Steady and Transient Flow of a Cement Slurry

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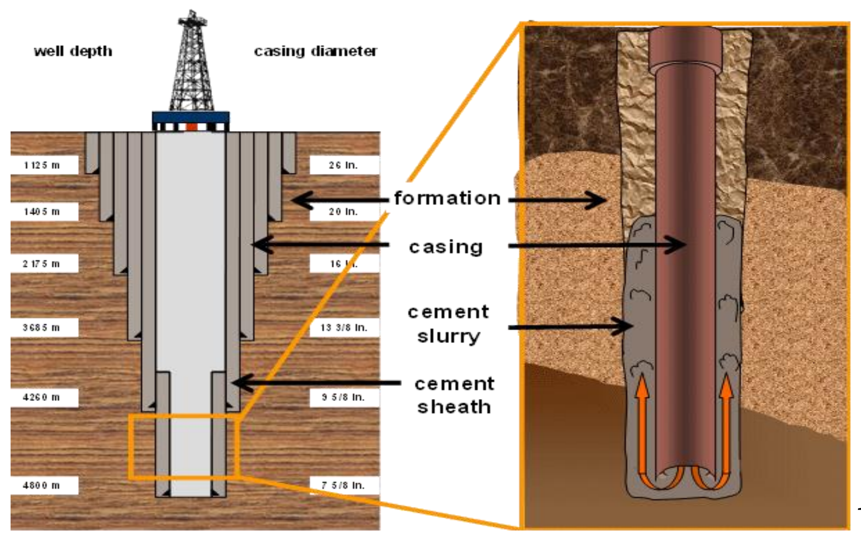
MOTIVATION & OBJECTIVES

Motivation

- Well cementing is the process of placing a cement slurry in the annulus space between the well casing and the surrounding for zonal isolation
- Rheological behavior of oil well cement slurries is significant in well cementing operation

Objectives

- Study the impact of constitutive parameters on behavior of cement slurry
- Perform parametric study for various dimensionless numbers



Piot, B. (2009) [1]

MATHEMATICAL MODEL

- In this paper, we assume that the cement slurry behaves as a non-Newtonian fluid.
- We use a constitutive relation for the viscous stress tensor which is based on a modified form of the second grade (Rivlin-Ericksen) fluid [2].
- Steady motion and unsteady motion are analyzed with one-dimensional flow.

Conservation of mass

$$\frac{\partial \rho}{\partial t} + div(\rho \boldsymbol{v}) = 0$$

ρ: density of cement slurry, which is related to ρ_f (density of pure fluid);
$\boldsymbol{v}$: velocity vector; $\rho = (1-\phi)\,\rho_f$; ϕ: volume fraction

Conservation of linear momentum

$$\rho \frac{d\boldsymbol{v}}{dt} = div\boldsymbol{T} + \rho \boldsymbol{b}$$

d/dt: total time derivative, given by $\frac{d(.)}{dt} = \frac{\partial(.)}{\partial t} + [grad(.)]\boldsymbol{v}$
$\boldsymbol{T}$: stress tensor; $\boldsymbol{b}$: body force vector

Conservation of angular momentum

$$\boldsymbol{T} = \boldsymbol{T}^{\boldsymbol{T}}$$

Stress tensor of fluid [3]:

$$\boldsymbol{T}_v = -p\boldsymbol{I} + \mu_{eff}(\phi, \boldsymbol{A_1})\boldsymbol{A_1} + \alpha_1 \boldsymbol{A_2} + \alpha_2 {\boldsymbol{A_1}}^2$$

p: pressure; α_1, α_2 are the normal stress coefficients; μ_{eff} is the effective viscosity, which is dependent on volume fraction and shear rate [4]:

$$\mu_{eff}(\phi, \boldsymbol{A_1}) = \mu_0 \left(1 - \frac{\phi}{\phi_m}\right)^{-\beta} \left[1 + \alpha \mathrm{tr} {\boldsymbol{A_1}}^2\right]^m$$

ϕ_m: maximum solid concentration packing;
$\boldsymbol{A_n}$: n-th order Rivlin-Ericksen tensors,
where $\boldsymbol{A_1} = \nabla \boldsymbol{v} + \nabla \boldsymbol{v}^T$; $\boldsymbol{A_2} = \frac{d\boldsymbol{A_1}}{dt} + \boldsymbol{A_1}\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^T \boldsymbol{A_1}$

Convection-diffusion equation

$$\frac{\partial \phi}{\partial t} + div(\phi \boldsymbol{v}) = -\mathrm{div}\boldsymbol{N}$$

