SAND2019-0855C

Modal Projection Matching

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Motivation

Laboratory vibration tests typically try to mimic field environment dynamics

If there is <u>any</u> difference between the device under test's boundary conditions in the laboratory compared to the field environment, its dynamic characteristics have been modified

Field Environment

https://share-ng.sandia.gov/news/resources/news_releases/images/2017/TTR_FlyBy.jpg

Laboratory Test

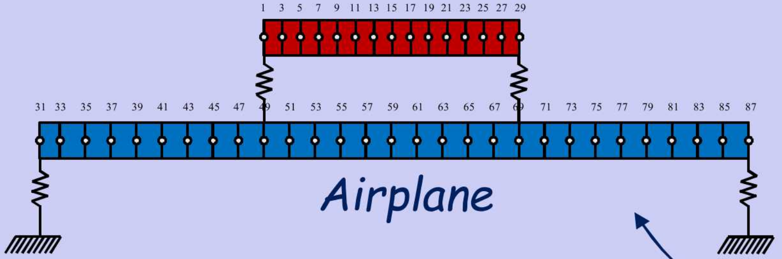
https://www.sandia.gov/news/publications/lab_accomplishments/articles/2016/nuclear-weapons-engineering.html

Recent Approaches

Field Environment

https://share-ng.sandia.gov/news/resources/news_releases/images/2017/TTR_FlyBy.jpg

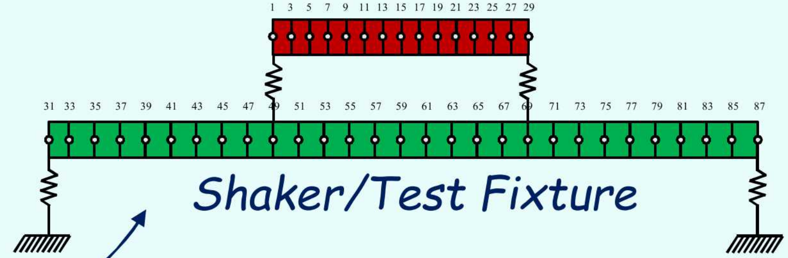
Bomb



Laboratory Test

https://www.sandia.gov/news/publications/lab_accomplishments/articles/2016/nuclear-weapons-engineering.html

Bomb

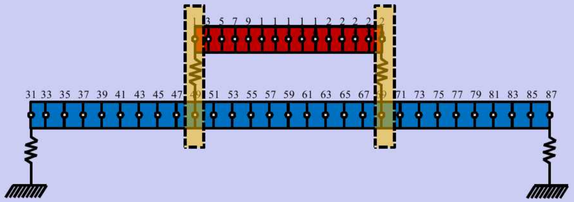


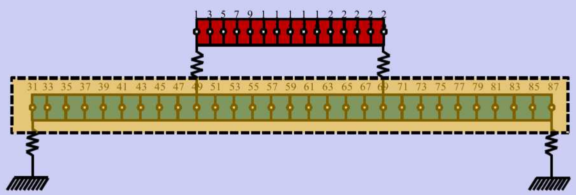
Different Dynamic Characteristics

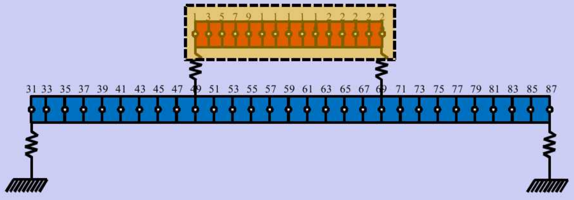
Recent Approaches

How Are the Dynamics Translated Between Configurations?

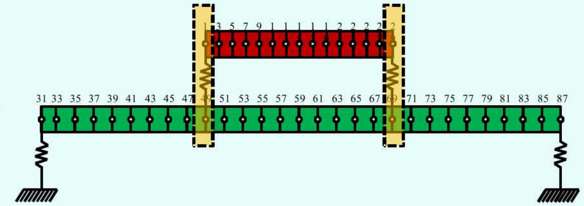
Field Environment

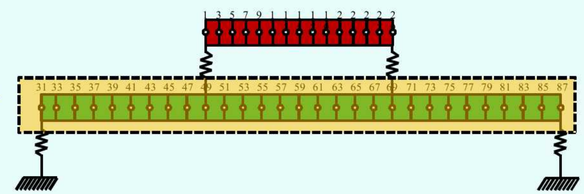


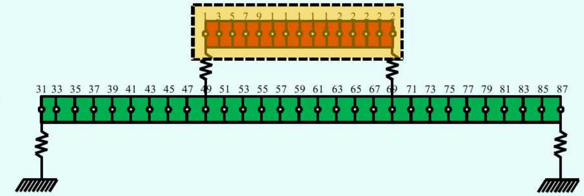




Laboratory Test







FIxture NEutralization (FINE)

Jesus Reyes

Modal Craig Bampton (MCB)

Matt Allen, Randy Mayes, Julie Harvie, Tyler Schoneherr

Impedance-Matched Multi-Axis Test (IMMAT)

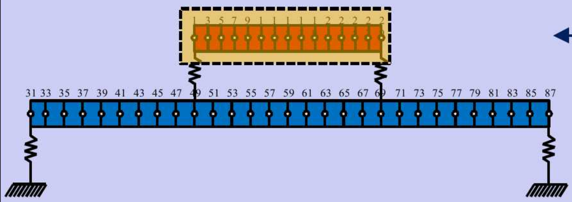
Philip Daborn

Theory

The dynamic response of any linear system can be expressed as a sum of the system's modes (modal superposition)

Field Environment

(System 1)

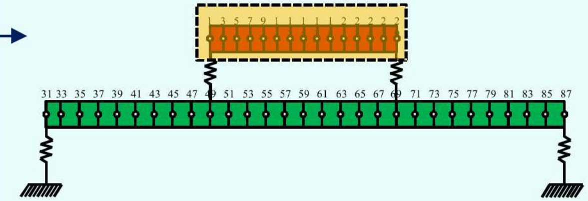


$$x_1 = U_1 p_1$$

Modal Projection Matching

Test Laboratory

(System 2)



$$x_2 = U_2 p_2$$

$$x_{1,d} = x_{2,d}$$

$$p_2 = U_{2,d}^{\dagger} U_{1,d} p_1$$

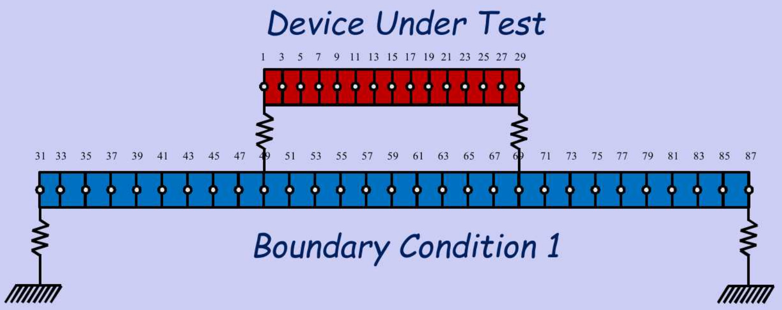
$$p_2 = U_{21} p_1$$

d - degrees of freedom only on the device under test
x – Displacement
p – Modal displacement
U – Mode Shapes

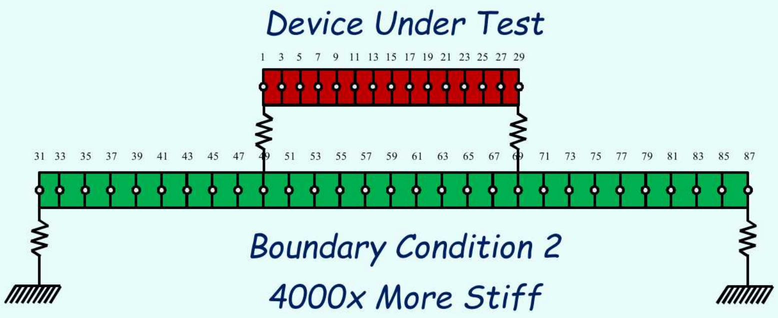
Model

2 finite element models were created with two beams each. The "boundary condition" beam was modified for the second model

