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Hierarchical Material Properties in Finite Element Analysis: Application to Oilfield Situation Awareness

PRESENTED BY

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THE PAIN OF VOLUMETRIC DISCRETIZATION

Example problem: discretization of steel casing in an oil well

0.2 m outer diameter, 0.025 m wall thickness, electrical conductivity 5e6 S/m

regular tet with edge length 0.025 m occupies a volume $(0.025\ \text{m})^3 / (6\sqrt{2}) = 1.84\text{e-}6\ \text{m}^3$

1 km of casing requires 7.4e6 tets

Over a 1 km^3 Earth model discretized at, say 10 m, 7.4/(7.4 + 8.5)*100% = 46.5% of the tets are devoted to 0.0000014% of the mesh volume.

This is **computationally explosive**, especially for realistic oilfield settings where there are 10s of km of steel casing + surface pipelines + storage tanks + electric cable + …

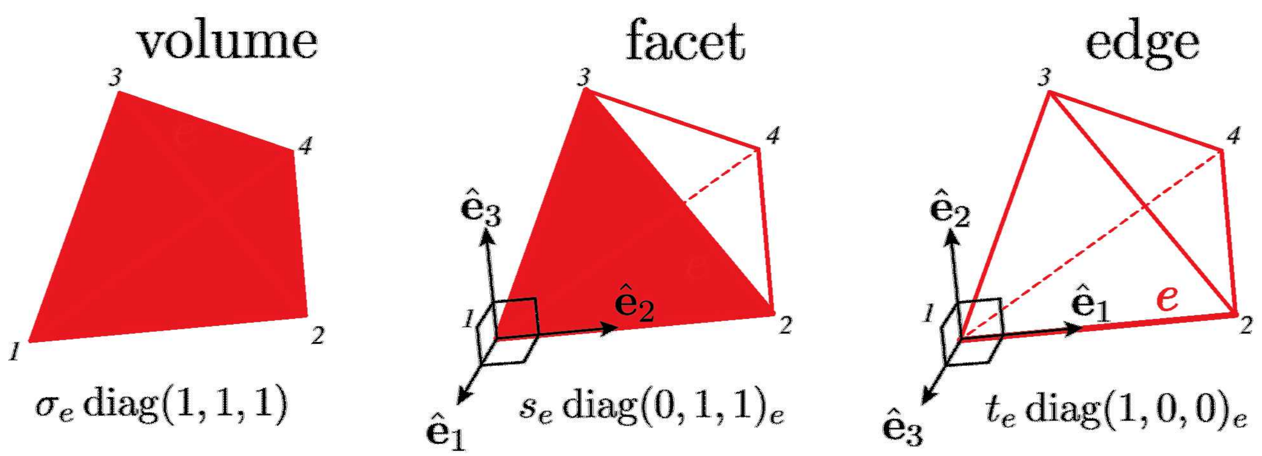
Typical approaches to the problem are

- specialized algorithms for parallel compute architectures (Commer et al., 2015, Hoversten et al., 2015, Um et al., 2015)
- Discretization of slightly "fatter" casing, whose large size reduces the element count with an acceptable reduction in accuracy (Haber et al., 2016; Weiss et al., 2016).

A NEW HOPE

Hanging the material properties on the tets, faces and edges of the unstructured tetrahedral mesh allows for thin conductors to be economically represented by facets and edges, rather than 100s of millions of tiny tets.

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) + \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) + \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$$



HIERARCHICAL BASIS FUNCTIONS FOR MATERIALS

This hierarchy of material distributions is made possible by using rank-2 tensor basis functions – an extension of the early work in 2D anisotropy by Weiss and Newman (Geophysics, 2002, 2003)

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) + \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) + \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$$

$$\boldsymbol{\psi}_e^V(\boldsymbol{x}) = \mathrm{diag}(1,1,1) \begin{cases} 1 & \text{if } \boldsymbol{x} \in \text{volume } e \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{\psi}_e^F(\boldsymbol{x}) = \mathrm{diag}(0,1,1)_e \begin{cases} 1 & \text{if } \boldsymbol{x} \in \text{facet } e \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{\psi}_e^E(\boldsymbol{x}) = \mathrm{diag}(1,0,0)_e \begin{cases} 1 & \text{if } \boldsymbol{x} \in \text{edge } e \\ 0 & \text{otherwise} \end{cases}$$

The tensor representation keeps the material properties local to the edges and facets in the Finite Element weak formulation / bilinear form.

