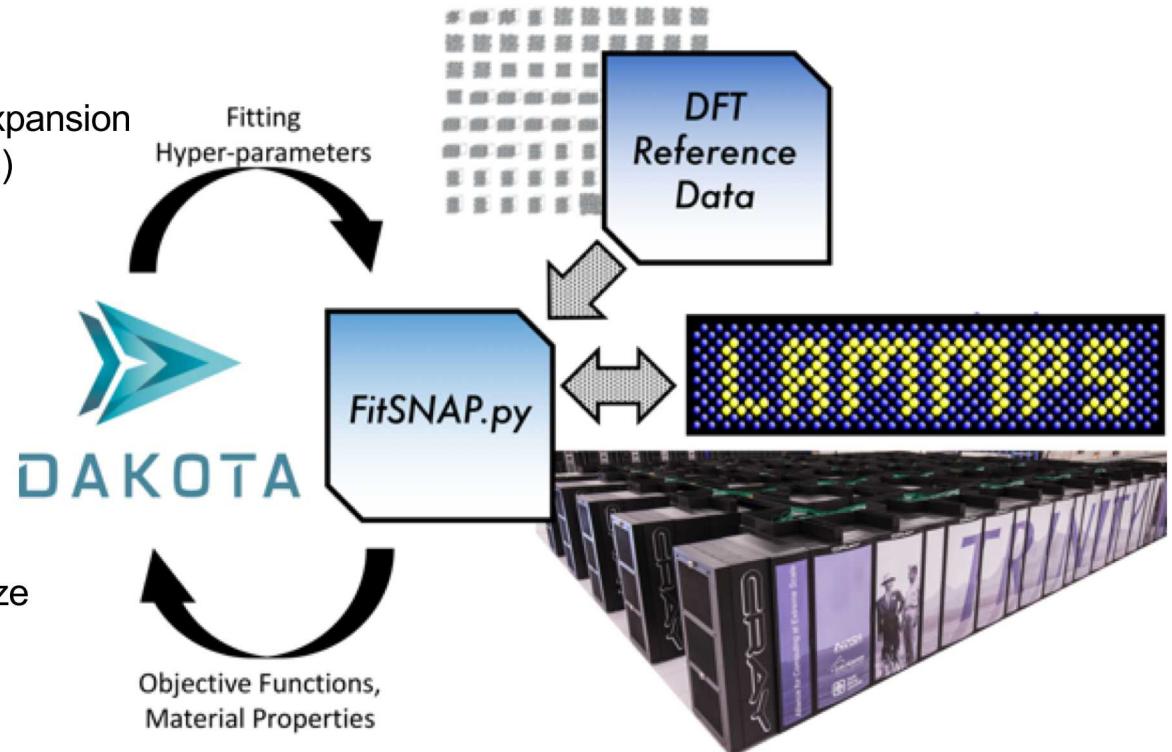
$$E_{SNAP}^i = \beta_0 + \sum_{k=1}^K \beta_k (B_k^i - B_{k0}^i)$$

## Regression Method

- $\beta$  vector fully describes a SNAP potential
- Decouples MD speed from training set size

