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How linear is a linear system?

PRESENTED BY

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IMAC XXXVII 2019

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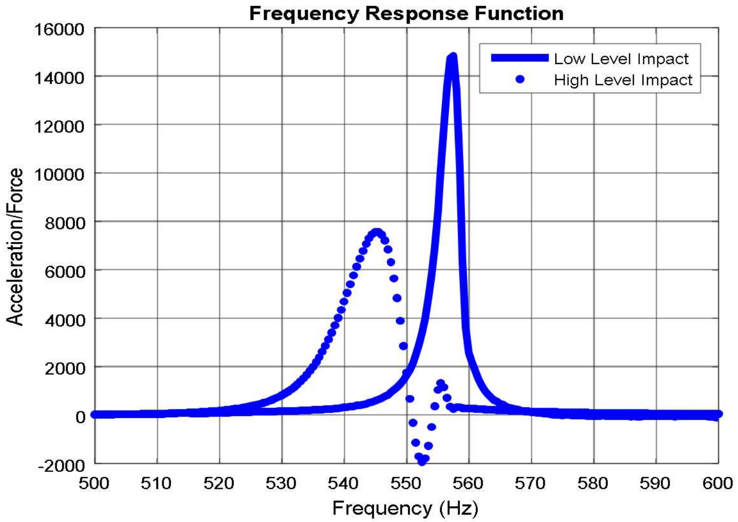
Nonlinear Modal Model Motivation

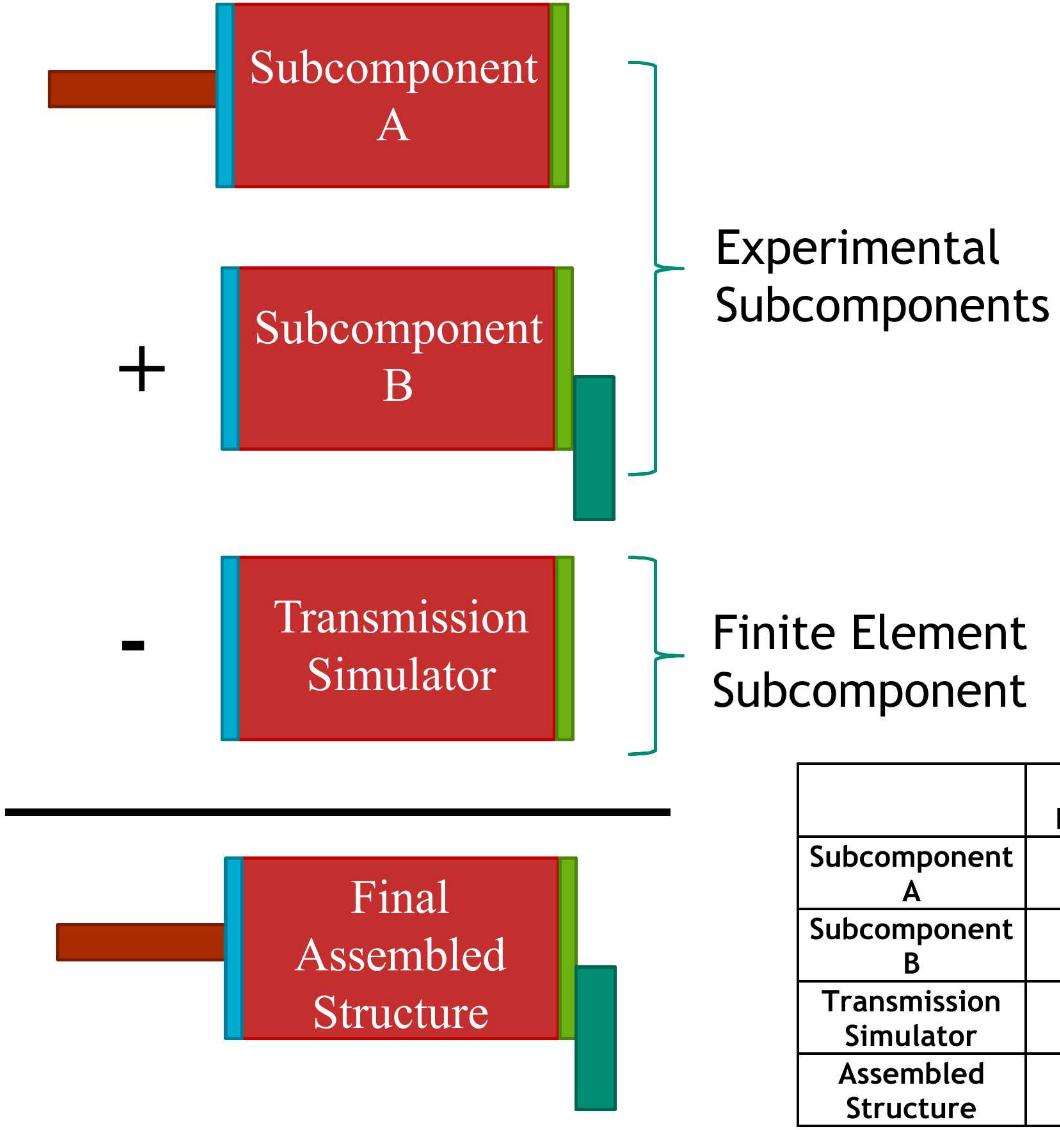
- Often we treat a structures dynamic response as linear, which means the response scales with forcing amplitude.
- Many industries rely on bolted joints to connect subcomponents. The frictional interfaces at these joints cause an otherwise linear system to have a nonlinear response, observed as a change in damping and stiffness with response amplitude

