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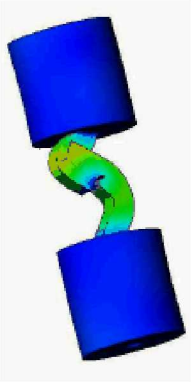

# Influences of Modal Coupling on Nonlinear Modal Models

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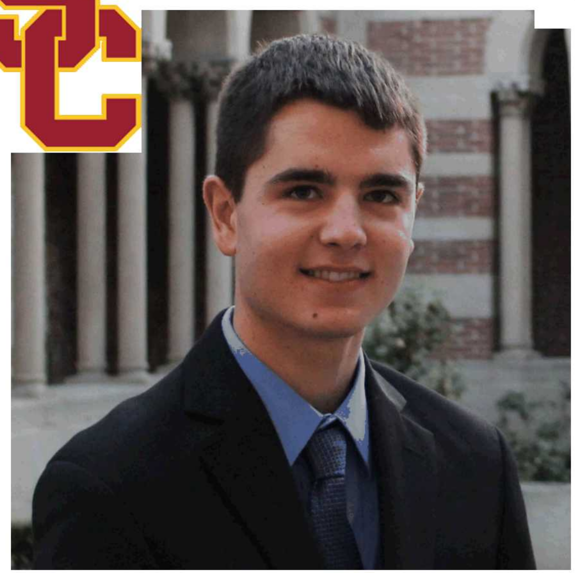
# Research Team

**Aabhas Singh**

University of
Wisconsin – Madison

**Ben Moldenhauer**

University of
Wisconsin - Madison



**Phil Thoenen**

University of Southern California

# Mentor Team

**Matt Allen**

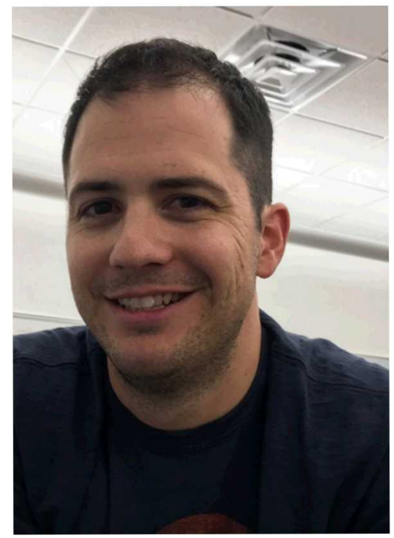
University of Wisconsin – Madison

**Dan Roettgen**

Sandia National Laboratories

**Rob Kuether**

Sandia National Laboratories

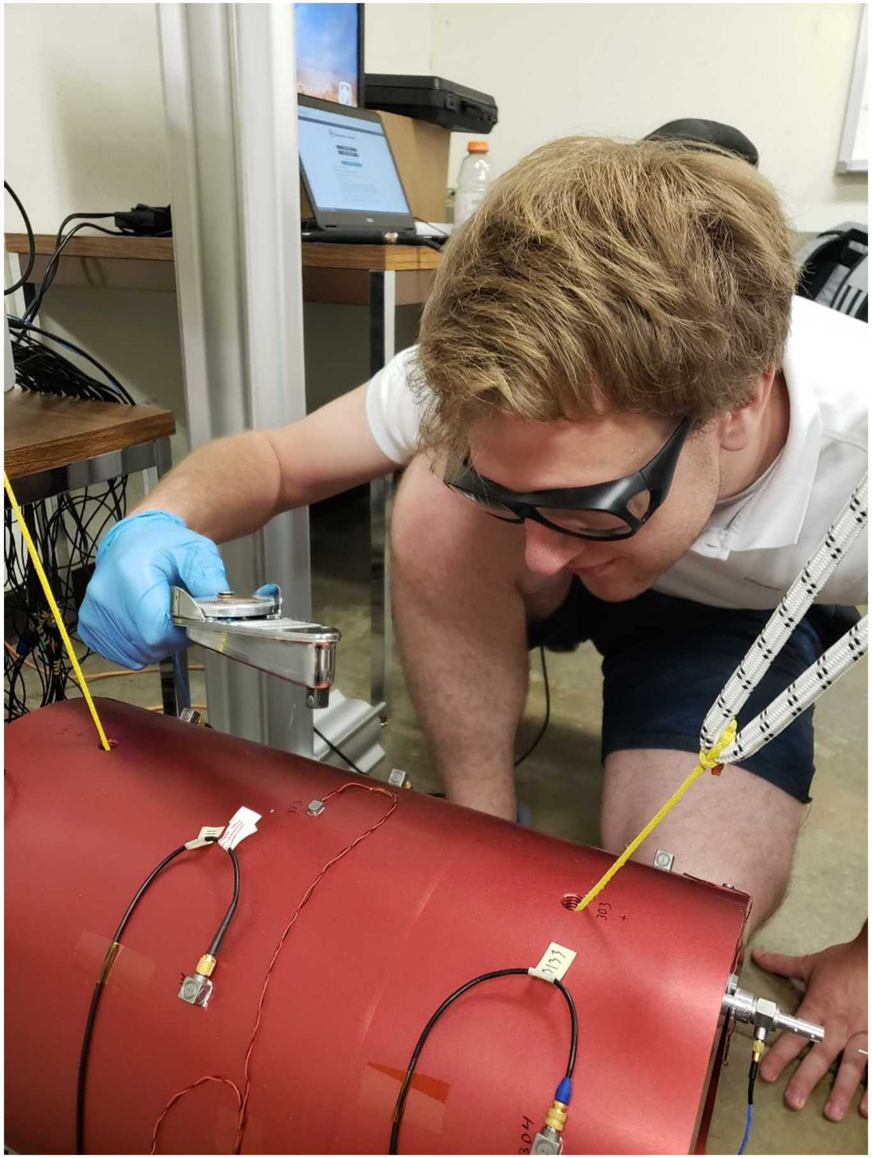


**Ben Pacini**

Sandia National Laboratories

# Agenda

1. Motivation
2. Test Structure
3. Experimental Process
   a. Linear Testing
   b. Nonlinear Testing
4. Nonlinear Coupling Data
5. Closing Remarks



# Linear vs. Nonlinear Systems

- Linear analysis assumes:
  - Small deformations
  - Response is amplitude independent
  - Modal responses can be superimposed
  - $\ddot{q}_r + 2\zeta_r\omega_r\dot{q}_r + {\omega_r}^2 q_r = \Phi^T F_{ext}$

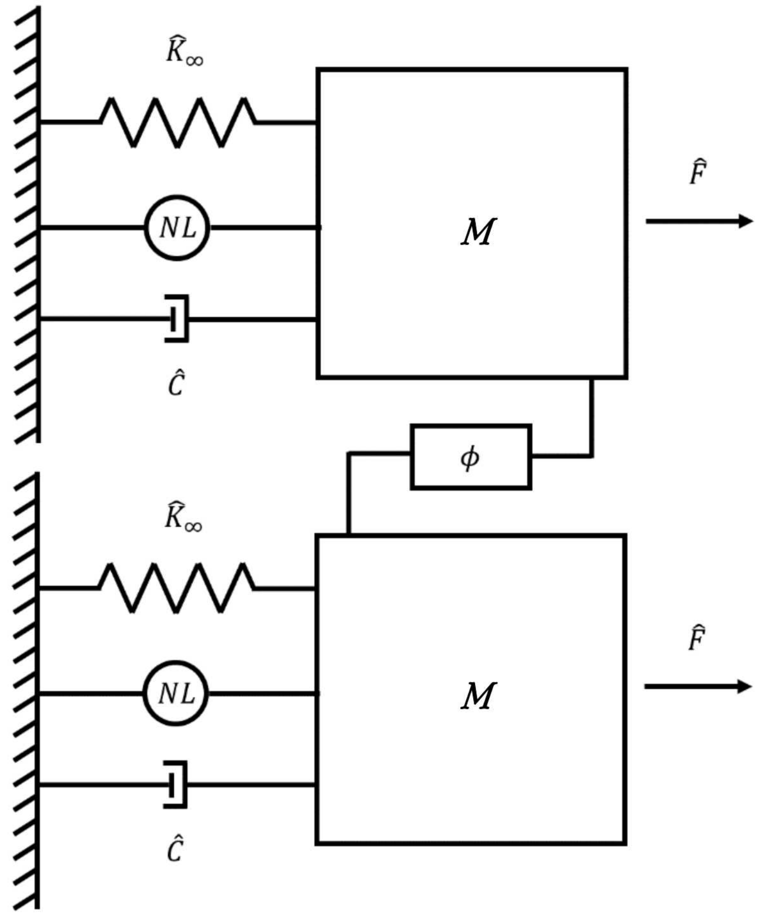
- Pseudo – Nonlinear analysis assumes:
  - Shapes of the linear modes are preserved
  - Linear modes can decouple nonlinear data
  - Coupling between modes is negligible and no energy is transferred between modes
  - $\ddot{q}_r + 2\zeta_r\omega_r\dot{q}_r + {\omega_r}^2 q_r + F_{nl}(q_r, \dot{q}_r) \ = \Phi^T F_{ext}$

What happens if there is coupling between the modes?

# Modal Coupling

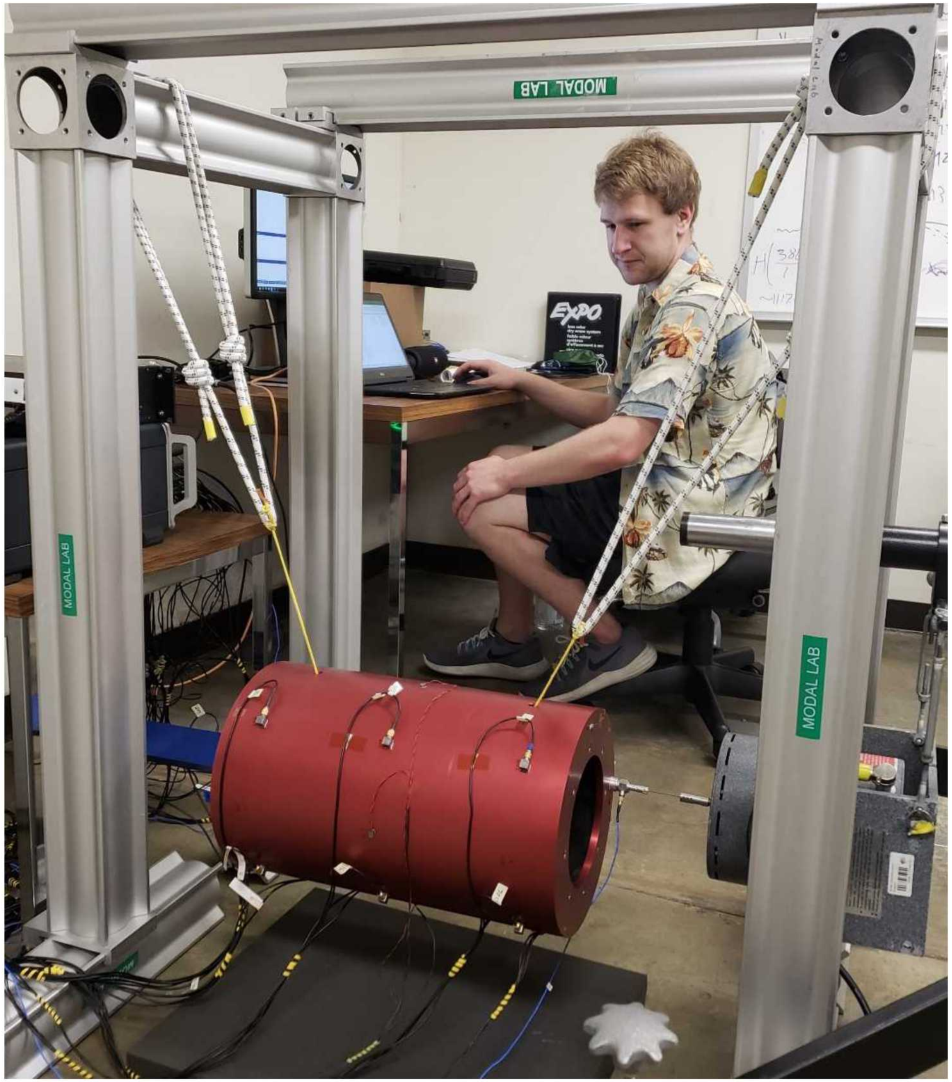
- Excitation of one mode causes a transfer of energy that perturbs another mode

