LA-UR-20-23604
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Title: Numerical Plasma Physics

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Intended for: Invited Colloquium Talk

Issued: 2020-05-14

# Numerical Plasma Physics

## Why, how, and what am I missing?

**Patrick Kilian**

2020-05-15

Managed by Triad National Security, LLC for the U.S. Department of Energy's NNSA

# Our Model: The Vlasov-Maxwell-System

Plasma Physics is reasonably described by the Vlasov(-Boltzmann) equation

$$\frac{\mathrm{d}}{\mathrm{dt}} f_s = \frac{\partial}{\partial t} f_s + \vec{v} \cdot \nabla_x f_s + \frac{q_s}{m_s} \left( \vec{E} + \frac{\vec{v}}{\mathrm{c}} \times \vec{B} \right) \cdot \nabla_v f_s = \cancel{(\partial_t f_s)_{coll}} \approx 0$$

and Maxwell's equations

$$\nabla \times \vec{E} = -\frac{1}{\mathrm{c}} \frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{E} = 4\pi \rho$$

$$\nabla \times \vec{B} = \frac{1}{\mathrm{c}} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{\mathrm{c}} \vec{\jmath} \qquad \nabla \cdot \vec{B} = 0$$

To close the system we use

$$\rho = \sum_s q_s \int f_s \, \mathrm{d}^3 v \qquad \vec{\jmath} = \sum_s q_s \int \vec{v} \, f_s \, \mathrm{d}^3 v$$

# Variations of the Problem

- Keep the collision term $\rightarrow$ Vlasov-Boltzmann-Maxwell-System
- Add source / sink term for e.g. fusion
- Electrostatic approximation $\rightarrow$ Vlasov-Poisson-System
- Remove radiation $\rightarrow$ Vlasov-Darwin-System
- Treat dielectrics $\rightarrow$ treat $\vec{E}/\vec{D}$ and $\vec{B}/\vec{H}$ using materials model

We have a complete and self-consistent mathematical model. Can we go home now?
No! The equations are unsolvably hard in the general case.
This is why we (have to) do numerics.

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# Euler vs Lagrange

There is two opposing world views

**Eulerian** (*Stream gauges*)

- There is a fixed spatial reference
- We impose a (structured) grid
- Changes is time are for a given location …
- …but might be because a feature moves, or …
- …because the feature changes in time.

**Lagrangian** (*Rubber ducks*)

- Particles have defined identities
- and we just follow them around and look at their immediate environment
- Changes are for a given particle…
- …and are due to temporal evolution of features, but …
- …the particle might be at different locations over time.

Mixtures such as moving mesh codes and semi-Lagrangian codes of course exist.

# Magneto-Hydro-Dynamics (MHD)

- Assume a distribution function with a bunch of parameters (e.g. $\mathcal{M}(n, \vec{u}, T)$)
- Take moments of the Vlasov equation
- Sum over all species (at least electrons and ions)
- Evolution equation for $n^{th}$ moment ($\rho$, $\vec{u}$) contains $n+1^{th}$ moment ($\vec{u}$, $\overleftrightarrow{P}$)
- Stop after finite number of moments (5, 10) and close with some closure scheme
- Discretize resulting equations (using finite volume or spectral methods)

Biggest advantages:

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Biggest advantages:

- Able to describe large scale system
- Most tractable analytically
- Mature codes available

Biggest limitations:

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Biggest advantages:

Biggest limitations:

- Changes in the (type of the) distribution function are lost
- All species are the same temperature
- Needs good closure model

# Particle-in-Cell (PiC)

- Sample distribution function $f_s$ using macro particles
- Derive evolution equations for those
- Discretize Maxwell's equations using Finite-Difference-Time-Domain (FDTD)
- Close using deposition schemes that compute $\rho$ and $\vec{\jmath}$ from the macro particles

Biggest advantages:

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Biggest advantages:

- Captures changes in the distribution functions
- Captures the micro physics
- Simple code

Biggest limitations:

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Biggest advantages:

Biggest limitations:

- Has to resolve micro-physical scales for stability
- Noise level due to limited particle number
- Huge computational effort

# Full Vlasov

- Discretize phase space
- Find evolution equation for the discrete representation of $f_s$
- Discretize Maxwell's equations in a "compatible" way

Biggest advantages:

- Captures changes in the distribution functions
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- Very low noise level

Biggest limitations:

- 6d grids require tons of memory, even at poor resolution
- Tends to generate fine structures in phase space that need to be removed without being too diffusive

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- The $\vec{B} \to 0$ limit of MHD

Biggest advantages:

- When you don't have to worry about $\nabla \cdot \vec{B} = 0$, you can treat the remaining physics much better
- Good for very large systems or low degree of ionization
- Many mature codes

Biggest limitations:

- *Ignore magnetic fields, hope for the best*

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- Include some model for the electromagnetic waves that are not directly resolved
- Needs numerical schemes for radiation transport in the optically thin/thick regime
- Popular at LANL

Biggest advantages:

- Include the energy transport and pressure of radiation
- Automatically produces synthetic radiation observables

Biggest limitations:

- Treatment of $I(\vec{x}, \vec{\Omega}, \nu, t)$ is hard
- Codes tend to be export controlled

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# Two-Fluid MHD / Multi-Fluid MHD

- Retain separate sets of fluid equations for electrons, protons. (heavier) ions, dust, ...
- Compute (net) $\rho$ and (total) $\vec{\jmath}$
- Species are coupled through $\vec{E}$ and $\vec{B}$ that is seen by all species
- Possibly source/sink terms for reactions (chemical, fusion, dust aggregation, ...)

Biggest advantages:

- Useful for analytic calculations
- Better electric field model
- Includes the Hall effect and diamagnetic drift

Biggest limitations:

- Electrons set small spatial and temporal scale
- Interactions between species can be very complicated
  (entire reaction networks with high rates)

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- Fully-kinetic for other species (using PiC methods)
- Maxwell's equations usually radiation-free (Darwin or Electrostatic)
- Field Solver often implicit, $\vec{E}(t^{n+1/2})$ problem

Biggest advantages:

- Retains all the kinetic physics of (multiple) ion species
- Much more efficient then PiC, especially in cold plasma
- Longer time scale, large spatial scales than PiC

Biggest limitations:

- Whistler waves are $\omega \propto k^2$ without inertia, make problem stiff
- Problems at plasma-vacuum boundaries and large density jumps
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- Derive equations of motion for these “particles”
- Estimate density by averaging over all neighboring particles within a smoothing length
- Might use a grid or kd-tree to quickly find neighboring particles

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- When particles “stack” you get pairing instability
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- Electron inertia is retained
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- Extension to 1d3v Darwin should be useful, relatively easy and worth a publication

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- Able to simulate VHF propagation with realistic parameters
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- Assume that particles are very magnetized
- Average over the fast gyration
- Phase space reduces to 5d

Biggest advantages:

- Very good for fusion research, up to full device simulations
- Allows 3d2v simulation, which have feasible memory usage
- Allows arbitrary gyrotropic distribution functions

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Biggest advantages:

- Very good at rarefied gas flows, contaminant transport, space craft charging
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- Can handle vacuum and changing ionization fraction

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- Unnecessary in the collisionless regime
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# Specialized Codes

- Dispersion solver to find $\omega(\vec{k})$
- Equilibrium solver

I am sure there is **other methods** I did not mention.
Sorry if I missed your favorite one.
Please tell me about it. Seriously!

# Time for Questions

Thanks for listening!

Are there questions or comments?