TRY IT OUT WITH ELECTRO/MAGNETOSTATICS

Poisson Eq for electro/magnetostatics

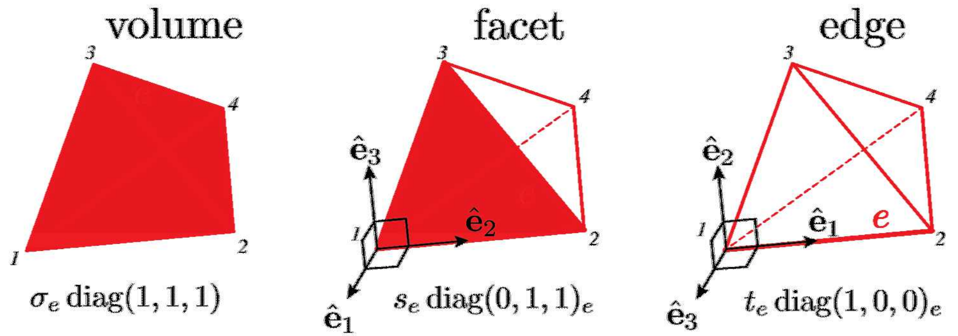
$$-\nabla \cdot (\boldsymbol{\sigma} \cdot \nabla u) = f \qquad \int_\Omega \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u)\ dx^3 = \int_\Omega vf\ dx^3$$

Sparse anisotropic conductivity collapses 3D gradients to 2D and 1D gradients…

$$\boldsymbol{\sigma} = \mathrm{diag}(0, \sigma, \sigma) \qquad \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u) = \sigma \nabla_{23} v \cdot \nabla_{23} u$$

$$\boldsymbol{\sigma} = \mathrm{diag}(\sigma, 0, 0) \qquad \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u) = \sigma \nabla_1 v \cdot \nabla_1 u$$

…thus ensuring that the facet and edge material properties are local and not distributed over the tetrahedral volume.



ASSEMBLY AND SOLUTION OF THE LINEAR SYSTEM

Variational formulation:

$$\int_\Omega \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u)\ dx^3 = \int_\Omega vf\ dx^3$$

Hierarchical model:

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) + \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) + \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$$

3D inner products collapse to 2D and 1D inner products

$$\int_\Omega \nabla v \cdot \left[\sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x})\right] \nabla u\ dx^3 = \sum_{e=1}^{N_V} \sigma_e \int_{V_e} \nabla v \cdot \nabla u\ dx^3 = \sum_{e=1}^{N_V} \sigma_e\, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^4 \mathbf{u}_e$$

$$\int_\Omega \nabla v \cdot \left[\sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x})\right] \nabla u\ dx^3 = \sum_{e=1}^{N_F} s_e \int_{F_e} \nabla_{23} v \cdot \nabla_{23} u\ dx^2 = \sum_{e=1}^{N_F} s_e\, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^3 \mathbf{u}_e$$

$$\int_\Omega \nabla v \cdot \left[\sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})\right] \nabla u\ dx^3 = \sum_{e=1}^{N_E} t_e \int_{E_e} \nabla_1 v \cdot \nabla_1 u\ dx = \sum_{e=1}^{N_E} t_e\, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^2 \mathbf{u}_e$$

Global stiffness matrix is a sum of 3D, 2D and 1D element stiffness matrices.

$$\mathbf{K}\mathbf{u} = \mathbf{b}$$

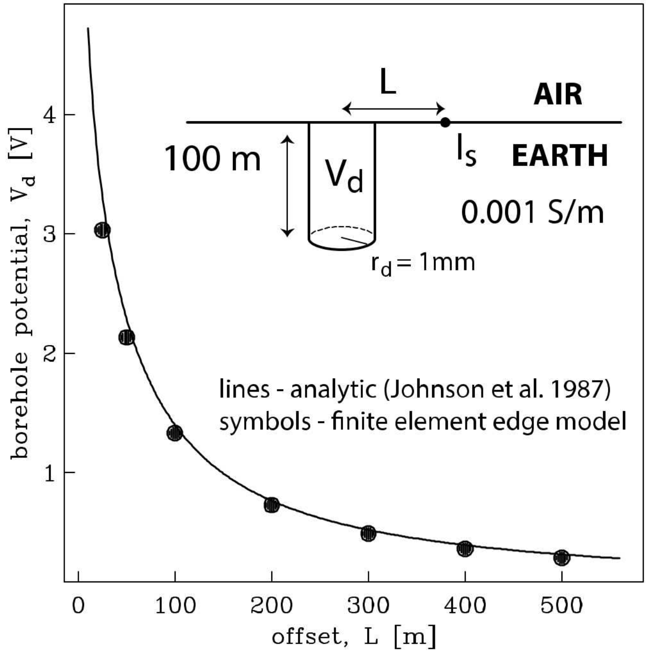
$$\mathbf{K} = \sum_{e=1}^{N_V} \sigma_e \mathbf{K}_e^4 + \sum_{e=1}^{N_F} s_e \mathbf{K}_e^3 + \sum_{e=1}^{N_E} t_e \mathbf{K}_e^2$$

Solve iteratively with Jacobi scaled conjugate gradients and on-the-fly matrix assembly (Weiss, 2001)

