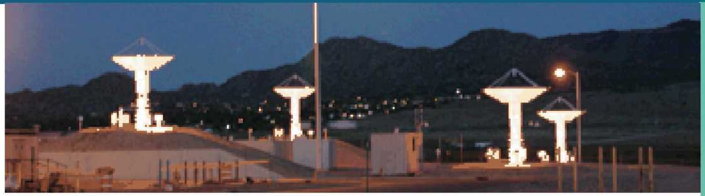
SAND2019-0908C

Techniques for Nonlinear Identification and Maximizing Modal Response



PRESENTED BY

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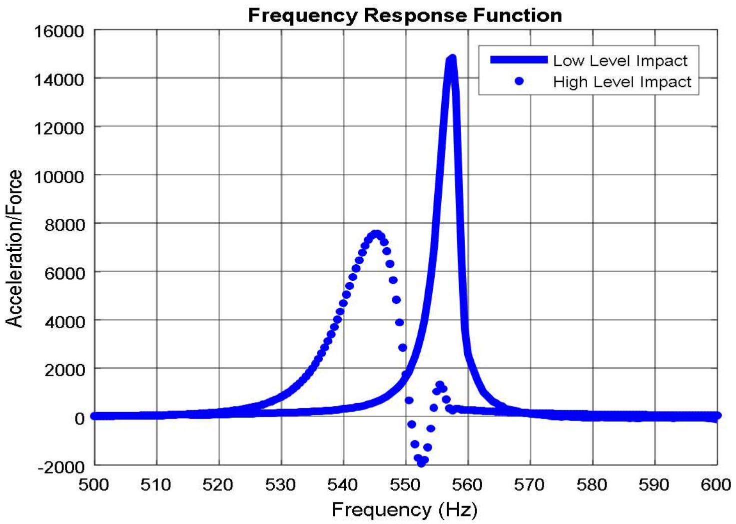
IMAC XXXVII 2019

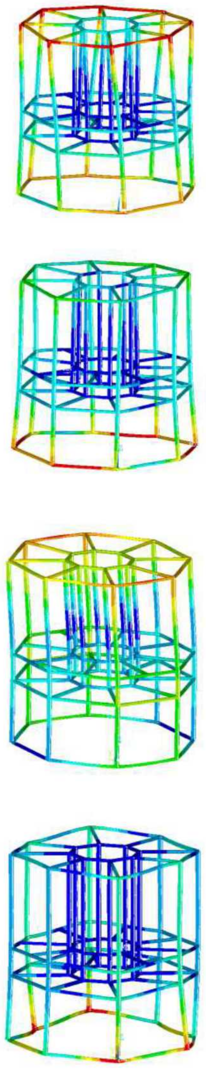
Nonlinear Modal Model Motivation

• Often we treat a structures dynamic response as linear, which means the response scales with forcing amplitude.

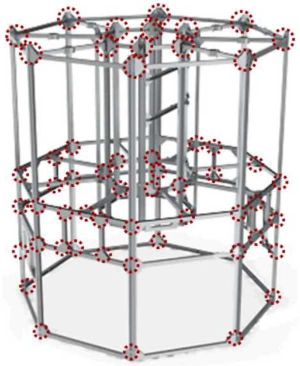
• Many industries rely on bolted joints to connect subcomponents. The frictional interfaces at these joints cause an otherwise linear system to have a nonlinear response, observed as a change in damping and stiffness with response amplitude



• Many constitutive elements have been formed to characterize these responses when the nonlinearity is caused by joints (Iwan[1], Palmov, Smallwood, etc.)



• If we have many joints, it becomes cumbersome to identify the parameters of each joint separately!



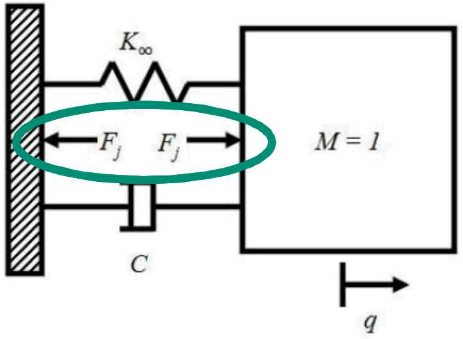
$$\begin{Bmatrix} F_S^1, K_T^1, \chi^1, \beta^1 \\ F_S^2, K_T^2, \chi^2, \beta^2 \\ F_S^3, K_T^3, \chi^3, \beta^3 \\ \vdots \end{Bmatrix}$$

• Experimental evidence has shown that many jointed structures can be tested and represented with uncoupled weakly nonlinear modes.[2]

• This comes with two main assumptions
  - Energy transfer between modes remains negligible
  - The mode shapes of the nonlinear system are preserved at all amplitudes

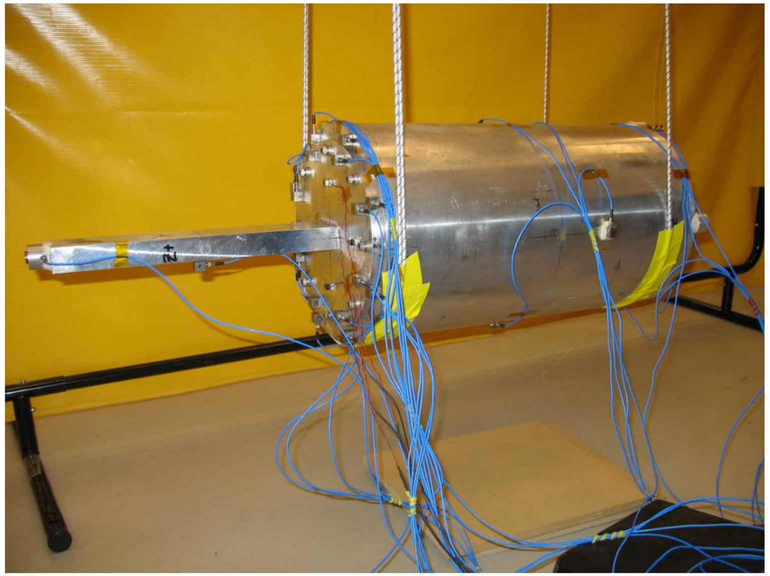
[1] D. J. Segalman, "An Initial Overview of Iwan Modeling for Mechanical Joints," Sandia National Laboratories, Albuquerque, New Mexico SAND2001-0811, 2001.
[2] M. Eriten, et al., "Nonlinear system identification of frictional effects in a beam with a bolted joint connection," Mechanical Systems and Signal Processing, vol. 39, pp. 245-264, 2013.

History of the Nonlinear Modal Modeling at Sandia National Labs

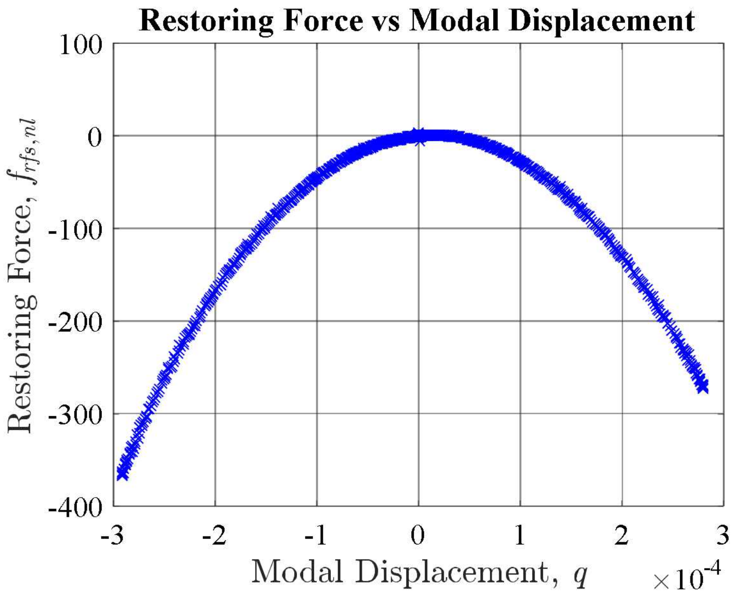


- Current Sandia research has been focused on nonlinear modal modeling
- This theory augments a traditional modal model with the addition of a nonlinear forcing element to capture nonlinear response on a mode-by-mode basis

- Previous works SNL using nonlinear modal modeling:
  - Study use of modal Iwan models to represent nonlinear joint dynamics (2001)
  - Establishing nonlinear modal modeling technique using impact hammer excitations (2015)
  - Updating finite element models using nonlinear modal elements derived with windowed sinusoidal excitations (2016)
  - Nonlinear substructuring using elements derived from shaker excitations (2017)



