# A model with cosmological Bell inequalities


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**Abstract**

We discuss the possibility of devising cosmological observables which violate Bell's inequalities. Such observables could be used to argue that cosmic scale features were produced by quantum mechanical effects in the very early universe. As a proof of principle, we propose a somewhat elaborate inflationary model where a Bell inequality violating observable can be constructed.


*Dedicated to Andy Strominger on the occasion of his* $60^{th}$ *birthday.*

# 1 Introduction

According to the theory of inflation, primordial density fluctuations have a quantum mechanical origin [1, 2, 3, 4, 5]. An important problem is to find compelling evidence for their quantum nature. In other words, one would like to rule out alternative scenarios where the fluctuations originated through classical statistical mechanics during an inflationary phase. This can happen in models where there is a form of friction converting the inflaton energy into other forms of energy that then produce the fluctuations as in [6, 7, 8, 9]. In such theories, the correlations between the fluctuations are classical in origin. One can compare detailed predictions for higher point correlation functions, or even put bounds on the two and three point functions, see e.g. [10].

Formally, the wavefunction of the universe produced by inflation is highly entangled. Therefore one expects that it should be possible to perform a Bell inequality violating experiment [11]. Such an experiment would conclusively demonstrate the quantum origin of the fluctuations. Here we will ask the conceptual question of whether, and in what sense, could we ever perform such a cosmological Bell experiment. A Bell experiment involves making measurements at two distant locations, call them Alice's and Bob's location. At each of these locations one should be able to measure two non-commuting operators. In cosmology, we can make observations on two spatially separated cosmic patches that have been causally disconnected since the time of reheating. However, it is not possible to measure two non-commuting operators for the following reason. The standard observables involve measuring the values of the cosmological curvature fluctuations (or adiabatic density fluctuations), $\zeta(x)$. However, it is not possible to measure its conjugate momentum, $\pi_\zeta$. [1]

At this stage one could conclude that it is impossible to perform a Bell type measurement in cosmology. But this would be premature. First notice that *any* observation we make consists of commuting observables, once we consider only the final decohered observables [12]. In order to run the Bell experiment one repeats the experiment many times, interpreting each run of the experiment as occurring on the same quantum state, and putting the boundary between classical and quantum just after the measurements. We can view cosmology in a similar way. We can view separate patches of the sky as running different cosmological experiments on the same underlying quantum state. We can further divide these patches into a pair of smaller subregions, which were causaly disconnected at some earlier time. By suitably observing properties of these subregions one could construct an observable subject to a Bell inequality. There is a Bell inequality if one assumes that the probability distribution that we observe today was generated by an inflationary process leading to a relation between scale and time. Namely short distance features were created after the long distance features. In this case, we can translate the standard causality constraints in the approximately de-Sitter space into constraints on the spatial structure of the wavefunction.

[1] Bell inequalities were discussed previously in [13], but with the assumption that one can indeed measure the momentum $\pi_\zeta$.

We have been unable to find an observable of this kind using the simplest inflationary theory consisting of the metric plus a single scalar field. However, we will present a more baroque inflationary scenario where one can prove the quantum origin of some fluctuations. This scenario was solely designed to make a Bell inequality violating experiment possible and it seems unlikely that Nature would choose it. We think it is nevertheless valuable to have a fairly concrete model where one can clearly understand the various issues involved. Hopefully, a clever reader (or non-reader) will find a Bell inequality violating observable in a more realistic model.

This paper is organized as follows. First, in section two, we review the standard discussion of Bell inequality experiments. In section three we review a few features of inflation and discuss the conceptual set up for a cosmological Bell experiment. In section four we present a baroque inflationary model where a Bell inequality violating experiment is possible. We conclude with a discussion.

## 2 Review of Bell inequality experiments

In order to set up an experiment with a Bell inequality it is necessary to have the following elements [11]. See figure 1.

- Two separate spatial locations where measurements are performed. Call them Alice's location and Bob's location.
- An entangled quantum state, with components at these two locations.
- At each location we should be able to perform two possible measurements that are described by two non-commuting operators. Call them $A$ and $A'$ for Alice's location and $B$, $B'$ for Bob's location, with $[A, A'] \neq 0$ and $[B, B'] \neq 0$.
- Alice should have the "free will" to select randomly between the $A$ and $A'$. The same holds for Bob for his choice of $B$ and $B'$ These choices are made locally and are uncorrelated with each other. These choices are made by physics outside the quantum system under consideration. In practice this is done by looking at local random variables that are assumed to be independent of the quantum system in question.[2]
- We should have a quantum measurement of these operators with definite answers.
- We classically transmit the results of these measurements to a central location where we correlate the results.

[2] There is a Bell inequality violating measurement involving $B\,\bar{B}$ oscillations [14] where the validity of this assumption has been called into question [15, 16].

Let us review the simplest and most discussed example. Here the entangled state corresponds to a pair of spins, one at each location. The operators correspond to measuring the spin along various axes and have eigenvalues $\pm 1$. In other words, we have that $A = \vec{n}.\vec{\sigma} = n^i \sigma^i$, with $\sigma^i$ the Pauli matrices. And $A' = \vec{n}'.\vec{\sigma}$. We have similar expressions for $B$ and $B'$ acting on the second spin.

In this situation is it useful to consider the quantity introduced in [17]

$$\langle C \rangle = \langle AB \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle \tag{2.1}$$

This is a particular linear combination of expectation values for different choices of operators or detector settings.

In a local classical hidden variable theory one can prove the Bell inequality $|\langle C \rangle| \leq 2$ as follows. For each value of the hidden variables we have a well defined answer for each of the two possible measurements at each side. Namely, a unique value for $A$ and also for $A'$, similarly for $B$ and $B'$. Furthermore, causality implies that the answer for $B$ and $B'$ does not depend on whether we measure $A$ or $A'$. Therefore, for each value of the hidden variable we can have either $B = B'$ or $B = -B'$. And in each of the two cases either the first two terms in (2.1) cancel or the last two terms cancel. Therefore the maximum value of $|C|$ is two.

In quantum mechanics, the expectation value of $C$ can be bigger. In fact, in quantum mechanics we can view (2.1) as the expectation value of the quantum operator $C = AB + AB' + A'B - A'B'$. It is easy to check that its square is

$$C^2 = 4 - [A, A'][B, B'] \tag{2.2}$$

where we used that the square of each of the measured operators is one, $A^2 = 1$, $A'^2 = 1$, etc. Now the commutator term can make $C^2$ larger than four. Only when this commutator is non-zero can we have $|\langle C \rangle|$ larger than two, violating the Bell inequality. Notice also that $|[A, A']| \leq 2$.[3]. Therefore it is easy to see that $\langle C^2 \rangle \leq 8$ or $|\langle C \rangle| \leq 2\sqrt{2}$ [18]. Choosing

$$A = \sigma_x \ , \quad A' = \sigma_y \ , \quad B = \sin\theta \sigma_x + \cos\theta \sigma_y \ , \qquad B' = \cos\theta \sigma_x - \sin\theta \sigma_y \tag{2.3}$$

we can check that on a spin singlet state we get $C = -2\sqrt{2}\cos(\theta - \frac{\pi}{4})$. For $\theta = \pi/4$ we get the maximal violation which has the extra $\sqrt{2}$ factor.

There has also been discussion of Bell inequalities for harmonic oscillator degrees of freedom. For example, [19] considers squeezed states and defines operators that are translation conjugates of $(-1)^n$ where $n$ is the occupation number operator. These operators are not easy to measure in the cosmological context. They certainly cannot be measured after reheating [13], due to the impossibility of measuring $\pi_\zeta$. For a more realistic inflationary scenario, it will be necessary to consider entangled states and measurements of the harmonic oscillator degrees of freedom that describe the scalar or tensor fluctuations. However, for our baroque model this discussion in terms of spins will be enough.

[3]This inequality is saturated for Pauli matrices. For example, consider $A = \sigma_x$ and $A' = \sigma_y$, which leads to $[\sigma_x, \sigma_y] = 2i\sigma_z$.

