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Recent developments in a 10-Node Composite Tetrahderal Finite Element for Solid Mechanics

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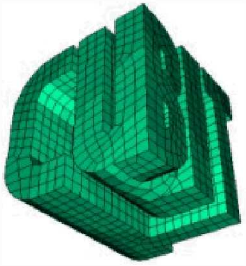
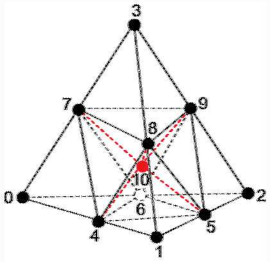
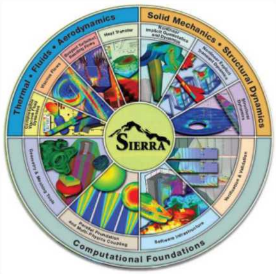
Recent developments

- We make the following assertions
  - Status quo hexahedral workflow is unacceptable
  - Meshing is no fun
  - Hypothesize a new tetrahedral workflow for solid mechanics
- We recently devoted 18 months to prototyping an improved workflow
- Developments include
  - Hardening of Cubit with interface to Distene's MeshGems
  - Extension (and robustification) of Sierra/SM for tet10 technology
  - *New formulation of the composite tet with kinked edges*
  - Analyst driven workflow – modification/discretization/visualization
- Customers achieving 100X reduction in design to analysis
- Continuous improvement (quality/remeshing/error/anisotropy)



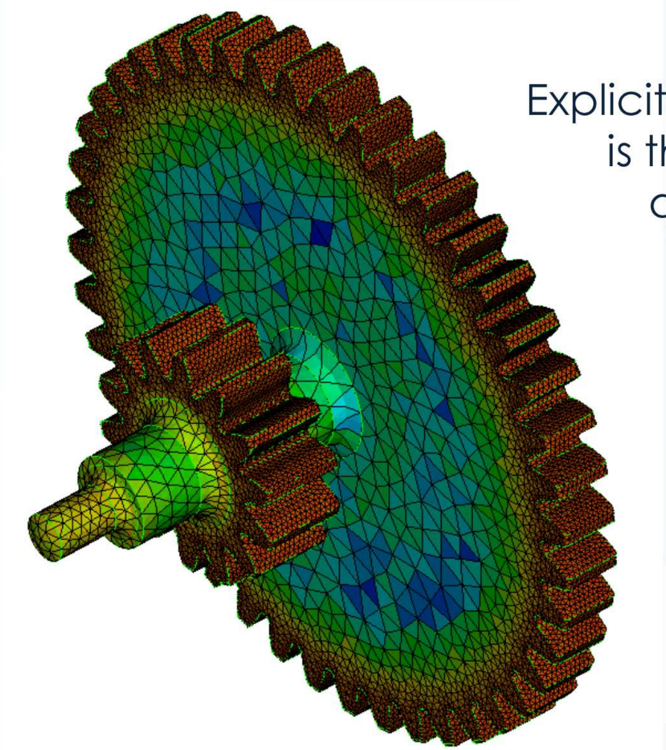




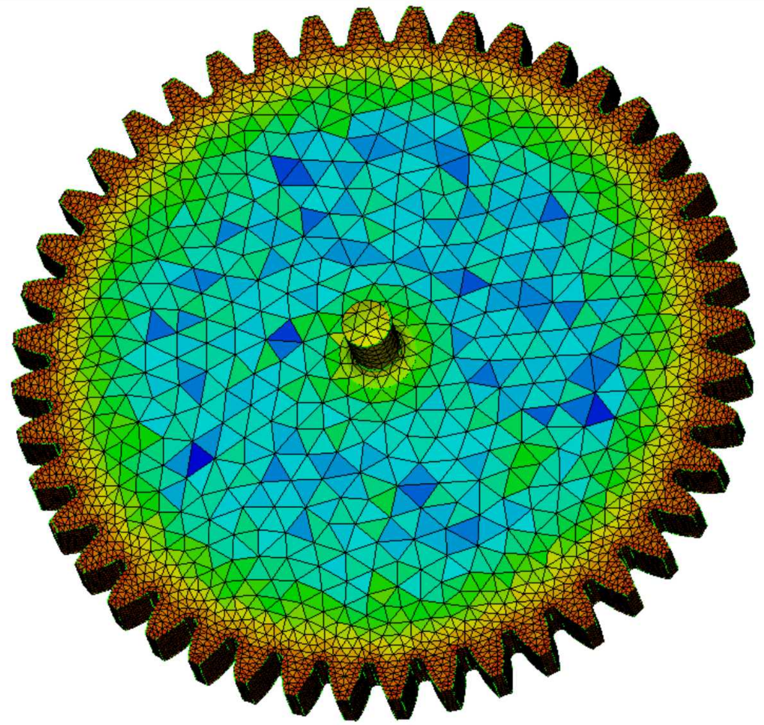
Motivation for a tetrahedral workflow

Hex meshing complicated components
is the bottleneck for decision making

Analysts seek geometry modifications
- Fine features can drive performance
- Engineering decisions provide impetus



Explicit transient dynamics
is the workhorse of
analysis efforts



Stable time step

Rapid discretization strategies
balance geometric resolution
and stable time step

Element technology should support large,
isochoric deformations, contact, and explicit
transient dynamics

Variational formulation

Motivated by prior work in IJNME: Thoutireddy, et al., (2002), Ostien, et al. (2016)

$$\Phi[\boldsymbol{\varphi}, \bar{\boldsymbol{F}}, \bar{\boldsymbol{P}}] := \int_B A(\bar{\boldsymbol{F}})\,\mathrm{d}V + \int_B \bar{\boldsymbol{P}} : (\boldsymbol{F} - \bar{\boldsymbol{F}})\,\mathrm{d}V - \int_B R\boldsymbol{B}\cdot\boldsymbol{\varphi}\,\mathrm{d}V - \int_{\partial_T B} \boldsymbol{T}\cdot\boldsymbol{\varphi}\,\mathrm{d}S$$

$$\bar{\boldsymbol{P}} = \lambda_\alpha \left(\int_\Omega \lambda_\alpha\lambda_\beta \boldsymbol{I}\,\mathrm{d}V\right)^{-1} \int_\Omega \lambda_\beta \boldsymbol{P}\,\mathrm{d}V,$$

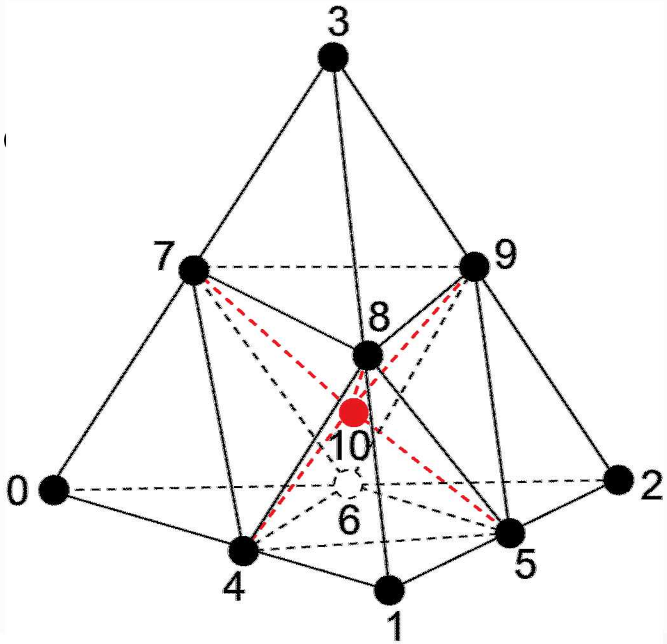
$$\bar{\boldsymbol{F}} = \lambda_\alpha \left(\int_\Omega \lambda_\alpha\lambda_\beta \boldsymbol{I}\,\mathrm{d}V\right)^{-1} \int_\Omega \lambda_\beta \boldsymbol{F}\,\mathrm{d}V$$