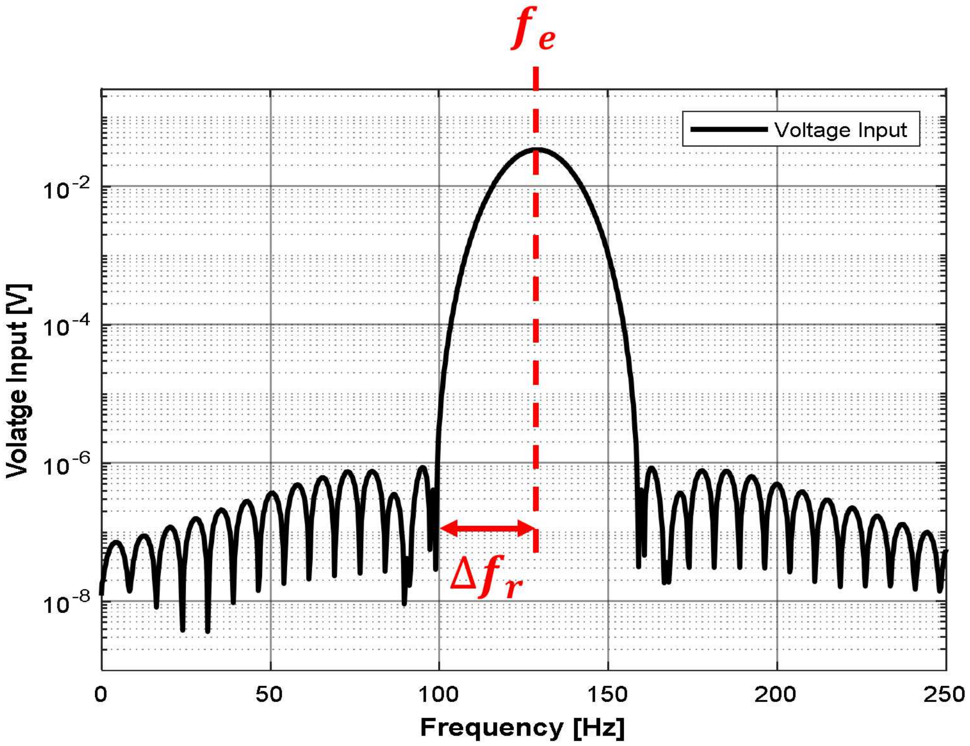
Motivation for Maximizing Modal Amplitude

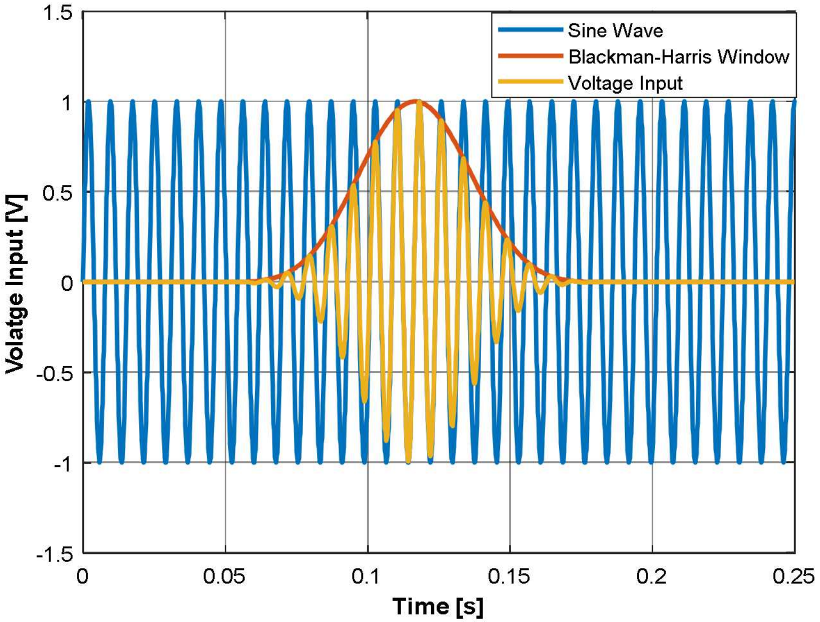


- Nonlinear modal models are often characterized by the amplitude dependent natural frequency and damping observed in the system
- These measurements are only accurate within the amplitude range tested
- Extrapolation with nonlinear models is inadvisable, so higher responses must be achieved in testing

In order to reach operational vibration levels the modal amplitude excited needs to be increased

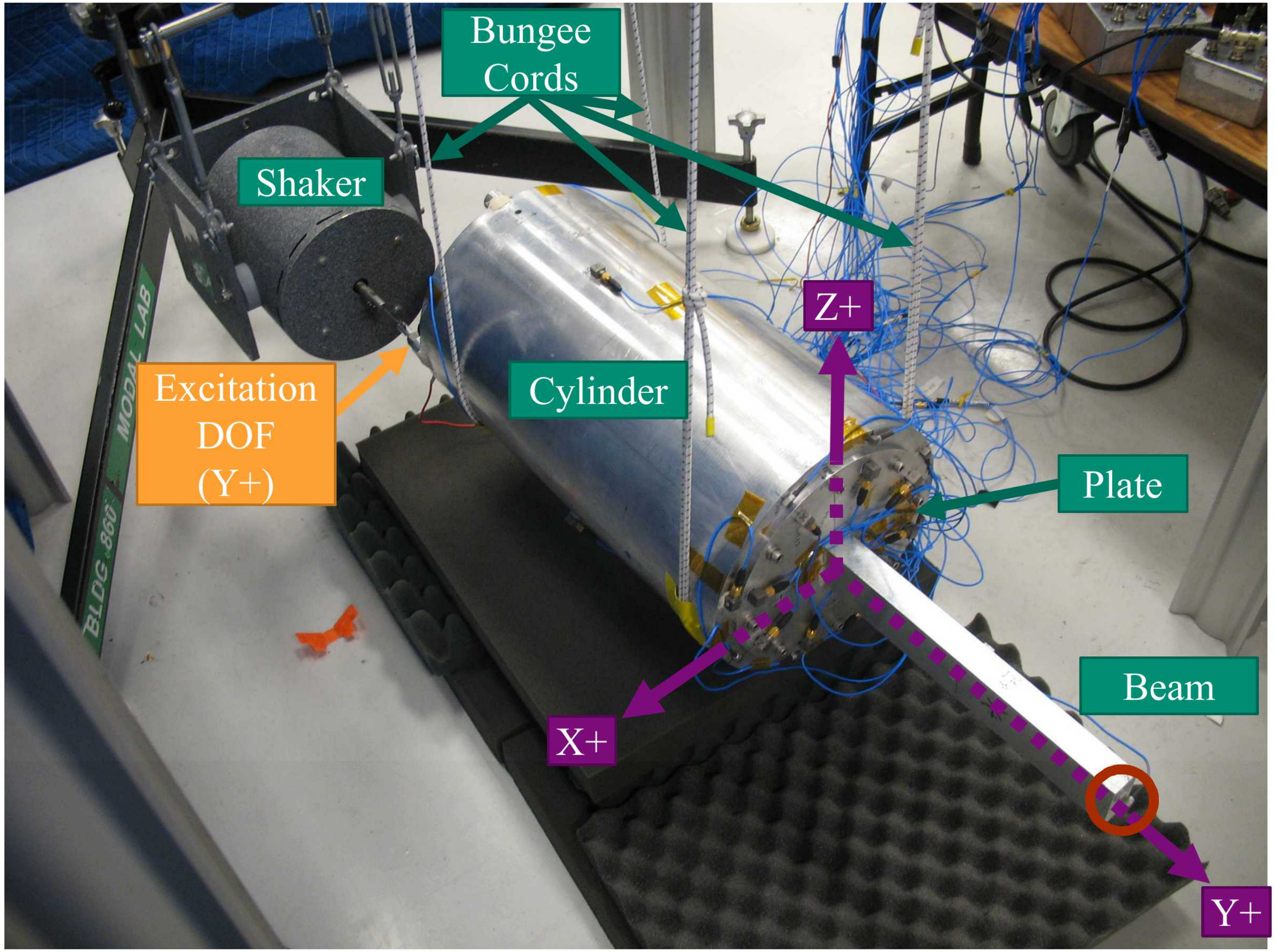
Windowed Sinusoid aka the Sine Beat





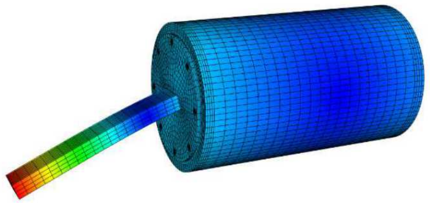
- The objective of this project is to maximize the modal response of a nonlinear structure using variations on the previously studied sine beat excitation technique
- Windowed sinusoids were adjusted by changing the center excitation frequency f_e and frequency band range Δf_r