Field Environment

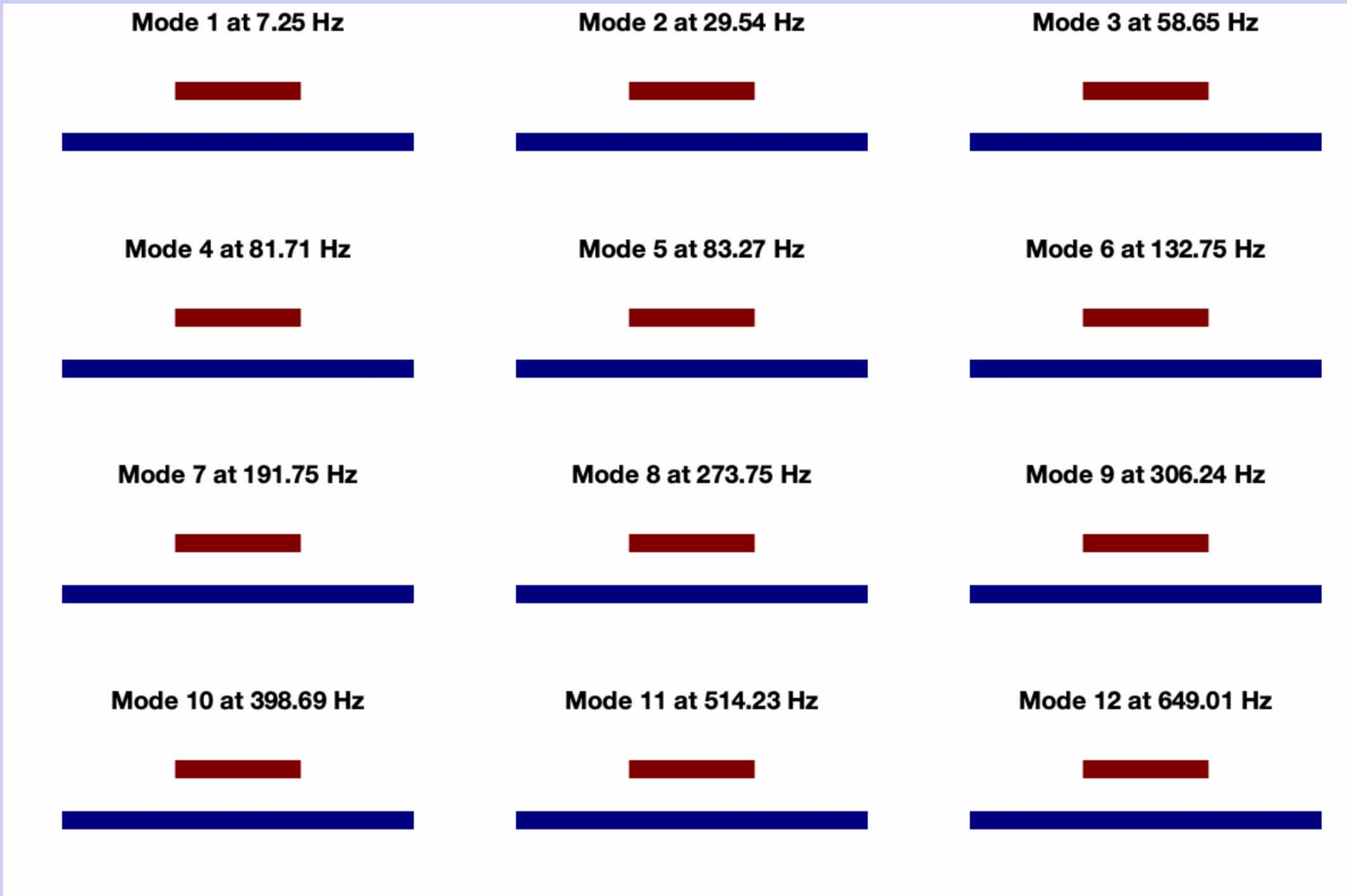


Test Laboratory



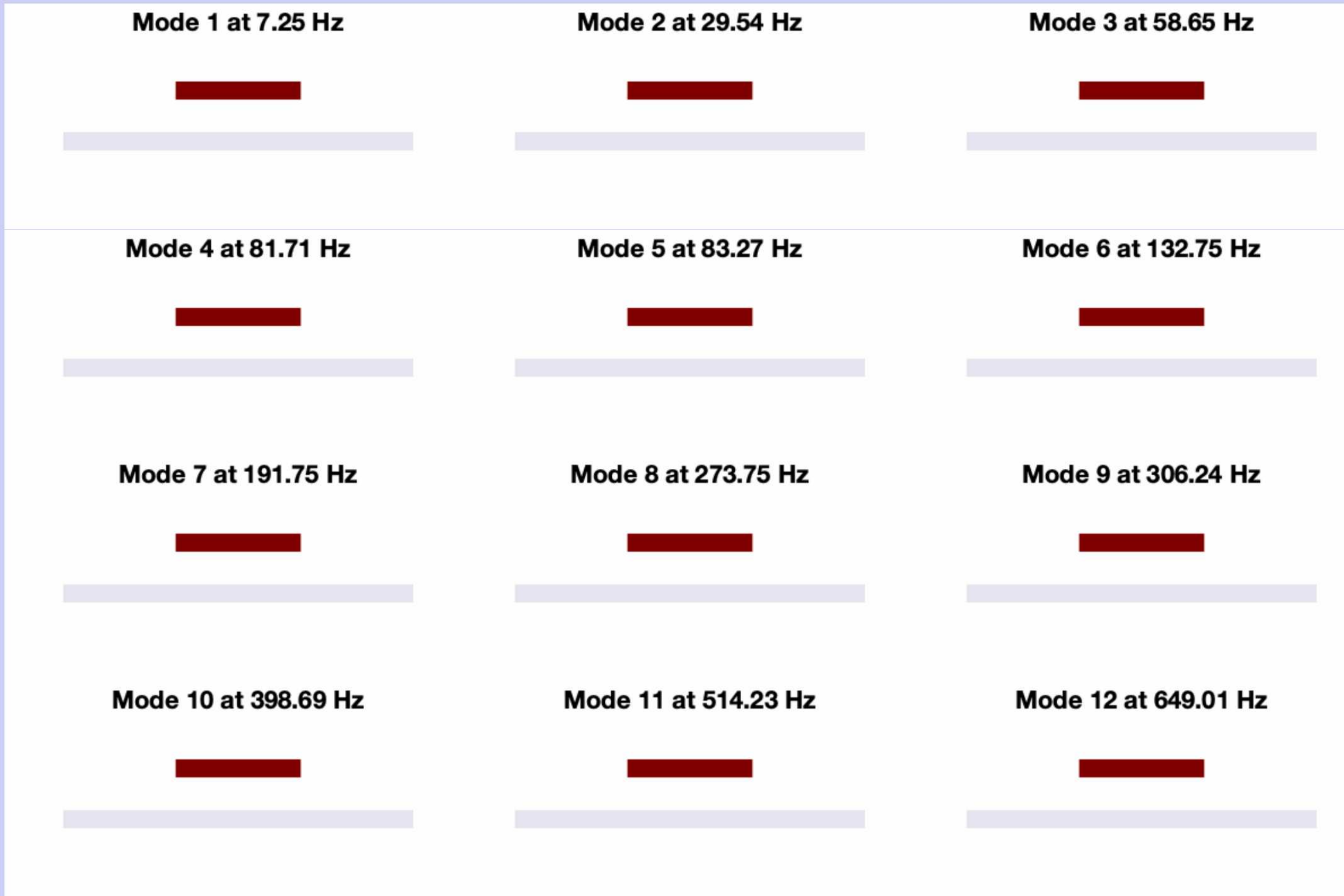
Model

Field Environment Model Modes



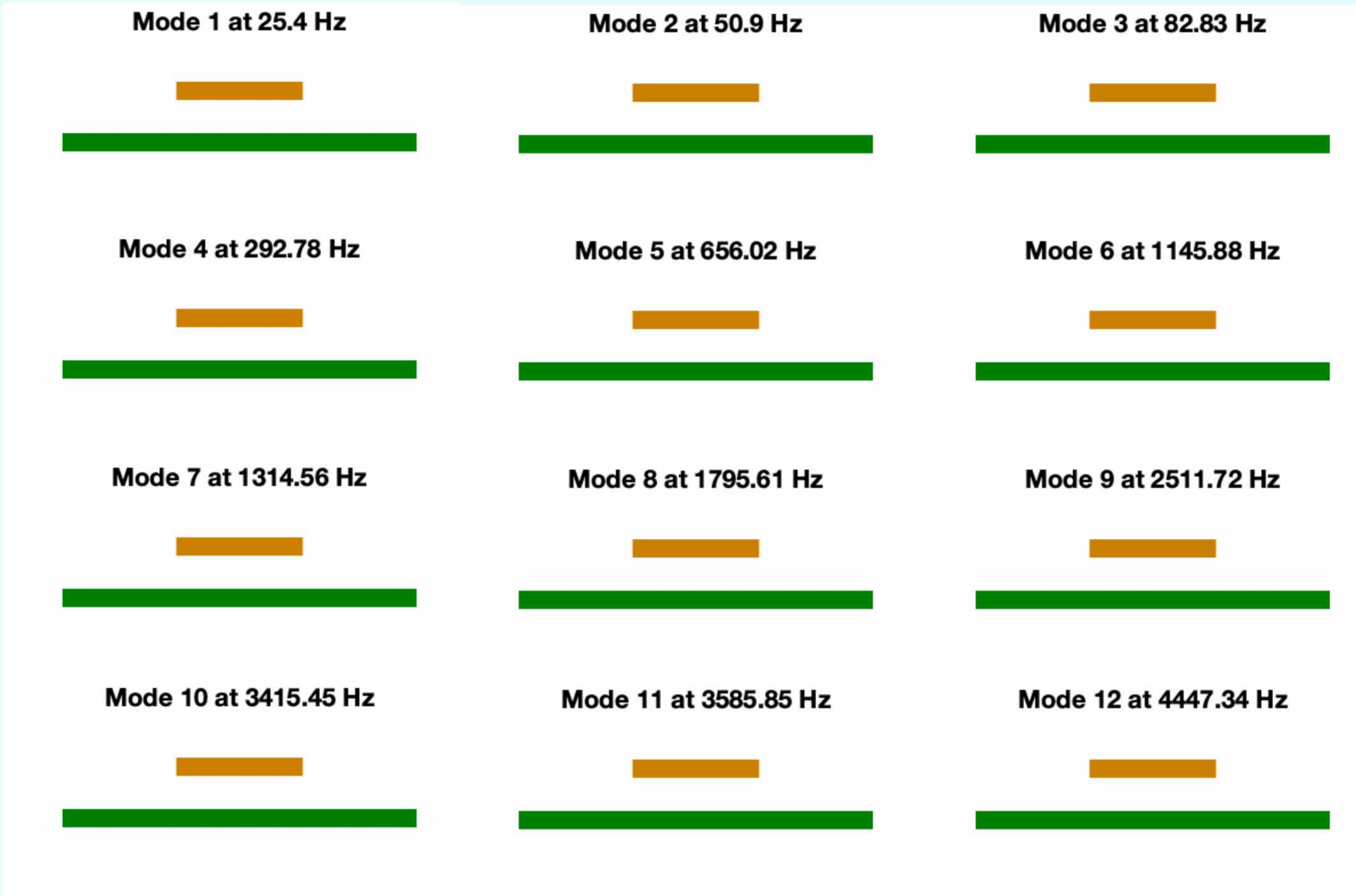
Model

Field Environment Model Modes



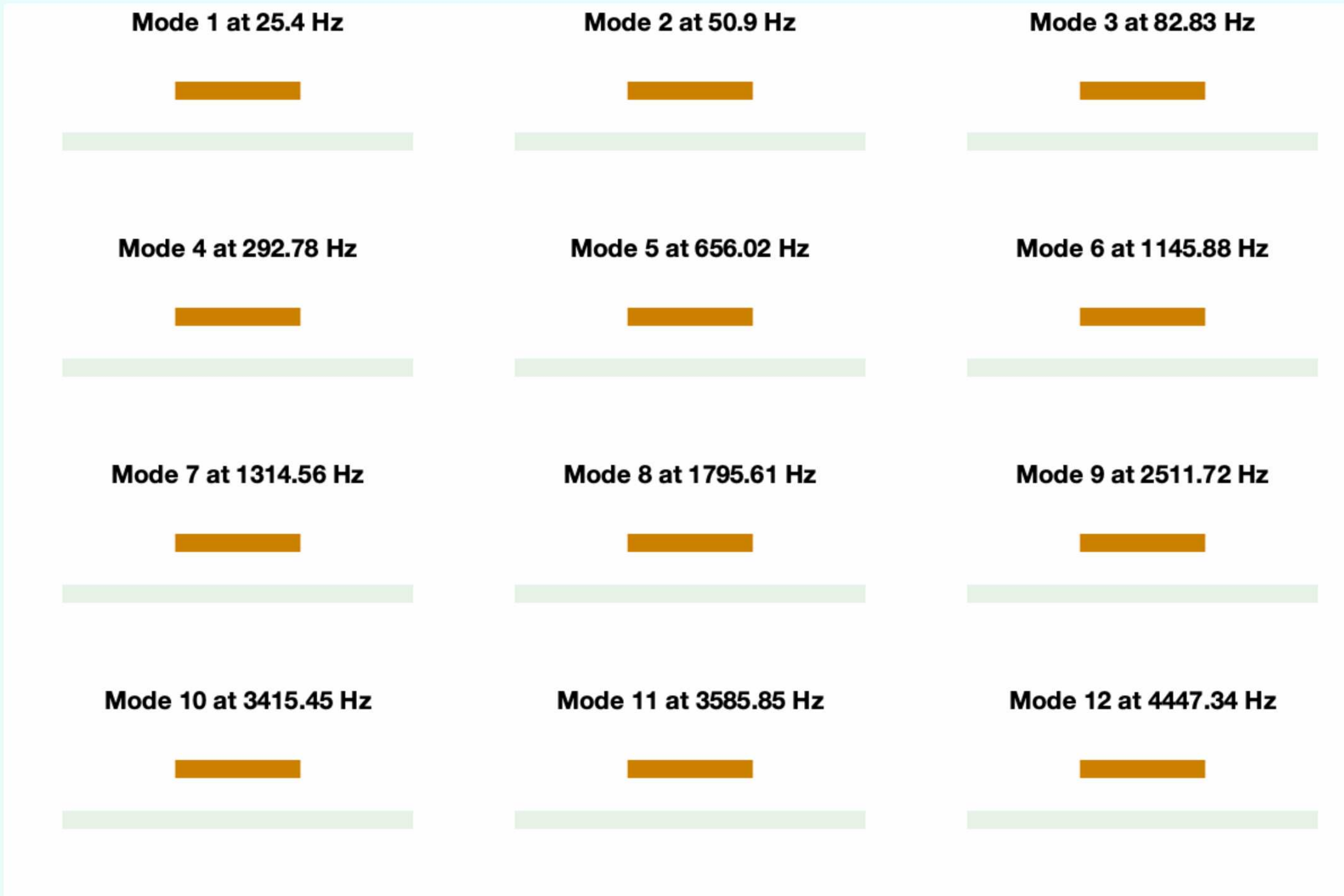
Model

Test Laboratory Model Modes

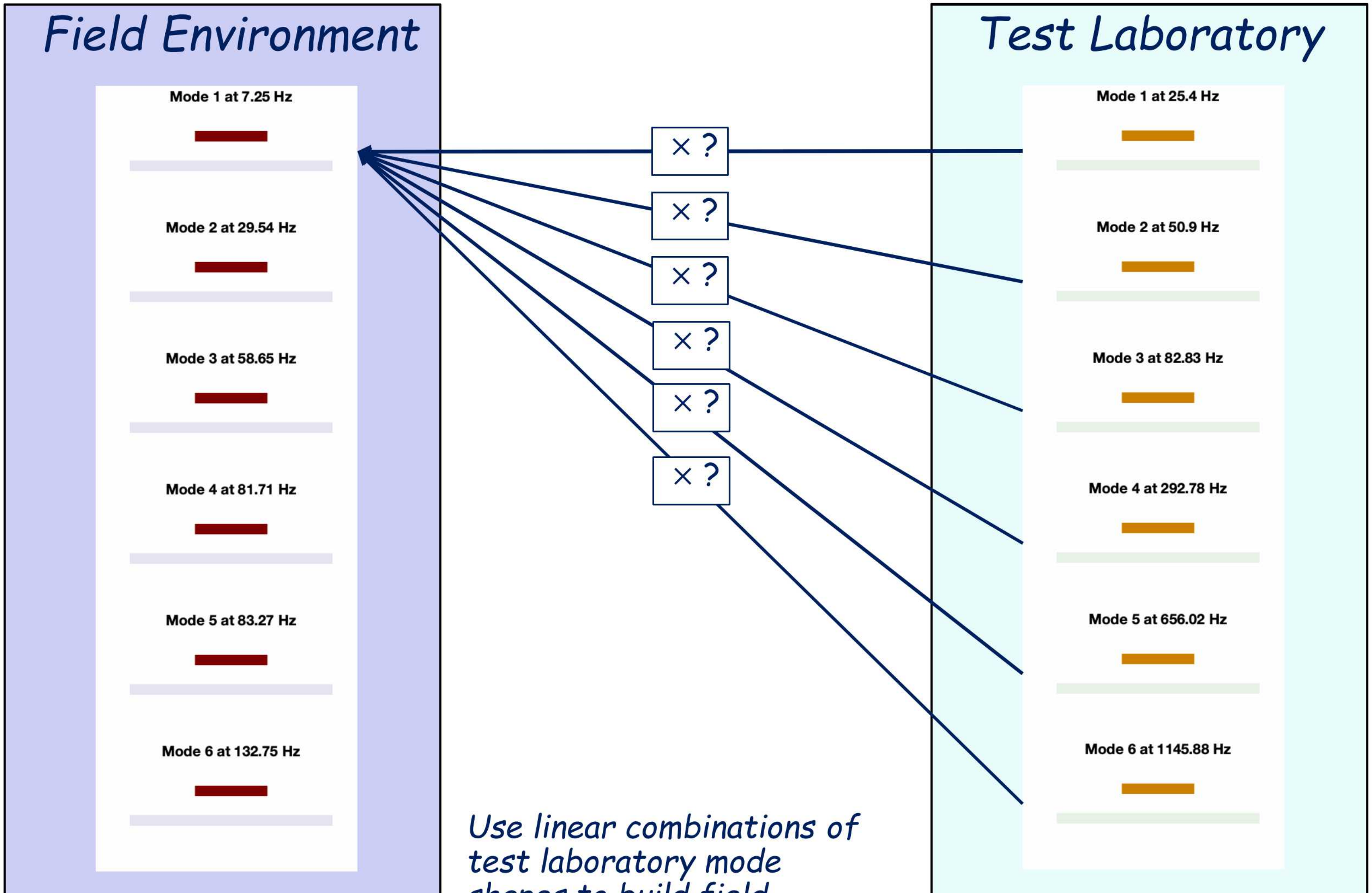


Model

Test Laboratory Model Modes

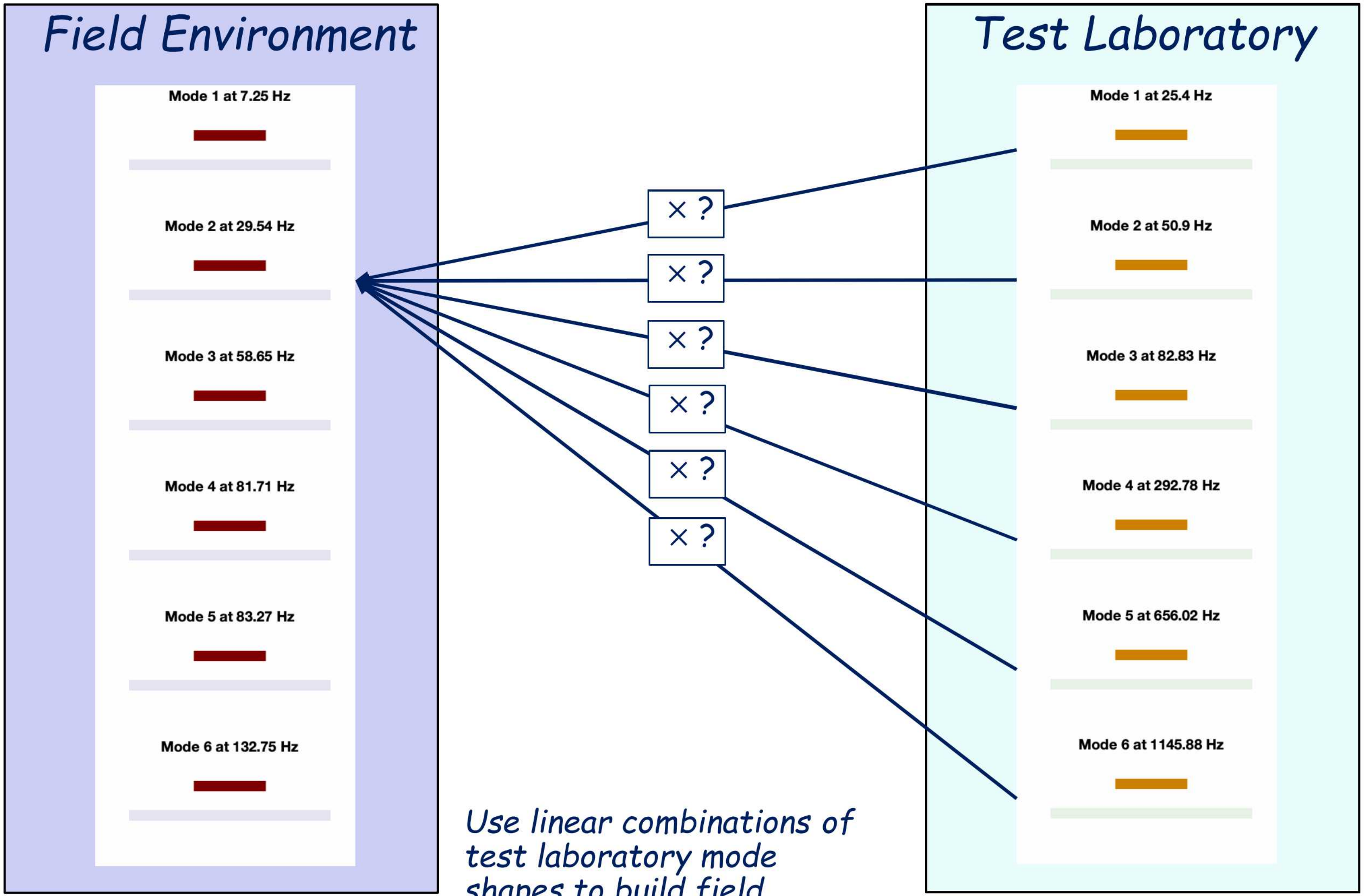


Analysis



Use linear combinations of test laboratory mode shapes to build field environment mode shapes

Analysis

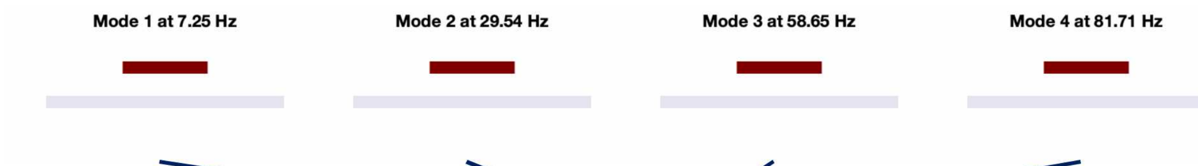


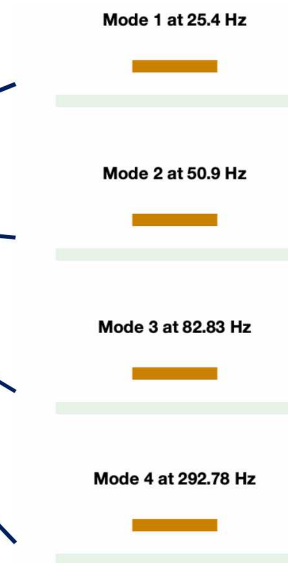
Use linear combinations of test laboratory mode shapes to build field environment mode shapes

