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Derivation of Shaker Shock Input of an Oscillatory Decaying Shock to Optimize High Frequency SRS

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Abstract

Shock Response Spectra (SRS) are commonly used to characterize transient shock field environments, and a common means to reproducing the field environment in the laboratory is with electrodynamic shaker tests consisting of sums of decayed sinusoids. Owing to the nonlinear nature of the SRS transformation, there can be considerable variation in the energy associated with the decayed sine waveform used to replicate the desired SRS. However, for the case where the SRS has a hump at low frequency and a lower magnitude "flat" response at high frequency, it is possible to replicate the high frequency SRS with little or no energy in that region. This class of SRS is common with transportation environments.

The purpose of this paper is to study the range of possible high frequency inputs associated with this flatline SRS and present a technique for optimizing the high frequency spectral content.

Introduction

The analysis will utilize Monte Carlo techniques to scope out the range of possible high frequency content associated with an actual laboratory test specification. A multi-degree-of-freedom spring mass model will be used to assess the damage potential for several cases of interest.

SRS Theory

The root cause of the issue comes from the fact that the magnitude of the SRS at any given frequency is a function of the entire spectral content of the input waveform based on the Transmissibility Response Function (TRF) of a Single Degree of Freedom (SDOF) oscillator shown in Figure 1.

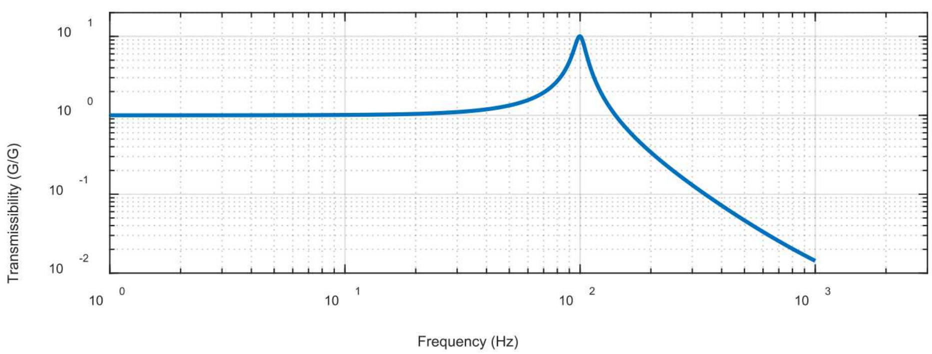

Figure 1: Transmissibility Response Function for SDOF Oscillator

When the spectral content of the input is predominantly in the low frequency region, the response of the higher frequency SDOF oscillators can be dominated by the peak amplitude of the low frequency waveform instead of the

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actual high frequency spectral content, which in turn produces the flatline SRS. The problem with this is that it hides the true high frequency spectral content.

Field Data

Figure 2 presents the acceleration history and Fourier Transform (FT) of the field data used in this study. Figure 3 presents the Shock Response Spectra (SRS). Both the FT and the SRS indicate that the spectral content is greatly reduced above 300 Hz. In keeping with standard practices, a test specification was created that enveloped the raw field SRS using a series of straight line segments on a log-log plot spanning the frequency range from 7 Hz to 1 kHz. The resulting test specification SRS is also shown in Figure 3.