$$\boldsymbol{R}_a(\boldsymbol{\varphi}) := \int_\Omega \bar{\boldsymbol{P}}\cdot\boldsymbol{\mathcal{B}}_a\,\mathrm{d}V - \int_\Omega R\boldsymbol{B}N_a\,\mathrm{d}V - \int_{\partial_T\Omega} \boldsymbol{T}N_a$$

$$\boldsymbol{\mathcal{B}}_a(\boldsymbol{X}) := \delta_{ik}\frac{\partial N_a(\boldsymbol{X})}{\partial X_J}\,\boldsymbol{e}_i \otimes \boldsymbol{E}_J \otimes \boldsymbol{e}_k$$

- $\boldsymbol{\varphi}$ C[0] piecewise linear
- $\boldsymbol{F}$ C[-1] linear over parent element
- $\boldsymbol{P}$ C[-1] linear over parent element

12 sub-tets



Analytic gradient operator

Develop an exact gradient operator that projects and interpolates sub-tet gradients

$$\bar{\boldsymbol{F}}(\boldsymbol{X}) := \bar{\boldsymbol{\mathcal{B}}}_a(\boldsymbol{X})\boldsymbol{x}_a$$

$$\bar{\boldsymbol{\mathcal{B}}}_a(\boldsymbol{X}) := \lambda_\alpha(\boldsymbol{X})\left[\int_\Omega \delta_{ik}\lambda_\alpha(\boldsymbol{X})\lambda_\beta(\boldsymbol{X})\,\mathrm{d}V\right]^{-1}\int_\Omega \lambda_\beta(\boldsymbol{X})\frac{\partial N_a(\boldsymbol{X})}{\partial X_J}\,\mathrm{d}V\,\boldsymbol{e}_i\otimes\boldsymbol{E}_J\otimes\boldsymbol{e}_k$$

$$\bar{\boldsymbol{\mathcal{B}}}_a(\boldsymbol{\xi}) = \lambda_\alpha(\boldsymbol{\xi})\left[\int_{\Omega_{\boldsymbol{\xi}}} \delta_{ik}\lambda_\alpha(\boldsymbol{\xi})\lambda_\beta(\boldsymbol{\xi})\,\mathrm{d}V_{\boldsymbol{\xi}}\right]^{-1}\int_{\Omega_{\boldsymbol{\xi}}} \lambda_\beta(\boldsymbol{\xi})\frac{\partial N_a(\boldsymbol{\xi})}{\partial \boldsymbol{\xi}}\,\mathrm{d}V_{\boldsymbol{\xi}}\left(\frac{\partial\boldsymbol{\xi}}{\partial X_J}\right)\boldsymbol{e}_i\otimes\boldsymbol{E}_J\otimes\boldsymbol{e}_k$$

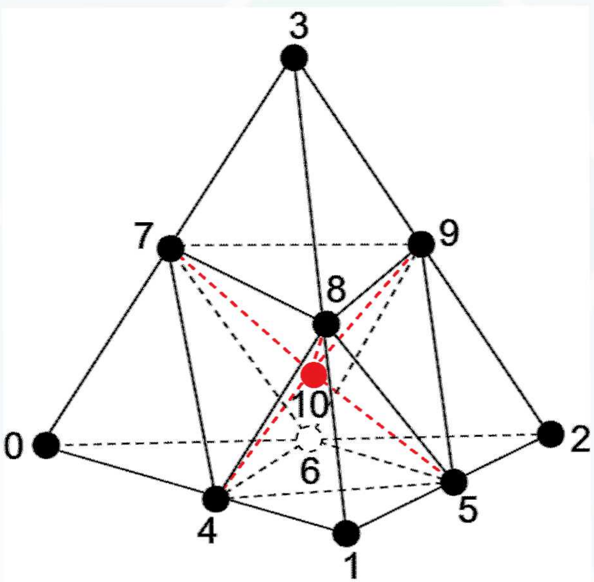
$$\bar{\boldsymbol{\mathcal{B}}}_a(\boldsymbol{\xi}) = \bar{\mathcal{L}}_{a;ilk}(\boldsymbol{\xi})\left(\frac{\partial\xi_l}{\partial X_J}\right)\boldsymbol{e}_i\otimes\boldsymbol{E}_J\otimes\boldsymbol{e}_k$$

$$\bar{\boldsymbol{\mathcal{L}}}_a(\boldsymbol{\xi}) = \lambda_\alpha(\boldsymbol{\xi})\delta_{ik}(M_{\alpha\beta})^{-1}\sum_{S=0}^{11}\frac{\partial N_a}{\partial\xi_l}\int_{E_S}\lambda_\beta(\boldsymbol{\xi})\,\mathrm{d}V_{\boldsymbol{\xi}}\,\boldsymbol{e}_i\otimes\boldsymbol{a}_l\otimes\boldsymbol{e}_k$$

$$\bar{\boldsymbol{\mathcal{B}}}_a(\boldsymbol{\xi}) = \bar{\mathcal{L}}_{a;ilk}(\boldsymbol{\xi})\left[\bar{\mathcal{L}}_{b;JlM}(\boldsymbol{\xi})X_{b;M}\right]^{-1}\boldsymbol{e}_i\otimes\boldsymbol{E}_J\otimes\boldsymbol{e}_k$$

$$\bar{B}_{aJ}(\boldsymbol{\xi}) = \bar{L}_{al}(\boldsymbol{\xi})\left[X_{Jb}\bar{L}_{bl}(\boldsymbol{\xi})\right]^{-1}$$