- Usually occurs due to interactions at joints shared by different mode shapes



# Project Objectives

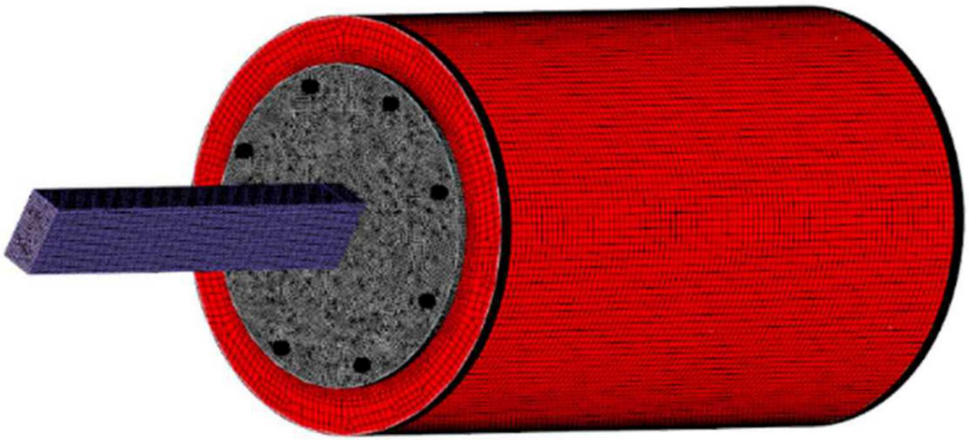
- Excite combinations of modes on a nonlinear structure
- Experimentally identify the presence modal coupling
- Collect data for use in validating computer models of coupling



# Test Structure

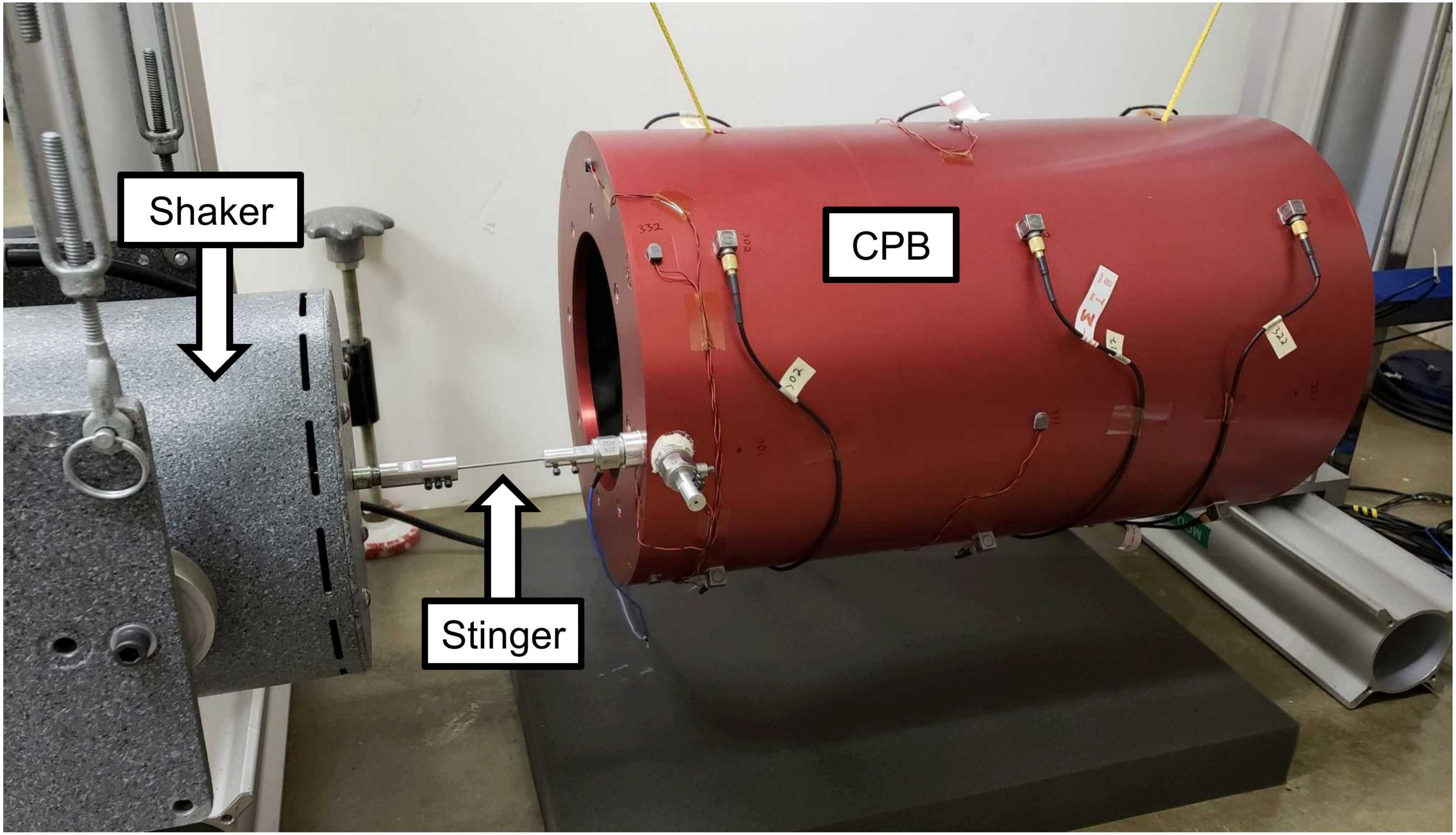
Cylinder – Plate – Beam (CPB)

- Plate bolted to cylinder
- Beam bolted and glued to plate

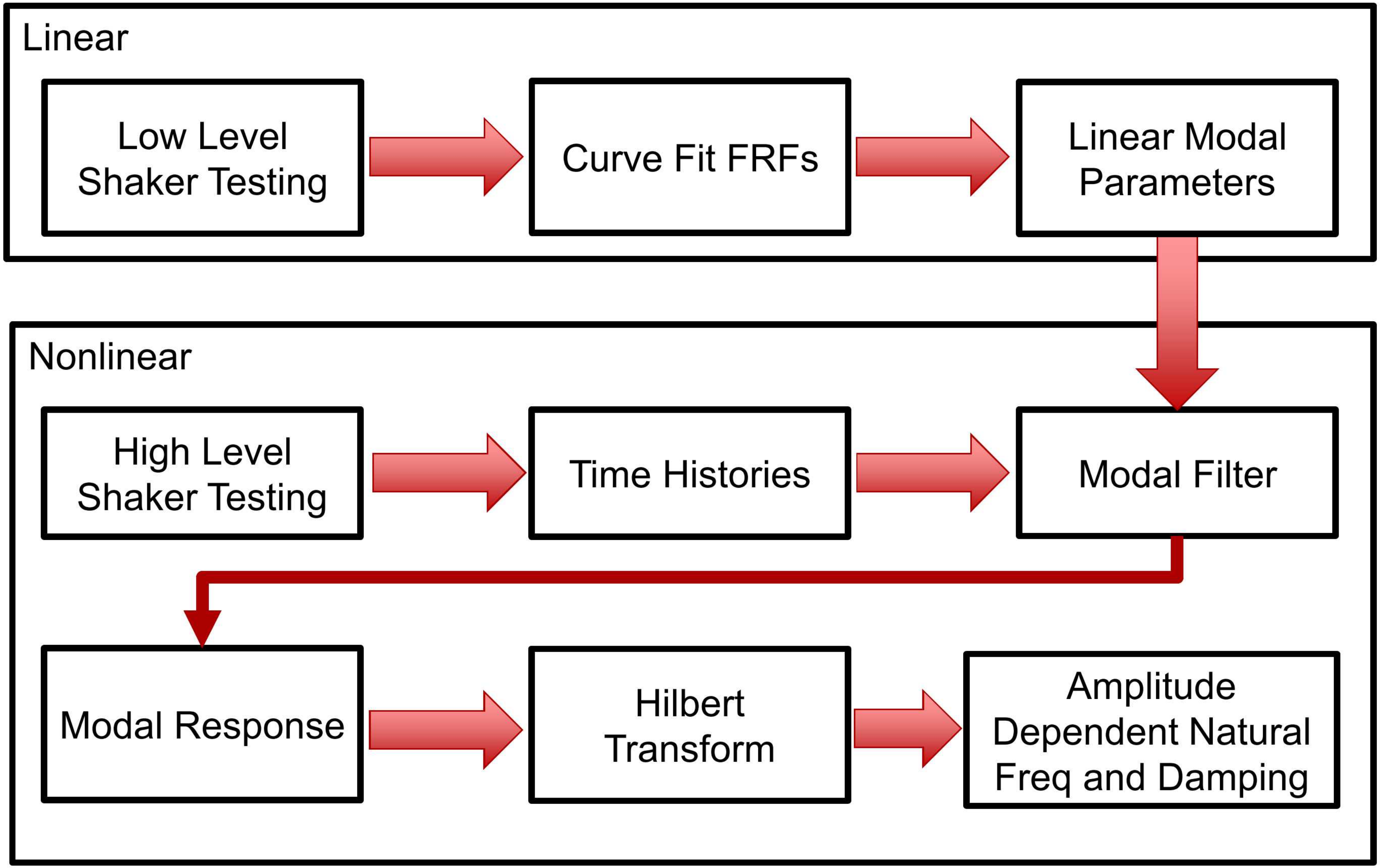


18 triaxial + 8 uniaxial accelerometers

# Experimental Setup

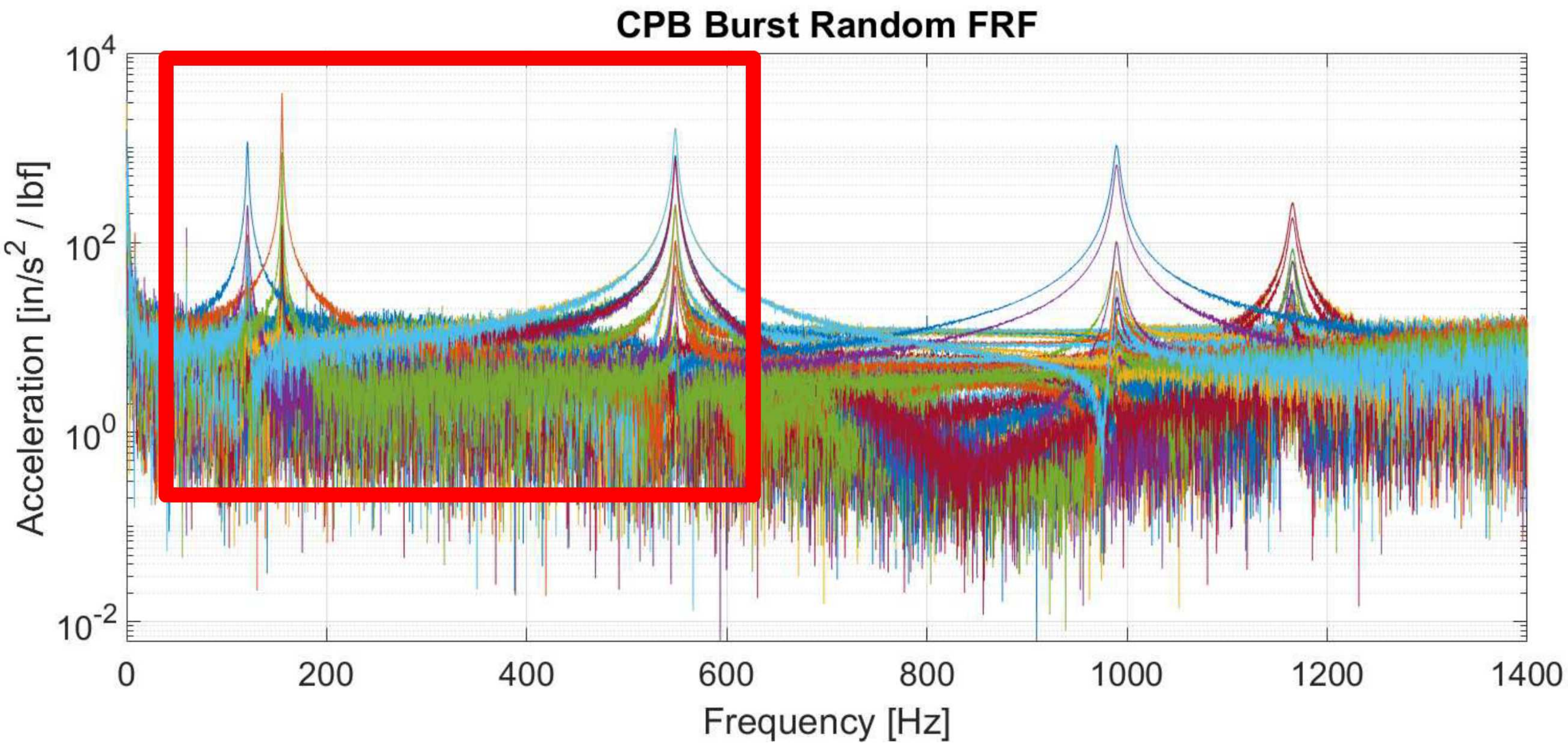


# Experimental Process



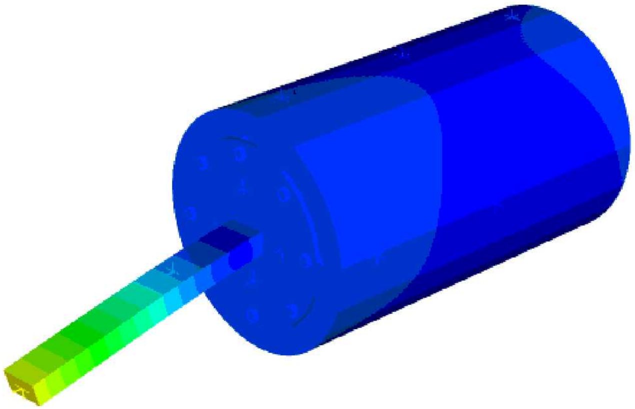
# Linear Experimental Data

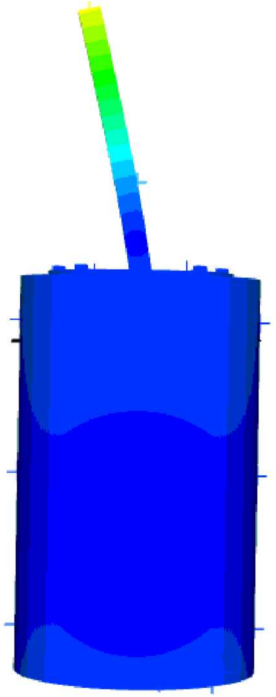
- Employed low level burst random excitation from the shaker
  - Curve fit linear FRFs for modeshapes to use as a modal filter



