Restoring the Grand Canyon Ecosystem: Evidence from Dynamic Systems Theory

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Abstract

Constructing and operating dams for hydropower generation can impact downstream ecosystems by altering natural flow regimes and interfering with the lifecycles of aquatic and riparian species. Increased concern from environmental interests, coupled with federal relicensing requirements that aim to minimize environmental impacts, have put pressure on dam operators to quantify the flow regime changes that have resulted from dam construction and to evaluate how future modifications to dam releases may impact river systems. In this study, we present a method for analyzing the impact of dam operations by reconstructing system trajectories from hydrologic flow time series for different periods of dam operation (pre-dam, post-dam, and post-modification). We apply these methods to flow data from Lees Ferry, Colorado, just downstream of Glen Canyon Dam on the Colorado River, to determine how the dimensionality of the river system has changed over time and to evaluate the effectiveness of a new operating policy put into place after the 1992 Grand Canyon Protection Act was passed. While the post-dam period governed by hydropower operations is characterized by high dimensional, stochastic behavior, the pre-dam and post-modification periods are low dimensional and less complex, demonstrating the success of the new operating policy in mirroring the natural flow regime. Our analysis also suggests that applying a strictly deterministic or stochastic modeling approach to historical flow data does not appropriately capture the varying complexity of the Colorado River system during the three distinct periods of hydrologic behavior.

1. Introduction

Dams play an integral role in the management of water resources and provide multiple benefits to society by increasing water supply reliability, enhancing flood control, generating hydropower, and providing recreational opportunities (Graf 1999). However, dams can also have detrimental effects on riverine ecosystems. Construction of large dams and reservoirs reduces the longitudinal connectivity of river systems, preventing fish passage upstream and interrupting important migration patterns. Dam operations also cause significant changes in natural flow regimes by adjusting the timing and frequency of low flow and high flow events. These new flow patterns can disrupt the timing of fish spawning and migration, vegetative growth, and other lifecycle processes of aquatic and riparian species. Dam operations can also impact sediment and temperature regimes within a river system. Much of the sediment that would normally flow downstream and deposit in the floodplain is trapped behind the dam instead, causing major changes in the channel morphology and reducing the dynamic character of the instream habitat. In addition, low flows that occur when water is held back to fill reservoirs can lead to increased water temperatures and harm fish species, such as salmon, that rely on cooler water temperatures during their migration periods. Low flow periods can also disconnect the river from its floodplain, cutting off species from their former habitat. The net effect of all of these modifications is to weaken the native plant and animal populations and pave the way for the growth of invasive species that can take advantage of the modified flow regimes (Graf 1999; Richter and Thomas 2007; Nilsson and Berggren 2000).

As awareness of the degraded nature of rivers in the US grows, there is now more discussion of the benefits of modifying dam operations to mitigate the effects on fluvial systems. All hydroelectric projects that undergo relicensing through the Federal Energy Regulatory Commission (FERC) must now comply with National Environmental Policy Act requirements and incorporate flow recommendations from national and state fisheries agencies to minimize project impacts on aquatic species (DOE 2004). In addition, the US Army Corps of Engineers (USACE), one of the largest reservoir operators in the country, has partnered with The Nature Conservancy (TNC) on an initiative known as the Sustainable Rivers Project. Through this initiative, USACE and TNC are studying ways to modify dam operations to mimic pre-dam flow regimes and improve the economic, social, and environmental sustainability of dams (TNC 2013). In recent years, there has also been increased support for removing dams to restore natural flow patterns and pre-development ecosystems. However, removal of large dams that are critical components of major water systems in the western US is unlikely in the foreseeable future. As a result, dam reoperation is an important component of multi-objective water resources management because it not only provides benefits for the river ecosystem, but also allows for the continued use of reservoirs to benefit society.