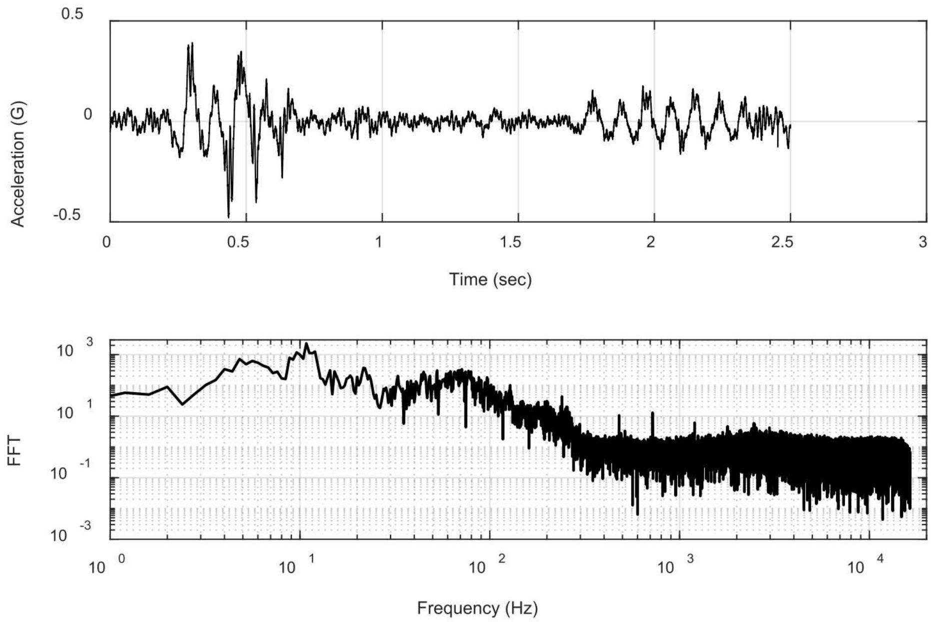

Figure 2: Field Data

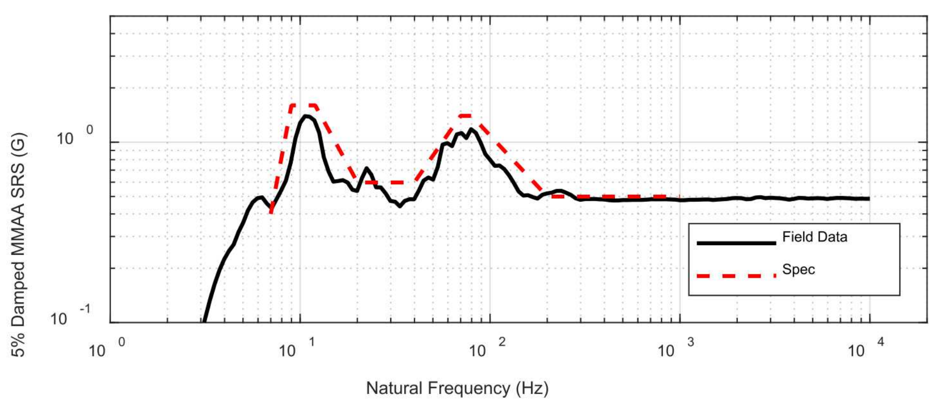

Figure 3: Field Data and Test Specification SRS

Sum of Decayed Sines

The laboratory test specification is defined using a set of decayed sine tones as defined in equation (1) where A_n, ζ_n, and f_n are the amplitude, damping and tonal frequencies respectively.

$$A(t) = \sum_{n=1}^{N} A_n e^{-2\pi\zeta_n f_n t} sin(2\pi f_n t) \quad (1)$$

The tonal amplitudes are iterated upon until the resulting SRS matches the desired SRS within a specified tolerance. Matlab codes designed to implement this process were developed at Sandia by David Smallwood [1]. Smallwood's algorithm derives the tonal amplitudes from low to high frequency. This order of operation is best suited to take advantage of the contribution of the low frequency tones on the high frequency response, but it also tends to minimize the amplitudes of the high frequency tones.

Monte Carlo

The next step was to generate hundreds of decayed sine waveforms consisting of sine tones in the 7 Hz to 200 Hz range, each of which produce an SRS that approximates the desired SRS. The goal was to look at the range of possible peak accelerations based on the low frequency spectral content.

The Monte Carlo approach has been used successfully at Sandia to generate ensembles of decayed sine waveforms [2, 3]. In summary, the Monte Carlo algorithm chooses a set of initial tonal frequencies, f_{CENT}, typically using a uniform octal spacing (i.e., N tones/octave). Every tonal frequency is then varied randomly about its corresponding initial frequency value using a uniform random distribution. A guard band equal to 1% of the frequency range between the initial tones is imposed to prevent adjacent random tones from overlapping. Figure 4 presents this concept graphically for N=4 tones/octave and 50 Monte Carlo simulations. The first simulation in any set of N is always defined using the initial values of f_{CENT} (simulation #1 in the plot).

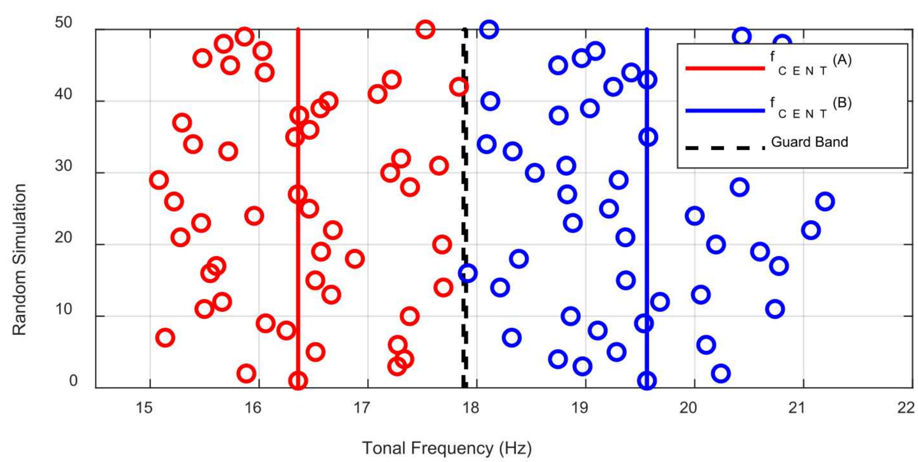

Figure 4: Randomly Selected Sine Tone Frequencies

By varying the tonal density (tones/octave) and generating 30-40 scenarios per tonal density, it is possible to generate several hundred sets of decayed sine waveforms and their corresponding SRS. While more simulations is always better, one can typically tell from the resulting scatter plot whether a nominally minimal solution has been achieved.

