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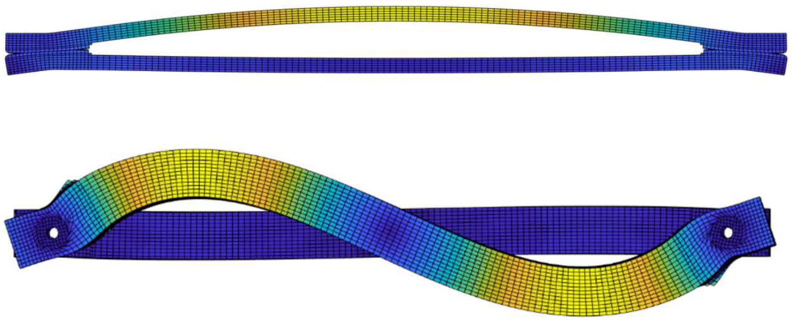

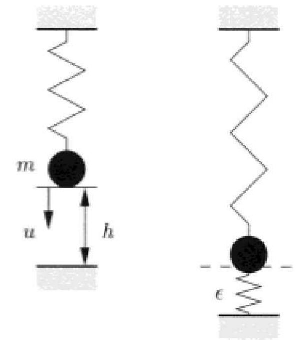

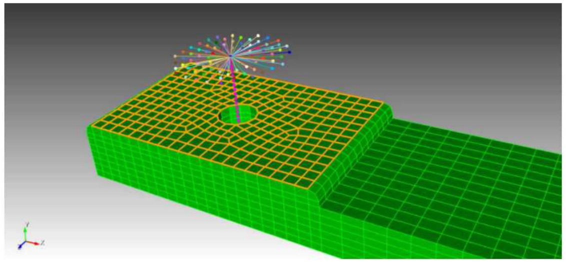

Reduction of Nonlinear Degrees of Freedom in Jointed Hurty/Craig-Bampton Substructures

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Agenda

- Motivation
- Theory overview: system-level interface reduction vs. proposed method
- Modal basis
  - Joint interface (JI) modes
  - Approximate residual interface mode contributions (ARIMCs)
  - Trial vector derivatives (TVDs)
- Results of transient simulations (full interface vs. reduced interface)
- Computational savings
- Conclusions & future work

Motivation

- Inter-component contact strongly influences system-level stiffness and damping
- Modern finite element models with contacting parts are too computationally demanding for most machines
  - Accurate prediction of interface forces requires a highly refined mesh at the contact area(s)
- Many interface reduction techniques only apply to linear systems (rigidly-connected interfaces)
- Nonlinear interface reduction methods:
  - often require transformation between full model & reduced model
  - are usually not concerned with capturing local interface kinematics
- **Proposed solution:** reduce **non-interface DOF** with HCB transformation + reduce **interface DOF** w/ state-of-the-art interface modes

Theory Overview

- Hurty/Craig-Bampton (HCB) transformation

$$\begin{Bmatrix} \mathbf{u}_{\text{interior}} \\ \mathbf{u}_{\text{interface}} \end{Bmatrix} \xrightarrow{\text{HCB}} \begin{Bmatrix} \mathbf{q}_{\text{interior}} \\ \mathbf{u}_{\text{interface}} \end{Bmatrix}$$

(full) **(HCB)**

- Linear interface reduction (LIR)

$$\begin{Bmatrix} \mathbf{q}_{\text{interior}} \\ \mathbf{u}_{\text{interface}} \end{Bmatrix} \xrightarrow{\text{LIR}} \begin{Bmatrix} \mathbf{q}_{\text{interior}} \\ \mathbf{q}_{\text{interface}} \end{Bmatrix}$$

(HCB) **(LIR)**

- Proposed method: nonlinear interface reduction (NLIR)

$$\begin{Bmatrix} \mathbf{q}_{\text{interior}} \\ \mathbf{u}_{\text{interface}} \end{Bmatrix} \xrightarrow{\text{re-partition}} \begin{Bmatrix} \mathbf{q}_{\text{interior}} \\ \mathbf{u}_{\text{interface-red}} \\ \mathbf{u}_{\text{interface-phys}} \end{Bmatrix} \xrightarrow{\text{NLIR}} \begin{Bmatrix} \mathbf{q}_{\text{interior}} \\ \mathbf{q}_{\text{interface-red}} \\ \mathbf{u}_{\text{interface-phys}} \\ \mathbf{q}_{\text{TVD}} \\ \mathbf{q}_{\text{preload}} \end{Bmatrix}$$