# Mode 1

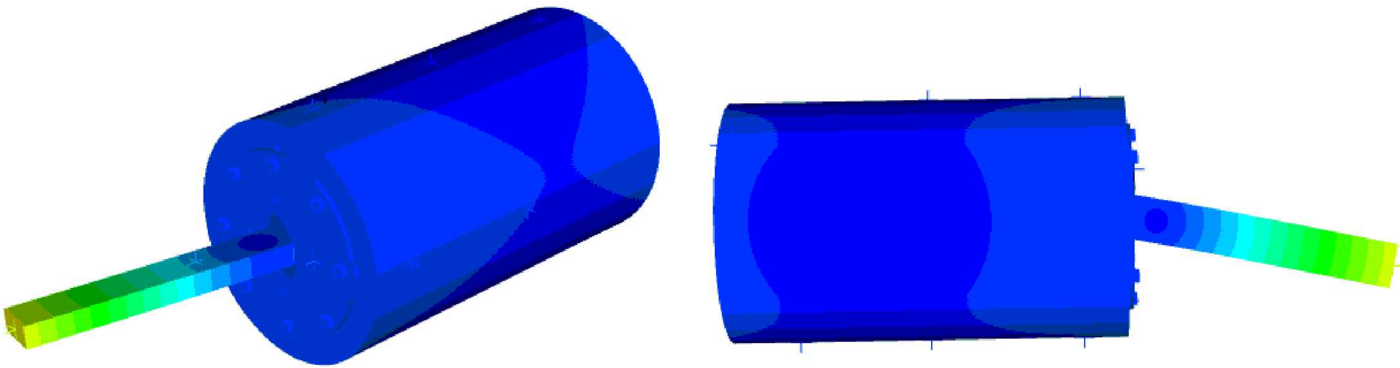
| Mode Description | Experimental $\omega_n$ (Hz) |
|---|---|
| 1st Beam Bending X | 120.8 |





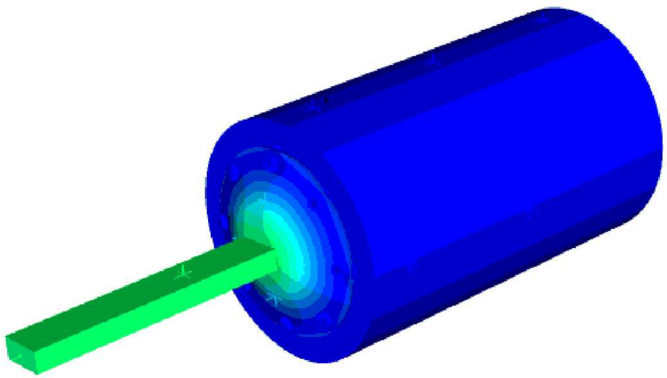
# Mode 2

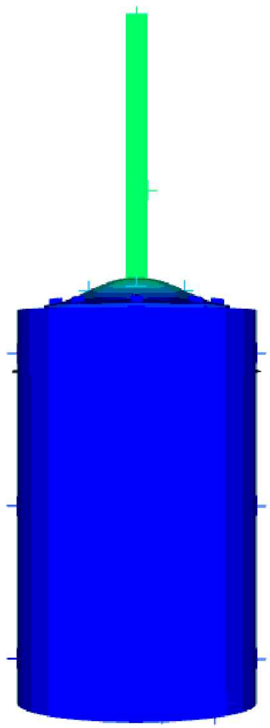
| Mode Description | Experimental $\omega_n$ (Hz) |
|---|---|
| 1st Beam Bending Y | 155.3 |



# Mode 3

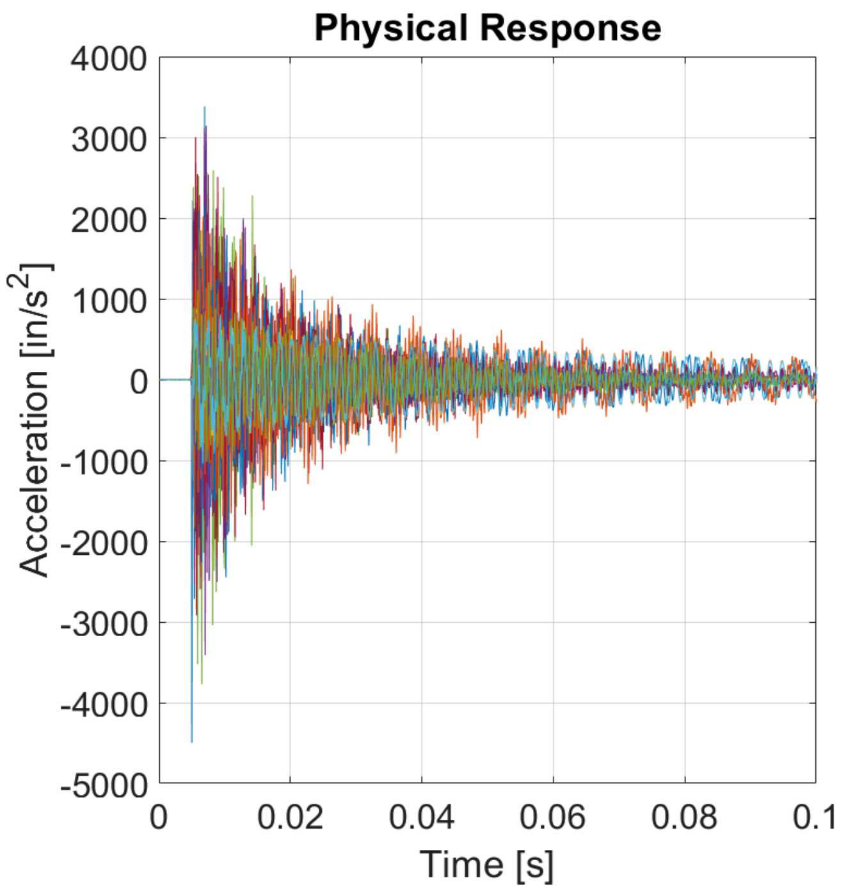
| Mode Description | Experimental $\omega_n$ (Hz) |
|---|---|
| Plate Drum | 548.43 |

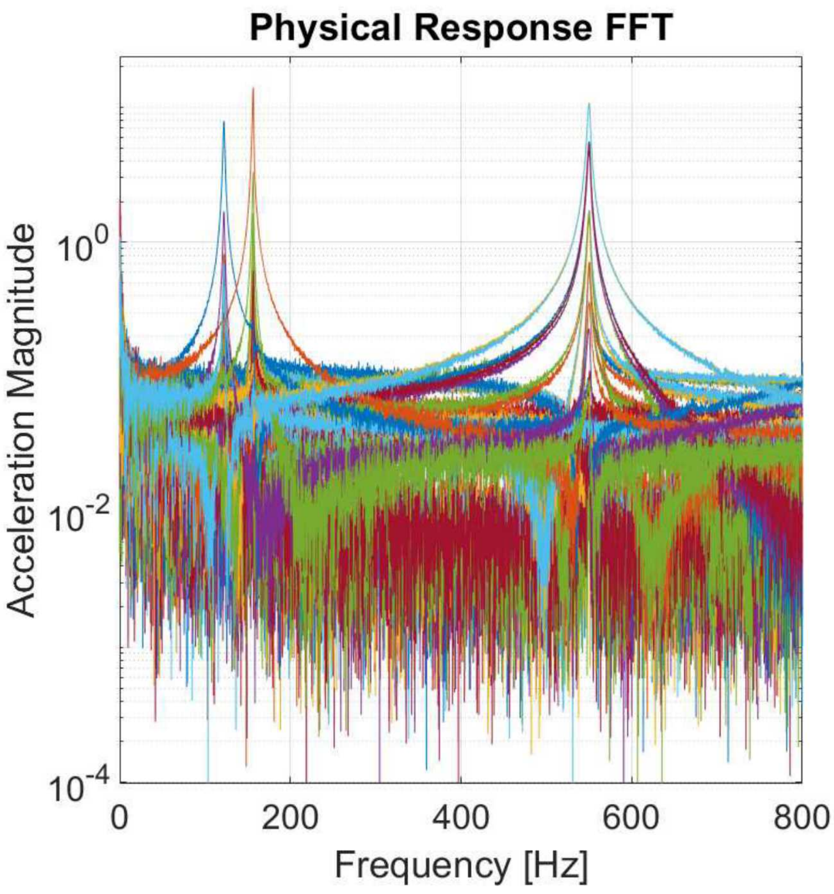




# Modal Filtering

- Linear mode shapes allow for filtering of physical response into modal coordinates