$$\bar{F}_{iJ}(\boldsymbol{\xi}) = x_{ia}\bar{B}_{aJ}(\boldsymbol{\xi})$$

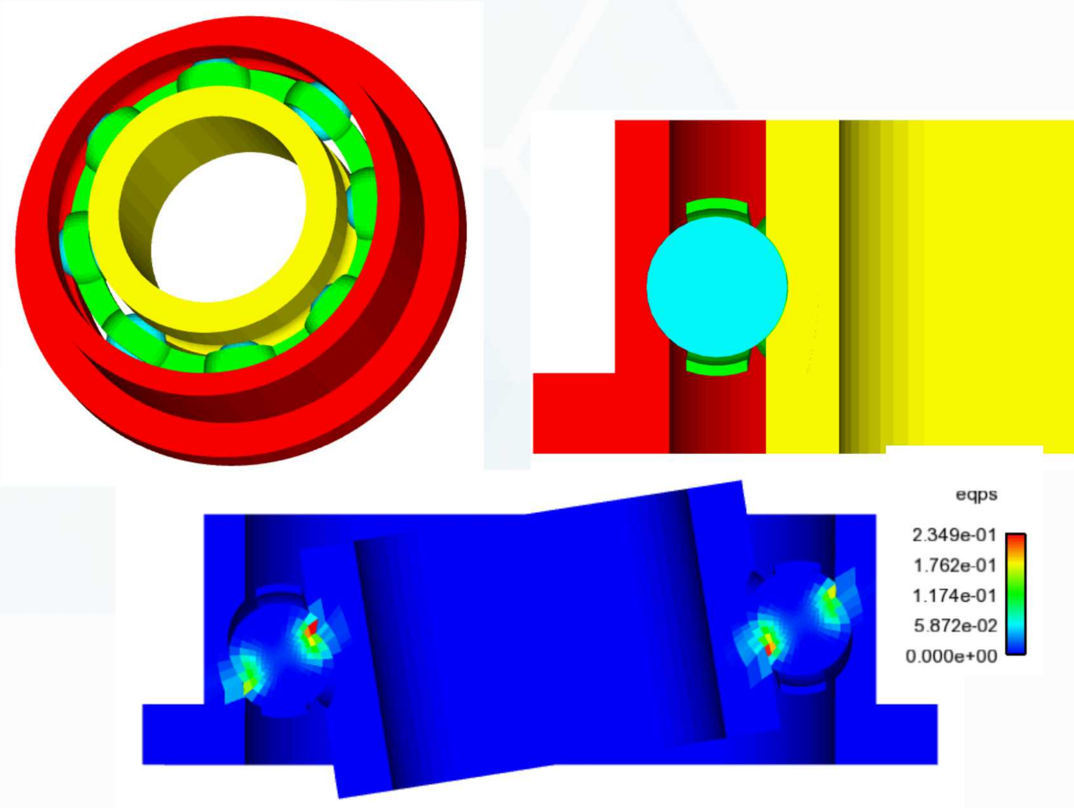


$$\bar{L}_{al}(\boldsymbol{\xi}) \equiv \bar{L}_{10\times3} = \frac{1}{24}\begin{pmatrix} 9-60\xi_0 & 9-60\xi_0 & 9-60\xi_0 \\ -9+60\xi_1 & 0 & 0 \\ 0 & -9+60\xi_2 & 0 \\ 0 & 0 & -9+60\xi_3 \\ 70(\xi_0-\xi_1) & 2(-4-35\xi_1+5\xi_2+10\xi_3) & 2(-4-35\xi_1+10\xi_2+5\xi_3) \\ 2(-1+5\xi_1+40\xi_2-5\xi_3) & 2(-1+40\xi_1+5\xi_2-5\xi_3) & 10(\xi_0-\xi_3) \\ 2(-4+5\xi_1-35\xi_2+10\xi_3) & 70(\xi_0-\xi_2) & 2(-4+10\xi_1-35\xi_2+5\xi_3) \\ 2(-4+5\xi_1+10\xi_2-35\xi_3) & 2(-4+10\xi_1+5\xi_2-35\xi_3) & 70(\xi_0-\xi_3) \\ 2(-1+5\xi_1-5\xi_2+40\xi_3) & 10(\xi_0-\xi_2) & 2(-1+40\xi_1-5\xi_2+5\xi_3) \\ 10(\xi_0-\xi_1) & 2(-1-5\xi_1+5\xi_2+40\xi_3) & 2(-1-5\xi_1+40\xi_2+5\xi_3) \end{pmatrix}$$

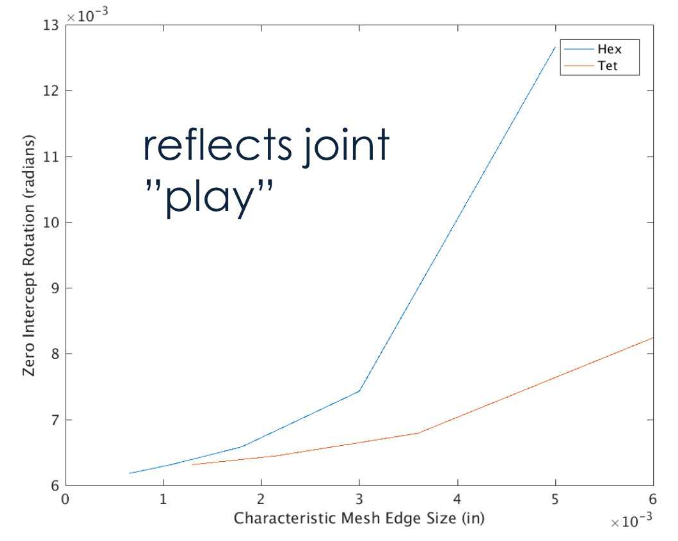
Evaluate for your integration scheme (Ostien, IJNME,2016)

Gaps provides technology pull

Analysis objective. Employ submodel to find the force vs. displacement and torque vs. rotation curves that feed into cylindrical joint parameters

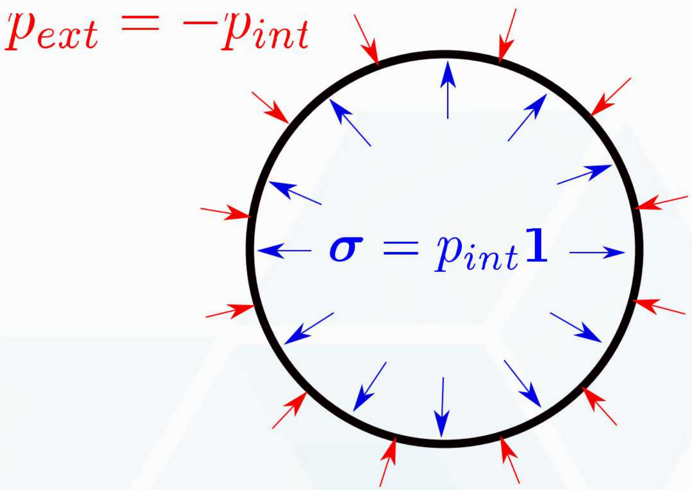


Technology pull. Composite tet formulation only approximate for curved edges. Fix it.

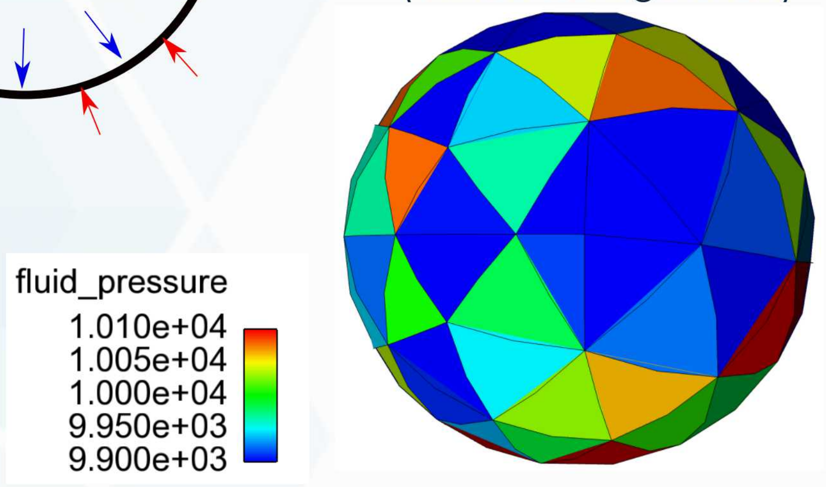


- Hex/tet models appear to converge to same result.
- At high refinement, uncertainty from dynamics exceeds discretization uncertainty
- *For a given edge length, curved tet mesh is more accurate and nearly equivalent to a 2X finer hex mesh*

