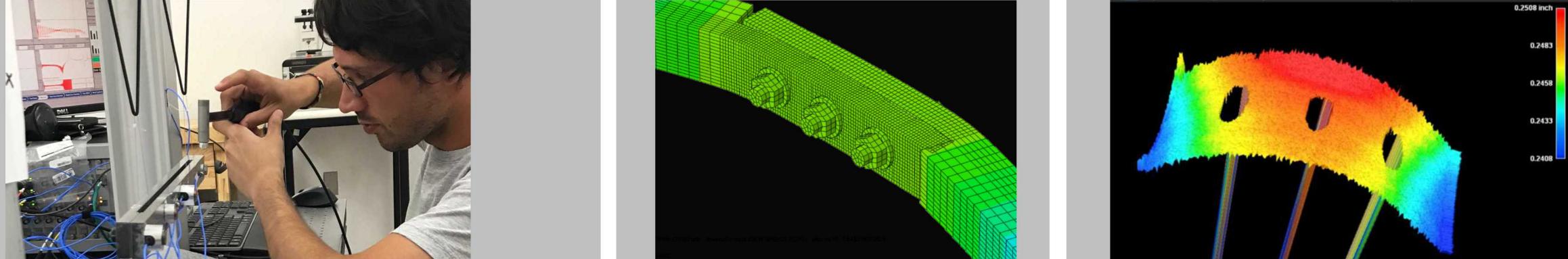


*Exceptional service in the national interest*



# A Priori Methods for Assessing the Nonlinearity of a Jointed Structure

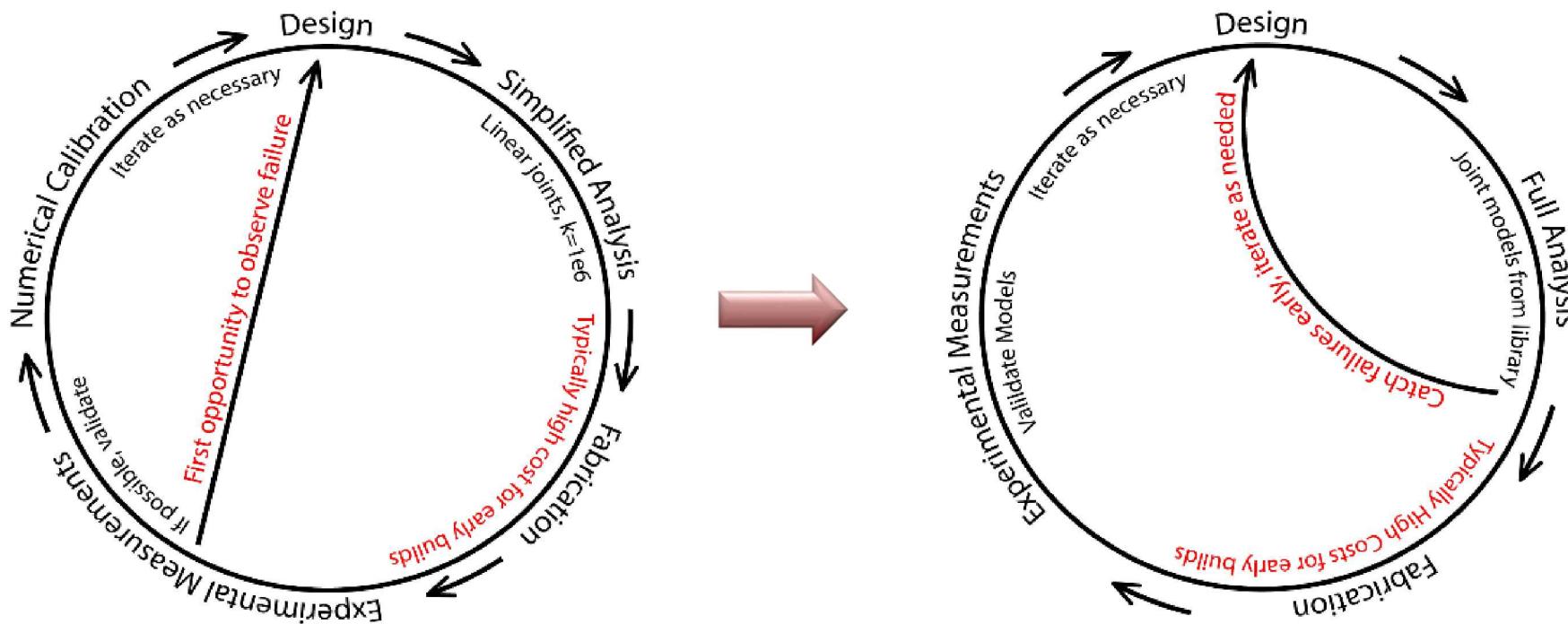
Craig Broadman, She'ifa Punla-Green, Edward Rojas, Matthew Brake,  
Dane Quinn, Rob Flicek, Ben Pacini, Eric Dodgen, and Christopher Schwingshackl



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# Overview

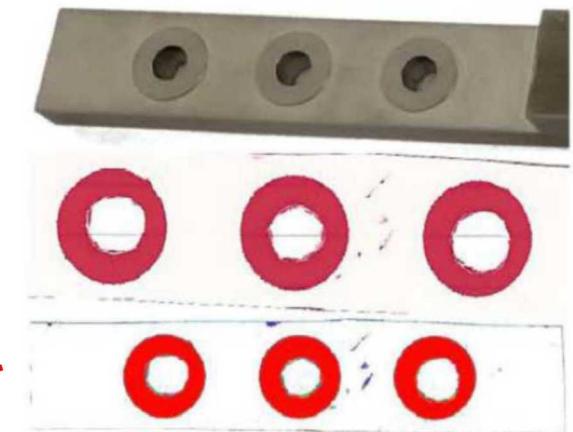
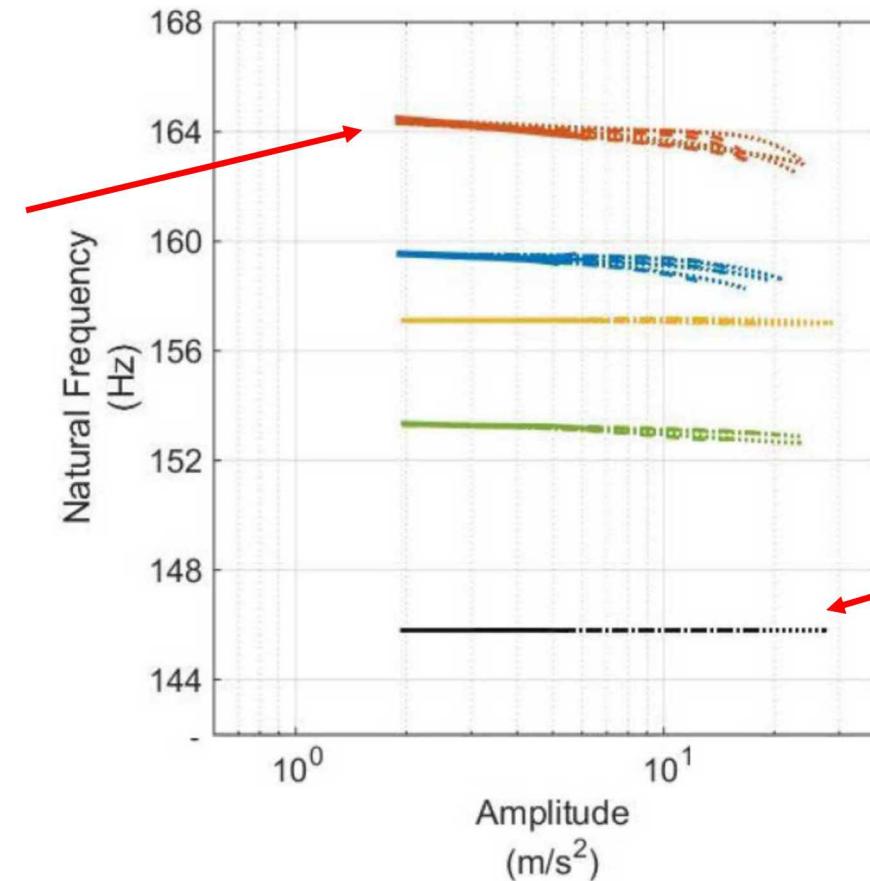
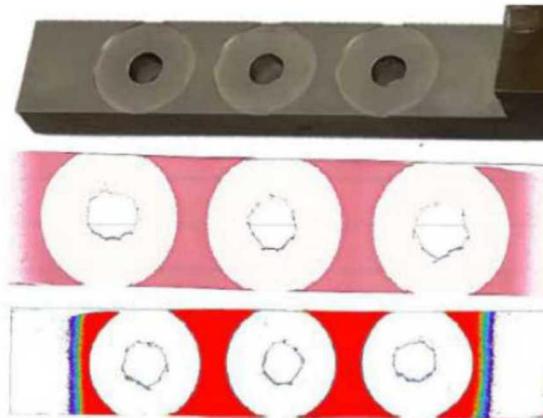
What if we could **remove the uncertainty** associated with the nonlinearity of a novel joint?



