

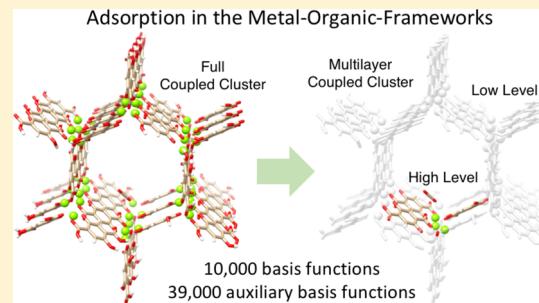
Multilayer Divide-Expand-Consolidate Coupled-Cluster Method: Demonstrative Calculations of the Adsorption Energy of Carbon Dioxide in the Mg-MOF-74 Metal–Organic Framework

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Supporting Information

ABSTRACT: The implementation and evaluation of a multilayer extension of the divide-expand-consolidate (DEC) scheme within the LSDalton program is presented. The DEC scheme is a linear-scaling, fragmentation-based local coupled-cluster (CC) method that provides a means of overcoming the scaling wall associated with canonical CC electronic structure calculations on large molecular systems. Taking advantage of the local nature of correlation effects, the correlation energy for the full molecule is calculated from a set of independent fragments using localized molecular orbitals. However, when only a small subsystem of a larger system is of interest, for example, adsorption sites or catalytically active sites, the majority of the computational time may be spent evaluating the correlation energy of fragments which have little effect on the properties in the area of interest (AOI). The multilayer DEC (ML-DEC) scheme addresses this by taking advantage of the independent nature of the fragments in order to evaluate the correlation energy of various regions of the system at different levels of theory. Regions far from the AOI are evaluated at lower (cheaper) levels of theory such as Hartree–Fock (HF) or Møller–Plesset second-order perturbation theory (MP2), while the area immediately surrounding the AOI is treated with a higher level CC model. Through the ML-DEC scheme, the computational cost of CC calculations on these types of systems can be significantly reduced while maintaining the accuracy of higher level calculations. Results from HF/RI-MP2 and RI-MP2/CCSD ML-DEC calculations of the binding energy of a fatty acid dimer are presented. We find that the ML-DEC scheme is capable of reproducing DEC energy differences at a target level of theory, provided that the region treated at the target level of theory is chosen to be sufficiently large. Time-to-solution is found to be significantly reduced, particularly in the RI-MP2/CCSD calculations. Finally, the ML-DEC scheme is applied to the calculation of CO₂ adsorption in a Mg-MOF-74 channel.



INTRODUCTION

In recent years, there has been a significant push to extend the applicability of high-accuracy coupled-cluster (CC) methods to larger systems, leading to the development of a number of local correlation schemes. Canonical CC methods treat correlation effects, which are mostly local by nature, within the delocalized Hartree–Fock (HF) basis resulting in $O(N^5)$ scaling with system size for the cheapest CC implementation, Møller–Plesset second-order perturbation theory (MP2), up to $O(N^7)$ for CC with singles, doubles, and perturbative triples excitations [CCSD(T)]. Local correlation schemes, however, utilize a localized basis for the calculation of correlation energies in order to overcome the scaling wall associated with canonical CC methods. These schemes are generally based on either the approximations of the wavefunction originating from the work of Pulay and Sæbo,^{1–7} including the most efficient approximations based on pair natural orbital (PNO),^{8–15} as well as other schemes based on local natural orbital (LNO)^{16–18} or orbital-specific approximations,¹⁹ or the approximations based on explicit fragmentation of the system.²⁰ Fragmentation-based approaches, which include the fragment

molecular orbital (FMO) method,^{20,21} cluster-in-molecule (CIM) approach,^{22–24} incremental method,^{25–27} and divide-and-conquer scheme,^{28,29} to name a few, are based on the work of Förner and co-workers^{30,31} where the full-system CC correlation energy calculation is reduced into a set of smaller independent fragment CC calculations.⁵⁵

The divide-expand-consolidate (DEC) scheme^{32–35} is one such fragmentation approach which relies on local occupied and virtual molecular orbitals (LMOs)³⁶ to calculate the correlation energy from a set of independent fragments. Unlike methods which rely on physical fragmentation of the system, such as FMO, the orbital space is only fragmented for the CC correlation calculation, while exchange interactions are still evaluated using canonical HF, therefore eliminating the need for many-body exchange calculations between fragments.⁶³ Specifics of the DEC scheme will be discussed in the following section; however, the key feature that makes the DEC scheme unique among the fragmentation methods is that the fragments

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are determined dynamically in an automated manner in order to ensure error control to a desired tolerance. The number of fragments increases linearly with the system size, making the DEC scheme linear-scaling overall. Fragment energies are evaluated independently, allowing for an embarrassingly parallel implementation with multiple levels of parallelism in order to improve load balancing. The end result is an algorithm poised to make efficient use of modern and upcoming supercomputing architectures.

In large systems, however, it is often the case that only a small region of the full system is of primary interest—for example, defects, adsorption sites, or catalytically active sites. In these cases, much of the computational effort in the DEC scheme is spent on evaluating the correlation energy of fragments which make a relatively small contribution to the property of interest. In these cases, the DEC scheme can be made more efficient by only applying high-level correlation methods, for example, CCSD or CCSD(T), to the regions of interest, while the remaining regions can be treated with lower level methods such as MP2 or HF.

Multilevel correlation approaches were first explored in the late 1990s. Perhaps the earliest example is the integrated molecular orbital + molecular orbital (IMOMO)³⁷ scheme introduced by Humbel, Sieber, and Morokuma, which allows for the combination of ab initio, density functional theory (DFT), or semiempirical MO methods for structure and energy prediction. This method was quickly followed by the ONIOM method,³⁸ which extended the two-layer IMOMO method to a three-layer approach combining CC, HF, or MP2 and force field methods.