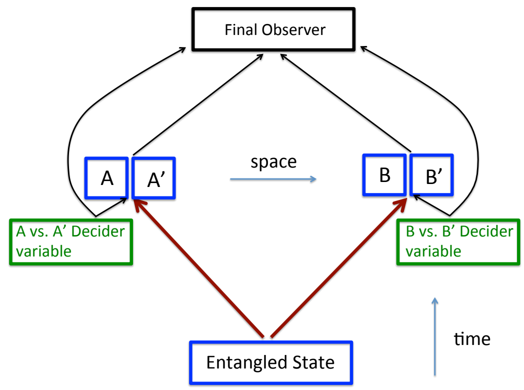



Figure 1: Set up for a Bell inequality violating experiment. An entangled state is produced in the past and its two parts are transmitted to Alice and Bob who perform measurements on $A$ or $A'$ or $B$ or $B'$. The choice of experiment ($A$ vs. $A'$) is determined by a local variable, which we can call Alice's "free will" or decider variable. The results of the experiments and the values of the decider variables are classically transmitted to a central observer who computes the statistical averages. All classical communications have been denoted here by black lines.

# 3 Set up for a cosmological Bell experiment

## 3.1 Review of inflation

We can write the metric of a uniform, spatially flat, FLRW space as

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 = a^2(\eta)[-d\eta^2 + d\vec{x}^2] \tag{3.4}$$

where $t$ is proper time and $\eta$ is conformal time. The $\vec{x}$ coordinates are called "comoving coordinates". Conformal time is particularly useful to display causal relations. During the inflationary period the scale factor grows exponentially, $\frac{\dot{a}}{a} \sim H(t)$, with $H(t)$ slowly varying. The variation of $H(t)$ is due to the slow evolution of a scalar field, which in the classical approximation is a function of time only $\phi = \phi_0(t)$. Inflation ends when $\eta \sim 0$, see figure 2. For a review, see e.g. [20].

Quantum mechanics produces spatially dependent fluctuations in the values of the scalar field [1, 2, 3, 4, 5]. These give rise to adiabatic curvature fluctuations in the late universe. In the leading approximation, we can independently follow the evolution of each Fourier mode, $\phi_{\vec{k}}(\eta)$. Each of these Fourier modes behaves as a harmonic oscillator with a time dependent mass and fixed frequency. Each mode corresponds to a wave whose wavelength is fixed in the $\vec{x}$ coordinates of (3.4). Their physical wavelength is very small at early times and very large towards the end of inflation. When $k|\eta| \sim 1$ the fluctuations are created and the value of $\phi_k$, or more properly that of $\zeta_k = -\frac{H}{\dot{\phi}_0 M_{pl}} \phi_k$, is fixed until the mode reenters the horizon during the Big Bang phase, see figure 2. The non-constant part of $\zeta$ decays exponentially in proper time after we exit the horizon. Furthermore, the amplitude of the second independent solution decays as $(\eta k)^3$ after horizon exit. This is also the order of magnitude of the commutator of $k^3[\zeta_{\vec{k}}, \dot{\zeta}_{-\vec{k}}] \propto i(\eta k)^3$. This goes as $e^{-3N_k}$ where $N_k$ is the number of e-folds that remain from the time the mode exited the horizon till the end of inflation[4]. For cosmological size modes this is a number bigger than about $N_k > 30-40$. Therefore, if we wanted to make a measurement of the momentum we would need a precision greater than $10^{-90}$ which, even for a theorist, looks impossible. Moreover, the measurement of any cosmological observable is limited by cosmic variance which goes as $1/\sqrt{N_{mod}}$ where $N_{mod}$ is the number of modes we observe. Even if we observe all modes up to the size of a galaxy, this gives us an ultimate precision of about $10^{-10}$.

In summary, inflation gives us a probability distribution for $\zeta(x)$ at the time of reheating of the form $\rho[\zeta(x)] = |\Psi[\zeta(x)]|^2$. Since we cannot measure the decaying mode, we can view the state of the universe at the reheating surface as characterized by the classical probability distribution $\rho[\zeta(x)]$. If we consider two well separated points $x_A$ and $x_B$ then the operators $\zeta(x_A)$ and $\zeta(x_B)$ commute with each other. Therefore it is impossible to obtain a Bell inequality out of these operators.[5]

If more than one field is involved, then we can also have isocurvature fluctuations, but the conclusion is the same. The probability distribution is classical and has the form $\rho[\zeta(x), \theta(x)] = |\Psi(\zeta(x), \theta(x))|^2$, where $\theta(x)$ is the second field.

Therefore, if we view Alice and Bob as doing experiments after the end of inflation, then we will not be able to set up a Bell inequality for primordial perturbations. This is true if we make the realistic assumption that we cannot measure the conjugate momentum. This is disappointing! But fortunately this is not the end of the story.

## 3.2 Setting up a cosmological Bell experiment

Implicit in any discussion of the Bell inequality is the assumption of when we make the quantum to classical transition. In other words, when the measurement occurs. For example, even in the standard setup of figure 1 it is important that the quantum to classical transitions happen in such a way that we can view all black lines as classical. Similarly, in cosmology, we can avoid the previous conclusion if we imagine an Alice and a Bob who did their experiments during inflation and “wrote” their results on the classical distribution $\rho[\zeta(x), a(x)]$. In other words, this classical distribution is viewed as the classical message which is transmitting to us the result of experiments which happened during inflation, see figure 3(b).

Now, what are suitable Alices and Bobs?. Of course, they do not need to be actual people. More important than Alice and Bob are the measurements that they do. A

[4] In a general FLRW background this commutator goes as $[\zeta, \dot{\zeta}] \propto a^{-3}$ so that it becomes even smaller after the end of inflation and subsequent horizon reentry, since the universe continues expanding.

[5] See [21, 22] for a related discussion.

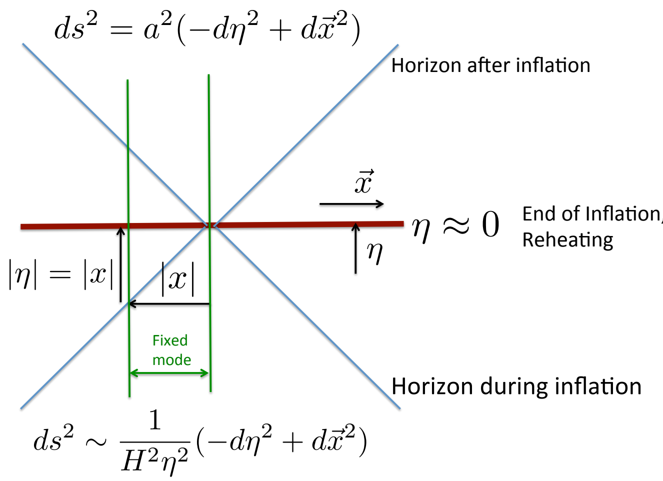



Figure 2: Sketch of the evolution of the universe. The vertical direction is conformal time, $\eta$, and the horizontal is space. We have an early period of inflation ending at $\eta \sim 0$ followed by an ordinary radiation/matter dominated universe. A comoving distance $x$ crosses the horizon during inflation at time $\eta \sim -|x|$. So scales correspond to time. The vertical green line follows a given wavelength mode as it crosses the apparent horizons given by the diagonal lines.

