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FUEL CELL MICROTURBINE HYBRID SYSTEM ANALYSIS THROUGH DIFFERENT UNCERTAINTY QUANTIFICATION METHODS

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ABSTRACT

The analysis of different energy systems has shown various sources of variability and uncertainty; hence the necessity to quantify and take these into account is becoming more and more important. In this paper, a steady state, off-design model of a solid oxide fuel cell and turbocharger hybrid system with recuperator has been developed. Performances of such stiff systems are affected significantly by uncertainties both in component performance and operating parameters. This work started with the application of Monte Carlo Simulation method, as a reference sampling method, and then compared it with two different approximated methods. The first one is the Response Sensitivity Analysis, based on Taylor series expansion, and the latter is the Polynomial Chaos, based on a linear combination of different polynomials. These are non-intrusive methods, thus the model is treated as a black-box, with the uncertainty propagation method staying at an upper level. The work is focused on the application on highly non-linear complex systems, such as the hybrid systems, without any optimization process included. Hence, only the uncertainty propagation is considered. Uncertainties in the fuel utilization, ohmic resistance of the fuel cell, and efficiency of the recuperator are taken into account. In particular, their effects on fuel cell lifetime and some simple economic parameters are evaluated. The analysis distinguishes the specific features of each approach and identifies the strongest influencing inputs to the monitored output. Both approximated methods allow an important reduction in the number of evaluations while maintaining a good accuracy compared to Monte Carlo Simulation.

NOMENCLATURE

ANOVA	Analysis of variance
ASR	Area specific resistance [ohm/cm^2]
CFN	Annual cash flow [$]
COV	Coefficient of variation [%]
DOE	Design of experiment
EOL	End of life [yr]
FC	Fuel cell
FPI	Fast probability integration
FU	Overall fuel utilization [%]
HS	Hybrid system
IRR	Internal rate of return [%]
LSM	Lanthanum strontium manganite
MCS	Monte Carlo simulation
NBCR	Net benefit to cost ratio [%]
NPV	Net present value [$]
PBP	Pay-back period [yr]
PC	Polynomial chaos
PDF	Probability density function
PE	Percentage error [%]
PEM	Proton exchange membrane
RSA	Response sensitivity analysis
SOFC	Solid oxide fuel cell
TCI	Total capital investment [€]
TG	Turbogas
TPB	Triple phase boundary
YSZ	Yttria-stabilized zirconia
A	Area [m^2]
C_{el}	Electricity price [$ kWh^{-1}]
C_f	Fuel cost [$ kg^{-1}]
C_{main}	Maintenance cost [$]
c_p	Specific heat [J kg^{-1} K^{-1}]
F	Faraday's constant [C mol^{-1}]
G	Gibbs free energy [kJ]

$g_{Mj}(Z)$	Functional relationship between j-th output parameter and the inputs Z
h	Specific enthalpy variation from 298K condition [kJ kg^{-1}]
	Convection coefficient [W m^{-2} K^{-1}]
i	Current density [A cm^{-2}]
i_0	Exchange current density [A cm^{-2}]
K_p	Equilibrium constant
k	Conduction coefficient [W m^{-1} K^{-1}]
L	Cell length [m]
M_j	J-th parameter of output for the system
$\dot{m}$	Mass flow rate [kg s^{-1}]
n	Number of transferred electrons
P_{el}	Electricity production [kWh]
P_f	Fuel consumption [kg]
P_{GT}	Gas turbine power [kW]
p	Pressure [bar]
$\dot{Q}$	Fuel cell thermal output [kW]
q_{gen}	Generated heat [W m^{-1}]
R	Area specific resistance [Ω m^2]
R_g	Ideal gas constant [J mol^{-1} K^{-1}]
r_d	Degradation rate [% kh^{-1}]
T	Temperature [K]
t	Time [s]
V	Voltage, overpotential [V]
x	Mole fraction
Z_i	I-th parameter of input for the system
α	Charge transfer coefficient
δ	Discretization step
η	Efficiency
μ	Mean
ν	Variance
ρ	Density [kg m^{-3}]
σ	Standard deviation

Subscripts

act	Activation
an	Anode
ca	Cathode
dif	Diffusion
ohm	Ohmic

1 INTRODUCTION

Engineering design is increasingly supported by uncertainty quantification techniques. In fact, because of the complexity of some problems, or due to the variability caused by the manufacturing process, or because of limited information about new and unexplored fields, many solutions inevitably cannot be accurately predicted. Building detailed models for these systems helps in evaluating performance, including any reasonable variables, especially for energy systems such as the one described in this paper. Many are the ways to deal with this uncertainty and even more are the available techniques to analyze it, some of them have been already presented in other works such as [1] [2] [3]. Few examples of comparative analysis of such methods are also available in open literature. Padulo *et al.* [4] compared different UQ methods with a particular attention to robust engineering design problems. In particular, the new Sigma-Point approach was compared to MC, Gaussian Quadrature and higher-order MM. Lee *et al.* [5] showed the comparative study of several UQ methods for black-box type problems. The full factorial numerical integration, the univariate dimension reduction and the polynomial chaos expansion were applied to four illustrative examples.

During the present work, the sampling method Monte Carlo Simulation (MCS) and two different approximating methods, namely Response Sensitivity Analysis (RSA) and Polynomial Chaos (PC), have been explored as tools for uncertainty quantification. In modelling, when simulated system are computationally expensive, based on complex structures, or when the simulated period is extended over months or even years, a minimum computational effort and short calculation time become crucial. Despite its great reliability, MCS cannot be used when any single analysis requires too many or too long simulations. The analysis performed for this work on the fuel cell gas turbine hybrid system model focuses on the entire operating life of the plant. As introduced before, because of a long simulated interval and a large amount of variables that could be taken in consideration in this system, fast processing methods are needed. Therefore, beside the scientific importance of studying the effect of uncertainties on the degradation and then on system life and economic parameters, this paper aims to underline the advantages of having a well-calibrated approximated method instead of using one based on samplings, such as MCS. In such a context, this work can be seen as a follow up of [1] [2] [6] [7]. The continue research in this field can help industries developing new tools or optimize the existent ones able to manage stochastic information and to carry out useful analysis and solution. It is important to clarify the influence of some parameters usually affected by variability or uncertainty on the life of a hybrid system and the consequences induced on economical parameters that usually lead to the final choice of investment. As for any other probabilistic study like this, a set of parameters affected by uncertainty must be chosen, and propagation of this variability can be estimated monitoring the outputs of interest.

The research available in the open literature related to uncertainties in energy systems is mainly focused on steady-state models [8] [9] [10]. Probabilistic methods are mostly applied for optimization purposes and design performance evaluation [2] [9] and very few cases are related to dynamic energy system analyses. Model uncertainties, materials variability, and uncertainty in operating parameters were considered in SOFC systems and the effects on the performance were evaluated [8] [9] [10] [11] [12]. RSA was applied to a PEM in order to count for the uncertainty in load profile and costs, evaluate the impact on fuel cell performance, and optimize the design and the operating strategy [2]. Model uncertainties were taken into account in a multi-objective optimization approach for a SOFC based system [11].

The Monte Carlo approach was used by Thomas *et al.* to predict the life of a lithium-ion cell with a degradation model

[13] while Placca *et al.* studied the effect of temperature uncertainty on cell voltage and degradation rate in a PEM through ANOVA [14].

For this work, typical ranges of variability have been founded for FU, ASR and η_{cycle}, based on literature information, industrial information, and author's background; while fuel cell life, PBP, IRR and NBCR are set as monitored outputs. The different approaches for uncertainty propagation have been tested in parallel on two configurations of a hybrid system model. Results carried out from MCS and approximated methods have been compared, highlighting pros and cons for each of them.

