Flow of a Cement Slurry Modeled as a Generalized Second Grade Fluid

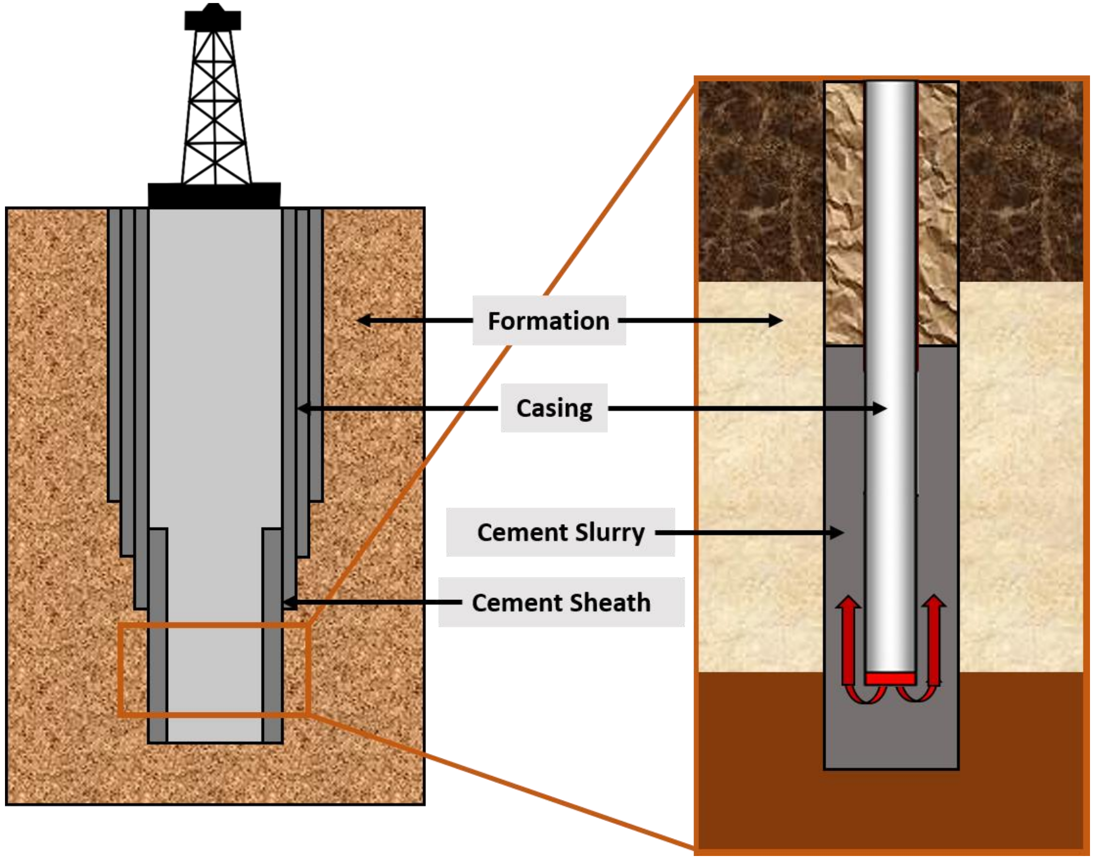
Chengcheng Tao, Eilis Rosenbaum, Barbara G. Kutchko, Mehrdad Massoudi
U.S. Department of Energy, National Energy Technology Laboratory (NETL)
Chengcheng.Tao@netl.doe.gov

Outline of Presentation

- Introduction to well cementing
- Scope of the study
- Mathematical modeling
- Parametric study
- Concluding remarks and future work

Motivation

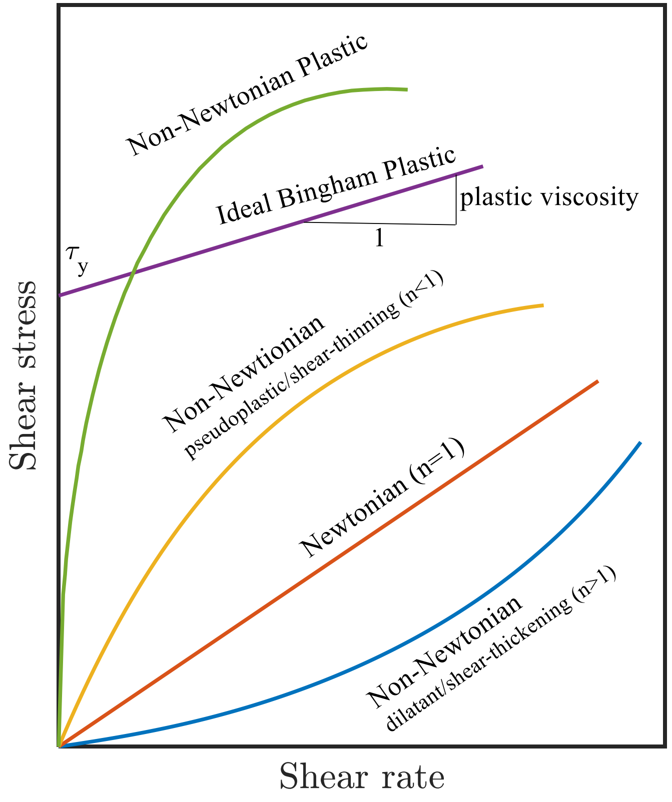
- **Well cementing** is the process of placing a cement slurry in the annulus space between the well casing and the surrounding for zonal isolation
- **Goal** is to eliminate fluids migration in the well
- **Challenges** in oil well cementing operations:
  - High temperature, high pressure, weak or porous formations, corrosive fluids
- **Cement slurry design** for the oil well is a function of various parameters:
  - Well bore geometry, casing hardware, drilling mud characteristics, filtration and mixing conditions etc.
- **Rheological behavior** of oil well cement slurries is significant in well cementing operation