$$\min(||\mathbf{w} \cdot D\beta - T||^2 - \gamma_n ||\beta||^n)$$

Weights      Set of Descriptors      DFT Training

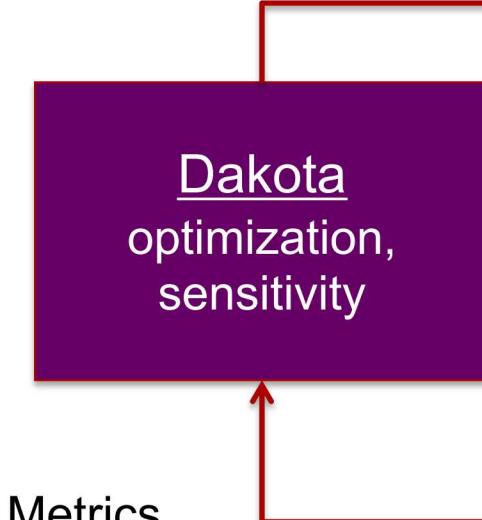


# SNAP Fitting Process

## FitSnap.py

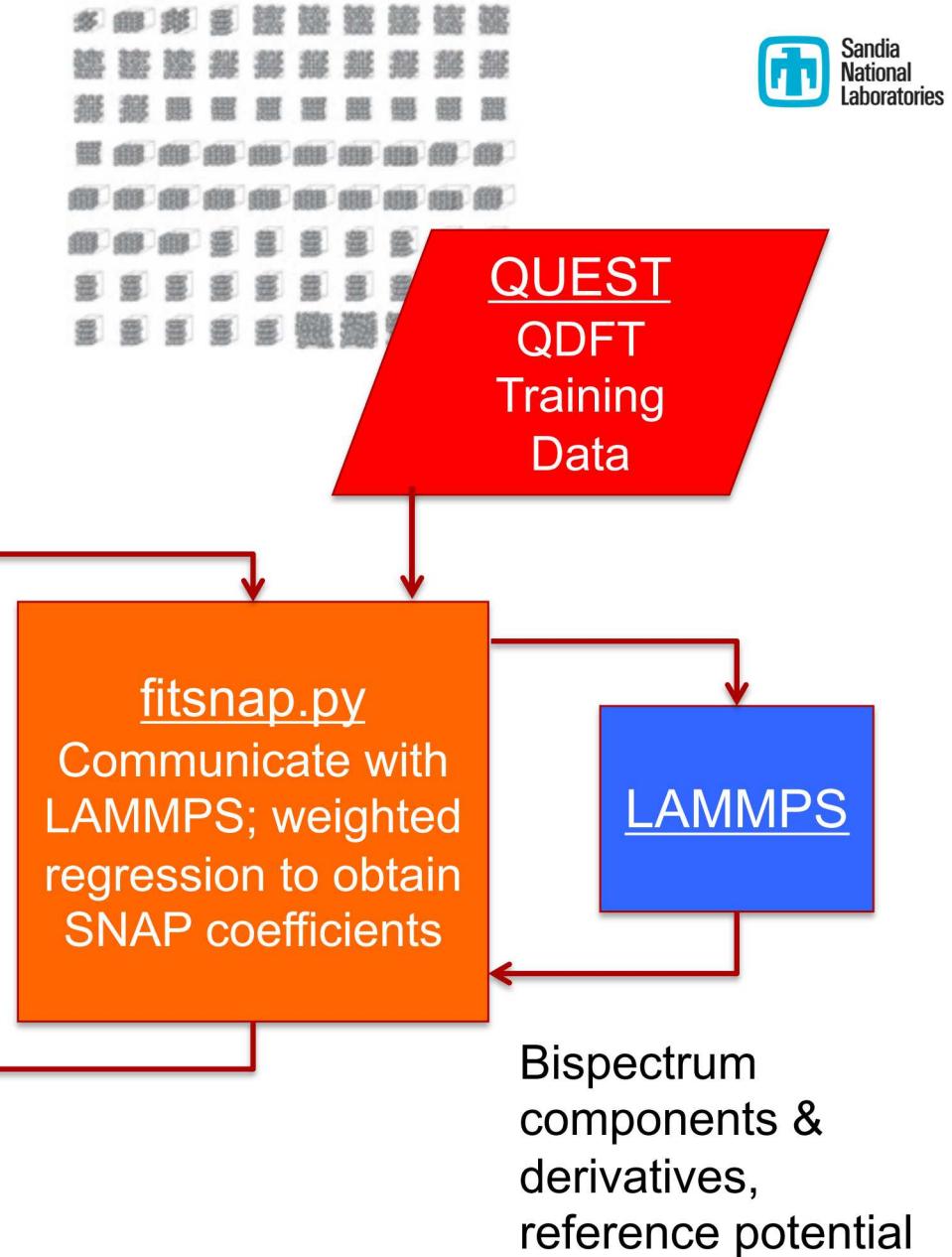
“Hyper-parameters”

- Cutoff distance
- Group Weights
- Number of Terms
- Etc.



Metrics

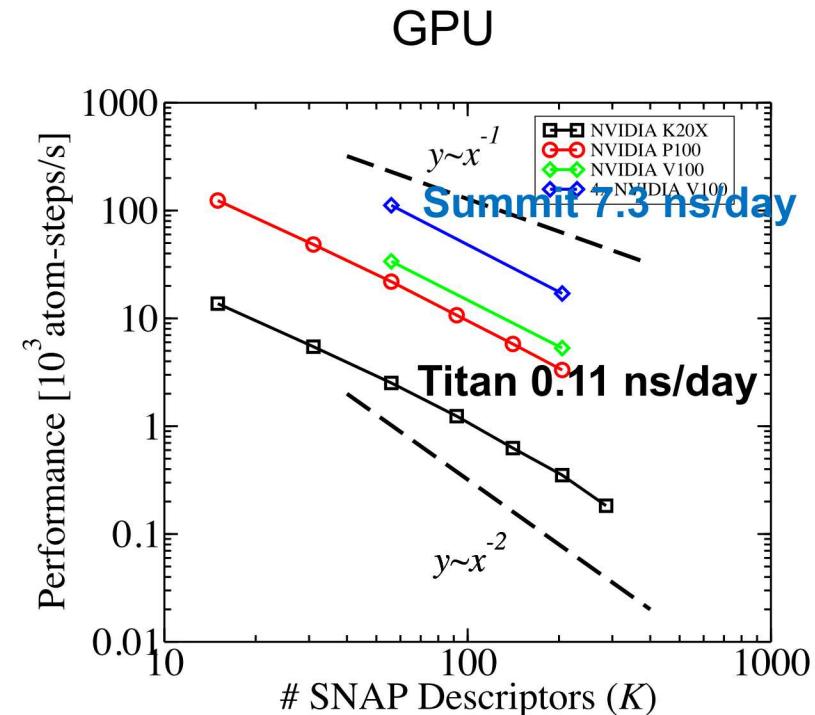
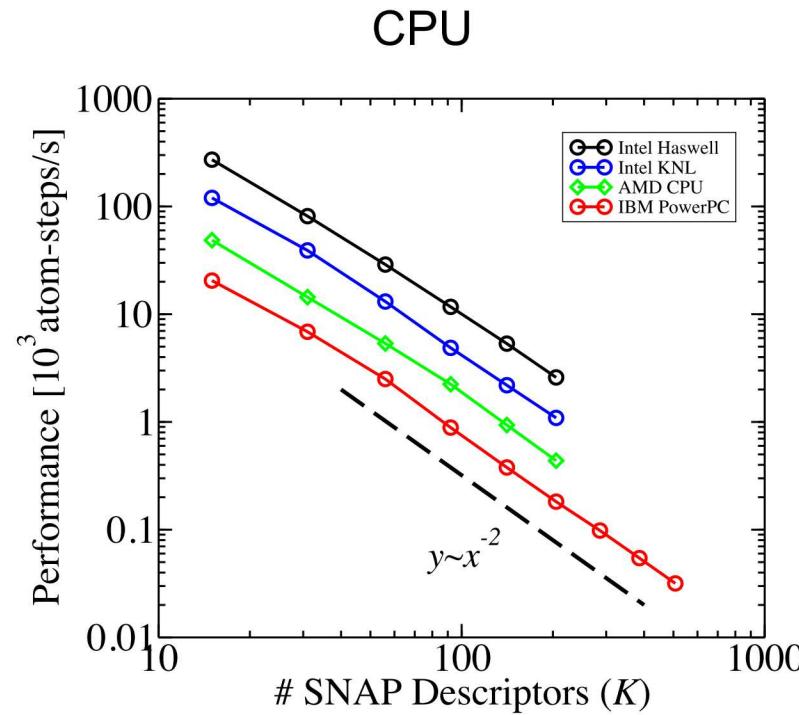
- Force residuals
- Energy residuals
- Elastic constants
- Etc.