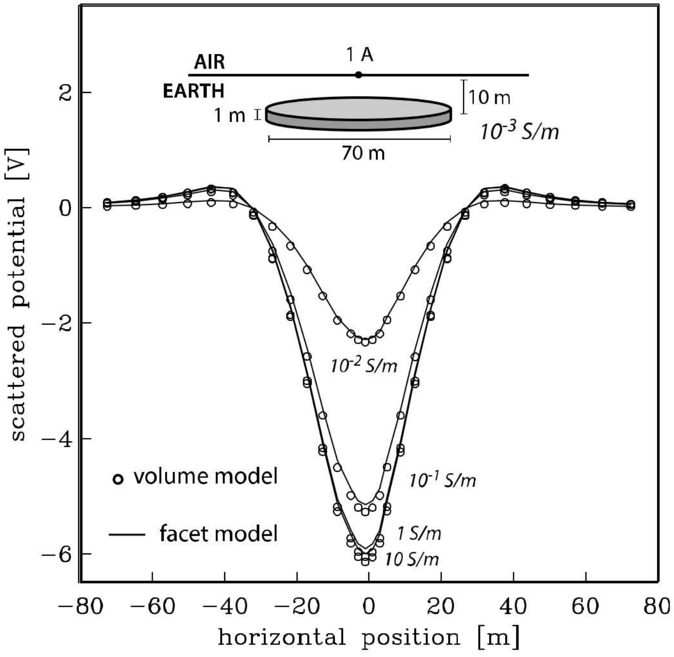
BENCHMARKING AND INTERNAL CONSISTENCY

Benchmarking and internal consistency checks show that for thin conductors, the facet/edge representation achieves acceptable accuracy over a range of geometries and material properties.

Thin Cylinder Benchmark



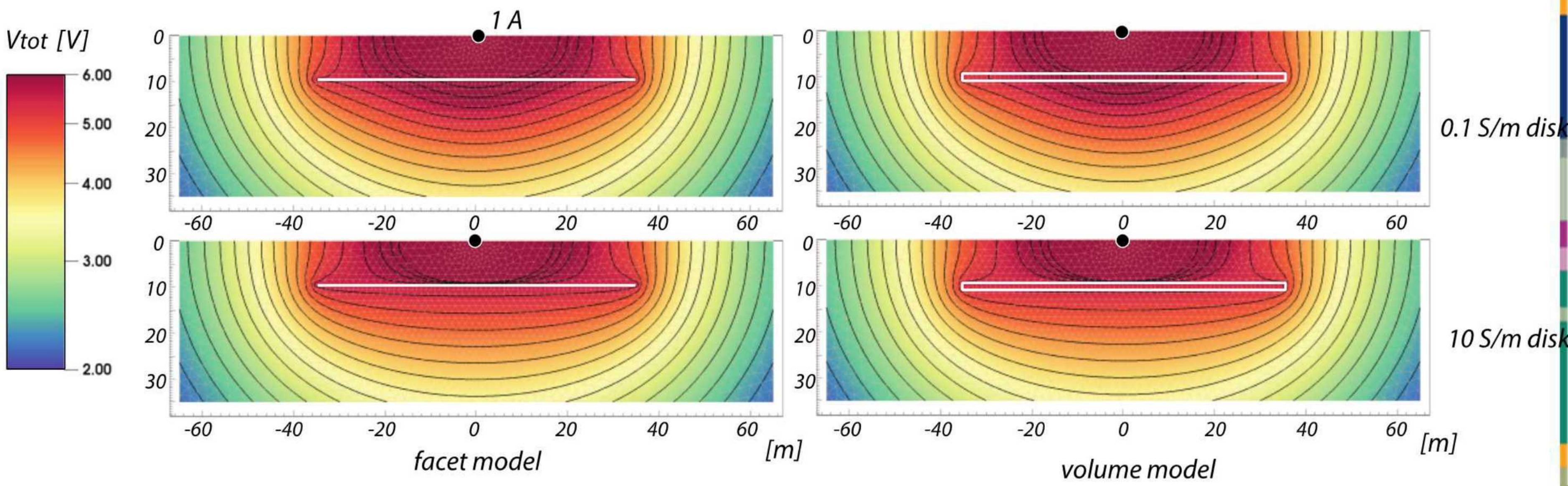
Volume/Facet Consistency Check



MORE INTERNAL CONSISTENCY

Visual inspection of thin disk results for facet elements (left) and many small tetrahedral elements (right).