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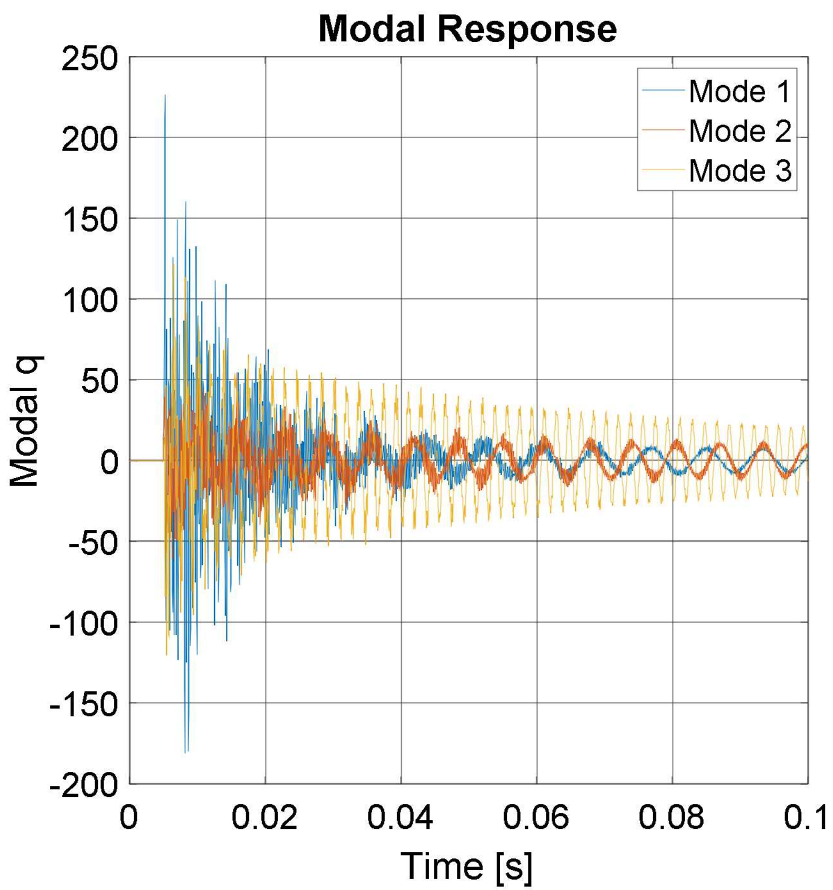
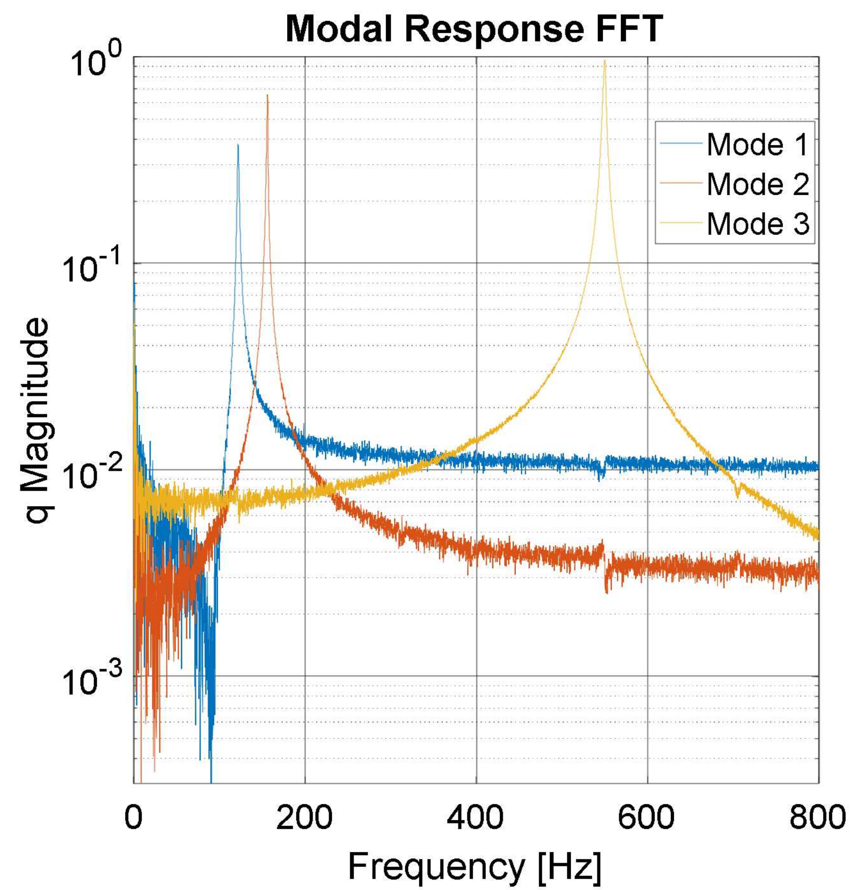
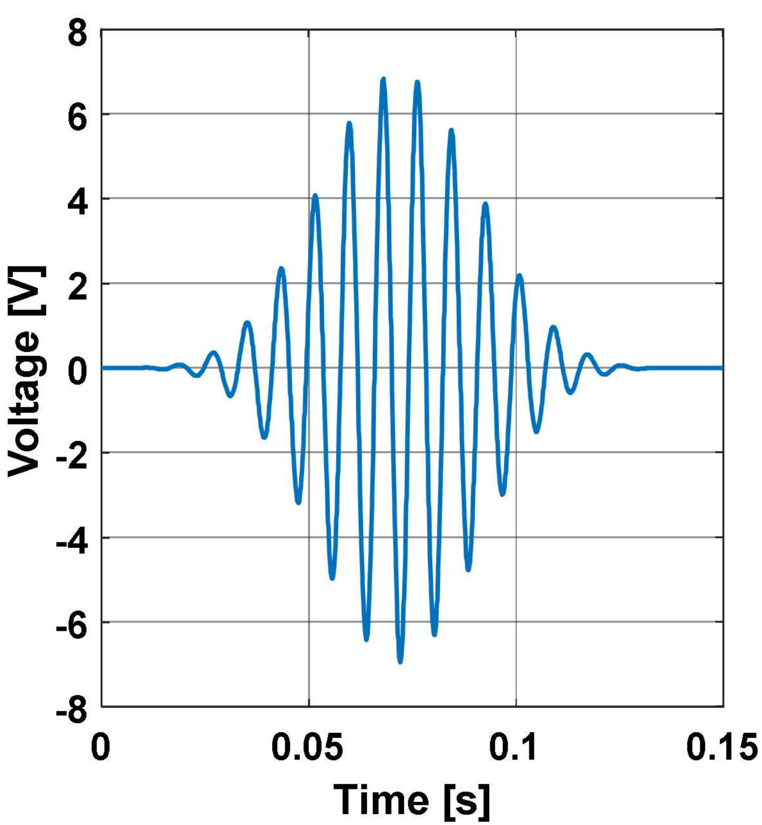
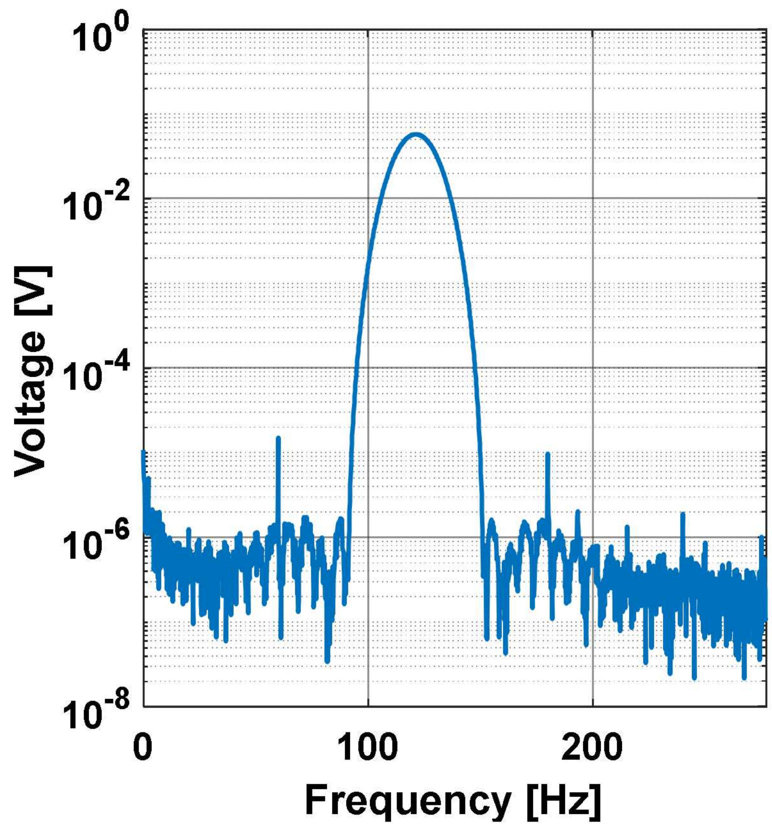


# Nonlinear Data

- Shaker delivers definable force input – able to create a voltage signal with specific frequency content



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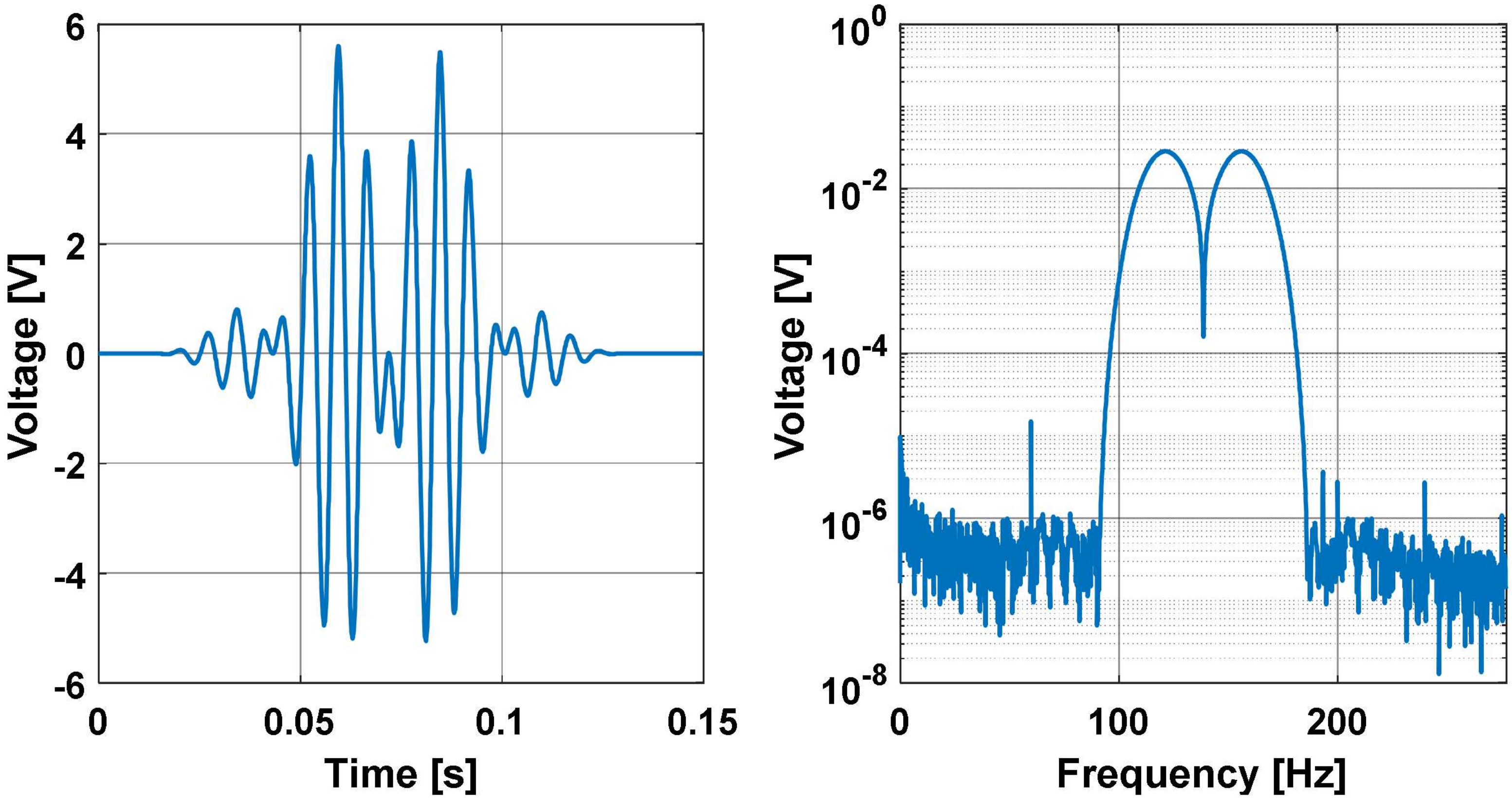
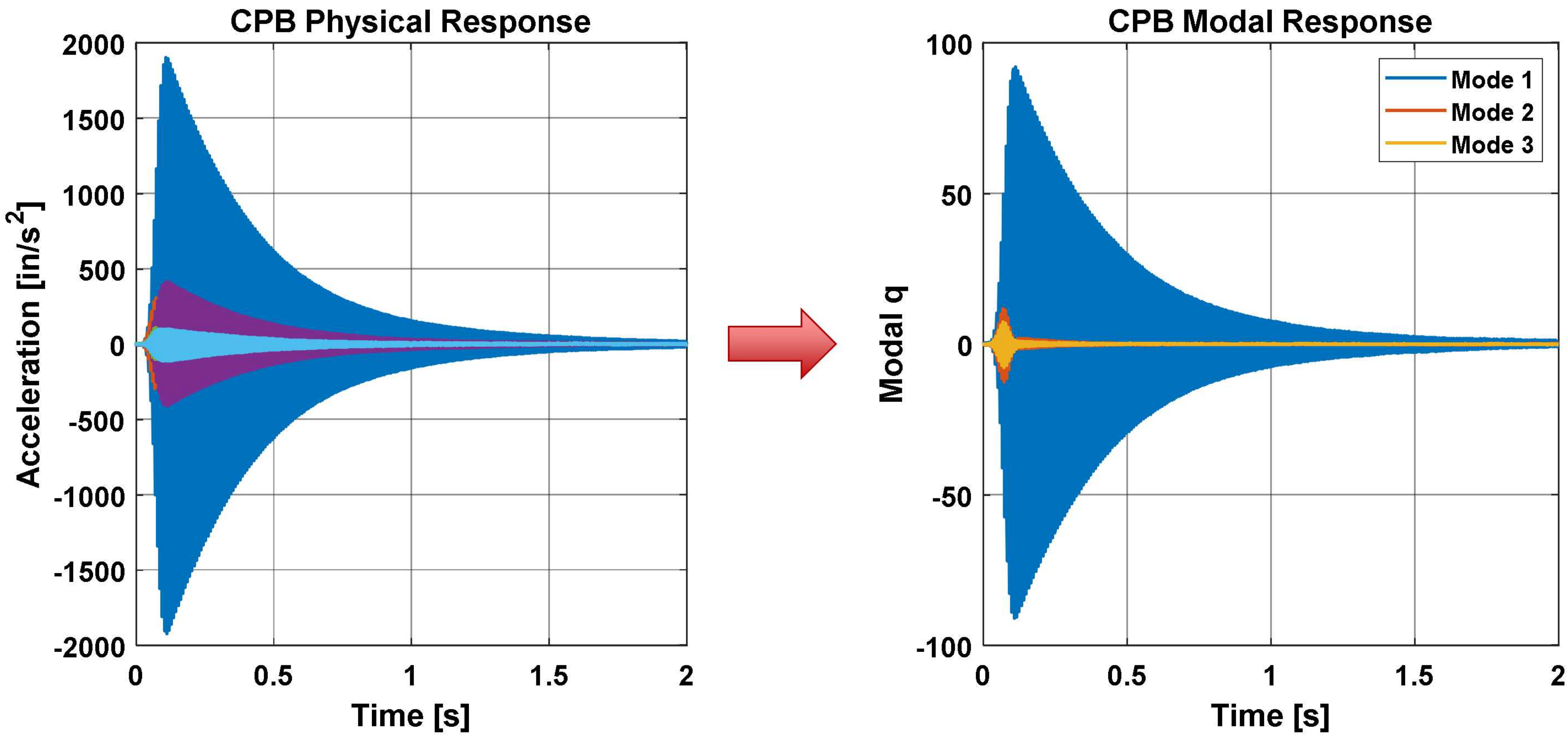