# Observations from Previous Research

## 1. 2017 study on interfacial effects

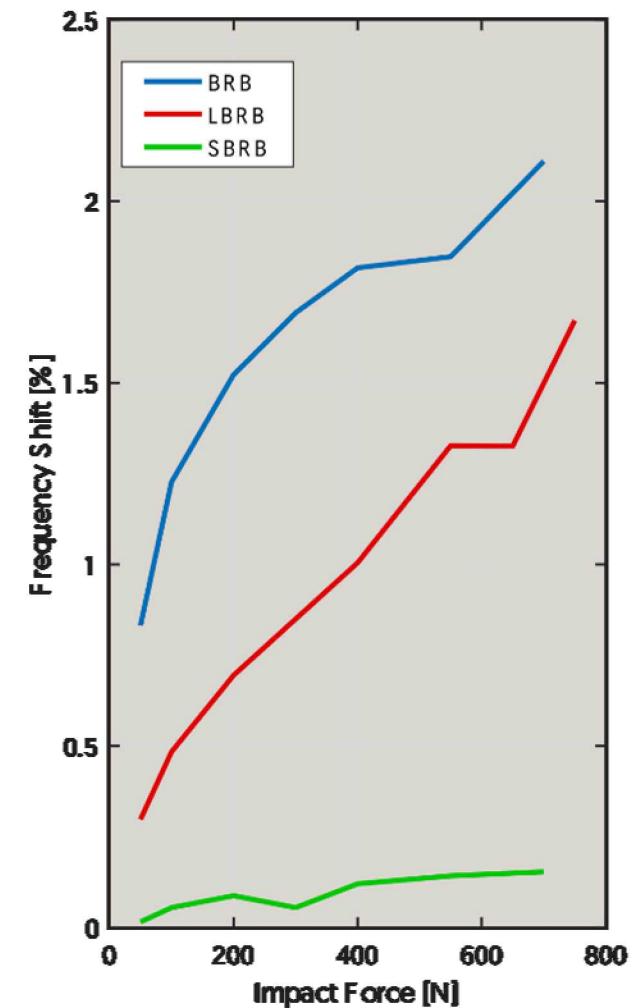
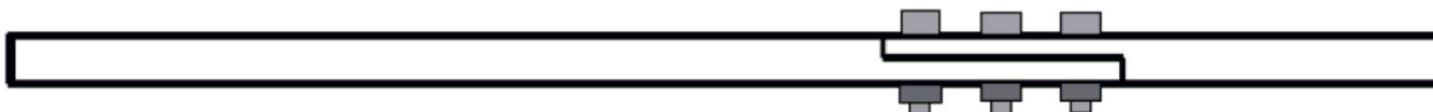
- Interfaces with high, uniform contact pressure → “Linear” dynamics
- Interfaces with transitions from high to low contact pressure → Strongly nonlinear



# Observations from Previous Research

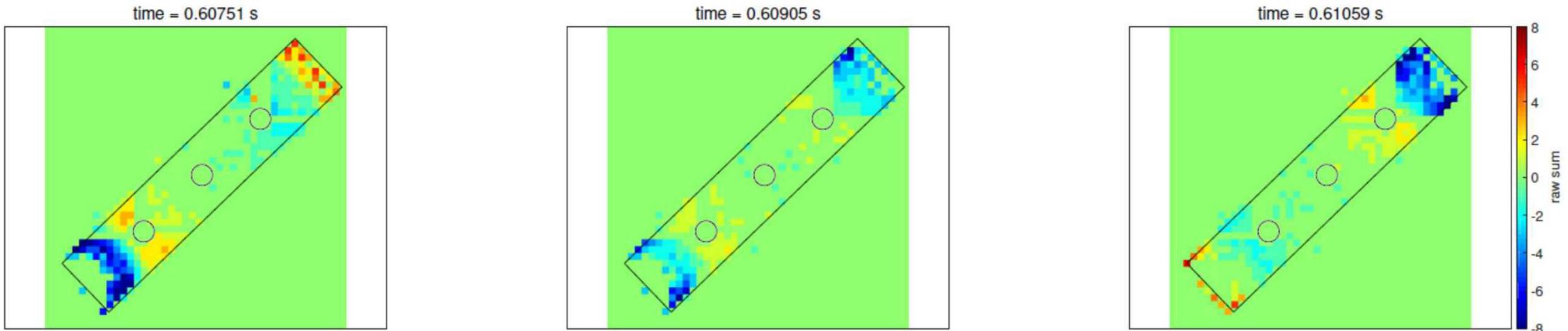
## 2. 2017 study on far-field effects

- “Identical” interfaces have very different responses
- Far-field structure affects how interface is loaded



# Observations from Previous Research

3. 2018 study on interfacial pressure during dynamic loading
  - Moderate contact pressure regions → fluctuate during dynamic loading



4. 2018 study on interface roughness as a variability
  - Low contact pressure regions → Source of energy dissipation

# *A Priori* Methods Hypothesis

We hypothesize that the strength of nonlinearity for a given mode of an assembly can be determined by:

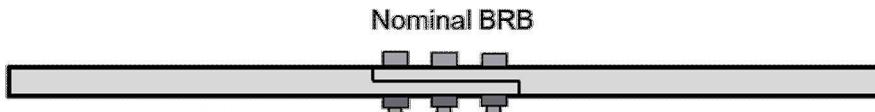
1. The interfacial contact pressure
2. The modal strain energy

To test this hypothesis, we:

1. Measured the nonlinear characteristics of multiple modes for 11 different systems
2. Simulated the interfacial contact pressure and modal strain energy of each system

# Benchmark Systems: The Brake-Reuß Beam and Variations

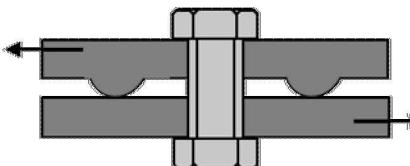
- BRB
- Spring (SBRB)



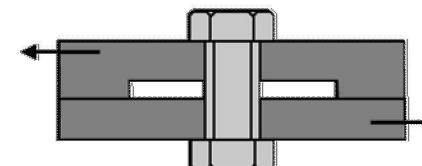
- Long (LBRB)