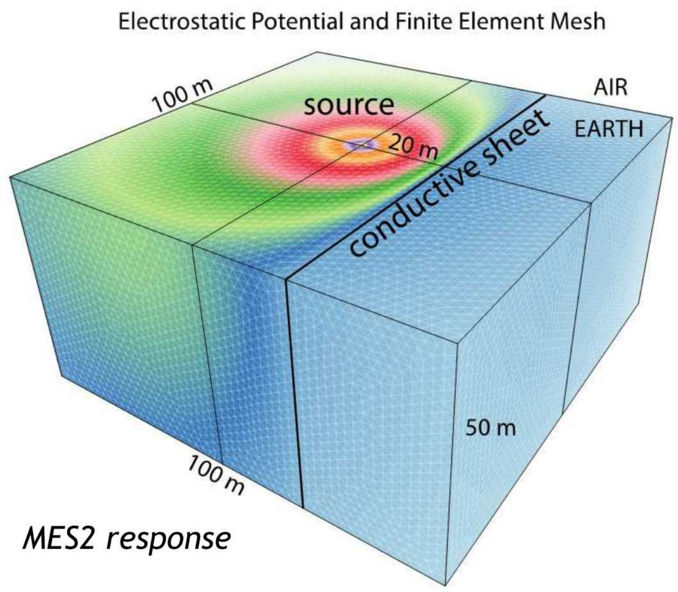
Shown is a cross section of electric potential through the disk and surrounding geology for a weak conductor (top) and strong conductor (bottow). Background conductivity is 0.001 S/m.



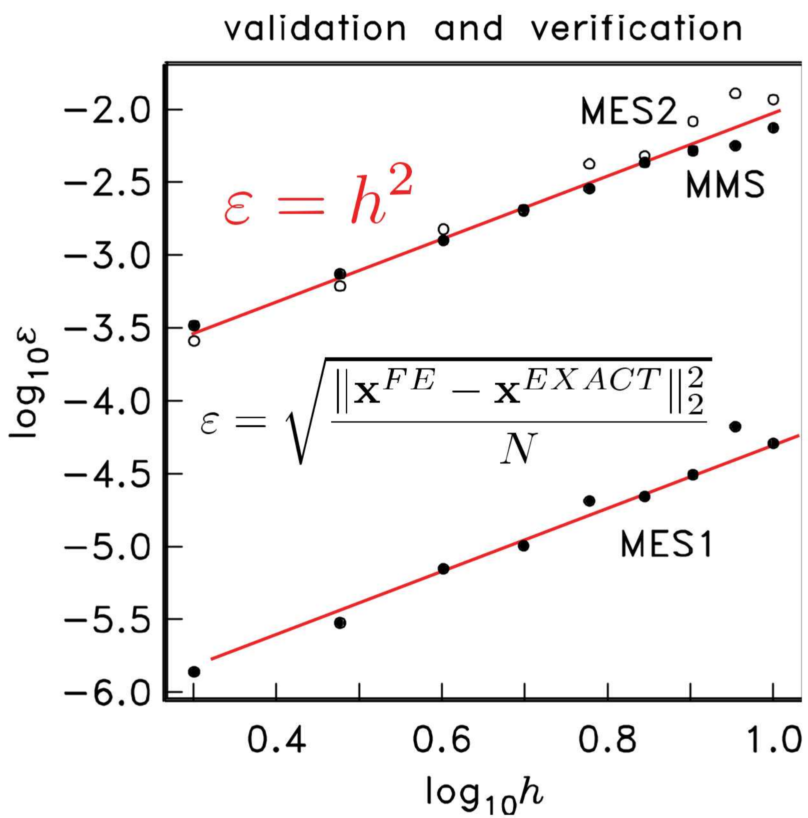
VALIDATION AND VERIFICATION

Method of Exact Solution
When the exact solution is known for a given Earth model and source, compare it with FE solution.

MES1: dipole in a wholespace
MES2: dipole on a halfspace with a thin conductive sheet.