# Nonlinear Data

- Use shaker to excite specific modes



# Nonlinear Data

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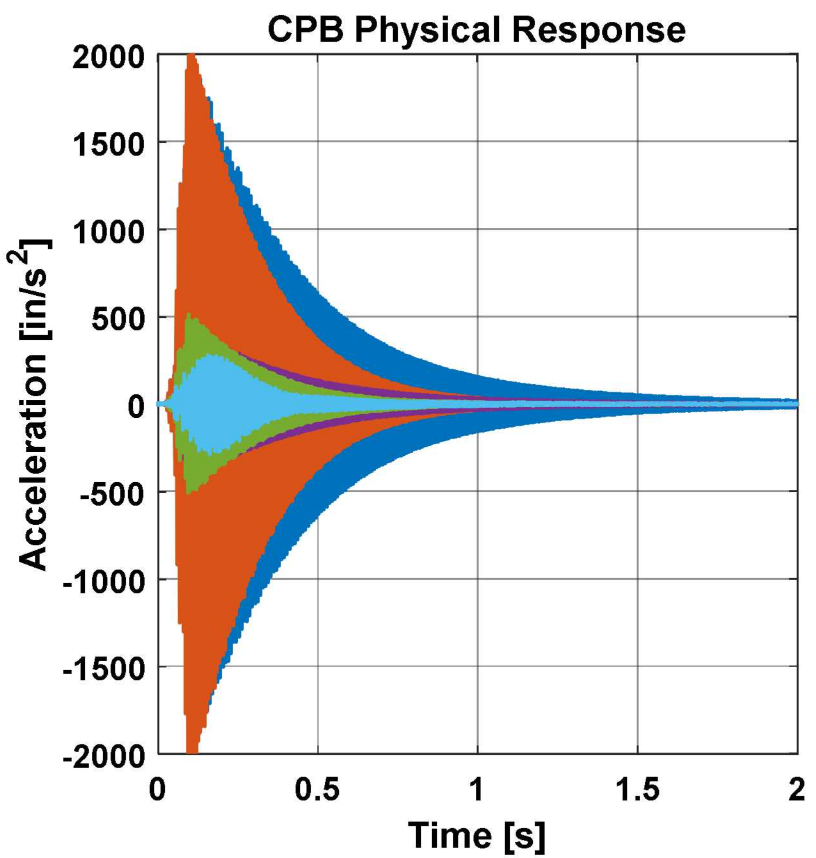
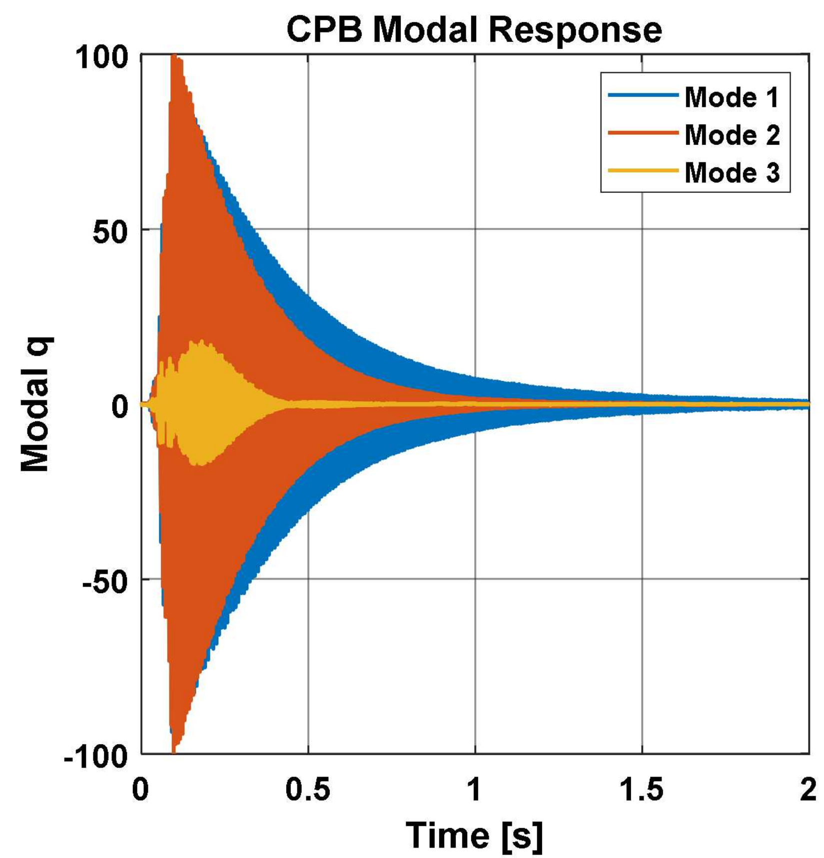
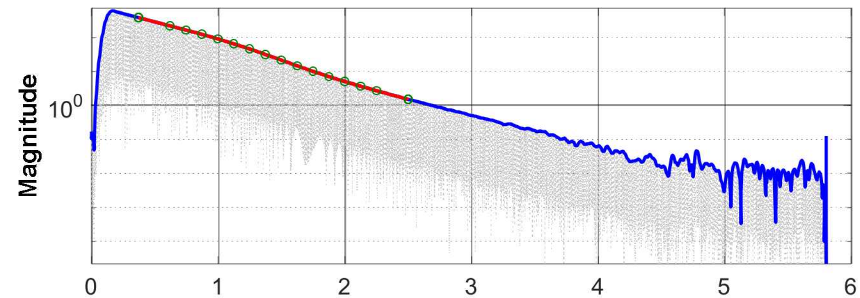


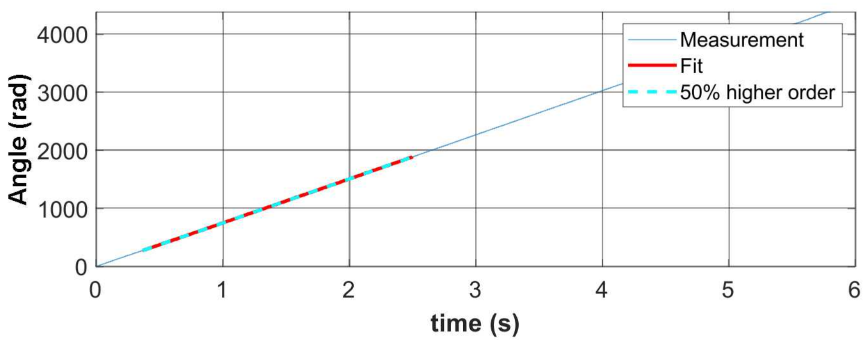



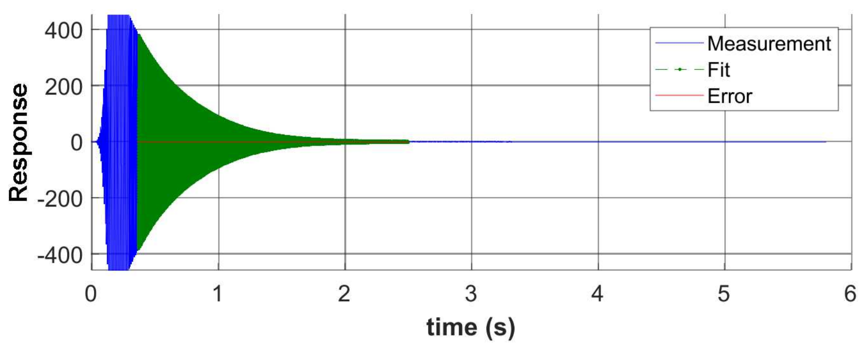
# Hilbert Analysis

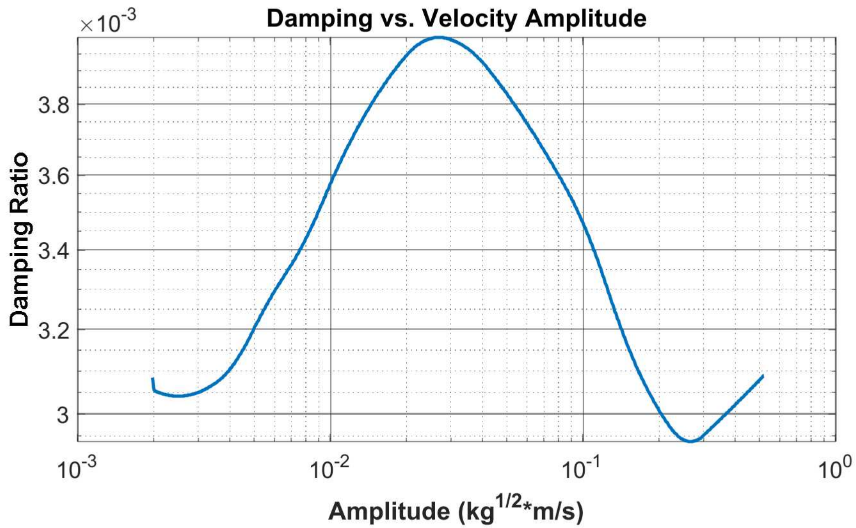
- Requires that each response be uncoupled such that it can be represented by a SDOF system
  - Signal can be represented by a decaying harmonic
  - $\ddot{\eta} = Re\left[\exp\left(\psi_1(t) + i\,\psi_2(t)\right)\right]$
- Compute Hilbert Transformation ($\mathcal{H}(t)$) for an amplitude dependent representation of damping and frequency
- $\omega_{d,r} = \frac{d\psi_2}{dt}$
- $\zeta_r \triangleq \frac{d\psi_1}{dt} / \omega_r$