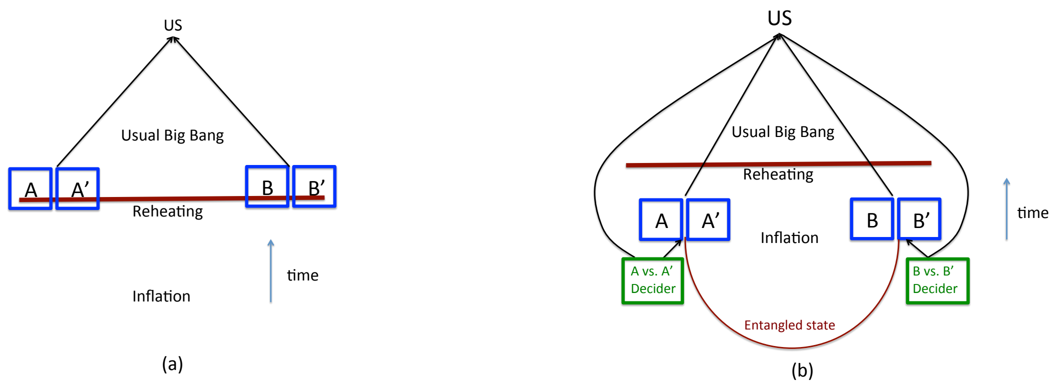



Figure 3: (a) An unsuccessful set up for a cosmological Bell inequality. There are no non-commuting operators that we can measure after the end of inflation. (b) A diagram of a more successful set up where the whole process occurs during inflation. We generate an entangled state. Some time later we generate the variables that will decide whether we make an $A$ or $A'$ measurement and similarly for $B$ and $B'$. These decider variables as well as the result of the measurement should remain as classical variables for the rest of the evolution and be visible to us.

measurement is a particular unitary evolution of the combined system plus measuring apparatus, whose state can be viewed as classical. We need to produce all the elements of the Bell inequality discussion out of fluctuations. The initial entangled state would be a quantum fluctuation, the measurement apparatus would be another quantum fluctuation that has already become classical. It should have shorter wavelength than the one corresponding to the entangled state. This shorter wavelength fluctuation should act both as the decider variable as well as measuring apparatus. The measurement should be some process which depends on the quantum state of one of the pieces of the entangled sate. The result of the measurement should be transmitted to us. Therefore the measurement should be some process which produces a large effect on the fluctuations so that we can see it today. The state of the shorter wavelength fluctuations that acted as "decider" variables should also be preserved and transmitted to us. We know one mechanism for transmitting this information. Namely, through the inflationary evolution of massless (or nearly massless) scalar fields where small fluctuations are amplified and stretched to cosmic scales. In figure 3(b) we sketch the type of setup that we have in mind.

Unfortunately, we have not been able to produce a suitable observable using the simplest single scalar field model. One difficulty is the following. We mentioned above that the fluctuations become classical as they exit the horizon. The fluctuations which will serve as the detector and decider variables are necessarily of shorter wavelength that the ones in the entangled state. This is in order to ensure that the values of the decider variables are determined locally, independently for Alice and Bob. Unfortunately this also means that the entangled state we are attempting to measure is actually more classical than the measuring device, which is the opposite of what we want.

# 4 A baroque model that leads to a Bell inequality measurement

Instead of giving up, we will imagine that we have a more complicated model of inflation where we can indeed set up a Bell inequality. Simply as a matter of principle, we would like to ask whether there is an inflationary model that is Bell-friendly. Namely, one that spontaneously creates, from the vacuum, all the necessary elements for the Bell-inequality experiment, performs the measurement and records the results for the post-inflationary observer. One can imagine several ways of doing this, but we will concentrate on one particular example in order to display a concrete model.

The model builds the various elements in the Bell experiment as follows

- The entangled state consists of a pair of massive particles that carry an isospin degree of freedom. The isospin is entangled in the singlet state. These particles are very massive at the beginning of inflation, then they get lighter at a specific time and then they become heavier again. Therefore they are created at a specific time during inflation, the time when they become lighter.

- The decider variables or detector settings correspond to an axion field. The axion field has a variable decay constant, $f_a$. Again, this is large for most of the time, but it becomes smaller, comparable to $H$ for a few efolds. This means that the dimensionless axion angular variable has larger fluctuations at a particular scale. The axion field survives beyond the end of inflation and produces isocurvature fluctuations. These isocurvature fluctuations retain the information of the detector settings for the post-inflationary observer.

- The measurement occurs as follows. We postulate an isospin dependent contribution to the mass of the particles. This contribution arises from a term that becomes important a few Hubble times after that pair is created. This coupling depends on the axion value. The measurement consists in an interaction between the mass of this particle and the inflaton.

- The result of the measurement is preserved for the post-inflationary observer as follows. The massive particles classically modifies the evolution of the inflaton so as to produce a discernible fluctuation, or hot spot, larger than the quantum fluctuations. These hot spots are centered where the massive particles are located. Their amplitude depends on the projection of the isospin of the massive particle along an axis whose direction is axion dependent.

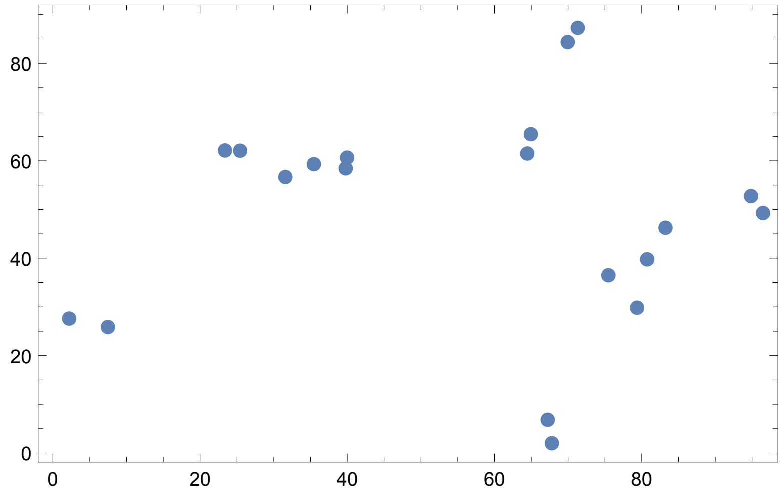



Figure 4: One instance of the particle creation process with the statistics that follows from a time dependent mass during inflation. The center of mass position of the pair has a uniform random distribution. The distribution of distances between the pairs peaks at a distance that is set by the time during inflation where the particles became less massive. The axes are comoving coordinates. We see that it is reasonably easy to recognize the members of a particular pair.

The final conclusion is that this baroque universe has produced a very particular pattern of curvature and isocurvature fluctuations. The pattern of curvature fluctuations is mainly the usual almost scale invariant one with additional hot spots where there is a significant deviation. These spots come in well separated pairs. Each separated pair constitutes a particular instance of an entangled pair together with a measurement. The measurement operation depends on the value of the axion field, which can be read off from the isocurvature fluctuations.

In order to have a more clear example we now discuss these elements in more detail.

## 4.1 Creation of well separated pairs of massive particles

We imagine massive particles whose mass depends on the inflaton $\phi$, $m(\phi)$. The particles are generically very massive, $m \gg H$. But we also imagine that there is a particular value of the inflaton, $\phi_0$, where the particles become relatively light $H \sim m$, but somewhat bigger than $H$ so that we do not produce too many of them. As the inflaton evolves, there is a particular time when it passes through $\phi_0$. At this time particle pairs are created. We are interested in a situation where the pair creation is rare enough that particles are well separated, but strong enough that we produce lots of pairs in the observable universe. It is useful to think of the background metric as approximated by

$$ds^2 = H^{-2}\frac{-d\eta^2 + d\vec{x}^2}{\eta^2} \tag{4.5}$$

The classical time dependence of the inflaton leads to a time dependent mass $m(\eta)$. The equation of motion for the massive field is

$$h'' - \frac{2}{\eta}h' + (k^2 + \frac{m^2(\eta)}{\eta^2 H^2})h = 0 \tag{4.6}$$

We imagine a situation where the WKB approximation is approximately valid for all times. If the WKB approximation were exactly valid, there would be no particle creation. We can consider a small amount of particle creation which is characterized by a Bogoliubov coefficient $\beta$ which tells us the mixing between the two WKB solutions. Here $\beta(k)$ is small in the region $k|\eta_0| < 1$ and it is very small for larger values. This leads to a probability distribution for the relative comoving distance $x$ between the two pairs which peaks at $x \sim |\eta_0|$. Of course, the probability distribution for the center of mass of the pair is completely uniform. A simulation of such a distribution is given in figure 4. We give a bit more details in appendix A. The important feature here is that the typical distance $x$ (in comoving coordinates) is of the order of the time at which the pair is created.