2 METHODOLOGY

Simulation of a deterministic model provides a set of outputs, which may give an incomplete and frequently incorrect representation of the system. A deterministic single-point simulation gives no value of the range of potential values for given parameter or their respective probability of occurrences, which may be expected in the system. To complete this information, the sensitivity and uncertainties in the results are needed. Uncertainty quantification methods can be classified into sampling methods (e.g., Monte Carlo simulation (MCS), Latin hypercube simulation, etc.) and approximate methods (e.g. response sensitivity analysis (RSA) method, fast probability integration (FPI) methods, polynomial chaos (PC) and metamodeling) [15]. In open literature, all of these approximated methods were applied to different applications in engineering. Kim *et al.* [16] and Abrassi *et al.* [6] applied Response Sensitivity analysis for a fuel cell gas turbine hybrid system optimization with more than 20 uncertainty inputs and for the performance analysis of a micro gas turbine respectively. In the design field of energy systems, a lot of examples for the application of Polynomial Chaos are available, such as [17] and [18], even if very few applied this method to fuel cell and hybrid system models.

2.1 MONTE CARLO SIMULATION

Among the sampling methods, MCS is the most common traditional probabilistic simulation technique for performing a probabilistic analysis on a model via a very large number of repeated simulations [19].

MCS is particularly distinguished as a probabilistic simulation technique since it can solve extremely complex and discontinuous problems precisely, provided the model is simulated with a high enough sampling number. Once the probabilistic information of a variable such as the mean value, the variance, and/or the probability distribution are known, the randomness of the variable can be simulated close to its true or real randomness using a random number generator. Repeating the simulation changing the set of randomly generated input variable values and storing the system output values every step, a set of probabilistic values and probability distribution functions (PDFs) of the output variables are obtained. This type of approach explicitly results in exact uncertainty (relatively speaking) propagation from the input variables to the system response, of course, assuming that the sampling number is high enough. Moreover, the probabilistic information of the system output quantifies the range within performance falls relatively to the objective limits set on the system.

2.2 RESPONSE SENSITIVITY ANALYSIS

Response Sensitivity Analysis (RSA) is a sensitivity-based approximation approach; it is utilized to estimate probabilistic information on the outputs of an analyzed system through a Taylor series expansion based calculation and with few information on the characteristics of input data [1] [6] [16].

The RSA algorithm returns probabilistic information on the output such as their first order moment (mean value) and second order moment (standard deviation). As explained in [16], when the mean and variance of each system input (Z_i) are known and an implicit nonlinear functional relationship $g_{M_j}(\vec{Z})$ between each system output M_j and the inputs $\vec{Z}$ is available, it is possible to use Taylor series expansion in order to estimate approximated value of mean and variance for each system output M_j, i.e.,

$$M_j \cong g_{M_j}(\mu_{\vec{Z}}) + \sum_{i=1}^{n}(Z_i - \mu_{Z_i})\left.\frac{\partial g_{M_j}(\vec{Z})}{\partial Z_i}\right|_{\mu_{Z_i}} + \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}(Z_i - \mu_{Z_i})(Z_k - \mu_{Z_k})\left.\frac{\partial^2 g_{M_j}(\vec{Z})}{\partial Z_i \partial Z_k}\right|_{\mu_{Z_{i\,o\,k}}} + \cdots \tag{1}$$

where μ_{Z_i} is the mean of the each i-th input.
Depending on where the expansion is truncated, the first and second order approximated means μ_{M_j} of each output M_j, will appear as shown respectively in equation (2) and (3).

$$\mu_{M_j} = \mu(M_j) \cong g_{M_j}(\mu_{Z_1}, \mu_{Z_2}, \dots, \mu_{Z_n}) \tag{2}$$

$$\mu_{M_j} = \mu(M_j) \cong g_{M_j}(\mu_{Z_1}, \mu_{Z_2}, \dots, \mu_{Z_n}) + \frac{1}{2}\sum_{i=1}^{n}\left(\frac{\partial^2 g_{M_j}}{\partial {Z_i}^2}\right)\nu(Z_i) \tag{3}$$

While the first order approximated variance ν_{M_j} will be:

$$\nu_{M_j} = \nu(M_j) \cong \sum_{i=1}^{n}\left(\frac{\partial g_{M_j}}{\partial Z_i}\right)^2 \nu(Z_i) \tag{4}$$

For practical purposes, the first-order variance (Eq. 4) and the second-order mean (Eq. 3) are generally used. If there are no explicit functional relationships between the system responses and inputs, the partial derivatives in the previous formulas cannot be determined analytically. However, numerical solution can be obtained using finite difference schemes [2], [6].

In Equation 4, the derivative term $\frac{\partial g_{M_j}}{\partial z_i}$ is called the system response sensitivity for M_j associated to Z_i. This is an important parameter, since, if properly converted into a dimensionless variable (Eq. 5), it can estimate the impact of each single input uncertainty on the monitored outputs at the same time of the probabilistic analysis.

$$Sensitivity = \frac{\partial g_{M_j}}{\partial Z_i}\frac{Z_{i,nom}}{g_{M_{j,nom}}} \tag{5}$$

RSA is easily applicable to dynamic system simulation because it is a computationally inexpensive method compared to MCS. When system responses show piecewise linear characteristics, the RSA method provides high fidelity analysis results very close to those provided by MCS. Moreover, to reduce the error due to truncation of Taylor series, a proper finite difference scheme for the resolution of derivatives has to be chosen [6]. The discretization step, usually called delta (δ), utilized in the definition of finite differences, is crucial for the approximation of results [6]. An example of finite difference scheme for the second order truncation used to approximate derivatives is shown in equation (6).

$$\frac{\partial g_{M_j}}{\partial Z_i} \cong \frac{M_{ji}^{+} - M_{ji}^{-}}{2\delta} \tag{6}$$

2.3 POLYNOMIAL CHAOS

Among the approximated methods for uncertainty quantification, the Polynomial Chaos (PC) is well known. It is based on the work by Wiener [20], who originally was concerned with stochastic processes with Gaussian random variables. More recently, the generalized PC (gPC) approach was introduced by Xiu [21]. In addition, it has the potential to achieve a significant reduction in computational cost (number of evaluations) with respect to traditional techniques such as Monte Carlo approaches [22] [23]. Moment estimation (i.e. mean and standard deviation) and sensitivity analysis (Sobol indices) can be extracted without significant additional costs from the PC expansion.

In the PC framework, both intrusive and non-intrusive PC methods are available [24]. The intrusive approach requires the governing equations to be rewritten. This means altering the model source code, which is used for the computations. This type of approach is only possible when the source code is available. On the other hand, the non-intrusive approach (NIPC) treats the model as a black-box. The way to gain information about the system is by running simulations with some specific sampling data points. For this reason, the non-intrusive approach is much more common for engineering applications.

Regardless the distinction between intrusive and non-intrusive methods, all Polynomial Chaos (PC) work is based on the same principle. An approximation of the model is constructed using an orthogonal set of polynomials, which serve as basis functions for an N-dimensional parameter space.

In a general definition, a polynomial chaos expansion can be written as:

$$Y(X) = \sum_{j=0}^{\infty} a_j \Phi_j\,(X) \tag{7}$$

where Y is the model response and X contains the input variables, both of which are affected by uncertainty. Therefore, the solution is split into a deterministic part, coefficients a_j, and a stochastic part, the polynomial basis Φ_j.