As a result of dam reoperation, hydropower generation capacity is often reduced, leading to a loss of low-cost, high-reliability power. Other energy sources, such as coal and natural gas, are required to replace the lost generation capacity. This is problematic for energy providers, who are now under more pressure to increase their clean energy production in order to reduce emissions and meet renewable portfolio standards (EPA 2015). Thus, when faced with the need to develop new operating policies that consider downstream ecosystems, dam operators want to ensure that the new release schedules are actually achieving their stated goals.

Dynamic systems theory has been used in a variety of applications to characterize the multi-dimensional aspects of complex systems. Previous work related to rivers has focused on the fractal nature of the rivers themselves and how channel networks evolve over time (Tarboton et al. 1988; Rinaldo et al. 1993). More recently, fractal dimensions have been used to highlight the potential benefits of using chaos theory to serve as a bridge between deterministic and stochastic modeling of environmental systems, including river flows (Sivakumar 2012). However, to our knowledge, a system dynamics approach has not been applied to rigorously study watersheds and the effects of changes in dam operations on a river's flow regime. In this study, we use attractor reconstructions and their corresponding fractal dimensions to characterize flow regime changes that have occurred on the Colorado River at Lees Ferry as a result of the construction and operation of Glen Canyon Dam. We then use these metrics to determine the dimensionality of three distinct flow regimes and to evaluate the effectiveness of operational changes meant to mirror the natural system. We also discuss the implications for modeling the river system and its relevant variables.

2. Study Area

Glen Canyon Dam began operation in 1964 and provides flood control, hydropower generation, and water storage for agricultural, municipal, and industrial users. It is a 178-m (583-ft) concrete arch dam with eight hydropower generating units that provide a total capacity of 1.32 GW. The dam is a major component of the Colorado River Storage Project, providing year-to-year carryover storage in Lake Powell, which has a total capacity of 32.3 billion m^3 (26.2 million acre-ft) (USBR 2008).

Figure 1 shows the time series of flow data for the Colorado River at Lees Ferry, AZ. From 1965 to 1992, releases from Glen Canyon Dam were based primarily on the combined power demands of approximately 100 entities in Arizona, Colorado, New Mexico, Nevada, Utah, and Wyoming that are serviced by the Western Area Power Administration (Harpman 1999). Changes in peaking power demands and the need to generate power during system emergencies resulted in large fluctuations in releases. However, in 1992, the Grand Canyon Protection Act was passed, requiring modification to the operations at Glen Canyon Dam to reduce the impact on downstream ecosystems. After consideration of various operating policies, the Modified Low Fluctuating Flow option was selected as the preferred alternative. This alternative sets minimum releases of 227 cubic meters per second (cms) (8,000 cubic feet per second [cfs]) from 7am to 7pm and 142 cms (5,000 cfs) during the rest of the day, with a maximum instantaneous release of 708 cms (25,000 cfs). It also restricts ramping rates, permitting a maximum increase of 113 cms/hr (4,000 cfs/hr) and a maximum decrease of 42.5 cms/hr (1,500 cfs/hr). The total allowable daily flow fluctuation in a given month ranges from 142-227 cms (5,000-8,000 cfs) per 24-hour period, depending on the monthly release volume. Overall, the policy has reduced daily and hourly flow fluctuations and has provided more consistent flows for downstream wildlife. This alternative also includes beach- and habitat-building flows, which are infrequent, high-flow releases that exceed the maximum allowable release. These flows are intended to reverse the erosion of sandbars by introducing more sediment into the river and increasing sand deposition along the riverbanks, thus providing more riparian habitat and larger areas for camping (USBR 1995). High flow releases, which measured between 1,161 cms (41,000 cfs) and 1,274 cms (45,000 cfs), occurred in 1996, 2004, 2008, 2012, and 2013 and typically lasted between three and eight days (USBR 2014). Although these releases were higher than those allowed prior to

1992, the high flow experiments still did not match the magnitude and frequency of pre-dam high flows, which frequently exceeded 1,416 cms (50,000 cfs) under natural conditions.