# Outline of Current Research Areas

- **SNAP Computational Speed in LAMMPS**
  - SNAP with OpenMP (Good for CPUs)
  - **SNAP with KOKKOS (CPU, GPU,...) [ECP CoPA project]**
  - Exploring new GPU algorithms [NERSC/NESAP]
- **Plasma-Facing Materials (SciDAC-4)**
  - **Tungsten/Beryllium (complete)**
  - W/Be/H (in progress)
  - Nitrogen, Neon (future work)
- **Phase Transitions in Extreme Environments**
- **Radiation Damage in III-V Semiconductors**
  - **New Multi-element SNAP formulation**
- **SNAP Accuracy**
  - **Quadratic SNAP**
  - SNAP + Neural Networks
  - Better descriptors

# Adding Descriptors Increases Cost A Lot

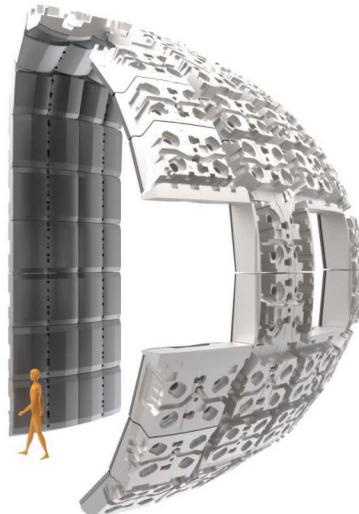


- Benchmarks for Exascale Computing Project
- Short MD simulation of BCC tungsten @ 300K
- GPU and KNL use the LAMMPS Kokkos package
- 2000 atoms, 1 node

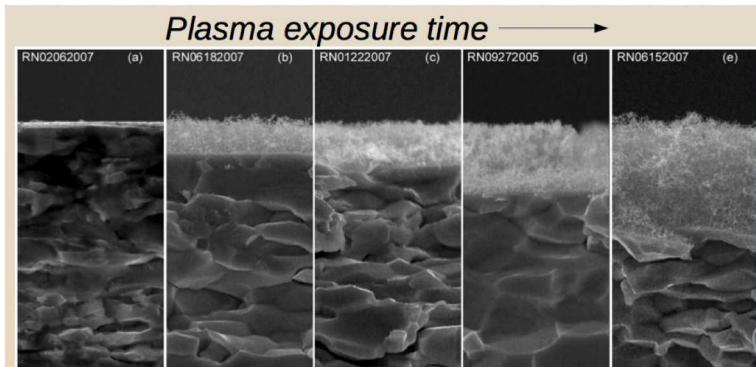
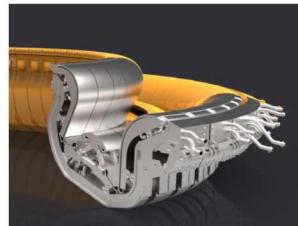
# Tungsten and Beryllium as Plasma-Facing Material



**First Wall**  
PFM = Beryllium



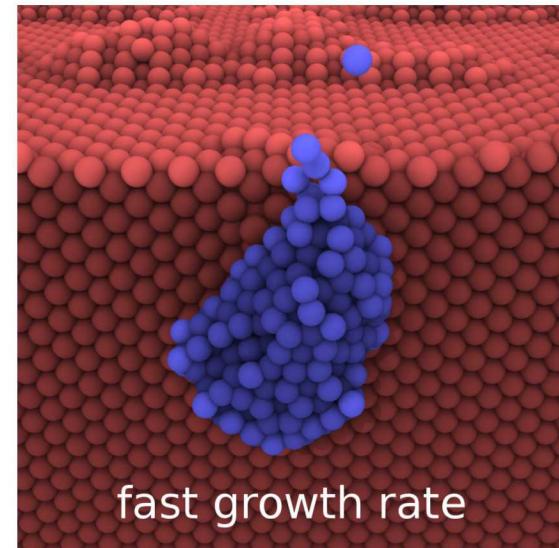
**Divertor**  
PFM = Tungsten  
10-20 MW/m<sup>2</sup>



*Nanostructured layer growth (fuzz) is observed at  $T=1120$  K and a flux of  $\sim 5 \times 10^{22}$  He  $m^{-2}s^{-1}$  [2].*

## ITER fusion reactor:

- Putting the sun in a box
- Plasma-facing material is tungsten
- Exposed to He and H at elevated temperature
- Fuzz buildup limits power output and useful life of divertor elements