(HCB) **(HCB)** **(NLIR)**

- **Interior** DOF unchanged (still modal)
- **Interface** DOF split into **reduced** and **physical** partitions
  - **Reduced** partition approximated using **joint interface (JI) modes** and **approximate residual interface (ARI) modes**
  - **Physical** partition unchanged
- **Trial vector derivatives (TVDs)** and **preload mode** included in reduction space

Modal Basis

$$\mathbf{u}_{\mathrm{HCB}} = \mathbf{T}^{\mathrm{NLIR}}\mathbf{u}_{\mathrm{NLIR}}$$

$$\mathbf{u}_{\mathrm{HCB}} = \left[\left(\mathbf{T}^{\mathrm{JI}} + \mathbf{T}^{\mathrm{ARI}}\right) \quad \mathbf{\Theta}^{\mathrm{TVD}} \quad \mathbf{u}^{\mathrm{PL}}\right]\mathbf{u}_{\mathrm{NLIR}}$$

$$\begin{Bmatrix} \mathbf{q}_{\mathrm{i}} \\ \mathbf{u}_{\mathrm{r}} \\ \mathbf{u}_{\mathrm{p}} \end{Bmatrix} = \left[\left(\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}^{\mathrm{JI}} & \mathbf{\Psi}^{\mathrm{JI}} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{\Phi}_i^{\mathrm{ARI}} & \mathbf{\Phi}_r^{\mathrm{ARI}} & \mathbf{\Phi}_p^{\mathrm{ARI}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}\right) \quad \mathbf{\Theta}^{\mathrm{TVD}} \quad \begin{matrix} \mathbf{u}_i^{\mathrm{PL}} \\ \mathbf{u}_r^{\mathrm{PL}} \\ \mathbf{u}_p^{\mathrm{PL}} \end{matrix}\right] \begin{Bmatrix} \mathbf{q}_{\mathrm{i}} \\ \mathbf{q}_{\mathrm{r}} \\ \mathbf{u}_{\mathrm{p}} \\ \mathbf{q}_{\mathrm{t}} \\ q_{\mathrm{PL}} \end{Bmatrix}$$

n_{HCB} x 1 | n_{HCB} x $(n_{JI} + n_p)$ | n_{HCB} x $(n_{JI} + n_p)$ | n_{HCB} x n_{TVD} | $(n_{JI} + n_p + n_{HCB} + 1)$ x 1

n_{HCB} x 1

n_{HCB} x $(n_{JI} + n_p + n_{TVD} + 1)$

JI modes, ARI modes, and TVDs

Joint Interface (JI) Modes

1. Apply Newton's Second Law at the interface:

$$\mathbf{f}_{r_1}^{\mathrm{HCB}} = \mathbf{f}_{r_2}^{\mathrm{HCB}}$$

2. Compute eigenmodes ($\boldsymbol{\Phi}^{\mathrm{JI}}$) & constraint modes ($\boldsymbol{\Psi}^{\mathrm{JI}}$) in the system constrained by (1).

5. Assemble:

$$\mathrm{T}^{\mathrm{JI}} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}^{\mathrm{JI}} & \boldsymbol{\Psi}^{\mathrm{JI}} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$

Approximate Residual Interface (ARI) Modes

1. Compute the static residual flexibility matrix:

$$\mathbf{F}^{\mathrm{RS}} = \left(\mathbf{K}_{rr}^{\mathrm{HCB}}\right)^{-1} - \boldsymbol{\Phi}^{\mathrm{JI}}\left(\boldsymbol{\Lambda}^{\mathrm{JI}}\right)^{-1}\left(\boldsymbol{\Phi}^{\mathrm{JI}}\right)^{\mathrm{T}}$$