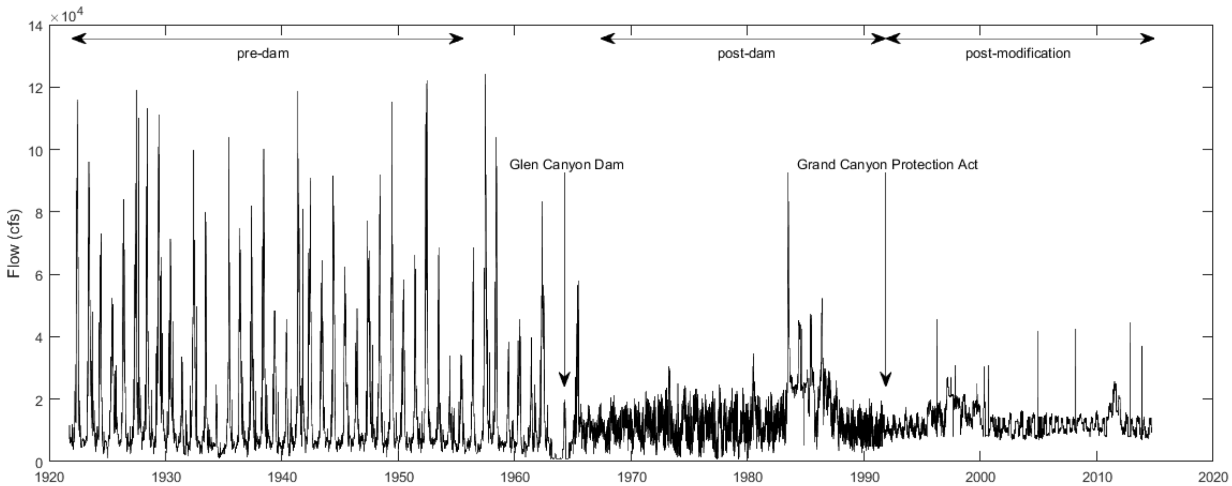

Figure 1. Historical daily stream flow record for USGS 09380000: Colorado River at Lees Ferry, AZ (USGS 2015)

3. Methods

For this study, historical daily streamflow data was obtained from the USGS National Water Information System website for USGS 09380000: Colorado River at Lees Ferry, AZ (USGS 2015). We performed our analysis in three parts, first creating an m-dimensional (m>1) reconstructed attractor from the flow time series, then determining the fractal dimension of the reconstructed attractor, and finally iterating in successively higher embedding dimensions until the fractal dimension leveled off. These steps were accomplished using a Matlab code modified from Henry et al. (2001). Our methodology is described in more detail below.

3.1 Attractor Reconstruction

According to dynamic systems theory, the evolution of a dynamic system, such as a river basin, can be displayed by its trajectory, or attractor, in the phase space that represents its variables. The dimensionality of the phase space (i.e., the number of relevant system variables) is typically unknown *a priori*. For a complicated system, its dimensionality can be very high, effectively precluding any deterministic analysis and modeling. However, if a dynamic system can be represented by a smaller number of variables (hence having a phase space with smaller dimensionality), it can potentially be analyzed mechanistically. More importantly, if the system undergoes structural modifications, such as dam construction on a river, the dimensionality of the reconstructed attractor can shed light on the effects of such changes.