Test Set-Up and Linear Modal Analysis Results

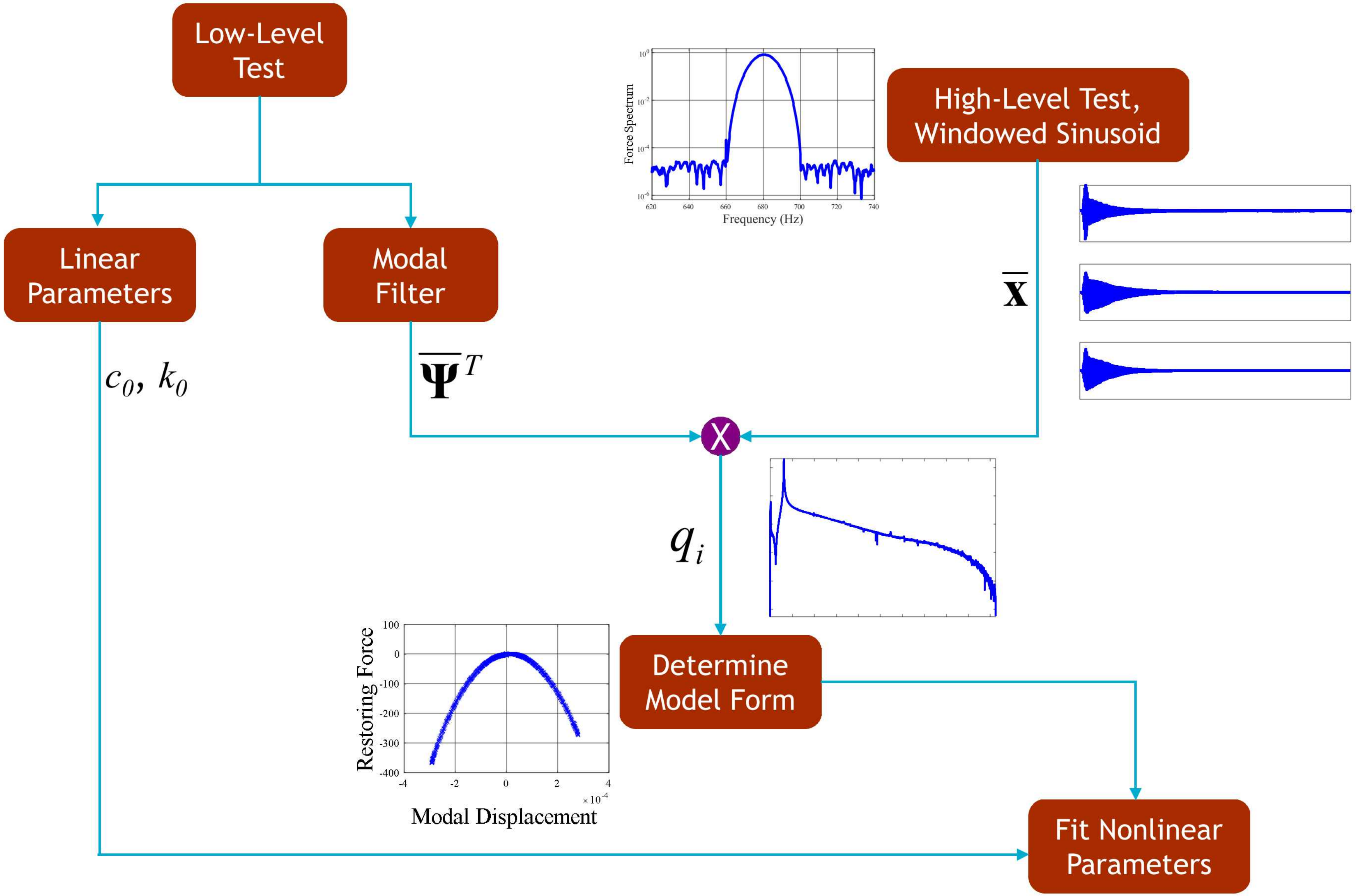


Mode*	Frequency (Hz)	Damping (%)	Description
7	130.0	0.397 %	1st bend of Beam in X-direction
8	172.6	0.322 %	1st bend of Beam in Z-direction
9	385.8	0.069 %	Ovaling of Cylinder
10	391.9	0.083 %	Ovaling of Cylinder
11	551.6	0.278 %	Axial mode
12	945.4	0.413 %	Ovaling of Cylinder
13	948.3	0.513 %	Ovaling of Cylinder
14	1025.7	0.076 %	2nd bend of Beam in X-direction