$$p_2 = \boxed{U_{2,d}^{\dagger} U_{1,d}} p_1$$

$$p_2 = \boxed{U_{21}} p_1$$

Analysis





U21 Matrix

Rows: Laboratory Test Model Mode; Columns: Field Environment Model Mode

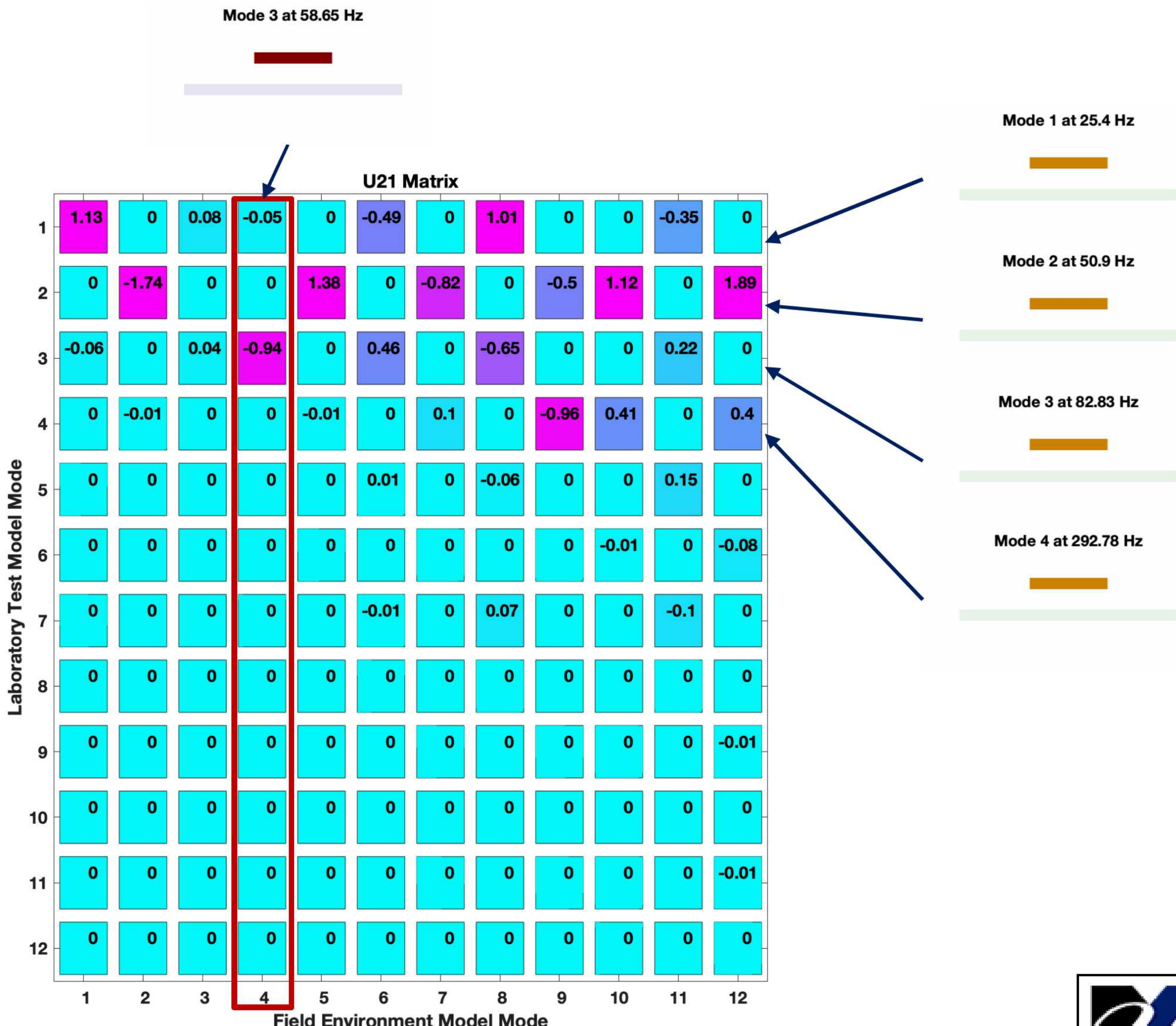
	1	2	3	4	5	6	7	8	9	10	11	12
1	1.13	0	0.08	-0.05	0	-0.49	0	1.01	0	0	-0.35	0
2	0	-1.74	0	0	1.38	0	-0.82	0	-0.5	1.12	0	1.89
3	-0.06	0	0.04	-0.94	0	0.46	0	-0.65	0	0	0.22	0
4	0	-0.01	0	0	-0.01	0	0.1	0	-0.96	0.41	0	0.4
5	0	0	0	0	0	0.01	0	-0.06	0	0	0.15	0
6	0	0	0	0	0	0	0	0	0	-0.01	0	-0.08
7	0	0	0	0	0	-0.01	0	0.07	0	0	-0.1	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	-0.01
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	-0.01
12	0	0	0	0	0	0	0	0	0	0	0	0