Similarly, multilevel extensions to local correlation methods were pioneered in the local correlation schemes of Pulay and Sæbø and have since been implemented in a number of other local correlation schemes. For example, the local molecular orbital: molecular orbital method,³⁹ implemented by Mata, Werner, and Schütz, evaluates the correlation within regions of LMOs using different levels of Pulay's local correlation methods, for example, LCCSD(T) or LMP2. In this scheme, orbital pairs are assigned to the high or low level based on the atomic center to which they belong, and only strong pairs in which both orbitals fall within the high level region are evaluated at a high level. Another such multilevel scheme has been reported within the CIM approach²⁴ in which reactive sites within a system were treated using the completely renormalized CC method with singles, doubles, and non-iterative triples [CR-CC(2,3)]^{40,41} while remaining regions were treated with MP2. More recently, a multilevel approach was also implemented within the DLPNO-CCSD(T) method by imposing tighter accuracy thresholds for the local CCSD(T) calculations in regions of interest.⁴² Also, a density-functional/wave-function embedding scheme based on LNOs was introduced by Kállay and co-workers.⁴³

In this article, we report the implementation of a multilayer DEC (ML-DEC) scheme. The independent nature of the fragments within the DEC scheme allows for facile assignment of the fragments to different "layers" in which the correlation energy is evaluated at different levels of theory while maintaining the dynamic optimization of the fragments within user-controlled tolerances. To date, the ML-DEC scheme has been implemented and evaluated for any combination of HF, MP2, and CCSD (though it is recommended that the low level be only one step below the high level). In addition, the DEC scheme has recently been extended to include the resolution-

of-the-identity MP2 (RI-MP2) model (DEC-RI-MP2)⁴⁴ as well as Laplace transformed RI-MP2 (DEC-LT-RIMP2).⁴⁵ These models significantly reduce the time-to-solution (TTS) of local MP2 calculations and are also available for use within the ML-DEC scheme.

As a proof of concept, we apply the ML-DEC scheme to the calculation of interaction energies in two different systems: (a) the palmitic acid ($C_{16}H_{32}O_2$) dimer and (b) CO_2 adsorbed on the open metal sites (OMS) of the metal–organic framework (MOF) Mg-MOF-74⁴⁶ (also known as CPO-27-Mg). The fatty acid dimer constitutes a simple test case to demonstrate the computational efficiency of the ML-DEC scheme as well as convergence of the ML-DEC correlation energy to the standard DEC energy as the high level layer expands. The interaction between the monomers is localized to the two carboxylic acid groups, making the initial assignment of the high-level layer straight forward, and expansion of the high level layer requires only progressing along the carbon chains.

The MOF system, on the other hand, provides a test of the ML-DEC scheme in a larger and more complex environment. The choice of this test system is motivated by recent investigations into the effects of acid gases on MOFs in the context of carbon sequestration from industrial flue gas. MOFs with OMSs such as Mg-MOF-74 have been studied for this purpose due to their high adsorption capacity for CO_2 . However, acid gases such as SO_x and NO_x , along with H_2O , are found as impurities in flue gas, and these compete heavily for adsorption sites within the MOF, effectively poisoning the material. Understanding the effects of these acid gases requires the characterization of their interactions with MOFs through both experimental and computational means. Traditionally, these interactions have been studied computationally in periodic MOF systems using plane-wave DFT; however, the results of these calculations are highly dependent on the choice of pseudopotential and additional parameters such as Hubbard- U corrections and van der Waals dispersion corrections. In the absence of corresponding experimental data, it is impossible to know with certainty which parameterizations provide the most realistic description of the MOF-adsorbate interaction. It is therefore desirable to use methods within the hierarchy of CC theory which account for dispersion and can be systematically improved by increasing the level at which correlation is treated. However, because of the high computational cost of the CC methods, this typically limits the computational investigations to small clusters which suffer from errors due to finite size effects.

In order to capture the chemical environment within the MOF, larger clusters than those accessible to canonical CC methods must be used, and therefore, these systems stand to benefit greatly from the linear-scaling, embarrassingly parallel DEC scheme. In addition, it is well established from previous experimental and theoretical investigations that the strongest interactions between Mg-MOF-74 and the adsorbate gas molecules generally occur at the OMSs.^{47–51} This suggests that we can efficiently evaluate the adsorption energies of these small gas molecules within the MOF using the ML-DEC scheme by treating only the fragments centered on the adsorbate, OMSs, and the immediate neighbors at the high level of theory while all other fragments are evaluated at the low level. However, we wish to emphasize that the calculations presented here are intended to demonstrate the capability of the ML-DEC scheme to efficiently treat large systems with the accuracy of standard DEC calculations rather than to obtain

194 adsorption energies with experimental accuracy. We have
 195 therefore used a relatively simple method to optimize the
 196 placement of the CO₂ molecule within the MOF channel
 197 rather than performing extensive optimizations on the periodic
 198 system. For the same reason, basis set superposition error
 199 (BSSE) corrections have also not been included.

200 This manuscript is organized as follows: we first briefly
 201 review the DEC scheme and describe the implementation of
 202 the ML-DEC scheme. Next, results of HF/RI-MP2 and RI-
 203 MP2/CCSD ML-DEC test calculations on the palmitic acid
 204 dimer are reported, which are compared to standard DEC
 205 calculations. We then discuss the application of the ML-DEC
 206 scheme to the calculation of the CO₂ adsorption energy in a
 207 793-atom Mg-MOF-74 channel. Finally, conclusions regarding
 208 the efficiency and accuracy of the ML-DEC scheme are
 209 provided.