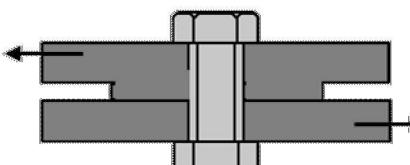
- Hertzian Contact (HZ)



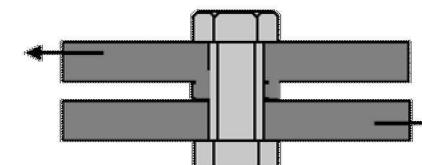
- Reverse Pad Contact (RPD)



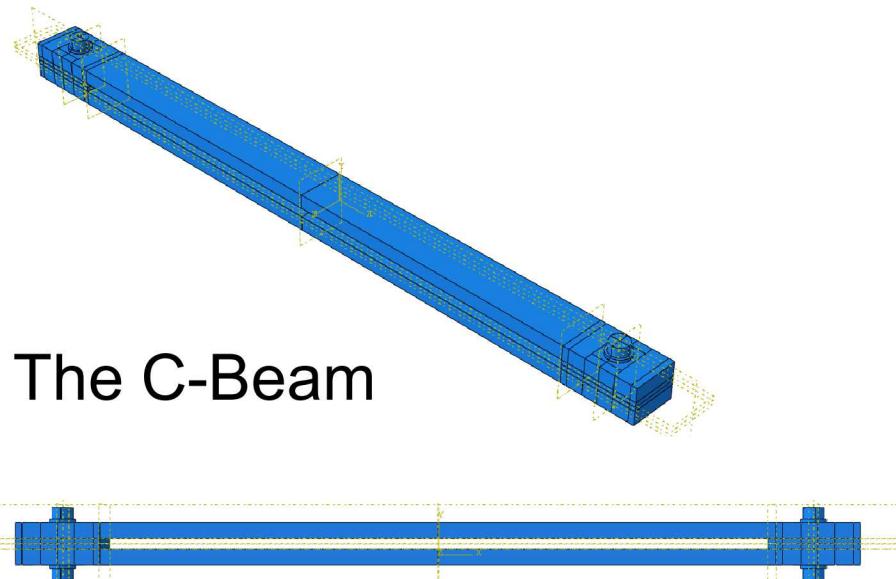
- Large Pad Contact (LPD)



- Small Pad Contact (SPD)