MES2 response



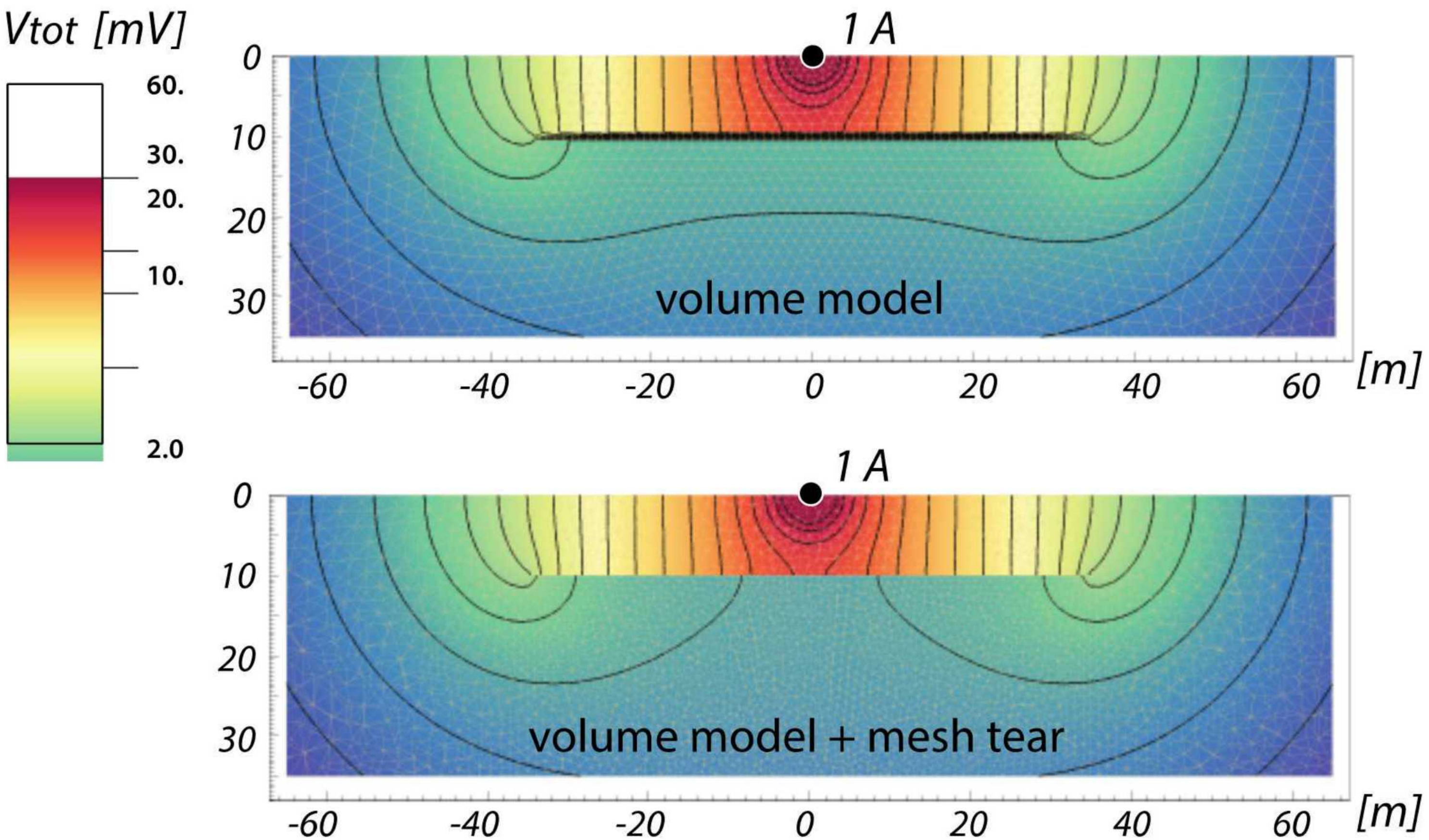
Method of Manufactured Solutions
Posit an analytic solution and then algebraically solve for the sourcing term. Compare it with FE solution.

MMS: choose: $\phi = \exp\left[-(r/a)^2\right]$ and $\sigma = \text{constant}$

Convergence Analysis: hierarchical FE error convergence consistent with classical FE.

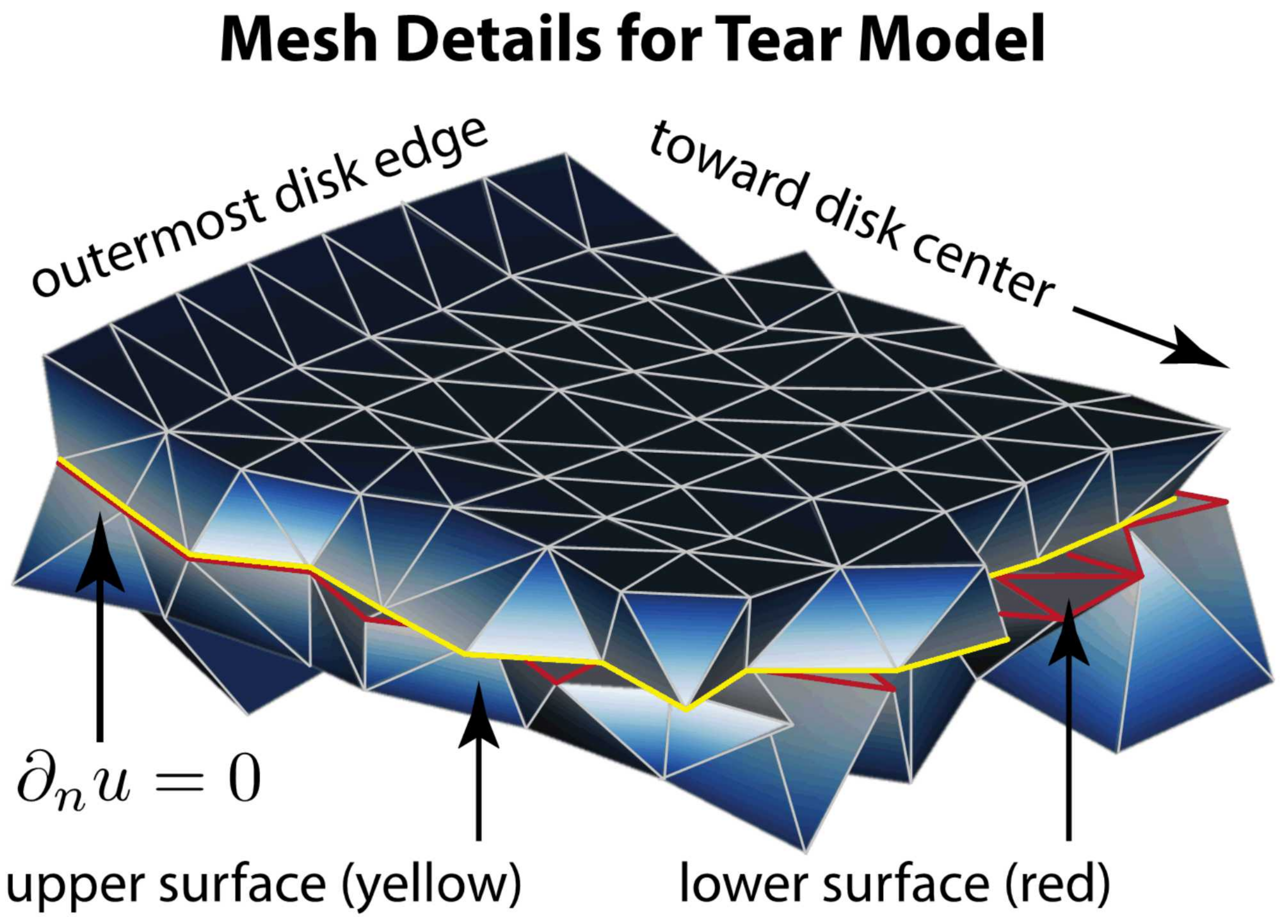
WHAT ABOUT THIN RESISTIVE ELEMENTS?

