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Quantification of Modal Truncation Error with Respect to a Test Fixture's Ability to Replicate a Structural Dynamic Environment

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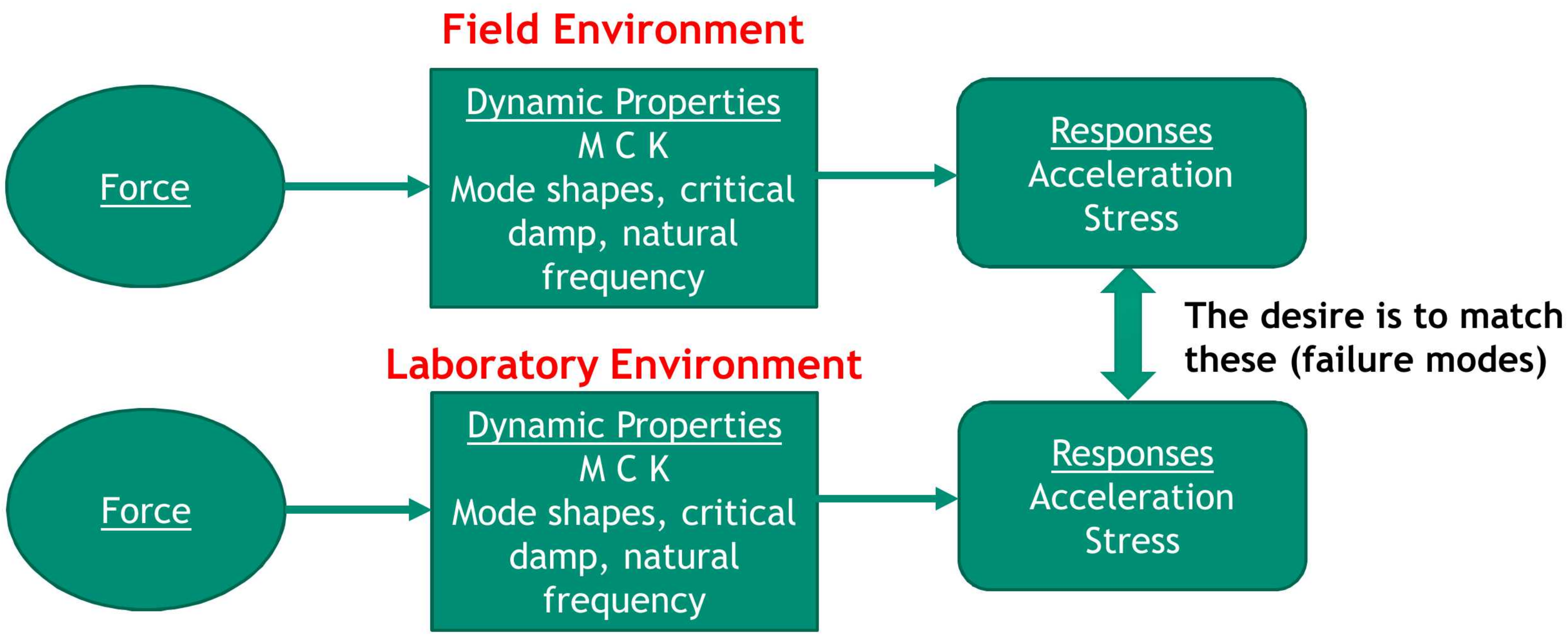
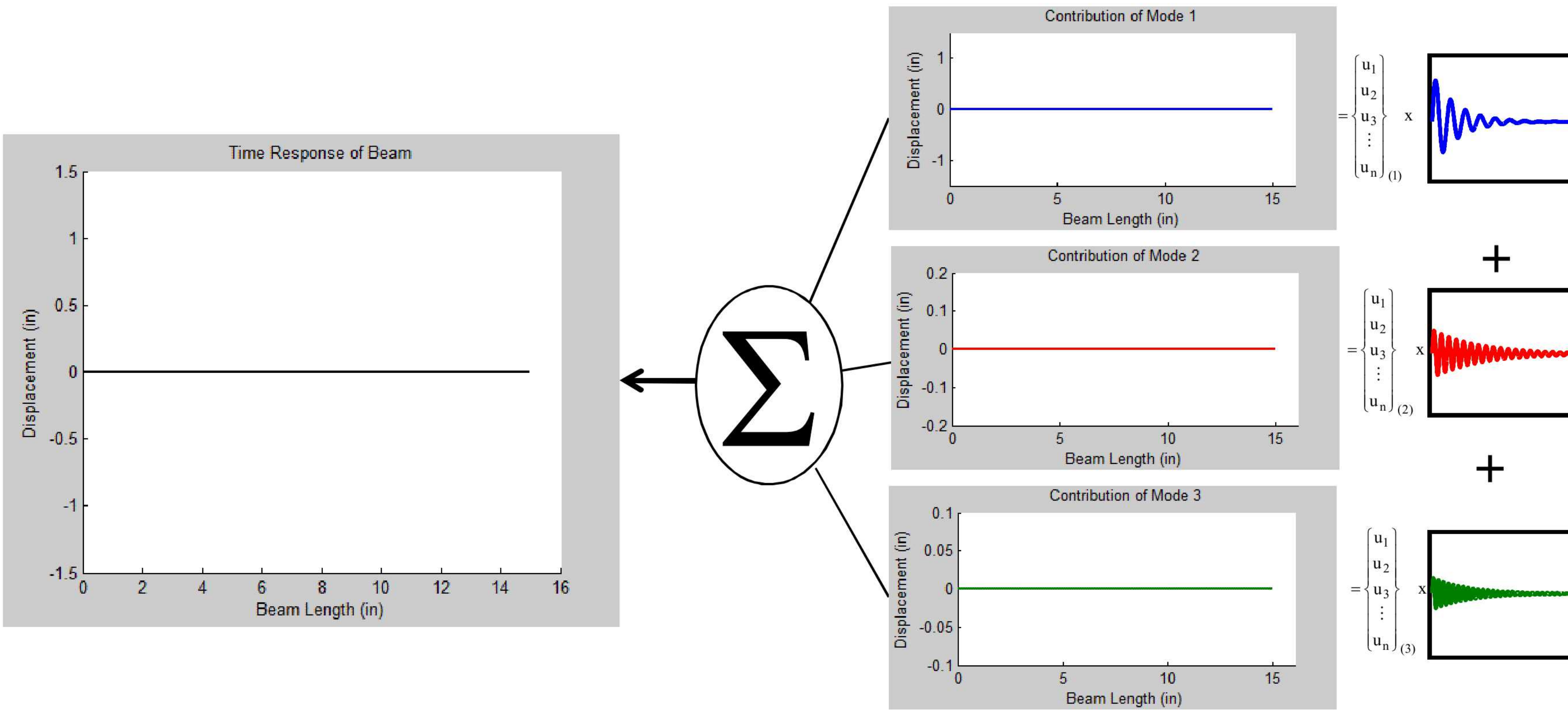
What is needed for Laboratory Testing?