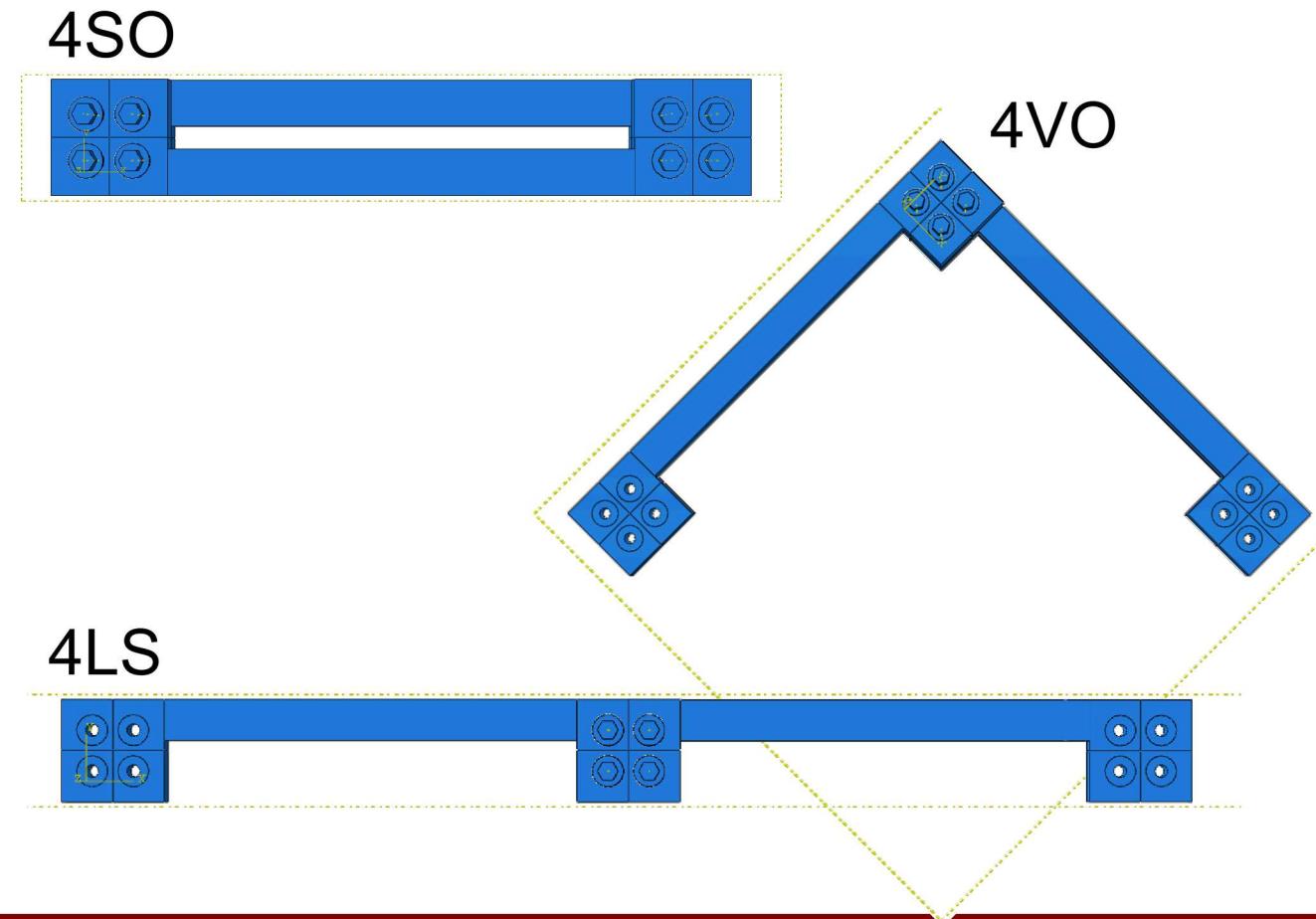


# Benchmark Systems: The C-Beam and 4-Bolt Interface



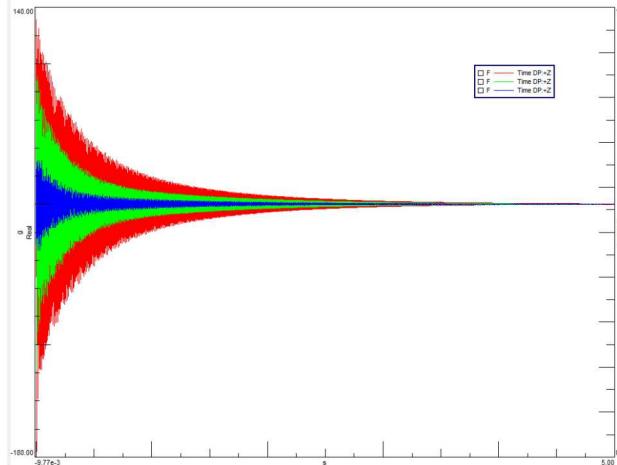
The C-Beam

The 4-Bolt Interface

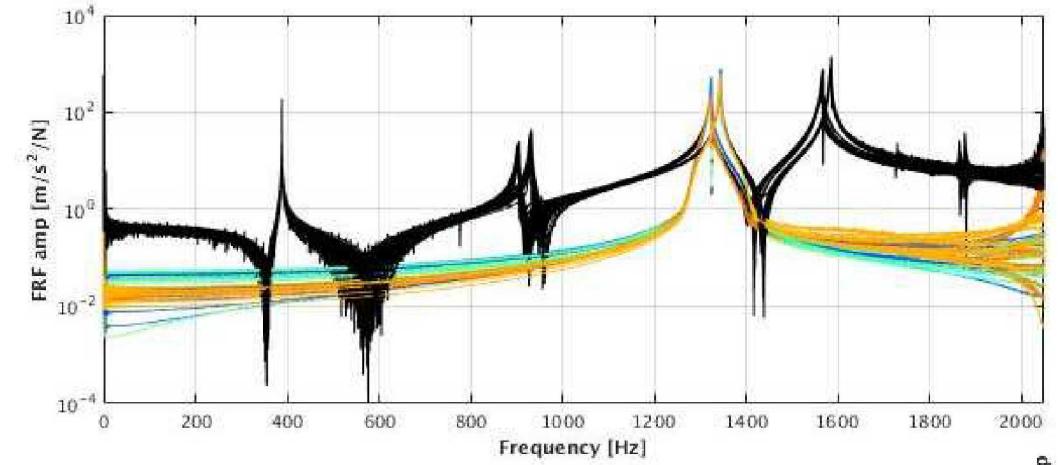


# Experimental Methods

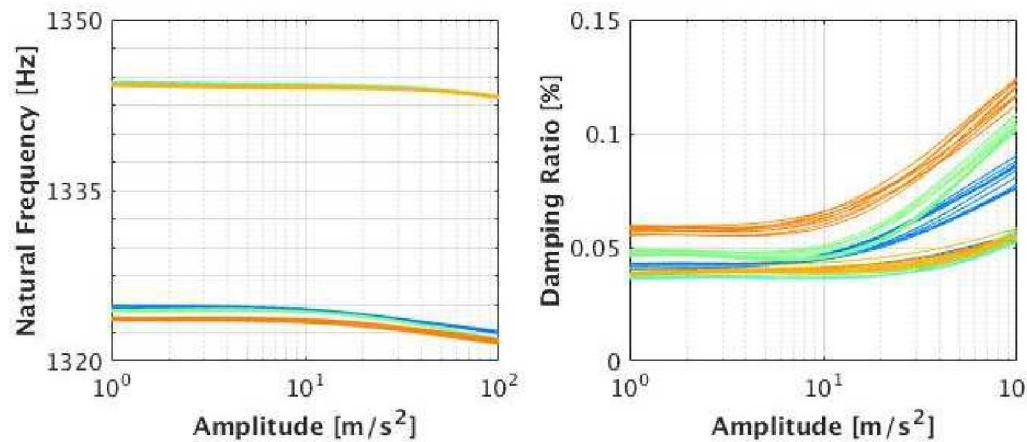
## 1. Impact Testing



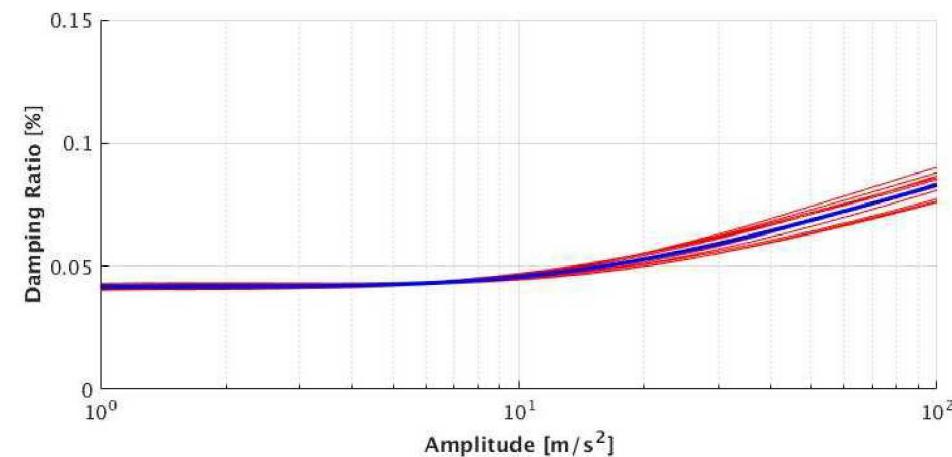
## 2. Bandpass Filtering and Hilbert Transform



## 3. Frequency and Damping Extraction

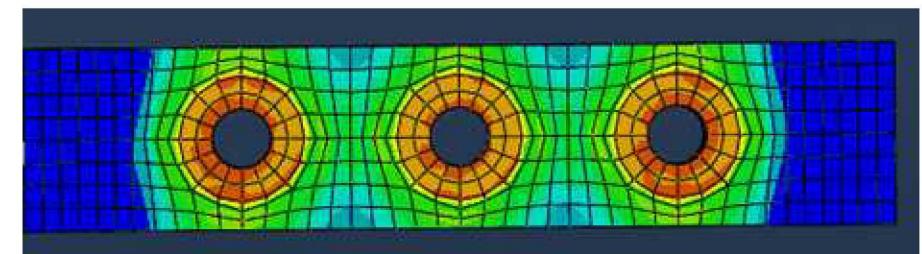
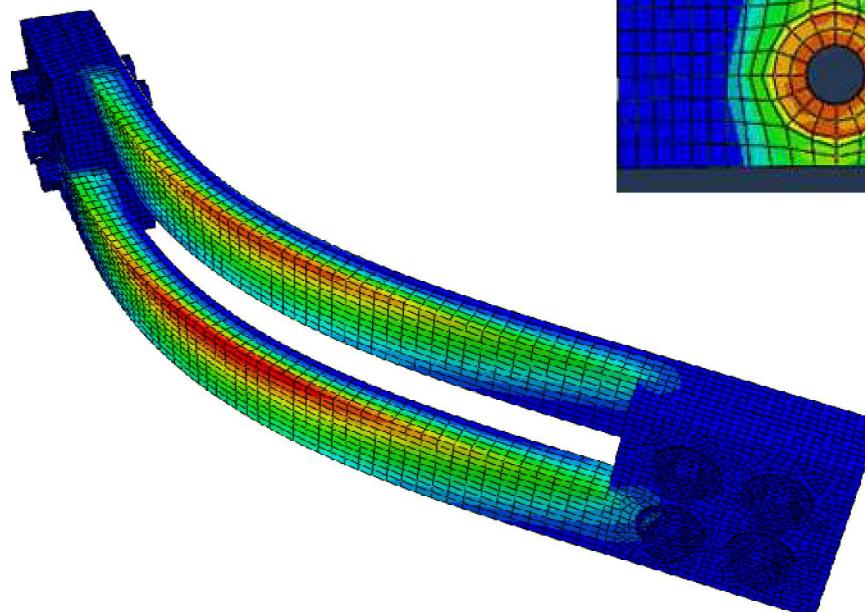