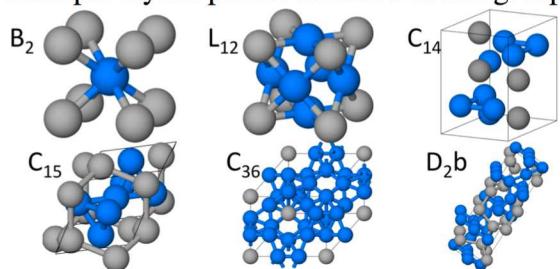
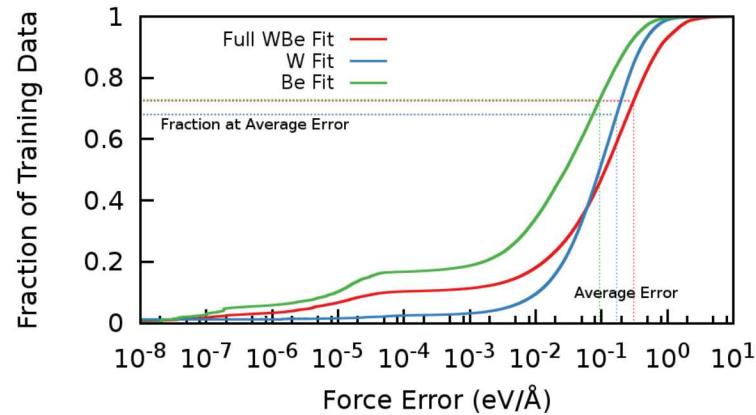
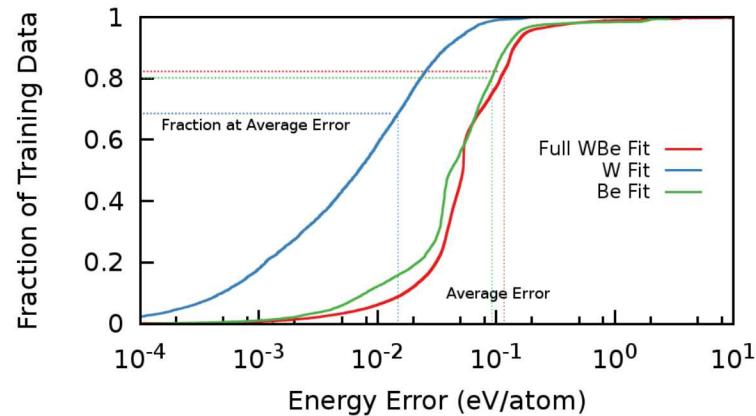
Luis Sandoval, Blas Uberuaga,  
Danny Perez, Art Voter, Phys.  
Rev. Lett. (2015)

## Training SNAP for Transferability – Tungsten+Beryllium

- Making a multi-element SNAP potential does sacrifice some accuracy from either pure component fit.
- Looking at which training data was weighted heavily

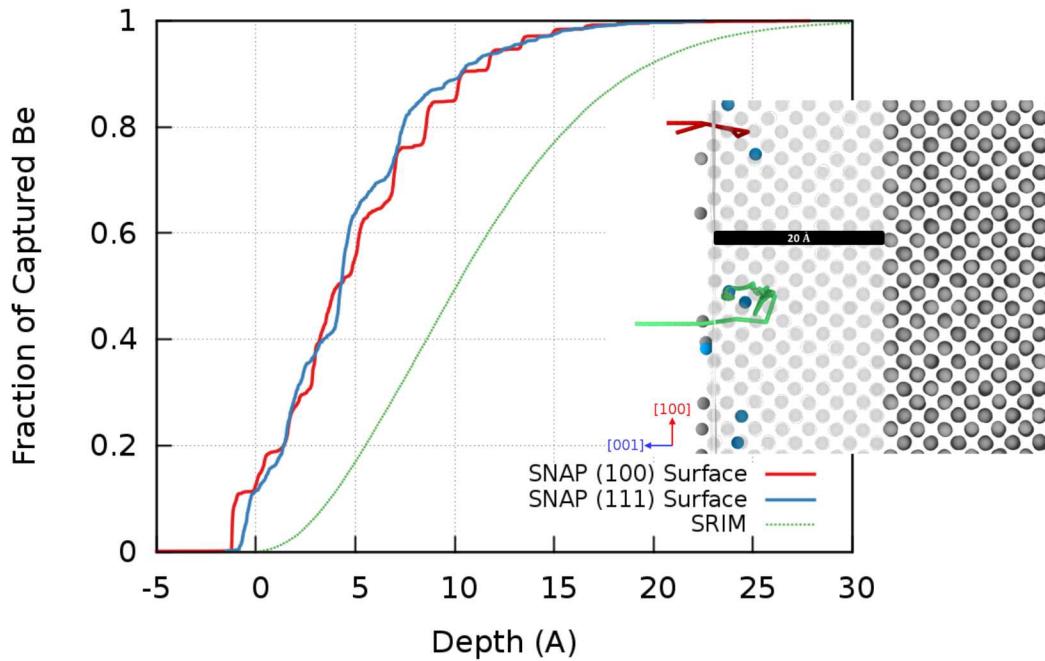
Description	$N_E$	$N_F$	$\sigma_E$	$\sigma_F$
W-Be:				
Elastic Deform <sup>†</sup>	3946	68040	$3 \cdot 10^5$	$2 \cdot 10^3$
Equation of State <sup>†</sup>	1113	39627	$2 \cdot 10^5$	$4 \cdot 10^4$
DFT-MD <sup>†</sup>	3360	497124	$7 \cdot 10^4$	$6 \cdot 10^2$
Surface Adhesion	381	112527	$2 \cdot 10^4$	$9 \cdot 10^4$

† Multiple crystal phases included in this group:

## Single Implantation Simulations

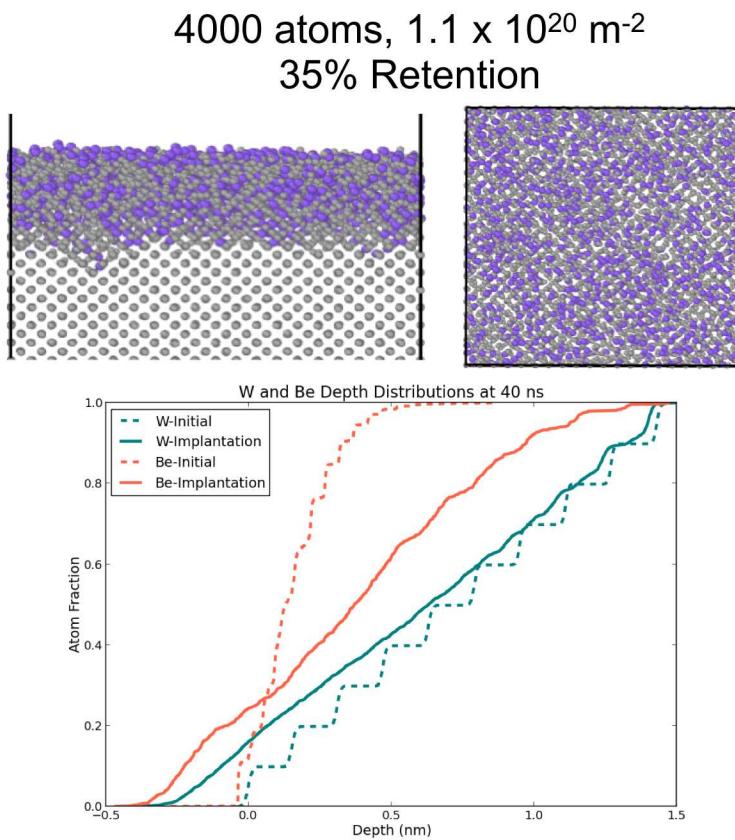
- MD depth profile is more shallow than binary collision models predict
- Capture rate is lower than BC model at 40% (versus 60%)