## 4.2 Axion with time dependent $f_a$

In this subsection we imagine an axion field with an action

$$S = \int f_a^2(\nabla\theta)^2 = \int d\eta d^3x \frac{f_a^2(\eta)}{H^2}\frac{[(\partial_\eta\theta)^2 - (\partial_i\theta)^2]}{\eta^2} \tag{4.7}$$

where $\theta$ is a periodic field, $\theta = \theta + 2\pi$.[6] We assume that the axion "decay constant" $f_a$ depends on the inflaton $\phi$. Since $\phi$ is time dependent, then $f_a$ becomes time dependent. We assume that $f_a$ starts out large, $f_a \gg H$, and becomes smaller, but larger than $H$ at $\phi_1$ and that then it rises and becomes large again. The net effect of this is that the fluctuations of the angular variable $\theta$ are larger at distances corresponding to time $\eta_1$, $x \sim |\eta_1|$, and then they become much smaller at shorter distances. Simulated axion fluctuations with these properties can be found in figure 5. It is conceptually cleaner to imagine that $\phi_1$ comes at a slightly later time than $\phi_0$ discussed in section 4.1, but no great harm is done if they happen together. But it is important that $f_a$ rises for later times. In figure 5 we show both the axion field and the created particle pairs, zooming on a particular pair. We see that each member of the pair can be in regions with different values of the axion field. The evolution of the axion was fine-tuned to generate rather different values of $\theta$ at

[6] We are using the word "axion" to describe a periodic field, but this field does not have to be the QCD axion.

the locations of each member of the pair. The increase of $f_a$ has allowed us to suppress quantum fluctuations at shorter distances so that for the next step we have a well defined value for the axion field at the location of each particle.

Also, since the axion has fluctuations at distances shorter than the separation between the massive pairs, we can view the fluctuations around each pair as being produced locally. In other words, since the axion becomes the decider variable, we want to ensure that it is chosen locally around each massive particle, in a way that independent from what happens around the other massive particle of the pair. This happens in this model if we view the generation of the fluctuations as occurring when the modes cross the horizon.[7]

We imagine that the axion field survives beyond the end of inflation and that there is a small potential $V \sim \Lambda^4 \cos\theta$ which gives it a mass and causes it to oscillate at late times. This contributes as a dark matter component. The axion fluctuations give rise to isocurvature fluctuations in this dark matter component. In our universe we do not see such fluctuations in the overall dark matter density, but this could be a subdominant contribution of the dark matter. Here the point is that, in principle, by observing the size of these isocurvature fluctuations we can determine the initial amplitude of the field $\theta$ in the corresponding regions of the universe. Now in order not to produce domain walls we want that $f_a$ remains always smaller than $H$ so that the axion field is always around the same minimum of the potential. Indeed, in figure (5) we see that fluctuations are smaller than $\pi$.

## 4.3 Seeing the massive particles after the end of inflation

The particle pairs that we discussed above will be diluted by the expansion of the universe and one can wonder how they will ever be observable. First we will discuss a mechanism that makes them observable. Later we will modify the mechanism to include the isospin degree of freedom.

We have postulated the existence of massive particles whose mass depends on the inflaton. This implies that there is a coupling between the inflaton and these massive particles. We can think of the massive particles as a classical source for the inflaton field. This produces a perturbation of the inflaton around the location of the particles [25, 26]. In other words, the massive particles "pull" on the inflaton, locally delaying its evolution. This in turn delays the end of inflation, causing a further expansion in this region. In order to have an observable "classical" signal, we want the net effect of this pull to be larger than the quantum fluctuations. In the approximation that $H$ and the slow roll parameter $\epsilon$ are constant, there is a surprisingly simple expression for the effect of the massive particle. We find that the late time expectation value of the curvature fluctuations $\zeta(x)$ due to the

[7] Of course, since the whole region under consideration started out in a subhorizon region, one can call this independence assumption into question. Of course, this assumption can also be called into question, for the same reason, in present day Bell experiments (even [23]) since the whole observable universe was initially contained in a small Hubble region. In other words, we are applying for inflation the same kind of assumptions we are used to applying for present day experiments.

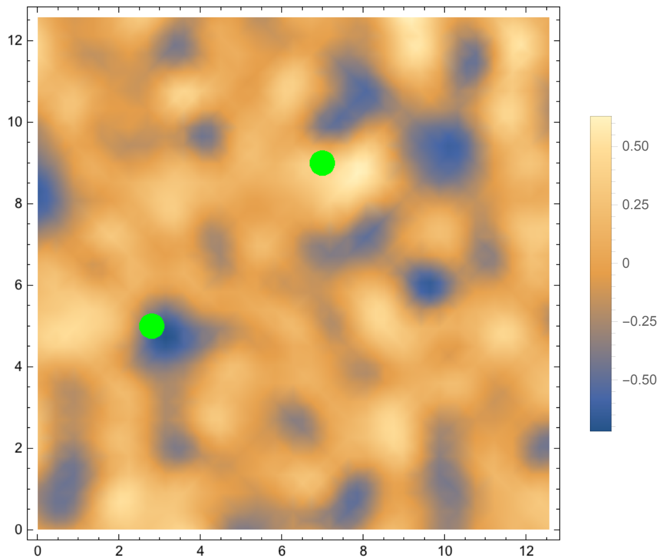


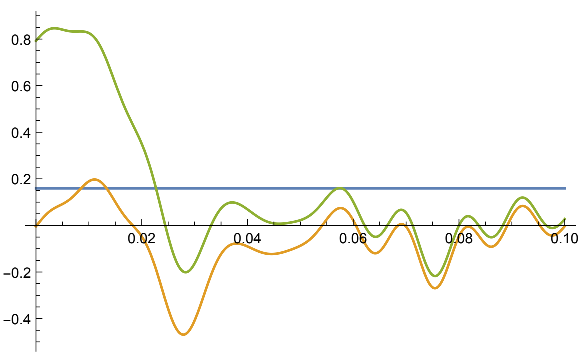

Figure 5: Here we see a profile of the generated axion field. It has features on characteristic scales which are small compared to the separation between the massive particles, so that each massive particle sees a different value of the axion when the "isospin" of each particle is measured. The green dots are two members of a particular pair of created particles. This figure is zoomed relative to figure 4.

presence a particle at $\vec{x} = 0$ is given by (see appendix C)

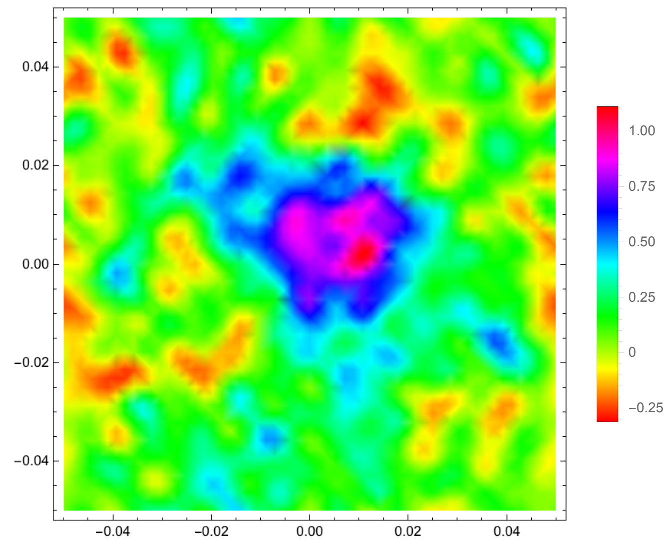
$$\langle \zeta_{part}(x) \rangle = \frac{m(\eta = -|x|)}{2\sqrt{2\epsilon} M_{pl}} \times \left( \frac{1}{2\pi\sqrt{2\epsilon}} \frac{H}{M_{pl}} \right) \tag{4.8}$$

where $M_{pl}$ is the reduced Planck mass. Notice that the mass is evaluated at a conformal time equal to the distance in comoving coordinates from the location of the particle. Note that the time dependence of the mass is translated to the spatial dependence of the profile of the field. The last factor in (4.8) is the amplitude of the quantum fluctuations. Therefore we want the first term to be larger than one. This can be achieved if $m \sim M_{pl}$ and $\epsilon$ is small. It would be unreasonable to postulate a mass much larger than $M_{pl}$ since that would become a black hole. But if $\epsilon$ is $10^{-3}$, then we can have a classical effect which is ten times larger than the quantum fluctuations and should be visible. See figure 6 for simulations of this classical solution plus the quantum fluctuations. We call this region which has a value of the primordial curvature fluctuation $\zeta(x)$ larger than the average a "hot spot", regardless of how how it will appear in the CMB or other probe of the primordial fluctuations[8]. We want a situation where the hot spot is recognizable on an individual basis. This is the reason we required (4.8) to be larger than the quantum fluctuations.