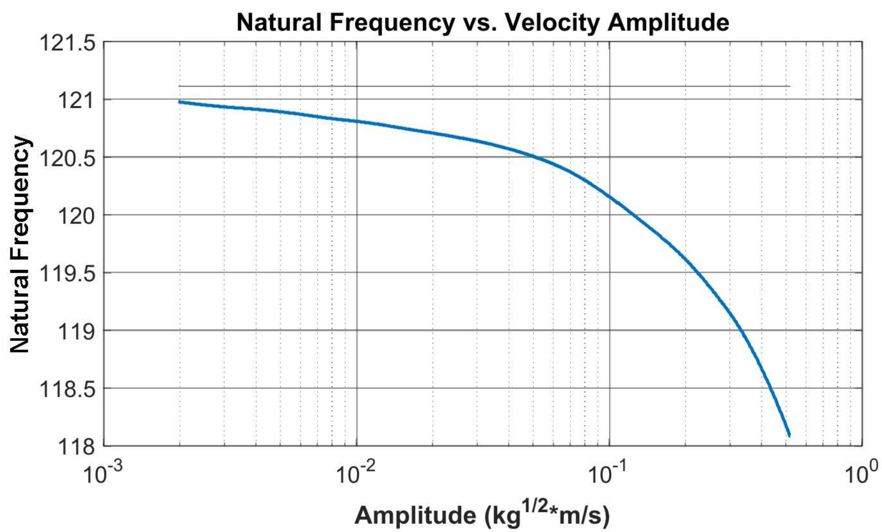
# Hilbert Analysis

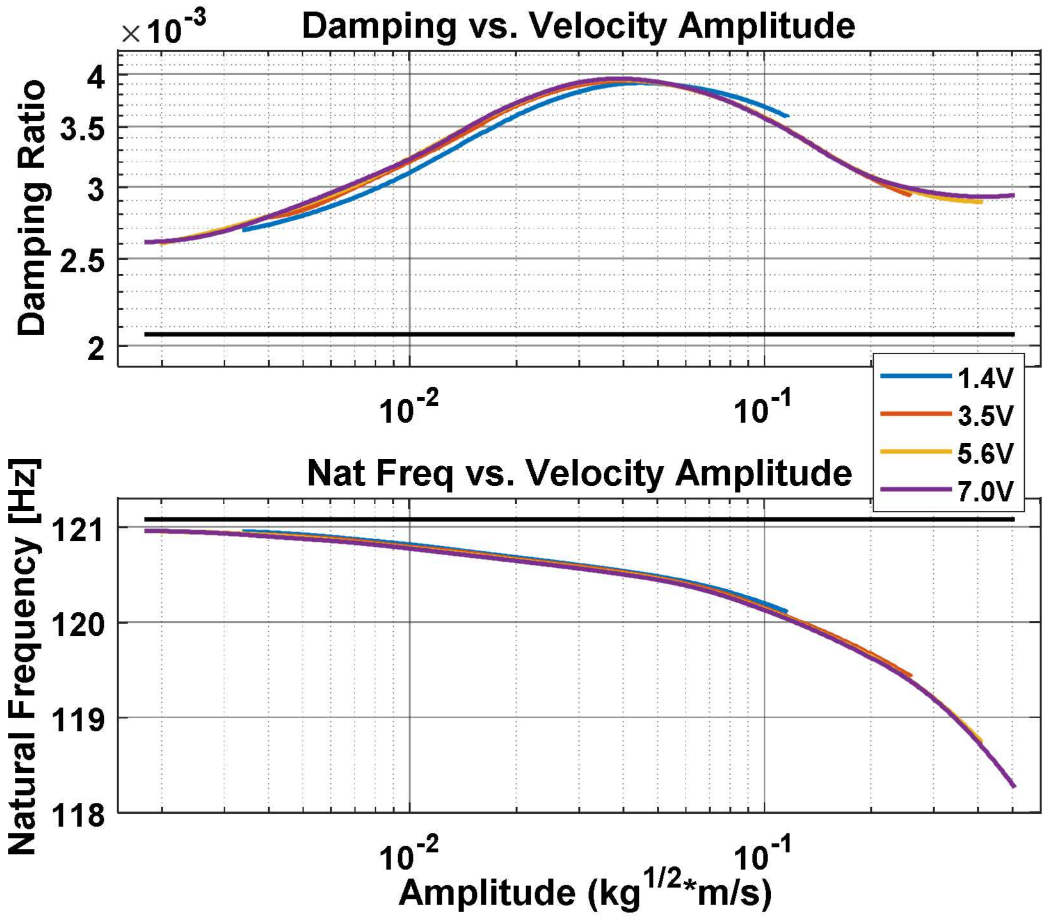


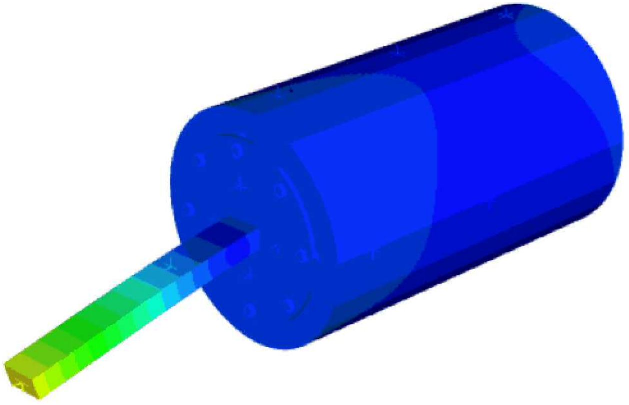



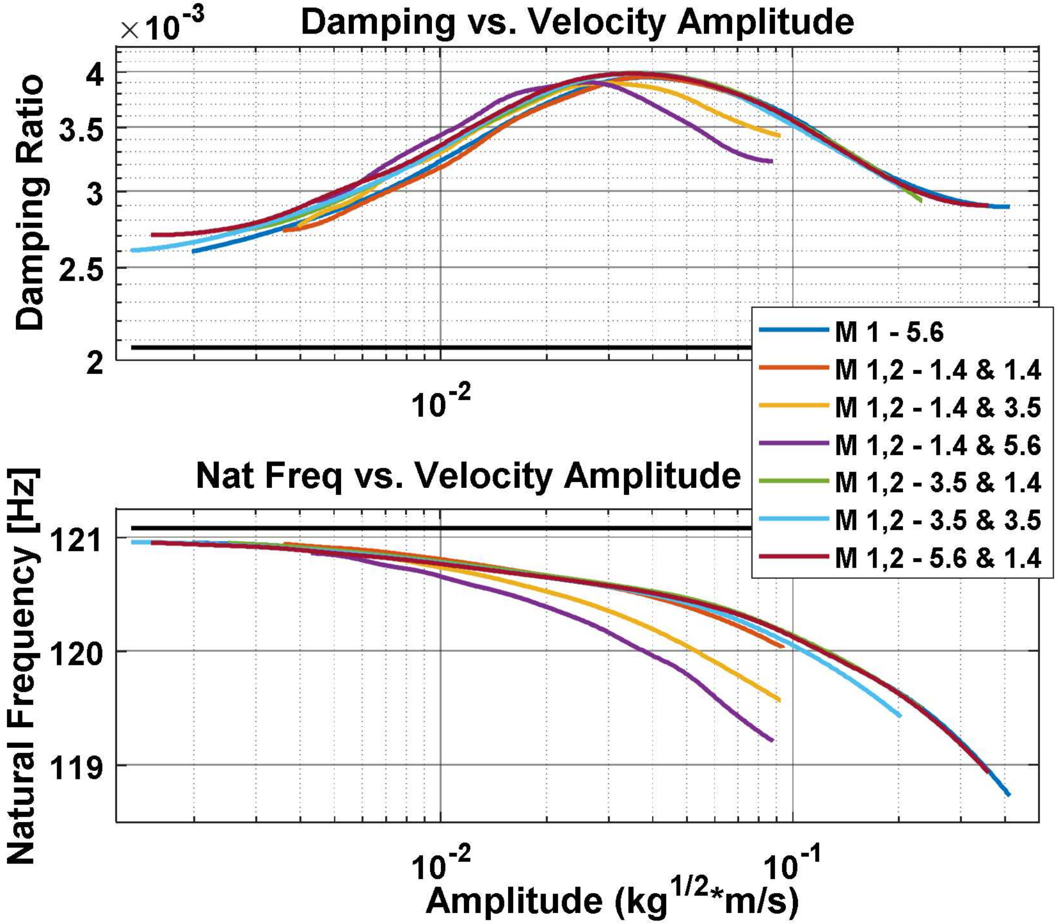





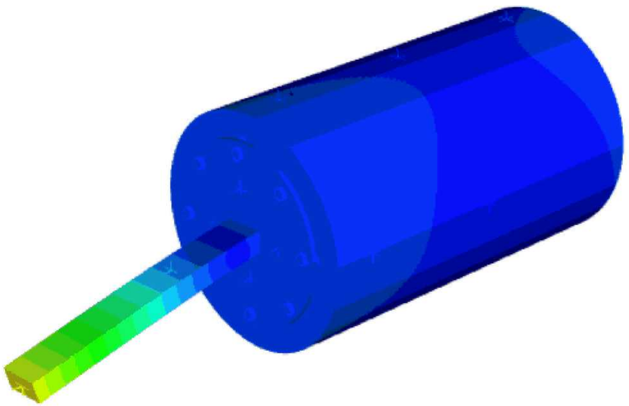

# Mode 1



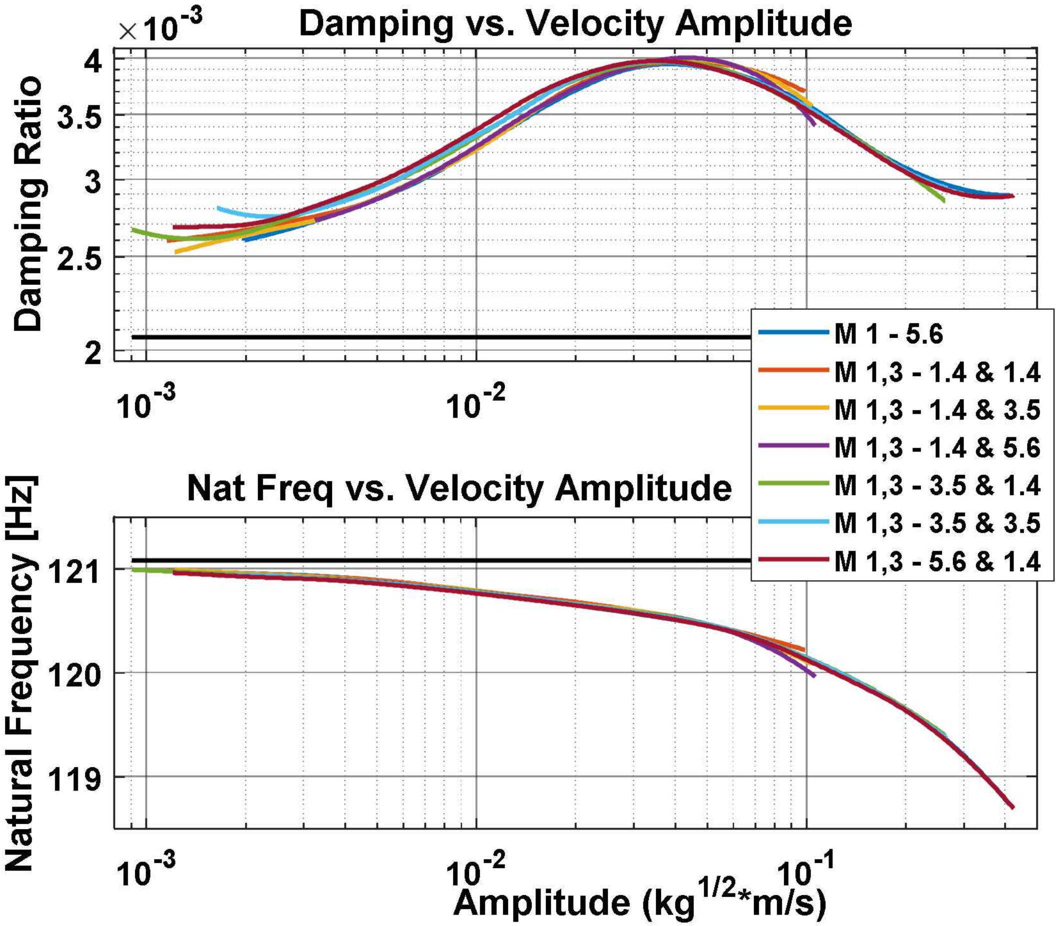
- When excited alone at various levels, frequency and damping overlay with increasing amplitude



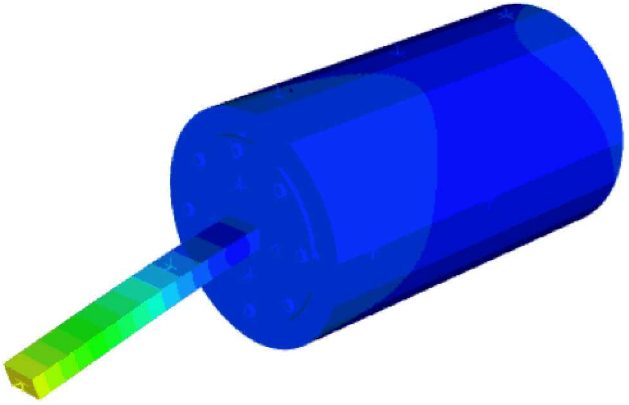

# Mode 1 & 2



- Coupling visible as a frequency and damping shift when mode 2 is excited to a higher level than mode 1