Defect Type	Percent of Implanted Be	
	(100) Surface	(111) Surface
[111] Dumbbell	41.2	23.9
Substitution	22.2	34.6
[100] Surf. Hollow Site	12.3	8.3
Tetrahedral Interstitial	10.4	12.4
[110] Dumbbell	8.4	11.3
Octahedral Interstitial	5.3	4.1
Other	0.4	2.8
Surf. Bridge Site	0.03	2.6

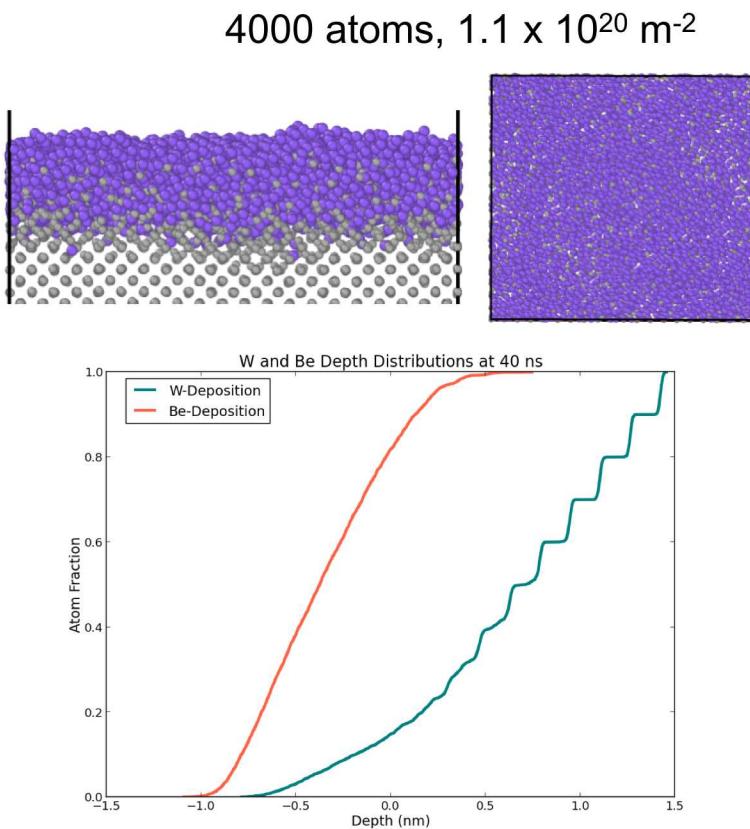
Defect Type	Formation Energy (eV)		
	DFT	SNAP	BOP
[111] Dumbbell	4.30	3.66	0.67
Substitution	3.11	3.29	-2.00
[100] Surf. Hollow Site	-1.05	-1.39	-3.52
Tetrahedral Interstitial	4.13	4.20	-0.28
[110] Dumbbell	4.86	4.29	-0.03
Octahedral Interstitial	3.00	5.11	0.34
[100] Surf. Bridge Site	1.01	0.44	-1.30

# Cumulative Energetic Implantation



- 75 eV cumulative Be implantation in W
- Implanted every 10 ps
- 1000 K, (100) surface, 6 nm x 6 nm x 12 nm box
- Initially Be implants into material and resides at defects like <111> dumbbell or substitutional sites
- At higher fluences, layer becomes amorphous
  - Exchange of W and Be with no obvious crystal structure
- Amorphous layer extends from about 1.5 nm in the surface to 0.2-0.3 nm above the original surface
- Tungsten loses crystal structure
- Be penetrates deeper (diffusion?)

# Cumulative Thermal Deposition



- Be now randomly placed on surface with no energy
- Rest of simulation parameters are the same
- Initially Be resides at hollow sites but once those become occupied, Be starts to exchange with W
- Similar results of amorphous layer that forms at higher fluences
- Layer is thicker, extending from 0.5 nm below the surface to 1 nm above the surface
- Be mainly remains near surface
- Almost 20% of the W in the first 1.5 nm is located above the original surface

# Can We Improve Multi-Element SNAP?



## Current Multi-element SNAP

$$u_{jmm'} = U_{jmm'}(0, 0, 0) + \sum_{r_{i'j} < R_{cut}} f_c(r_{i'}) w_\delta U_{jmm'}(\theta_0, \theta, \phi)$$
$$B_{j_1, j_2, j} = \sum_{m_1, m'_1 = -j_1}^{j_1} \sum_{m_2, m'_2 = -j_2}^{j_2} \sum_{m, m' = -j} (u_{m, m'}^j)^* H_{j_1 m_1 m'_1, j_2 m_2 m'_2}^{jmm'} u_{m_1, m'_1}^{j_1} u_{m_2, m'_2}^{j_2}$$