210 ■ THEORY

211 **Review of the DEC Scheme.** Here, we present a brief
 212 review of the DEC scheme as implemented in the LSDalton
 213 program⁵² within the Dalton suite.⁵³ For a more detailed
 214 description of the DEC scheme, the reader is referred to
 215 refs.^{32,34}

216 As mentioned above, the DEC scheme and other local
 217 correlation methods rely on LMOs in order to overcome the
 218 steep scaling of canonical CC methods. In the DEC scheme,
 219 canonical occupied and virtual MOs (CMOs) are first obtained
 220 from a canonical HF calculation on the full system.
 221 Fragmentation of the orbital space is only introduced in the
 222 correlation calculation. The CMOs are then localized to obtain
 223 the set of LMOs which are assigned to the nearest atom
 224 relative to the center of charge of each LMO. By default,
 225 orbitals centered on light atoms (i.e., hydrogen) are assigned to
 226 the nearest heavy atom. A number of localization schemes have
 227 been implemented in LSDalton, including Pipek-Mezey,⁵⁴
 228 Boys,⁵⁵ and powers of the second⁵⁶ and fourth⁵⁷ central
 229 moments (PSM and PFM, respectively). The calculations
 230 presented here utilized the PSM method with powers of 2 for
 231 both occupied and virtual orbital localization. A review of these
 232 localization schemes can be found in ref 36.

233 The DEC scheme is based on the decomposition of the
 234 correlation energy of a molecular system into the sum of
 235 atomic and pair correlation energies within the system

$$236 E_{\text{corr}} = \sum_P^{N_{\text{frag}}} \left[E_P + \frac{1}{2} \sum_{Q \neq P}^{N_{\text{frag}}} \Delta E_{PQ} \right] \quad (1)$$

237 Letting *ij* represent occupied orbitals and *a,b* represent
 238 virtual orbitals, *E_P* and ΔE_{PQ} in the above equation can be
 239 written in terms of electron repulsion integrals (*g*) and CC
 240 amplitudes (*t*) according to eqs 2 and 3 below.

$$241 E_P = \sum_{ij \in P}^{N_{\text{frag}}} (t_{ij}^{ab} + t_i^a t_j^b) (2g_{iajb} - g_{ibja}) \quad (2)$$

$$242 \Delta E_{PQ} = \sum_{i \in P, j \in Q}^{ab} (t_{ij}^{ab} + t_i^a t_j^b) (2g_{iajb} - g_{ibja}) \\ + \sum_{i \in Q, j \in P}^{ab} (t_{ij}^{ab} + t_i^a t_j^b) (2g_{iajb} - g_{ibja}) \quad (3)$$

The above formulations of the correlation energy introduce
 243 no approximations. However, in order to achieve linear scaling,
 244 we truncate the virtual orbital space for each fragment and
 245 introduce a distance cutoff, *R_{cut}*, to eliminate pairs of fragments
 246 separated by large distances where correlation would be
 247 negligible. With these approximations, eqs 2 and 3 become
 248

$$E_P = \sum_{ij \in P}^{ab} (t_{ij}^{ab} + t_i^a t_j^b) (2g_{iajb} - g_{ibja}) \\ ab \in [\bar{P}] \quad (4) \quad 249$$

$$\Delta E_{PQ} = \sum_{i \in P, j \in Q}^{ab} (t_{ij}^{ab} + t_i^a t_j^b) (2g_{iajb} - g_{ibja}) \\ ab \in [\bar{P}] \cup [\bar{Q}] \\ R_{PQ} < R_{\text{cut}} \\ + \sum_{i \in Q, j \in P}^{ab} (t_{ij}^{ab} + t_i^a t_j^b) (2g_{iajb} - g_{ibja}) \\ ab \in [\bar{P}] \cup [\bar{Q}] \\ R_{PQ} < R_{\text{cut}} \quad (5) \quad 250$$

251 where $[\bar{P}]$ and $[\bar{Q}]$ represent the truncated virtual orbital
 252 spaces (in the LMO basis) of fragments *P* and *Q* respectively,
 253 and *R_{PQ}* is the distance between the atomic centers of the two
 254 fragments. $[\bar{P}]$ and $[\bar{Q}]$ are optimized self-consistently in order
 255 to ensure error control to a user-defined tolerance, denoted the
 256 fragment optimization threshold or FOT (default value: 1×257
 $10^{-4} E_{\text{h}}$).

258 Reference 58 provides a detailed description of the stages of
 259 fragment optimization which we briefly discuss here. The
 260 optimization of an atomic fragment's virtual orbital space $[\bar{P}]$ is
 261 accomplished in two steps: fragment expansion and fragment
 262 reduction. During fragment expansion, sets of virtual orbitals
 263 localized to neighboring atomic sites are sequentially added to
 264 $[\bar{P}]$ according to a priority list based on the distance between
 265 the atomic center of fragment *P* and the center of charge of a
 266 given orbital. As groups of orbitals are added, the atomic
 267 fragment energy is evaluated and expansion continues until the
 268 change in energy from the previous expansion step is less than
 269 the FOT. Fragment reduction is then performed to determine
 270 if any orbitals can be removed from $[\bar{P}]$ without introducing
 271 errors larger than the FOT. This step is necessary in order to
 272 improve the computational efficiency of the pair fragment
 273 calculations, which will be discussed next. Both fragment
 274 expansion and reduction are iterative processes that require the
 275 evaluation of the atomic fragment correlation energy after each
 276 iteration and therefore fragment optimization is typically
 277 performed using a less expensive method such as MP2 or RI-
 278 MP2. In the final step of fragment reduction, the MP2 or RI-
 279 MP2 optimized atomic fragment energies are obtained. If this
 280 corresponds to the DEC target level of theory, no further
 281 calculations of the atomic fragments are required. Otherwise, if
 282 a higher level of theory is requested, the atomic fragment
 283 correlation energies are reevaluated according to eq 2 using the
 284 optimized $[\bar{P}]$ at the target level of theory.