The original work by Wiener [20] uses Hermite polynomials as the basis functions to represent Gaussian random variables. Different types of polynomials could be used, depending on the probability distributions of the random inputs [16].

Once the stochastic system response has been determined as a PC expansion, the determination of the relevant statistics is straightforward, thanks to the orthogonality of the basis terms. In fact, from the evaluation of the coefficients of the expansion the mean and variance could be easily evaluate, as explain in equations (8) and (9):

$$\mu = a_0 \tag{8}$$

$$\sigma^2 = \sum_{i=1}^{p} a_j{}^2 \langle \Phi_j{}^2 \rangle \tag{9}$$

The integral $<\Phi_i^2>$ is reported for standard expansions in [26] or can be calculated numerically. Sensitivity analysis, evaluated through the Sobol indices, can be also performed without significant efforts. Sobol coefficients s_k can be calculated as follows:

$$s_k = \frac{var[Y_{X_k}(X)]}{var[Y(X)]} = \frac{\sum_{k=I_k} a_k^2 \langle \Phi_k^2 \rangle}{\sum a_k^2 \langle \Phi_k^2 \rangle} \tag{10}$$

where the index *k* sums over all the polynomials dependent only on X_k.

Hence, a method for calculating the coefficients of the expansion is required. Between the different NIPC methods, several quadrature rules, for the definition of the sampling data points (DOE points), with different accuracy level and sparsity exist. The most common one is the full tensor quadrature. A full tensor product quadrature is an effective approach for calculating multidimensional integrals when the number of dimensions is relatively small, but since the number of DOE points grows exponentially with the number of random dimensions; its effectiveness decrease rapidly for larger-dimensionality problems [27].

In problems with a moderately large number of variables, sparse tensor product grids (first proposed by Smolyak [28]) can be used to reduce the number of DOE points, while preserving a high level of accuracy. Panizza *et al.* [29] applied the sparse-grid approach on the uncertainty quantification of centrifugal compressor performance, and proved its effectiveness compared to MCS. A further improvement can be achieved also with the adaptive sparse grids [30] [31], where the number of DOE points in each dimension is chosen adaptively based on the difference between approximations of successive orders. Another approach that recently grows in the analysis of complex systems in engineering field is the use of least squares approximations (LSA) within a pseudo-spectral approximation concept. The idea is to construct a polynomial chaos approximation with an appropriate set of orthogonal polynomials and then calculate the coefficients of the expansion so that they provide the best fitting (in a least square sense) to some data [32] [33].

3.1 HYBRID SYSTEM MODEL

The model used in this analysis is a 1D model to simulate a co-flow, planar anode-supported SOFC, composed of a Ni doped yttria-stabilized zirconia (Ni-YSZ) anode, an YSZ-lanthanum strontium magnetite (LSM) cathode, and YSZ electrolyte [34]. The model was previously developed with the aim of integrating it into a SOFC gas turbine hybrid system emulator [34] [35] [36] taking into consideration the degradation effects. The full description of the model is reported in [7] [34] [35] and it is here summarized. An overview of the hybrid system model is depicted in Figure 1.

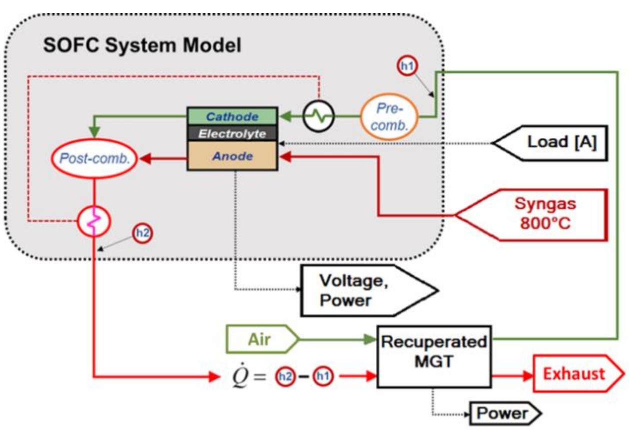

Figure 1: Schematic layout of the hybrid system model

The model employs a coupled approach of finite difference and finite volume, respectively, for thermal and electrochemical equations. Since the electrochemistry models do not use differential equations, the finite volume approach is more appropriate, while for heat transfer a finite difference method is applied. Originally, the SOFC model was created to simulate the effects of voltage degradation in the cell, including different mechanisms in a simple expression that relates the degradation rate to cell operating parameters (current density, fuel utilization and temperature), as explained in [7]. This model was here used for an uncertainty quantification analysis with the aim to evaluate the sensitivity of different outputs of interest to stochastic variations in the inputs, such as FC life and economic parameters.

More details on equations, parameters, and model validation can be found in Hughes *et al.* [34]. In Table 1 the main equations are summarized.

Table 1: Main equations of the SOFC model

Nernst potential	$V_{NERNST} = -\frac{\Delta G^0_{H_2O}}{nF} + \frac{R_gT}{nF} ln\left(\frac{p_{H_2}\sqrt{p_{O_2}}}{p_{H_2O}}\right)$	(11)
Cell voltage	$V_{cell} = V_{NERNST} - V_{dif} - V_{act} - V_{ohm,deg}$	(12)
Activation polarization	$V_{act} = \frac{R_gT}{\alpha nF} sinh^{-1}\left(\frac{i}{2i_0}\right)$	(13)
Diffusion polarization	$V_{dif} = \frac{R_gT}{2F}\left(ln\left(\frac{x_{H_2,bulk} \cdot x_{H_2O,TPB}}{x_{H_2O,bulk} \cdot x_{H_2,TPB}}\right) + \frac{1}{2} ln\left(\frac{x_{O_2,bulk}}{x_{O_2,TPB}}\right)\right)$	(14)
Ohmic polarization	$V_{ohm} = R \cdot i$	(15)
Resistance	$R = (R_{ohm} + R_{irr}) \cdot \left(1 + r_d \cdot \frac{t}{1000}\right)$	(16)
Irreversible contribution of degradation	$R_{irr} = \sum_{time}(R_{PEN} + R''_{oxide}) \cdot \frac{r_d}{1000}$	(17)
Degradation contribution	$V_{ohm,deg} = V_{ohm}\left(1 + r_d \cdot \frac{hours}{1000}\right)$	(18)
Anode exchange current density	$i_{0,an} = 5.5 \cdot 10^8 \frac{p_{H_2}}{p_{amb}} \frac{p_{H_2O}}{p_{amb}} exp(-\frac{50 \cdot 10^3}{R_gT})$	(19)
Cathode exchange current density	$i_{0,ca} = 7 \cdot 10^8 \left(\frac{p_{O_2}}{p_{amb}}\right)^{0.25} exp(-\frac{100 \cdot 10^3}{R_gT})$	(20)
Water-gas shifting (WGS) reaction	$H_2 + CO \leftrightarrow CO_2 + H_2O$	(21)
WGS equilibrium constant	$K_{p,shift} = \frac{p_{H_2}p_{CO_2}}{p_{H_2O}p_{CO}} = exp\left(\frac{4276}{T} - 3.961\right)$	(22)
Temperature distribution	$kA_{channel}\frac{\partial^2 T}{\partial x^2} + \frac{hA_{gas}}{L}(T_\infty - T) + q_{gen} = \rho c_p A_{channel}\frac{\partial T}{\partial t}$	(23)
Thermal output	$\dot{Q} = \dot{m}_{out}h_{out} - \dot{m}_{in}h_{in}$	(24)

In the hybrid system model, the fuel cell thermal output was recovered by a gas turbine to generate electrical power, according to Equation 25. The gas turbine model, purposely simplified, included a map of the recuperated cycle efficiency as function of generated power. Therefore, only the turbine power was calculated regarding the turbomachinery, while compressor and recuperator were not modelled.

$$P_{GT} = \eta_{GT} \cdot \dot{Q} \tag{25}$$

In the open literature, different control approach could be applied to hybrid system with different aim. For example, Ferrari M. [37] applied an advanced control approach to an hybrid system plant to prevent thermal stress in the fuel cell and to reduce the peak values of cathode/anode pressure difference and STCR. In this paper, a different control approach was implemented. In particular, two different configurations for the hybrid system model were taken into account, simulating two hybrid systems with a pressurized fuel cell, considering two different operating strategies. The aim was to evaluate the different impact of model uncertainty on a hybrid system and to understand if a different strategy can have a significant impact in the results.