Elemental  
Weight

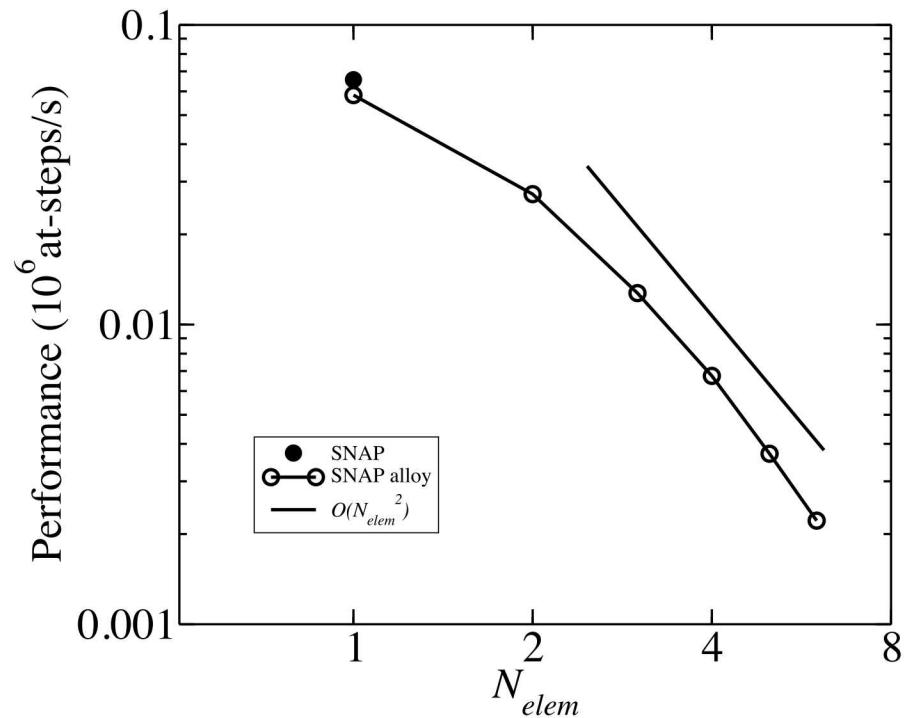
## Explicit Multi-Element SNAP

**Elemental Basis Function**  $u_{jmm'}^\delta = U_{jmm'}(0, 0, 0) + \sum_{\substack{r_{ii'} < R_{cut} \\ i' \in \delta}} f_c(r_{ii'}) w_\delta U_{jmm'}(\theta_0, \theta, \phi)$

**Three-Element Bispectrum Component**  $B_{j_1 j_2 j}^{\beta \gamma \delta} = \sum_{m_1, m'_1 = -j_1}^{j_1} \sum_{m_2, m'_2 = -j_2}^{j_2} \sum_{m, m' = -j}^j (u_{jmm'}^\beta)^* H_{j_1 m_1 m'_1, j_2 m_2 m'_2}^{jmm'} u_{j_1 m_1 m'_1}^\gamma u_{j_2 m_2 m'_2}^\delta$

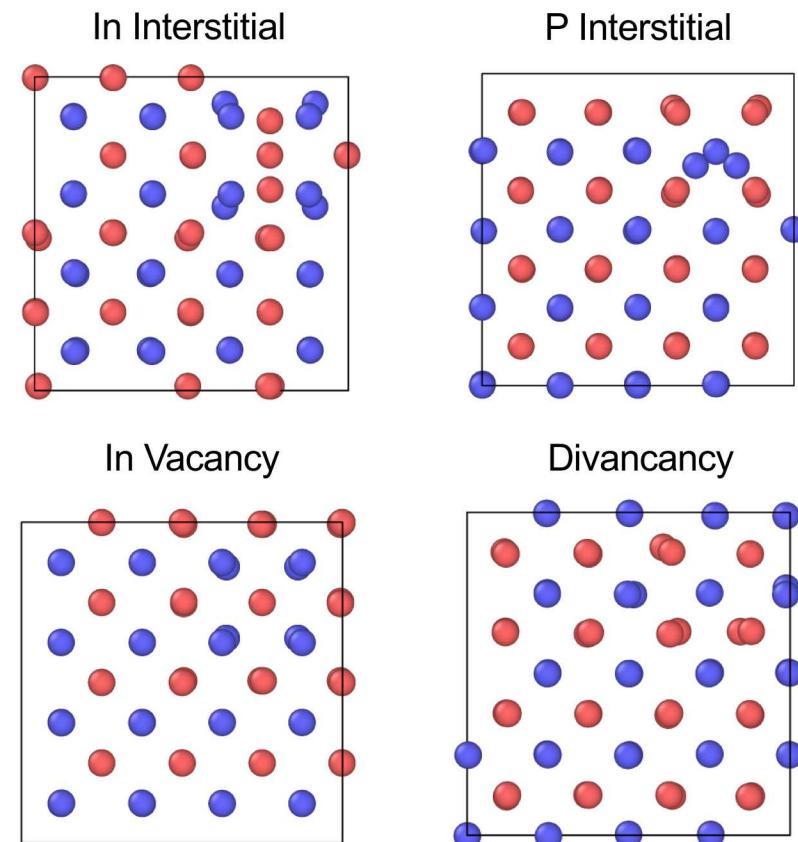
## Computational Cost of Multi-Element

- Cost dominated by  $dBi/dRj$
- For each bispectrum component there are  $Nelem^3$  labellings
- But neighbor  $j$  only contributes to the ones that have at least one label matching element of  $j$
- $Nelem^2$  labellings of remaining two labels