2. Compute the ARI matrix:

$$\mathrm{T}^{\mathrm{ARI}^*} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{F}^{\mathrm{RS}}\mathbf{M}_{ri}^{\mathrm{HCB}} & \mathbf{0} & \mathbf{F}^{\mathrm{RS}}\left(\mathbf{M}_{rp}^{\mathrm{HCB}} + \mathbf{M}_{rr}^{\mathrm{HCB}}\boldsymbol{\Psi}^{\mathrm{JI}}\right) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \times \left[\left(\tilde{\mathrm{T}}^{\mathrm{JI}}\right)^{\mathrm{T}}\mathbf{M}^{\mathrm{HCB}}\tilde{\mathrm{T}}^{\mathrm{JI}}\right]^{-1}\left[\left(\tilde{\mathrm{T}}^{\mathrm{JI}}\right)^{\mathrm{T}}\mathbf{K}^{\mathrm{HCB}}\tilde{\mathrm{T}}^{\mathrm{JI}}\right] = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\Phi}_i^{\mathrm{ARI}} & \boldsymbol{\Phi}_r^{\mathrm{ARI}} & \boldsymbol{\Phi}_p^{\mathrm{ARI}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

($\tilde{\mathrm{T}}^{\mathrm{JI}}$ contains dominant and residual JI eigenmodes)

Trial Vector Derivatives (TVDs)

1. Compute the initial TVDs & assemble:

$$\boldsymbol{\theta}_{j,\mathrm{PL}}^{\mathrm{V}^*} = \frac{\partial \boldsymbol{\phi}_j^{\mathrm{JI}}}{\partial \mathrm{q}_{\mathrm{PL}}} \quad ; \quad \theta_{\mathrm{PL},j}^{\mathrm{V}} = \frac{\partial \mathbf{u}_r^{\mathrm{PL}}}{\partial \mathrm{q}_{rj}}$$

$$\boldsymbol{\theta}_{j,\mathrm{PL}}^{\mathrm{S}^*} = \frac{\partial \boldsymbol{\psi}_j^{\mathrm{JI}}}{\partial \mathrm{q}_{\mathrm{PL}}} \quad ; \quad \theta_{\mathrm{PL},j}^{\mathrm{V}} = \frac{\partial \mathbf{u}_r^{\mathrm{PL}}}{\partial \mathrm{u}_{pj}}$$

$$\boldsymbol{\Theta}^{\mathrm{TVD}^*} = \left[\boldsymbol{\Theta}^{\mathrm{V}^*} \quad \boldsymbol{\Theta}^{\mathrm{S}^*}\right]$$

2. Orthogonalize & reassemble:

$$\left[\left(\boldsymbol{\Theta}^{\mathrm{TVD}^*}\right)^{\mathrm{T}}\mathbf{K}_{rr}^{\mathrm{cHCB}}\boldsymbol{\Theta}^{\mathrm{TVD}^*}\right]\boldsymbol{\chi}_j = \lambda_j\boldsymbol{\chi}_j$$

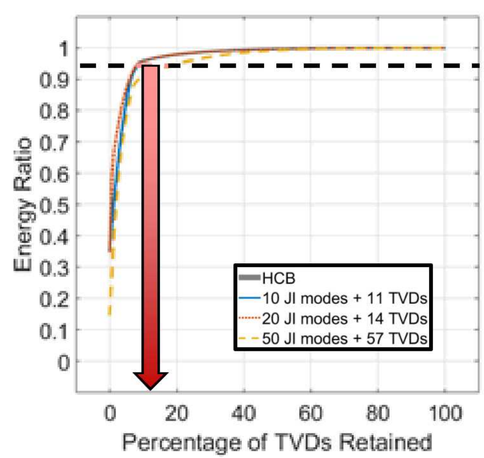
($\mathbf{K}_{rr}^{\mathrm{cHCB}}$ contains contact stiffness contributions)