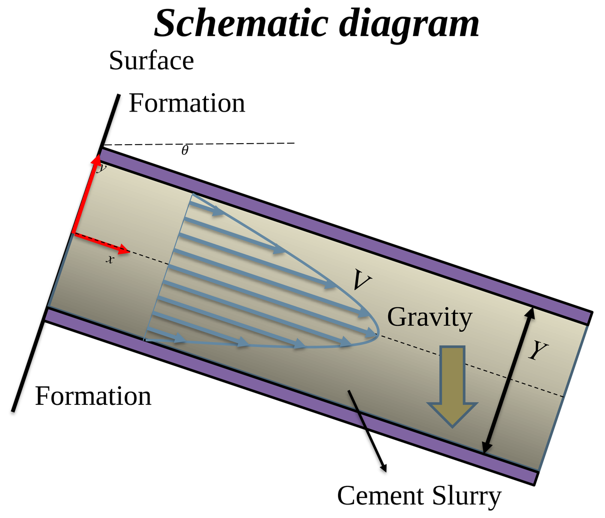
Particle flux [5]: particles collisions; spatially varying viscosity; Brownian diffusive flux

$$\boldsymbol{N} = -a^2 \phi K_c \nabla(\dot{\gamma}\phi) - a^2 \phi^2 \dot{\gamma} K_\mu \nabla\left(ln\mu_{eff}\right) - D\nabla\phi$$

where, $D\nabla\phi = \eta \left\| {\boldsymbol{A_1}}^2 \right\| \cdot D_0\, [K_1 + K_2(1-\phi)^2 + K_3(\phi_m - \phi)^2 H(\phi_m - \phi)]^2 \nabla\phi$ [6]

Geometry of the problem

Schematic diagram



Governing equations are made dimessionless:

$$\bar{y} = \frac{y}{H} \qquad \bar{v} = \frac{v}{V} \qquad \bar{r} = \frac{r}{R}$$

$$\begin{cases} \frac{\partial}{\partial \bar{y}} \left\{ \left(1 - \frac{\phi}{\phi_m}\right)^{-\beta} \left[1 + R_0 \left(\frac{dv}{dy}\right)^2\right]^m \frac{d\bar{v}}{d\bar{y}} \right\} = R_1 - R_2 \sin\theta \\ \frac{K_c}{K_\mu}\left(\phi^2 \frac{d}{d\bar{y}}\left|\frac{d\bar{v}}{d\bar{y}}\right| + \phi \left|\frac{d\bar{v}}{d\bar{y}}\right| \frac{d\phi}{d\bar{y}}\right) + \frac{m\phi^2 \left|\frac{d\bar{v}}{d\bar{y}}\right|}{1 + R_0\left(\frac{d\bar{v}}{d\bar{y}}\right)^2} \cdot 2R_0 \frac{d\bar{v}}{d\bar{y}} \frac{d^2\bar{v}}{d\bar{y}^2} + \frac{\beta}{\phi_m}\phi^2 (1 - \frac{\phi}{\phi_m})^{-1} \frac{d\phi}{d\bar{y}} \left|\frac{d\bar{v}}{d\bar{y}}\right| \\ + [R_3 + R_4(1-\phi)^2 + R_5(\phi_m - \phi)^2 H(\phi_m - \phi)] \frac{d\phi}{d\bar{y}} \left(\frac{d\bar{v}}{d\bar{y}}\right)^2 = 0 \end{cases}$$

where, $R_0 = 2\alpha \frac{V^2}{H^2}$; $R_1 = \frac{\partial \hat{p}}{\partial x} \frac{H^2}{\mu_0 V}$; $R_2 = \frac{\rho_f g H^2}{\mu_0 V}$;
$R_3 = \frac{2\eta D_0 V K_1}{a^2 K_\mu H}$; $R_4 = \frac{2\eta D_0 V K_2}{a^2 K_\mu H}$; $R_5 = \frac{2\eta D_0 V K_3}{a^2 K_\mu H}$

Steady flow

$$\begin{cases} \phi = \phi(y) \\ \boldsymbol{v} = v(y)\boldsymbol{e_x} \end{cases}$$

Unsteady flow

$$\begin{cases} \phi = \phi(r,t) \\ \boldsymbol{v} = v(r,t)\boldsymbol{e_z} \end{cases}$$

Boundary conditions

Steady flow $\bar{v}(\bar{y} = -1) = 0; \bar{v}(\bar{y} = 1) = 0; \int_{-1}^{1} \bar{\phi}\, d\bar{y} = \bar{\phi}_{avg}$