Optimization Metrics

To determine which Monte Carlo simulation is the optimal solution, it is necessary to define a set of scalar metrics. The peak amplitude of the waveform was the first metric chosen because it most closely associated with the SRS flatline. However, a metric was needed to insure that the optimal input waveform produced an SRS that matched the desired SRS over the frequency range of interest. Therefore, the second metric chosen was the "rms dB error" developed at Los Alamos National Laboratory and shown in equation (2) where N denotes the number of target SRS, Δf is the bandwidth associated with each SRS value, and f_B is the overall frequency bandwidth.

$$E_{RMSDB} = \left(\frac{1}{N}\right)\sum_{n=1}^{N}\left(\sqrt{\frac{\sum_{k=1}^{K}\left[\left(20*\log\left(\frac{S(k)_M}{S(k)_R}\right)\right)^2 \Delta f(k)\right]}{f_B}}\right) \tag{2}$$

The rms dB error treats a positive and negative error equal so while an error of 0 dB is perfect, one cannot say whether the given response SRS was high or low. However, there are times when one wishes to know if a given SRS was high (conservative) or low (under conservative). Therefore, a third error metric, denoted as the average dB error, was developed. The formula for the average dB error is shown in equation (3) where M is the number of spectral values in the SRS. The inner summation represents the mean of the N SRS as a function of frequency. The outer summation computes the average of the mean values across all frequencies.

$$E = \frac{1}{M}\sum_{m=1}^{M}\frac{1}{N}\sum_{n=1}^{N} S_n(f) \tag{3}$$

For the purposes of this study, the SRS were evaluated at 12 points/octave regardless of the tonal density of the underlying acceleration waveform.

Case Studies

Figure 5 plots the rms dB error and peak acceleration for 550 simulations (50 random tone sets for each of 11 tonal densities ranging from 2 to 12 tones/octave). These metrics were computed for just the SRS values below 200 Hz. The peak acceleration associated with the field SRS test specifications is included on the plot.

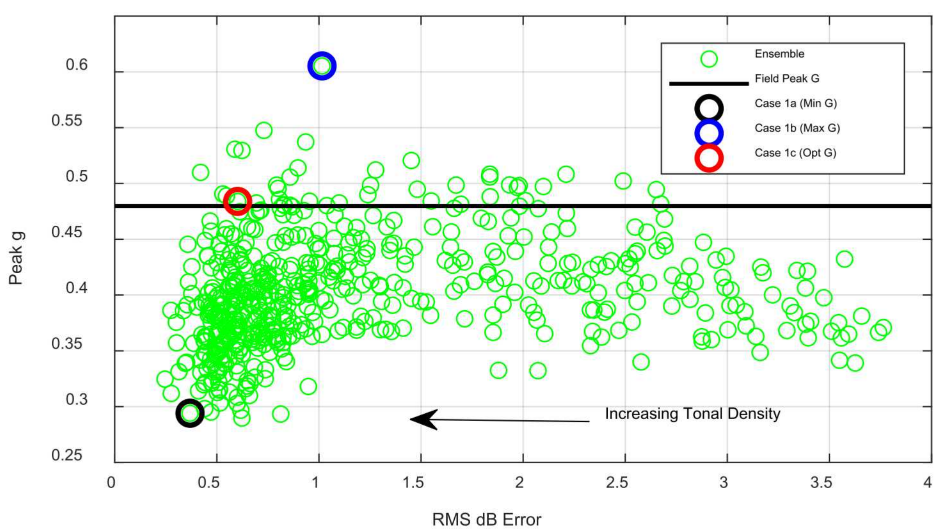

Figure 5: Monte Carlo Error Metrics

An initial set of three case studies were chosen to demonstrate the range of possible outcomes. These are described below and identified in Figure 5.

Case 1a looked at the smallest low frequency peak acceleration (the min G case). Since it was necessary to add high frequency tones to this case to match the test specification out to 1 kHz, it was assumed that this case represented the greatest amount of high frequency content.

Case 1b looked at the largest low frequency peak acceleration (the max G case).

Case 1c looked at a scenario in which the low frequency peak acceleration equaled the test specification high frequency SRS (the opt G case).

By default, no additional high frequency tones were needed to match the high frequency SRS for cases 1b and 1c.

Figure 6 overlays the SRS for the three low frequency cases with the test specification SRS. The version of case 1a with high frequency tones, denoted as "1a (WB)", is also shown.