Each column describes how a field environment mode is created using linear combinations of laboratory test modes

Analysis

$$p_2 = \boxed{U_{2,d}^{\dagger} U_{1,d}} p_1$$

$$p_2 = \boxed{U_{21}} p_1$$



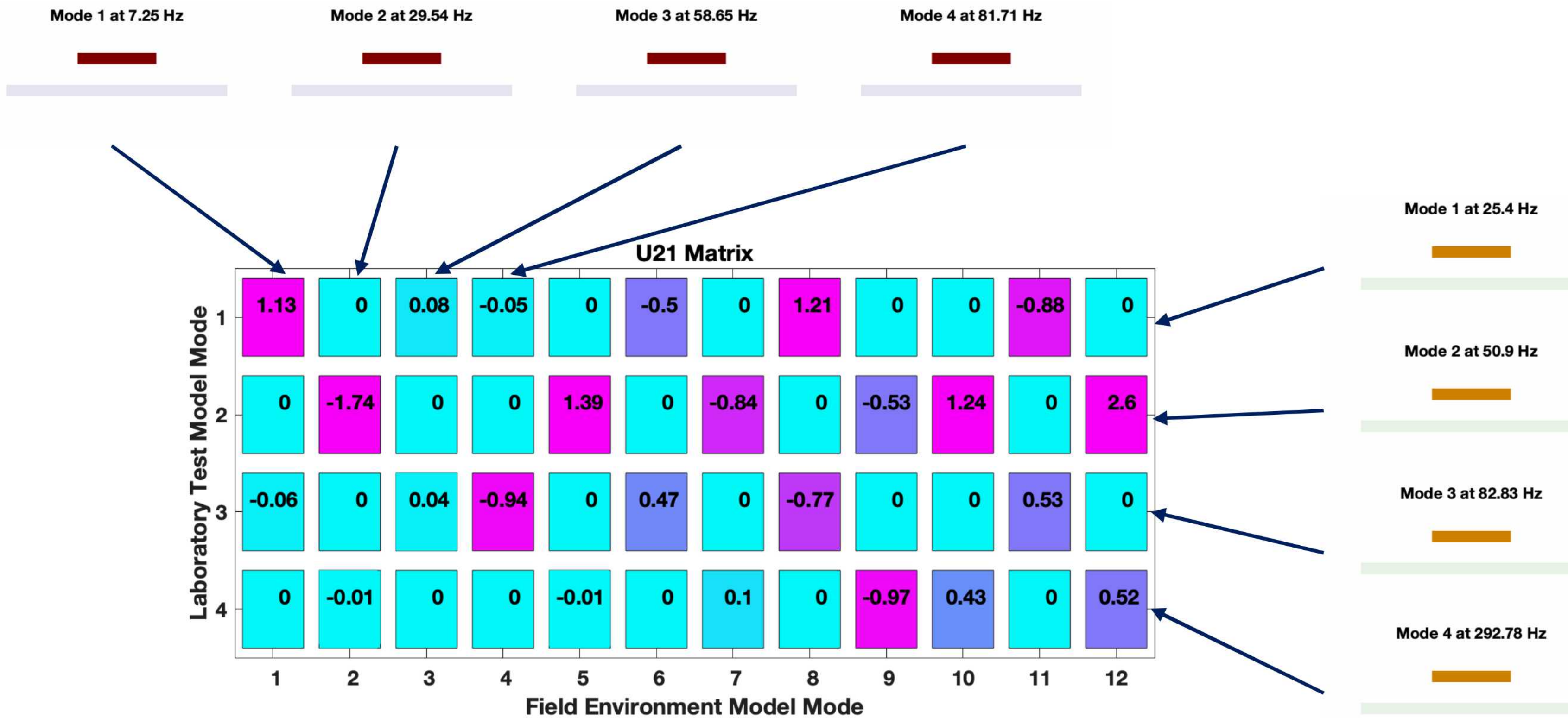
Laboratory Test Model Mode \ Field Environment Model Mode	1	2	3	4	5	6	7	8	9	10	11	12
1	1.13	0	0.08	-0.05	0	-0.49	0	1.01	0	0	-0.35	0
2	0	-1.74	0	0	1.38	0	-0.82	0	-0.5	1.12	0	1.89
3	-0.06	0	0.04	-0.94	0	0.46	0	-0.65	0	0	0.22	0
4	0	-0.01	0	0	-0.01	0	0.1	0	-0.96	0.41	0	0.4
5	0	0	0	0	0	0.01	0	-0.06	0	0	0.15	0
6	0	0	0	0	0	0	0	0	0	-0.01	0	-0.08
7	0	0	0	0	0	-0.01	0	0.07	0	0	-0.1	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	-0.01
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	-0.01
12	0	0	0	0	0	0	0	0	0	0	0	0

Each column describes how a field environment mode is created using linear combinations of laboratory test modes

Analysis

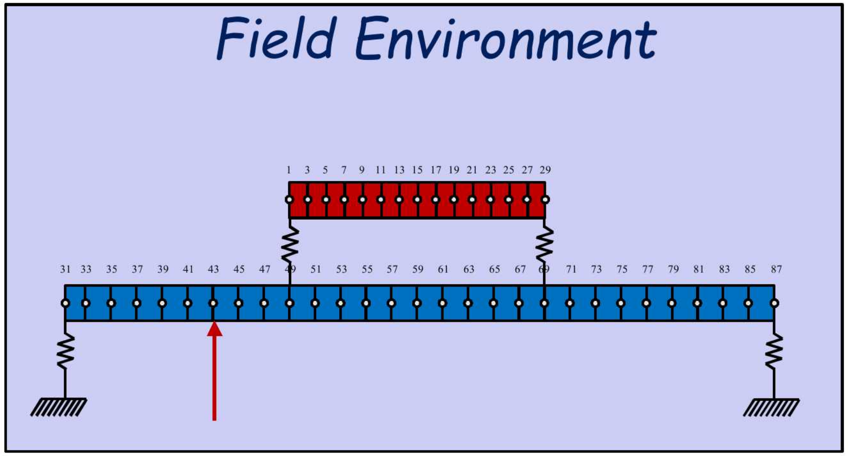
$$p_2 = \boxed{U_{2,d}^{\dagger} U_{1,d}} p_1$$

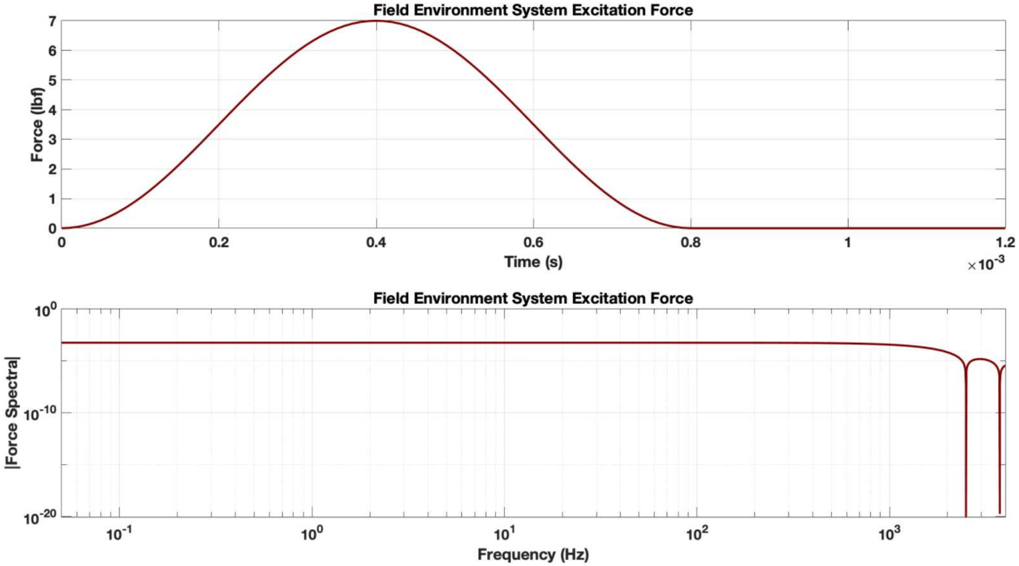
$$p_2 = \boxed{U_{21}} p_1$$



Each column describes how a field environment mode is created using linear combinations of laboratory test modes

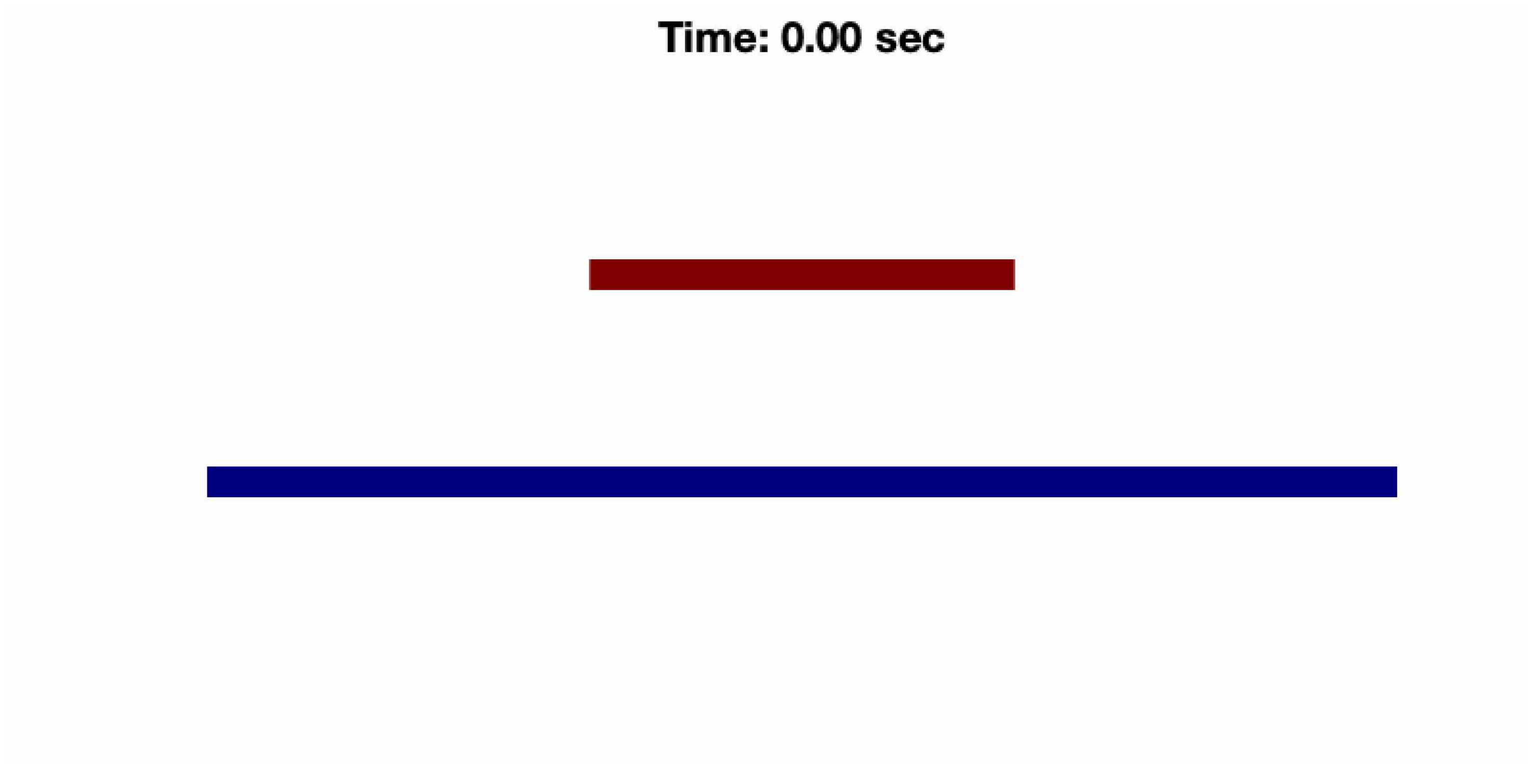
Results


Field Environment
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29
31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71 73 75 77 79 81 83 85 87