In both configurations, cell voltage was kept constant, to reduce degradation over time and extend cell durability [38]. In the first hybrid configuration (1), "HS Power", the total system power was maintained constant, shifting load to the turbine as the fuel cell degraded. For this purpose, fuel flow increased in order to provide more heat in the off-gas burner to drive the turbine. Hence, the turbine worked in part-load condition at the beginning of the operating life and turbine power was incremented over time.

The second considered control strategy (2) named "HS Voltage", where the total power of the system was let decrease during time while maintaining constant voltage. The gas turbine size was considered optimized for the fuel cell initial conditions and operating at design point throughout the entire life of the plant. Thus, the fuel decreases together with the current to ensure that the power of the turbine remains constant while FU decrease. More in detail, since fuel and current do not decrease with the same rate the overall effect is a weak decreasing in FU. The two cases are briefly summarized in Table 2.

Table 2: Summary of the operating strategies

	Voltage	Stack power	FU	Turbine power	Total power
HS Power	Constant	↓	↓	↑	Constant
HS Voltage	Constant	↓	↓	Constant	↓

The initial conditions considered in this analysis for the two cases are illustrated in Table 3.

Table 3: Initial conditions

	HS Power	HS Voltage
Current density [A cm^{-2}]	0.5	
Cathode inlet flow [kg s^{-1}]	1	
Average cell temperature [°C]	800	
Anode fuel flow [kg s^{-1}]	0.09	
Fuel utilization [%]	80%	
Fuel composition*	28.6% CO, 12% CO2, 29.1% H2, 27.1% H2O, 3.2% N2	
Fuel cell pressure ratio (p/p_{amb})	3.5	
Cell voltage [V]	0.82	
Stack power [kW]	333	
Turbine power [kW]	160	130

*fuel is assumed to be syngas from coal gasification and cleaning process

The conditions for the End of Life (EOL) were considered as follows:

- For the hybrid system with "HS Power" strategy, since voltage and total system power were both constant, EOL was determined when the stack power was approximately 25% and the turbine power reached the considered design condition
- For the hybrid system with "HS Voltage" strategy, EOL was determined when the total system power reached 50%

Different criteria could be considered for the power reduction to optimize the economic performance of the systems, for instance allowing the power to decrease further in the constant voltage scenarios. However, here the purpose is focused on the uncertainty evaluation and propagation through economic parameters.

3.2 ECONOMIC MODEL

A simple economic analysis was implemented to evaluate the impact of uncertainty in degradation rate on some key economic parameters, such as Pay Back Period (PBP), Internal Rate of Return (IRR), and Net Benefit Cost Ratio (NBCR). Those were calculated as function of annual cash flow (CFN) and total capital investment (TCI), according to Equations 26-30. The considered variable costs or profits were fuel consumption, net electricity and a maintenance factor.

$$TCI = \sum_{j=1}^{PBP} CFN_j \tag{26}$$

$$NPV = \sum_{j=1}^{EOL} \frac{CFN_j}{(1+r)^j} - TCI \tag{27}$$

$$NBCR = \frac{NPV}{TCI} \tag{28}$$

$$\sum_{j=1}^{EOL} \frac{CFN_j}{(1+IRR)^j} - TCI = 0 \tag{29}$$

Where:

$$CFN_j = C_{el}P_{el} - C_f P_f - C_{main} \tag{30}$$

The assumptions for the calculation of the economic parameters were the following, assuming that the plant is installed in USA:

- an initial investment of $/kW 1000 for the FC stack, considering a 330 kW stack and 1% of the stack cost for ancillaries [36]
- an initial investment of $/kW 750 for the gas turbine (with a nominal power of 350kW in the "HS Power" configuration and 130kW in the "HS Voltage" configuration) [39]
- an initial investment of $129,500 for the recuperator in the hybrid system configurations [40]

- revenue of \$/kWh 0.15 for electric power produced considering a feed-in tariff in order to favour SOFC early penetration in the market
- a cost of \$/kg 0.1 (\$/kJ 2.4·10-6) of fuel considering the price of natural gas [41]; although the simulated fuel composition was a typical syngas employed to extrapolate the degradation model, for simplicity in this study a gasifier was not considered in the economic analysis
- 3% of the total investment cost for the annual maintenance cost
- an internal rate of 1% to actualize the cash flows

The work does not aim to evaluate accurately the economic performance of the systems, but to assess the uncertainty propagation from the inputs onto fuel cell lifetime and economic parameters. A simplified economic analysis was carried out, taking into consideration only the capital cost of the hybrid systems (mGT+SOFC) and variable cost (fuel consumption and electrical power production). Hence, auxiliaries and gasification process were not taken into account in this work. In addition, a fixed maintenance cost was considered for simplicity rather than a function of the fuel cell lifetime: in fact, the economic analysis was performed on the life cycle of the first SOFC stack, and not on the whole lifetime of the plant. In other words, the replacement of stacks onto the same balance of plant has not been considered. For this reason, NPV was calculated for costs occurring up to the moment of SOFC failure (EOL in the formulas) and not on the whole lifetime of the balance of plant. Thus, the economic results are indicative of how the probability distribution of NBCR, PBP and IRR is affected by uncertainties in the model.

4 RESULTS

This analysis was conducted focusing on the variability of system performance and economics due to uncertainties of single component performance. A fuel cell degradation model with fixed coefficients is included; it means that no uncertainty is considered on how the system degrades.

4.1 DETERMINISTIC RESULTS

Firstly, a deterministic simulation was performed to understand, without uncertainty, the behaviour of the plant in terms of fuel cell lifetime (considering 1 year as 8760 hours) and economic outputs. The FC life is calculated based on the EOL definition explained in paragraph 3.1. In this case, all the model parameters subject to uncertainty have been set with constant values, equal to the average values of their distributions.

The results of the deterministic simulation for both configurations are reported in Table 4. From these results, some important aspects need to be highlighted. In particular, the choice of control strategy for hybrid system has an important effect both on fuel cell lifetime and on economic parameters. In fact, the constant power strategy allows an important increase of the fuel cell life compared to the strategy with constant voltage. This leads to a significant difference also in the economic parameters, making this strategy more attractive from an economic point of view.

Table 4: Deterministic results

	HS Power	HS Voltage
FC life [year]	18.69	12.29
IRR [%]	23%	14%
PBP [year]	4.54	6.47
NBCR [-]	0.90	0.16

4.2 MCS RESULTS

After analysing the deterministic performance of the plants, uncertainty in some inputs was introduced. The stochastic analysis focused on design parameters that typically influence system performance: overall fuel utilization (FU), ohmic losses in the stack (ASR), and recuperated-TG cycle efficiency (η) were considered. The choice of these parameters is based on the availability of experimental data, knowing that other parameter like operating temperature could be taken into account since it is also a key variable for the lifetime. In addition, the coefficients of the degradation model were considered, in this paper, deterministic. A stochastic analysis related to the uncertainty of such coefficients could be found in [42].