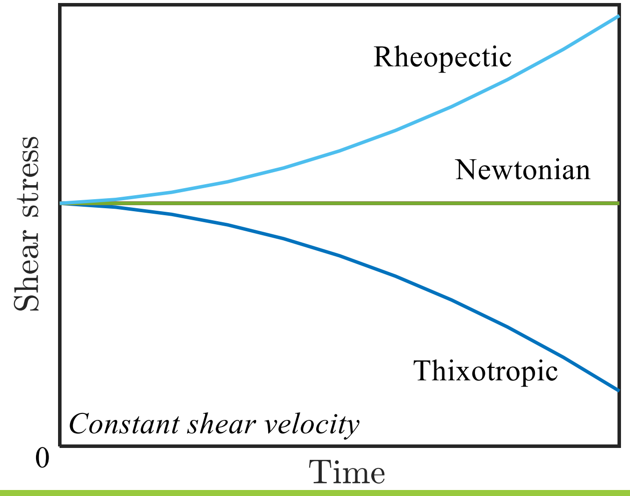


Schematic diagram of oil well cementing

Scope of the Study

- In general, non-Newtonian (non-linear) fluids exhibit one or all behaviors:
  - The ability to shear-thin or shear-thicken
  - The ability to creep
  - The ability to relax stresses
  - The presence of normal stress differences in simple shear flows
  - The presence of yield stress
- For cement slurry:
  - Viscosity depends on the **shear rate, volume fraction**
  - Cement has a **yield stress**
  - Cement shows thixotropic behavior





Mathematical Model-Governing Equations

- **Conservation of mass**

$$\frac{\partial \rho}{\partial t} + div(\rho \boldsymbol{v}) = 0 \qquad (1)$$

ρ: density of cement slurry

$\boldsymbol{v}$: velocity vector, $div(\boldsymbol{v}) = 0$ for an isochoric motion

- **Conservation of linear momentum**

$$\rho \frac{d\boldsymbol{v}}{dt} = div\boldsymbol{T} + \rho \boldsymbol{b} \qquad (2)$$

d/dt: total time derivative, given by $\frac{d(.)}{dt} = \frac{\partial(.)}{\partial t} + [grad(.)]\boldsymbol{v}$

$\boldsymbol{b}$: body force vector

$\boldsymbol{T}$: Cauchy stress tensor given by the constitutive equation

- **Conservation of angular momentum**

$$\boldsymbol{T} = \boldsymbol{T}^T \qquad (3)$$

- **Convection - diffusion equation**

$$\frac{\partial \phi}{\partial t} + div(\phi \boldsymbol{v}) = \mathrm{f} \qquad (4)$$

ϕ: volume fraction

f: diffusive particle flux that is to be determined by the constitutive theory

Mathematical Model-Constitutive Relations

I. For the viscous stress tensor T

$$\boldsymbol{T} = \boldsymbol{T}_y + \boldsymbol{T}_v$$

$\boldsymbol{T}_y$: yield stress – future work

$\boldsymbol{T}_v$: viscous stress, which is dependent on shear rate, particle volume fraction, temperature, pressure, cement hydration, etc.

A modified second grade (Rivlin-Ericksen) fluid model is applied for viscous stress of cement slurry (Massoudi & Tran, 2016)

$$\boldsymbol{T}_v = -p\boldsymbol{I} + \mu_{eff}(\phi, \boldsymbol{A_1})\boldsymbol{A_1} + \alpha_1 \boldsymbol{A_2} + \alpha_2 \boldsymbol{A_1}^{\boldsymbol{2}} \qquad (5)$$

p: pressure

ϕ: volume fraction

$\boldsymbol{A_n}$: n-th order Rivlin-Ericksen tensors

where $\boldsymbol{A_1} = \nabla\boldsymbol{v} + \nabla\boldsymbol{v}^T$ $\quad \boldsymbol{A_2} = \frac{dA_1}{dt} + \boldsymbol{A_1}\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^T\boldsymbol{A_1}$

α_1, α_2: normal stress coefficients

μ_{eff}: effective viscosity, which is dependent on volume fraction (Krieger 1959) and shear rate

$$\mu_{eff}(\phi, \boldsymbol{A_1}) = \mu_0 \left(1 - \frac{\phi}{\phi_m}\right)^{-\beta} \left[1 + \alpha \mathrm{tr}\boldsymbol{A_1}^2\right]^m$$

μ_0: viscosity of the cement slurry without particles; ϕ_m: maximum volume concentration of solids; β, m: material parameters

Mathematical Model-Constitutive Relations

II. For the diffusive particle flux f

$$\mathrm{f} = -\mathrm{div}\boldsymbol{N} \qquad (6)$$

$\boldsymbol{N}$: flux vector, related to the movement of the particles (Philips et al, 1992)

$$\boldsymbol{N} = \boldsymbol{N_c} + \boldsymbol{N_\mu} + \boldsymbol{N_b} = -a^2\phi K_c \nabla(\dot{\gamma}\phi) - a^2\phi^2\dot{\gamma}K_\mu \nabla(\ln\mu_{eff}) - D\nabla\phi$$

particles collision **spatially varying viscosity** **Brownian diffusive flux**

D is the diffusion coefficient (diffusivity), which is the function of $\dot{\gamma}$ and ϕ

$$D(\dot{\gamma},\phi) = \eta\left\|\boldsymbol{A_1}^2\right\| \cdot D_0[K_1 + K_2(1-\phi)^2 + K_3(\phi_m - \phi)^2 H(\phi_m - \phi)]$$

(Bridges and Rajagopal 2006; Garboczi and Bentz 1992)