- The goal of the laboratory test is to reproduce the component stresses in the laboratory that were experienced in the field
- How do we know if a test fixture will allow us to observe the desired response of the unit under test? Spoiler alert: We need to look at the component's mode shapes in the field and laboratory configurations



Stress State and Modal Superposition

- Relative displacement in a structure defines the strain and stress in an object.
- The displacement and stress can be calculated as a sum of the modes



Video taken from Ms. Julie Harvie

Derivation of the Modal Projection Error

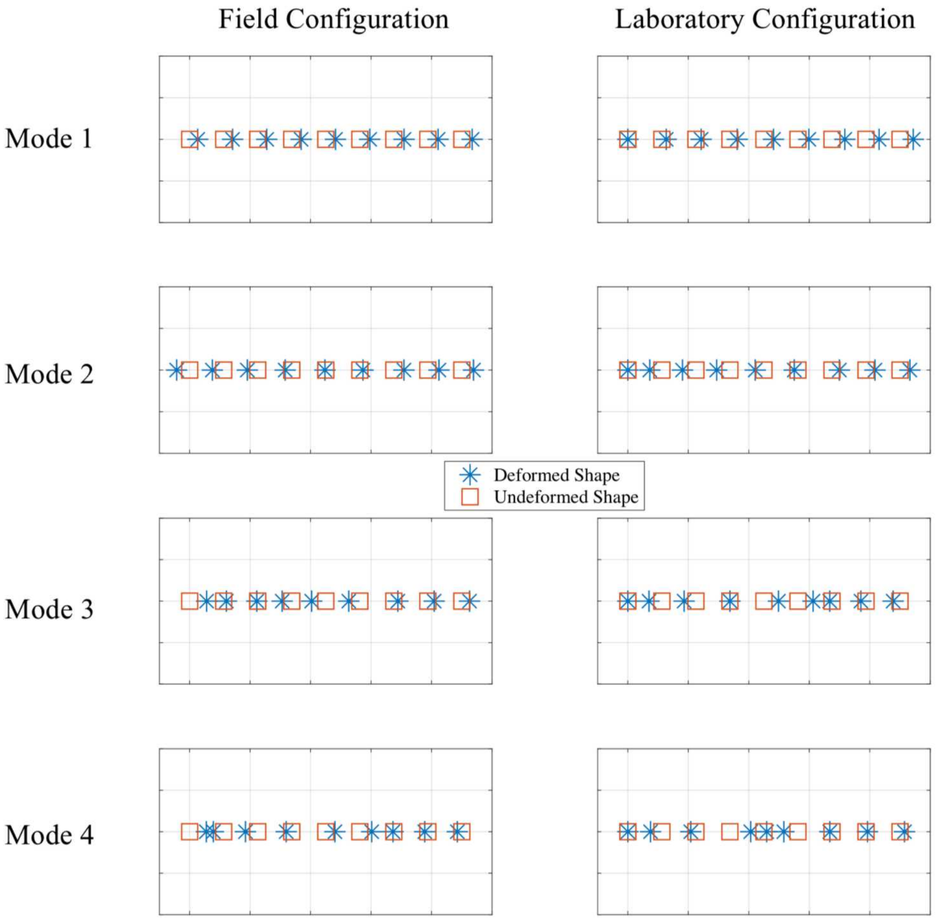
$\bar{x}_L = \bar{x}_F$	(Eq1) We want the displacement field to be the same between the field and test environments
$x_{iL} \approx \sum_{m=1}^{n} \phi_{imL} q_{mL}$	(Eq2) Modal representation of the displacement during the test environment with a finite number of modes
$x_{iF} \approx \sum_{m=1}^{n} \phi_{imF} q_{mF}$	(Eq3) Modal representation of the displacement during the field environment with a finite number of modes
$\phi_L \bar{q}_L = \phi_F \bar{q}_F$	(Eq4) Equation 1 transformed into truncated modal space
$\bar{q}_L = \phi_L^+ \phi_F \bar{q}_F$	(Eq5) With the modal coordinates known from the field, the motion from the field is projected onto the laboratory mode shape space and the lab modal coordinates are calculated in a least squared solution.