## 4. Smooth and Average Curves



# Numerical Methods

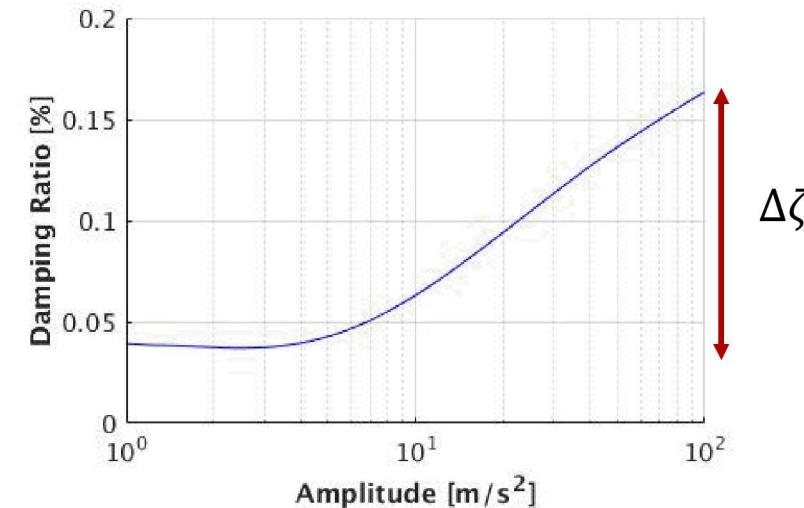
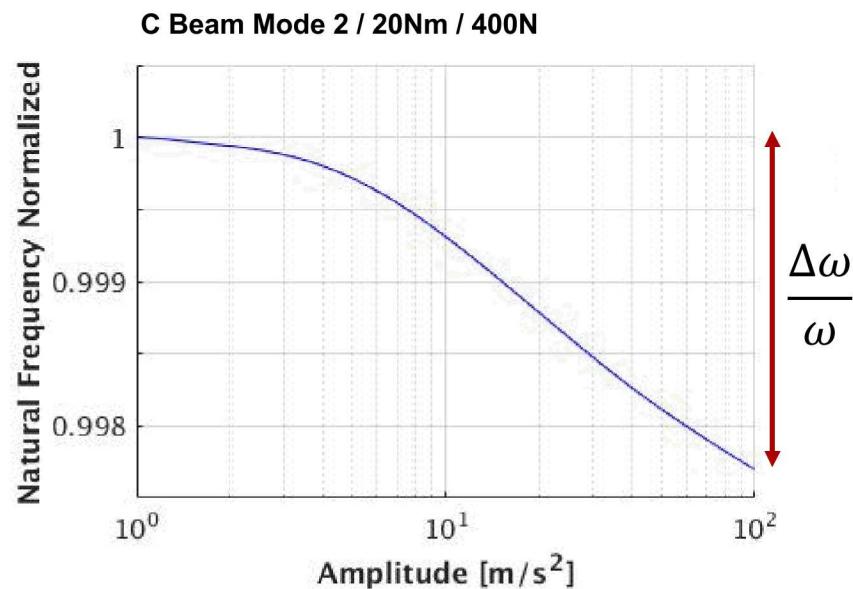
- Goal: easy-to-access data from FEA model
- Contact pressure and area calculated from a nonlinear frictionless static loading
- Modal strain calculated from a linear modal analysis (tied interface)
- Statistical metrics used to characterize results
  - Mean
  - Max
  - Standard Deviation
  - Skew
  - Kurtosis



# Defining Strength of Nonlinearity (SNL)

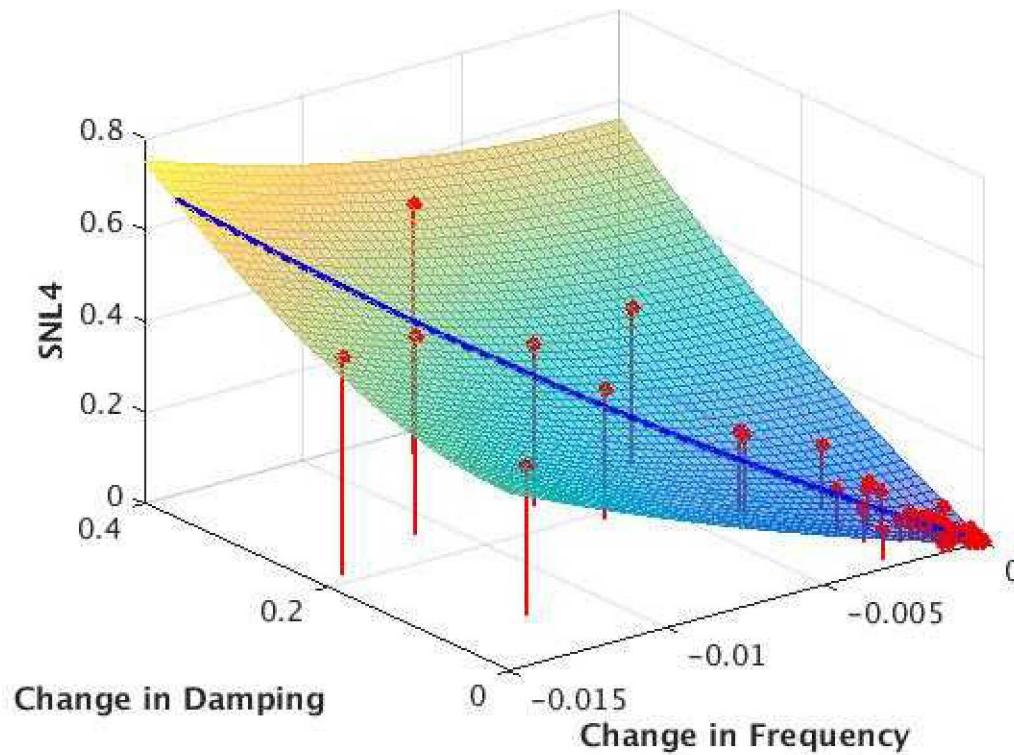
- Method 1: frequency and damping shift

- $$SNL = \alpha \frac{\Delta\omega}{\omega} + \beta \Delta\zeta$$



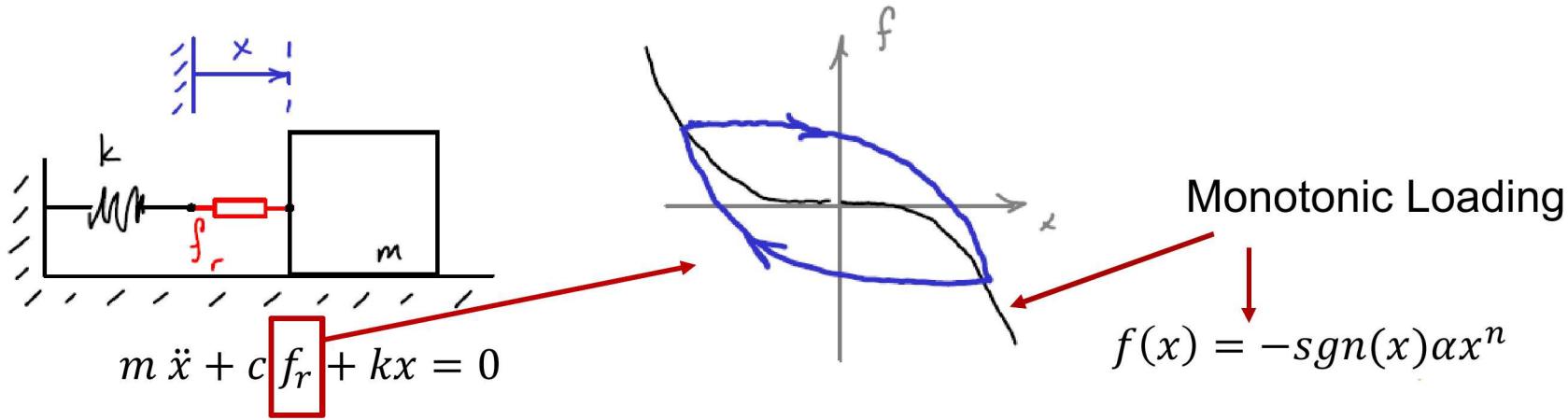
# Defining SNL: Frequency and Damping Shift

$$SNL4 = \left( \left( 20 * \frac{\Delta\omega}{\omega} \right)^2 + (\Delta\zeta)^2 \right)^{\frac{1}{2}} + \left( 20 * \frac{\Delta\omega}{\omega} \right)^2 + (\Delta\zeta)^2$$