As outlined by Takens (1981), the system trajectory can be reconstructed from a single observational series (e.g., daily flow data) using a delay embedding theorem, even without knowing the number of system variables. The reconstructed trajectory is invariant under continuous transformation of the system variables and is thus similar to the actual system trajectory (Takens 1981; Eckmann and Ruelle 1985). This reconstruction process can reveal the number of variables involved but cannot identify them. The essence of the process is to take observations from a single available time series with a delay τ and build a series of m-dimensional points, or vectors, that form the attractor. Thus, two parameters, the delay τ and the dimensionality m, must be recovered. An m-dimensional embedding X_i can then be constructed from a series of N discrete observations of x, as shown below.

$$X_i = \left(x_i, x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+(m-1)\tau}\right) \qquad i = 1, 2, \ldots, N-(m-1)\tau$$

An appropriate delay for the flow data can be determined using an autocorrelation function, as shown in Figure 2, to ensure that x_i and $x_{i+\tau}$ are not highly correlated. Other methods, such as the mutual information method (Fraser and Swinney 1986), have also been proposed. For the flow data, the delay was set by selecting the value of τ at which the autocorrelation function (Eq. 1) first equaled zero, as suggested by Addison (1997).

$$R(\tau) = \frac{\sum_{i=1}^{N-\tau}(x_i')(x_{i+\tau}')}{\sum_{i=1}^{N-\tau}(x_i')^2} \qquad (1)$$

$$where\ x_i' = x_i - \overline{x_\iota}$$

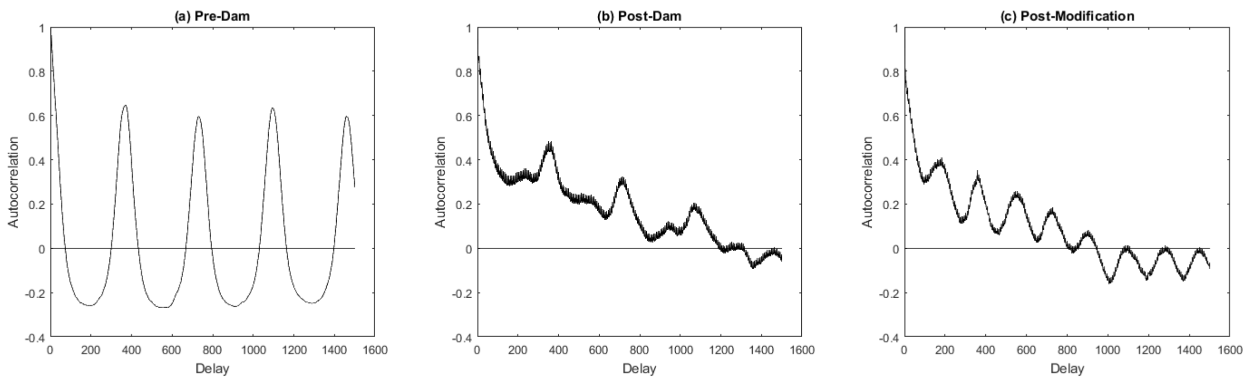

Figure 2. Autocorrelation function for (a) pre-dam period (time delay = 73 days), (b) post-dam period (time delay = 1,186 days), and (c) post-modification period (time delay = 801 days)

As Figure 2 illustrates, the time delays used to construct the system trajectories are not consistent between the three periods of analysis. While the pre-dam system only required a delay of 73 days, the values for the post-dam and post-modification periods were much higher, requiring 1,186 days and 801 days, respectively. As a result, the number of data points used in the fractal dimension calculation varied greatly between the operating periods. To determine whether this would impact the results, the analysis was repeated with an equal delay (τ = 100 days) for each period of operation. Both analyses produced similar fractal dimensions and attractor reconstructions, indicating that the choice of time delay did not significantly alter the results.

3.2 Fractal Dimension Calculation

For each embedding dimension, the fractal dimension was determined by calculating the correlation dimension C_r of the attractor following the method of Grassberger and Procaccia (1983). C_r is determined using Eq. 2, where θ is the Heaviside function and r is a specified distance in the phase space. The summation term counts all point pairs, X_i and X_j, on the attractor trajectory that are separated by a distance less than r.

$$C_r = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \theta(r - |X_i - X_j|) \qquad (2)$$