Because of the continuity condition, thin resistors require a little more intervention. Specifically, we require that the infinitely thin resistor be multi-valued on the disk surface. THIS IS A REQUIREMENT IMPOSED ON THE FUNCTION SPACE FROM WHICH THE FE SOLUTION IS DRAWN, not a 'defect' in the hierarchical concept.



TEARING UP THE MESH

The ”tear” representing the thin resistor is doubly discretized, with one set of nodes corresponding to tets on one side of the tear, and second set for tets on the other side. Still, the surface is infinitely thin and we avoid extreme discretization of a thin, but finite thickness “slab” filled with millions of tiny tets.



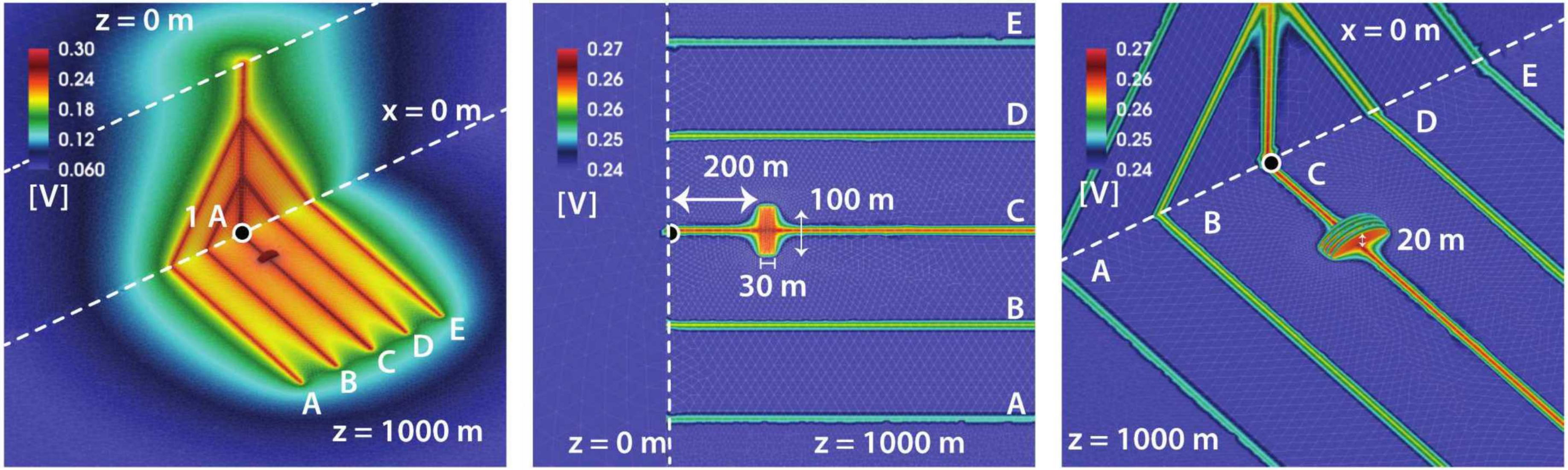
OBSERVATIONS ON WELL CASING RESPONSE

With the mathematical framework in place along with favorable benchmarking results, we're now emboldened to investigate oilfield problems.