[8] In [26] it was argued that depending on whether their size is larger than the size of the horizon at recombination they can appears as hot or cold spots in the CMB.




(a)




(b)

Figure 6: For these figures we assumed a particular time dependence of the mass where $\langle \zeta_{part} \rangle$ becomes a factor of about five above the background value of the fluctuations (see (C.25)). We see the standard gaussian random field plus a hot spot created by the coupling to a massive particle. (a) A slice of the distribution centered on the center of the hotspot. The orange line represents an instance of the fluctuating field with no hotspot. The green line shows the hot spot becoming larger than the mean value of the fluctuations, represented by the horizontal blue line. (b) Two dimensional plot of the hot spot plus the quantum fluctuations. We clearly see the hot spot standing out over the background. This figure is zoomed relative to figure 5.

## 4.4 The measurement

Here we want to consider a process whose outcome depends on the isospin of the particle. Of course, we need to introduce an isospin breaking interaction. We postulate that the mass of the massive particle has a component that depends on the isospin projection along an axis that depends on the axion field. More precisely, we imagine that the field $h$ describing the massive particles has mass terms of the form

$$\begin{aligned} m_1^2(\phi)h^\dagger h + \lambda_2(\phi)h^\dagger(\sigma_x \cos n\theta + \sigma_y \sin n\theta)h = \\ = m_1^2(\phi)\left[|h_1|^2 + |h_2|^2\right] + \left[\lambda_2(\phi)e^{in\theta}h_1^* h_2 + c.c.\right] \end{aligned} \tag{4.9}$$

In the first line we view $h = (h_1, h_2)$ as an isospin doublet bosonic field with the $\sigma$ matrices are acting on the isospin indices of $h$. In the second line we wrote the lagrangian in terms of the two complex component fields.[9] . The number $n$ is an integer and a value of about $n \sim 10$ is reasonable to amplify the fluctuations shown in figure 5. We introduced it with the sole purpose of amplifying the effects of the fluctuations of the axion so that the the vector along which we are projecting the spin ranges over all possible orientations.[10] The two eigenvalues of the mass are

$$m_\pm = \sqrt{m_1^2(\phi) \pm |\lambda_2(\phi)|} \tag{4.10}$$

We want $|\lambda_2| \leq m_1^2$ so as not to have an instability. We also want that at late times $|\lambda_2|$ is similar to $m_1^2$ so that the two eigenvalues of the mass differ by, say, a factor of two. In addition, we also want that $m_\pm$ are of order $M_{pl}$. This is required so that both values of the mass are observable and distinguishable, as in the discussion in subsection 4.3. In addition, we would like that $\lambda_2$ is negligible when the particles are created so that they are really created in an isosspin singlet. Then, as $m_1$ rises, then $\lambda_2$ should also rise and become large.

We recognize that this subsection seems the most contrived part of the model.

Notice that in this model the field $h$ is a complex field and when we produce a particle pair one member of the pair will be a particle and the other an antiparticle. The complex conjugate field is, of course, also a doublet $(h_1^*, h_2^*) = (\tilde{h}_1, -\tilde{h}_2)$, where the $\tilde{h}$ field transforms as a standard doublet, in the same way as $(h_1, h_2)$ under $SU(2)$. This means, in particular, that the eigenvalues of the masses are given by the projection of $-(\sigma.\vec{n})$ acting on the $\tilde{h}$ doublet. If we consider the pair of fields $h$ and $\tilde{h}$, we have an ordinary spin singlet state.

---

[9] This is the most general term that we can write down that is consistent with the $O(2)$ symmetry generated by phase rotations of the doublet, together with the "reflection" $h_1 \to h_2^*$ , $\quad h_2 \to h_1^*$. Also the coupling to the axion preserves a common symmetry under changing $h_1$ and $h_2$ by opposite phases together with a shift of the axion.

[10] We could have set $n = 1$ and then produced larger fluctuations of the axion than in figure 5, by decreasing the overall value of $f_a$. This would have the problem of producing domain walls after the end of inflation. However, we could set the post-inflationary axion potential to zero and imagine that the axion is observable as a variation of the some of the fundamental constants in the late universe.

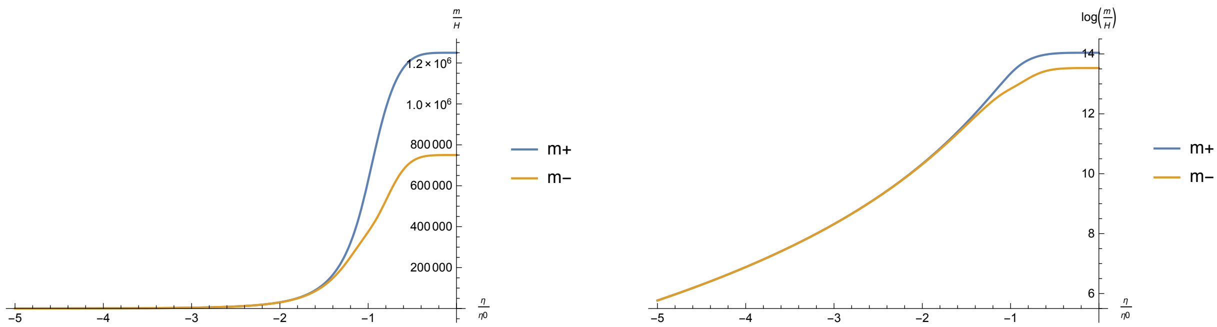



Figure 7: (a) Plot of the type of $m_\pm$ functions that we want for the model.(b) Same plot in logarithmic scale. We plot times after the particles are created, when the masses are equal and of order $m/H \sim$ few, they then rise to large values of order $M_{pl}$ and, in addition, they become different from each other due to a non-zero $\lambda_2$ in (4.9).

If $\theta$ was constant in space, then if the particle member of the pair has mass $m_+$, then the antiparticle member *also* has mass $m_+$. In other words, the $\pm$ of the mass of the particle is equal to the sign of the $\sigma.\vec{n}$ operator. For the antiparticle the $\pm$ of the mass is equal to the sign of $-(\sigma.\vec{n})$ acting on $\tilde{h}$. This extra minus sign has trivial consequence, it reverses the sign of the quantum mechanical expectation value for the $C$ observable defined in (2.1) relative to what is expected for an ordinary spin singlet state.

## 4.5 Post inflationary observations

We imagine that we, as late time observers, can measure both the primordial scalar fluctuations as well as the primordial axion fluctuations. Of course, neither an axion, nor its primordial fluctuations have been seen. If we want to make the model consistent with present day data we can imagine that the axion we are discussing corresponds to a subleading component of the dark matter. This dark matter density depends on the value of $\theta$ left over from the end of inflation, since this determines the deviation from the minimum of the axion potential. Therefore, fluctuations in $\theta$ translate into fluctuation of this component of the dark matter. This is an isocurvature fluctuation. These have a characteristic scale which is set by the comoving scale corresponding to the time during inflation where $f_a$ was small. This is the scale of the features in figure 10. In summary, by looking at the fluctuations in the subdominant matter distribution we could make a plot of the primordial axion position at the end of inflation. The plot would look as in figure 10.