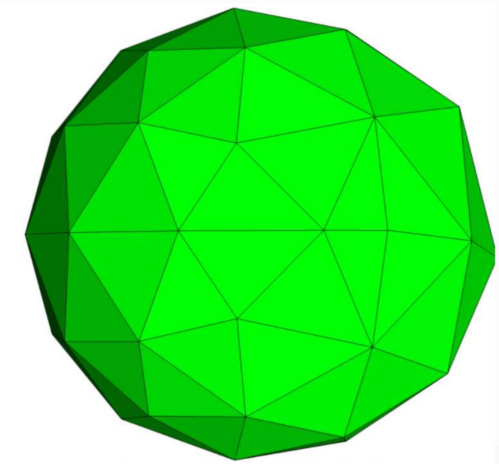
Clear need to revisit formulation



mid-edge nodes on geometry
(kinked configuration)



mid-edge nodes at midpoints
(straight edges)



contours of pressure should be constant
(not exactly)

New formulation

Technology pull: Mid-edge nodes must snap to boundary to increase geometric representation for the contact of bearings and components with curved edges

Improved composite tet10 formulation

- Gradient operator
- Jacobian (volume)
- Stress projection
- Mass matrix

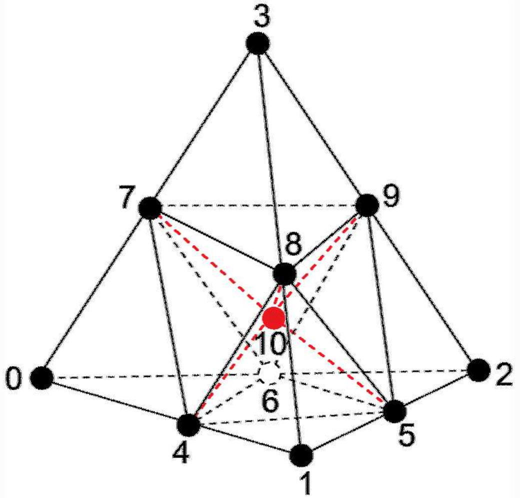
$$\bar{B}_{aJ}(\boldsymbol{\xi}) = \lambda_{1\times 4}\left[\sum_{S=0}^{11} J_{\boldsymbol{\xi}}^S \Psi_{4\times 4}^S\right]^{-1} \sum_{S=0}^{11} L_{al}^S [O_{Jl}^S]^{-1} J_{\boldsymbol{\xi}}^S \Lambda_{4\times 1}^S$$

$$J_c(\boldsymbol{\xi}) = \lambda_{1\times 4}(M_{4\times 4})^{-1} \sum_{S=0}^{11} \det[O_{Jl}^S]\Lambda_{4\times 1}^S$$

$$\bar{P}_{iJ}(\boldsymbol{\xi}) = \lambda_{1\times 4}(M_{4\times 4})^{-1} \int_{\Omega_{\boldsymbol{\xi}}} (\lambda_{1\times 4})^{\mathrm{T}} P_{iJ}(\boldsymbol{\xi}) J_c(\boldsymbol{\xi})\, \mathrm{d}V_{\boldsymbol{\xi}}$$

$$\mathcal{M}_{ab} = \rho_0 \textstyle\sum_{S=0}^{11} J_{\boldsymbol{\xi}}^S \int_{E_S} (N_{1\times 10}^S)^{\mathrm{T}} N_{1\times 10}^S\, \mathrm{d}V_{\boldsymbol{\xi}}$$

12 sub-tets



Gradient operator

$$\bar{\boldsymbol{F}}(\boldsymbol{X}) := \bar{\boldsymbol{\mathcal{B}}}_a(\boldsymbol{X})\boldsymbol{x}_a$$ *goal*

$$\bar{\boldsymbol{\mathcal{B}}}_a(\boldsymbol{X}) := \lambda_\alpha(\boldsymbol{X}) \left[\int_\Omega \delta_{ik}\lambda_\alpha(\boldsymbol{X})\lambda_\beta(\boldsymbol{X})\,\mathrm{d}V\right]^{-1} \int_\Omega \lambda_\beta(\boldsymbol{X})\frac{\partial N_a(\boldsymbol{X})}{\partial X_J}\,\mathrm{d}V\,\boldsymbol{e}_i\otimes\boldsymbol{E}_J\otimes\boldsymbol{e}_k$$

$$\boldsymbol{X}(\boldsymbol{\xi}) = N_a(\boldsymbol{\xi})\boldsymbol{X}_a$$ *isoparametric assumption*

$$\bar{\boldsymbol{\mathcal{B}}}_a(\boldsymbol{\xi}) := \lambda_\alpha(\boldsymbol{\xi}) \left[\int_{\Omega_{\boldsymbol{\xi}}} \delta_{ik}\lambda_\alpha(\boldsymbol{\xi})\lambda_\beta(\boldsymbol{\xi})J_{\boldsymbol{\xi}}(\boldsymbol{\xi})\,\mathrm{d}V_{\boldsymbol{\xi}}\right]^{-1} \int_{\Omega_{\boldsymbol{\xi}}} \lambda_\beta(\boldsymbol{\xi})\frac{\partial N_a(\boldsymbol{\xi})}{\partial\boldsymbol{\xi}}\frac{\partial\boldsymbol{\xi}(\boldsymbol{X})}{\partial X_J}J_{\boldsymbol{\xi}}(\boldsymbol{\xi})\,\mathrm{d}V_{\boldsymbol{\xi}}\,\boldsymbol{e}_i\otimes\boldsymbol{E}_J\otimes\boldsymbol{e}_k$$

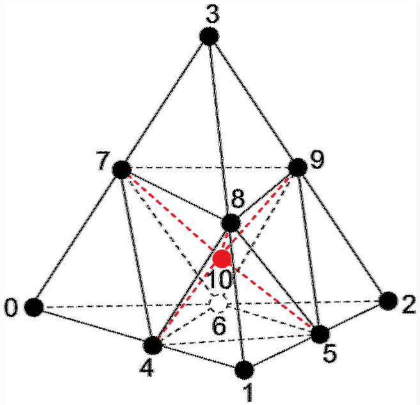
$$\bar{\boldsymbol{\mathcal{B}}}_a(\boldsymbol{\xi}) = \lambda_\alpha(\boldsymbol{\xi})\delta_{ik}(M_{\alpha\beta})^{-1}\sum_{S=0}^{11}\frac{\partial N_a}{\partial\xi_l}\left[\frac{\partial N_b}{\partial\xi_l}X_{b;J}\right]^{-1}J^S_{\boldsymbol{\xi}}\int_{E_S}\lambda_\beta(\boldsymbol{\xi})\,\mathrm{d}V_{\boldsymbol{\xi}}\,\boldsymbol{e}_i\otimes\boldsymbol{E}_J\otimes\boldsymbol{e}_k.$$

$$\bar{B}_{aJ}(\boldsymbol{\xi}) = \lambda_{1\times4}(M_{4\times4})^{-1}\sum_{S=0}^{11}\frac{\partial N_a}{\partial\xi_l}\left[\frac{\partial N_b}{\partial\xi_l}X_{b;J}\right]^{-1}J^S_{\boldsymbol{\xi}}\int_{E_S}(\lambda_{1\times4})^{\mathrm{T}}\,\mathrm{d}V_{\boldsymbol{\xi}}$$

$$\frac{\partial X_J}{\partial\xi_l} = X_{Jb}L_{bl}(\boldsymbol{\xi}) \qquad [O^S_{Jl}]^{-1} = [X_{Jb}L^S_{bl}]^{-1} \qquad J^S_{\boldsymbol{\xi}} = \det[\boldsymbol{O}^S]$$

$$\Lambda^S_{4\times1} = \int_{E_S}(\lambda_{1\times4})^{\mathrm{T}}\,\mathrm{d}V_{\boldsymbol{\xi}} \qquad \Psi^S_{4\times4} = \int_{E_S}(\lambda_{1\times4})^{\mathrm{T}}\lambda_{1\times4}\,\mathrm{d}V_{\boldsymbol{\xi}}$$

$$\bar{B}_{aJ}(\boldsymbol{\xi}) = \lambda_{1\times4}\left[\sum_{S=0}^{11}J^S_{\boldsymbol{\xi}}\Psi^S_{4\times4}\right]^{-1}\sum_{S=0}^{11}L^S_{al}[O^S_{Jl}]^{-1}J^S_{\boldsymbol{\xi}}\Lambda^S_{4\times1}$$