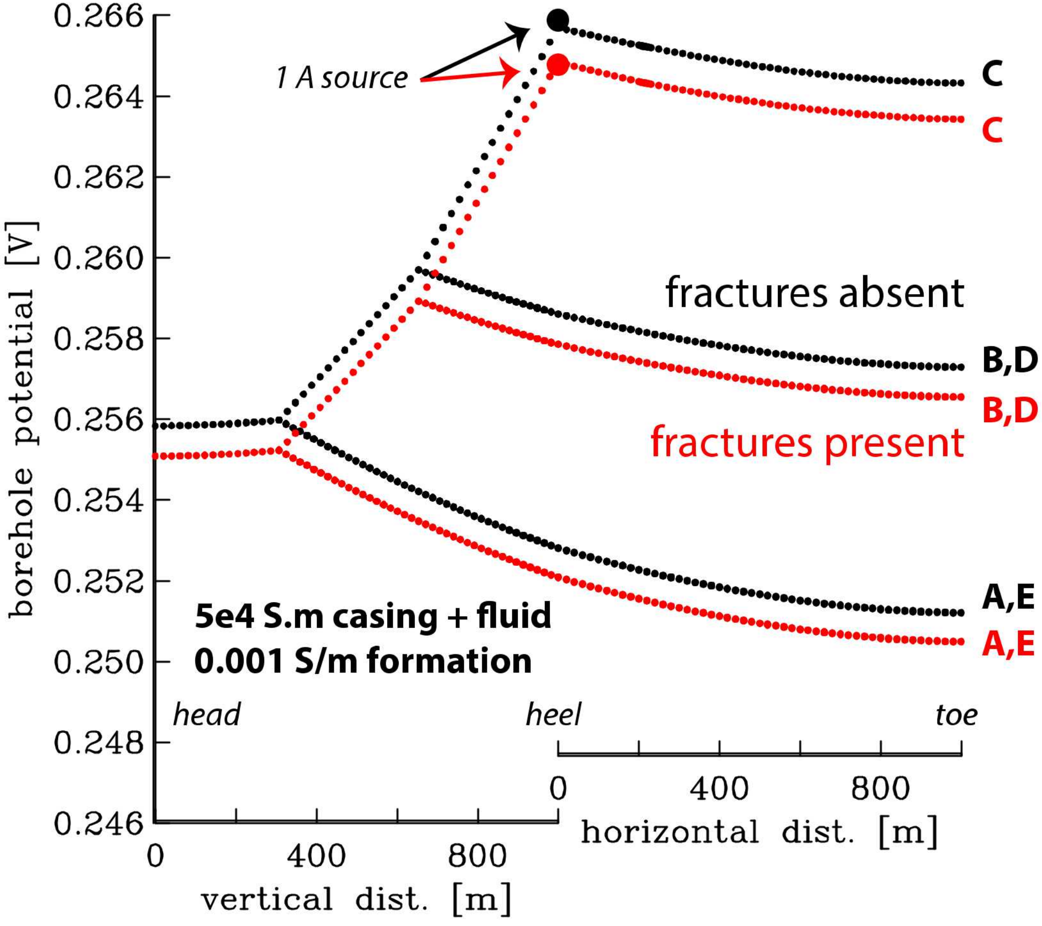
EXAMPLE PROBLEM: an idealized multi-lateral, now with a fracture.

What effect does a conductive fracture have on the casing system?