The standard deviation of the considered input is reported in Table 5, together with their initial mean values, these latter change during simulation. The uncertainties on these values were considered inputs for the model.

Table 5: Design input parameters at initial conditions

	μ	σ
FU [%]	80	2
ASR [ohm/cm^2]	0.024	0.00072
η [%]	25	1.5

The mean and the standard deviation of the parameters are assumed according to known information, actual measurements, or Authors' knowledge and experience [43]. In particular, since the needed data of standard deviation for the recuperated-GT cycle efficiency was related to the standard deviation in recuperator effectiveness, a prior estimation was necessary. Hence, the evaluation of a σ of 1.5% for the entire GT cycle efficiency was evaluated with the same model and approach described in [6], starting from a standard deviation of 2.5% for the effectiveness of heat exchanger given by manufacturers [43]. The mean value of efficiency was chosen according to the technical features of the emulator on which the model was based.

In addition, each parameter is assumed to be distributed as Gaussian PDF. For most engineering problems, a clear probability inference of parameters usually requires a large amount of experimental data, which is often impractical due to expense considerations or experimental limitations. Thus, a normal (Gaussian) distribution is popularly adopted without losing the generality, which is, under such circumstance, more appropriate than other distributions. This is due to the facts that: (a) normal distributions are often found in engineering problems when the data collection is adequate and; (b) in many cases, a detailed description of probability distributions is not required since only the means and variances are sought [44]; (c) normal is an easier type of distribution to handle with approximated method used in this work.

With this assumption, a stochastic analysis was applied to the systems considering the uncertainty in the aforementioned input parameters.

A convergence analysis was performed on MCS in order to understand the minimum number of sampling needed to obtain a result with a high level of reliability. Then a simulation with 1000 samples has been adopted as reference case for the following considerations. Starting from the data series obtained from the MCS, a PDFs fitting was performed. It results that each output parameter could be graphically described as a Gaussian distribution.

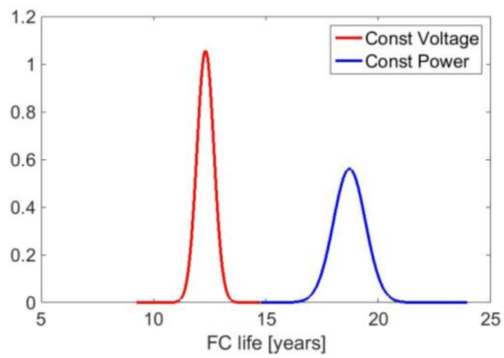

Figure 2: Comparison of FC life between the two strategies

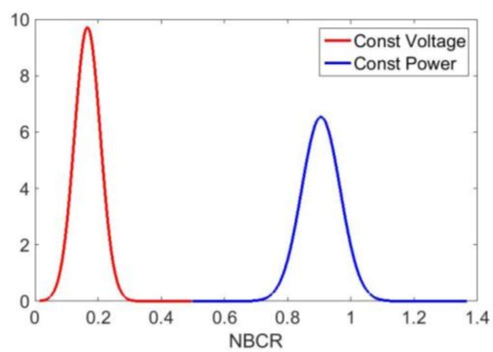

Figure 3: Comparison of NBCR between the two strategies

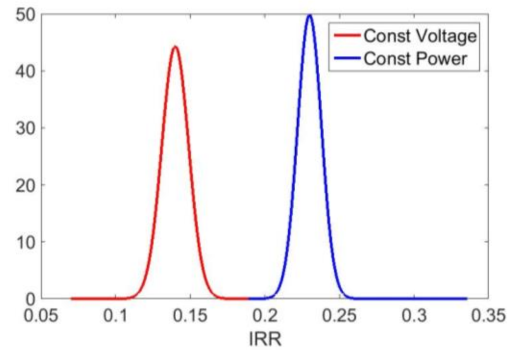

Figure 4: Comparison of IRR between the two strategies

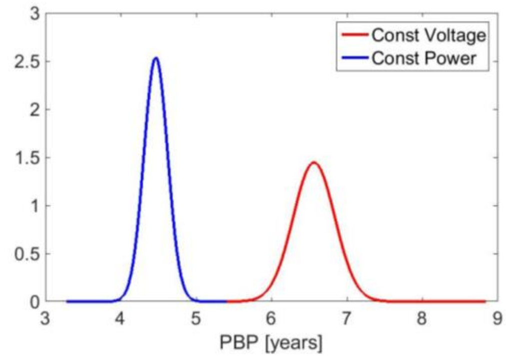

Figure 5: Comparison of PBP between the two strategies

In Figures 2-5 the distributions obtained from the MCS simulations are shown, with a particular attention on the different behaviour of the outputs between the two configurations. A Gaussian distribution was considered to represent the outputs after a pdf fitting analysis. With the given assumptions, the constant power control strategy guarantees a longer life for the system, reducing the payback time and resulting in higher NBCR and IRR, making this solution economically more attractive. In addition, it is clear that the Gaussian distributions never overlapped. This means that, once again, considering these particular assumptions and introducing uncertainties, the constant power strategy behaves better than the constant voltage one in terms of FC life and economic parameters. All these consideration, already observed in the deterministic scenario, are confirmed in uncertainty regime too. It is important to underline how these results are influenced by modelling assumptions and different criteria for definition of EOL.

4.3 COMPARATIVE ANALYSIS

In this case, simulations were performed with RSA and PC in order to compare the two approximated methods with MCS and to find their strengths and weaknesses, when applied to highly non-linear complex systems. In particular, for the RSA, a second order for both mean and standard deviation was considered, while for the PC, the order 4 and 6 for the polynomial for each input was implemented. Since both PC and RSA give information only on mean and variance values of the outputs, those can be used to evaluate different PDFs. Hence, a Gaussian distribution was assumed based on previous MCS results.

As first analysis, the deviation from MCS results for both approximated methods was analysed; the adopted indicator is the percentage difference (or error, assuming the MCS result as the correct result). With this parameter, it is possible to understand how much any single value of the mean and standard deviation carried out from approximated methods deviates from references.

$$P.E. = \frac{Reference - Approximated\ result}{Reference} \cdot 100 \quad (31)$$

These calculations are summarized in Table 6 and Table 7. Both RSA and PC have a very good level of approximation related to the evaluation of the mean values, as the percentage error never exceeds 2.2% for every output.