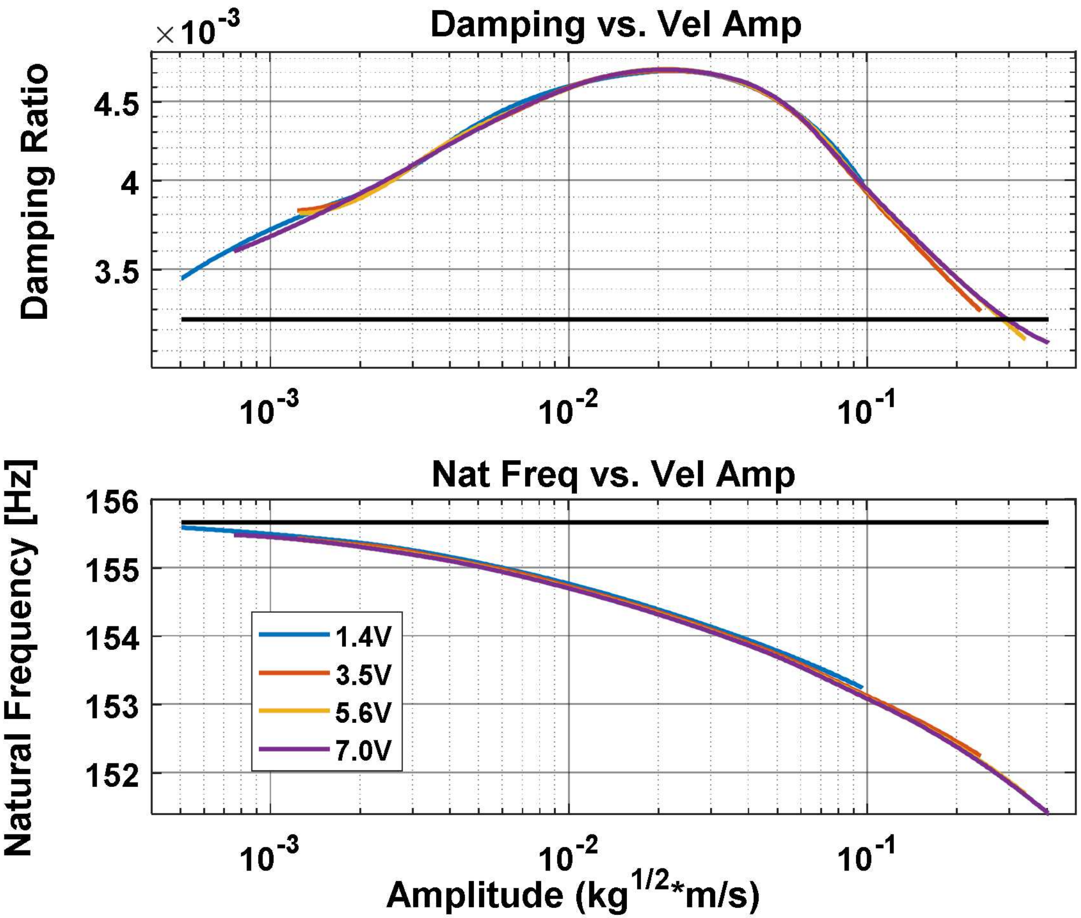


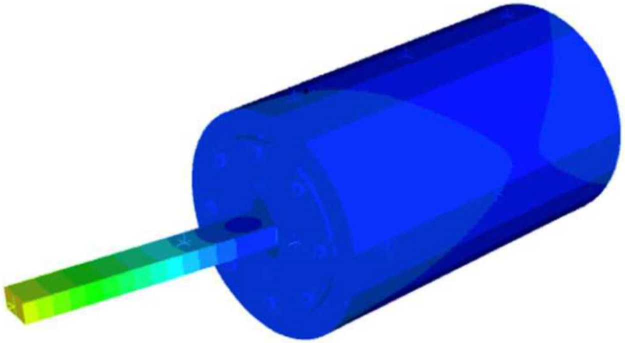

# Mode 1 & 3



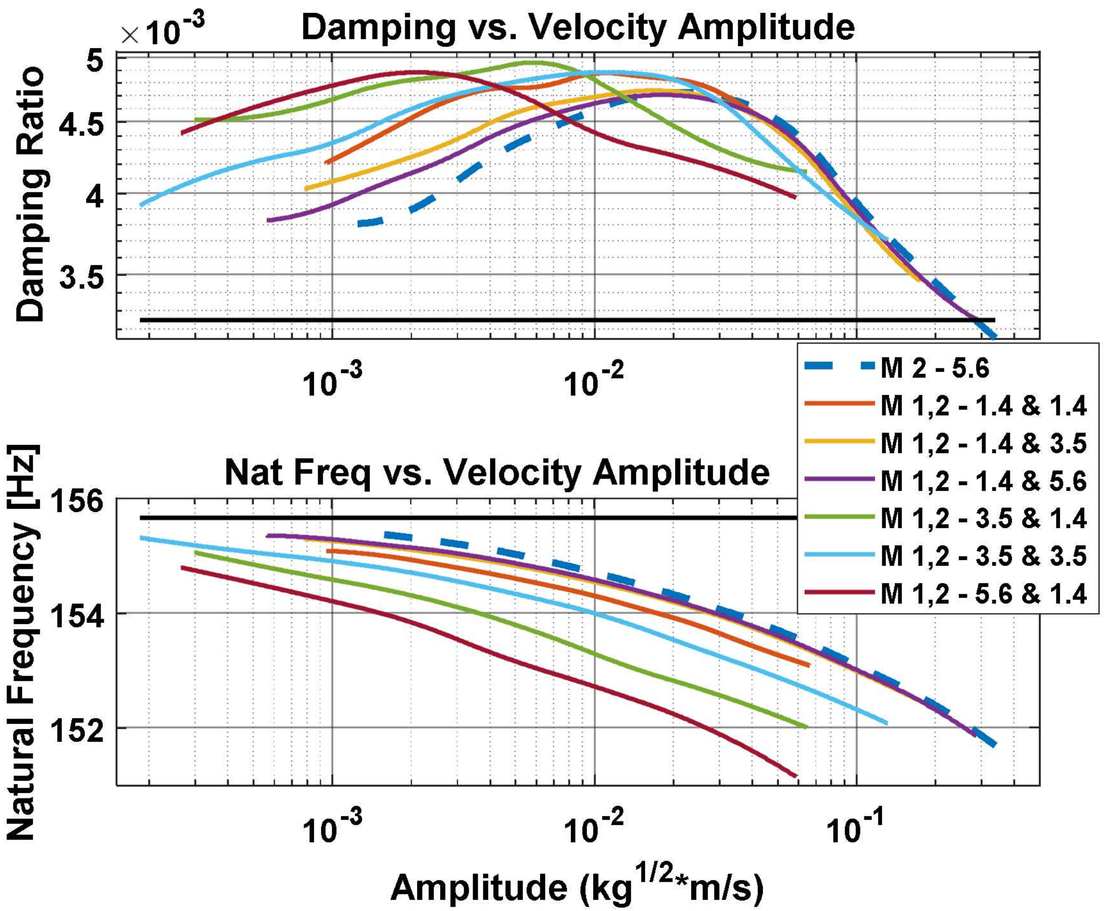
- Mode 1 appears to be completely unaffected by mode 3.

# Mode 2

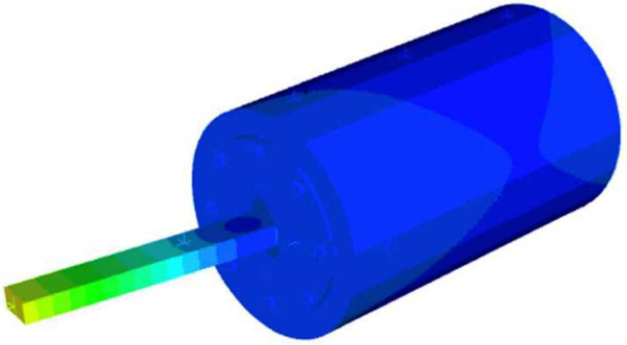


- As with Mode 1, when Mode 2 is excited alone, frequency and damping overlay with increasing amplitude.

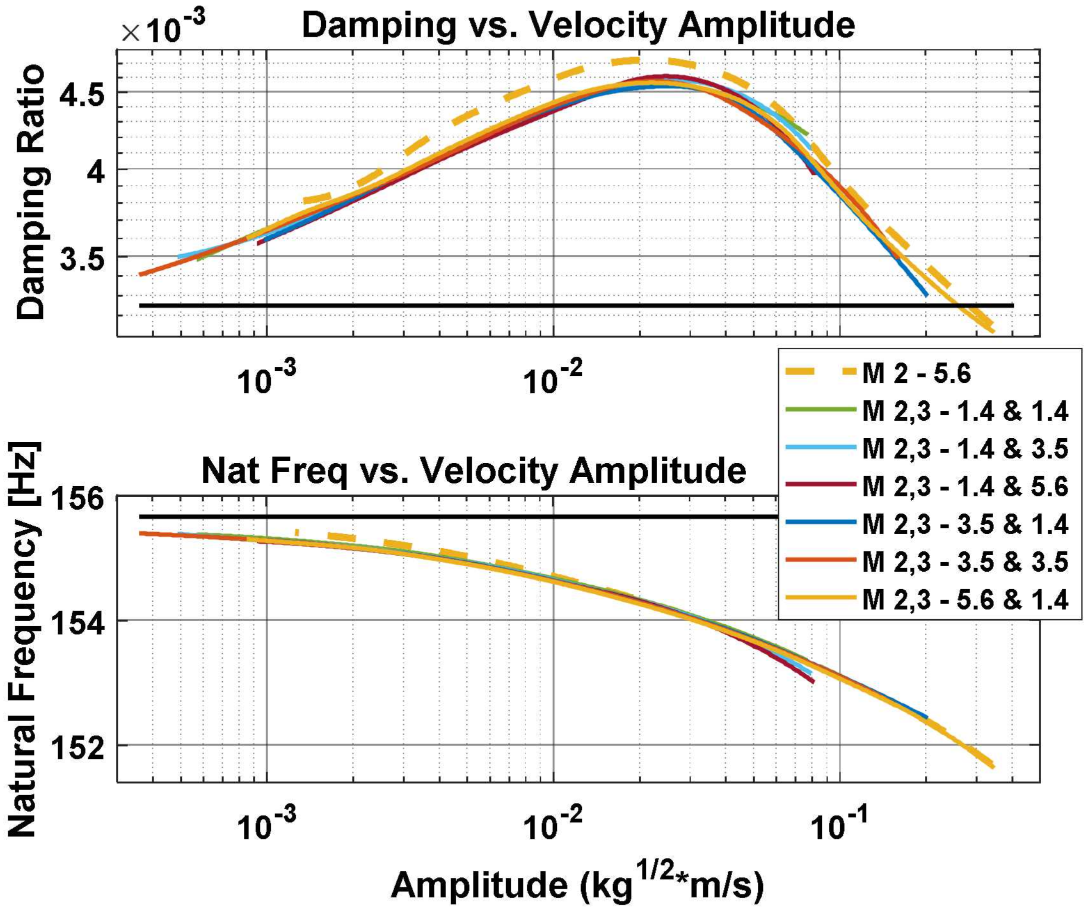


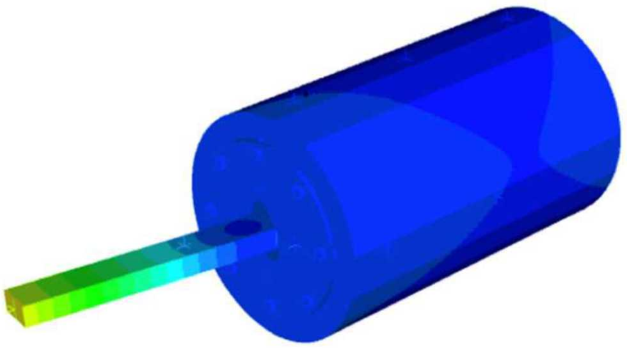

# Mode 2 & 1



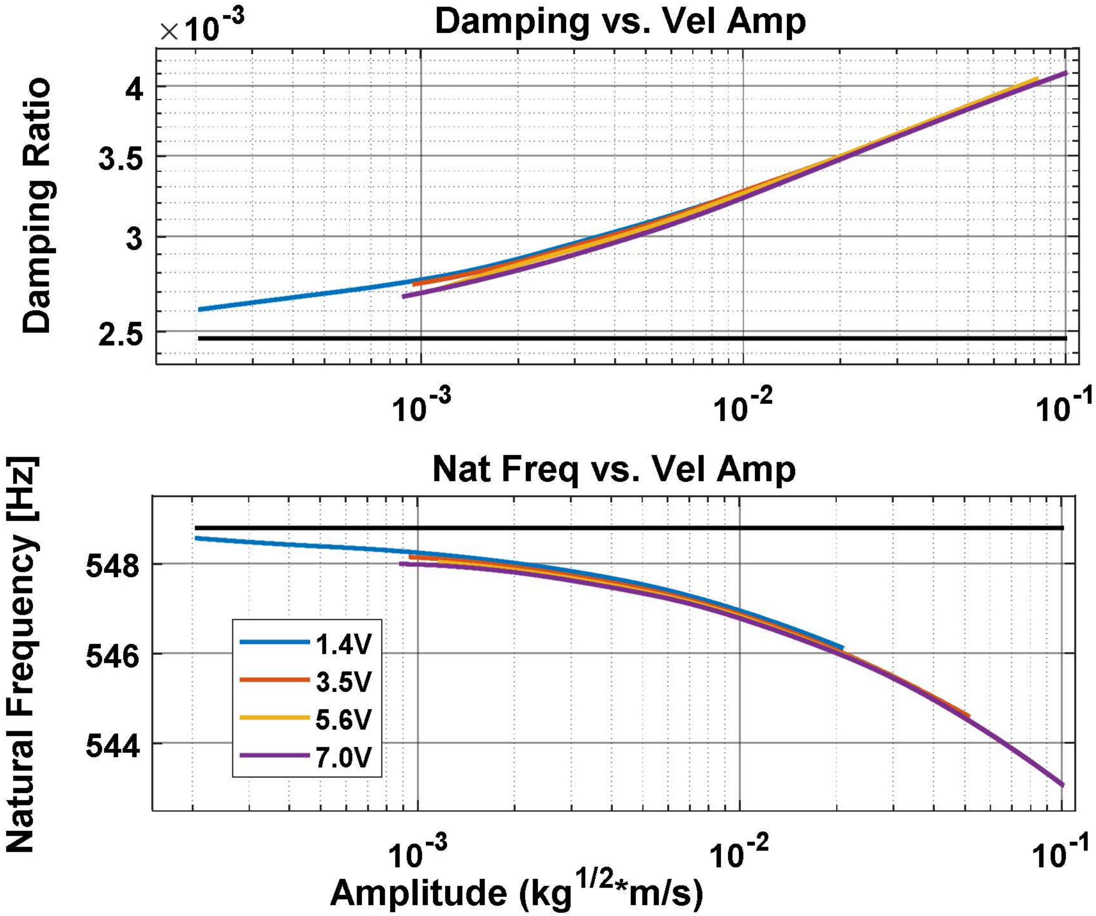
- Observe the same frequency and damping shift, but now to a greater degree in all cases where mode 1 is also excited.