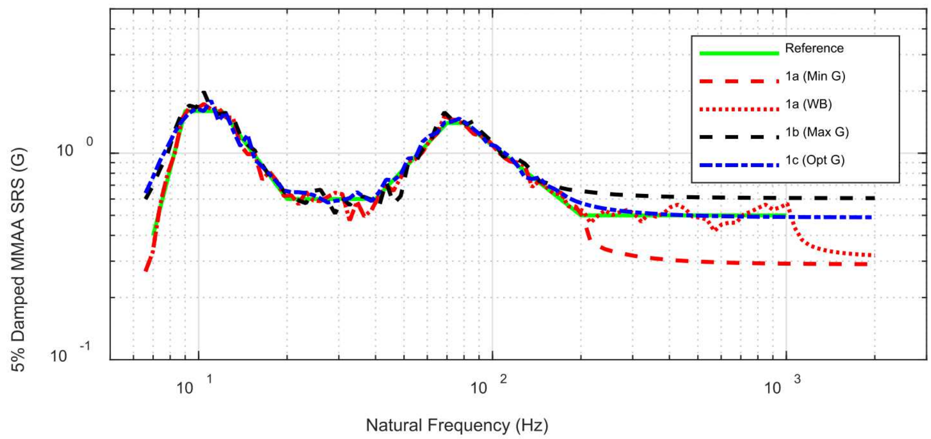

Figure 6: Input SRS for Three Case Studies (No Tones > 200 Hz) and Case 1a with High Frequency Tones

Damage Model

Up to this point we have used SRS simply as a measure of the spectral content. However, the original definition of the SRS was as a damage model where any given SRS value represents the equivalent static G load on an SDOF oscillator having that particular natural frequency and damping. For an oscillator with a natural frequency of 500 Hz, the SRS damage model would indicate that case 1b is a 20% over test, and case 1a (without high frequency tones) is a 50% undertest, while case 1a (with high frequency tones) and case 1c produce essentially the same level of damage as the desired test specification.

However, it is recognized that real world structures do not necessarily behave like a series of SDOF oscillators. Therefore, the decision was made to use a multi-degree-of-freedom (MODF) model to assess the damage potential. Figure 7 presents the model.

The MDOF resonant frequencies were chosen to lie in the frequency range between 400 Hz and 3000 Hz so as to primarily couple with the high frequency content in the input waveform. The responses for the uppermost three masses were computed by convolving the input acceleration waveform with the transfer functions for the model. SRS were then computed for each response.

Figure 8 presents the TRFs for the upper three masses (M1, M2, M3).

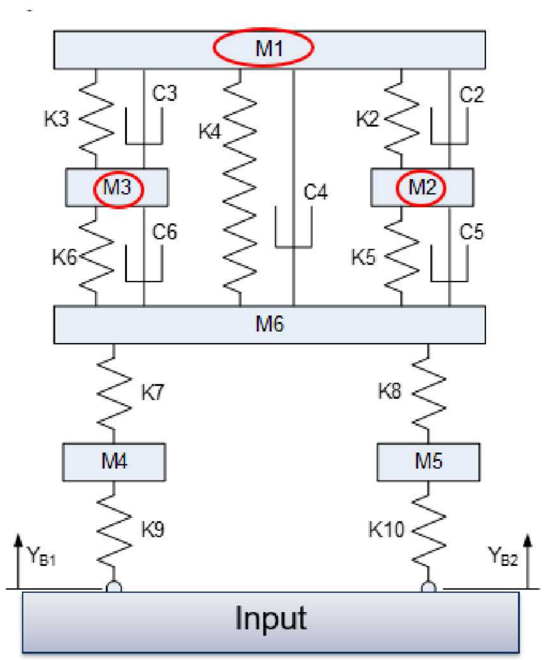

Figure 7: MDOF Model

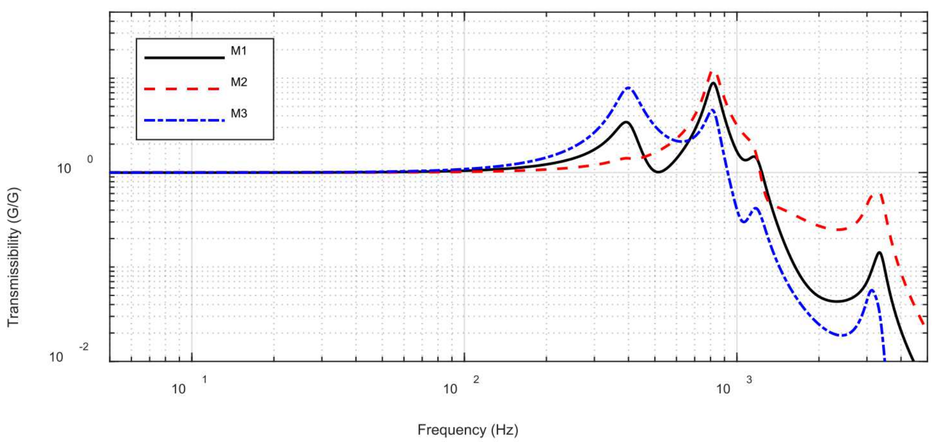

Figure 8: MDOF Transmissibility Response Functions

Case 1 Results

Figure 9 compares the input and response SRS for cases 1a (WB) , 1b, and 1c respectively against the corresponding field input and responses.