Unsteady flow $\bar{v}(\bar{r} = 1, \bar{t}) = 0; \frac{\partial \phi}{\partial \bar{r}}(\bar{r} = 1, \bar{t}) = 0; \frac{\partial \bar{v}}{\partial \bar{r}}(\bar{r} = 0, \bar{t}) = 0; \frac{\partial \phi}{\partial \bar{r}}(\bar{r} = 0, \bar{t}) = 0$ [7]

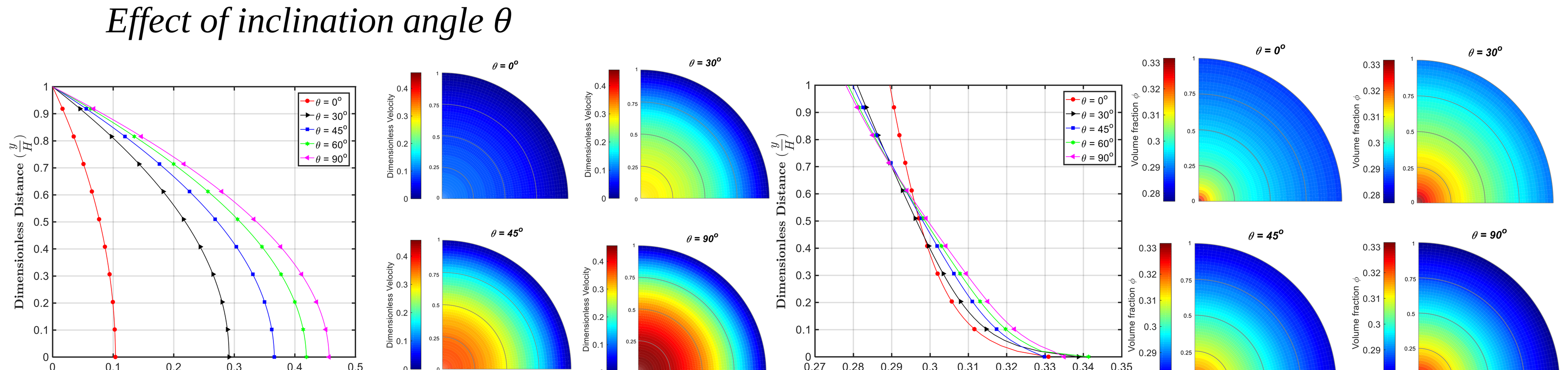
RESULTS

Parametric study with designated value of dimensionless numbers

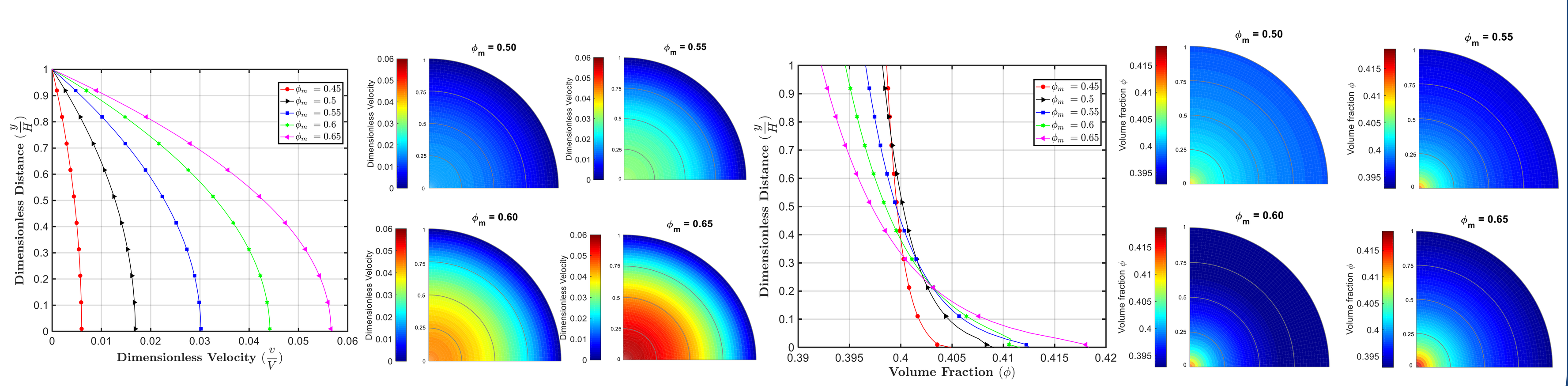
Dimensionless #	Value
ϕ_m	0.45, 0.5, 0.55, 0.6, 0.65
K_c/K_μ	0, 0.02, 0.04, 0.06, 0.08
θ	0º, 30º, 45º, 60º, 90º
m	-0.5, 0, 0.5, 1
R_0	0.01, 0.1, 1, 10
R_1	0, -1.5, -2.5, -3.5
R_2	0, 0.5, 1, 1.5
R_3	0.01, 0.1, 1
R_4	0.01, 0.1, 1
R_5	0.01, 0.1, 1