The fractal dimension for the m-dimensional embedding is then calculated by finding the slope of the linear region of the log C_r versus log r plot, shown in Figure 3. This procedure is repeated for consecutively higher embedding dimensions. For dynamic systems of limited (small) dimensionality, the fractal dimension of the reconstructed attractor increases linearly with increasing dimension of the embedding phase space until it saturates (Eckmann and Ruelle 1985; Addison 1997). This saturation level is the "true" fractal dimension of the attractor, and the corresponding dimension of the embedding space is taken as the number of variables describing the system. Dynamic systems that have a high dimensionality, very often due to dominant stochastic elements, typically show a continued increase of the attractor fractal dimension with increasing dimension of the embedding space, without signs of saturation. The expected behavior for such a dynamic system is shown in Figure 4.

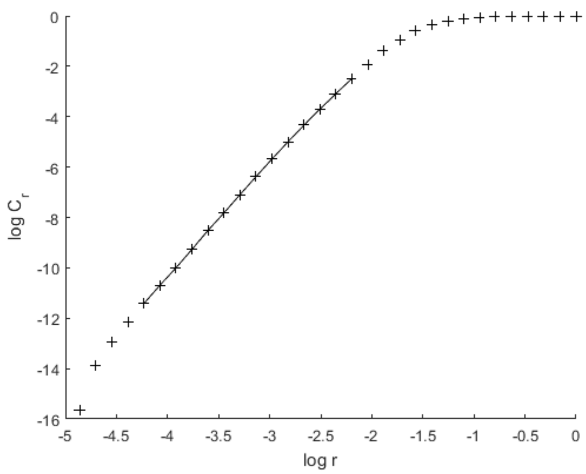

Figure 3. Example of correlation dimension method for finding fractal dimensions

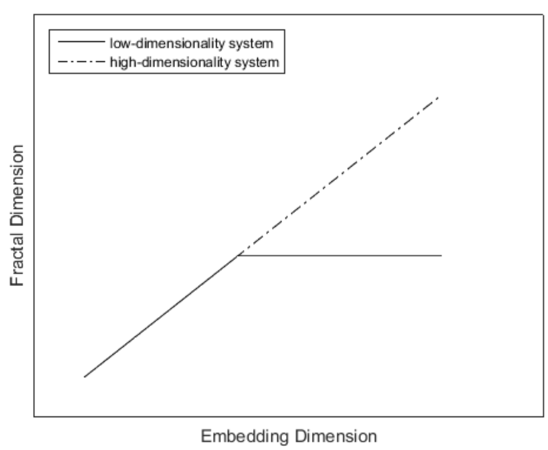

Figure 4. Expected behavior of low- and high-dimensionality systems

Three-dimensional attractor footprints were also plotted for each period of dam operation, using the embedding dimension corresponding to the saturation level. Because the post-dam period did not show signs of saturation, the highest embedding dimension was used. The fractal dimensions and attractor footprints for each operating regime (pre-dam, post-dam, and post-modification) were compared to analyze how Glen Canyon Dam impacted the natural flow regime of the Colorado River and to evaluate the effectiveness of operational changes at pushing the system back toward its pre-development, natural flow regime.

4. Results

Figure 5 shows the fractal dimension results for the Colorado River under natural conditions (Water Year [WY] 1922-1955), after construction of Glen Canyon Dam (WY 1967-1992), and after operational modifications implemented as a result of the Grand Canyon Protection Act (WY 1993-2014). Under the natural flow regime, the system displayed low dimensionality, with a fractal dimension of 2.9. After construction of the dam, the fractal dimension of the system increased to at least 5. The graph of the fractal dimension versus the embedding dimension did not show a clear plateau consistent with inherent system dimensionality, but instead suggests a strong stochastic component during this period. When operational modifications were put into place in 1992, the system returned to a lower fractal dimension of approximately 2.3, although this value increased slightly for embedding dimensions greater than 6.