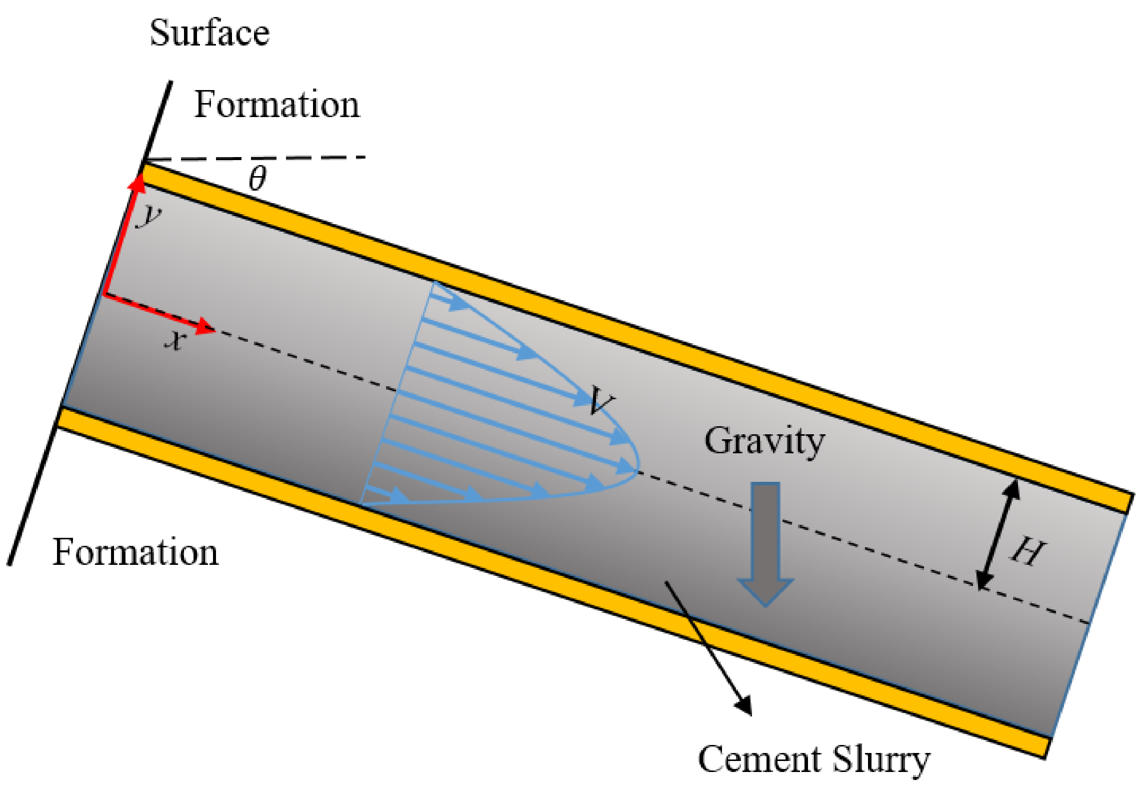
a: particle radus; K_c and K_μ: empirically coefficients; D_0: the diffusivity parameter

K_1,K_2 and K_3: fitting coefficients,H: Heaviside function, $H(x) = 1$ for $x > 0$, $H(x) = 0$ for $x \leq 0$

Substitute two constitutive relations (5) (6) into convection-diffusion equation (4)

Problem Statement

Steady Flow of a Cement Slurry



- The motion is steady and fully developed
- The flow is assumed to be one-dimensional
- The velocity and the volume fraction forms:

$$\begin{cases} \phi = \phi(y) \\ \boldsymbol{v} = v(y)\boldsymbol{e}_x \end{cases} \quad (7)$$

Non-dimensionalization:

$$\bar{y} = \frac{y}{H}; \ \bar{v} = \frac{v}{V}$$

Boundary conditions:

- $\bar{v}(\bar{y} = -1) = 0; \ \bar{v}(\bar{y} = 1) = 0$ (no-slip)
- $\int_{-1}^{1} \phi d\bar{y} = \phi_{avg}$

ϕ_{avg}: average value of ϕ integrated over the cross section

Numerical schemes

Conservation of mass (Eqn. (1)) is satisfied automatically with the form of ϕ and $\boldsymbol{v}$ in Eqn. (7)

Substitute viscous stress tensor $\boldsymbol{T}$ (Eqn. (5)) into conservation of linear momentum (Eqn. (2)) with non-dimensionalization

$$\frac{\partial}{\partial \bar{y}}\left\{\left(1-\frac{\phi}{\phi_m}\right)^{-\beta}\left[1+R_0\left(\frac{dv}{dy}\right)^2\right]^m \frac{d\bar{v}}{d\bar{y}}\right\} = R_1 - R_2 \sin\theta \quad (8)$$

Substitute concentration flux f (Eqn. (6)) into convection - diffusion equation (Eqn. (4)) with non-dimensionalization

$$\frac{K_c}{K_\mu}\left(\phi^2 \frac{d}{d\bar{y}}\left|\frac{d\bar{v}}{d\bar{y}}\right| + \phi\left|\frac{d\bar{v}}{d\bar{y}}\right|\frac{d\phi}{d\bar{y}}\right) + m\phi^2\left|\frac{d\bar{v}}{d\bar{y}}\right|\left[1+R_0\left(\frac{d\bar{v}}{d\bar{y}}\right)^2\right]^{-1} \cdot 2R_0 \frac{d\bar{v}}{d\bar{y}}\frac{d^2\bar{v}}{d\bar{y}^2} + \frac{\beta}{\phi_m}\phi^2\left(1-\frac{\phi}{\phi_m}\right)^{-1}\frac{d\phi}{d\bar{y}}\left|\frac{d\bar{v}}{d\bar{y}}\right| + [R_3 + R_4(1-\phi)^2 + R_5(\phi_m-\phi)^2 H(\phi_m -$$

Numerical analysis

- The dimensionless differential equations are solved using the MATLAB solver bvp4c

bvp4c

Solve boundary value problem — fourth-order method

Syntax

```
sol = bvp4c(odefun,bcfun,solinit)
sol = bvp4c(odefun,bcfun,solinit,options)
```

- The step size is automatically adjusted by the solver
- The tolerance for the maximum residue is 0.001
- The constrain boundary condition for ϕ_{avg} was applied by shooting method