# Defining SNL: Perturbations Approach

- Method 2: Based off of a mass-spring-damper system



$$\omega = 1 + C_1 A^{n-1}$$
$$\zeta = C_3 + C_2 A^{n-1}$$

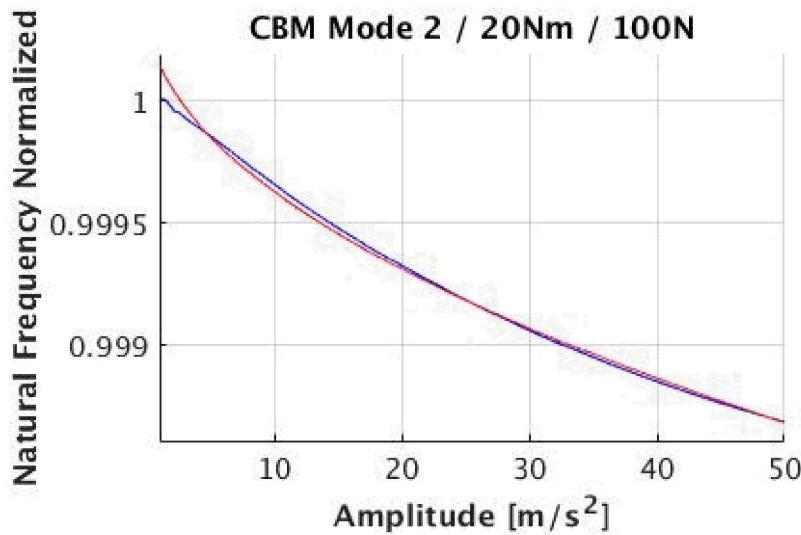
Linear Frequency (Normalized)

Interface properties

Linear Damping

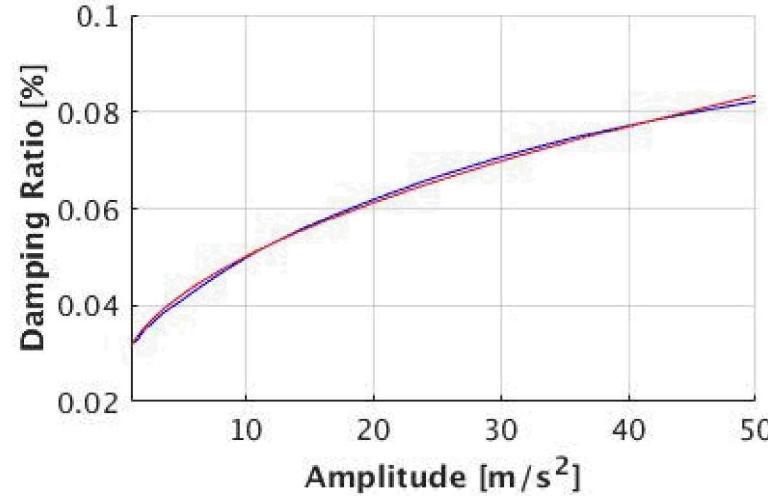
Influence of joint on structure

# Defining SNL: Perturbations Approach



$$\omega = 1.003 - 9.703 * 10^{-4} A^{0.25}$$

$$R^2 = 0.9757$$



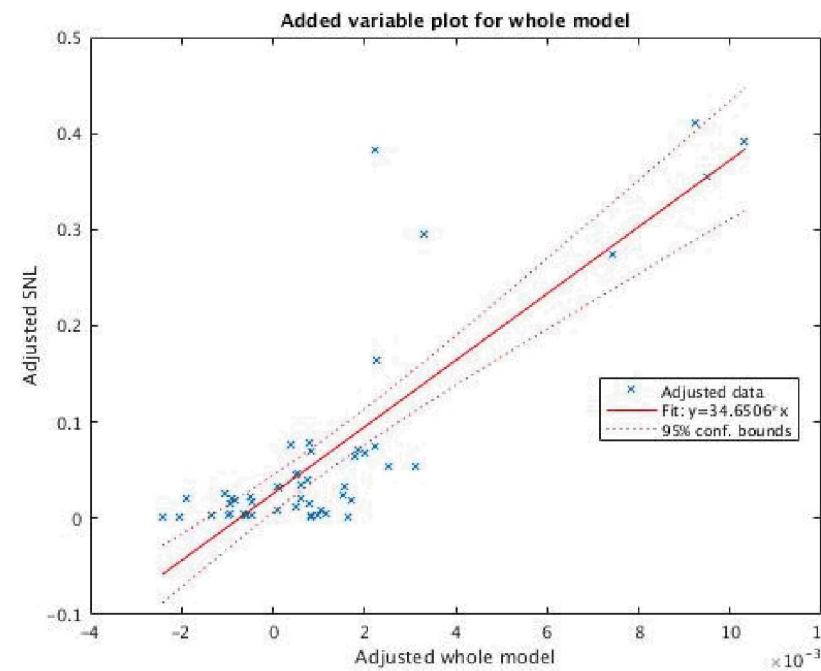
$$\zeta = -0.01 + 0.0345 A^{0.25}$$

$$R^2 = 0.9931$$

- Frequency  $R^2 = 0.8434$
- Damping  $R^2 = 0.7841$

# Analysis

- Two analyses
  - Machine learning
  - Statistical hypothesis testing
- MATLAB's built-in neural network used to develop a predictive equation for SNL
- 70% of variance explained by
  - Mean modal strain
  - Standard deviation of modal strain
  - Skewness of modal strain
  - Standard deviation of contact pressure
  - Contact area



# Analysis

- 82% of stiffness nonlinearity can be explained by modal strain and contact pressure
- 55% of damping variance can be explained by same analysis

## For stiffness variance

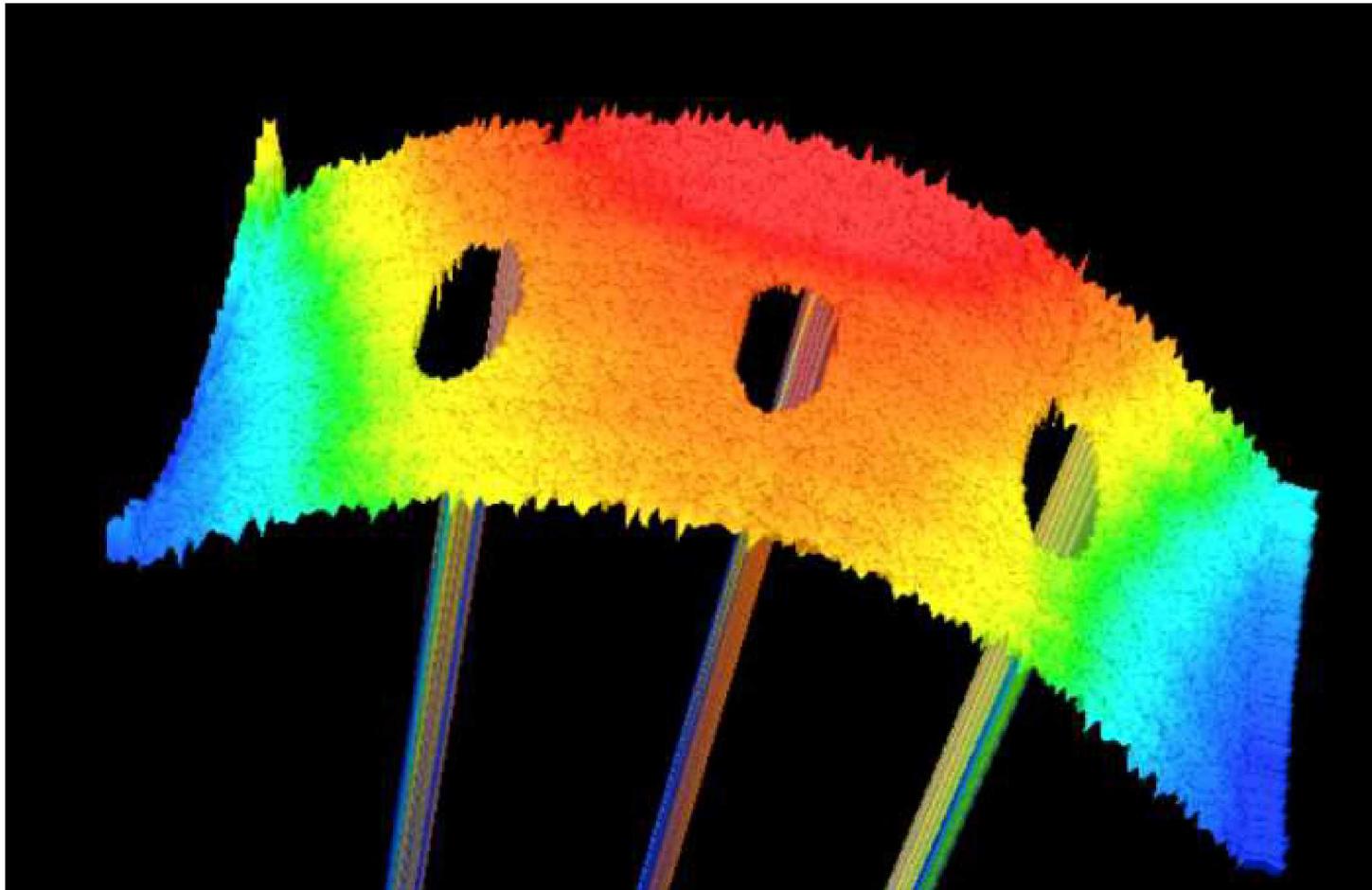
Significant Factors	Insignificant Factors
1. Mean of modal strain 2. Standard deviation of modal strain 3. Skewness of modal strain 4. Skewness of contact pressure 5. Contact area 6. Kurtosis of modal strain 7. Mean of contact pressure	13. Kurtosis of contact pressure 12. Maximum of contact pressure 11. Standard deviation of contact pressure 10. Maximum of modal strain