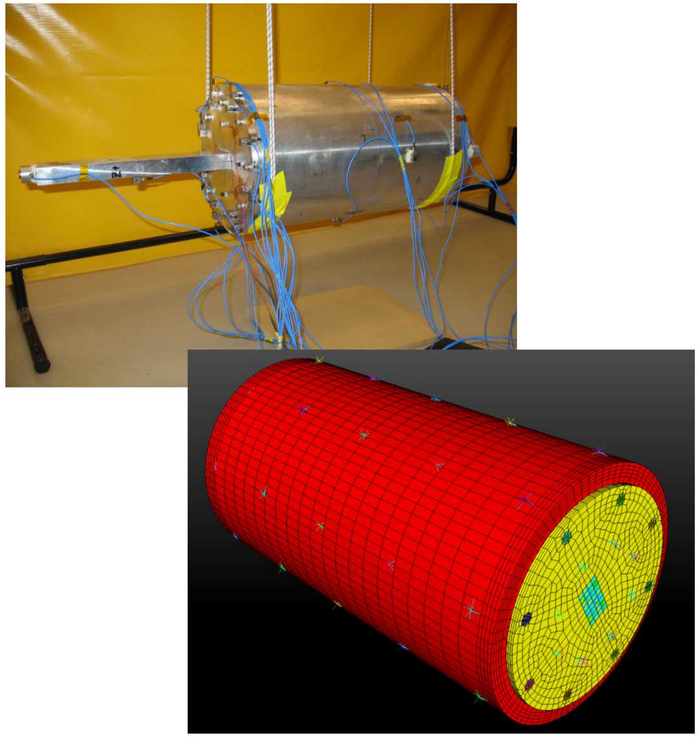


- Many constitutive elements have been formed to characterize these responses when the nonlinearity is caused by joints (Iwan[1], Palmov, Smallwood, etc.)
- At IMAC XXXVI we presented nonlinear substructuring results with really low damping ratio errors… they weren't always so low

[1] D. J. Segalman, "An Initial Overview of Iwan Modeling for Mechanical Joints," Sandia National Laboratories, Albuquerque, New Mexico SAND2001-0811, 2001.

Substructuring Schematic





	Nose Beam	Plate	Cylinder	Aft Plug	Tail Beam
Subcomponent A	X	X	X	X	
Subcomponent B		X	X	X	X
Transmission Simulator		X	X	X	
Assembled Structure	X	X	X	X	X

Linear Substructuring Theory

- The traditional Transmission Simulator method adds two components and a negative copy of the fixture.
- Begin with uncoupled modal equations of motion for each subcomponent:

$$\begin{bmatrix} I_A & 0 & 0 \\ 0 & I_B & 0 \\ 0 & 0 & -I_{TS} \end{bmatrix} \begin{Bmatrix} \ddot{q}_A \\ \ddot{q}_B \\ \ddot{q}_{TS} \end{Bmatrix} + \begin{bmatrix} \left[\ddots 2\zeta_A\omega_A \ddots\right] & 0 & 0 \\ 0 & \left[\ddots 2\zeta_B\omega_B \ddots\right] & 0 \\ 0 & 0 & -\left[\ddots 2\zeta_{TS}\omega_{TS} \ddots\right] \end{bmatrix} \begin{Bmatrix} \dot{q}_A \\ \dot{q}_B \\ \dot{q}_{TS} \end{Bmatrix} + \begin{bmatrix} \left[\ddots \omega_A^2 \ddots\right] & 0 & 0 \\ 0 & \left[\ddots \omega_B^2 \ddots\right] & 0 \\ 0 & 0 & -\left[\ddots \omega_{TS}^2 \ddots\right] \end{bmatrix} \begin{Bmatrix} q_A \\ q_B \\ q_{TS} \end{Bmatrix} = \begin{Bmatrix} \phi_A^T F_A \\ \phi_B^T F_B \\ \phi_{TS}^T F_{TS} \end{Bmatrix}$$

- Set-up physical constraints and convert to the modal domain:

$$B \begin{Bmatrix} x_A \\ x_B \\ x_{TS} \end{Bmatrix} = 0 \quad \Rightarrow \quad \begin{bmatrix} \phi_{TS}^+ & 0 \\ 0 & \phi_{TS}^+ \end{bmatrix} \begin{bmatrix} \phi_A & 0 & -\phi_{TS} \\ 0 & \phi_B & -\phi_{TS} \end{bmatrix} \begin{Bmatrix} q_A \\ q_B \\ q_{TS} \end{Bmatrix} = \tilde{B} \begin{Bmatrix} q_A \\ q_B \\ q_{TS} \end{Bmatrix} = 0$$

- Create synthesized coordinates and formulate a transformation matrix:

$$\tilde{B} L \eta = 0 \quad \Rightarrow \quad L = null(\tilde{B})$$

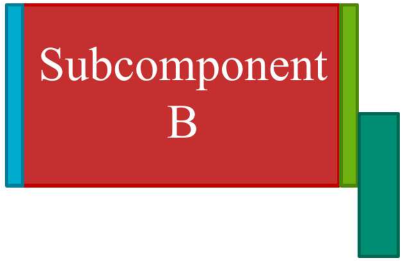
- Apply transformation to predict assembly level dynamics:

$$L^T \begin{bmatrix} I_A & 0 & 0 \\ 0 & I_B & 0 \\ 0 & 0 & -I_{TS} \end{bmatrix} L \begin{Bmatrix} \ddot{\eta}_A \\ \ddot{\eta}_B \\ \ddot{\eta}_{TS} \end{Bmatrix} + L^T \begin{bmatrix} \left[\ddots 2\zeta_A\omega_A \ddots\right] & 0 & 0 \\ 0 & \left[\ddots 2\zeta_B\omega_B \ddots\right] & 0 \\ 0 & 0 & -\left[\ddots 2\zeta_{TS}\omega_{TS} \ddots\right] \end{bmatrix} L \begin{Bmatrix} \dot{\eta}_a \\ \dot{\eta}_B \\ \dot{\eta}_{TS} \end{Bmatrix} + L^T \begin{bmatrix} \left[\ddots \omega_A^2 \ddots\right] & 0 & 0 \\ 0 & \left[\ddots \omega_B^2 \ddots\right] & 0 \\ 0 & 0 & -\left[\ddots \omega_{TS}^2 \ddots\right] \end{bmatrix} L \begin{Bmatrix} \eta_A \\ \eta_B \\ \eta_{TS} \end{Bmatrix} = L^T \begin{Bmatrix} \phi_A^T F_A \\ \phi_B^T F_B \\ \phi_{TS}^T F_{TS} \end{Bmatrix}$$

The Transmission Simulator method has performed well in the past on many linear testbed systems

Linear Subcomponent Test Results

- "Low-level" burst random testing used to capture linear subcomponent modal parameters



Mode	$f_{n,o}$ [Hz]	ζ_o [%cr]	Shape Description
7	128.03	0.38	1st bend of Beam in soft direction
8	170.00	0.28	1st bend of Beam in stiff direction
9	548.36	0.37	Plate axial mode
10	863.75	1.15	(2,0) ovaling of Cylinder
11	878.25	1.11	(2,0) ovaling of Cylinder
12	980.80	0.45	(3,0) ovaling of Cylinder
13	987.70	0.46	(3,0) ovaling of Cylinder
14	1084.20	0.12	2nd bend of Beam in soft direction