Steady flow

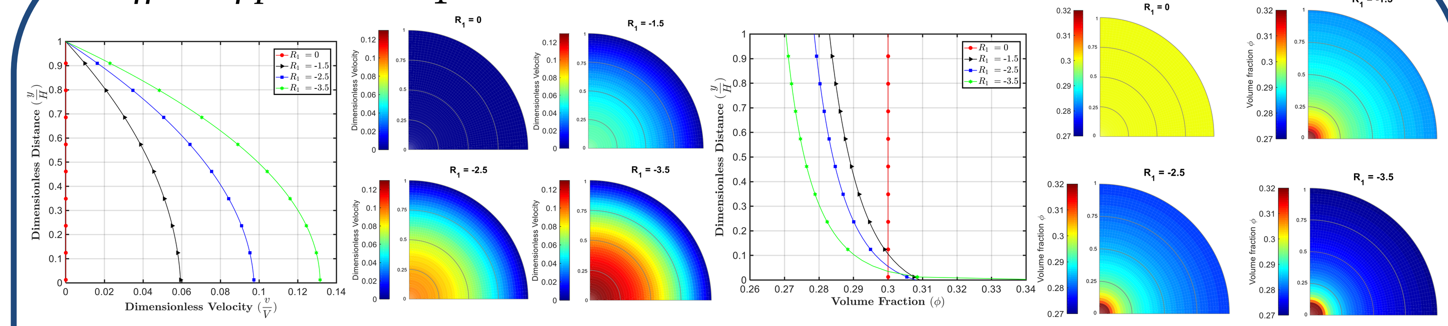
Effect of inclination angle θ



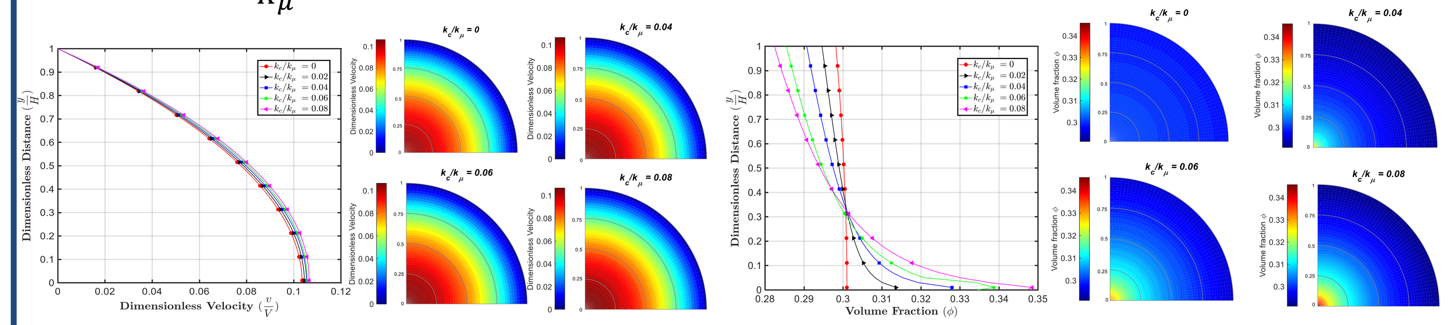
Effect of ϕ_m



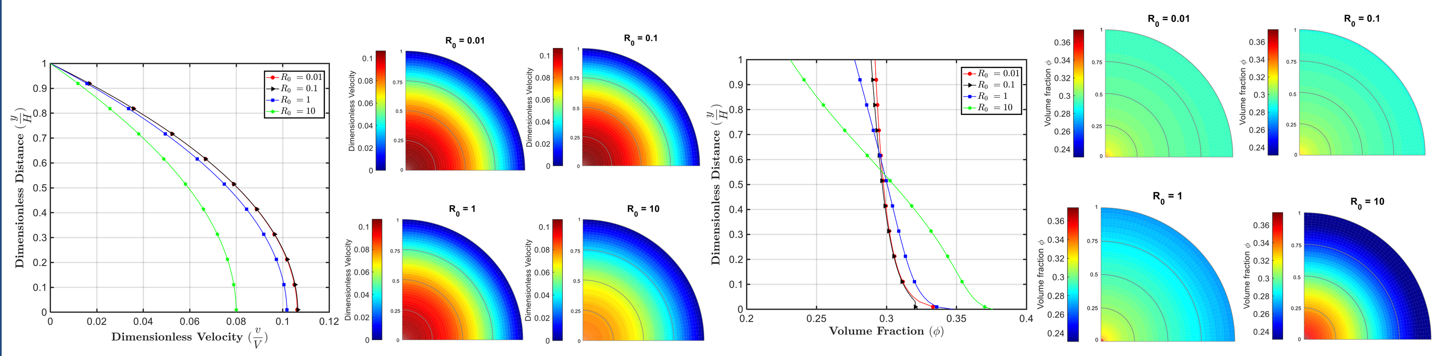
Effect of pressure R_1



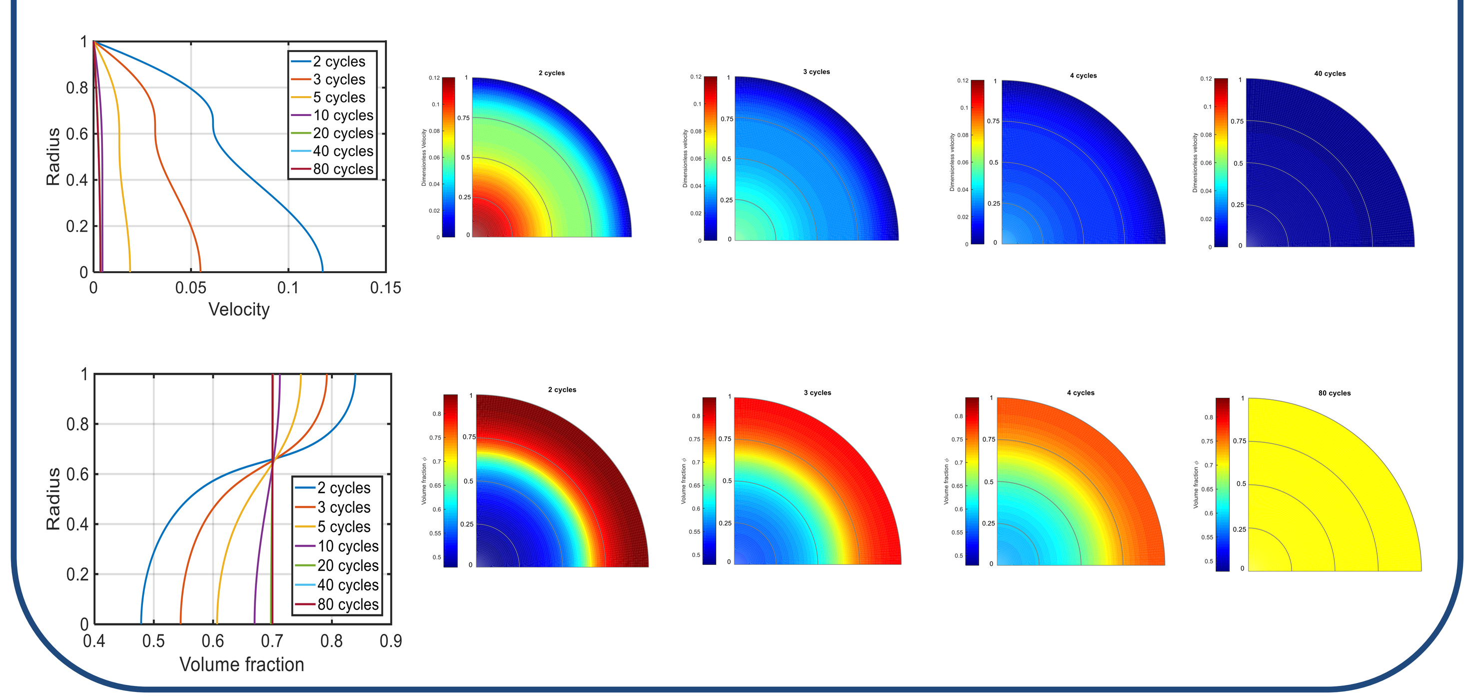
Effect of $\frac{K_c}{K_\mu}$



Effect of R_0



Unsteady flow



CONCLUSIONS

- The parametric study indicates that maximum packing ϕ_m, concentration flux parameters $\frac{K_c}{K_\mu}$, the angle of inclination θ, pressure and gravity terms affect the velocity and particle distribution significantly.
- This study is simply a preliminary case and further studies will be performed where the effects of diffusion, heat transfer, such as viscous dissipation and yield stresses will be considered.

References

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