Optimal determinants

In general, we seek to integrate a function over the element

$$g(\boldsymbol{X}) = \int_\Omega f(\boldsymbol{X})\,\mathrm{d}V \qquad g(\boldsymbol{\xi}) = \int_\Omega f(\boldsymbol{X})J(\boldsymbol{\xi})\,\mathrm{d}V_{\boldsymbol{\xi}} \qquad g(\boldsymbol{\xi}) = \sum_{i=1}^{ip} w_i f(\boldsymbol{\xi}_i)J(\boldsymbol{\xi}_i)$$

Yikes, the Jacobian from the parametric to reference configuration is piecewise constant

What do we do? Minimize error through L_2 projection:

$$J_c(\boldsymbol{\xi}) = \lambda_\alpha(\boldsymbol{\xi})\left[\int_{E_P}\lambda_\alpha(\boldsymbol{\xi})\lambda_\beta(\boldsymbol{\xi})\,\mathrm{d}V\right]^{-1}\int_{E_P}\lambda_\beta(\boldsymbol{\xi})\det\left[\frac{\partial\boldsymbol{X}}{\partial\boldsymbol{\xi}}\right]\mathrm{d}V$$

$$J_c(\boldsymbol{\xi}) = \lambda_\alpha(\boldsymbol{\xi})\left[\int_{\Omega_{\boldsymbol{\xi}}}\lambda_\alpha(\boldsymbol{\xi})\lambda_\beta(\boldsymbol{\xi})J_{\boldsymbol{\xi}}\,\mathrm{d}V_{\boldsymbol{\xi}}\right]^{-1}\int_{\Omega_{\boldsymbol{\xi}}}\lambda_\beta(\boldsymbol{\xi})\det\left[\frac{\partial\boldsymbol{X}}{\partial\boldsymbol{\xi}}\right]J_{\boldsymbol{\xi}}\,\mathrm{d}V_{\boldsymbol{\xi}}$$

$$J_c(\boldsymbol{\xi}) = \lambda_\alpha(\boldsymbol{\xi})\left[\int_{\Omega_{\boldsymbol{\xi}}}\lambda_\alpha(\boldsymbol{\xi})\lambda_\beta(\boldsymbol{\xi})\,\mathrm{d}V_{\boldsymbol{\xi}}\right]^{-1}\sum_{S=0}^{11}\det[O^S_{Jl}]\int_{E_S}\lambda_\beta(\boldsymbol{\xi})\,\mathrm{d}V_{\boldsymbol{\xi}}$$

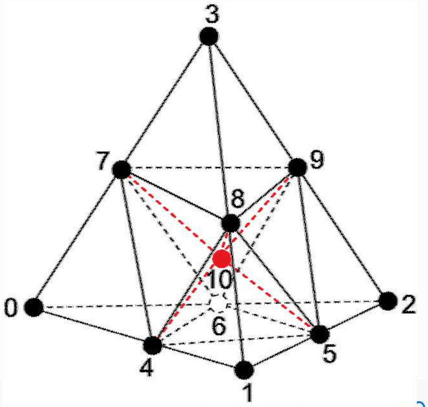
$$J_c(\boldsymbol{\xi}) = \lambda_{1\times 4}(\hat{M}_{4\times 4})^{-1}\sum_{S=0}^{11}\det[O^S_{Jl}]\Lambda^S_{4\times 1}$$

What if I only know information at integration points?

For kinked edges

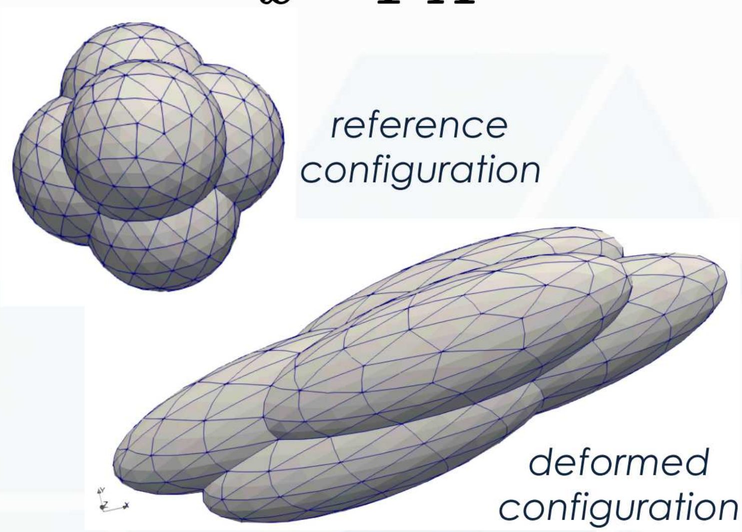
$$V_c = \int_{\Omega_{\boldsymbol{\xi}}} J_c(\boldsymbol{\xi})\,\mathrm{d}V_{\boldsymbol{\xi}} = \sum_{i=1}^{4}\frac{1}{24}J_c(\boldsymbol{\xi}_i)$$