Field Environment System Excitation Force
Force (lbf)
Time (s)
×10^-3
Field Environment System Excitation Force
|Force Spectra|
Frequency (Hz)

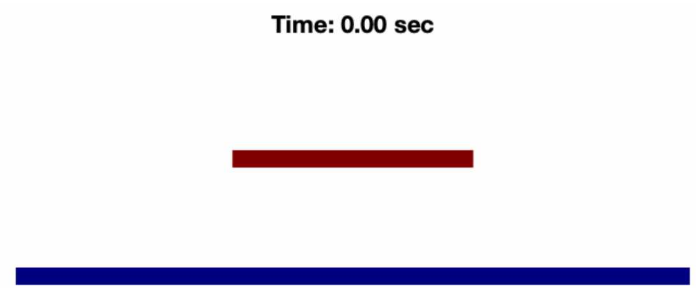
Results

Field Environment Response to Impulse Force



Results

Field Environment Response to Impulse Force



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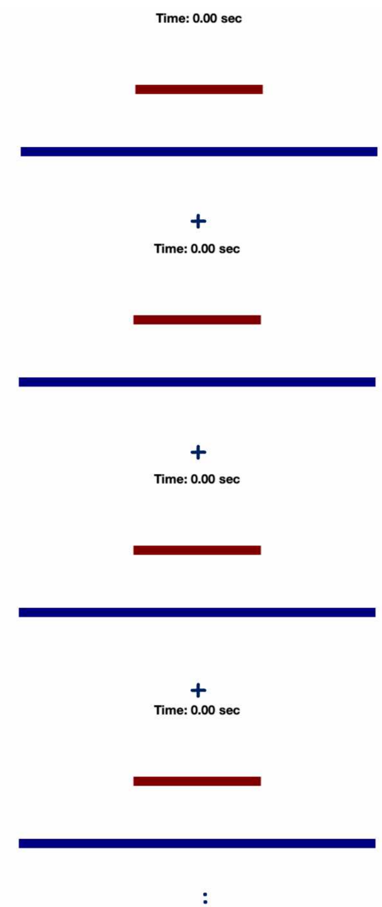


Mode 1

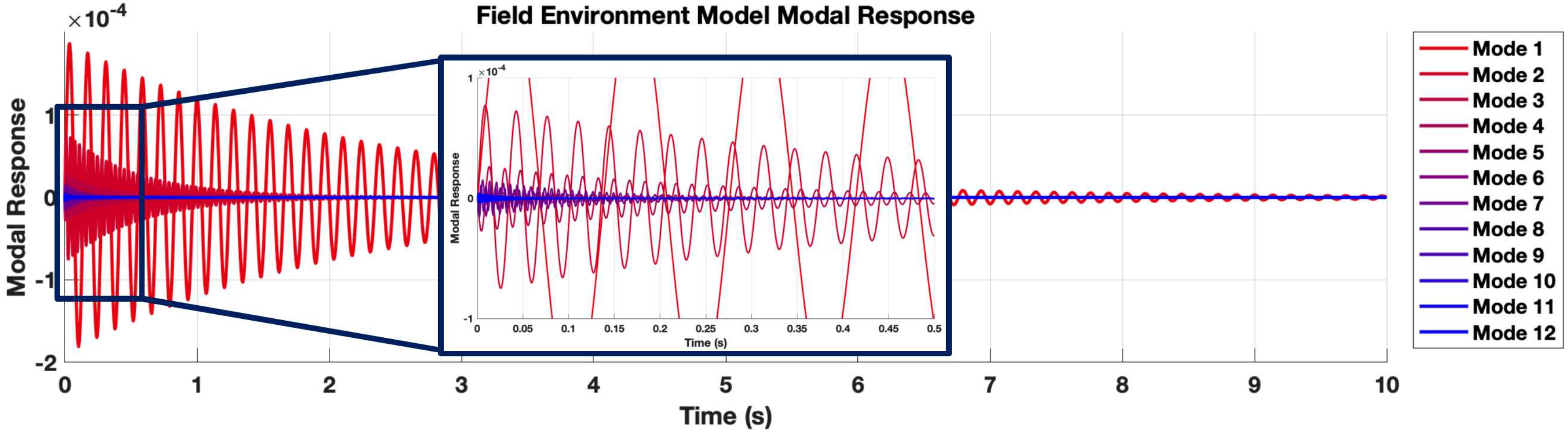
Mode 2

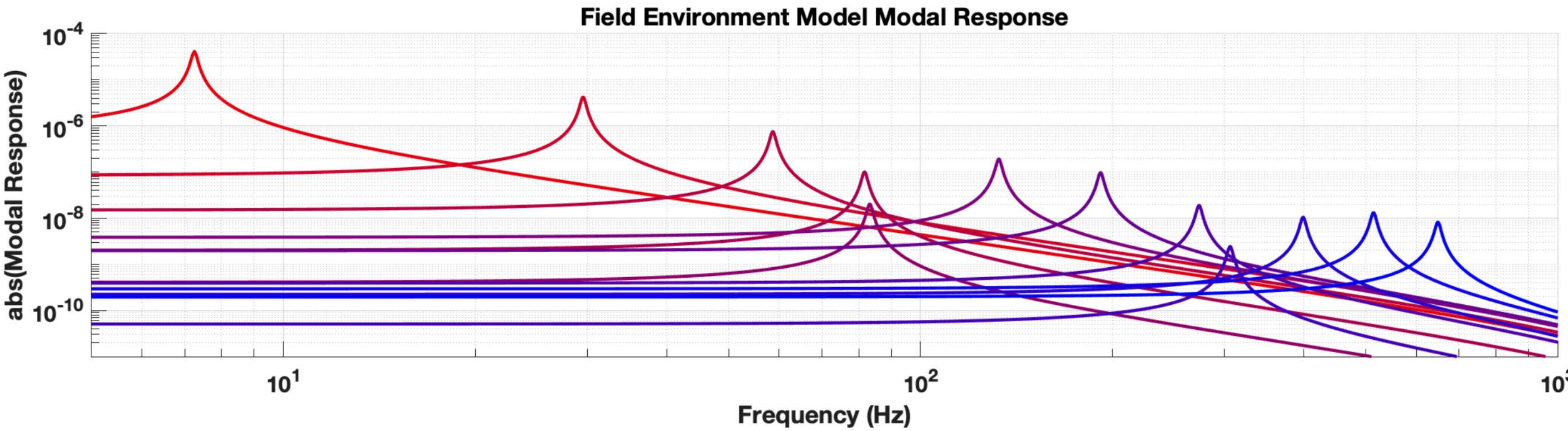
Mode 3

Mode 4

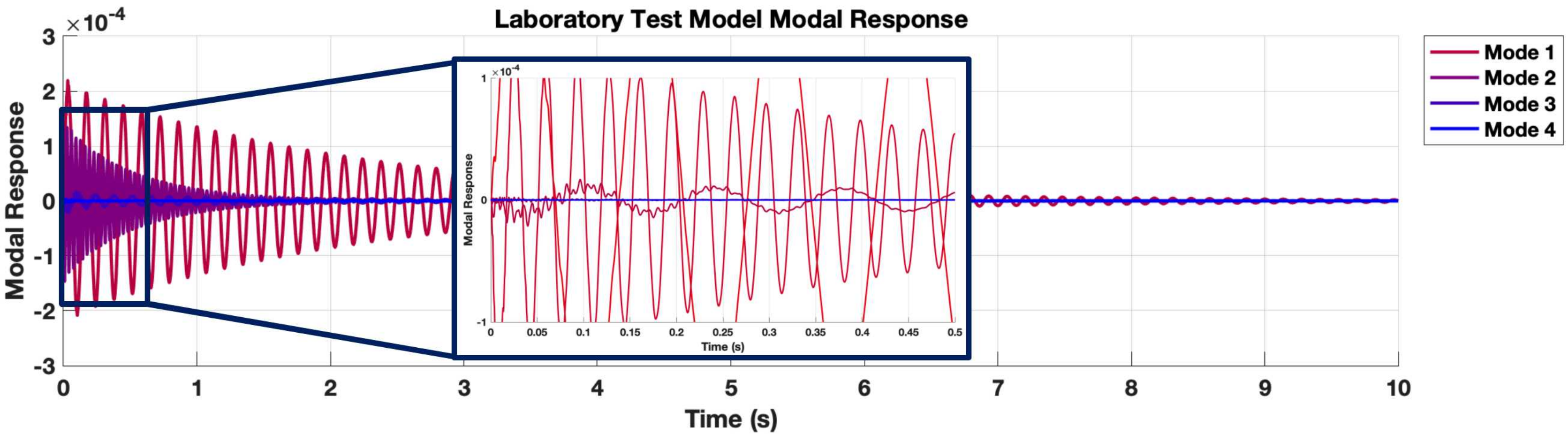
The dynamic response of any system can be expressed as a sum of it's modes (modal superposition)

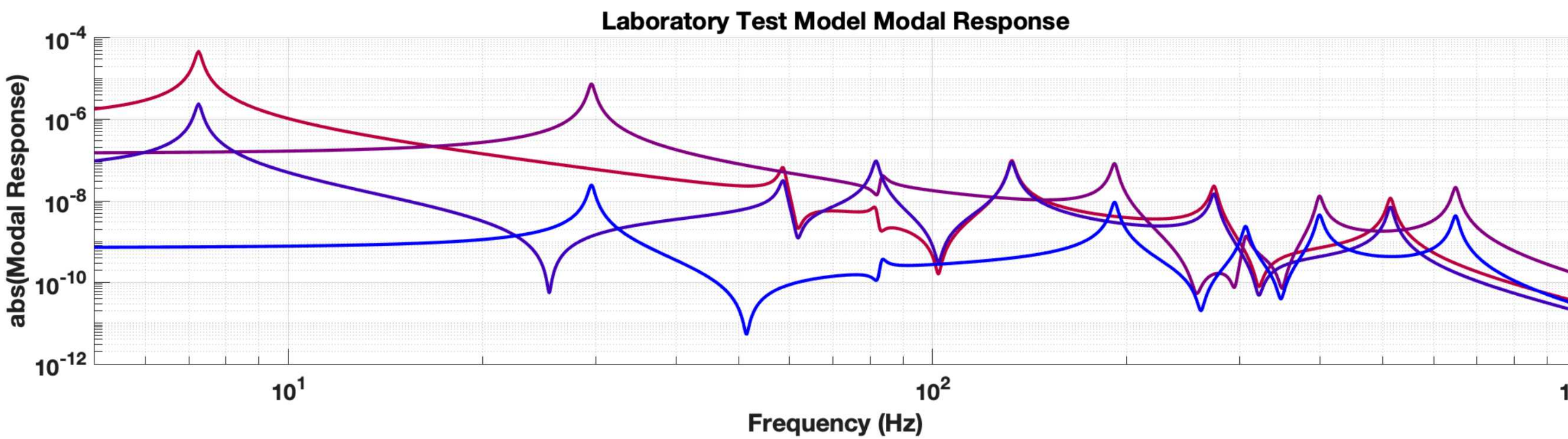
Results – Field Environment Model Response