285 The truncated virtual orbital spaces for pair fragments are
 286 constructed from the union of the constituent optimized
 287 atomic fragments, $[\bar{P}] \cup [\bar{Q}]$. This highlights the need for the
 288 reduction step in the fragment optimization procedure as any
 289 reduction of $[\bar{P}]$ and $[\bar{Q}]$ similarly reduces the virtual orbital
 290 space for every pair involving those atoms. As there are many
 291 more pair fragment calculations than atomic fragment
 292 calculations to perform and pair fragments inherently involve
 293 much larger orbital spaces, reduction in the cost of evaluating
 294

294 the pair correlation energies can provide significant improvements 295 in the overall computational efficiency as well as 296 memory requirements for individual fragments. The cost of the 297 pair calculations is also reduced through a screening method in 298 which pair energies are estimated using partially optimized 299 atomic virtual orbital spaces. The lowest energy pairs whose 300 total estimated energy does not exceed the FOT are omitted 301 from the optimized pair fragment calculations. The pair energy 302 estimations are typically performed at the same level of theory 303 as fragment expansion and reduction.

304 Following the optimization steps and pair energy estimation, 305 atomic and pair fragment energies are evaluated for the 306 optimized fragments at the target level of theory as necessary. 307 These individual fragment energies are added according to eq 308 1 to provide the total correlation energy correction to the 309 canonical HF energy. Errors bound by the FOT per atomic 310 fragment are introduced in both the fragment optimization 311 step and pair energy estimate calculations, and therefore, the 312 overall error in the DEC scheme is approximated by

$$313 \quad \delta E_{\text{DEC}} = 2 \times \text{FOT} \times N_{\text{frag}} \quad (6)$$

314 Coarse, medium, and fine-grained parallelism is utilized in 315 order to achieve computational efficiency in the DEC scheme. 316 Fragments are evaluated independently by groups of 317 processors which distribute the calculation over multiple 318 nodes using MPI. Within the nodes, work is further divided 319 using OpenMP or OpenACC.

320 **ML-DEC Scheme.** In this section, we discuss the 321 implementation of the ML-DEC scheme, which is included 322 in the LSDalton2018.0 release. The ML-DEC scheme takes 323 advantage of the independent nature of the fragment 324 calculations which allows for the evaluation of fragment 325 correlation energies at different levels of theory. The layers are 326 defined by the user by assigning atoms in the MOLECULE.- 327 INP file to level = high or level = low. This is visualized in 328 Figure 1, in which a large molecular system has been

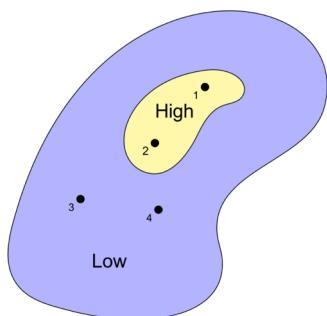


Figure 1. Visualization of high (yellow) and low (blue) level layers in the ML-DEC scheme. Black dots labeled 1–4 denote centers of representative atomic fragments.

329 subdivided into two layers: a high level layer (yellow) and a 330 low level layer (blue). Correlation energies of the atomic 331 fragments are evaluated based on the level into which their 332 atomic center has been assigned. It is important to note that 333 the user-defined layers are not required to be contiguous as 334 shown in Figure 1, making the ML-DEC scheme suited to treat 335 more complex problems such as multisite catalysis. Although 336 chemical intuition on the part of the user is required to assign 337 atoms to the high- and low-level layers, this method maintains

338 the user-friendly, automatic fragment optimization governed 338 by the FOT as in the standard DEC scheme. 339

340 The ML-DEC scheme begins with the same atomic 340 fragment optimization and pair energy estimation steps as in 341 the standard DEC scheme, described above. All atomic 342 fragments are optimized at the same level of theory, regardless 343 of their assignment to the high or low level layers. After 344 fragment optimization, target CC models are assigned to each 345 atomic fragment according to the user specifications. Pair 346 fragments are evaluated at the higher of the levels of theory 347 requested for the two constituent atomic fragments. For 348 example, in Figure 1 above, the pair between atoms 1 and 2 349 would be evaluated at the high level of theory, pair between 350 atoms 2 and 3 would be evaluated at the high level of theory, 351 and pair between atoms 3 and 4 would be evaluated at the low 352 level of theory. As in the standard DEC scheme, computational 353 efficiency is improved by omitting pair fragment calculations 354 whose correlation energies do not contribute significantly to 355 the total correlation energy based on the pair energy estimates. 356 However, pair fragments from the high level layer which 357 should be skipped based on their energy estimates are instead 358 reduced to the low level layer. Finally, all atomic and pair 359 fragments whose energies have not been evaluated at their 360 target levels of theory during the optimization process are 361 evaluated, and the total correlation energy is obtained 362 according to eq 1 as before. 363

364 Example LSDALTON.INP and MOLECULE.INP input 364 files for a number of ML-DEC calculations are provided in the 365 Supporting Information along with descriptions of parameters 366 which users may adjust in order to improve computational 367 efficiency of the ML-DEC calculations. Current CC models 368 integrated within the ML-DEC scheme are MP2, RI-MP2 369 [with and without the use of Laplace transformations (LT)], 370 CCSD, and CCSD(T), though ML-DEC calculations utilizing 371 CCSD(T) have not yet been evaluated for efficiency and 372 accuracy against standard DEC-CCSD(T) calculations. In 373 addition, the low-level layer may be treated at the HF level of 374 theory, ignoring correlation effects. It is recommended that 375 only one level of theory separate the high and low levels. 376

