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A multigrid approach to solve hypersonic flow problems

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Outline

1 **Modeling the problem**

2 Associated linear system

3 Preconditioning strategy
- Blunt wedge problem
- Hifire problem

4 Conclusion

Thermal Navier-Stokes flow

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}_i(\mathbf{U})}{\partial x_i} - \frac{\partial \mathbf{G}_i(\mathbf{U})}{\partial x_i} = \mathbf{0} \tag{1}$$

with

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho v_j \\ \rho E \end{pmatrix}, \ \mathbf{F}_i(\mathbf{U}) = \begin{pmatrix} \rho v_i \\ \rho v_i v_j + P\delta_{ij} \\ \rho E v_i + P v_i \end{pmatrix} \text{ and } \mathbf{G}_i(\mathbf{U}) = \begin{pmatrix} 0 \\ \tau_{ij} \\ \tau_{ij} v_j - q_i \end{pmatrix} \tag{2}$$

where ρ is the fluid density, v is the fluid velocity and E the fluid energy per unit of mass which is expressed as $E = \frac{1}{2} v_i v_i + e$ the sum of the kinetic and internal energy e. P is the fluid pressure, τ_{ij} is the viscous stress tensor. $q_i = -\kappa \frac{\partial T}{\partial x_i}$ is the heat flux, T the temperature and κ the thermal conductivity of the gas.

Thermal Navier-Stokes flow

For a Newtonian fluid (linear stress/strain) the stress can be expressed as

$$\tau_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) + \lambda \delta_{ij} \left(\frac{\partial v_k}{\partial x_k} \right) \tag{3}$$

with μ the viscosity and λ the bulk viscosity (often $\lambda = -\frac{2}{3}\mu$ for Newtonian fluid). Finally the pressure is given by the equation of state (EOS) of the fluid. For a perfect gas we have

$$P = \rho R T \tag{4}$$

with R perfect gas constant. More details on the mechanical formulation used to represent the fluid behavior can be found in [5] and [6].

Time and spacial discretization

1. The time integration is performed using an implicit Euler scheme
2. Non-linear problem resulting from the time integration are inexactly solved with 1 Newton iteration.
3. The system is discretized using a cell-centered finite volume scheme [5], stabilization is added to the operator with a SUPG formulation for the fluid pressure [1].

We are interested in a steady state problems so a pseudo-time integration scheme is used to ramp-up the CFL number associated with the time integration problem.

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Linear problem structure

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Denoting R the nonlinear residual, x the incremental solution update and A the Jacobian associated with (1), we get the linear system:

$$Ax = -R \tag{5}$$

$$A = \begin{bmatrix} A_{\rho\rho} & A_{\rho\nu} & 0 \\ A_{\nu\rho} & A_{\nu\nu} & A_{\nu E} \\ A_{E\rho} & A_{E\nu} & A_{EE} \end{bmatrix} \tag{6}$$

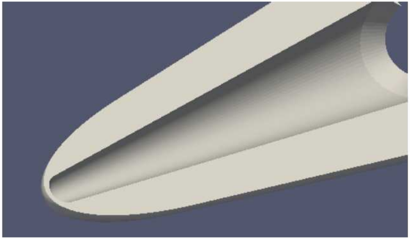
Physical coupling between equations leads to non-symmetry.

Mesh structure

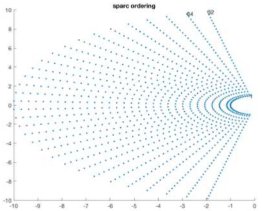
Hyper-sonic objects generate strong shock-waves leading to:

1. strong flow directionality
2. low dissipation
3. hard to resolve transition

To help with these we use structured and aligned meshes.