Table 6: Percentage error for each method compared to MCS (HS Voltage configuration)

	Percentage error (const Voltage)							
	FC life		PBP		NBCR		IRR	
	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
RSA	-1.8%	-2.0%	1.2%	13.9%	4.3%	6.4%	1.0%	3.9%
PC 4th	-0.1%	-40.1%	-1.8%	-111.5%	0.4%	-17.6%	0.1%	-28.0%
PC 6th	-0.1%	-40.4%	-1.4%	-101.9%	1.4%	-17.4%	0.3%	-28.5%

Table 7: Percentage error for each method compared to MCS (HS Power configuration)

	Percentage error (const Power)							
	FC life		PBP		NBCR		IRR	
	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
RSA	0.3%	16.0%	-2.2%	-20.5%	0.4%	8.6%	0.1%	12.3%
PC 4th	-0.1%	2.4%	-0.6%	-77.4%	1.3%	-31.0%	0.2%	-61.7%
PC 6th	-0.1%	2.5%	-0.7%	-72.6%	1.2%	-26.5%	0.2%	-60.4%

Regarding the calculation of the standard deviation, globally RSA behaves better than PC, except for the estimation of σ of lifetime in the constant voltage strategy. Due to the high complexity of the system and the wide time extension simulated, a value of error up to 10% was considered acceptable: this was the criterion to distinguish accurate solutions from the others. For a better comprehension, values that exceed this limit are highlighted in the tables. As easily noticeable, a great amount of cases overcomes this limit for the standard deviation; the solution to increase accuracy is different for each method. For RSA, the choice of accurate values for the delta parameter (δ) could be the solution; this is a variable somehow related to the resolution of the method. Firstly, different values of delta were taken into account in order to find the one that fit better with MC results. A good compromise was found between mean and standard deviation. However, other investigations on this parameter are needed. For example, a minimization algorithm was suggested, which automatically fits the delta to reduce such an error, with an early fitting of RSA on MCS results (when available). Until now, the delta value has always been set manually basing on experience. The introduction of this further passage in RSA codification could help both to reach better level of approximation and to achieve quicker a good set-up for the RSA method.

For PC, it was expected that increasing the polynomial degree could help to improve the approximation, however, as reported in *Table 6* and *Table 7*, passing from a 4^{th} to a 6^{th} order, it did not show the expected benefit. It is possible to conclude that for this particular version of the problem another tuning strategy has to be considered for PC. For instance, extending the range of mixed terms included in the PC algorithm could certainly help in decreasing the error.

The most powerful aspect of approximated methods is the great saving in computational time they guarantee. The number of calls to the model during each probabilistic analysis can well quantify the computational efforts; this is proportional to the calculation time. *Table 8* shows the advantage, in terms of number of model runs, to compute an approximated method (PC or RSA) rather than a sampling one (MCS).

Table 8: Number of model calls per simulation for each method

	MCS	PC	RSA
n° of model runs	1000	256	13

A more direct comprehension of the level of approximation given by these two methods is visible in Figure 8 and Figure 9 where the complete PDFs carried out by each method are depicted. The standard deviation is related to the shape of the Gaussian curves, in particular, the higher is the σ and the wider the bell-shape curve appears, while the mean value fixes the center of the PDF. Observing these figures, the good level of approximation of the means is highlighted by the fact that the PDFs are well centered. In all cases, RSA still has a good level of approximation; the error on the standard deviation has less influence if the σ/μ ratio, or coefficient of variation (COV), is small. This is an important aspect to be taken into account during the analysis of results, in fact, this observation seems to indicate that a correct estimation of variance is less important than a good approximation of means in order to obtain an overall good achievement of results. According to this argument, different values of acceptable percentage errors could be chosen to identify what is an acceptable level of error. Anyway, even if Figure 8 and Figure 9 show a good alignment with MCS, which could push to consider the RSA results acceptable, in comparison, the criterion of percentage error introduced before still remain the more rigorous and reasonable one. COV parameter briefly introduced before represents another way to compare these methods. Figure 6 and Figure 7 show value of COVs calculated for each output distribution carried out with each method. Two remarkable observations come out from these diagrams, firstly, starting from the same distributions of inputs, compared to the other one, the constant power approach with returns a lower dispersion in outputs distribution, that means thinner bells for pdf and lower values of COVs. Secondly, a better approximation of RSA is confirmed by the lower difference between values of COVs coming from MCS and this latter approximated method (except for FC-lifetime in constant power case).

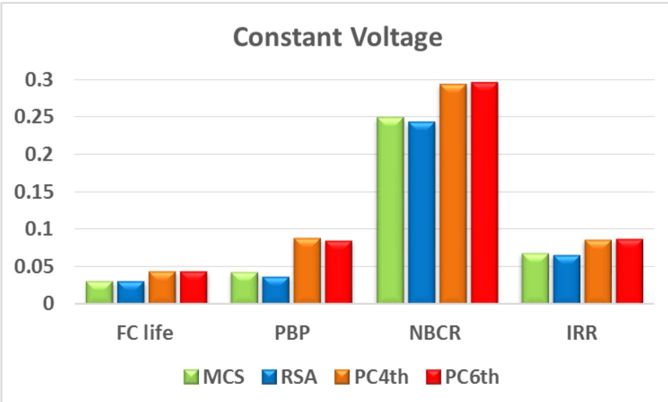

Figure 6: Comparison between COV obtained with different methods

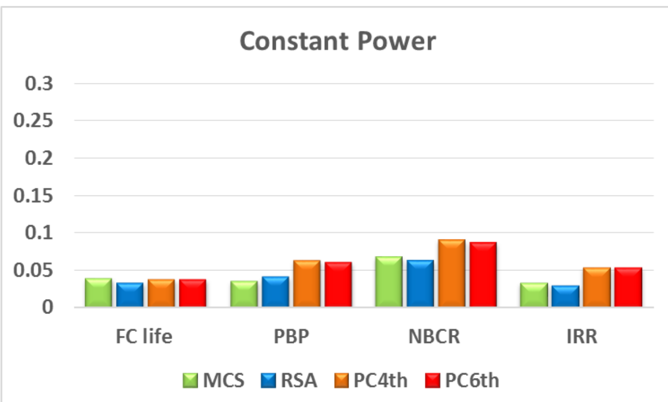

Figure 7: Comparison between COV obtained with different methods

A low value of this indicator denotes the low influence that the percentage error on the standard deviation has on the PDF's shape.

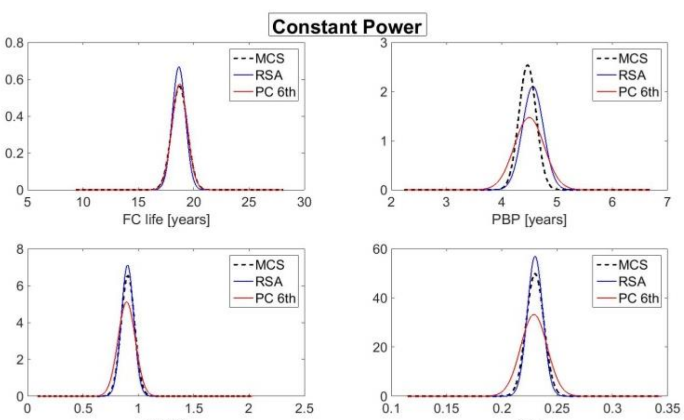

Figure 8: Probability density functions of the outputs from MCS (black), RSA (blue), PC 6th (red) in the Constant Power configuration

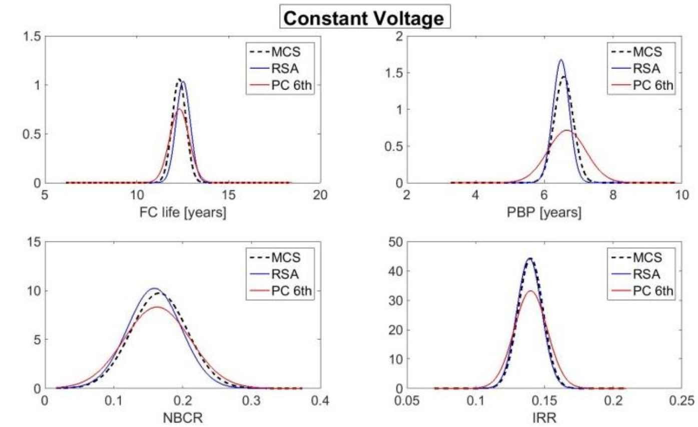

Figure 9: Probability density functions of the outputs from MCS (black), RSA (blue), PC 6th (red) in the Constant Voltage configuration

Stochastic analysis favour the constant power strategy also because the dispersion in results is limited compared to the other solution. This means that system performance, and consequently the economic analysis, are globally influenced more by the variability of inputs when the power degrades during life. Full analysis of results is reported in Table 9 and Table 10.