Results – Laboratory Test Target Response

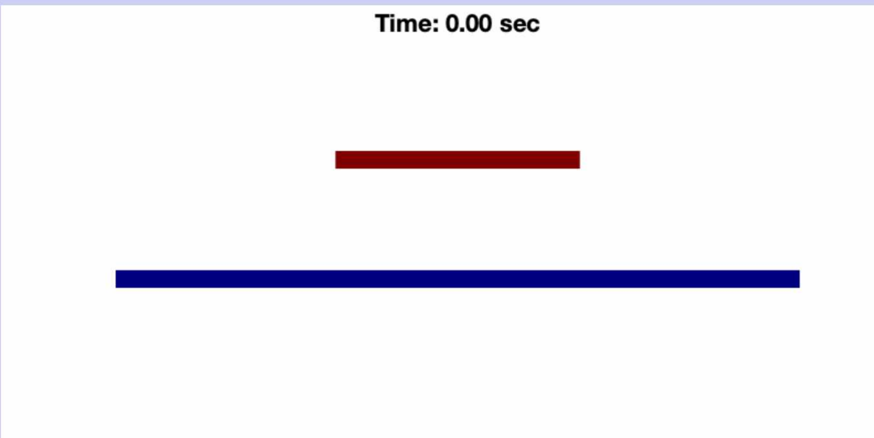




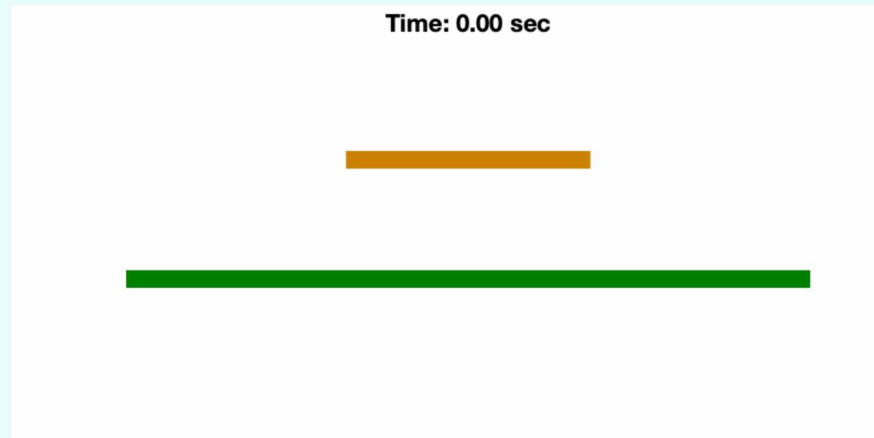
This is a set of <u>target</u> modal responses for the laboratory environment system

Results

Field Environment

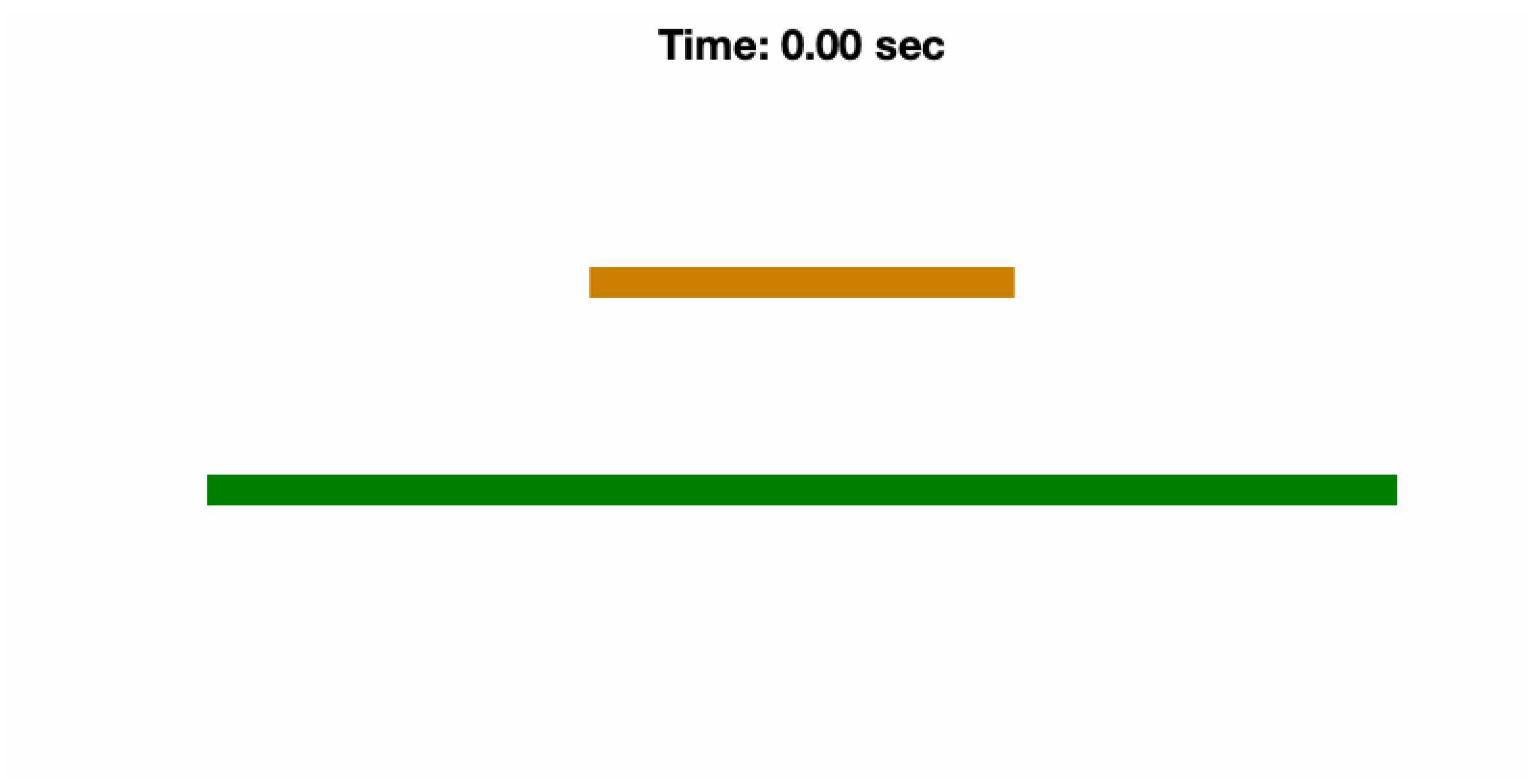


Test Laboratory Target



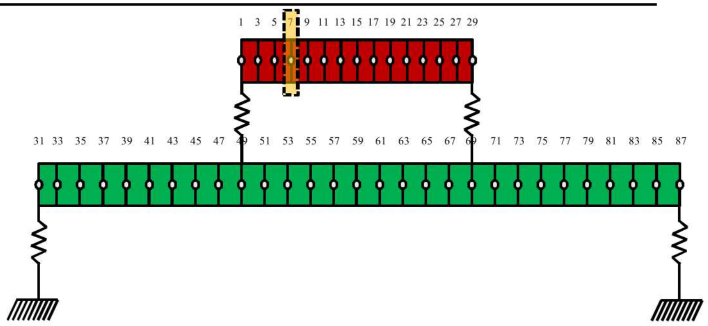
Results

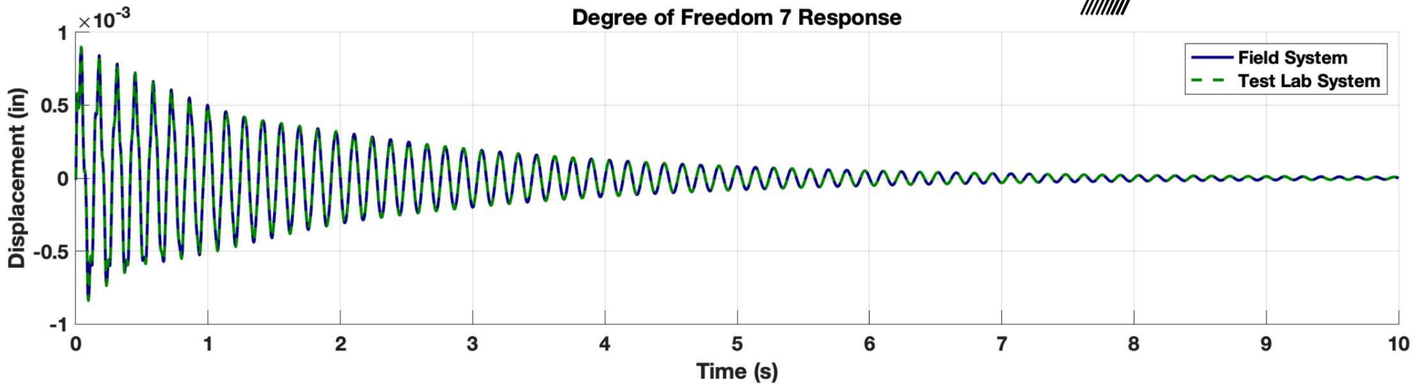
Field Environment and Laboratory Test Target Response Animated Together

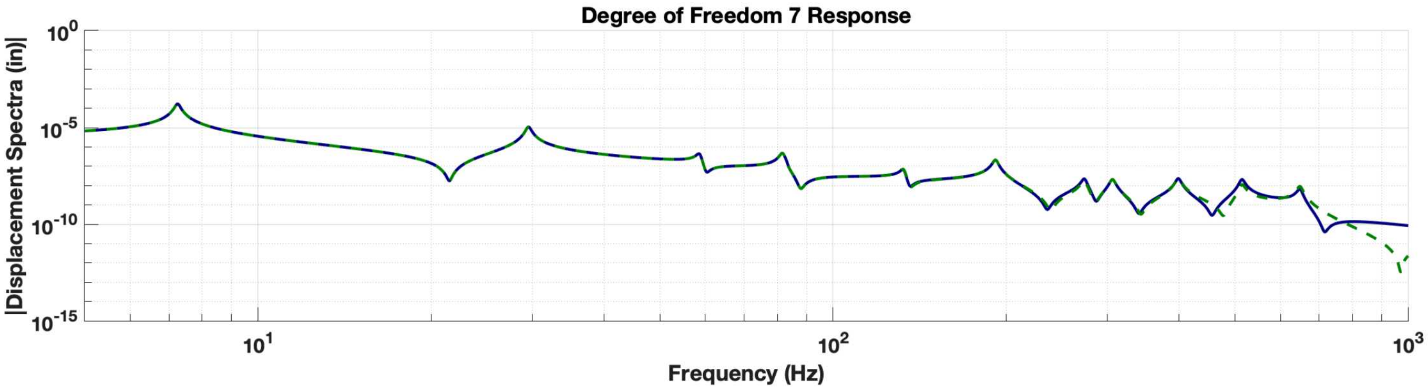


Results

Responses at DOF 7 for both Field Environment and Laboratory Test Target (typical)







Results

We now have target modal responses for the laboratory test system. To figure out how to best achieve those target responses, we need to introduce a new function:

$$\left[H_{o,i,k}\right] = \frac{x}{f} = U_{k,o} \frac{1}{\overline{m}_k\left(-\omega^2 + 2\omega_k j\zeta_k\omega + \omega_k^2\right)} U_{k,i}^T$$

FRF — *Physical displacement over physical force*

$$\left[\overline{H}_k\right] = \frac{\underline{p}}{\underline{f}} = \frac{1}{\overline{m}_k\left(-\omega^2 + 2\omega_k j\zeta_k\omega + \omega_k^2\right)}$$

Modal FRF — *Modal displacement over modal force*

i – Input Physical Degree of Freedom
o – Output Physical Degree of Freedom
k – Response Mode