Mode	$f_{n,o}$ [Hz]	ζ_o [%cr]	Shape Description
7	232.23	0.174	1st bend of Tail in soft direction
8	607.00	0.869	1st bend of Tail in stiff direction
9	879.75	0.996	(2,0) ovaling of Cylinder
10	886.06	1.058	(2,0) ovaling of Cylinder
11	981.82	0.437	(3,0) ovaling of Cylinder
12	990.10	0.390	(3,0) ovaling of Cylinder
13	1148.7	1.406	Plate axial Mode
14	1279.5	0.590	2nd bend of Tail in soft direction

Linear Substructuring Results

- Using the linear modes from each subcomponent a prediction for linear low level response can be obtained

Mode	Predicted f_n [Hz]	Measured f_n [Hz]	% Diff.	Predicted ζ [%cr]	Measured ζ [%cr]	% Diff.	MAC	Shape Description
7	128.50	128.22	0.22%	0.381	0.276	38.22%	1.00	1st bend of beam in soft direction
8	170.30	169.46	0.46%	0.282	0.176	60.53%	0.99	1st bend of beam in stiff direction
9	232.29	233.37	-0.46%	0.168	0.176	-0.70%	1.00	1st bend of tail in soft direction
10	548.37	551.39	-0.55%	0.375	0.245	52.97%	0.99	Axial mode
11	606.18	616.25	-1.63%	0.860	0.426	101.57%	1.00	1st bend of tail in stiff direction

[1]Rigid body modes not shown

- Predicted frequency and mode shape had small errors compared to a truth test but damping had more significant errors
- The goal of the original project was to characterize nonlinear frequency and damping in substructuring
- Perhaps the subcomponent models were obtained from tests exhibiting slightly nonlinear behavior…

We can use nonlinear experimental identification tools to asses the subcomponent nonlinearity!

Hilbert Transform Theory

- To use the Hilbert Transform to assess the situation the subcomponents were retested using a hammer impact
- Using a modal filter, the acceleration measurements are transformed into modal acceleration
- Next, we assume the modal response can be written in an exponential form

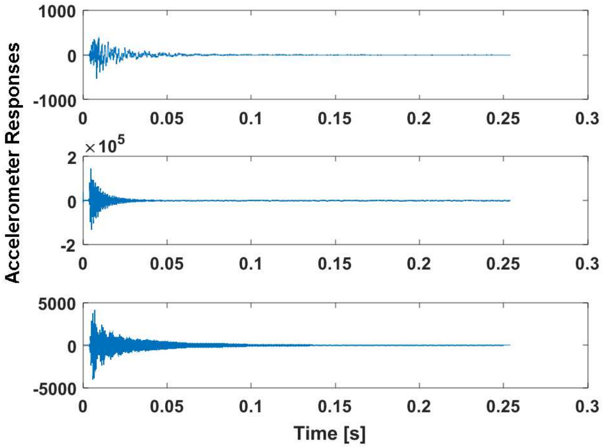
$$\ddot{q}(t) = e^{\psi_r(t)+i\psi_i(t)}$$

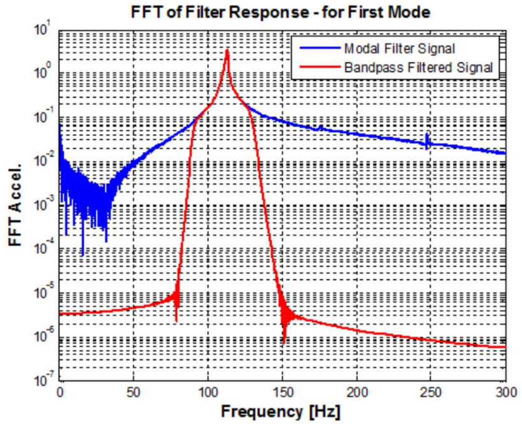
- Using the Hilbert transform we can obtain expressions for the Hilbert envelope and unwrapped phase
- The time derivatives of the Hilbert envelope and phase can be used to find expressions for the frequency and dissipation of the modal signal