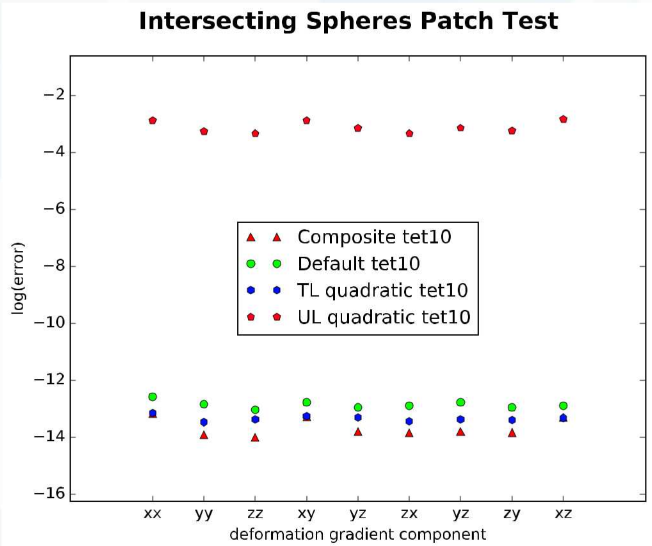
$$\boxed{V - V_c = 0}$$

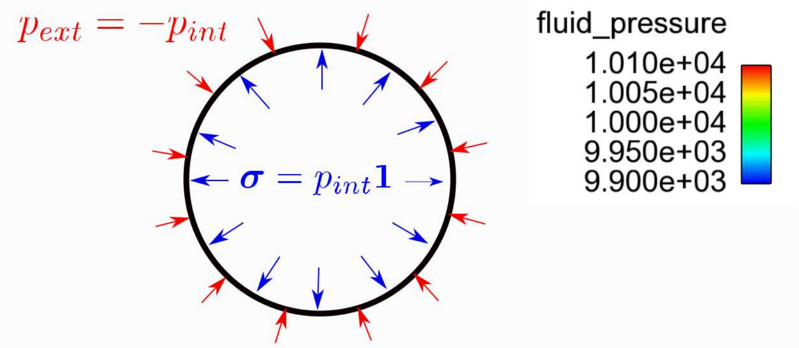


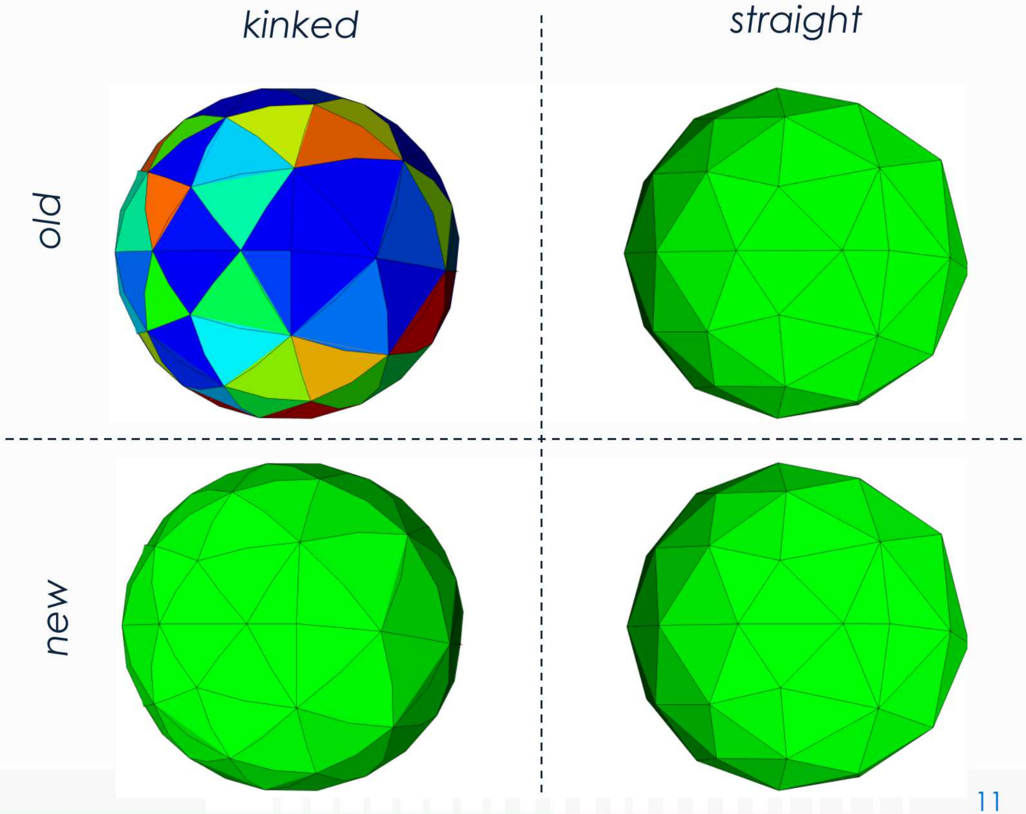
Verification of new formulation

$$\boldsymbol{x} = \boldsymbol{F}\boldsymbol{X}$$



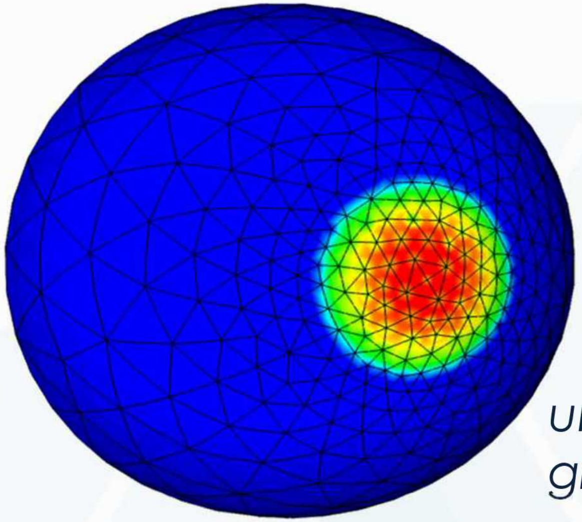




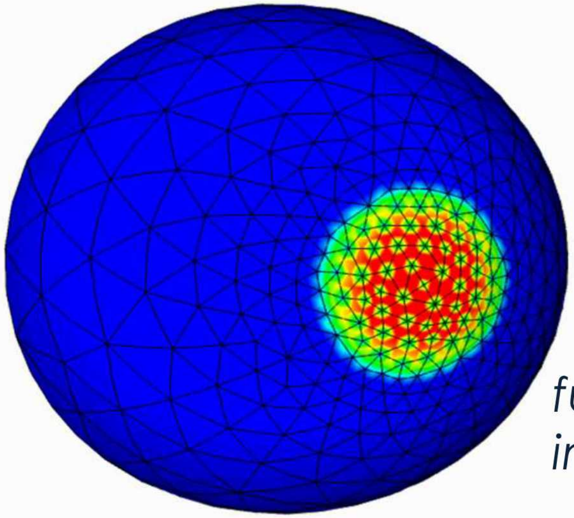


Example – Hertzian contact

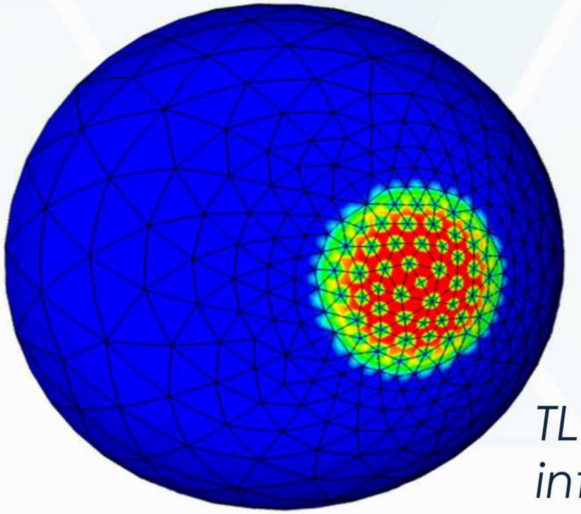
Hypoelasticity w/plots of the contact pressure