Results

Using just a portion of the equation to calculate the FRF, we can define a new function, Z. Z has a complex value for every input degree of freedom, mode, and frequency line

$$\left[H_{o,i,k}\right] = \frac{x}{f} = U_{k,o} \boxed{\frac{1}{\bar{m}_k\left(-\omega^2 + 2\omega_k j\zeta_k\omega + \omega_k^2\right)} U_{k,i}^T}$$

FRF

Physical displacement over physical force

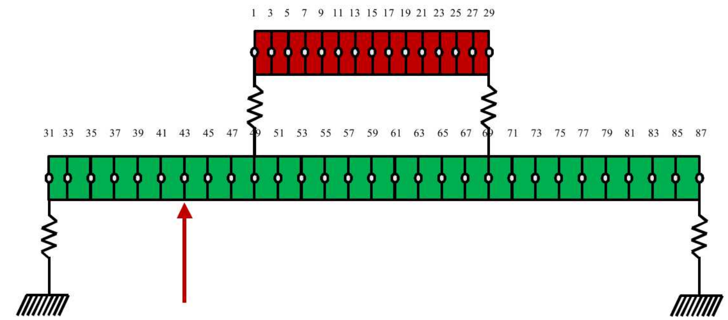
$$\left[Z_{i,k}\right] = \frac{p}{f} = \frac{1}{\bar{m}_k\left(-\omega^2 + 2\omega_k j\zeta_k\omega + \omega_k^2\right)} U_{k,i}^T$$

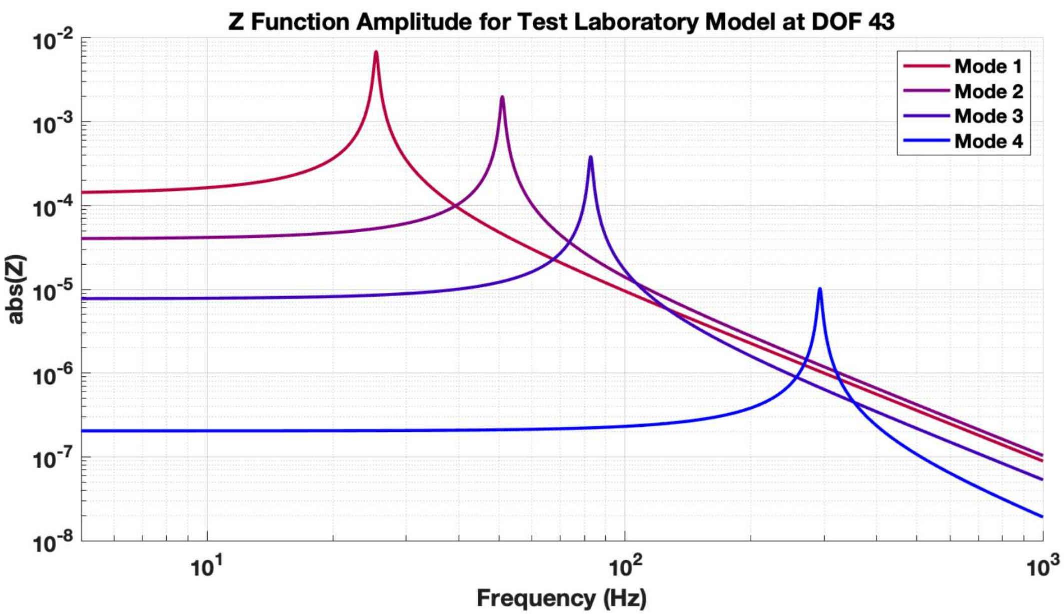
Z Function

Modal displacement over physical force

i – Input Physical Degree of Freedom
o – Output Physical Degree of Freedom
k – Response Mode

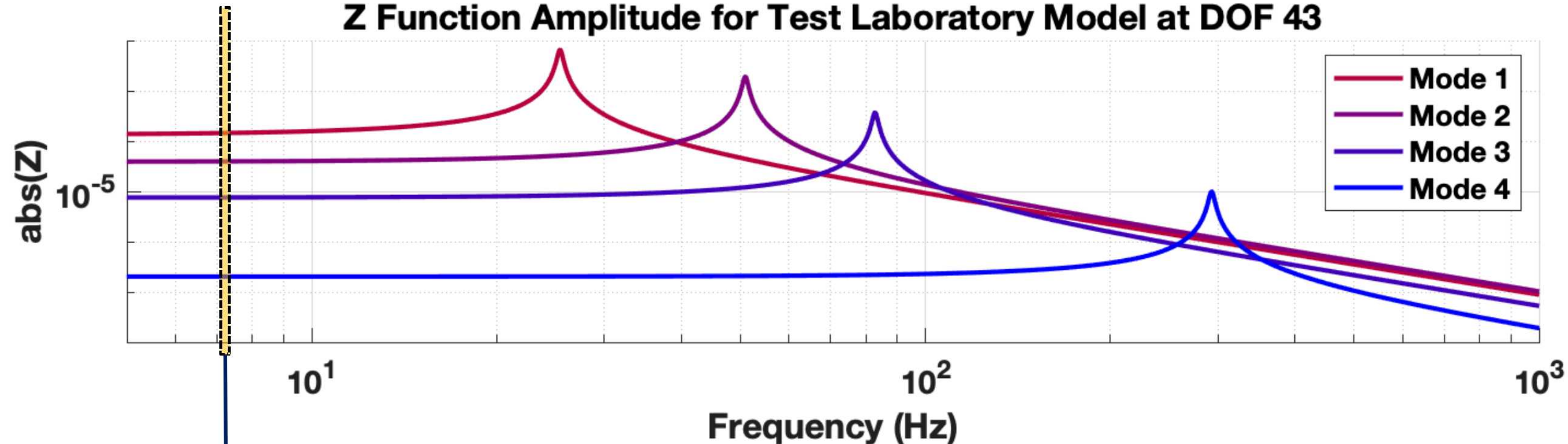
Results





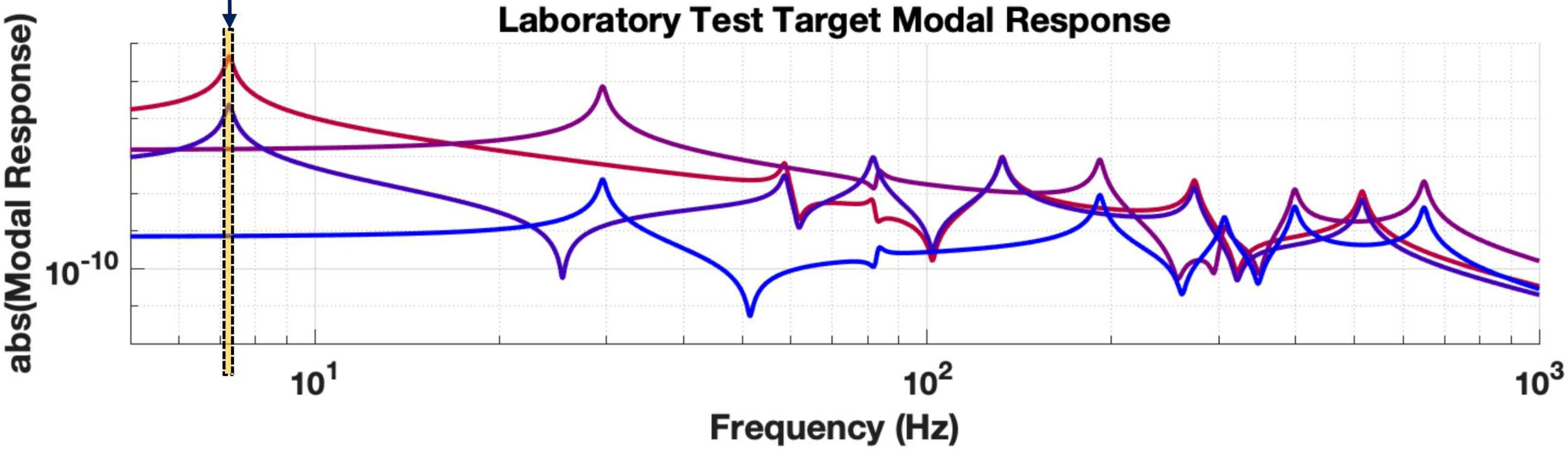
The Z function describes how an input force excites the modes of a system at each frequency line for a particular input force location.

Results



× ?

At each frequency line, we need to find a value to multiply Z function by to best achieve our target modal responses.



Results

This process is a compromise because we want to match 4 modes but we only have one input force.

To achieve the best compromise in a least-squared error sense, you can use this equation:

$$Z = \frac{p}{f}$$

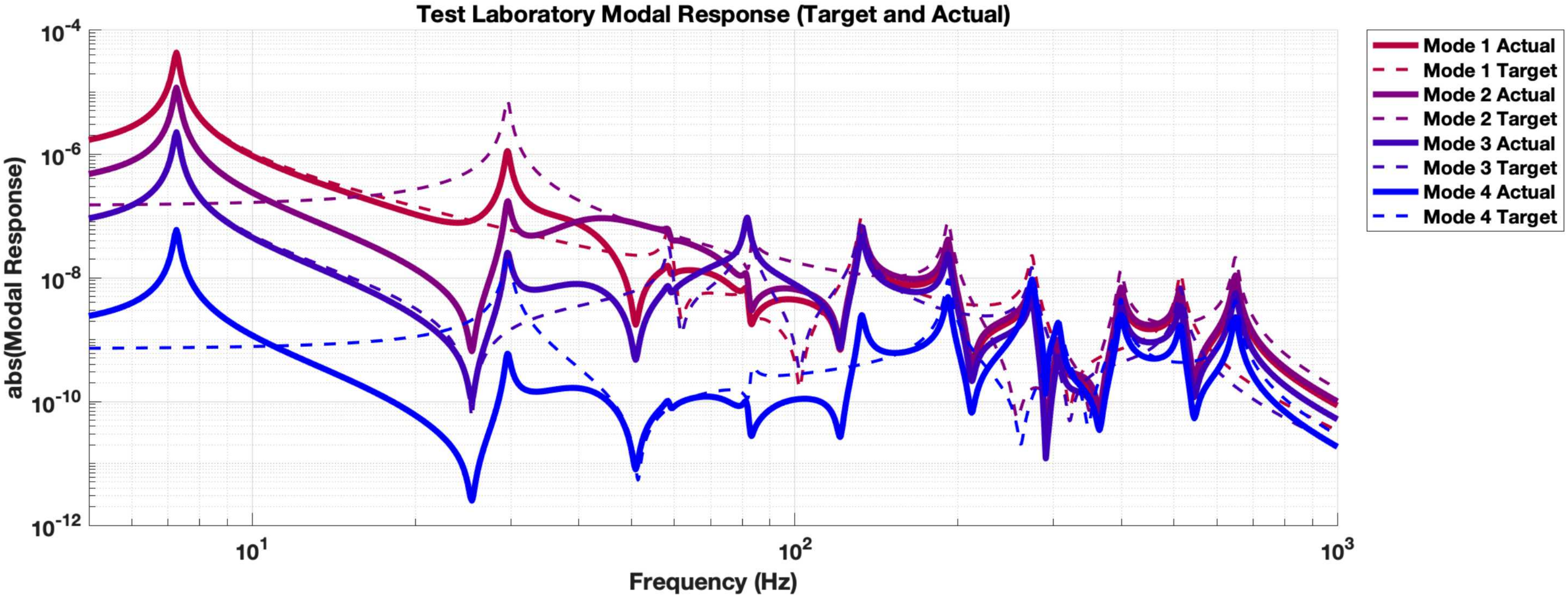
$$f = Z^{\dagger} p$$

Note that you don't have to use all the modes! This allows you to focus your compromised solution onto the dynamics that are important to you.

For example, rigid body modes don't induce strain. You can exclude them from this equation and find a solution that best matches the elastic modes that do induce strain on the device under test.

This same process can be used if you have multiple excitation locations (multiple shakers, 6-dof shaker etc.)