377 Simulations treating the low-level layer with HF require a 377 few special considerations in order to improve computational 378 efficiency. Although correlation contributions from atoms in 379 the low level should not be calculated, the atoms may still 380 contribute to pairs with high-level atoms, requiring optimiza- 381 tion of these atomic fragments. In order to reduce the number 382 of fragment optimizations that must be performed, we utilize 383 the existing pair threshold cutoff R_{cut} to determine which 384 atomic fragments can be safely omitted from the optimization 385 steps. All low-level atoms within R_{cut} of any high level atom will 386 be optimized while the remaining low-level atomic fragments 387 are skipped. For most applications, R_{cut} , corresponding to the 388 input keyword .PAIRTHR or .PAIRTHRANGSTROM, of 389 about 10.0 Å is sufficient as correlation effects fall off 390 significantly beyond this distance. Because the atomic fragment 391 correlation energies at the level of theory used for fragment 392 reduction are obtained automatically from the optimization 393 procedure, contributions from optimized low-level atomic 394 fragments in HF/RI-MP2 ML-DEC calculations are accounted 395 for at no additional computational cost. 396

397 The ML-DEC scheme is expected to be particularly 397 beneficial in the calculation of interactions localized to small, 398 isolated areas within the system, as errors from correlation 399 effects in the low-level layers far from regions of interest are 400

401 expected to cancel out when monomer energies are subtracted
 402 from the interacting system. Accurate results do of course
 403 depend on effective partitioning of the system into high- and
 404 low-level layers. In the following section, we will discuss
 405 convergence of calculated interaction energies with the size of
 406 the high level layer in the palmitic acid dimer test system, as
 407 well as resulting effects on the computational efficiency.

408 ■ RESULTS AND DISCUSSION

409 **Test Case 1: Palmitic Acid Dimer.** As a first test of the
 410 ML-DEC scheme we consider the palmitic acid dimer. The
 411 interaction between the two monomers in this system is
 412 localized to the carboxylic acid groups through the formation
 413 of hydrogen bonds. It is therefore a reasonable approach to
 414 assign the $-\text{COOH}$ group of each monomer to the high level
 415 layer, and this was taken as the initial configuration in both
 416 HF/RI-MP2 and RI-MP2/CCSD ML-DEC calculations as
 417 shown in Figure 2. This will be referred to as the minimal high-



418 **Figure 2.** Palmitic acid dimer optimized using DFT with the BP86
 419 functional and the cc-pVTZ basis set. Opaque atoms are those
 420 included in the high-level layer: red = oxygen, white = hydrogen, tan =
 421 carbon included in minimal high level layer, light blue = carbon
 422 included in first extension of the high level layer (ext₁), dark blue =
 423 carbon included in second extension of the high level layer (ext₂).

418 level layer. In order to demonstrate convergence to the
 419 standard DEC scheme reference energy, we also consider two
 420 extended high level layers which include one or two additional
 421 carbon atoms per monomer (shown in blue in Figure 2). Both
 422 the monomer and dimer systems used for these calculations
 423 were optimized at the DFT level of theory using the BP86
 424 functional^{59,60} and the cc-pVTZ basis set as implemented in
 425 NWChem.⁶¹

426 **HF/RI-MP2 Calculations.** DEC-RI-MP2 and HF/RI-MP2
 427 ML-DEC calculations on the palmitic acid dimer utilize the cc-
 428 pVTZ basis set for all atoms and $\text{FOT} = 1 \times 10^{-4} E_{\text{h}}$. The
 429 binding energy, ΔE is calculated from independent energy
 430 calculations of the optimized monomer and the optimized
 431 dimer system according to eq 7

$$432 \Delta E = E_{\text{D}}^{\text{D}}(\text{D}) - 2 \times E_{\text{M}}^{\text{M}}(\text{M}) \quad (7)$$

433 where D and M denote the dimer and monomer systems,
 434 respectively, and $E_{\text{X}}^{\text{Y}}(\text{Z})$ is the energy of system Z in the
 435 optimized geometry of system X in the basis of system Y. A
 436 consistent assignment of high level atoms is maintained in the
 437 monomer and dimer calculations, and a cutoff of $R_{\text{cut}} = 10.0 \text{ \AA}$
 438 is used to reduce the number of fragment optimization
 439 calculations. The number of RI-MP2 fragment optimization
 440 calculations as well as final pair energy calculations are
 441 summarized in Table 1 for the minimal high level layer
 442 ($-\text{COOH}$ on each monomer) and each extended high level
 443 layer. We note that BSSE corrections have not been applied to
 444 the interaction energies. However, their calculation via the
 445 counterpoise correction (CPC) is straightforward and does not
 446 differ from corrections calculated using the standard DEC
 447 scheme. The CPC requires an additional HF/localization
 448 calculation of each monomer in the presence of the ghost

Table 1. Fragment Optimization and Pair Energy Calculations Performed during Palmitic Acid Monomer and Dimer DEC-RI-MP2 and HF/RI-MP2 ML-DEC Calculations^a

model	fragment optimizations	pair energy calculations
Monomer, 988 Basis Functions		
ML-DEC	9	6
ML-DEC (ext ₁)	10	9
ML-DEC (ext ₂)	11	12
DEC-RI-MP2	17	51
Dimer, 1976 Basis Functions		
ML-DEC	20	32
ML-DEC (ext ₁)	22	42
ML-DEC (ext ₂)	24	50
DEC-RI-MP2	36	130

^aML-DEC (ext₁) refers to ML-DEC calculation utilizing a high level layer including one additional carbon atom per monomer (i.e., $-\text{CH}_2\text{COOH}$), and ML-DEC (ext₂) refers to a high level layer with two additional carbon atoms per monomer (i.e., $-\text{CH}_2\text{CH}_2\text{COOH}$).

atoms of the other monomer and, for the ML-DEC scheme, consistent assignment of atoms to high- and low-level layers.