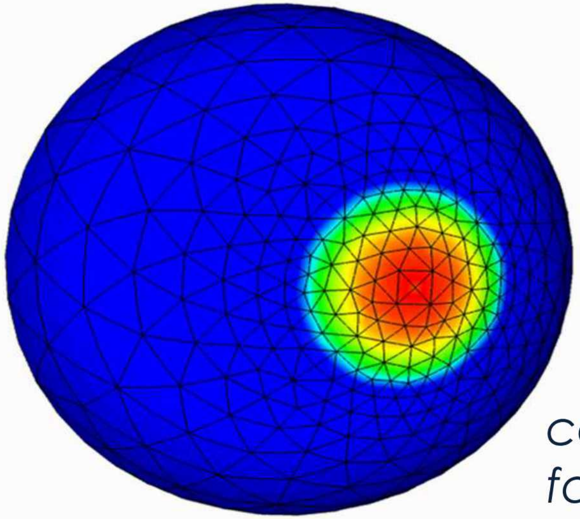
uniform gradient



full integration



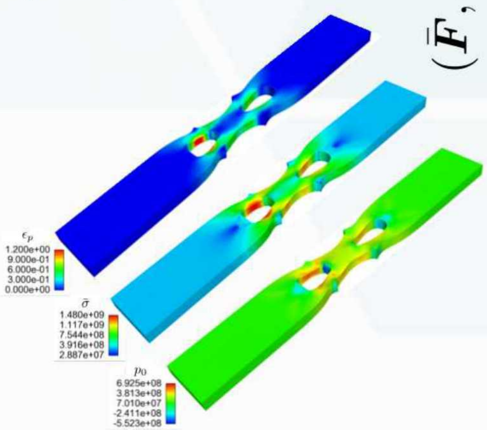
TL full integration

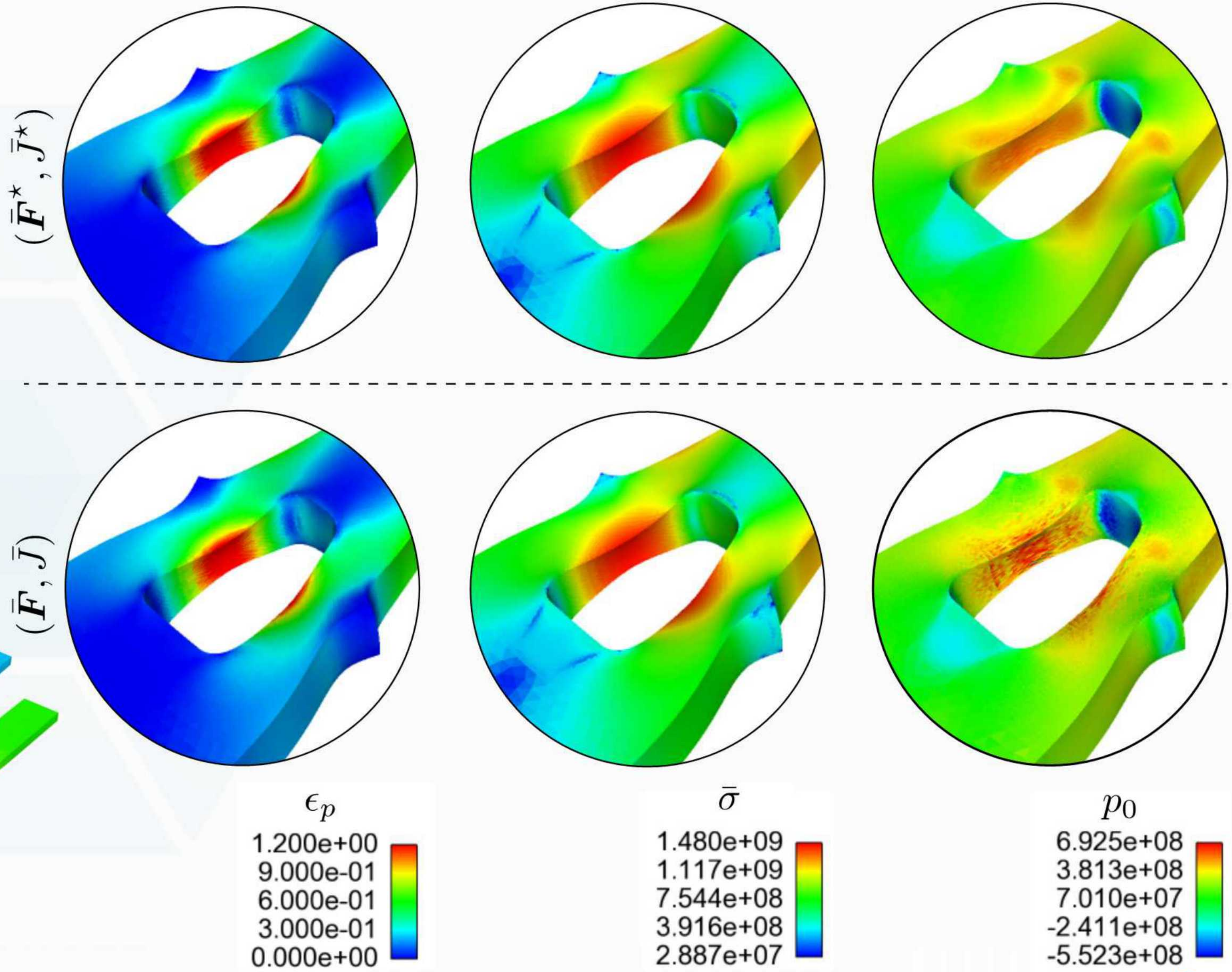


composite formulation

Example – Large, plastic deformations

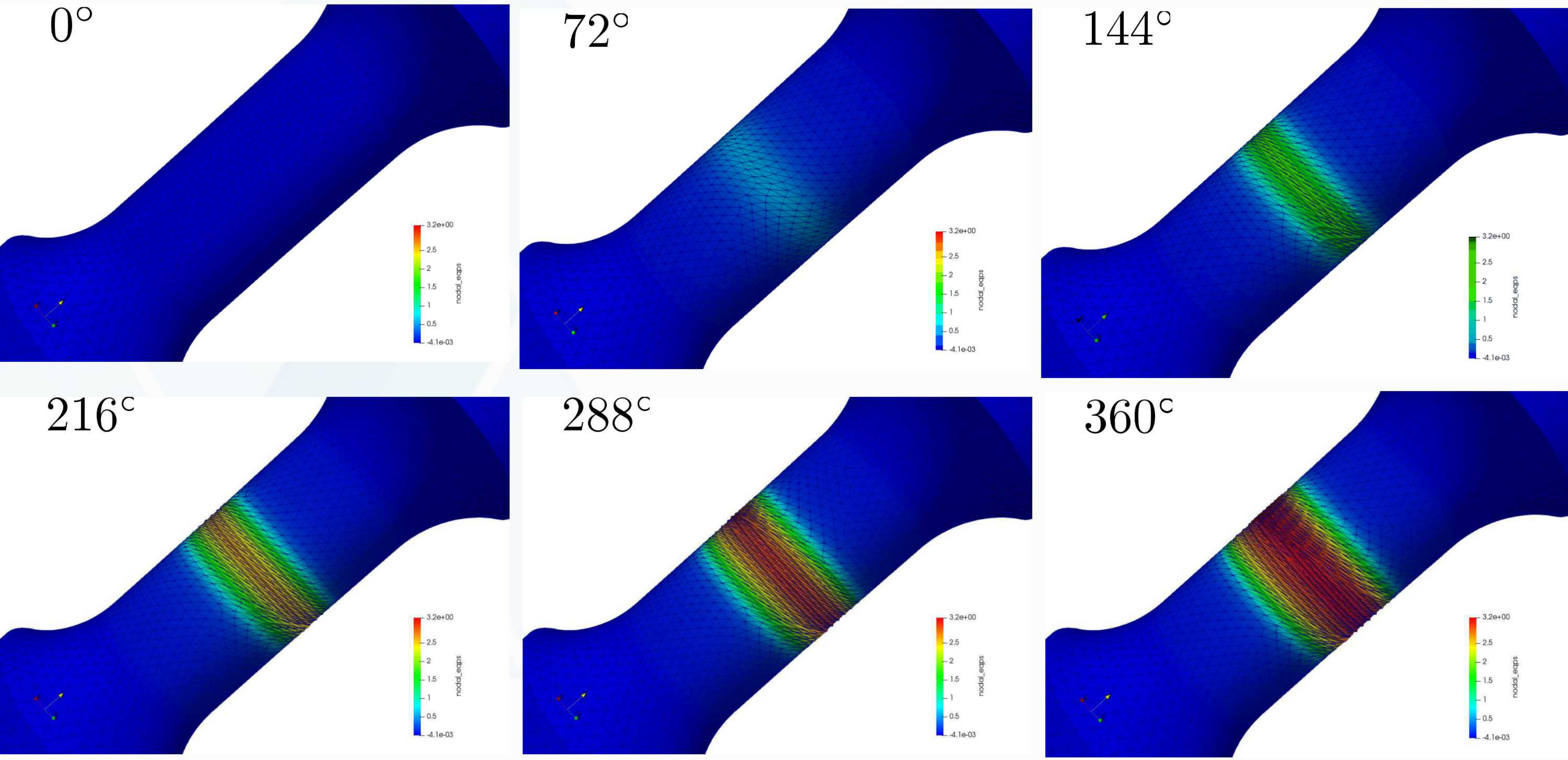
B. Talamini will develop a rigorous, 5-field formulation for volumetric locking in a few minutes (~12)