***Rigid body modes not listed**

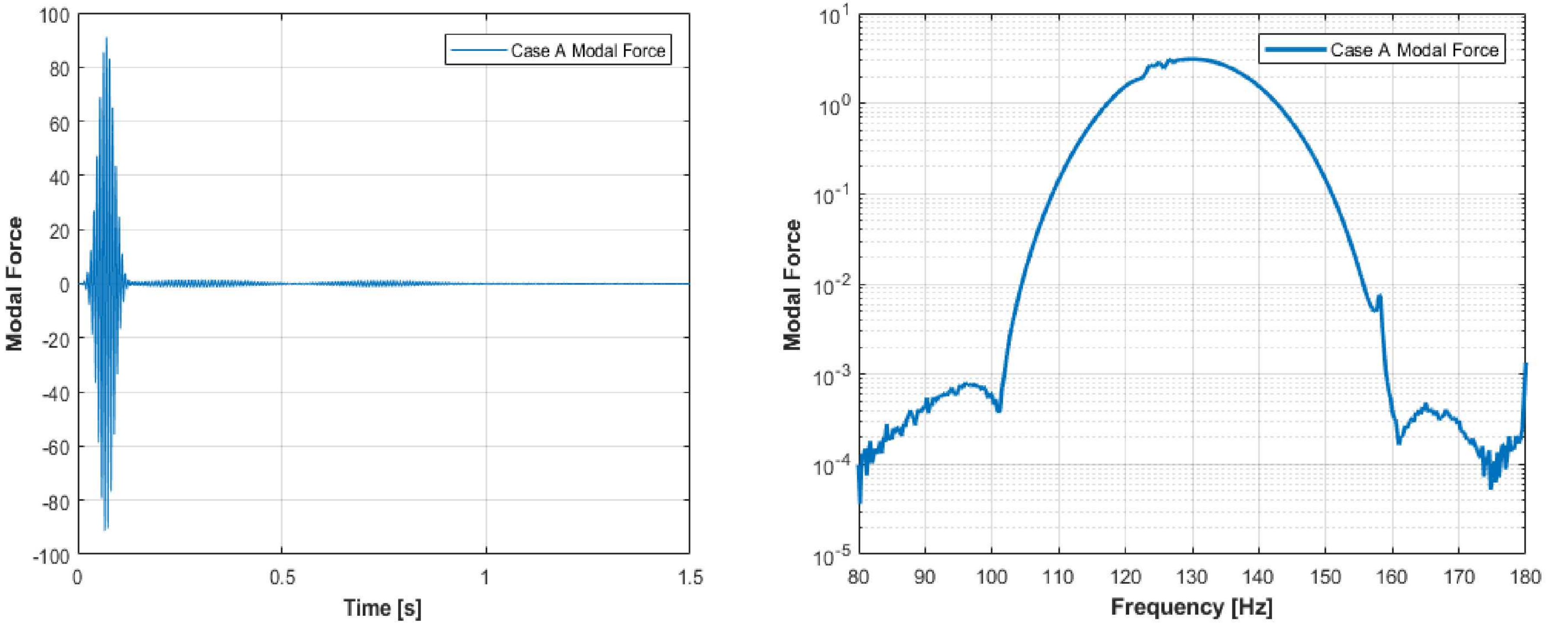


Nonlinear Modal Model Identification Process



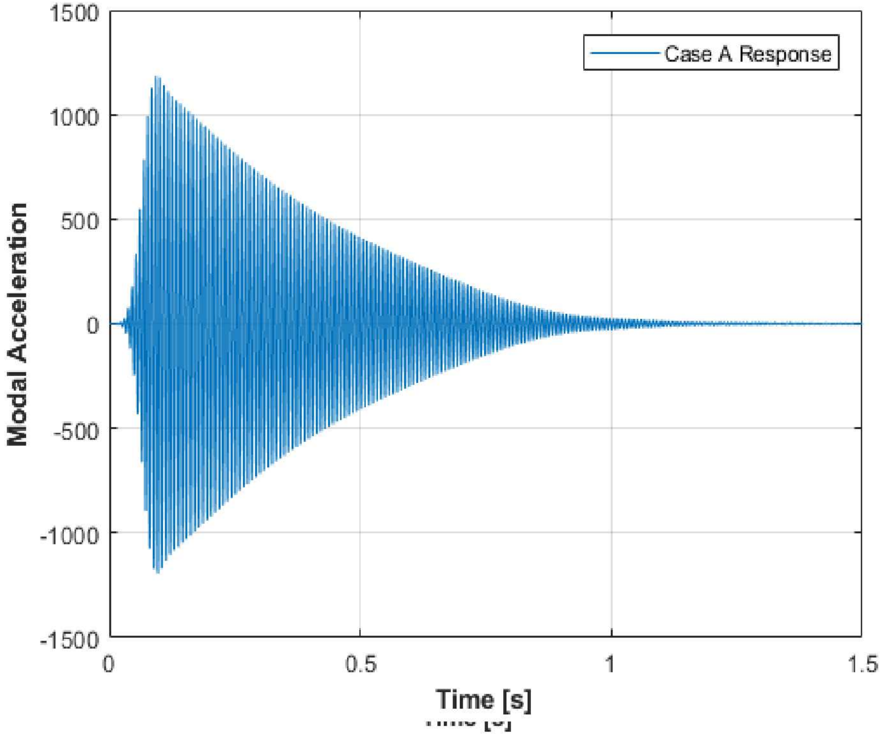
SNL Modal Model Example - Forcing

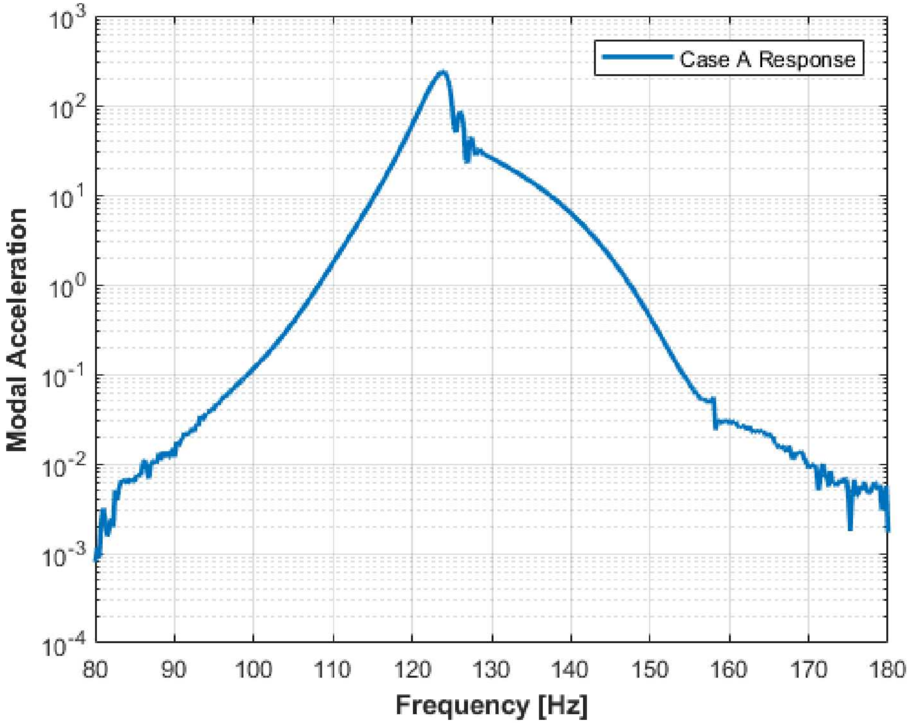
- A high-level force is applied in the form of a windowed sinusoid (sine beat)…



SNL Modal Model Example - Response

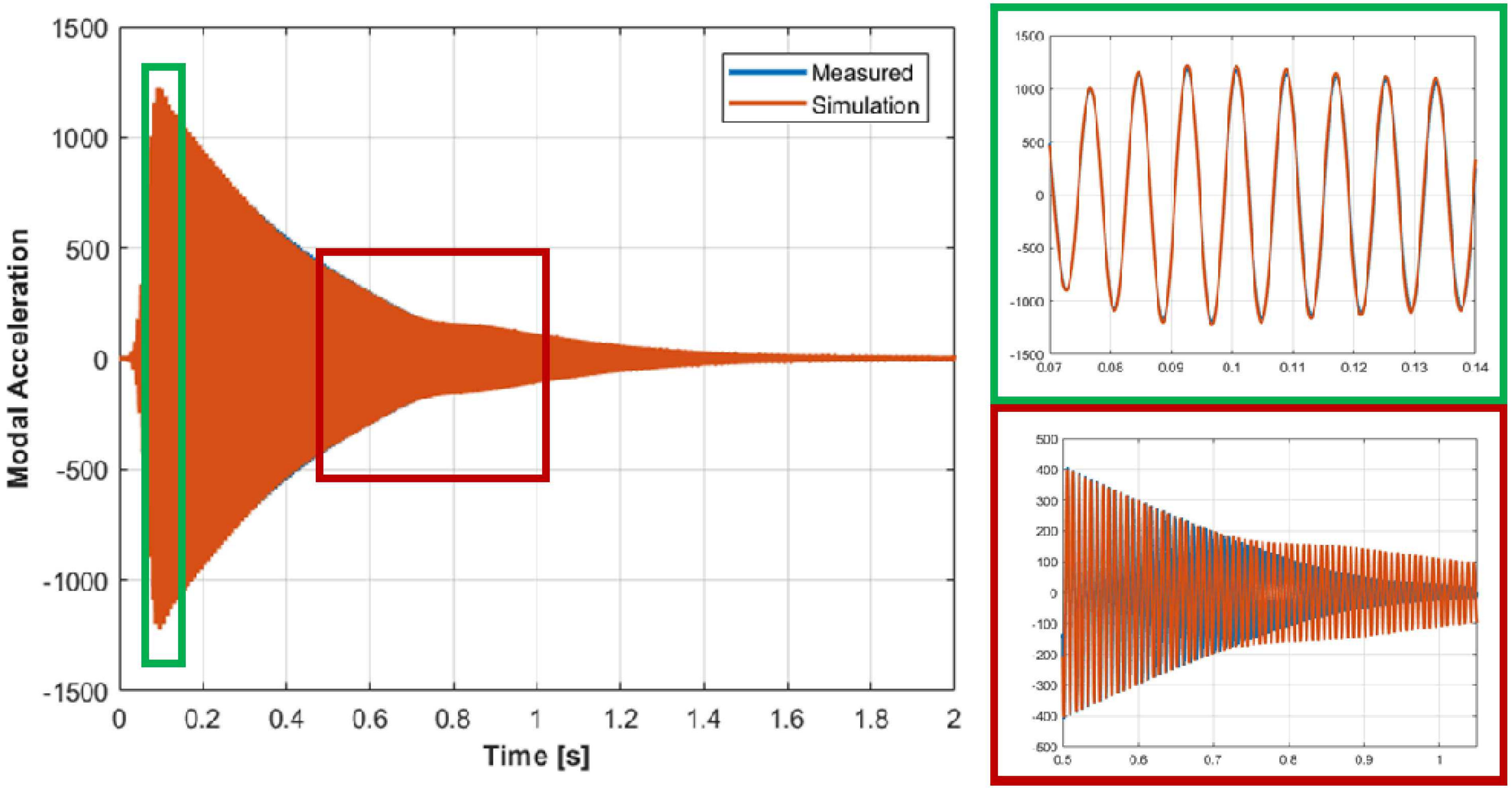
- A high-level force is applied in the form of a windowed sinusoid (sine beat)…