Now let us discuss the scalar fluctuations. In this model, the scalar fluctuations are given by the usual gaussian random field plus some characteristic hot spots which have a specific size in comoving coordinates. These hot spots are large enough to stand out from the gaussian field on an individual basis, as in figure 6. There are two types of hot spots that differ by their overall amplitude. Let us call them the superhot and the veryhot spots. These two possibilities correspond to the $\pm$ sign in (4.10). Identifying

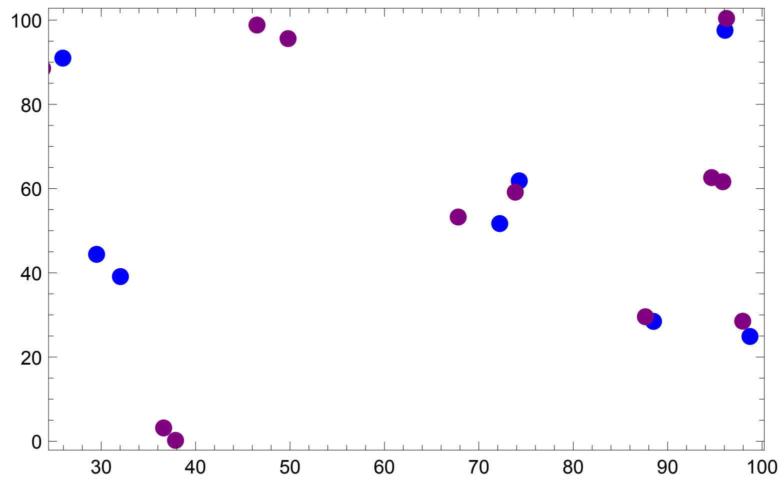



Figure 8: Performing an observation of the primordial scalar fluctuations, the observer identifies the superhot and the veryhot spots. Plotting only these hot spots we get the above map. Here purple is for very hot and blue for superhot. This is the same as the map for the created particles in figure 4 except that now we can associated a color (or a plus or minus sign) to each spot.

each hot spot individually and labelling it as a superhot or veryhot spot spot we can end up with a distribution of hot spots as in figure 8. Each hot spot can be assigned a $\pm 1$ depending on whether it is a superhot or veryhot spot. Note that in this model, each hotspot corresponds to an individual massive particle created during inflation. The final result of this procedure is a set of pairs. And for each member of the pair we have a plus or minus one. We interpret this plus or minus one as the measurement of the isospin along some axis.

Now we look at the map of the axion angle at the location of each spot. This axion map could look as in figure 5. In this way, we can assign an angle for each spot. So now we have a collection of pairs of $\pm 1$ measurements together with their corresponding angles. Let us call these pairs $(\pm 1_{\theta_A}, \pm 1_{\theta_B})$. These outcomes are similar to the ones obtained in the idealized Bell experiment discussed in section 2, where $\theta_{A,B}$ represents the values of the orientations of the axis along which the projection is made. In other words, we could define the observable $C$ as in (2.1), as $C(\theta_A, \theta'_A; \theta_B, \theta'_B)$ where $\theta_A$, and $\theta'_A$ correspond to the two choices of detector at Alice's location and $\theta_B$, $\theta'_B$ similarly at Bob's location. We could define Alice's location to be the location of the particle and Bob's location that of the antiparticle. However, in this model, the late time observer cannot distinguish between the particle and the antiparticle. Fortunately, this is not a problem. In the standard Bell inequality discussion, we still have a Bell inequality if we were to consider the new observable $\tilde{C} = \frac{1}{2}(C + C_{A \leftrightarrow B})$. And the quantum mechanical prediction for the singlet state is also invariant under $A \leftrightarrow B$, since the expectation value for given choices of orientations of detectors it is proportional to $\cos(\theta_A - \theta_B)$. We can consider the angles in (2.3) with

$\theta = \pi/4$, and looking at the observable $\tilde{C}$ we would observe a maximal violation of Bell's inequalities.[11]

# 5 Discussion

Violations of the Bell inequality are a key signature of quantum entanglement, displaying the weirdness of the quantum world. Given that the leading theory for the origin of fluctuations in our universe relies crucially on quantum mechanics, it is reasonable to ask whether a Bell experiment is possible in cosmology. In tabletop experiments we have the luxury of varying the initial conditions and manipulating various types of materials. In cosmology, we have just the one universe we live in. However, in theory, we also have the "luxury" of imagining alternative universes where other measurements are possible. Here we have imagined a universe where a cosmological Bell inequality experiment is possible. Though the model is somewhat contrived, it shows that it is in principle possible. Of course, it would be much more interesting to find such observables for the model that describes inflation in the real world (assuming that inflation does indeed describe the real world).

The exercise of constructing a model where the measurement is possible has exposed some of the assumptions that are necessary in order to formulate an inequality. In order for the observable to be subject to a Bell inequality, one needs to make several assumptions. We need to assume that the fluctuations are generated when modes cross the horizon and not earlier. In other words, we need to assume that the fluctuations that generated random values of the axion field at the location of the two particles are not correlated with the entangled isospin state of the massive particles. This is an assumption we always make when we perform a Bell inequality experiment. One can question this assumption, both for today's experiments as well as for experiments in the early universe, since the whole observable universe was once in a very small region of space. Nevertheless, the assumptions that go into the cosmological Bell inequality seem qualitatively similar to the ones that go into present day Bell inequality experiments.

Notice that a value of $|C| > 2$ implies a violation of Bell's inequalities, ruling out local classical hidden variables, while a value larger than $2\sqrt{2}$ would be a violation of quantum mechanics. Therefore one can also view it as a test of quantum mechanics.

In the model described in this paper, all the elements of the Bell experiment are constructed out of vacuum fluctuations during inflation. We have seen that this can be done considering several off-the shelf elements. In fact, particles whose mass depend on the inflaton were discussed in e.g. [24, 27], and they arise naturally in models with moving branes as open strings stretching between the branes [27]. Models with many fields are common, as well as models where an axion has quantum fluctuations during inflation, or

[11] Of course, in a practical experiment we would also want to average over all configurations of (2.3) where the quantum mechanical contributions are still the same. For example, we can perform an overall rotation or reflection of the vectors in (2.3).

a decay constant that is time dependent, see e.g [28]. Perhaps the most contrived aspect was the particular coupling assumed in section 4.4.

Independently of the motivation for this paper, it is also interesting that particles that become very massive can leave a discernible signal on the spectrum of primordial fluctuations [25, 26]. These are signals which appear on an *individual* basis. In other words, each hot spot is produced by an individual massive particle. The mass of the particle rises with the advance of the inflaton, slowing down the inflaton around the particle . The expansion of the universe imprints this signal over long distances, distances that are much larger than the Hubble radius at the end of inflation. The presence of particles whose mass varies so strongly with the inflaton makes one wonder whether the potential will remain flat after the quantum effects of this particles are taken into account. Particles that become light at some point during the evolution are natural in monodromy inflation models [30, 31]. For the purposes of this paper we are content with fine tuning the potential, since we are not trying to argue that this particular model is the most natural one. It is also tempting to speculate that this mechanism or some variation could be used to produce primordial black holes from suitably large hot spots.

Of course, researchers became convinced by quantum mechanics much before Bell inequality experiments were performed [32]. Similarly, there are many other features of the cosmological fluctuations that could be observed in the not so distant future which would give great evidence for a quantum origin of the cosmological fluctuations. These are quantities which we compute using the quantum theory such as the scalar three point function [33] (see [34] for a review), which could perhaps be observable using 21 cm observations [35] or other yet to be discovered way to measure a large number of primordial fluctuations. Other possibilities include seeing oscillations in the three point function with the patterns produced by the creation of massive particles [36, 37, 38, 39, 40]. Of course, it would be interesting to find other observables that harder to reproduce using non-quantum evolution.

Finally, note that nature has indeed produced a universe where Bell experiments are possible: they are certainly possible in the current era of accelerated expansion, though we do not know if our results will be seen by any "post-inflationary" observers!.

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# A Creation of massive particles with time dependent masses

We start from the equation for a massive particle (4.6). We then define $u = h/\eta$ to find the equation

$$u'' + p^2(\eta)u = 0 \ , \qquad p^2(u) = k^2 + \frac{m^2(\eta)/H^2 - 2}{\eta^2} \tag{A.11}$$

In the standard WKB approximation the solutions are

$$u = \frac{1}{\sqrt{2p}} \exp\left[i\int^{\eta} d\eta' p(\eta')\right] \ , \qquad \bar{u} = \frac{1}{\sqrt{2p}} \exp\left[-i\int^{\eta} d\eta' p(\eta')\right] \tag{A.12}$$

If the field is expanded with respect to these solutions we do not find any particle creation. The WKB approximation is correct a very early and very late times. The particle creation is described by finding the Bogoliubov $\beta$ coefficient which gives the amount of $\bar{u}$ solution at late times if we start purely with $u$ at early times. We will consider a situation where the WKB approximation is correct to leading order throughout the evolution. This happens when $p'/p^2 \ll 1$ at all times. In this situation the particle creation will be small and it can be computed approximately using the formula

$$\beta(k) = \int_{-\infty}^{0} d\eta \frac{p'^2}{4p^3} \exp\left[2i\int_{-\infty}^{\eta} d\eta' p(\eta')\right] \tag{A.13}$$

In the situation described in section 4.1 we find that $\beta$ is small. But for large $k|\eta_0|$ it is even smaller because the WKB approximation is very valid for all times. While for $k|\eta_0| < 1$ the WKB approximation is less strongly valid near $\eta \sim \eta_0$. But at this time we can neglect the $k$ dependence. This means that we will get a $\beta(k)$ which is small and $k$ independent for $k < |\eta_0|$ and which will become even smaller for larger values of $k$. A toy model for this is the function $\beta = \epsilon e^{-k^2\eta_0^2}$, with a small $\epsilon$. Here $\epsilon$ will characterize the distance between the pairs of created particles while $|\eta_0|$ characterizes their relative separation.