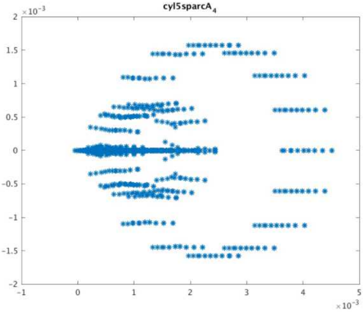


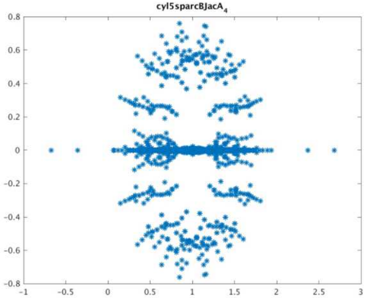
Eigenvalues structure



- Good clustering
- Scary values around the origin
- Block diagonal scaling leads to negative e.v.

Probably hard to precondition!





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Available options

1. Block tridiagonal inverse: native in application
2. Smoothed Aggregation (SA) AMG: usual work horse
3. Structured Aggregation AMG: preserves mesh structure
4. Petrov-Galerkin SA: handle non-symmetry

Most likely need a combination of some of the above... and maybe some tricks

Blunt wedge problem

Structured mesh: 72^3, 144^3 or 288^3 cells, 5 degrees of freedom per cell, supersonic input flow: Mach 3.

First attempt: use unstructured vs. structured aggregation, 1 sweep of ILU(0) as pre-smoother, 4 levels, coarsening rate: 3 per direction.

Mesh size	72^3	144^3	288^3
Unstructured	46	87	N/C
Structured	36	88	256

Table: Number of linear iterations (tol=1e-6)

Observations:

1. linear interpolation with structured aggregation diverges
2. three and four level methods give same convergence
3. no scaling for either structured/unstructured methods

Block tridiagonal smoother: serial 144^3

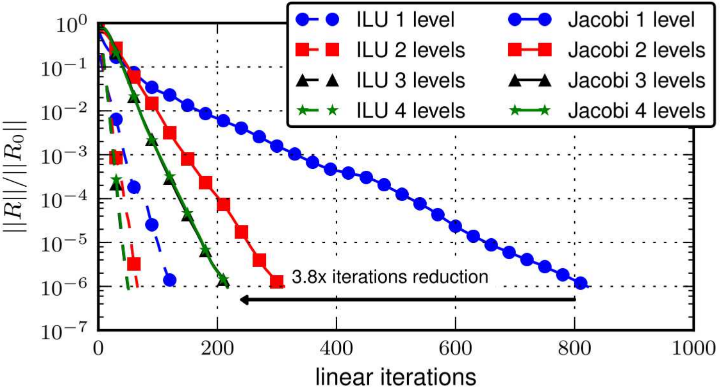

Figure: Unstable coarse operators forbid using to many levels and a direct solver on coarsest grid.

Block tridiagonal smoother: parallel 144^3

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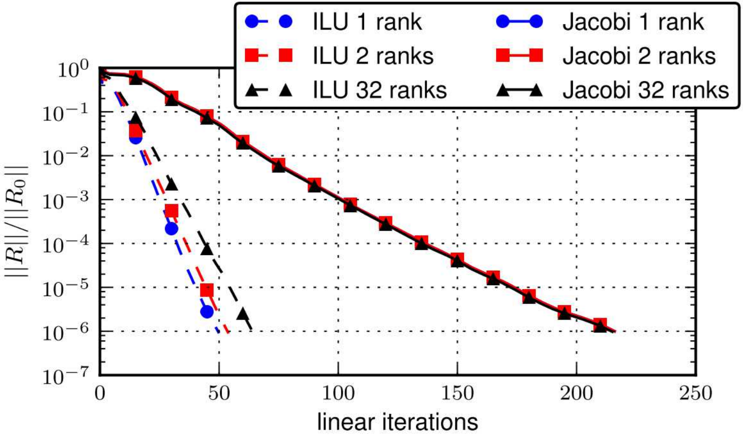

Figure: Due to line preserving partitioning the performance of the algorithm using the line block Jacobi smoother is independent of the number of processors used.