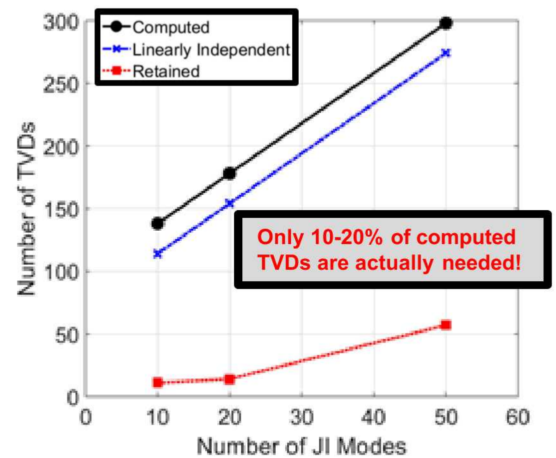
$$\boldsymbol{\theta}_j^{\mathrm{TVD}} = \lambda_j^{-0.5}\boldsymbol{\Theta}^{\mathrm{TVD}^*}\boldsymbol{\chi}_j$$

$$\boldsymbol{\Theta}^{\mathrm{TVD}} = \left[\boldsymbol{\theta}_1^{\mathrm{TVD}} \quad \boldsymbol{\theta}_2^{\mathrm{TVD}} \quad \dots \quad \boldsymbol{\theta}_{n_{\mathrm{TVD}}}^{\mathrm{TVD}}\right]$$

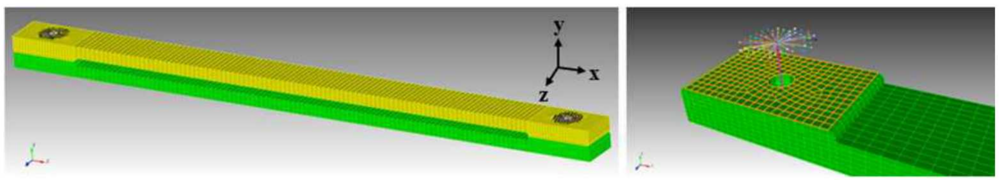
How many TVDs should be kept?

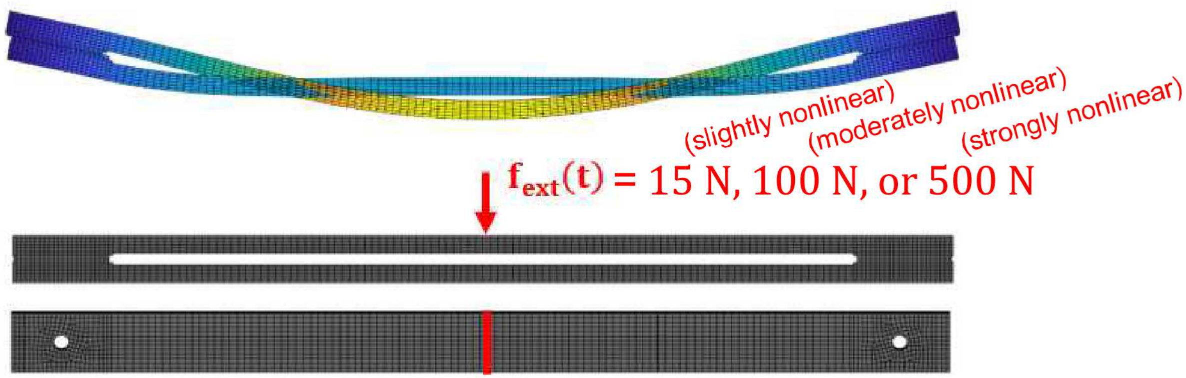
$$\text{Energy Ratio} = \frac{\sum_{j=1}^{n_{\mathrm{ret}}}\lambda_j}{\sum_{j=1}^{n_{\mathrm{TVD}}}\lambda_j} = \frac{\{\text{retained deformation energy}\}}{\{\text{total deformation energy}\}}$$