Performance of these simulations is evaluated based on TTS of the monomer and dimer calculations (TTS_m and TTS_d, respectively) for the ML-DEC scheme compared to the standard DEC scheme at the RI-MP2 level of theory. The same number of processors were used in the DEC and ML-DEC calculations, and care was taken to ensure that the number of MPI groups (an MPI group is a set of MPI processes assigned to one DEC fragment) did not exceed the number of fragment calculations in either the optimization or final energy calculation stages of the simulation so that processors were not sitting idle. Table 2 reports the energies and TTS obtained for both monomer and dimer DEC and ML-DEC calculations.

Table 2. Palmitic Acid Dimer Binding Energies from HF/RI-MP2 ML-DEC and DEC-RI-MP2 Simulations^a

model	TTS _m (min)	TTS _d (min)	ΔE (kJ/mol)
ML-DEC	6.71	5.51	-87.7
ML-DEC (ext ₁)	7.98	6.21	-69.6
ML-DEC (ext ₂)	9.27	6.84	-70.6
DEC-RI-MP2	17.51	10.19	-70.4

^aTTS is reported for both the monomer (TTS_m) and dimer (TTS_d). DEC and ML-DEC monomer calculations were run on 7 processors, while dimer calculations used 33. Energies are reported in kJ/mol.

From the results in Table 2, we can conclude that the ML-DEC scheme results in a significant decrease in TTS_m and TTS_d, reducing the full TTS by about half for the minimal high level layer. It is important to note that for RI-MP2 calculations, the majority of the simulation time is typically spent in fragment optimization, which requires multiple RI-MP2 level calculations on each atomic fragment through the expansion and reduction procedures. Therefore, reduction in the number of fragment optimizations (Table 1) has a more significant impact on the computational time than reductions in the number of pair fragments. For this reason, the ratio between TTS_m for the DEC and ML-DEC calculations lies closer to the ratio between their respective number of fragment optimizations rather than pair energy calculations.

478 In addition to this substantial reduction in computational
 479 time, we find that after the first extension of the high level layer
 480 [ML-DEC (ext₁)], HF/RI-MP2 ML-DEC calculations are able
 481 to reproduce the reference DEC-RI-MP2 interaction energies
 482 within 2%. Further extending the high level layer [ML-DEC
 483 (ext₂)] increases the TTS slightly while bringing the binding
 484 energy into nearly exact agreement with the DEC-RI-MP2
 485 reference (within 0.3%). This rapid convergence to the DEC-
 486 RI-MP2 reference value demonstrates that as we move away
 487 from the center of these local interactions, correlation effects
 488 from more remote fragments make a negligible contribution to
 489 the interaction.

490 **RI-MP2/CCSD Calculations.** It is expected that more rapid
 491 convergence to the reference DEC binding energy can be
 492 obtained when a low-level model which includes correlation
 493 effects is utilized. In order to verify this, the binding energy of
 494 the palmitic acid dimer has been evaluated using both RI-
 495 MP2/CCSD ML-DEC and standard DEC-CCSD calculations
 496 with the cc-pVDZ basis set and FOT = $1 \times 10^{-4} E_h$. For these
 497 simulations, the RI-MP2 level fragment optimizations no
 498 longer dominate the computational time and therefore
 499 reduction in the TTS can be better predicted by the reduction
 500 in CCSD-level pair energy calculations as shown in Table 3.
 501 Only the minimal high-level layer investigated above is used for
 502 the RI-MP2/CCSD ML-DEC calculations.

Table 3. Simulation Size and Performance Data for RI-MP2/CCSD ML-DEC and DEC-CCSD Calculations on the Palmitic Acid Dimer^a

model	CCSD pair fragment calculations		TTS		
	monomer	dimer	monomer (h)	dimer (h)	ΔE (kJ/mol)
ML-DEC	14	38	0.24	1.85	-70.2
DEC-CCSD	76	164	3.21	8.40	-69.4

^aAll simulations were run on 129 processors with 8 threads per process.

503 From the TTS data provided in Table 3, it is apparent that
 504 the computational savings are much more significant for RI-
 505 MP2/CCSD than HF/RI-MP2 because the bottleneck is no
 506 longer fragment optimization. In addition, even the smallest
 507 high level layer results in agreement within 2% of the reference
 508 DEC-CCSD energy. This is to be expected because the
 509 correlation energy of low level fragments is still being
 510 accounted for, though at the RI-MP2 level rather than
 511 CCSD, and the energy difference between CCSD and RI-
 512 MP2 is less than that between RI-MP2 and HF. Comparing to
 513 the DEC-RI-MP2 binding energy calculated with the cc-pVDZ
 514 basis set, -74.8 kJ/mol, we see that the RI-MP2/CCSD ML-
 515 DEC energy captures approximately 85% of the energy
 516 difference between the DEC-CCSD and DEC-RI-MP2 binding
 517 energies while reducing the TTS for the monomer and dimer
 518 calculations by 92.5 and 78.0%, respectively, compared to the
 519 full DEC-CCSD calculations.