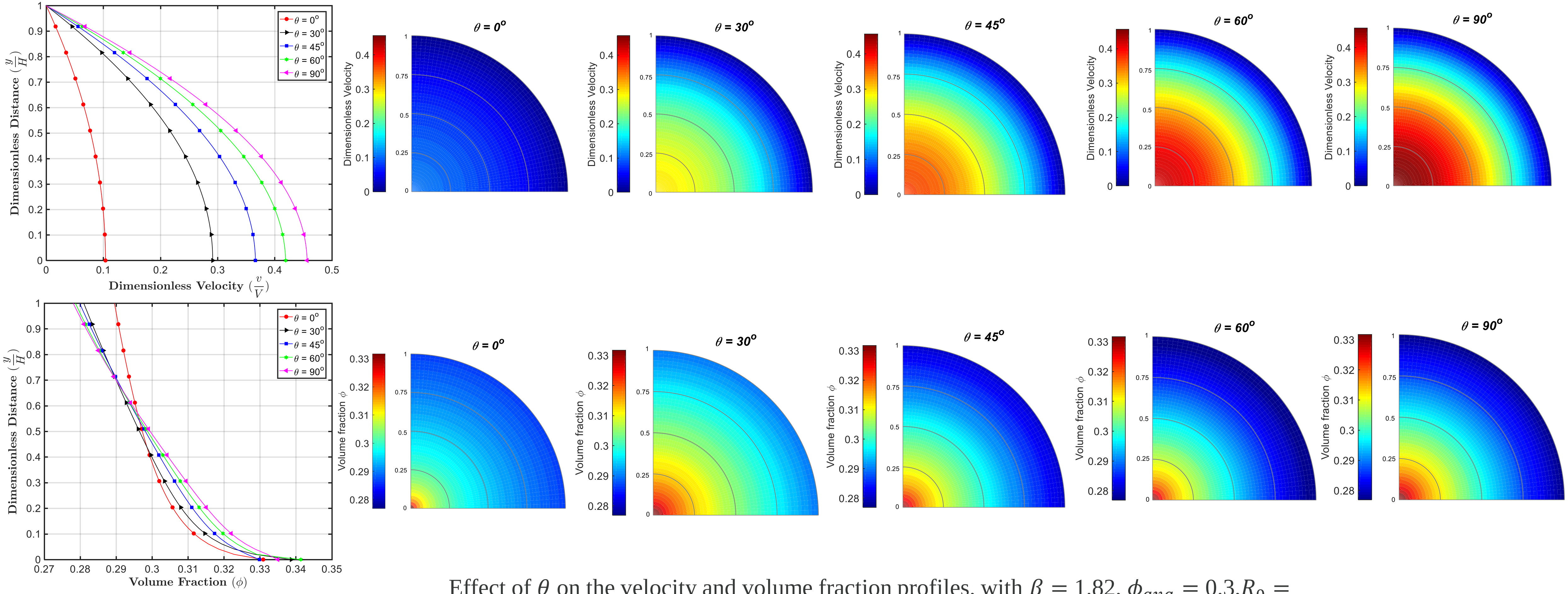
Parametric study

- The designed values for dimensionless numbers and parameters

Parameters	Range of Values
ϕ_m	0.45, 0.5, 0.55, 0.6, 0.65
K_c/K_μ	0, 0.02, 0.04, 0.06, 0.08
θ	0^o, 30^o, 45^o, 60^o, 90^o
m	-0.3, -0.1, 0, 0.1, 0.3, 0.7
R_0	0.01, 0.1, 1, 10
R_1	0, -1.5, -2.5, -3.5
R_2	0, 0.5, 1, 1.5
R_3	0.01, 0.1, 1
R_4	0.01, 0.1, 1
R_5	0.01, 0.1, 1

Parametric study

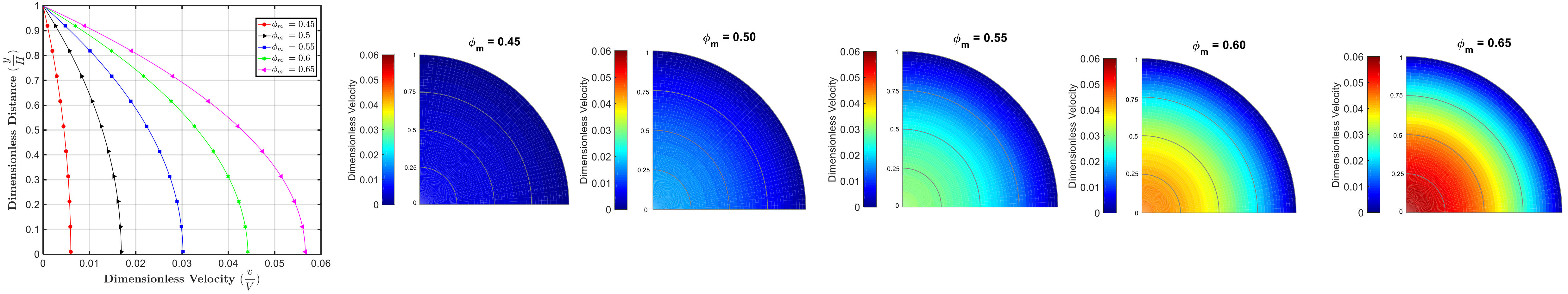
Effect of θ

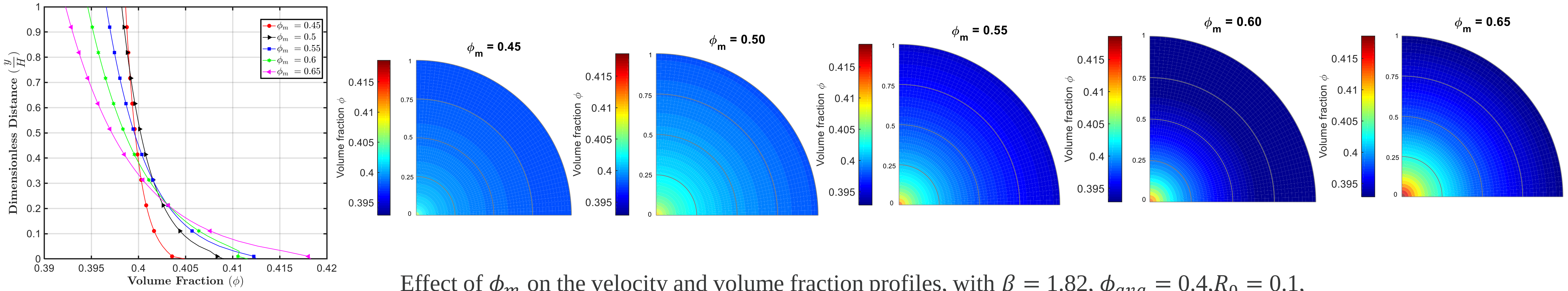


Effect of θ on the velocity and volume fraction profiles, with $\beta = 1.82$, $\phi_{avg} = 0.3$, $R_0 = 0.1$, $R_1 =$ -2.5, $R_2 = 10$, $R_3 = 0.01$, $R_4 = 0.07$, $R_5 = 1.8$, $\frac{K_c}{K_\mu} = 0.05$, $\phi_m = 0.65$, $m = 1$