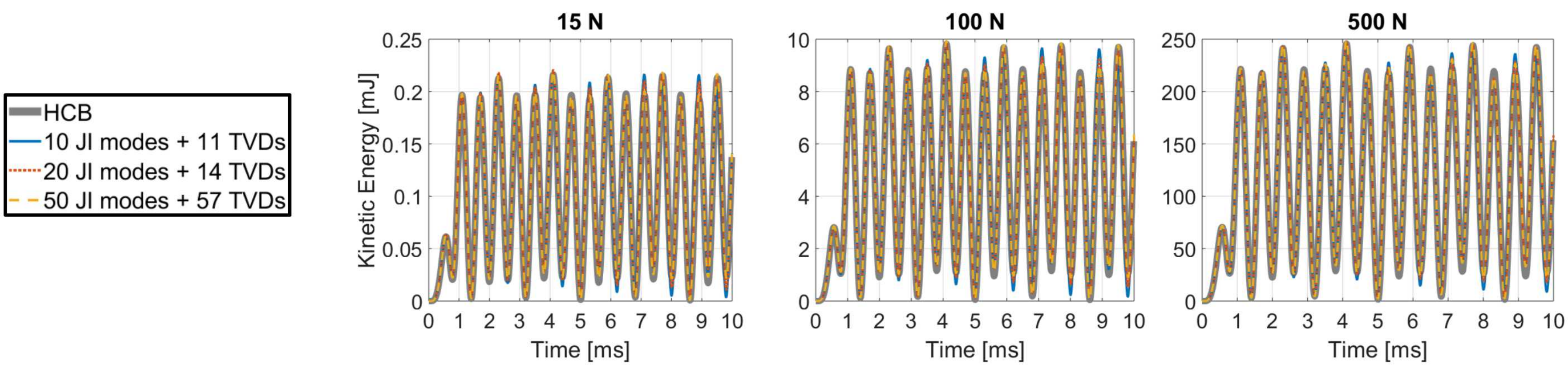


Application Example

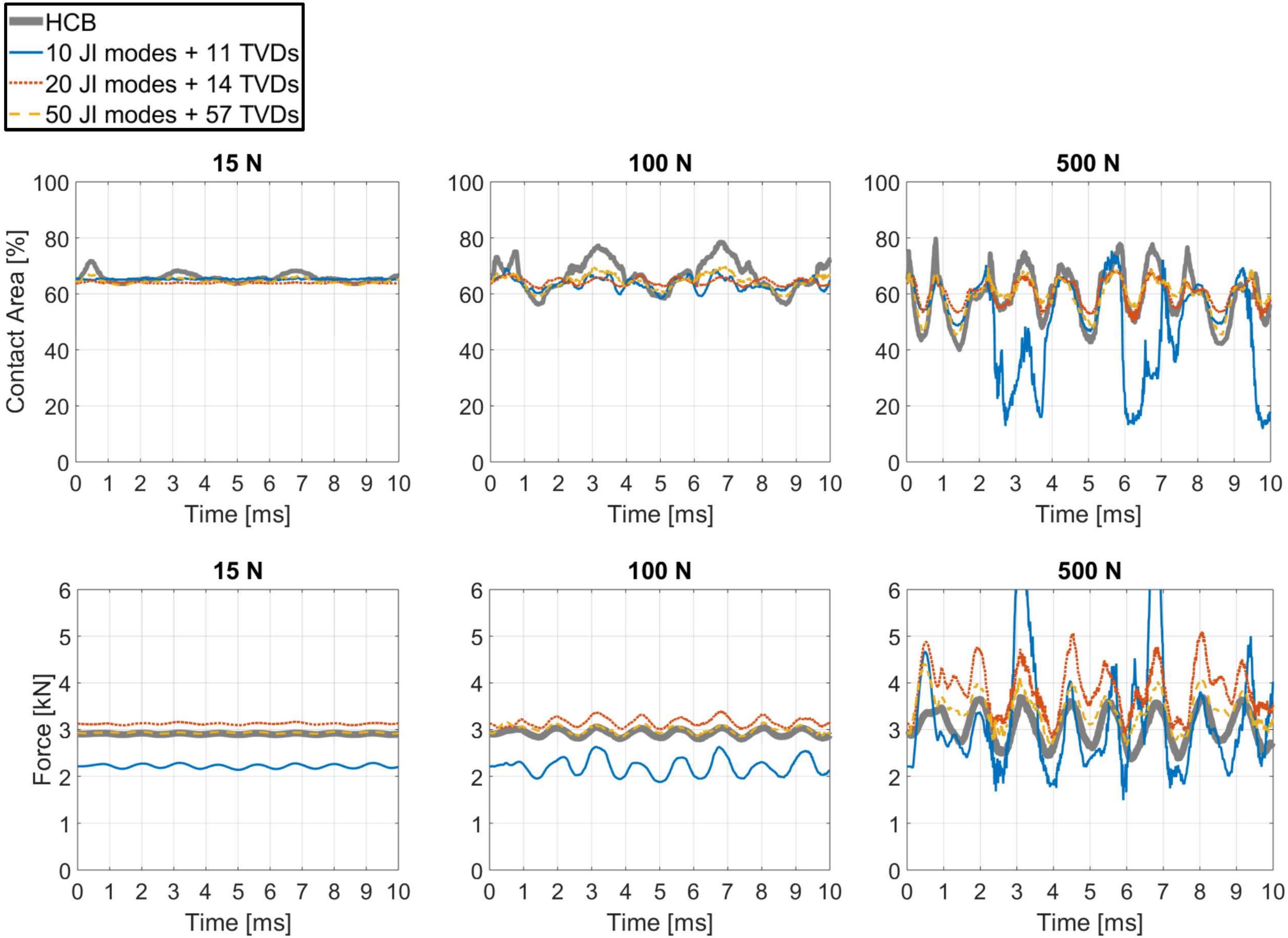




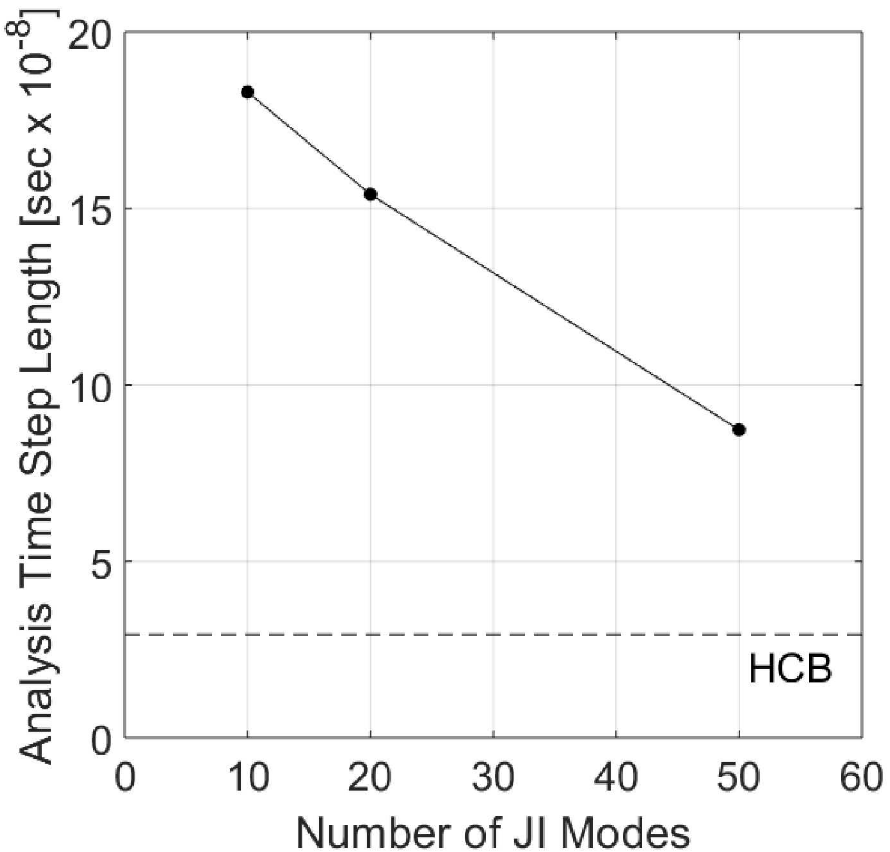
- Contact
  - Node-to-node penalty springs
  - Frictionless
- Full model:
  - 90,560 interior DOF
  - 3,684 interface DOF
- HCB model:
  - 16 fixed-interface modes
  - 3,684 interface DOF
- 1 ms haversine impulse
- Chung-Lee integration (central difference + numerical damping)