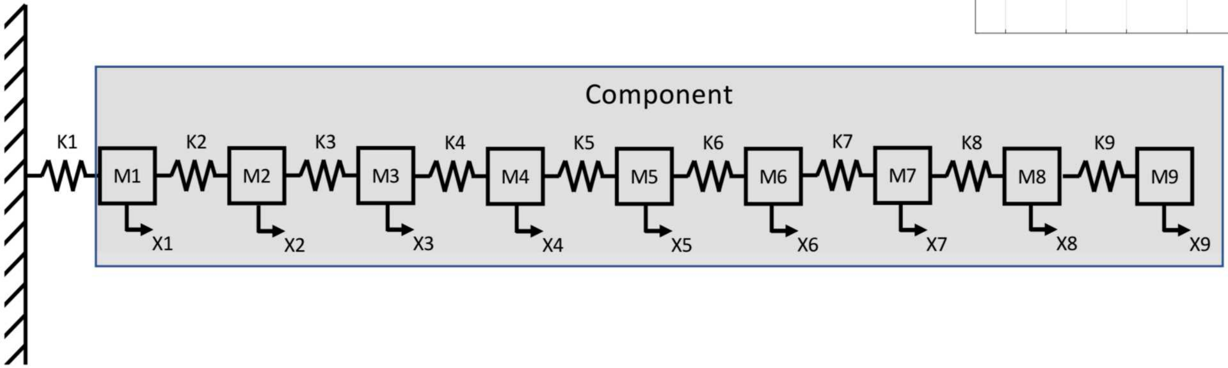
Derivation of the Modal Projection Error

$$\bar{q}_L = \boldsymbol{\phi}_L^{+} \bar{\phi}_{Fn} q_{Fn}$$	(Eq6) It is of interest to determine the error of reconstructing each field mode individually. The modal coordinates for the lab are calculated in a least squared sense.
$$\bar{\phi}_{Fn}^{+} \boldsymbol{\phi}_L \bar{q}_L = \tilde{q}_{Fn}$$ $$\bar{\phi}_{Fn}^{+} \boldsymbol{\phi}_L \boldsymbol{\phi}_L^{+} \bar{\phi}_{Fn} q_{Fn} = \tilde{q}_{Fn}$$	(Eq7) With the lab modal coordinates calculated from Eq6, the coordinates are projected back onto the space of the field environment. A reconstructed field modal coordinate is calculated.
$$\Gamma_n^2 = \frac{\tilde{q}_{Fn}}{q_{Fn}} = \bar{\phi}_{Fn}^{+} \boldsymbol{\phi}_L \boldsymbol{\phi}_L^{+} \bar{\phi}_{Fn}$$ $$\Psi_n^2 = 1 - \Gamma_n^2$$	(Eq8) The ratio between the reconstructed field coordinate and the original field coordinate is calculated and that can be used to define the modal error term. The error is squared because two projections took place to obtain the value.

Case Study - Setup

- A nine DOF component was connected to ground through a soft spring (field configuration) and a very stiff spring (lab configuration)
- How well do the laboratory mode shapes project to reconstruct the field mode shapes?





Case Study - Error Terms

- The laboratory modal coordinates were calculated to be

	Field Mode 1	Field Mode 2	Field Mode 3	Field Mode 4
Lab Mode 1	-0.93	-0.36	-0.10	-0.05
Lab Mode 2	0.24	-0.79	0.53	0.18
Lab Mode 3	0.08	-0.16	-0.56	0.80
Lab Mode 4	0.03	-0.05	-0.09	-0.10

- The error term per mode was calculated to be

Modal Truncation Error Term			
Mode 1	Mode 2	Mode 3	Mode 4
7.75%	21.1%	38.6%	30.5%