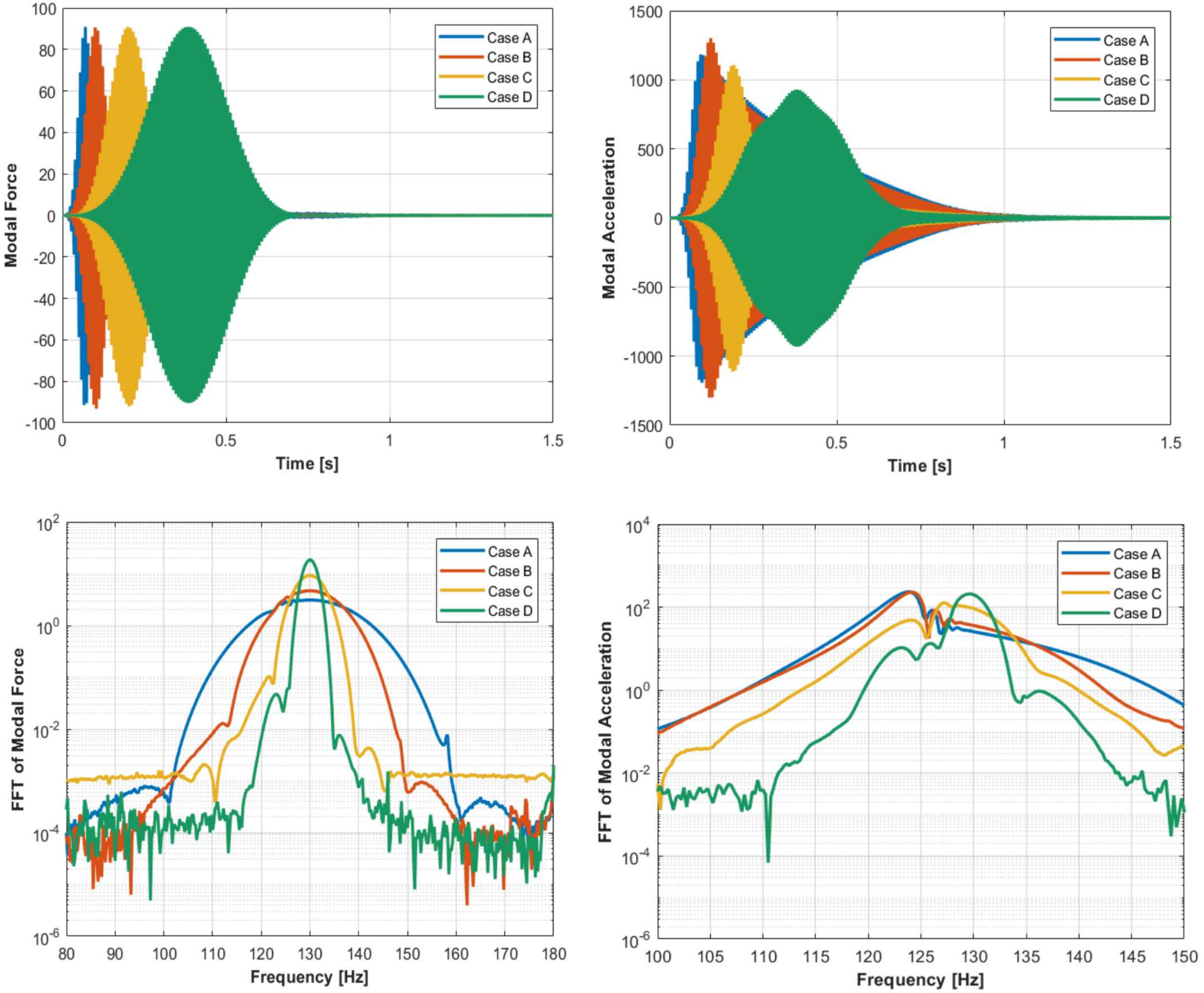


SNL Modal Model Example – Model and Simulation

- Captured Response



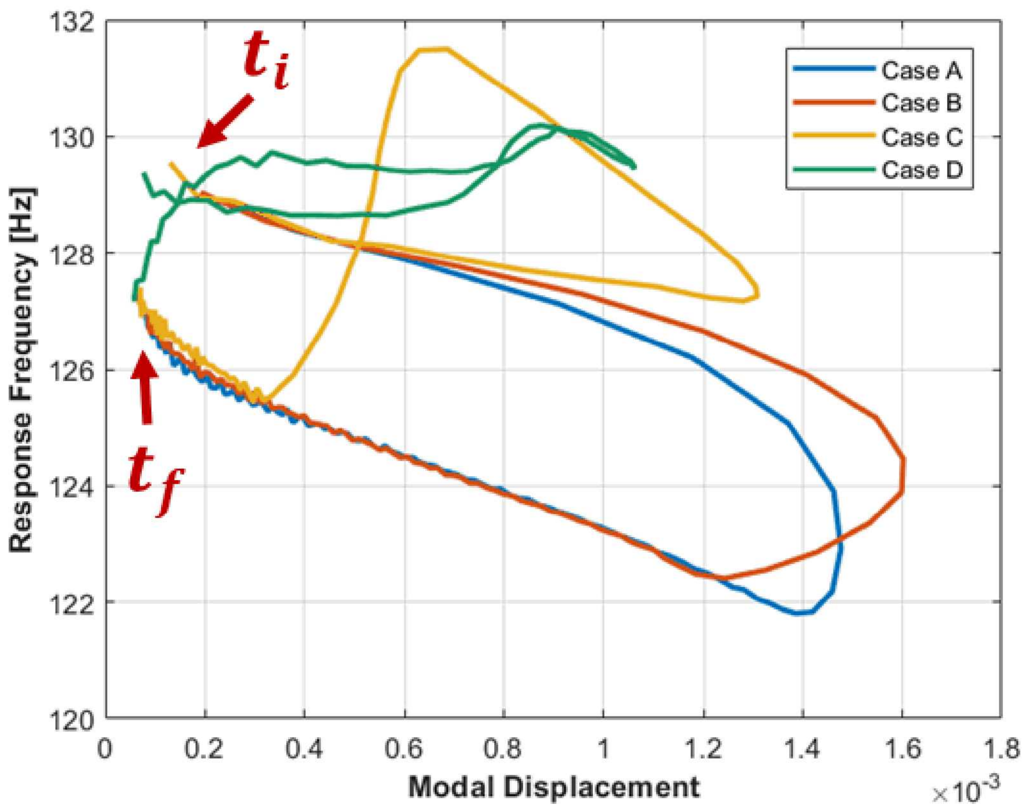
Narrow Window Width – Test Cases



Test Case	f_e [Hz]	Δf_r [Hz]	Amplifier Level
A	130	±30	70
B	130	±20	70
C	130	±10	70
D	130	±5	70

- **Case A** uses window parameters from previous studies
- **Cases B, C,** and **D** decrease the window width Δf_r
- The modal amplitude achieved by narrowing the window does not seem to increase from case to case and significantly lowers for **Case D**!
- In Cases **C** and **D** the system acts as a forced response due to the narrow window