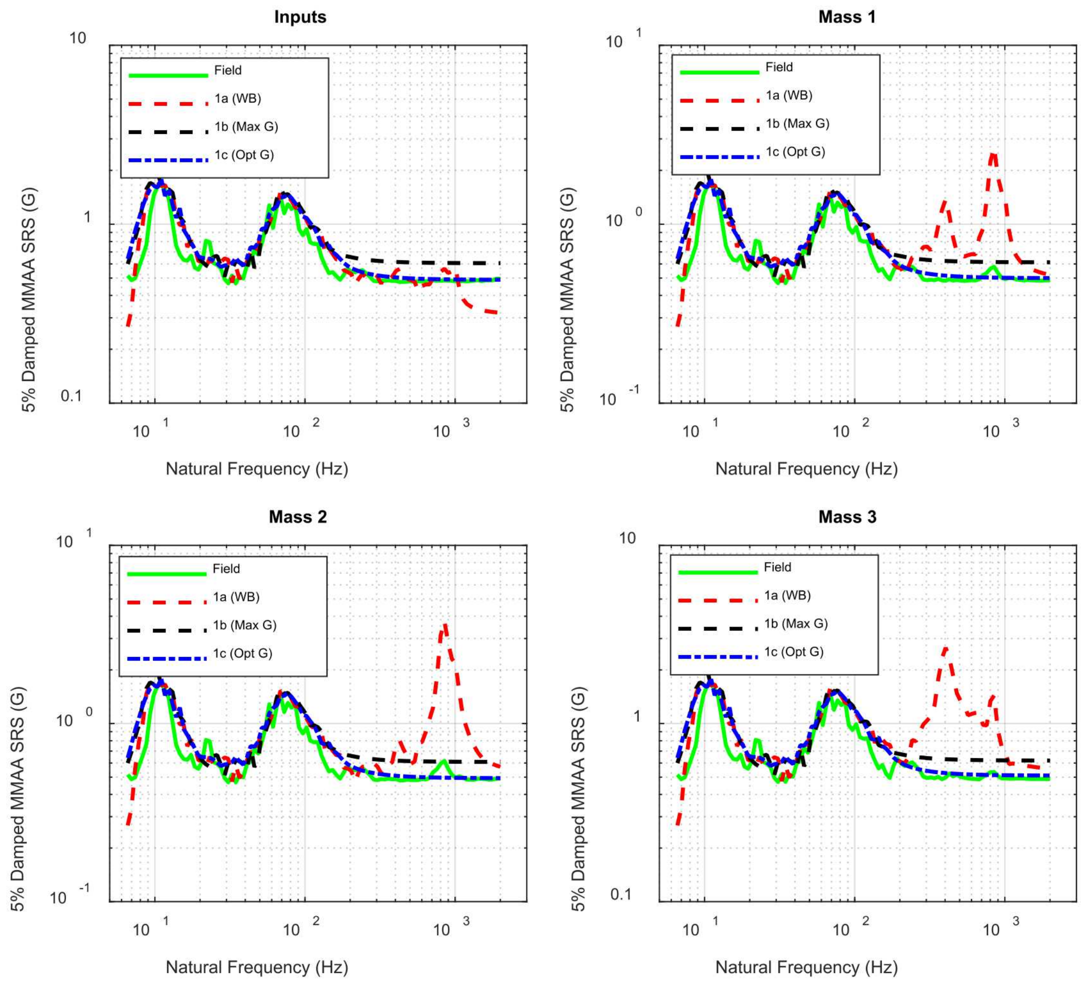

Figure 9: MDOF Results for Case 1a (Maximum High Frequency Content)

The results for case 1a (WB) represented a considerable over test, thereby confirming that it is possible to add too much high frequency content.

For cases 1b and 1c the responses for the MDOF masses essentially tracked the inputs at high frequency. Since the MDOF resonant frequencies all lie above the highest frequency tone for these cases, this result was also as expected, so Case 1b was a slight over test while case 1c was a slight undertest.

Case 2: Composite Input

The hypothesis is that if one could add just the right amount of high frequency energy to the low frequency tones for case 1c, then one would have the optimal waveform. The challenge was how to identify the high frequency spectral content and then replicate it in a way that was practical to implement.