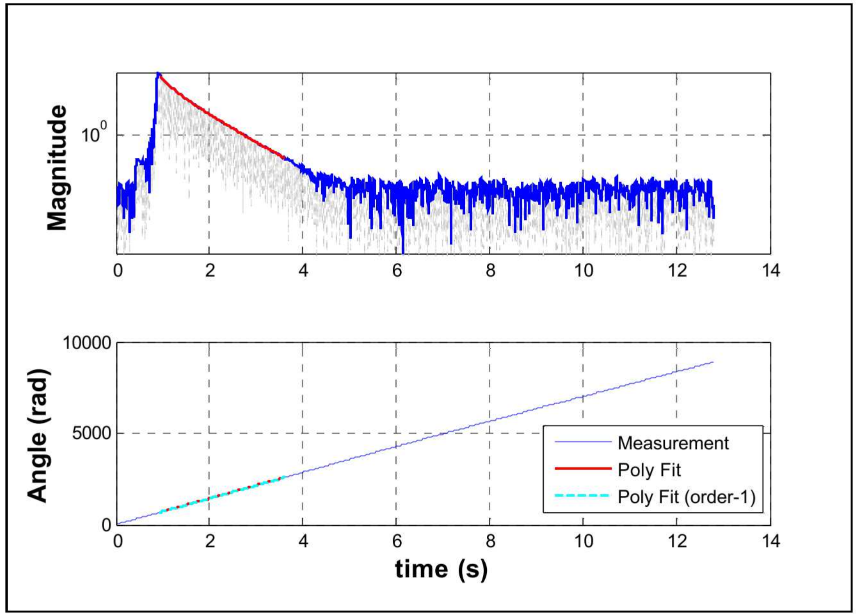
$$\omega_d(t) = \frac{d\psi_i}{dt} \qquad -\zeta\omega_n(t) = \frac{d\psi_r}{dt}$$

- These relationships can now be written as a function of amplitude

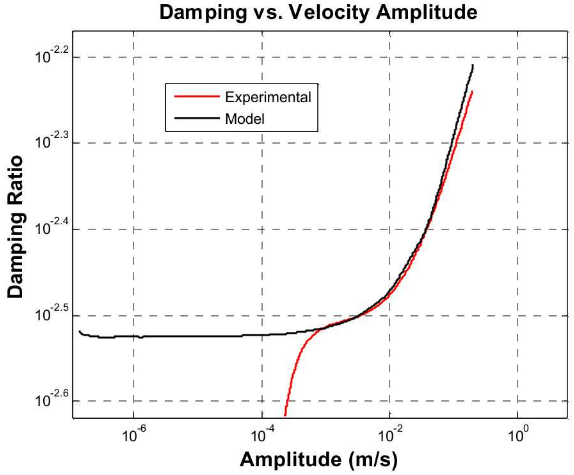
Hilbert Transform Example

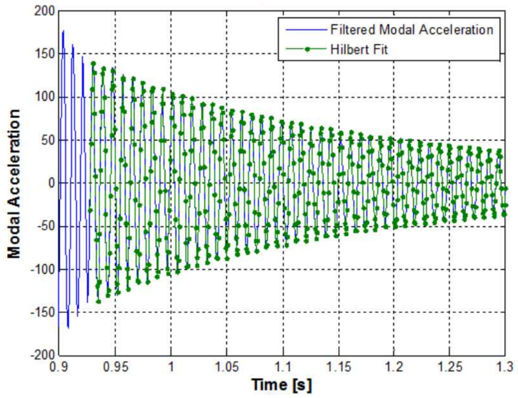






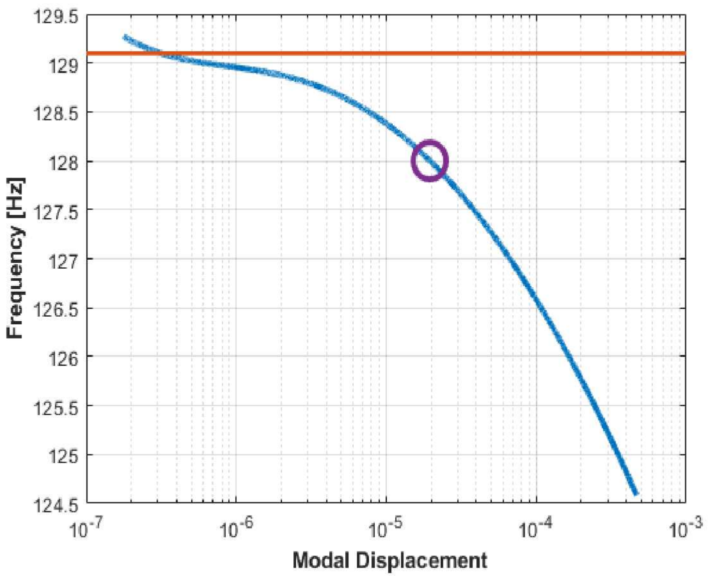
- Gather ringdown data
- Apply a modal filter
- Use the Hilbert Transform to fit an exponential form
- Check the Hilbert fit of experimental measurement
- Use SDOF approximations to fit damping/frequency