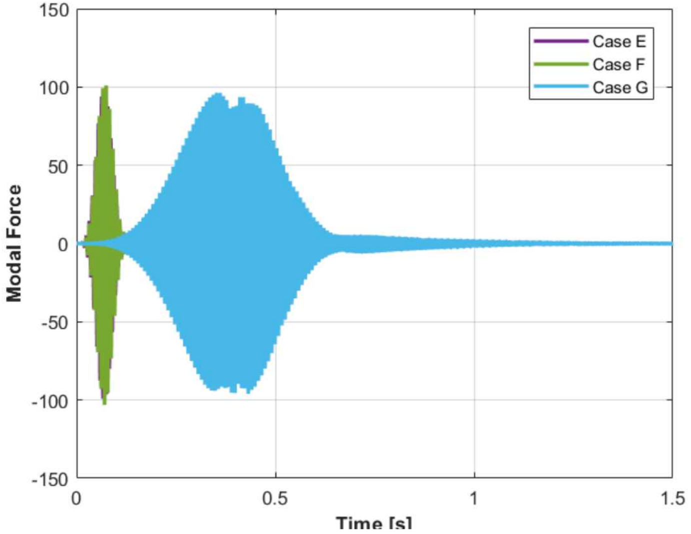
Narrow Window Width - Takeaways

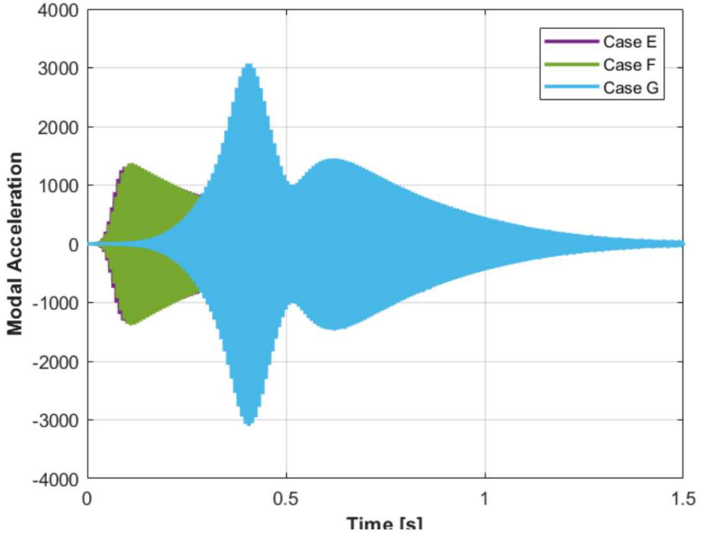


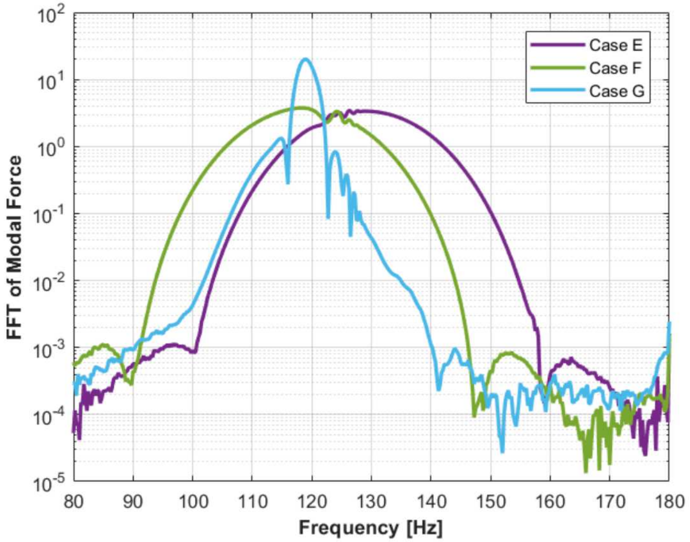
Test Case	f_e [Hz]	Δf_r [Hz]	Amplifier Level
A	130	±30	70
B	130	±20	70
C	130	±10	70
D	130	±5	70

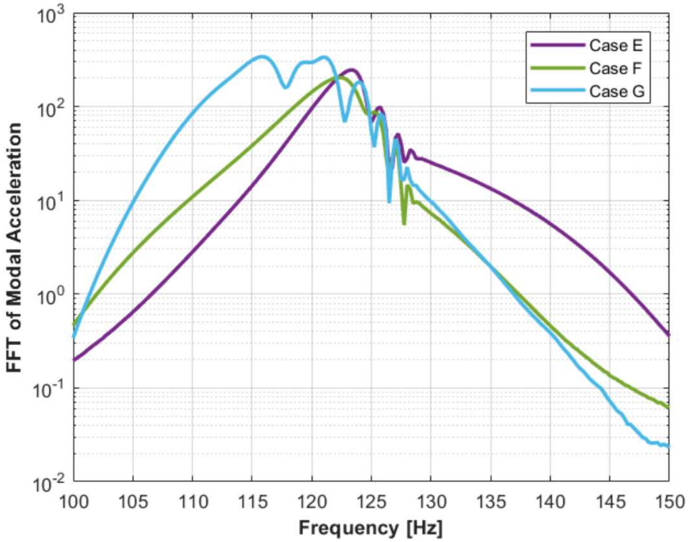
- The linear natural frequency for this mode is at 130 Hz but the energy levels input to the system force the frequency of the system much lower (122 Hz)
- Having too narrow of a window may cause the test to completely miss resonance once the system gets going!
- Shifting the center frequency may solve this issue….

Adjusting Center Frequency – Test Cases





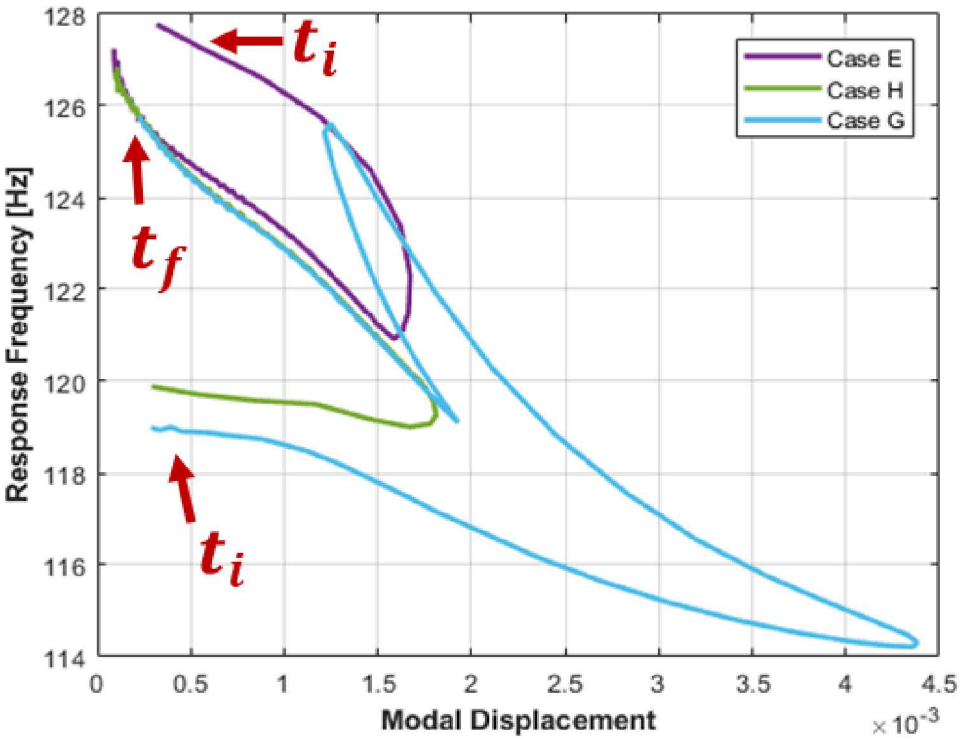




Test Case	f_e [Hz]	Δf_r [Hz]	Amplifier Level
E	**129**	**±30**	**70**
F	**119**	**±30**	**70**
G	**119**	**±5**	**70**

- **Case E** uses window parameters from previous studies
- **Case F** softens the center frequency f_e to 119 Hz maintaining a wide 30 Hz window width Δf_r. This generates an almost identical response to **Case E**
- **Case G** softens the center frequency, f_e, to 119 Hz and narrows the window width, Δf_r, to 5 Hz. This generates that reaches a modal amplitude more than double that of **Case E**