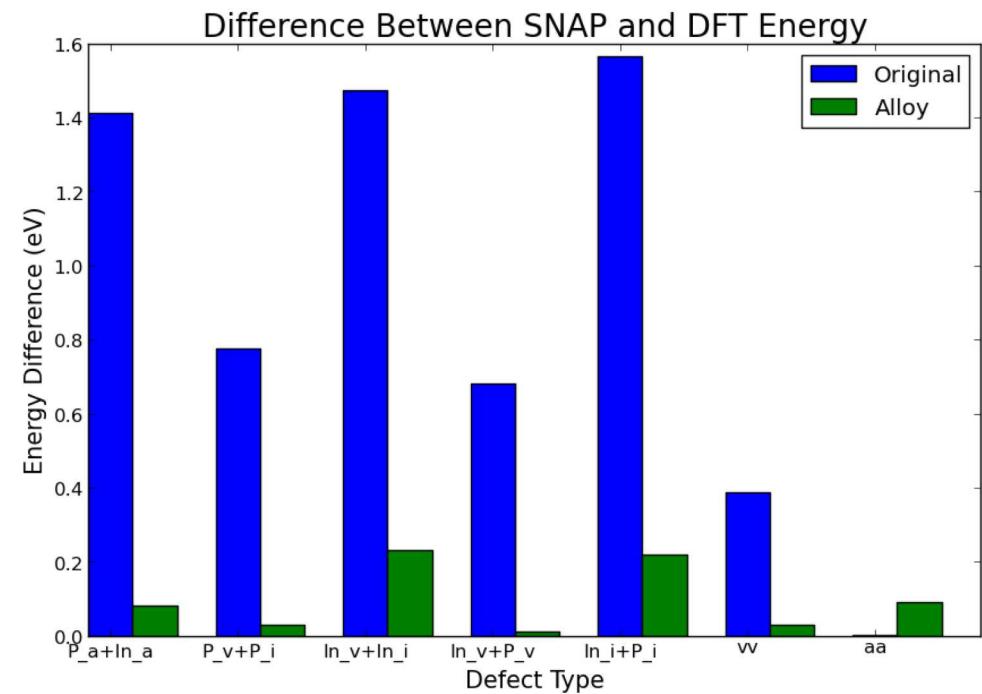
# Indium Phosphide SNAP Potential

- Interest in studying radiation damage in semiconductors such as indium phosphide
  - Potential needs to be able to reproduce dynamics of a high energy collision cascade
- For radiation damage, accurately reproducing defect formation energies is crucial
- SNAP needs to not only reproduce these important parameters but also be able to distinguish between 2 element types
- Initial multi-element version of SNAP replicated InP properties fairly well but failed in reproducing the defect formation energies
  - Difference between SNAP and DFT was greater than 1 eV for some defects
- Preliminary results of new multi-element alloy formulism of SNAP is showing much improvement



# InP SNAP Preliminary Results

- Compare original and alloy version of SNAP for InP
- Compare defect formation energies between SNAP and DFT
  - Defect formation energies were chosen to conserve stoichiometry
- Previous multi-element version of SNAP failed to reproduce defect formation energies
  - Still over 1 eV difference from DFT for best potential
- New alloy multi-element version of SNAP performs much better
  - Defect formation energies show much reduced error from DFT
  - Largest difference is 0.23 eV



# InP Elastic Constants

	Exp.*	SNAP
C11 (Gpa)	101.1	99.5
C12 (Gpa)	56.1	53.6
C44 (Gpa)	45.6	16.7
Bulk Modulus (Gpa)	71.1	66.8
Shear Modulus (Gpa)	22.5	17.7, 26.1
Poisson Ratio	0.36	0.33

- Elastic constants for InP are fairly well reproduced
- More consistent with expected values compared to previous iterations of this potential

\* D.N. Nichols, D.S. Rimai, and R.J. Sladek Solid State Commun. 667, 36 (1980)

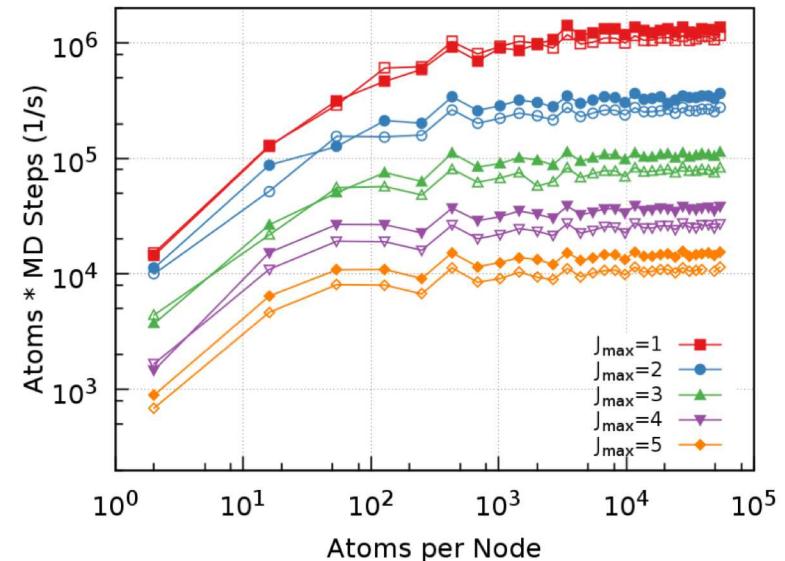
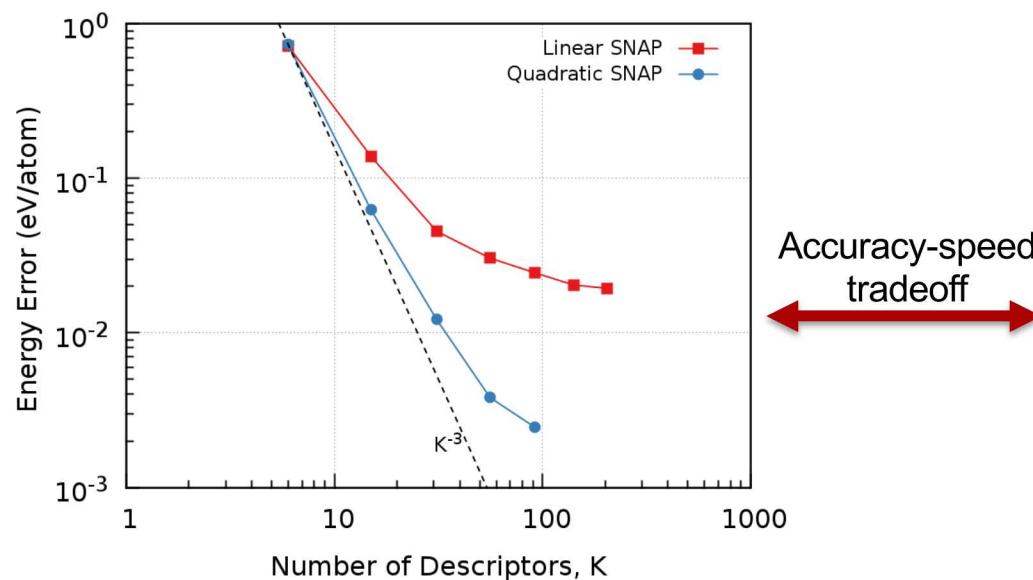
# What About Adding Quadratic Terms?