## For damping variance

Significant Factors
1. Contact area 2. Mean of modal strain 3. Standard deviation of contact pressure 4. Maximum of modal strain 5. Maximum of contact pressure 6. Skewness of modal strain 7. Kurtosis of contact pressure

# The Roughness Factor

- “Flat” interfaces are not flat

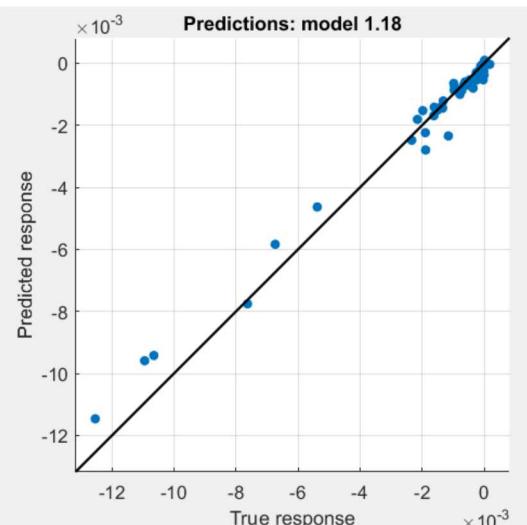


# The Roughness Factor

- We included microscale parameters in a set of nonlinear regression models
- 99% of stiffness variation can be explained using either a Gaussian or Stepwise Linear regression
- 97% of damping variation can be explained using a cubic support vector machine regression (90% can be explained via a Stepwise Linear regression)

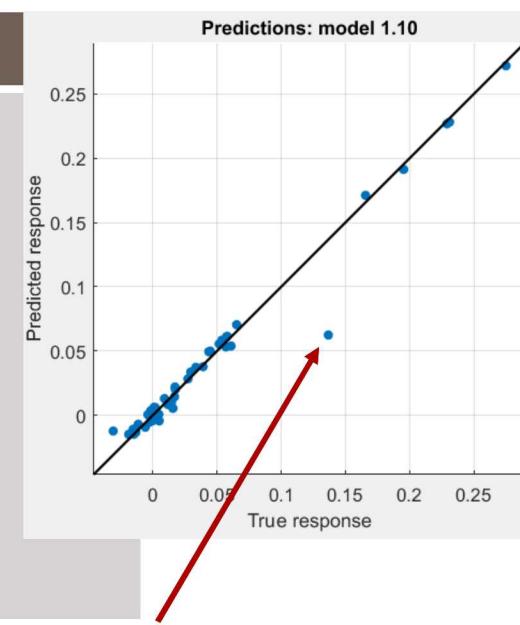
## Significant Factors for Stiffness

1. Std. dev. of contact pressure
2. Number of bolts
3. Macroscale curvature
4. Maximum asperity height
5. Kurtosis of roughness
6. Skewness of roughness
7. Mean modal strain
8. Mean contact pressure
9. Bolt torque
10. Maximum modal strain



## Significant Factors for Damping

1. Std. dev. of contact pressure
2. Mean modal strain
3. Maximum of contact pressure
4. Kurtosis of modal strain
5. Number of bolts
6. Skewness of contact pressure
7. Kurtosis of contact pressure
8. Maximum asperity height
9. Skewness of roughness
10. Kurtosis of roughness



C Beam, Mode 2, 5 Nm bolt torque

# Conclusion

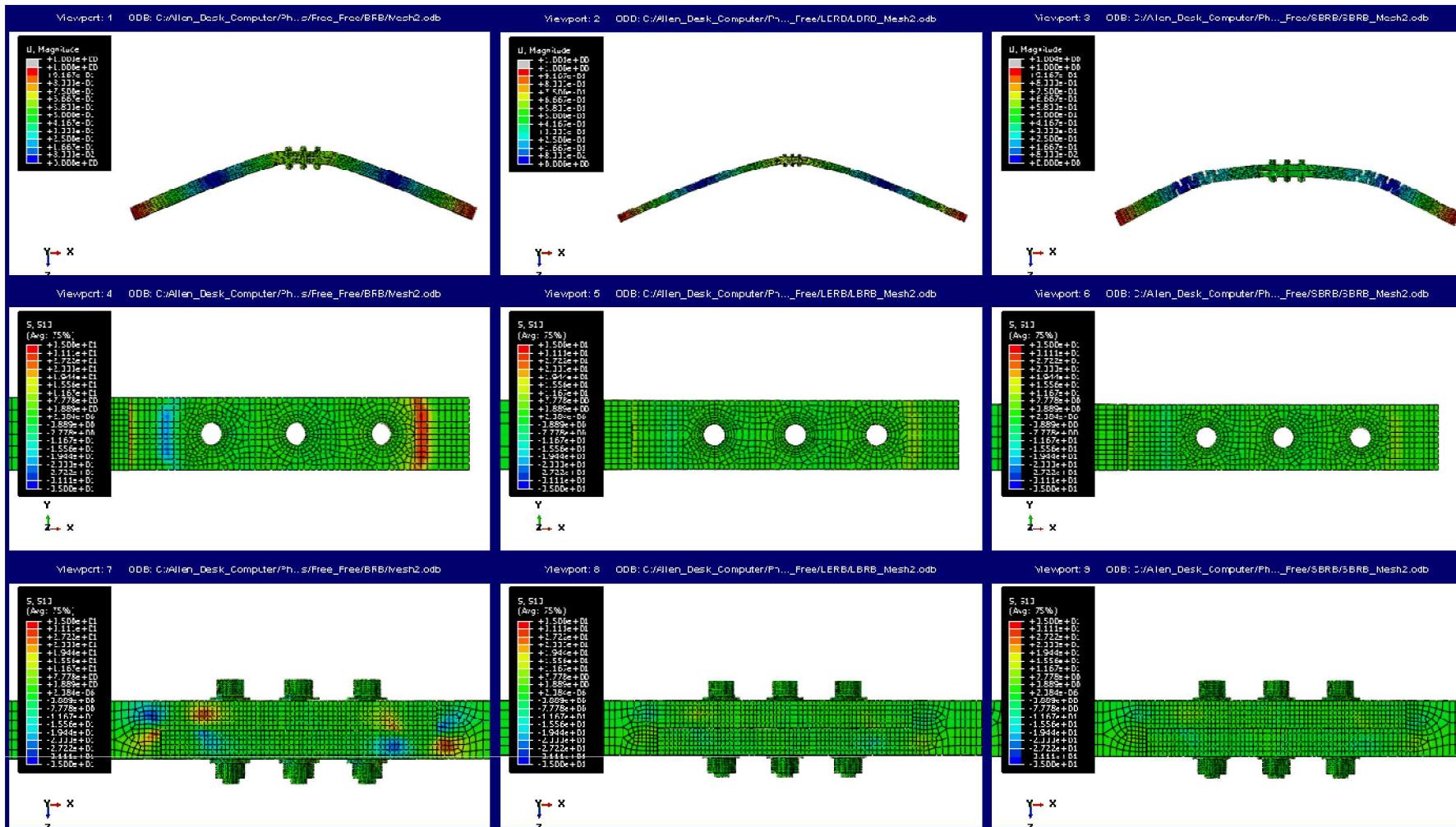
- The *A Priori* Hypothesis stated that the strength of nonlinearity for a given mode of an assembly can be determined by
  1. The interfacial contact pressure
  2. The modal strain energy
- The purely numerical *a priori* estimates can explain
  - 82% of stiffness
  - 55% of damping variance
- A combination of *a priori* estimates and microscale parameters can explain
  - 99% of stiffness
  - 97% of damping variance