Parametric study

Effect of ϕ_m

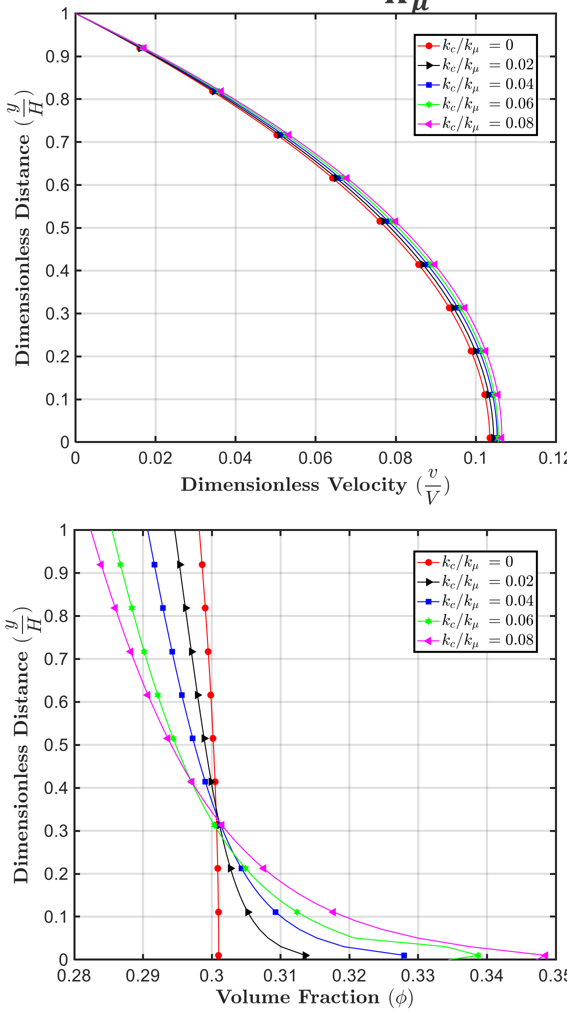




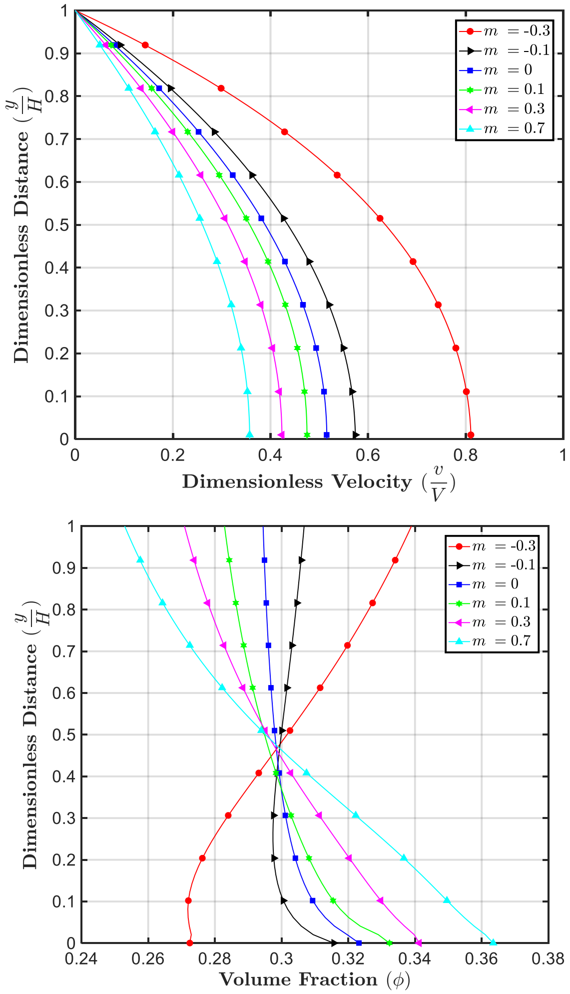
Effect of ϕ_m on the velocity and volume fraction profiles, with $\beta = 1.82$, $\phi_{avg} = 0.4$,$R_0 = 0.1$, $R_1 =$ -2.5, $R_2 = 0.1$, $R_3 = 0.01$, $R_4 = 0.07$,$R_5 = 1.8$, $\frac{K_c}{K_\mu} = 0.05$, $m = 1$, $\theta = 45^o$