Table 9: Means and standard deviations of outputs (Constant Voltage approach)

	Constant Voltage							
	FC life [year]		PBP [year]		NBCR [-]		IRR [%]	
	μ	σ	μ	σ	μ	σ	μ	σ
MCS	12.31	0.,40	6.57	0.28	0.16	0.04	0.14	0.01
RSA	12.53	0.39	6.49	0.24	0.16	0.04	0.14	0.01
PC	12.32	0.53	6.65	0.56	0.16	0.05	0.14	0.01

Table 10: Means and standard deviations of outputs (Constant Power approach)

	Constant Power							
	FC life [year]		PBP [year]		NBCR [-]		IRR [%]	
	μ	σ	μ	σ	μ	σ	μ	σ
MCS	18.71	0.60	4.47	0.16	0.91	0.06	0.23	0.01
RSA	18.66	0.60	4.56	0.19	0.90	0.06	0.23	0.01
PC	18.73	0.69	4.50	0.27	0.89	0.08	0.23	0.01

RSA is also able to investigate the effect that each single input has on each output, through the sensitivity parameter of equation (5). This is not possible to infer from the complete PDFs, where the influence of each single input is cumulated with the others. The comparison between Figure 10 and Figure 11 confirms the stronger impact of inputs when the degrading power strategy is chosen. Both graphs show there is a very weak effect of the FU variability on each monitored output, while the main influence is provided by the TG cycle efficiency (η - eta), which, once again, is larger for the constant voltage case. The ASR parameter is the only one that shows more impact on PBP and FC life in the first control strategy .

Therefore, in both control cases (constant and degrading power), sensitivity analysis suggests to reduce first the uncertainty on the microturbine efficiency in order to minimize variability on economic outputs, and second, to reduce the uncertainty on ASR.

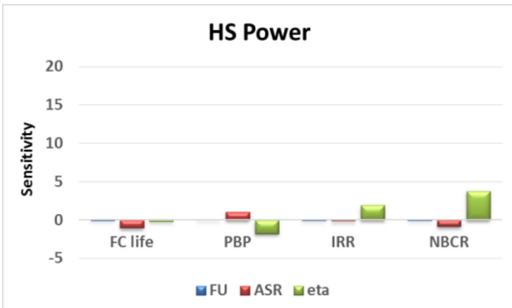

Figure 10: Sensitivities for Constant Power approach

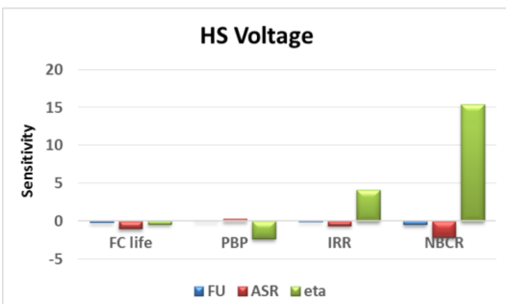

Figure 11: Sensitivities for Constant Voltage approach

6 CONCLUSION

In this paper, a stochastic analysis was applied to a fuel cell gas turbine hybrid system; the plant, represented with a model developed in Matlab Simulink, includes a degradation model for the SOFC. The main focus was to investigate the uncertainty effect related to some inputs on system performance, in particular on the lifetime of the fuel cell. Furthermore, a simple economic analysis was performed in order to understand the sensitivity on some economic indicators to the introduced variability. The same hybrid system model was tested with two different control strategies: the fuel cell voltage was kept constant in both cases, while the total power output was constant in the first configuration and was allowed to degrade along plant life in the second one. In addition, different probabilistic methods have been used for the stochastic analysis: MCS results were assumed as reference for the comparison between RSA and PC.

Overall, the hybrid system in deterministic conditions with constant power control showed better results if compared to the degrading power one. In particular, with the first strategy an increasing of 35% in lifetime and of about 81% in NBCR were obtained. Moreover, its sensitivity to the variation of the input seemed globally weaker than in the second case. The sensitivity analysis performed with RSA has shown that the uncertainty related to the microturbine cycle efficiency is the most critical parameter in both cases.

The comparison between the approximated methods returns a good level of accuracy to estimate the mean for both methods, with a maximum error of 2.2% and 1.8% using RSA and PC respectively. On the other hand, considering the standard deviation, RSA gives better results than PC respect to MCS ones. Possibility for improvement has been identified for both methods acting on their set up parameters, i.e. optimization of delta parameter for the RSA and higher optimization degree for the PC. Moreover, thanks to this work, a feasible strategy of calibration for the approximated methods has been suggested. This is based on optimization algorithms and it will be tested in future works.

A further investigation in this sense is strongly recommended and justified by the large computational savings that these approximated methods can provide: in this case, for the same calculation, PC and RSA cut the simulation efforts by a factor of 5 and 50, respectively, compared to MCS. In an application field where this kind of analysis is continuously needed with a lot of different configurations of the model to test, it is not practical to use Monte Carlo simulations.

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REFERENCES

[1] Abrassi, A., Cuneo, A., Traverso, T., 2015. "Off-design analysis of a micro gas turbine under stochastic conditions." Sustainable Places 2015, Savona, Italy.