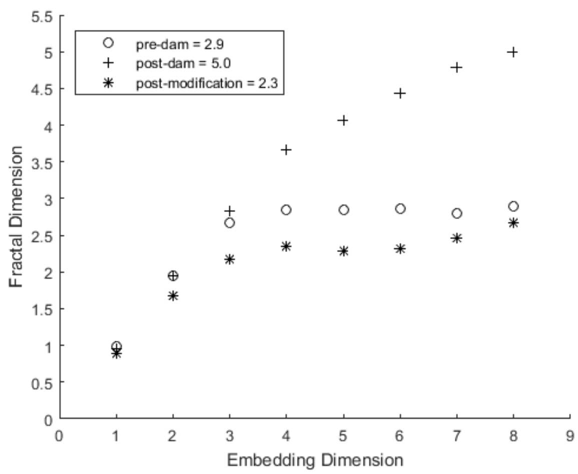

Figure 5. Fractal dimension versus embedding dimension for pre-dam, post-dam, and post-modification periods

The higher dimensionality seen under the original dam release schedule likely resulted from the variability in power demand across the service area of the Western Area Power Administration, which required frequent changes in release volumes in order to meet the electricity needs of the region. In contrast, the natural flow regime, which exhibited a lower dimensionality, was regulated by seasonal precipitation patterns and year-to-year variability in runoff volumes. These hydrologic variations occur over longer time periods than the hourly or daily flow fluctuations that are required for hydropower production. Similarly, the post-1992 operating plan aims to limit the impact of fluctuating electricity demands by restricting the short-term variability in flow releases. The new policy also damps out some of the naturally occurring, seasonal flow variation by cutting off peak flows during the wet season and maintaining a minimum flow during the dry season.

The attractor footprints reveal a similar pattern. Figure 6 shows three-dimensional cross-sections of the attractors for each period of dam operation. For the pre-dam and post-modification periods, a three-dimensional embedding was used. For the period immediately after construction of Glen Canyon Dam, an eight-dimensional embedding was used. Before dam construction, the attractor has a more predictable shape, with peaks stretching out along each axis but returning to the near-origin region. This suggests a structured system that mirrors the increases and decreases seen in a hydrograph after rainfall and runoff events. In contrast, the unpredictable, chaotic footprint seen during the post-dam period reflects the frequent flow variations that occurred for hydropower production. The lack of peaks in the trajectory reflects the restrictions on maximum flows that resulted from the operation of Glen Canyon Dam. After operational modifications in 1992, the footprint maintains features of both previous periods, with a main cluster of less-predictable structure as well as peaks forming along each axis. The peaks result from the high flow releases meant to move sediment and rebuild sandbars, while the clustered shape reflects the continued, although restricted, fluctuations that occur for hydropower production.