The solution considered in this study was to high pass filter the field response. A "high frequency" test specification was defined based on the SRS of the high pass filtered (HPF) waveform. Figure 10 presents the raw and filtered acceleration waveforms along with the SRS for the raw high pass filtered waveform (HPF Raw) and the corresponding high frequency specification (HPF Spec). A set of decayed sine tones were then generated to reproduce the high frequency test specification.

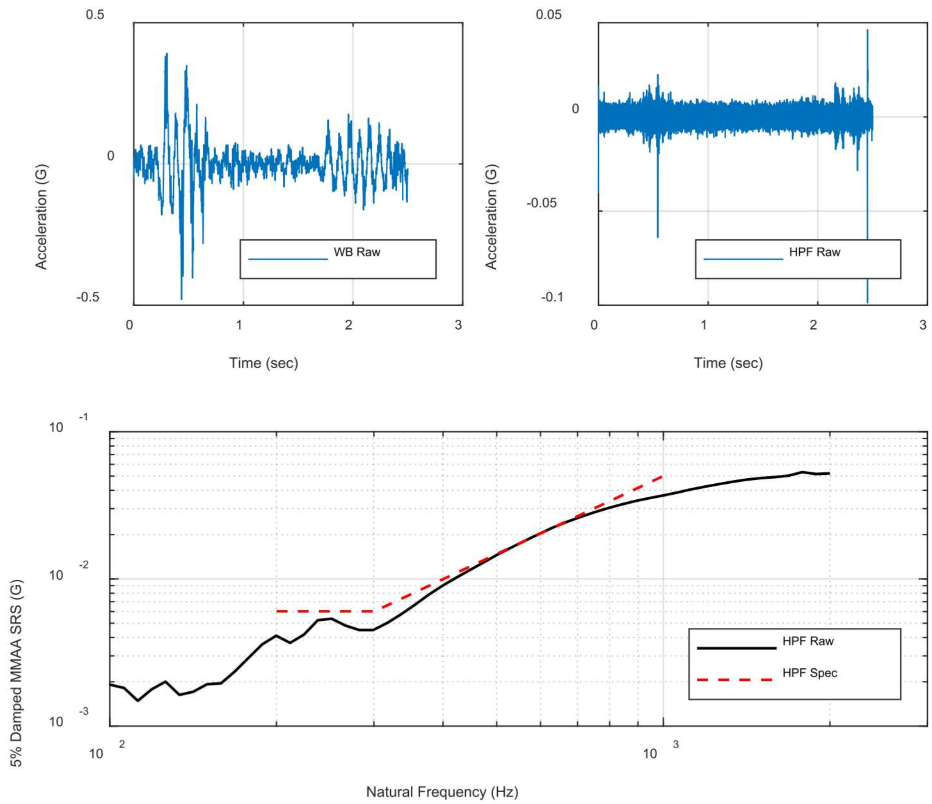

Figure 10: High Frequency Test Specification

The case 1c low frequency tones were then added to the high frequency tones to produce a composite waveform. Figure 11 compares the field input and MDOF SRS responses against the corresponding input and responses for the composite waveforms. The reader should note that the MDOF response SRS for this case now include the same small "humps" seen in the field response SRS.

To provide the reader with a quantitative scalar estimate of the error associated with each of the test cases, the decision was made to compile the LANL rms dB error and the average dB error for each case study. Since all four cases do a good job of replicating the spectral content below 200 Hz, the decision was made to compute the error metrics for just the spectral content above 200 Hz. Table 1 presents the error metrics for the four case studies (1a, 1b, 1c, and 2).

The results in Table 1 show that case 2 is the most accurate reproduction of the field response (smallest rms dB error) and is also slightly conservative (small positive average dB error).