# Acknowledgements

- This research was conducted at the 2018 Nonlinear Mechanics and Dynamics Research Institute hosted by Sandia National Laboratories and the University of New Mexico.
- This research was assisted by the research of previous NOMAD teams and research ongoing at Rice University.
- Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525.

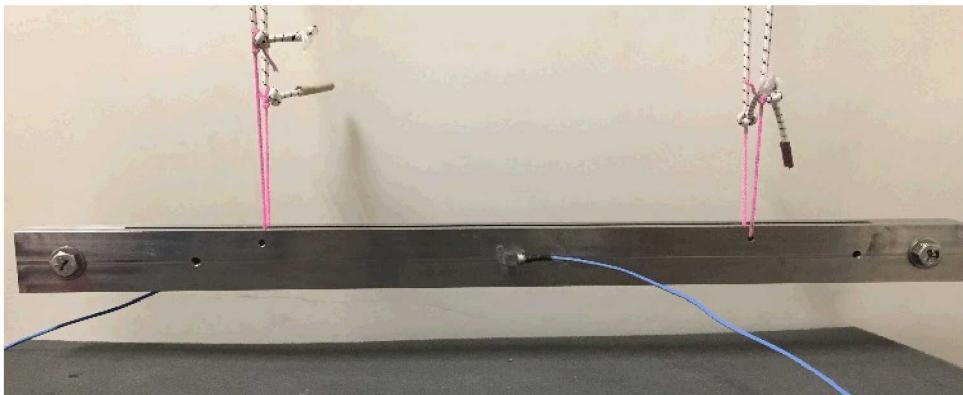


# Observation 2: Far-Field Effects

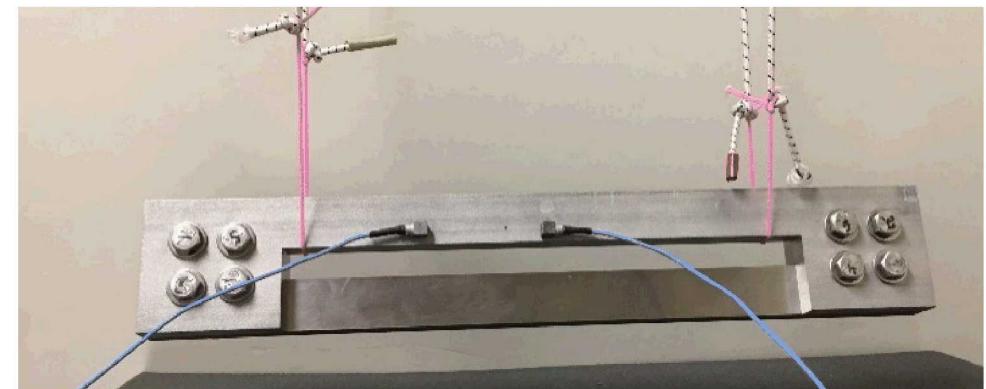


# Experimental Methodology

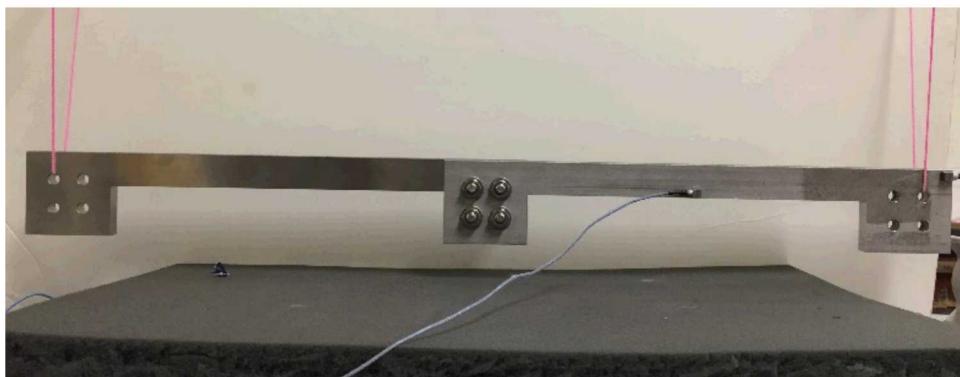
- CBM



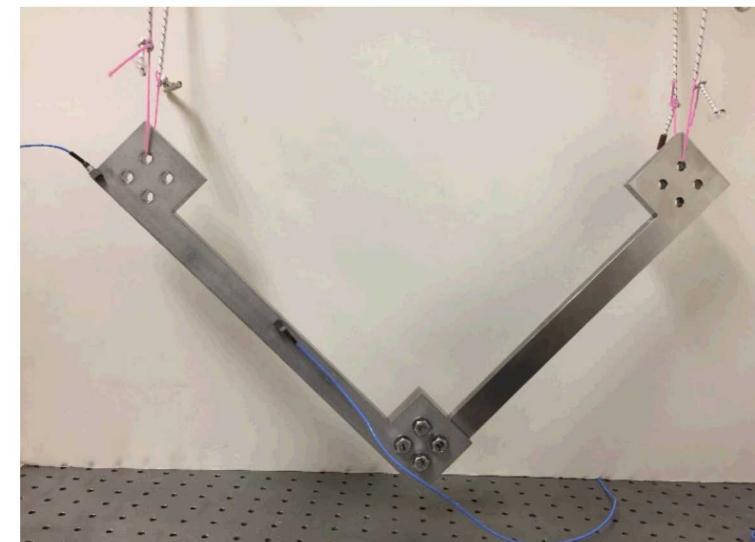
- 4SO



- 4LS



- 4VO



# Experimental Methodology

- Impact hammer tests using free-free boundary conditions
- Bolt torques ranging from 5Nm to 20Nm
- Impact levels from 60N to 900N
- Standardize by maximum mode shape