## A.1 An explicit example

Now let us work out an explicitly solvable example. Let us assume that we have a mass that varies as

$$\frac{m^2}{H^2} = \gamma\left(\frac{\eta}{\eta_0} - 1\right)^2 + \delta \tag{A.14}$$

We can write down the massive wave equation (4.6) and solve for the correctly normalized solutions with definite frequency in the far past

$$h = (-\eta)^{\frac{3}{2}} x^{-i\mu} e^{-ix} e^{\frac{\pi}{2}(\nu+\mu)} U(\frac{1}{2} - i\nu - i\mu, 1 - 2i\mu; 2ix)$$

$$\text{with} \qquad x \equiv \frac{\eta}{\eta_0}\sqrt{k^2\eta_0^2 + \gamma}\ , \qquad \mu^2 \equiv \gamma + \delta - \frac{9}{4}\ , \qquad \nu \equiv \frac{\gamma}{\sqrt{\gamma + k^2\eta_0^2}} \tag{A.15}$$

where $U$ is the function defined as HypergeometricU in *mathematica*. It behaves as $U(a, b; z) \sim z^{-a}$ for large $z$. For small values of $x$, (A.15) goes as

$$h \sim (-\eta)^{3/2}\left[x^{i\mu}e^{-\pi\mu}2^{2i\mu}\frac{\Gamma(-2i\mu)}{\Gamma(\frac{1}{2} - i\nu - i\mu)} + x^{-i\mu}\frac{\Gamma(2i\mu)}{\Gamma(\frac{1}{2} - i\nu + i\mu)}\right]e^{\frac{\pi}{2}(\nu+\mu)} \tag{A.16}$$

For large $\mu$ and $\nu$ we get (up to irrelevant phase factors in each term)

$$\frac{1}{\sqrt{2\mu}}e^{-\frac{3}{2}t}\left[e^{-i\mu t}e^{-\pi(\mu-\nu)} + e^{i\mu t}e^{-\pi/2(\mu-\nu-|\mu-\nu|)}\right] \tag{A.17}$$

with $\eta = -e^{-t}$. The first term reflects the possibility of particle creation so that the $\beta$ Bogoliubov coefficient is $\beta \sim e^{-\pi(\mu-\nu)}$. We are interested in the case where $\mu > \nu$ so that the first term is small. Notice that the only term that depends on $k$ is in $\nu$. And $\nu$ is maximal for $k = 0$ and it then decreases rather quickly as $k\eta_0 \sim \sqrt{\gamma}$. We can easily take $\gamma$ and $\delta$ to be of order one. In this way we can ensure that we produce well separated pairs. However, in this case, the mass does not grow enough to be visible to the late time observer. Therefore, after the particles are created we need another term in the mass that makes them grow more quickly so that they grow to values of order $M_{pl}$. This would require a modification of the time dependence of the mass, relative to (A.14) for times which are a few efolds after $\eta_0$. This new time dependence should make the mass rise to values of order $M_{pl}$ and be isospin dependent, as it is shown in figure 7.

# B Axion with varying decay constant

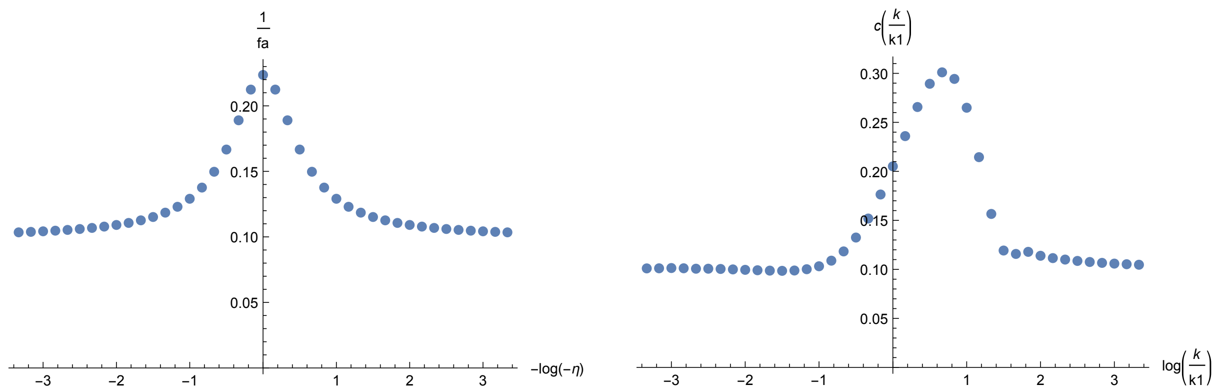
In this section we consider the fluctuations produced by an axion with varying $f_a$. We consider the action (4.7). Its equations of motion are

$$\partial_\eta\left[\frac{\tilde{f}_a^2}{\eta^2}\partial_\eta\theta\right] + \frac{k^2\tilde{f}_a^2}{\eta^2}\theta = 0 \tag{B.18}$$

where $\tilde{f}_a = f_a/H$. As a simple example, consider the following

$$\tilde{f}_a^2 = 100 - \frac{80}{1 + (\log\frac{\eta}{\eta_1})^2} \tag{B.19}$$

For very early or very late times this is constant and equal to $\tilde{f}_a \sim 10$, but $\tilde{f}_a$ dips to smaller values for time $\eta \sim \eta_1$. We can now compute the axion fluctuations as usual. Namely, we numerically solve (B.18) with boundary conditions at large $\eta$ corresponding




(a) (b)

Figure 9: (a) We plot of $1/\tilde{f}_a(\eta)$. We are plotting the inverse rather than the function itself since we expect that the axion fluctuations scale roughly as this value. (b) Numeric computation of the axion spectrum. We plot the function $c(k)$ defined in (B.20).

to the vacuum of a harmonic oscillator and then look at the value of the solution for very small $\eta$. Squaring it we obtain the the value of the axion fluctuations $c(k)$ defined by

$$\langle \theta(\vec{k})\theta(\vec{k}')\rangle_{\eta\to 0} = (2\pi)^3\delta^3(\vec{k}+\vec{k}')\frac{c(k)^2}{2k^3} \tag{B.20}$$

In figure 9 we see a plot of $c(k)$ for a few values of $k$. We also show in figure 10 the type of position space profile for the axion generated by this probability distribution. We see that it has features at characteristics scales but it is smooth at shorter distances.