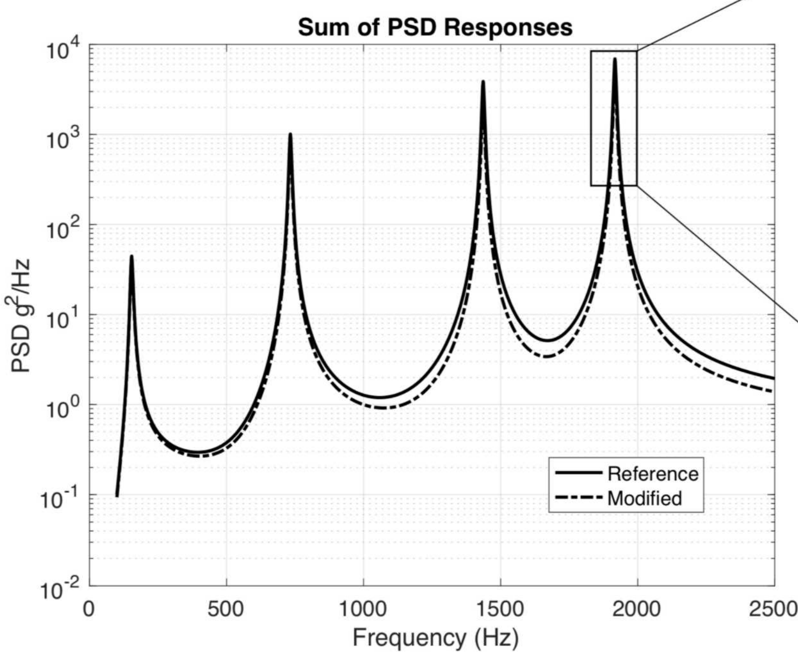
Case Study - Results

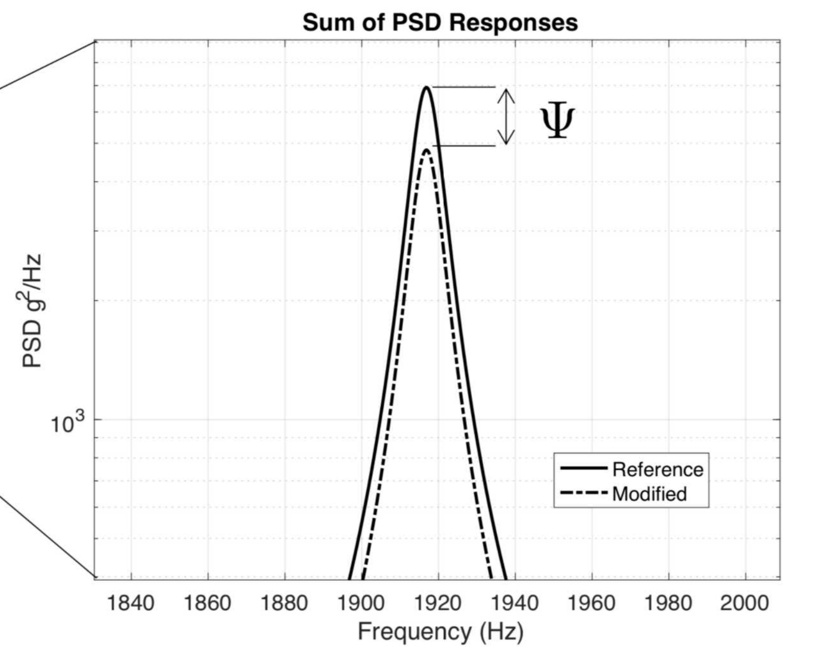
- A forcing function was applied to the field configuration and the response was calculated.

$$\mathbf{G_{xx\_field}} = \mathbf{H_F} \cdot \mathbf{G_{ff\_field}} \cdot \mathbf{H_F}^*$$

- An ideal forcing function was calculated for the laboratory configuration and the response was calculated. The sum of the PSD responses was calculated for the field and lab and the difference was equal to the modal truncation error.

$$\mathbf{G_{ff\_lab}} = [\mathbf{H_L}^*]^{-1} \cdot \mathbf{G_{xx\_field}} \cdot \mathbf{H_L}^{-1}$$





Summary

- This method of quantifying error is not limited to fixture design. It is applicable to model expansion and reduction, component mode synthesis substructuring, and load identification techniques such as SWAT. Operational deflection shapes can also be used.
- Linear combination of the mode shapes of the laboratory configuration will define the stress state of the laboratory environment.
- The error term is an inexpensive calculation that only requires knowledge of the component mode shapes in the field and laboratory configurations
- The error term can be used as a guide to determine how effective a test fixture is. The error term provides a lower bound on error for a laboratory test.
- Questions?