[2] Kim, K., Wang, M., von Spakovsky, M.R., Nelson, D.J., 2008. “Stochastic Modeling and Uncertainty Analysis with Multi-objective Optimization Strategies for the Synthesis/Design and Operation/Control of a PEMFC Fuel Processing Subsystem”, IMECE2008-68065.
[3] Wiener, N., 1938. "The Homogeneous Chaos", American Journal of Mathematics, 60(4), pp. 897-936.
[4] Padulo, M., Campobasso, M.S., Guenov, M.D., 2007., “Comparative analysis of uncertainty propagation methods for robust engineering design”, International conference on engineering design, ICED’07 28 – 31. Paris France
[5] Lee, S.H., Chen, W., 2008. “A comparative study of uncertainty propagation methods for black-box-type problems. Structural and Multidisciplinary Optimization”, 37(3)
[6] Abrassi, A., Cuneo, A., Traverso, T., 2016. “Off-design performance analysis of a micro-gas turbine under uncertainty” ISROMAC 16, Honululu, Hawaii.
[7] Zaccaria, V., Tucker, D., Traverso, T., 2016. “A distributed real-time model of degradation in a solid oxide fuel cell, part I: Model characterization”, Journal of Power Sources 311, pp.175-1816.
[8] He, Z., Li, H., Birgersson, E., 2014. “Correlating variability of modeling parameters with non-isothermal stack performance: Monte Carlo simulation of a portable 3D planar solid oxide fuel cell stack”, Applied Energy, 136, pp. 560–575
[9] Gorla, R.S.R., 2004. “Probabilistic analysis of a solid-oxide fuel-cell based hybrid gas-turbine system”, Applied Energy, 78, pp. 63–74
[10] Mawardi, A., Pitchumani, R., 2006. “Effects of parameter uncertainty on the performance variability of proton exchange membrane (PEM) fuel cells”, Journal of Power Sources, 160, pp.232–245
[11] Subramanyan, K., Diwekar, U.M., 2005, “Characterization and quantification of uncertainty in solid oxide fuel cell hybrid power plants”, Journal of Power Sources, 142, pp.103–116
[12] Biagiola, S.I., Schmidt, C., Figueroa, J.L., 2014. “Model uncertainty estimation of a solid oxide fuel cell using a Volterra-type model”, Journal of the Franklin Institute, 351, pp.4183–4197
[13] Thomas, E.V., Bloom, I., Christophersen, J.P., Battaglia, V.S., 2008. “Statistical methodology for predicting the life of lithium-ion cells via accelerated degradation testing”, Journal of Power Sources, 184, pp.312–317
[14] Placca, L., Kouta, R., Blachot, J.F., Charon, W., 2009. “Effects of temperature uncertainty on the performance of a degrading PEM fuel cell model”, Journal of Power Sources, 194, pp. 313–327
[15] Cuneo, A., Traverso, A., Shahpar, S., 2017. “Comparative analysis of methodologies to conduct uncertainty propagation and quantification”, ASME paper GT2017-63238.
[16] Kim, K., von Spakovsky, M.R., Wang, M., Nelson, D.J., 2012. “Dynamic optimization under uncertainty of t3he synthesis/design and operation/control of a proton exchange membrane fuel cell system”, Journal of Power Sources, 205, pp.252-263.
[17] Modgil, G., Crossley, G.M., Xiu, D., 2013. “Design Optimization of a High-Pressure Turbine Blade using Generalized Polynomial Chaos (gPC)”, 10th World Congress on Structural and Multidisciplinary Optimization May 19 -24, 2013, Orlando, Florida, USA.
[18] Seshadri, P., Parks, G.T., Jarret, J., Shahpar, S., 2013. “Towards robust design of axial compressors with uncertainty quantification”, AIAA/ ASME/ ASCE/ AHS/ ASC Structures, Structural Dynamics and Materials Conference · January 2013.
[19] Kalos, M.H., Whitlock, P.A., 1986. “Monte Carlo Methods, Volume I: Basics” John Wiley & Sons, New York.
[20] Wiener, N., 1938. "The Homogeneous Chaos", American Journal of Mathematics, 60(4), pp. 897 – 936.
[21] Xiu, D., 2010. "Numerical Methods for Stochastic Computation: A Spectral Method Approach", Princeton University Press.
[22] Eldred, M.S., 2009. “Recent Advances in Non-Intrusive Polynomial Chaos and Stochastic Collocation Methods for Uncertainty Analysis and Design”, 50th AIAA/ ASME/ ASCE/ AHS/ ASC Structures, Structural Dynamics, and Materials Conference, 4 - 7 May 2009, Palm Springs, California.
[23] Xiu, D., Lucor, D., Su, C.-H., and Karniadakis, G. E., 2002. “Stochastic modeling of flow-structure interaction using generalized polynomial chaos”, ICOSAHOM-01.
[24] Onoratory, G., Loeven, G.J.A., Ghorbaniasly, G., Bijl, H., Lacory, C., 2010. “Comparison of intrusive and non - intrusive polynomial chaos methods for cfd applications in aeronautics”, V European Conference on Computational Fluid Dynamics ECCOMAS CFD 2010 J. C. F. Pereira and A. Sequeira (Eds) Lisbon, Portugal,14-17 June 2010.
[25] Xiu, D., Karniadakis, G.E., 2002. "The Wiener-Askey Polynomial Chaos for Stochastic Differential Equations", Journal of Scientific Computing, 24(2), pp. 619-644.

[26] Ghanem, R., and Spanos, P., 1991. "Stochastic Finite Elements: A Spectral Approach" Springer-Verlag, New York.
[27] Eldred, M.S., Webstre, C.G., Constantine P.G., 2008. "Evaluation of Non-Intrusive Approaches for Wiener-Askey Generalized Polynomial Chaos", 49th AIAA/ ASME/ ASCE/ AHS/ ASC Structures, Structural Dynamics, and Materials Conference 16t 7 - 10 April 2008, Schaumburg, IL, AIAA 2008-1892.
[28] Smolyak, S. A., 1963. "Quadrature and Interpolation Formulas for Tensor Products of Certain Classes of Functions," Doklady Akademii Nauk SSSR, 148, pp. 1042-1045.
[29] Panizza, A., Rubino, D.T., Tapinassi, L., 2014. "Efficient uncertainty quantification of centrifugal compressor performance using polynomial chaos", Proceedings of ASME Turbo Expo 2014: Turbine Technical Conference and Exposition GT2014 June 16 – 20, 2014, Düsseldorf, Germany GT2014-25081.
[30] Gerstner, T., and Grieber, M., 2003. "Dimension-Adaptive Tensor-Product Quadrature," Computing, 71, pp. 65–87
[31] Ghisu, T., Parks, G., Jarrett, J., Clarkson, P.J., 2010, "Adaptive Polynomial Chaos for Gas Turbine Compression Systems Performance Analysis", AIAA Journal 48(6), pp. 1156-1170.
[32] Cohen, A., Davenport,M. A., and Leviatan, D., 2013. "On the Stability and Accuracy of Least Squares Approximations", Foundations of Computational Mathematics, 13(5), pp. 819-834.
[33] Seshadri, P., Narayan, A., and Mahadevan, S., 2016. "Optimal Quadrature Subsampling for Least Squares Polynomial Approximations", pp. 1-16.
[34] Hughes, D., Wepfer, W.J., Davies, K., Ford, J.C., Haynes, C., Tucker, D., 2011, "A real-time spatial SOFC model for hardware based simulation of hybrid systems" International ASME Conference on Fuel Cell Science, Engineering and Technology.
[35] Tucker, D., Shelton, M., Manivannan, A., 2009. "The role of solid oxide fuel cells in advanced hybrid power systems of the future", The Electrochemical Society Interface, Fall 2009, pp.25-28.
[36] Weimar, M., Chick, L., Gotthold, D., Whyatt, G., 2013. "Cost study for manufacturing of solid oxide fuel cell power systems", U.S. Department of Energy, Sep: 24–50, Report number PNNL-22732.
[37] Ferrari, M., 2015. "Advanced control approach for hybrid systems based on solid oxide fuel cells", Applied Energy, 145, pp.364–373
[38] Zaccaria, V., Traverso, A., Tucker, D., 2015. "A Real-Time Degradation Model for Hardware in the Loop Simulation of Fuel Cell Gas Turbine Hybrid Systems", ASME Paper GT2015-43604.
[39] Rosa do Nascimento, M.A., de Oliveira Rodrigues, L., Cruz dos Santos, E., Batista Gomes, E.E., Goulart Dias, F.L., Gutiérrez Velásques, E.I., Carrillo, R.A.M., 2014. "Micro Gas Turbine Engine: A Review", Micro Gas Turbine Engine, Brazil: InTech. 107–141.
[40] Arsalis, A., 2008. "Thermoeconomic modeling and parametric study of hybrid SOFC–gas turbine–steam turbine power plants ranging from 1.5 to 10MWe", Journal of Power Sources,181, pp.313-3.
[41] United States Natural Gas Industrial Prices, https://www.eia.gov/dnav/ng/hist/n3035us3m.html *(accessed on 04/11/2016)*
[42] Cuneo A., Zaccaria V., Tucker D., Traverso A., Probabilistic analysis of a fuel cell degradation model for solid oxide fuel cell and gas turbine hybrid systems, Energy Journal, *under review*
[43] HiFlux Ltd, Tanzi Besant, tel. +44 (0)20 8538 1651
[44] Friswell, M.I., Mottershead, J.E., Mares, C., 2007. "Stochastic model updating in structural dynamics", Proceedings of ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, Rethymno, Crete, Greece