Narrow Window Width - Takeaways

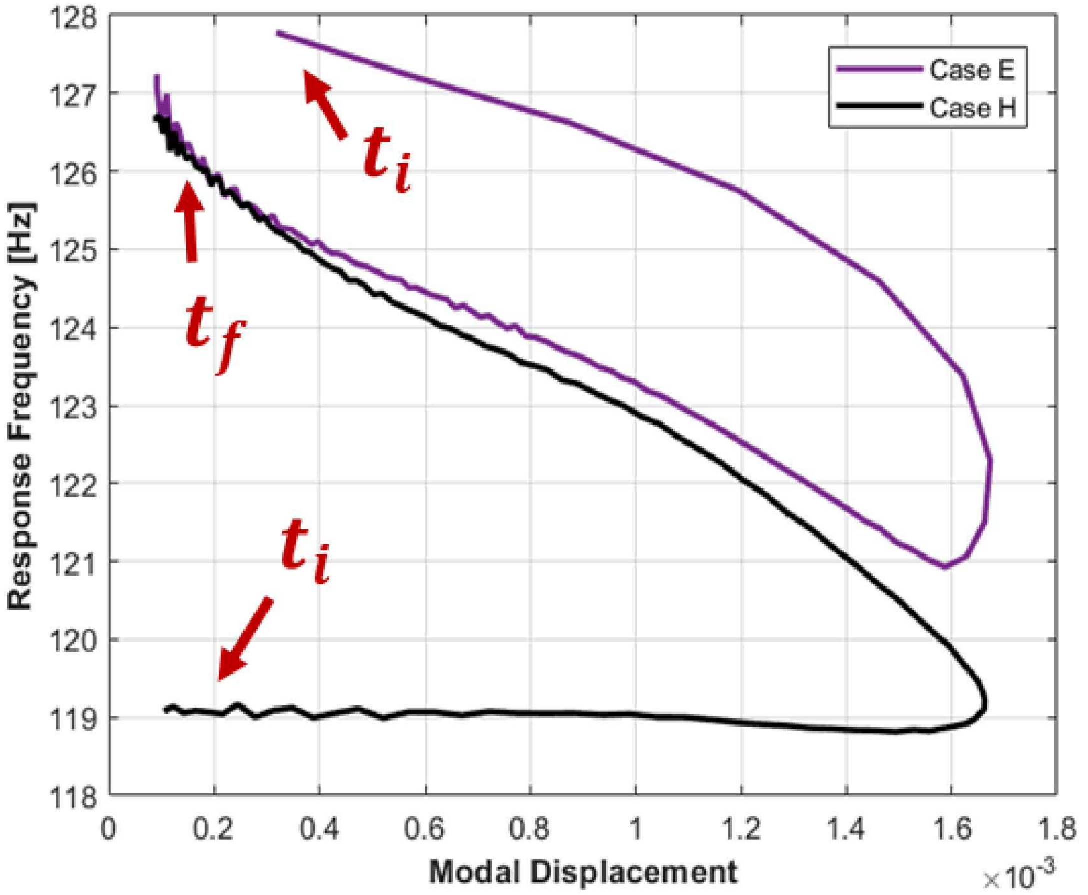


Test Case	f_e [Hz]	Δf_r [Hz]	Amplifier Level
E	129	±30	70
F	119	±30	70
G	119	±5	70

- All three cases converge to the same response frequency late in time but begin at different forcing frequencies
- The modal filtered response from **Case G** is much higher than that from **Case E**

Similar Modal Amplitudes

- A final case was tested with the shifted center frequency of 119 Hz and the narrow window width of 5 Hz and the amplifier level was lowered until similar modal amplitudes were observed to that of the standard window size



Test Case	f_e [Hz]	Δf_r [Hz]	Amplifier Level
E	129	±30	70
H	119	±5	43

- **Case H** provides forced response for much of the time history but converges to the same nonlinear response frequency
- In previous studies we have only fit models with data that is primarily free response

Impacts on Nonlinear Modal Model Fits

- These high-level excitations are ultimately used to fit a nonlinear modal model, in this study a cubic and quadradic polynomial form were implemented

$$\ddot{q} + c_0\dot{q} + k_0 q + \boxed{c_1\dot{q}|\dot{q}| + c_2\dot{q}^3 + k_1 q|q| + k_2 q^3} = \phi^T F$$

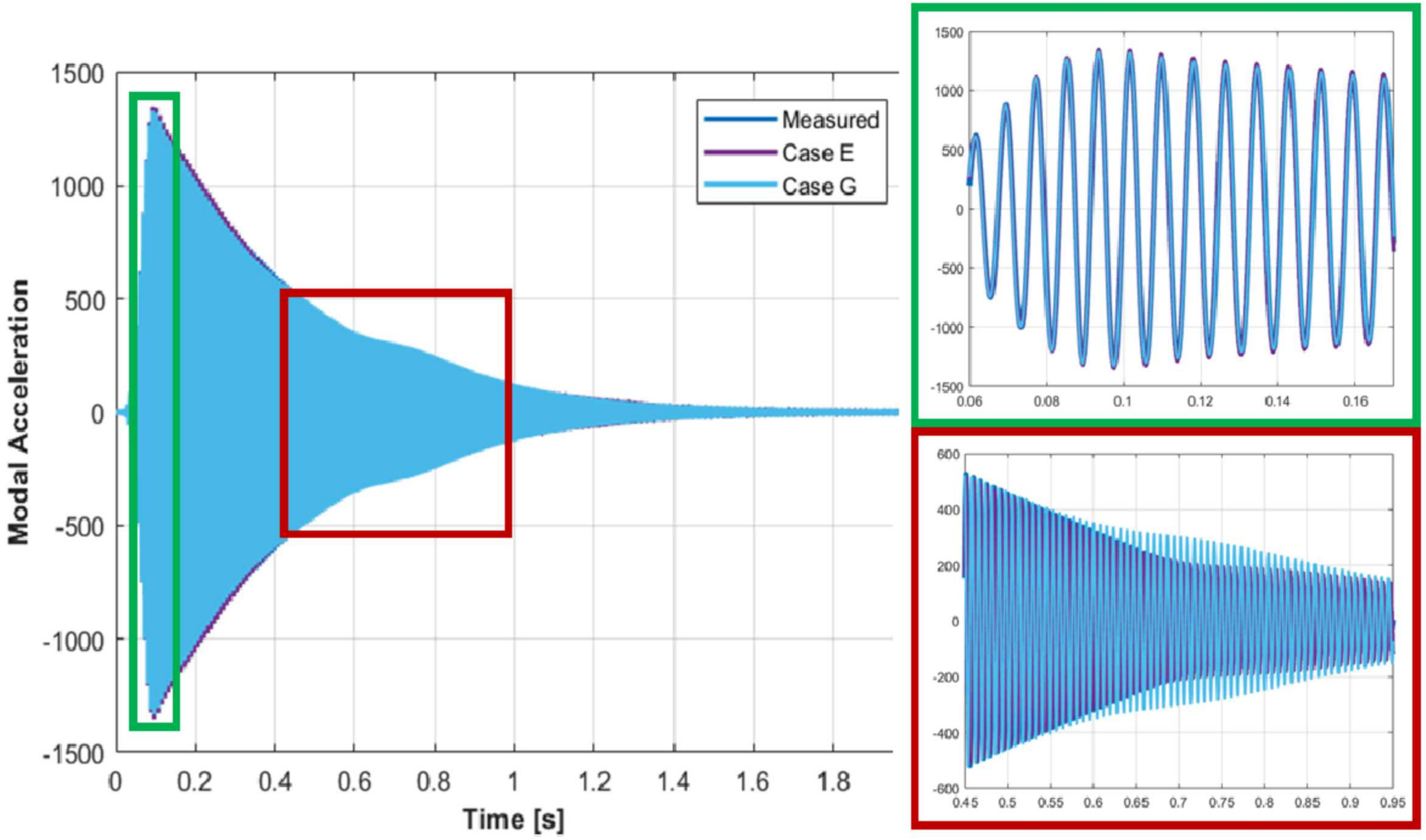
- To investigate these we will look at three cases:
  - The baseline **Case E**
  - The double modal amplitude **Case G**
  - The matched modal amplitude **Case H**

Test Case	f_e [Hz]	Δf_r [Hz]	Amplifier Level
E	**129**	**±30**	**70**
G	**119**	**±5**	**70**
H	**119**	**±5**	**43**

Test Case	Case E	Case G	$\%_{diff,EG}$	Case H	$\%_{diff,EH}$
k_1	-7.50E+07	-5.63E+07	-25%	-7.28E+07	-3%
k_2	1.54E+10	4.73E+09	-69%	1.37E+10	-11%
c_1	-1.489	-1.186	-20%	-1.403	-6%
c_2	-0.025	0.109	-536%	-0.034	36%