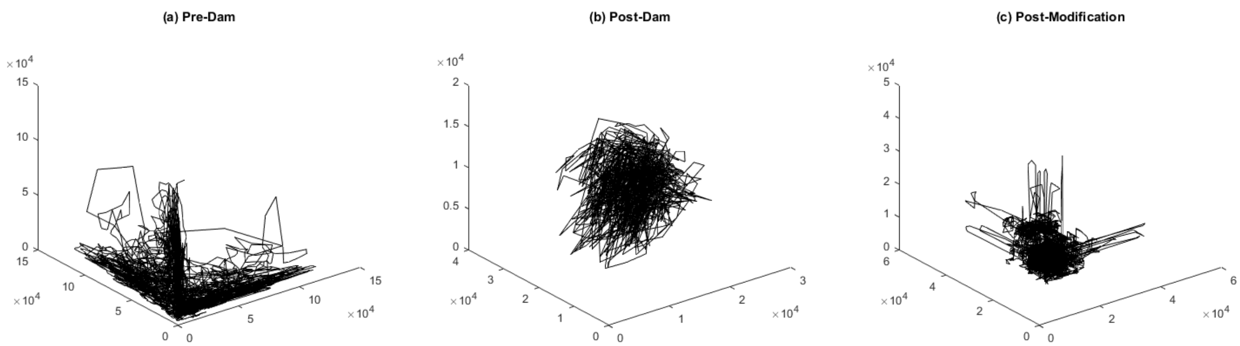

Figure 6. 3-dimensional attractors for each period of operation

5. Discussion

While previous approaches have used hydrologic or spectral analyses to study the impacts of dam operations on flow regimes, this study quantifies the fractal dimension of the Colorado River system, which provides an estimate of the number of dynamical variables that govern the system and would thus be required to model its behavior (Henry et al. 2001). During the pre-dam period, the flow regime is low dimensional and is governed by approximately three variables. In addition, the attractor has a predictable structure, indicating that it may be possible to model the pre-dam period using a lower-complexity, deterministic model. For the post-dam period, the lack of a clear plateau in the fractal dimension graph and the corresponding high dimensionality indicate that unrestricted hydropower operations exhibited a stochastic component. This, when combined with the clustered shape of the attractor, suggests that a higher-complexity, stochastic modeling approach would be required to properly model the flow regime during periods of hydropower operation. After operational modifications, the flow regime returns to a low dimensional state, with approximately three governing variables. However, despite recovering some characteristics of the pre-dam attractor, the post-modification attractor still has a clustered shape, indicating that a deterministic approach may not be appropriate for modeling this period. The variations across all three periods of operation demonstrate the importance of considering different flow regime behaviors when modeling river systems. In this case, using a strictly deterministic or stochastic modeling approach for the entire historical flow record does not adequately capture changes in the underlying dimensionality of the flow regime.

The attractor plots also give insight into more specific flow regime changes that have occurred, such as alterations in the magnitude and frequency of maximum flows and variability in daily flows. Before Glen Canyon Dam was constructed, flows regularly exceeded 1,416 cms (50,000 cfs), resulting in high peaks along each axis in the pre-dam attractor. After dam construction, high flows were eliminated, leading to a clustered attractor with values that rarely exceeded 850 cms (30,000 cfs). With the implementation of flow modifications and high-flow experiments, more frequent high flows were present, although the magnitude and frequency of these flows still did not match pre-dam conditions. These features of the attractors provide qualitative evidence that the new operating policy implemented in 1992 to regulate releases from Glen Canyon Dam has been successful at recovering aspects of the natural flow regime, such as high flows and less flow variability. This conclusion is supported by recent surveys of the Colorado River downstream of Glen Canyon Dam, which suggest that the modified operations have resulted in ecosystem improvements. Studies conducted after the 1996, 2004, and 2008 high flow experiments found that rehabilitation of sandbars occurred in most of Grand Canyon and in the lower part of Marble Canyon, through which the Colorado River flows before reaching Grand Canyon. These sandbars provide habitat for native species and campsites for rafters (USGS 2011). Populations of flannelmouth sucker, which prefer flowing streams (Mueller and Marsh 2002), appear to have stabilized since 1992 (USGS 2009). In addition, humpback chub, which depend on high spring flows for spawning (Mueller and Marsh 2002), increased in number from 2002 to 2008, despite a decreasing trend from 1989 to 2001 (Campbell et al. 2008).

Overall, the information obtained from the fractal dimensions and attractor reconstructions is useful to dam operators and environmental groups, who are interested in quantifying the effects of the flow regime changes that have occurred since 1992 in order to adaptively manage the Colorado River system. By determining the number of dimensions governing the flow regime, our approach provides a better understanding of the system's complexity, which informs the methods that are used to model the behavior of the system. In the future, this methodology will be extended to other rivers that have undergone modifications as a result of dam construction and reoperation to see if similar information can be obtained, with the ultimate goals of (1) allowing river managers to evaluate the effectiveness of specific operating policies in recovering natural flow characteristics and (2) enabling the development of more robust modeling methods that account for variations in flow regime during different parts of the historical flow record.

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Disclaimer

recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

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