520 **Test Case 2: CO₂ Adsorption in Mg-MOF-74.** The
 521 results discussed in the previous section demonstrate the
 522 computational efficiency of the ML-DEC scheme and its
 523 accuracy with respect to the standard DEC scheme, given a
 524 sufficiently large high level layer. We now apply the ML-DEC

scheme to a more industrially relevant real-world problem, CO₂ adsorption in Mg-MOF-74.

The MOF-74 isostructural family is made up of M²⁺ metal ions (M = Mg, Co, Ni, Zn) coordinated by dobdc = 2,5-dioxido-1,4-benzene-dicarboxylate ligands with an overall formula of M₂ (dobdc). Each metal center is coordinated by five dobdc oxygen atoms in an approximately square pyramidal configuration. The resulting open metal site has been shown to play a crucial role in strong binding affinities for a number of small gas molecules. In particular, Mg-MOF-74 has been shown experimentally to adsorb CO₂ with an uptake at 1.02 bar of 6.18 mmol/g.⁶² Adsorption energies have previously been calculated in periodic models of Mg-MOF-74 via plane-wave DFT⁶³ as well as in small cluster models via molecular DFT and MP2;⁶⁴ however, the calculated adsorption energies vary by over 10 kJ/mol depending on the method and system size. DFT results are particularly susceptible to influence from the choice of functionals, dispersion corrections, and plane-wave cutoff energies, among other adjustable parameters, while cluster models inevitably suffer from edge effects. By applying the ML-DEC scheme to the calculation of the CO₂ adsorption energy in a large, finite Mg-MOF-74 cluster, we can explicitly treat correlation near the adsorption site and minimize edge effects, thereby addressing many of the sources of variability listed above.

For these calculations, we use a 793-atom Mg-MOF-74 channel with a single adsorbed CO₂ molecule as shown in Figure 3. Details of the geometry optimization are provided in

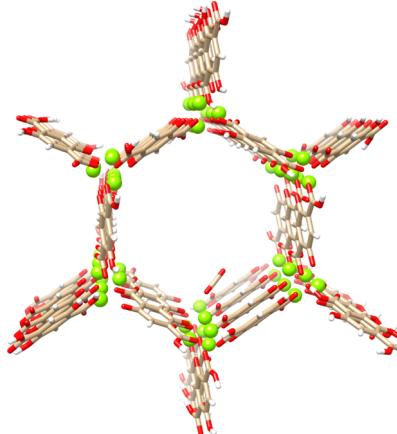


Figure 3. Mg-MOF-74 channel with a single adsorbed CO₂ molecule. Green = Mg, red = O, tan = C, white = H.

the Supporting Information. A minimal high level layer is first considered which contains only the Mg onto which the CO₂ is adsorbed, five coordinating oxygen atoms, and CO₂ molecule (Figure 4a). The high-level layer is expanded until the calculated interaction energy converges to the standard DEC-RI-MP2 energy. However, for this system, expansion of the high-level layer is not as straightforward as in the palmitic acid dimer test case as there are a number of different ways in which the high level layer can be expanded. As a first approach, we select all MOF atoms within 5 Å of the adsorbed CO₂ molecule to include in the expanded high level layer. Care is taken not to divide resonant structures across layers, and therefore, if any part of an aromatic ring falls within the 5 Å region, the full aromatic ring is included in the high-level layer. For the second expansion, terminal phenyl and carboxyl groups

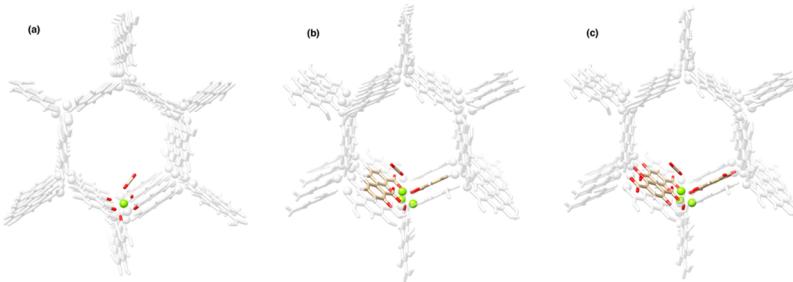


Figure 4. Mg-MOF-74 + CO₂ minimal high level layer (a), along with the first (b) and second (c) extensions to the high level layer for ML-DEC calculations. High-level atoms are shown in color (green = Mg, red = O, tan = C) and all low-level atoms are shown in white.

568 on the aromatic rings included in the first expansion are added
569 to the high-level layer. The two expanded high-level layers are
570 shown in Figure 4b,c.

571 Calculations on the MOF channel and its adsorbate utilize
572 the cc-pVQZ basis set on all atoms, a pair threshold cutoff of
573 10.0 Å and FOT = 1 × 10⁻⁵ E_h. The same localized HF MOs
574 are used in both the standard DEC and ML-DEC calculations.
575 As a comparison of the efficiency of these calculations, the
576 number of fragment optimization and pair energy calculations
577 required for the various HF/RI-MP2 ML-DEC models as well
578 as the DEC-RI-MP2 model are summarized in Table 4. From

Table 4. Atomic Fragment Optimizations and Optimized Pair Energy Calculations Performed during HF/RI-MP2 ML-DEC and Standard DEC-RI-MP2 Calculations on the Mg-MOF-74 + CO₂ System^a

model	fragment optimizations	pair energy calculations
ML-DEC	202	566
ML-DEC (ext ₁)	282	2005
ML-DEC (ext ₂)	323	2728
DEC-RI-MP2	568	14 720

^aThe model ML-DEC refers to the ML-DEC calculations using the minimal high level layer as shown in Figure 4a while ML-DEC (ext₁) and ML-DEC (ext₂) refer to the first and second extensions of the high level layer as shown in Figure 4b,c, respectively.