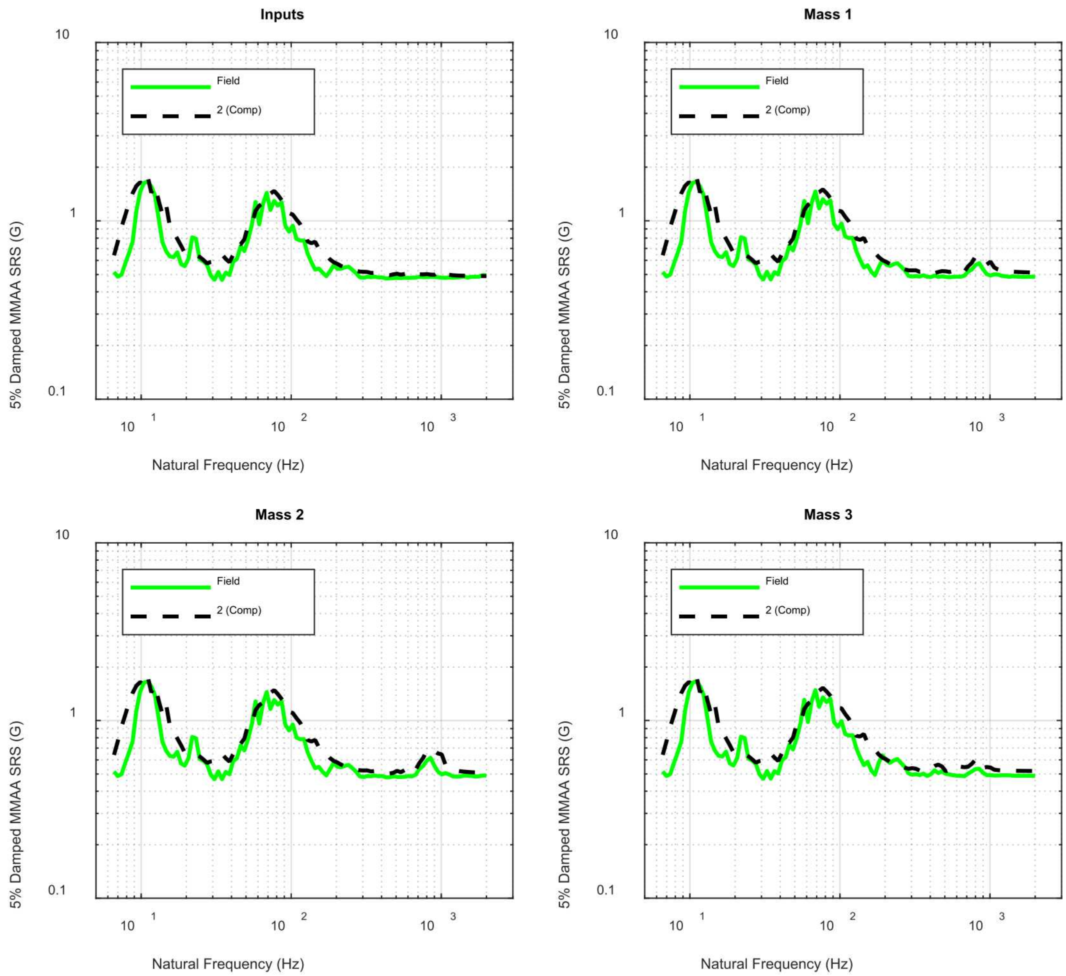

Figure 11: MDOF Results for Case 2 (Composite Waveform)

Table 1: Error Metrics for the High Frequency MDOF Results

Case	Rms dB Error	Average dB Error
1a (minimum low frequency G plus nominal high frequency content)	4.815	3.231
1b (maximum low frequency G)	2.033	1.177
1c (optimal low frequency G)	0.939	-0.049
2 (optimal low frequency G plus optimal high frequency G)	0.614	0.502

Truncated Low Frequency Content

One final case, which comes up quite often when simulating environments having large low frequency spectral content, was considered. In this case, it is often not possible to implement the necessary low frequency sine tones due to limitations in the peak displacement and/or velocity of the shaker table. If the item being tested does not have any low frequency resonant modes, the standard technique would be to truncate the test specification at a sufficiently high frequency to allow for the creation of an achievable waveform.

The loss of the low frequency content must be compensated for by increasing the high frequency content, so just as with case 1a, the potential is there for over driving the test article's higher frequency resonances. Figure 12 presents an example in which the wideband baseline (WB) specification is defined down to 7 Hz while the truncated (Trun) specification did not include tones below 20 Hz. The results show that the truncated waveform did indeed produce higher responses for the MDOF model.