# C Effect of massive particles on the scalar curvature fluctuations

Here we consider the Lagrangian

$$S = \frac{1}{2}\int d\eta d^3x \frac{2\epsilon M_{pl}^2}{H^2}\left[\frac{(\partial_\eta\zeta)^2 - (\partial_i\zeta)^2}{\eta^2}\right] - \int \frac{d\eta}{H} m(\eta)\partial_\eta\zeta(\eta,\vec{x}=0) \tag{C.21}$$

This is the lagrangian corresponding to the curvature fluctuations. The coupling to the the massive particle at rest is obtained from the coupling to $g_{00}$ which is through $\frac{\delta g_{00}}{g_{00}} = \dot{\zeta}/H$ (see equation (2.10) in [33]). We view the coupling to the mass as a perturbation, then we can apply the in-in formalism to compute the expectation value of $\zeta$ which is given in Fourier space by

$$\langle \zeta_{\vec{k}}(\eta=0)\rangle = -i\int_{-\infty}^{0} d\eta \frac{m(\eta)}{H}\langle \zeta_{\vec{k}}(0)\partial_\eta\zeta_{-\vec{k}}(\eta)\rangle + c.c.$$

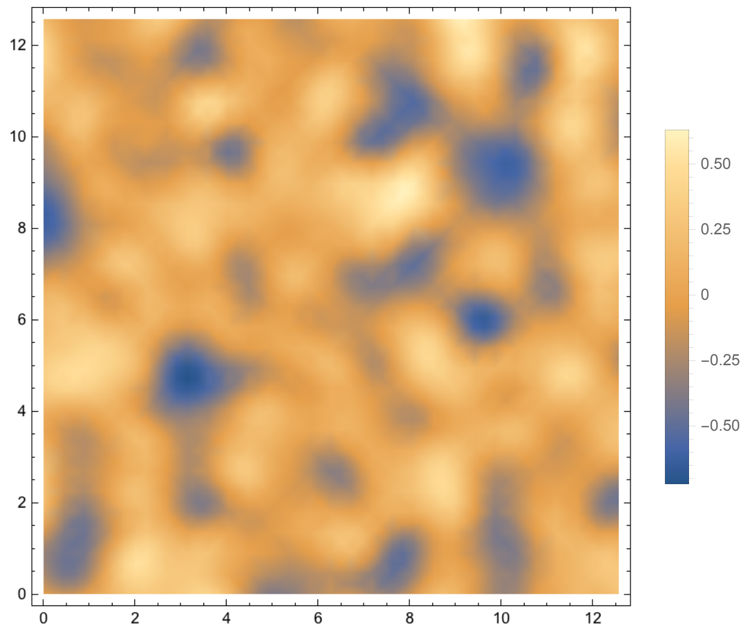



Figure 10: We see the position space profile for the axion generated by a particular instance of a distribution with statistics given by (B.20), with $c(k)$ as in figure 9.

$$= \frac{H}{2\epsilon M_{pl}^2} \int_{-\infty}^{0} d\eta m(\eta) \frac{\eta}{k} \sin k\eta \tag{C.22}$$

where we assumed that $H$ and $\epsilon$ were constant for simplicity. Then going to position space we find that the inverse fourier transform produces a $\delta(\eta + |\vec{x}|)$ so that the final answer is

$$\langle \zeta(\vec{x}) \rangle = \frac{1}{4\pi} m(\eta = -|\vec{x}|) \left( \frac{H}{2\epsilon M_{pl}^2} \right) \tag{C.23}$$

The standard gaussian two point function in position space is

$$\langle \zeta(\vec{x})\zeta(0) \rangle = \left( \frac{H^2}{2\epsilon M_{pl}^2} \right) \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}.\vec{x}} \frac{1}{2k^3} = \frac{1}{(2\pi)^2} \left( \frac{H^2}{2\epsilon M_{pl}^2} \right) \log(L/|\vec{x}|) \tag{C.24}$$

where $L$ is an IR cutoff. Which naturally inspires us to write (C.23) as in (4.8).

Figure 6 is based on choosing a classical profile

$$\langle \zeta(x) \rangle = \frac{1}{2\pi} \frac{5}{(1 + (50\eta)^2)} \tag{C.25}$$

This does not quite correspond to the time dependence of the mass discussed in (A.1), or figure 7. It is just an example.

# References


[1] V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33**, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981)].

[2] S. W. Hawking, Phys. Lett. B **115**, 295 (1982).

[3] A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982).

[4] A. A. Starobinsky, Phys. Lett. B **117**, 175 (1982).

[5] J. M. Bardeen, P. J. Steinhardt and M. S. Turner, Phys. Rev. D **28**, 679 (1983).

[6] A. Berera and L. Z. Fang, Phys. Rev. Lett. **74**, 1912 (1995) [astro-ph/9501024].

[7] A. Berera, Phys. Rev. Lett. **75**, 3218 (1995) [astro-ph/9509049].

[8] D. Lopez Nacir, R. A. Porto, L. Senatore and M. Zaldarriaga, JHEP **1201**, 075 (2012) [arXiv:1109.4192 [hep-th]].

[9] L. Senatore, E. Silverstein and M. Zaldarriaga, JCAP **1408**, 016 (2014) [arXiv:1109.0542 [hep-th]].

[10] M. Mirbabayi, L. Senatore, E. Silverstein and M. Zaldarriaga, Phys. Rev. D **91**, 063518 (2015) [arXiv:1412.0665 [hep-th]].

[11] J. S. Bell, Physics **1**, 195 (1964).

[12] J. B. Hartle, gr-qc/9304006.

[13] D. Campo and R. Parentani, Phys. Rev. D **74**, 025001 (2006) [astro-ph/0505376]; Braz. J. Phys. **35**, 1074 (2005) [astro-ph/0510445].

[14] A. Go [Belle Collaboration], J. Mod. Opt. **51**, 991 (2004) [quant-ph/0310192].

[15] R. A. Bertlmann, A. Bramon, G. Garbarino and B. C. Hiesmayr, Phys. Lett. A **332**, 355 (2004) [quant-ph/0409051].

[16] T. Ichikawa, S. Tamura and I. Tsutsui, Phys. Lett. A **373**, 39 (2008) [arXiv:0805.3632 [quant-ph]].

[17] J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt, Phys. Rev. Lett.**23**, 880 (1969).

[18] B.S. Cirelson, Lett. Math. Phys. **4**, 93 (1980).

[19] K. Banaszek and K. Wodkiewicz, Phys. Rev. A **58**, 4345 (1998).

[20] A. D. Linde, Contemp. Concepts Phys. **5**, 1 (1990) [hep-th/0503203].

[21] Y. Nambu and Y. Ohsumi, Phys. Rev. D **84**, 044028 (2011) [arXiv:1105.5212 [gr-qc]].

[22] S. P. de Alwis, arXiv:1504.05211 [hep-th].

[23] J. Gallicchio, A. S. Friedman and D. I. Kaiser, Phys. Rev. Lett. **112**, no. 11, 110405 (2014) [arXiv:1310.3288 [quant-ph]].

[24] D. J. H. Chung, E. W. Kolb, A. Riotto and I. I. Tkachev, Phys. Rev. D **62**, 043508 (2000) [hep-ph/9910437].

[25] N. Itzhaki, JHEP **0810**, 061 (2008) [arXiv:0807.3216 [hep-th]].

[26] A. Fialkov, N. Itzhaki and E. D. Kovetz, JCAP **1002**, 004 (2010) [arXiv:0911.2100 [astro-ph.CO]].

[27] L. Kofman, A. D. Linde, X. Liu, A. Maloney, L. McAllister and E. Silverstein, JHEP **0405**, 030 (2004) [hep-th/0403001].

[28] M. Fairbairn, R. Hogan and D. J. E. Marsh, Phys. Rev. D **91**, no. 2, 023509 (2015) [arXiv:1410.1752 [hep-ph]].

[29] N. Itzhaki and E. D. Kovetz, JHEP **0710**, 054 (2007) [arXiv:0708.2798 [hep-th]].

[30] L. McAllister, E. Silverstein and A. Westphal, Phys. Rev. D **82**, 046003 (2010) [arXiv:0808.0706 [hep-th]].

[31] E. Silverstein and A. Westphal, Phys. Rev. D **78**, 106003 (2008) [arXiv:0803.3085 [hep-th]].

[32] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).

[33] J. M. Maldacena, JHEP **0305**, 013 (2003) [astro-ph/0210603].

[34] X. Chen, Adv. Astron. **2010**, 638979 (2010) [arXiv:1002.1416 [astro-ph.CO]].

[35] A. Loeb and M. Zaldarriaga, Phys. Rev. Lett. **92**, 211301 (2004) [astro-ph/0312134].

[36] X. Chen and Y. Wang, JCAP **1004**, 027 (2010) [arXiv:0911.3380 [hep-th]].

[37] D. Baumann and D. Green, Phys. Rev. D **85**, 103520 (2012) [arXiv:1109.0292 [hep-th]].

[38] V. Assassi, D. Baumann and D. Green, JCAP **1211**, 047 (2012) [arXiv:1204.4207 [hep-th]].

[39] T. Noumi, M. Yamaguchi and D. Yokoyama, JHEP **1306**, 051 (2013) [arXiv:1211.1624 [hep-th]].

[40] N. Arkani-Hamed and J. Maldacena, arXiv:1503.08043 [hep-th].