Parametric study

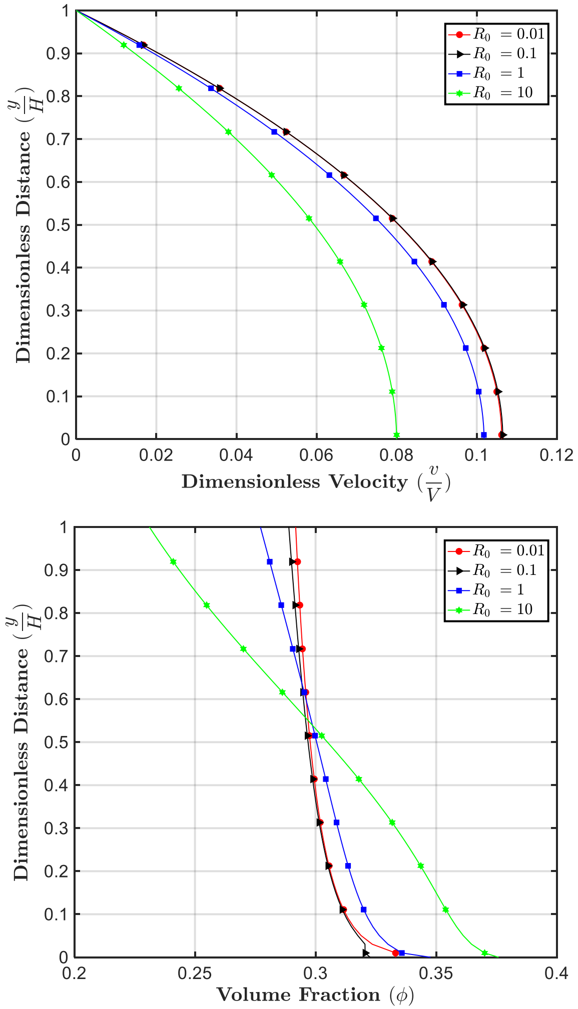
Effect of $\frac{K_c}{K_\mu}$



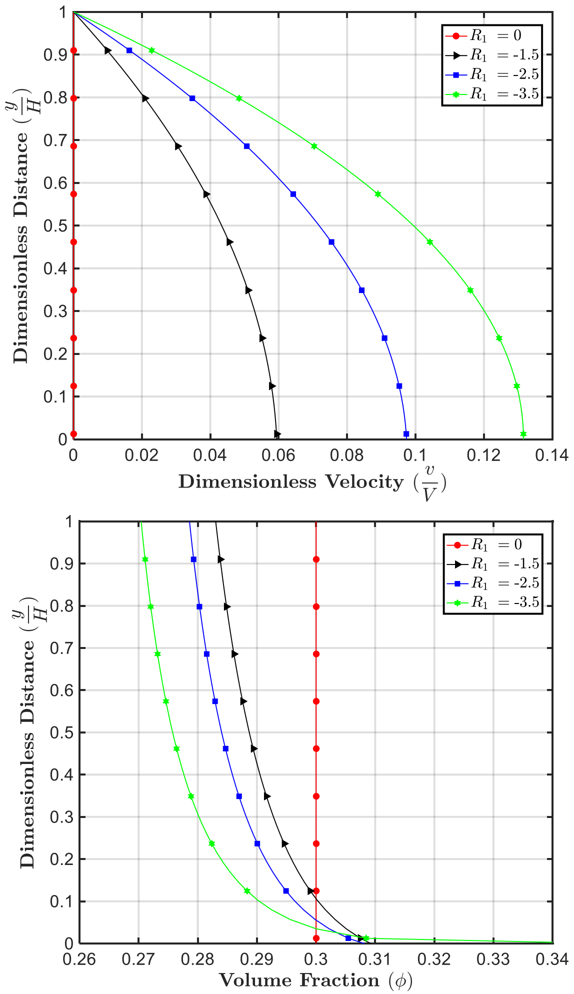
Effect of m



Effect of R_0

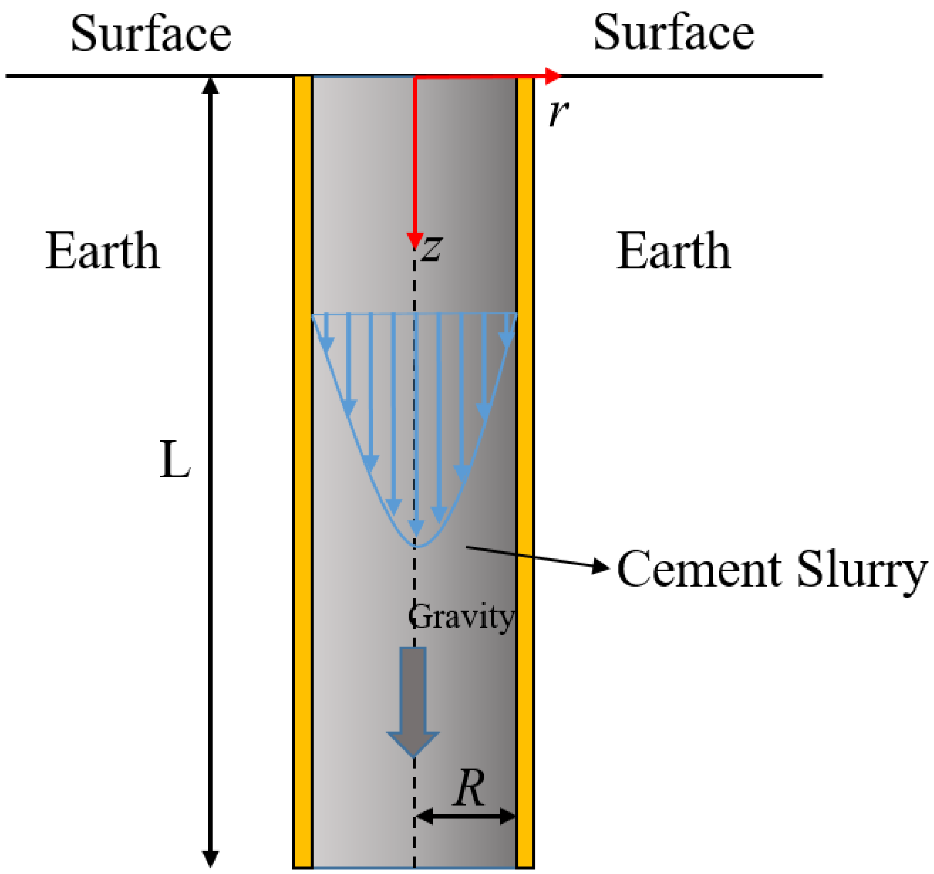


Effect of R_1



Problem statement

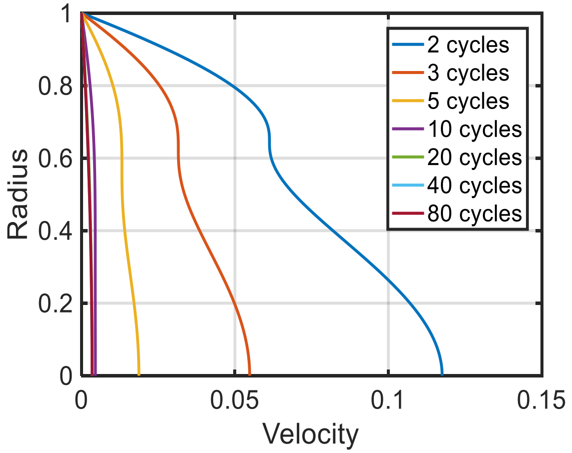
Unsteady Flow of a Cement Slurry

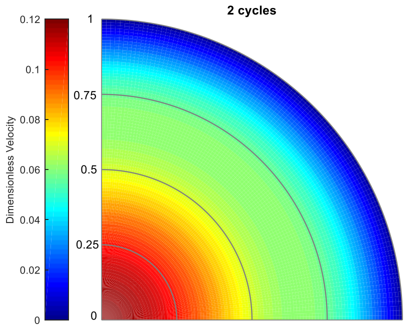


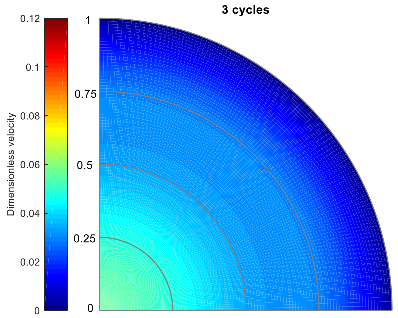
- The motion is unsteady and in transient state
- The flow is assumed to be one-dimensional
- The velocity and the volume fraction forms:

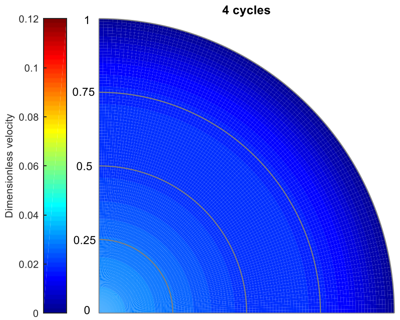
$$\begin{cases} \phi = \phi(\mathrm{r}, t) \\ \boldsymbol{v} = v(r, t)\boldsymbol{e_z} \end{cases}$$

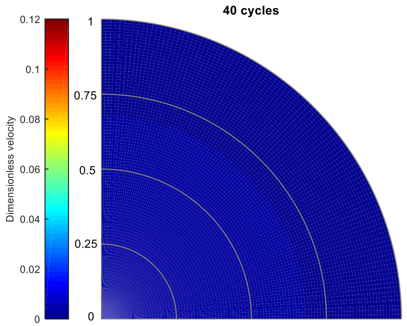
Parametric study

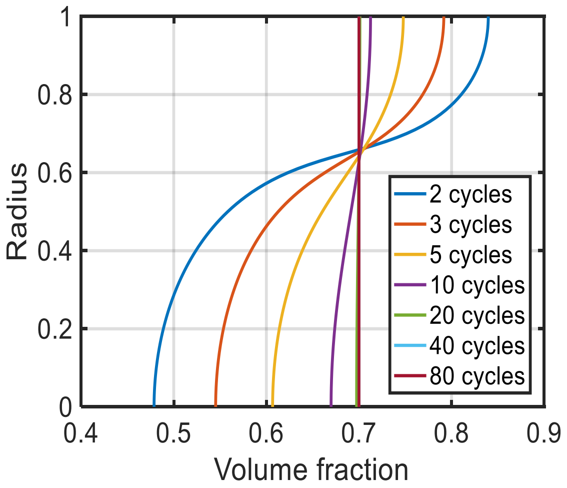


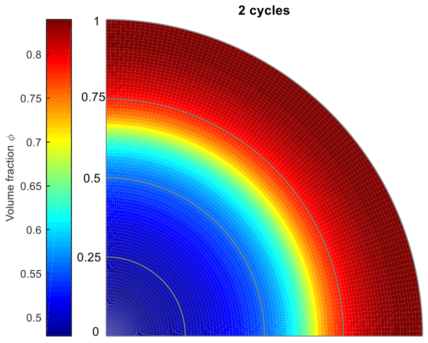


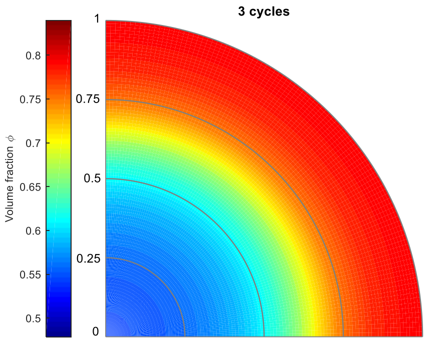


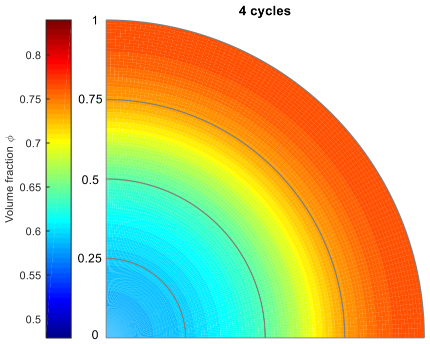


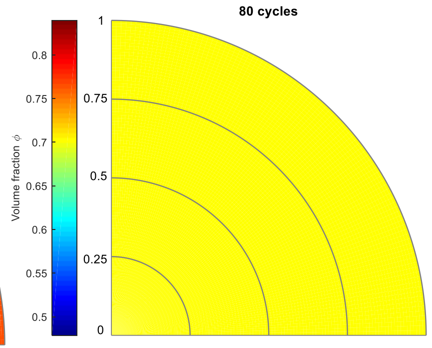


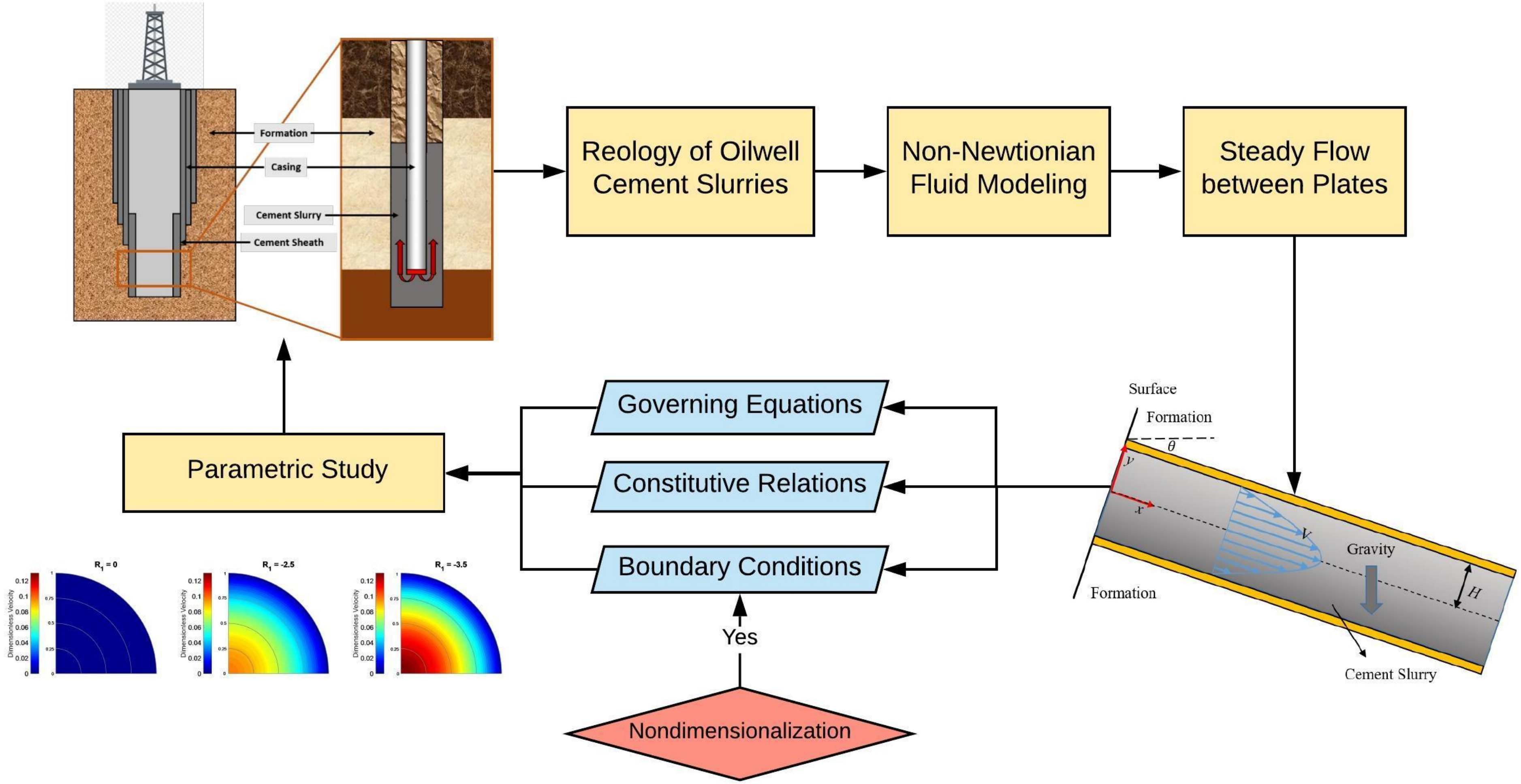











Formation
Casing
Cement Slurry
Cement Sheath
Reology of Oilwell Cement Slurries
Non-Newtionian Fluid Modeling
Steady Flow between Plates
Surface
Formation
Gravity
H
Formation
Cement Slurry
Governing Equations
Constitutive Relations
Boundary Conditions
Parametric Study
Yes
Nondimensionalization

Concluding remarks and future work

Concluding remarks:

- Studied the steady and unsteady flow of cement slurry with different inclinations
- Modeled the cement slurry as a non-Newtonian fluid with viscosity dependent on ***shear rate*** and ***volume fraction***
- Numerically solved the non-dimensionalized governing equations and boundary conditions
- Through parametric study, the velocity and volume fraction profiles are affected by shear rate dependent viscosity, particle flux parameters, angle of inclination, pressure and gravity term

Future work:

- Consider the **yield stress** portion $\boldsymbol{T}_y$ of the stress tensor $\boldsymbol{T}$ from experiment-based models in the non-Newtonian model
- Consider **viscosity** as a function of one or all of the following:
    - Shear rate $\dot{\gamma}$
    - Volume fraction ϕ
    - Temperature T
    - Pressure p
    - Thixotropic behavior (structural parameter describing the degree of flocculation/aggregation $\lambda(\mathrm{t})$)
    - Water-to-cement ratio *w/c*
    - Additives (Superplasticiser)
    - Mixing method …

$$\mu(\dot{\gamma}, \phi, T, p, \lambda(t), w/c, \dots)$$

Acknowledgments

- Oak Ridge Institute for Science and Education (ORISE)
- NETL Group:

  Dr. Barbara Kutchko, Dr. Mehrdad Massoudi, Dr. Eilis Rosenbaum, Dr. Kaushik Dayal
- Dr. Weitao Wu