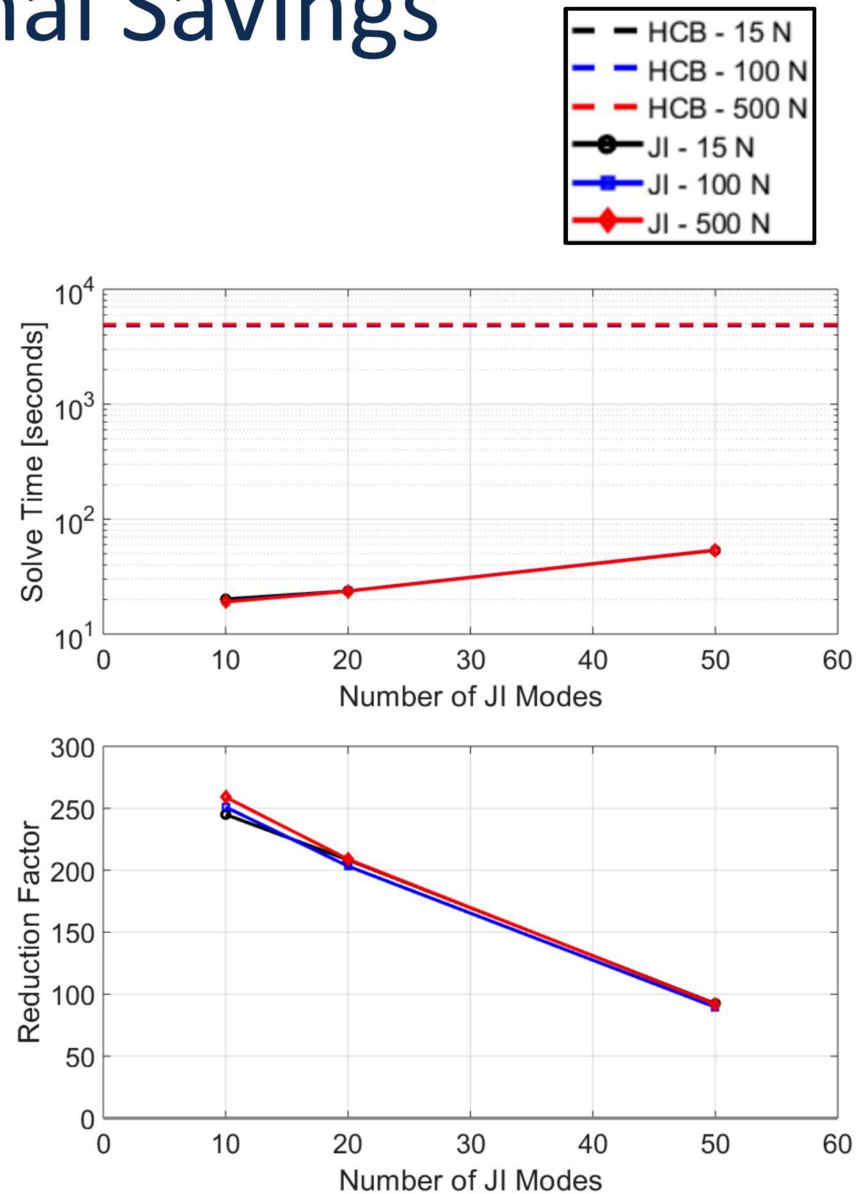


Dynamic Results: Local Response



Online Computational Savings





Conclusions & Future Work

- Conclusions
  - NLIR method captures global and local response with ~100 DOF
    - <5% of original 3700 DOF
    - Will be more difficult to obtain when friction is included
  - Critical timestep length increased by factor of 3
  - Simulations times reduced by factor of 100
  - Viable option when transformation between to full-order model is not feasible

- Future Work
  - Incorporate friction at contact surfaces
  - Examine application examples where friction is not a significant factor (e.g. normal impact)
  - Consider other dynamic loading cases (e.g. loading to excite other modes)

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