Problem stabilization

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SUPG stabilization depends on mesh size parameter:

- loss of stabilization on coarse grid
- mass/stiffness ratio is not physical

Potential fix:

1. damp the coarse grid correction
   - easy to implement and cheap
   - no mathematical background to support efficacy
2. fix stabilization on coarse grid
   - project mass and stiffness components separately
   - recombine on coarse grid using new stabilization parameter
   - mode complex and expensive
   - guarantees stability

Coarse grid correction damping

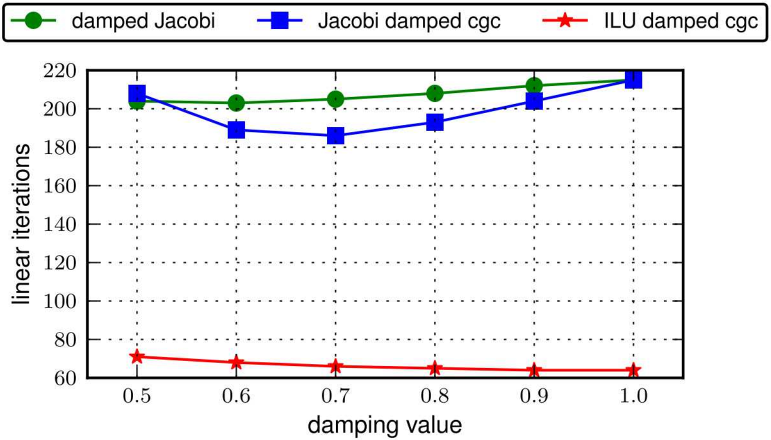

Figure: Interestingly damping the coarse grid correction is more effective than damping the preconditioner. This is indicative of instability in the coarse operators: $(\mathrm{I} - \alpha \mathrm{D}^{-1}\mathrm{A})$ vs. $\alpha(\mathrm{I} - \mathrm{D}^{-1}\mathrm{A})$.

Mass stabilization

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$$A_1 = RA_0P + (\alpha - 1)RM_0P \tag{7}$$

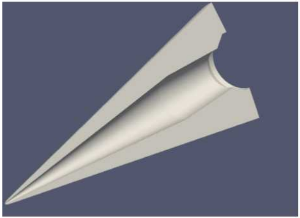
	Unstructured			Structured		
α	72^3	144^3	288^3	72^3	144^3	288^3
1	46	87	N/C	36	88	256
2	45	86	N/C	35	82	205
4	45	87	N/C	34	75	97
6	46	89	N/C	35	74	86
8	46	92	N/C	36	77	83
10	48	95	N/C	37	81	85

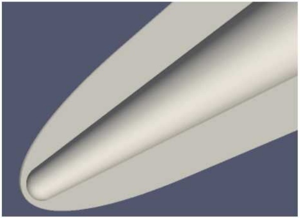
Observations:

- more helpful with structured grid
- $\alpha = 6$ near optimal for this problem
- fourth level improves convergence, coarse grid is more stable

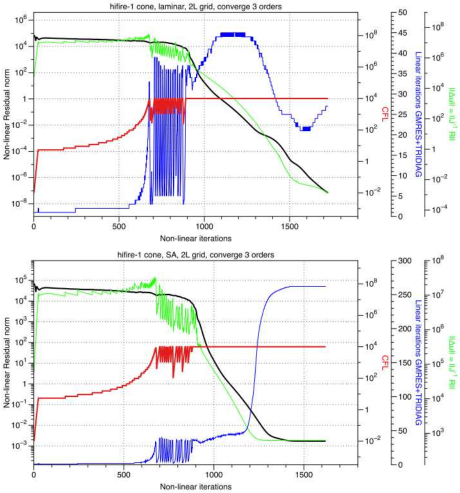
Hifire problem

Structured mesh: 20M cells, 5 degrees of freedom per cell, hyper-sonic input flow: Mach $6 \sim 8$.





Preliminary results



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Concluding remarks

We have achieved:

- implementation of structured grid and line detection algorithm
- stabilization of coarse grid operators using mass projection
- assessment of ILU/block tridiagonal smoother
- convergence on high resolution blunt-wedge problem

Concluding remarks

We are working on:

- improving non-linear convergence on Hifire
- Petrov-Galerkin experimentation on Blunt-Wedge and Hifire
- more complex physical problems including reaction gas and fluid-structure interaction

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