Example – Torsion (for fun)

Currently examining deformation of solid bars under torsion (6061-T6). Still many outstanding issues that will require remeshing. Large deformations. Implict.



Conclusions and future work

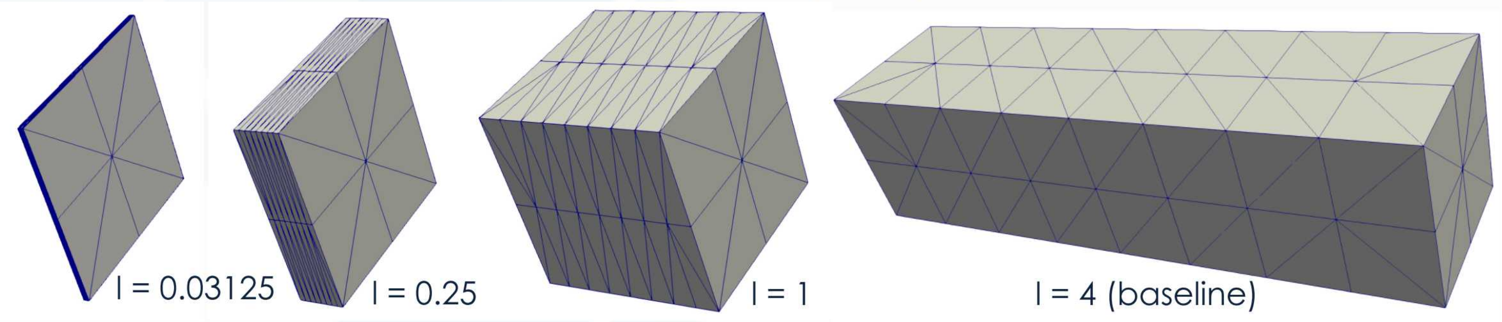
- Meshing is no fun
- Hypothesize a new tetrahedral workflow for solid mechanics
- New element formulation responds to analyst pull
- Able to employ analytic infrastructure + new integration
- New tetrahedral workflow improving design to analysis by 100X
- Future work will focus on
  - Applicable quality metrics
  - Lumping schemes for dynamics (w/application to contact)
- Future research efforts in
  - Local adaptivity and remeshing
  - Error estimation and the inclusion of anisotropic metrics

BACKUPS

Provide understanding. Fix it.

1. Initial work focused on usability guidelines for bearing contact
2. Analysts have established heuristics for the hexahedral workflow
   - Traceability of engineering basis is sometimes difficult
   - Tailored to specific technologies with a limited range of applicability
   - Necessary to get work done but does not encourage probing the limits of technologies
3. New technologies may require new guidelines for usability
   - We can tet mesh almost anything. How good does it need to be?
   - Requires fundamental studies of the impact of mesh quality on solutions (properties, loading)
4. Understanding requires a mesh. If there are bugs – fix them immediately

Role of mesh quality in uniaxial tension



8 elements along bar. Length varies from 0.03125 to 1024 (32,768X)

Aspect ratio varies between 1.5 and 160.

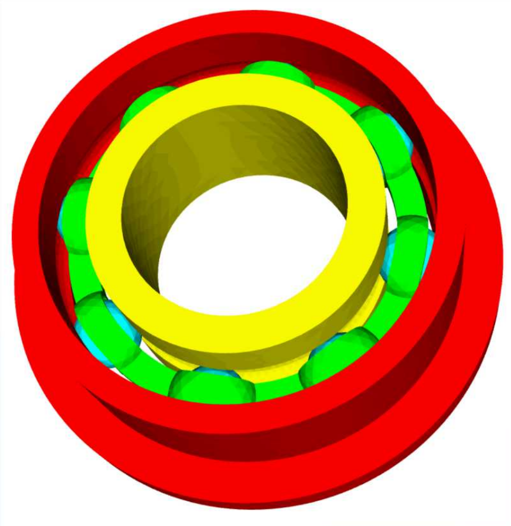
Scaled Jacobian varies between 0.6 and 0.0016.

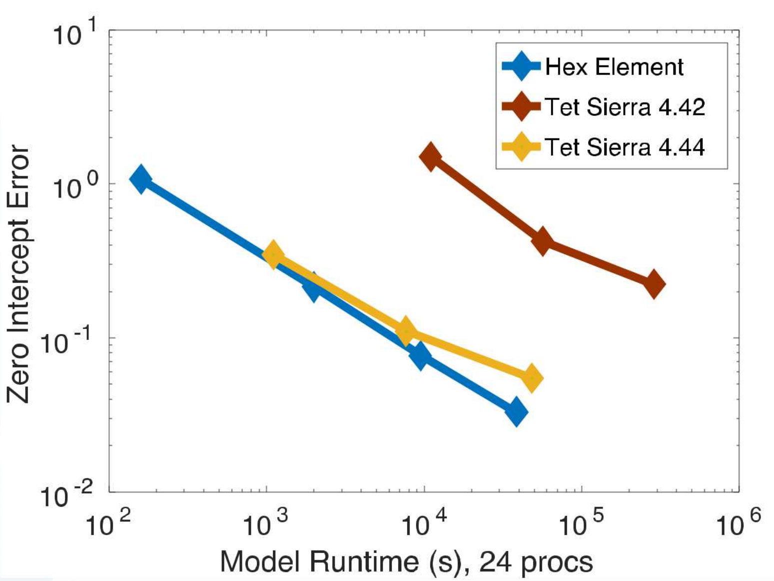
All meshes come to equilibrium.

For l = 0.03125
3 processors
ν = 0.3

RBM	LINEAR ITER	MP ITER	RESIDUAL	RELATIVE RESIDUAL	EXTERNAL REFERENCE	ENERGY	DISPLACEMENT
-	-	0	2.224e-02	1.000e+00	2.224e-02	-	-
0	U 94	1	7.820e-07	3.516e-05	2.224e-02	3.674e-12	7.207e-07
0	23	2	7.397e-11>M	3.325e-09	2.224e-02	1.178e-20	7.243e-17

Make it fast. Keep the accuracy.





Substantial impact on tetrahedral workflow

Analyses employed to fit a reduced order cylindrical joint model

Optimization	Speedup	Reason
Correct handling of curved edges	10.0X	Allows solving to same accuracy on a coarser mesh
Correct critical time step computation	3.0X	Achieves same result with less time steps
Code structure and mathematical	3.5X	Make more efficient use of machine hardware to do same computations

These optimizations are multiplicative, net speedup: **~100X!**