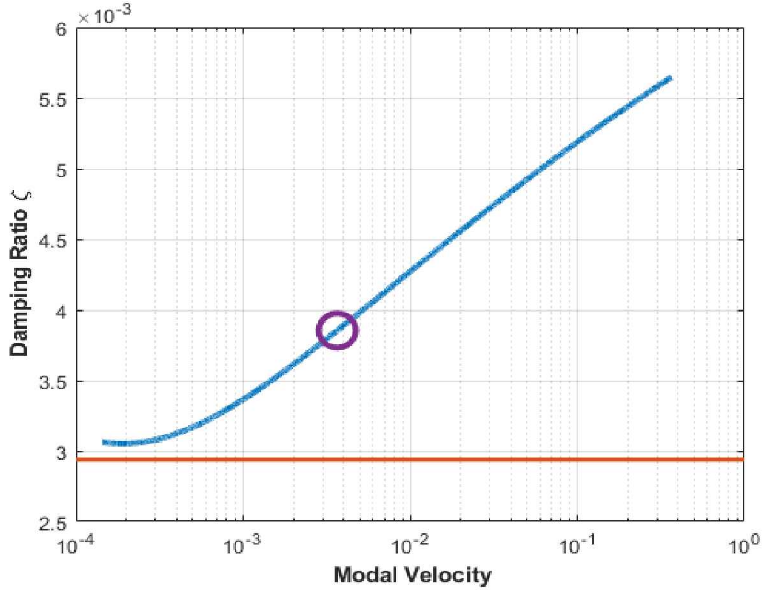




Hilbert Transform Findings

- Each experimental subcomponent was tested using impacts and fit using the Hilbert transform
- The linear frequency and damping ratio were found to be in the slightly nonlinear testing





Linear Subcomponent Test Results II

- Linear subcomponent testing was repeated and the amplifier settings were lowered until a frequency and damping from the Hilbert curve were obtained
- Frequencies changed less than 1% but large changes in damping ratio were found
- Similar errors were observed in Subcomponent B

Mode	$f_{n,o}$ [Hz]	$f_{n,u}$ [Hz]	f_n % Error	ζ_o [%cr]	ζ_u [%cr]	ζ % Error
7	128.03	129.2	-0.91%	0.384	0.294	30.61%
8	170.00	171.1	-0.64%	0.284	0.170	67.06%
9	548.36	552.0	-0.66%	0.373	0.241	54.77%
10	863.75	867.8	-0.47%	1.149	1.122	2.41%
11	878.25	883.3	-0.57%	1.113	0.930	19.68%
12	980.80	980.8	0.00%	0.451	0.447	0.89%
13	987.70	987.7	0.00%	0.456	0.462	-1.30%
14	1084.20	1031.8	-0.10%	0.123	0.090	37.17%

Subscript o for original testing
Subscript u for updated testing

Linear Substructuring Results II

- Using the **updated** linear modes from each subcomponent a prediction for linear response can be obtained