579 this information, we see that even for the largest high level
580 layer, ML-DEC (ext₂), the number of atomic fragment
581 optimization steps is reduced by more than 40% while the
582 pair energy calculations are reduced by over 80%. This
583 indicates the potential for even greater computational savings
584 in RI-MP2/CCSD and CCSD/CCSD(T) ML-DEC calcu-
585 lations on these systems.

586 The interaction energies (ΔE) obtained from each of the
587 models in Table 4 are calculated according to eq 8

$$\Delta E = E_{\text{MOF+CO}_2}^{\text{MOF}}(\text{MOF} + \text{CO}_2) - E_{\text{MOF+CO}_2}^{\text{MOF}}(\text{MOF}) - E_{\text{MOF+CO}_2}^{\text{CO}_2}(\text{CO}_2) \quad (8)$$

588 where as before $E_X^Y(Z)$ is the energy of system Z in the
589 optimized geometry of system X on the basis of system Y. We
590 note that this equation differs from eq 7 in that the energies of
591 the monomers are evaluated in the geometry of the complex,
592 thus providing the interaction energy rather than the binding
593 energy. For this study, no correction for BSSE was made. The
594 resulting interaction energies are reported in Table 5. It should
595 be noted that the results presented here are primarily intended
596 to demonstrate that ML-DEC interaction energies rapidly
597 converge to standard DEC values. Reproducing experimental
598 converge to standard DEC values. Reproducing experimental

Table 5. Interaction Energies (ΔE) from DEC-RI-MP2 and HF/RI-MP2 ML-DEC Calculations on the Mg-MOF-74 + CO₂ System^a

model	ΔE (kJ/mol)	lerrorl (kJ/mol)
DEC-RI-MP2	-68.5	
ML-DEC	-4147.0	4078.5
ML-DEC (ext ₁)	-647.7	579.2
ML-DEC (ext ₂)	-72.4	3.9

^aErrors in the ML-DEC values with respect to the standard DEC-RI-MP2 value are reported as well.

adsorption data would require either a larger basis set or BSSE 599 corrections, along with a more rigorous initial geometry 600 optimization of the system. Nevertheless, our results are in 601 qualitative agreement with previous theoretical studies at 602 DFT^{63,64} and MP2⁶⁴ levels of theory, which range from -45 to 603 -54 kJ/mol. 604

The results presented in Table 5 demonstrate that the 605 minimal high level layer consisting of the central Mg atom, its 606 five coordinating oxygen atoms, and the CO₂ molecule does 607 not sufficiently capture the important correlation effects in the 608 CO₂-MOF interaction. However, as the high level layer 609 expands, the ML-DEC interaction energy converges rapidly to 610 the DEC-RI-MP2 energy. Agreement within 6% between the 611 HF/RI-MP2 ML-DEC and DEC-RI-MP2 interaction energies 612 is obtained when the terminal carboxyl and phenyl functional 613 groups on these ligands are also included in the high level 614 layer. This model, ML-DEC (ext₂), is therefore considered to 615 be the optimal multilayer configuration for the MOF + CO₂ 616 system. Using this multilayer configuration, we are able to 617 calculate the RI-MP2 interaction energy of this approximately 618 10 000 basis function system with significantly less computa- 619 tional effort than required for a standard DEC-RI-MP2 620 calculation without loss of accuracy. The ML-DEC runs were 621 performed using smaller node count and thus direct 622 comparison of TTS is not possible. Nevertheless, based on 623 the number of fragment evaluations presented in Table 4, we 624 expect a factor of 3 performance gain from using the ML-DEC 625 approach. 626

CONCLUSIONS

Through the test cases presented above, we have demonstrated 628 that interaction energies calculated using the ML-DEC scheme 629 agree with the results obtained from standard DEC 630 calculations at the higher level of theory while reducing the 631 overall TTS. By exploiting the rapid drop-off of correlation 632 energy with distance, important contributions to a local 633 interaction can be captured at a high level of theory while 634 significantly decreasing the number of atomic and pair 635

636 fragment energies that must be evaluated at that level. The
 637 improved computational efficiency of the ML-DEC scheme
 638 allows for the treatment of much larger systems than those
 639 reasonably accessible with the standard DEC scheme, which
 640 has important implications for large systems in which the
 641 process or interaction of interest can be localized to isolated
 642 regions within the overall system, for example, defect sites,
 643 adsorption sites, or catalytically active sites.

644 We have shown that the accuracy of the ML-DEC scheme
 645 relative to standard DEC calculations depends on proper
 646 assignment of atoms to the high level layer which requires
 647 some chemical intuition; however, a conservative high level
 648 layer may be obtained through optimization at the less
 649 expensive HF/RI-MP2 ML-DEC level for use in subsequent
 650 RI-MP2/CCSD or, potentially, CCSD/CCSD(T) calculations.
 651 In addition, these optimized high-level layers can provide
 652 qualitative information on the extent of important correlation
 653 effects in a given system.

654 Overall, the ML-DEC scheme provides a powerful tool for
 655 the evaluation of interactions within large systems, such as
 656 biologically or industrially relevant materials, at CC levels of
 657 theory.

658 ■ ASSOCIATED CONTENT

659 ■ Supporting Information

660 The Supporting Information is available free of charge on the
 661 [ACS Publications website](#) at DOI: [10.1021/acs.jpca.9b08077](https://doi.org/10.1021/acs.jpca.9b08077).

662 Example LSDALTON.INP and MOLECULE.INP files
 663 as well as details of the optimization of the Mg-MOF-74
 664 channel ([PDF](#))

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