References

[1] Piot, B. (2009). Cement and Cementing: An Old Technique With a Future?. *SPE Distinguished Lecturer Program*, Society of Petroleum Engineers, Richardson, TX.

[2] Gandelman, R., Miranda, C., Teixeira, K., Martins, A. L., & Waldmann, A. (2004). On the rheological parameters governing oilwell cement slurry stability. *Annual transactions of the nordic rheology society*, 12, 85-91.

[3] Justnes, H., & Vikan, H. (2005). Viscosity of cement slurries as a function of solids content. *Ann. Trans. Nordic Rheology Soc*, 13, 75-82.

[4] Massoudi, M., & Tran, P. X. (2016). The Couette–Poiseuille flow of a suspension modeled as a modified third-grade fluid. *Archive of Applied Mechanics*, 86(5), 921-932.

[5] Bridges, C., & Rajagopal, K. R. (2006). Pulsatile flow of a chemically-reacting nonlinear fluid. *Computers & Mathematics with Applications*, 52(6-7), 1131-1144.

[6] Massoudi, M., & Uguz, A. K. (2012). Chemically-reacting fluids with variable transport properties. *Applied Mathematics and Computation*, 219(4), 1761-1775.

[7] Struble, L., & Sun, G. K. (1995). Viscosity of Portland cement paste as a function of concentration. *Advanced Cement Based Materials*, 2(2), 62-69.

[8] Justnes, H., & Vikan, H. (2005). Viscosity of cement slurries as a function of solids content. *Ann. Trans. Nordic Rheology Soc*, 13, 75-82.

[9] Bonen, D., & Shah, S. P. (2005). Fresh and hardened properties of self-consolidating concrete. *Progress in Structural Engineering and Materials*, 7(1), 14-26.

[10] Bentz, D. P., Ferraris, C. F., Galler, M. A., Hansen, A. S., & Guynn, J. M. (2012). Influence of particle size distributions on yield stress and viscosity of cement–fly ash pastes. *Cement and Concrete Research*, 42(2), 404-409.

[11] Ouyang, J., & Tan, Y. (2015). Rheology of fresh cement asphalt emulsion pastes. *Construction and Building Materials*, 80, 236-243.

[12] Zhou, Z., Wu, W. T., & Massoudi, M. (2016). Fully developed flow of a drilling fluid between two rotating cylinders. *Applied Mathematics and Computation*, 281, 266-277.

NATIONAL ENERGY TECHNOLOGY LABORATORY

Q&A