Impacts on Nonlinear Modal Model Simulations I of III

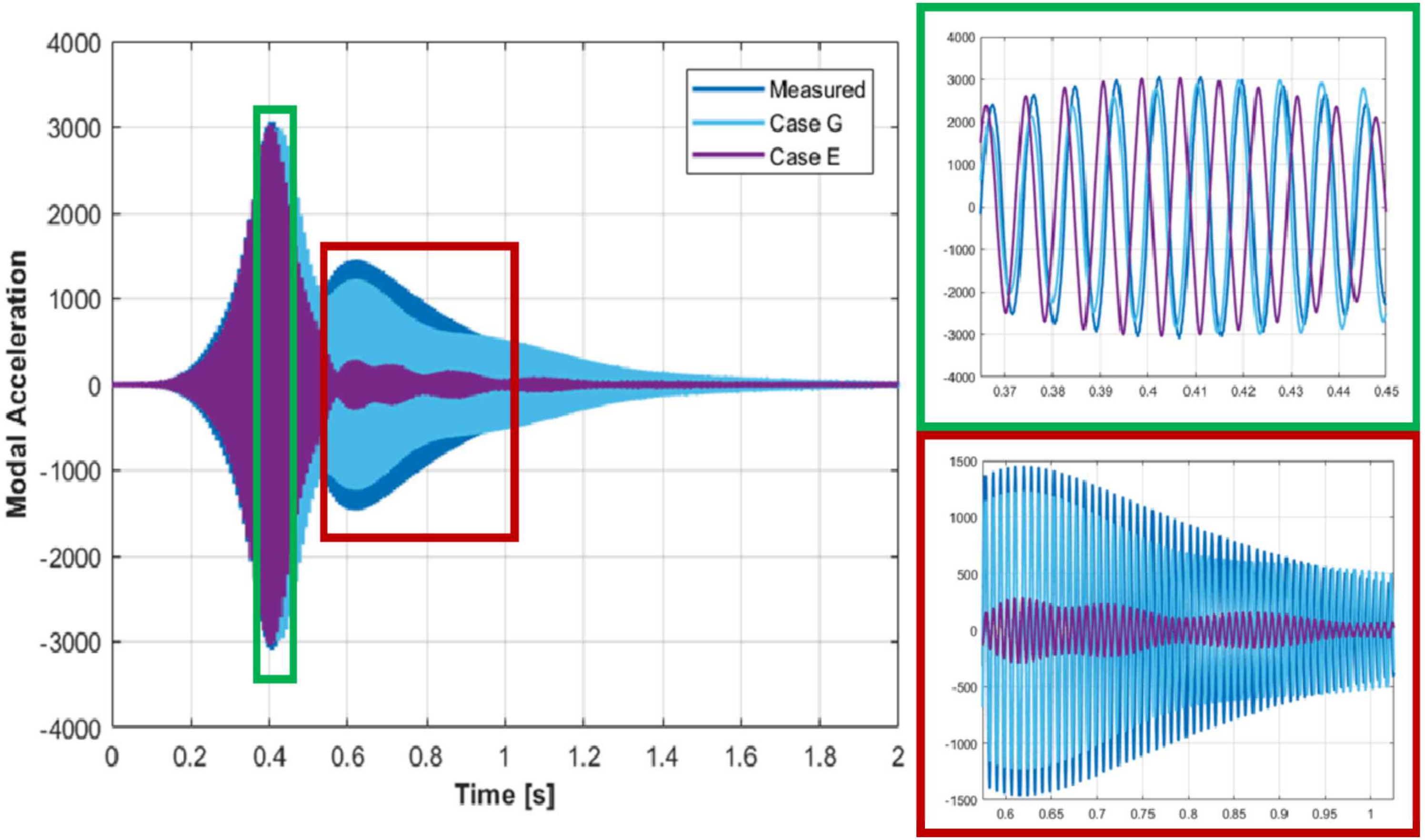
- Despite this difference the models obtained from baseline **Case E** and double modal amplitude **Case G** predict similar response to **Case E** loadings



Test Case	f_e [Hz]	Δf_r [Hz]	Amplifier Level
E	**129**	**±30**	**70**
G	**119**	**±5**	**70**
H	**119**	**±5**	**43**

Impacts on Nonlinear Modal Model Simulations II of III

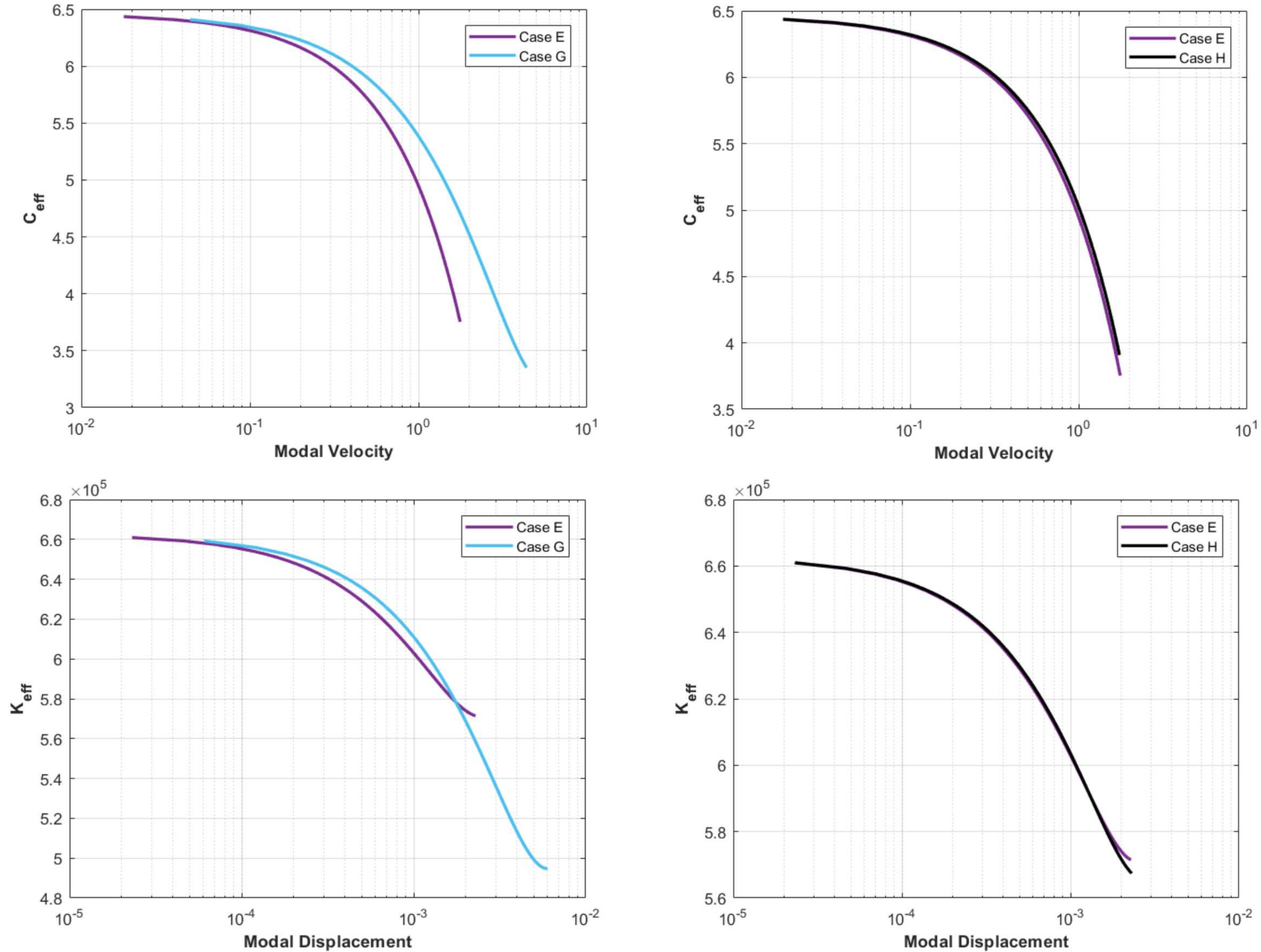
- As expected extrapolating with **Case E**'s model and the **Case G** forcing leads to erroneous results
- **Case G**'s model predicts more error than expected based on previous experience leaving this type of testing ripe for future research



Test Case	f_e [Hz]	Δf_r [Hz]	Amplifier Level
E	**129**	**±30**	**70**
G	**119**	**±5**	**70**
H	**119**	**±5**	**43**

Impacts on Nonlinear Modal Model Simulations I of III

- Plotting the effective stiffness and damping for these cases highlights the difference between the **Case G** model and similarities of the **Case H** model to the baseline



Test Case	f_e [Hz]	Δf_r [Hz]	Amplifier Level
E	**129**	**±30**	**70**
G	**119**	**±5**	**70**
H	**119**	**±5**	**43**

Remarks and Future Research

- You can effectively increase the capability of a amplifier-shaker configuration by adjusting the window and frequency parameters of a windowed sinusoidal excitation
  - Adjusting the window width alone can cause forced response at off resonance testing
  - Adjusting the center frequency puts the energy at the nonlinear shift of the mode allowing for a higher modal response once the window is narrowed
- With very little optimization the modal amplitude was doubled, more optimization could lead to even greater increases in achievable modal amplitude
- The models obtained at higher amplitude and within forced response could still simulate a baseline forcing case
- More work should be done to determine suitable model forms that can match structural response in free and forced response