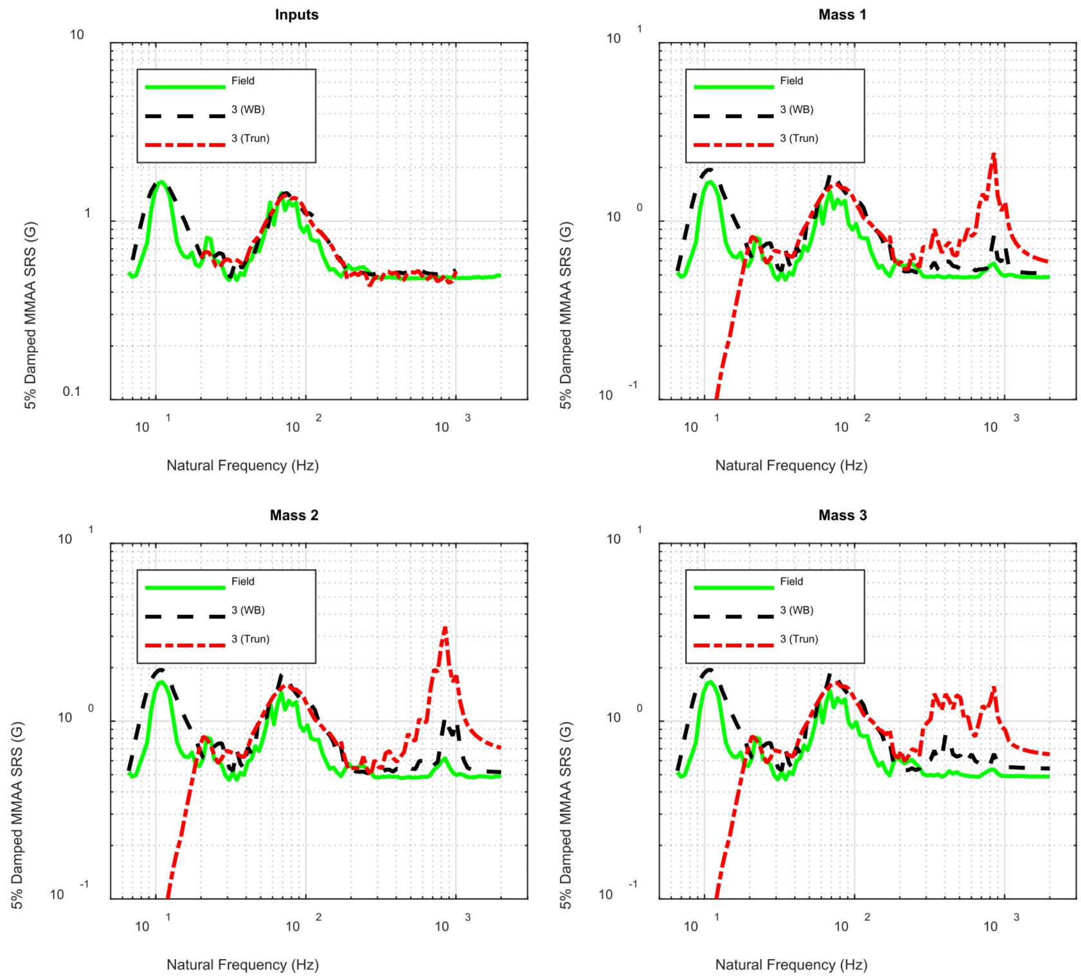

Figure 12: MDOF Results for Case 3 (Truncated Low Frequency Waveform)

Summary and Conclusions

This paper has demonstrated that the damage potential associated with a set of decayed sine tones developed to match a high frequency flatline SRS can vary significantly. To address this situation, a hybrid approach for more closely replicating the true high frequency content is also presented.

However, there are several outstanding issues that need to be resolved before one can implement the hybrid approach.

1) How does one preserve the high frequency content – the authors recommend saving a supplemental high frequency SRS specification along with the original broadband SRS specification. The specifications must include guidance for

the test engineer as to the desired tonal frequencies and peak accelerations associated with each specification. The most logical form of guidance would be to explicitly define the decayed sine parameters.

2) If the specification represents a compilation of multiple field events, it will be necessary to address the enveloping process. This would be somewhat more challenging if a statistical model is being used to combine the events [4].

Alternatives to a composite specification include:

In the situation where including no high frequency content is only a slight undertest, the reader could consider simply truncating the specification consistent with the frequency where the spectral content becomes insignificant. Such an approach would represent a small risk if other environments are more intense in the high frequency range.

In the situation where none of the low frequency Monte Carlo simulations can achieve the desired flatline SRS without adding high frequency content even though the spectral content of the field data is negligible, the reader should try and minimize the energy and/or peak G's associated with the high frequency tones (such an objective function is perfectly compatible with the Monte Carlo technique).

If a wideband SRS cannot be achieved in the laboratory without first removing the low frequency content, it is recommended that an analysis be done to understand if the true waveform (i.e., the correct proportion of high and low frequency spectral content) is damaging for the test article. If the test article's lowest resonant frequency is >> than the low frequency spectral content, then it is likely not damaging, and the reader could consider waiving the test altogether.

References

[1] Smallwood David O. "An Improved Recursive Formula for Calculating Shock Response Spectra". The Shock and Vibe Bulletin. May 1981.

[2] Heitman, Chad, et.al; "Monte Carlo Optimization of a Single Input Multiple Output (SIMO) Input Derivation for an Oscillatory Decaying Shock"; Proceedings of the 87th Shock and Vibration Symposium, 10/17-20/2016; New Orleans, LA.

[3] Cap. Jerome S., Heitman, Chad, and Raymer, Matthew K.; "Monte Carlo Optimization of a Hybrid Spectral / Temporal Single Input Multiple Output (SIMO) Input Derivation for an Oscillatory Decaying Shock"; Proceedings of the 88th Shock and Vibration Symposium; October 16-19, 2017, Jacksonville, FL.

[4] Cap, Jerome S., and C' de Baca, Melissa K.; "The Derivation of Appropriate Laboratory Vibration Test Durations and Number of Shock Hits from Non-Stationary Field Test Data"; Proceedings of the 86th Shock and Vibration Symposium; October 5-8, 2015; Orlando, FL