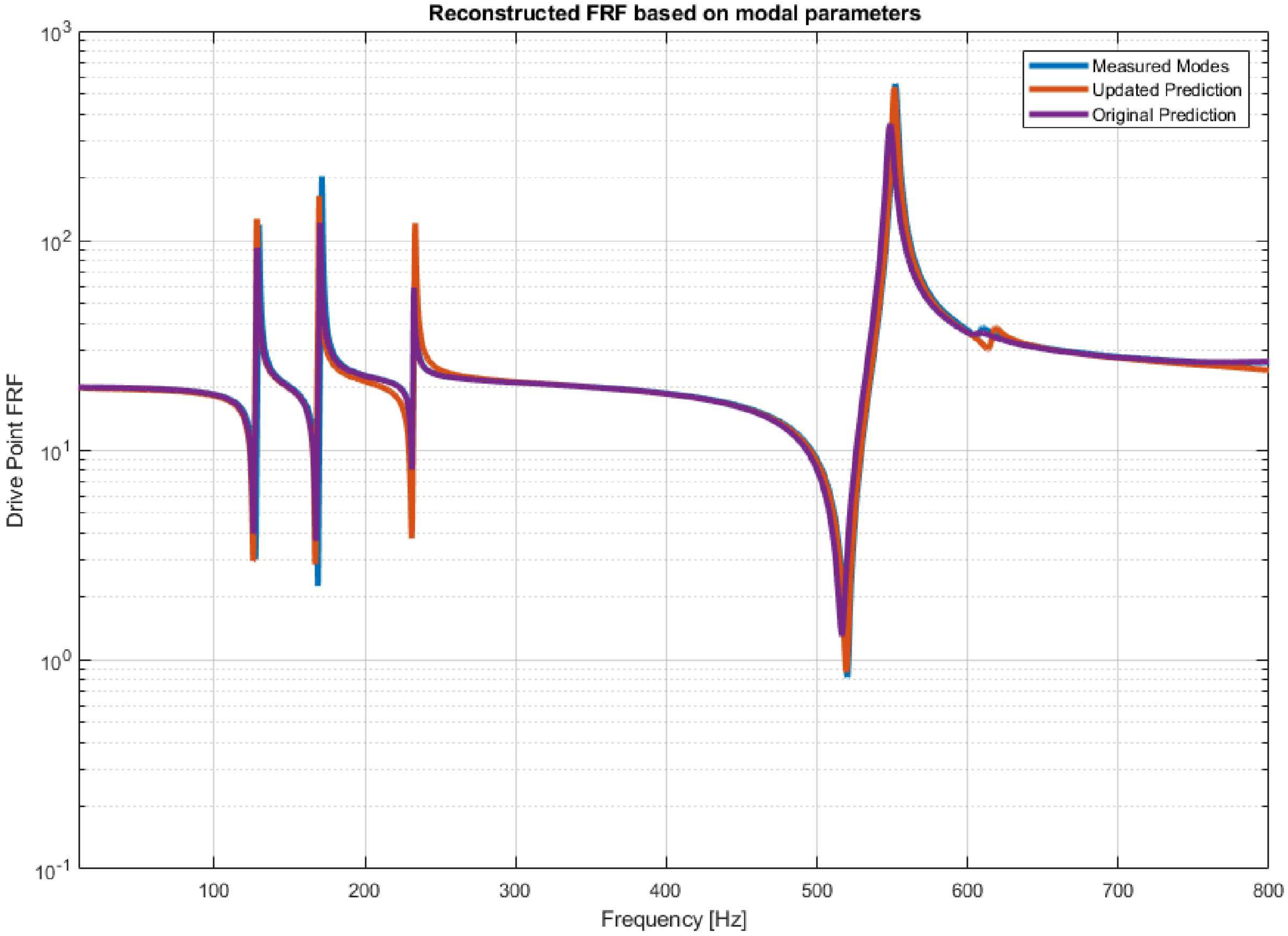
Mode	Predicted $f_{n,u}$ [Hz]	Measured f_n [Hz]	% Diff. Updated	% Diff. Original	Predicted ζ_u [%cr]	Measured ζ [%cr]	% Diff. Updated	% Diff. Original	MAC	MAC Original
7	129.62	128.22	1.09%	0.22%	0.295	0.276	6.94%	38.22%	1.00	1.00
8	171.47	169.46	1.19%	0.46%	0.171	0.176	-2.90%	60.53%	0.99	0.99
9	232.56	233.37	-0.35%	-0.46%	0.168	0.176	-4.47%	-0.70%	1.00	1.00
10	552.01	551.39	0.11%	-0.55%	0.241	0.245	-1.43%	52.97%	0.99	0.99
11	608.08	616.25	-1.32%	-1.63%	0.415	0.426	-2.56%	101.57%	1.00	1.00

[1]Rigid body modes not shown

- Predicted frequency errors remained below 2%
- Damping ratio errors decreased drastically
- MAC criteria remained unchanged

We can use nonlinear experimental identification tools to asses the subcomponent nonlinearity!

Linear Substructuring Results III



Conclusions

- Linear testing is important when studying the dynamics of a structure
- Low-level testing should be verified to ensure a linear test is not testing in a slightly nonlinear range
  - **Especially when testing jointed structures!**
- There are several tools that can be used to asses the nonlinearity in a test results
- In this case study, the Hilbert transform was used to assess subcomponent test results
- Assuming a result is linear when the system is tested in the nonlinear range can lead to large prediction errors, especially in damping!