# Mode 2 & 3

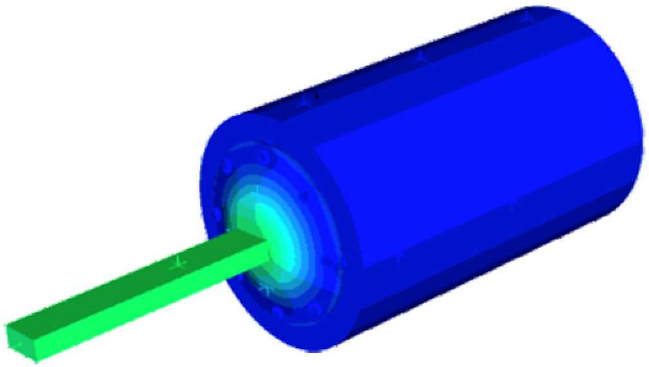


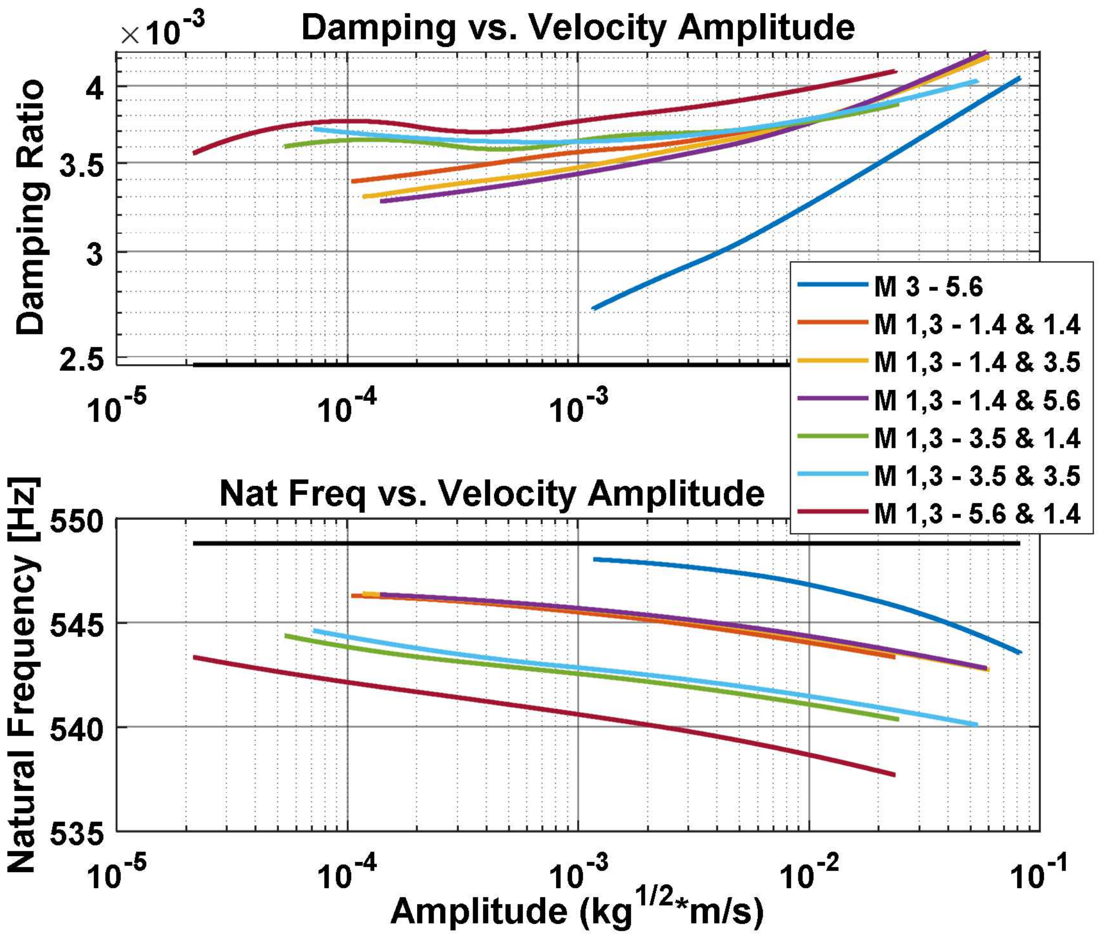
- Mode 2 also appears to be totally indifferent as to the presence of mode 3.



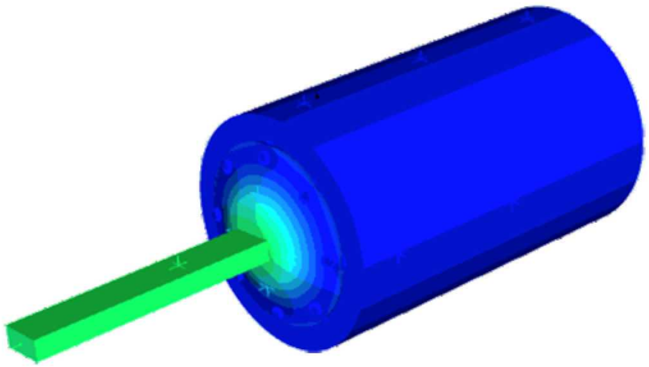

# Mode 3



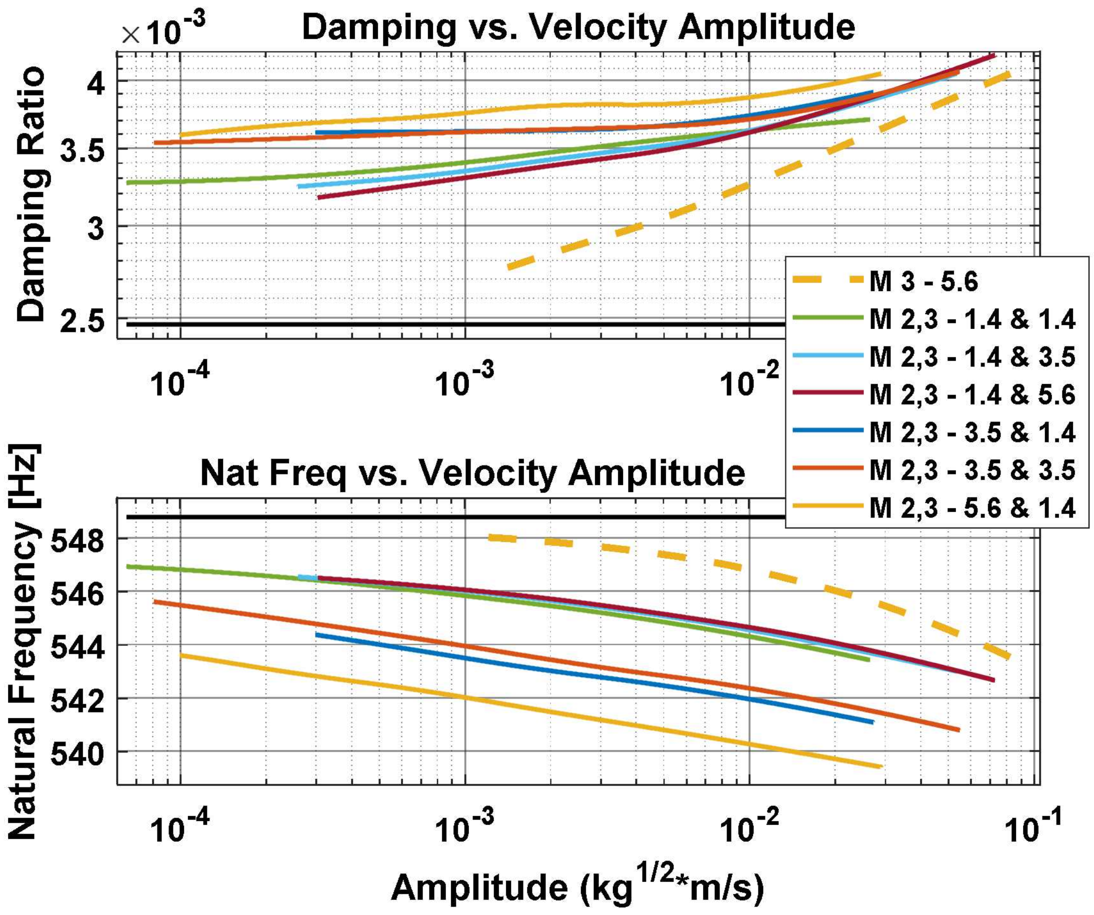
- When excited individually, all modes exhibit consistent frequency and damping shifts with increasing amplitude



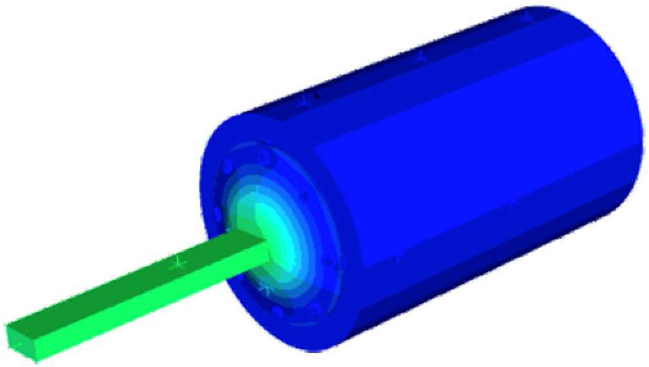

# Mode 3 & 1



- Mode 3 shows significant decreases in frequency and increases in damping when mode 1 is also excited at any level.



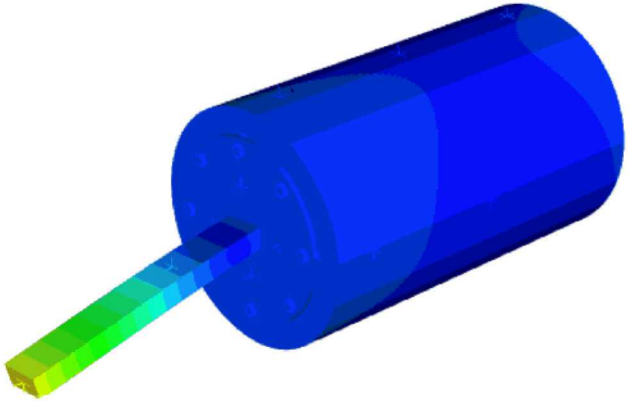

# Mode 3 & 2

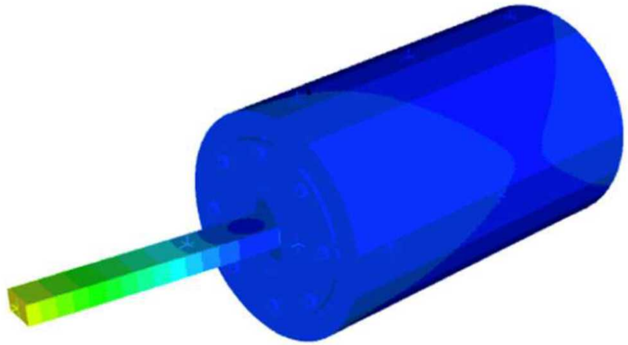


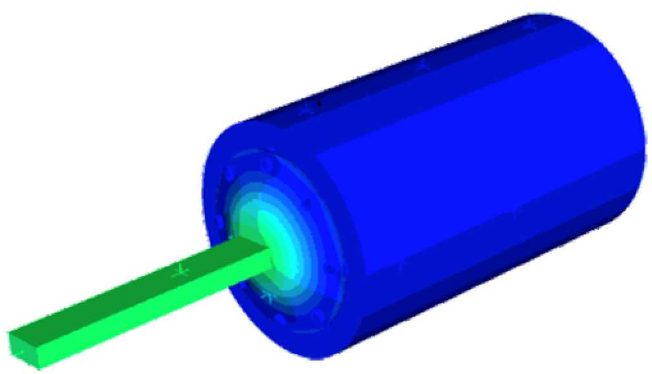
- When mode 3 is excited with mode 2, similar drastic shift in frequency and damping are observed.

# Closing Remarks

- When excited alone, all three modes have very consistent frequency and damping responses.
- Mode 1 and 2 show signs of modal coupling when simultaneously excited.
  - Mode 2 appears to be more significantly effected.
- Neither mode 1 or 2 display any indication of modal coupling with mode 3.
- However, mode 3 exhibits very strong coupling with modes 1 and 2.







# Acknowledgments

- This research was conducted at the 2018 Nonlinear Mechanics and Dynamics Research Institute hosted by Sandia National Laboratories and the University of New Mexico.

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