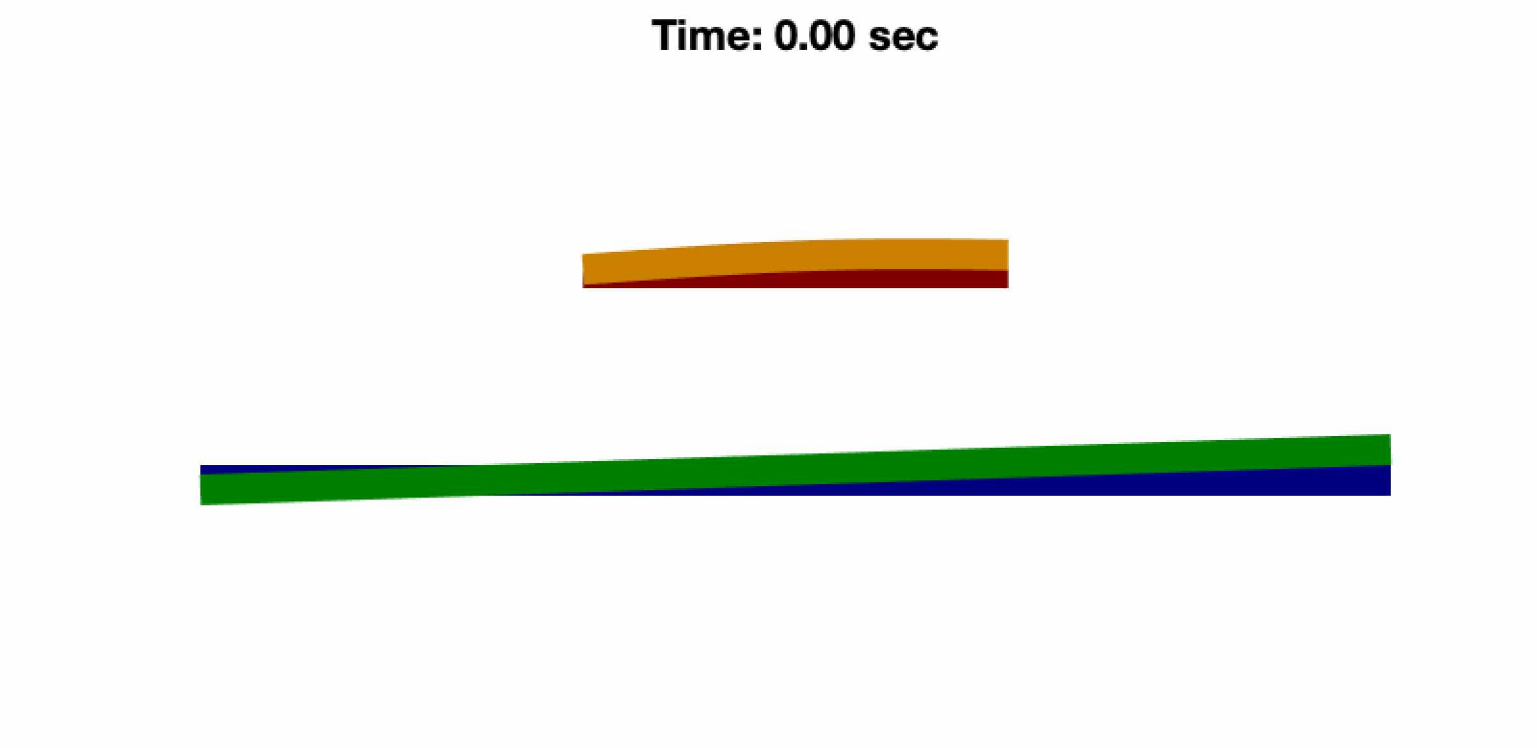
Results



Notice the compromises in trying to match the modal response. You would need four excitation locations to be able to achieve a perfect match on all four modes.

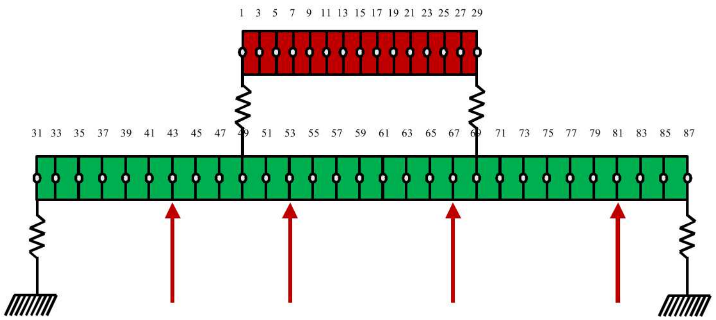
Results

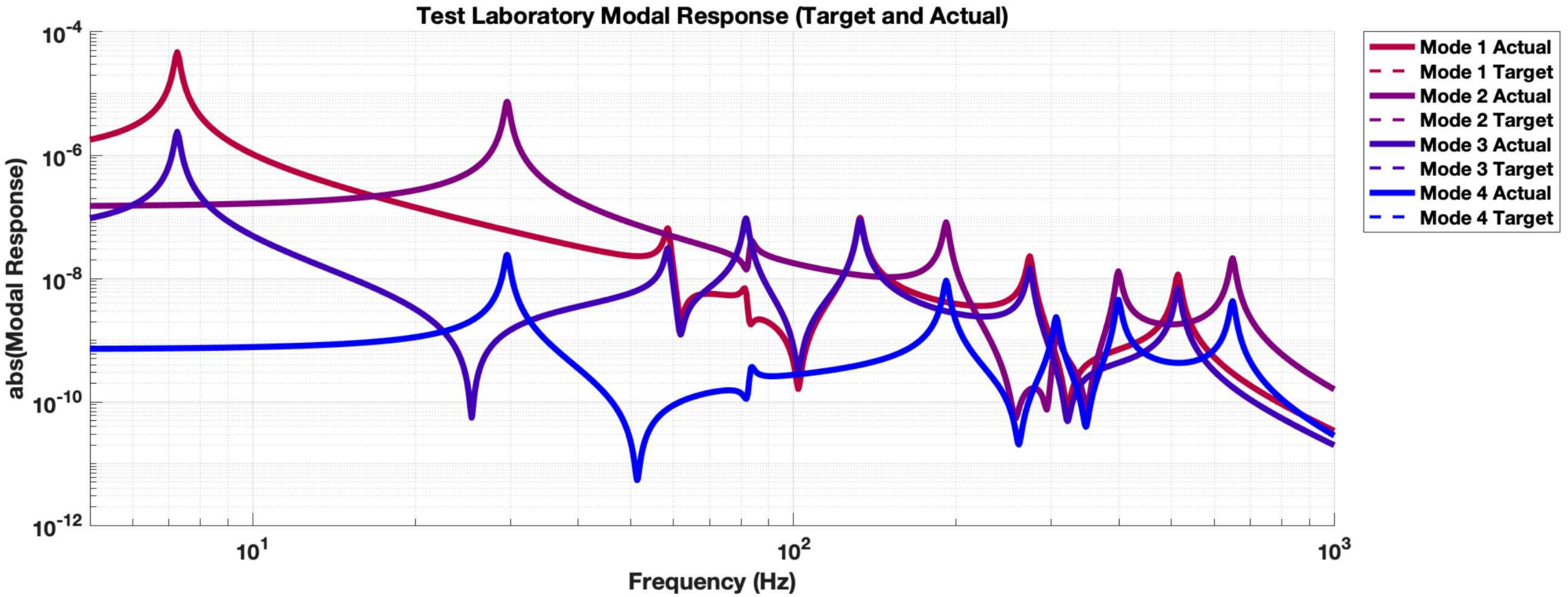
Field Environment and Laboratory Test Response Animated Together



Only one excitation location limits you to a solution that cannot fully match the dynamics of the device under test (top beam)

Results

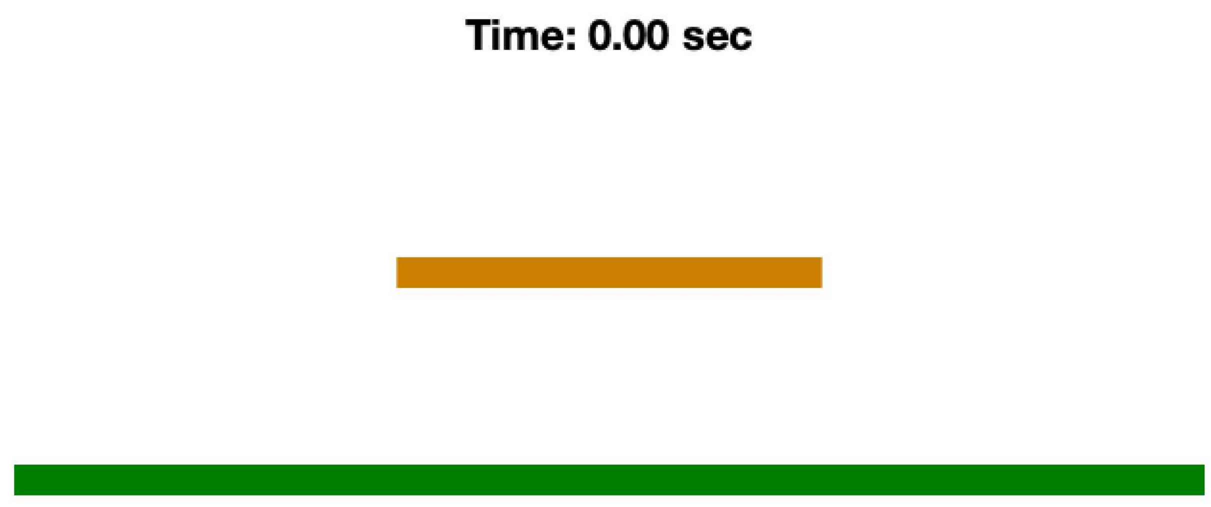




4 input forces allow you to achieve a specific modal response for four modes.

Results

Field Environment and Laboratory Test Response Animated Together



4 input forces allow you to achieve a specific modal response for four modes.

Conclusions

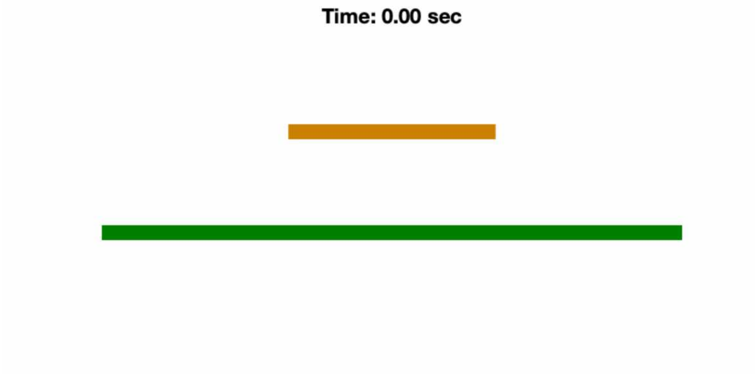
- The transformation process was well behaved and higly efficient using this method
- Few assumptions needed to be made which makes this method suitable to a wide range of configurations
- As with other modal based methods, modal truncation can be a problem if you are not careful

Future Work

- Further work needs to be done to help determine which force input locations are optimal for a given test
- This method needs to be compared to traditional methods for defining laboratory test environments and tested experimentally

Acknowledgements

Sandia National Laboratories provided funding for this research. I am extremely grateful for their support.



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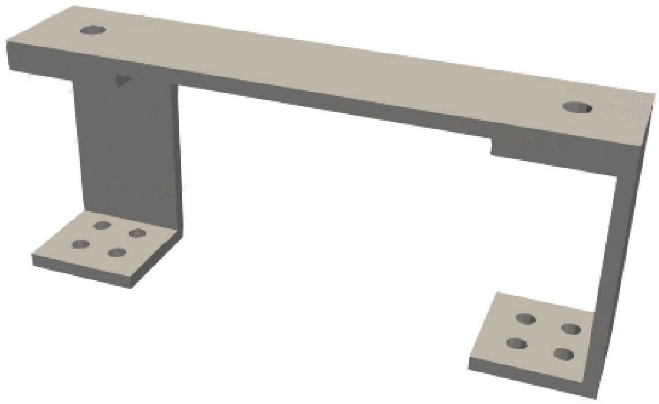
Field Environment



Test Laboratory