$$E_{SNAP}^i = \beta \cdot \mathbf{B}^i + \frac{1}{2}(\mathbf{B}^i)^T \cdot \alpha \cdot \mathbf{B}^i$$

- Linear terms are 4-body
- Quadratic terms are 7-body
- Number of linear coefficients grows as  $O(J^3)$
- Number of quadratic coefficients grows as  $= O(J^6)$
- Energy, force, stress remain **linear** in  $\beta$  and  $\alpha$
- Can still use linear least squares (SVD)
- Number of columns will increase from  $K$  to  $K(K+1)/2$

SNAP Tantalum

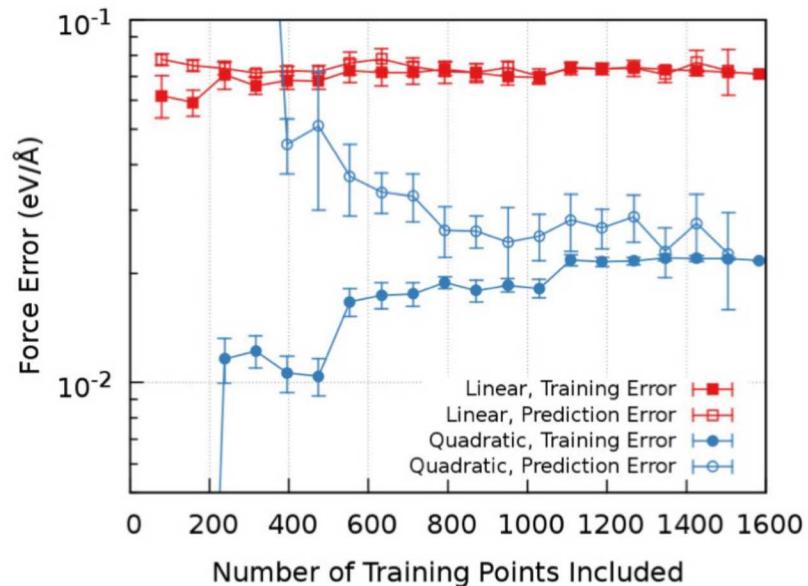
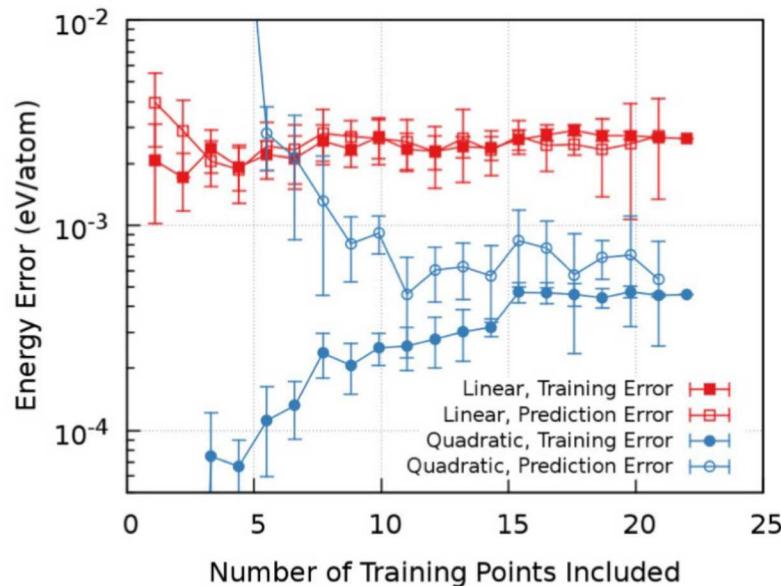
Wood and Thompson, *J. Chem.Phys.*, March, 2018  
Also <https://arxiv.org/abs/1711.11131>



## Quadratic SNAP – Cross Validation

- Concerned with overfitting now that there are MANY more free parameters during the fit.
- (Training Points) : (Descriptors) still  $\gg 1$  for assembled training sets

$$E_{SNAP}^i = \beta \cdot \mathbf{B}^i + \frac{1}{2}(\mathbf{B}^i)^T \cdot \alpha \cdot \mathbf{B}^i$$



# Conclusions

- Molecular dynamics is a powerful tool for exploring materials behavior
- Can access physics, chemistry, length, and timescale that is inaccessible to other methods (continuum, quantum)
- Applications are driving demand potentials of greater accuracy
- We have built a system for converting large amounts of quantum calculations into SNAP potentials for large-scale MD simulations of complex materials
- SNAP strikes a good balance between speed, accuracy, and robustness
- We still don't really understand why ML potentials work or don't work
- Still lots of room for improvement