casing: edge elements, fracture: facet elements

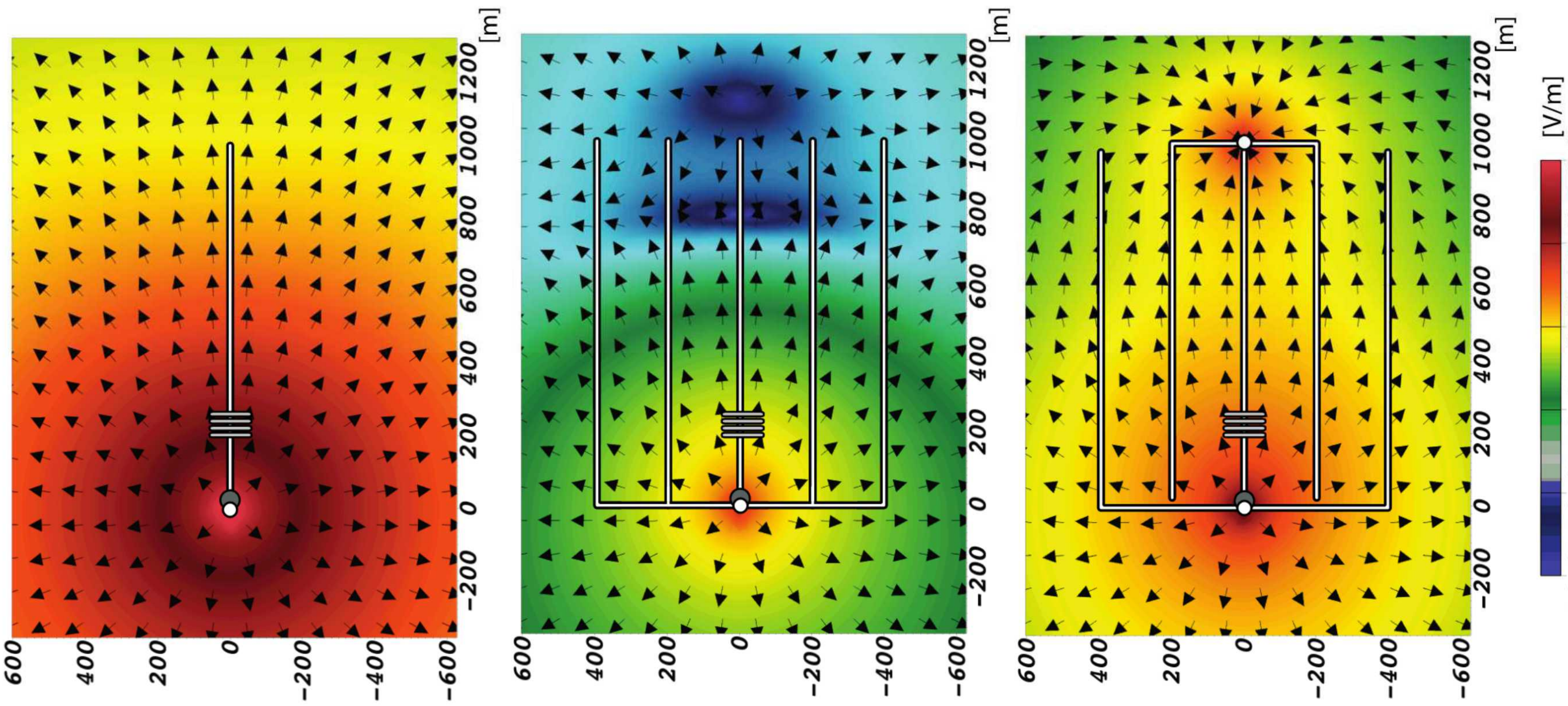


OBSERVATIONS ON WELL CASING RESPONSE



4D time-lapse response of casing response with conductive fractures introduced into the system

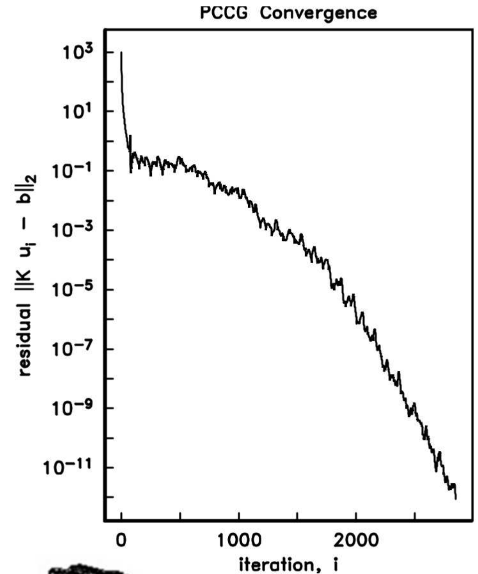
SURFACE FIELDS FOR DIFFERENT CASING SCENARIOS

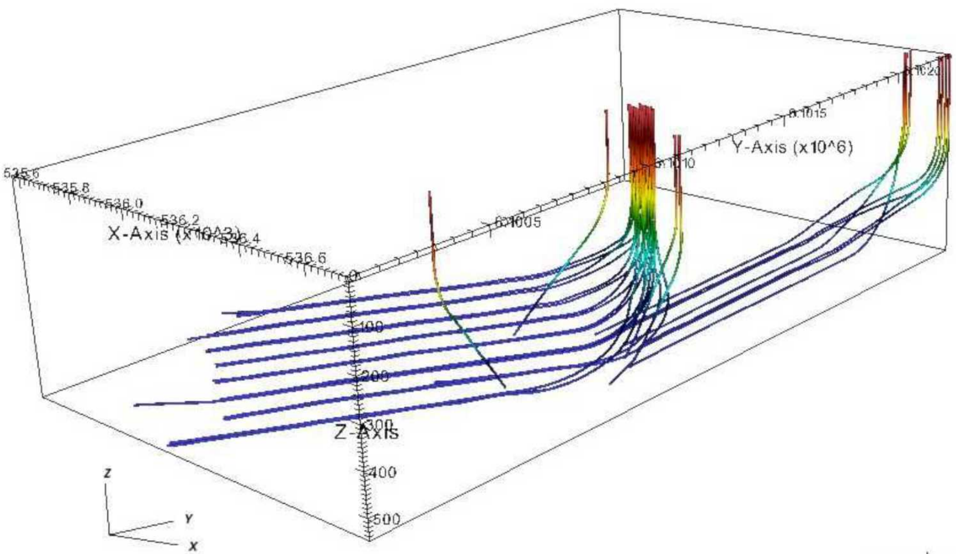


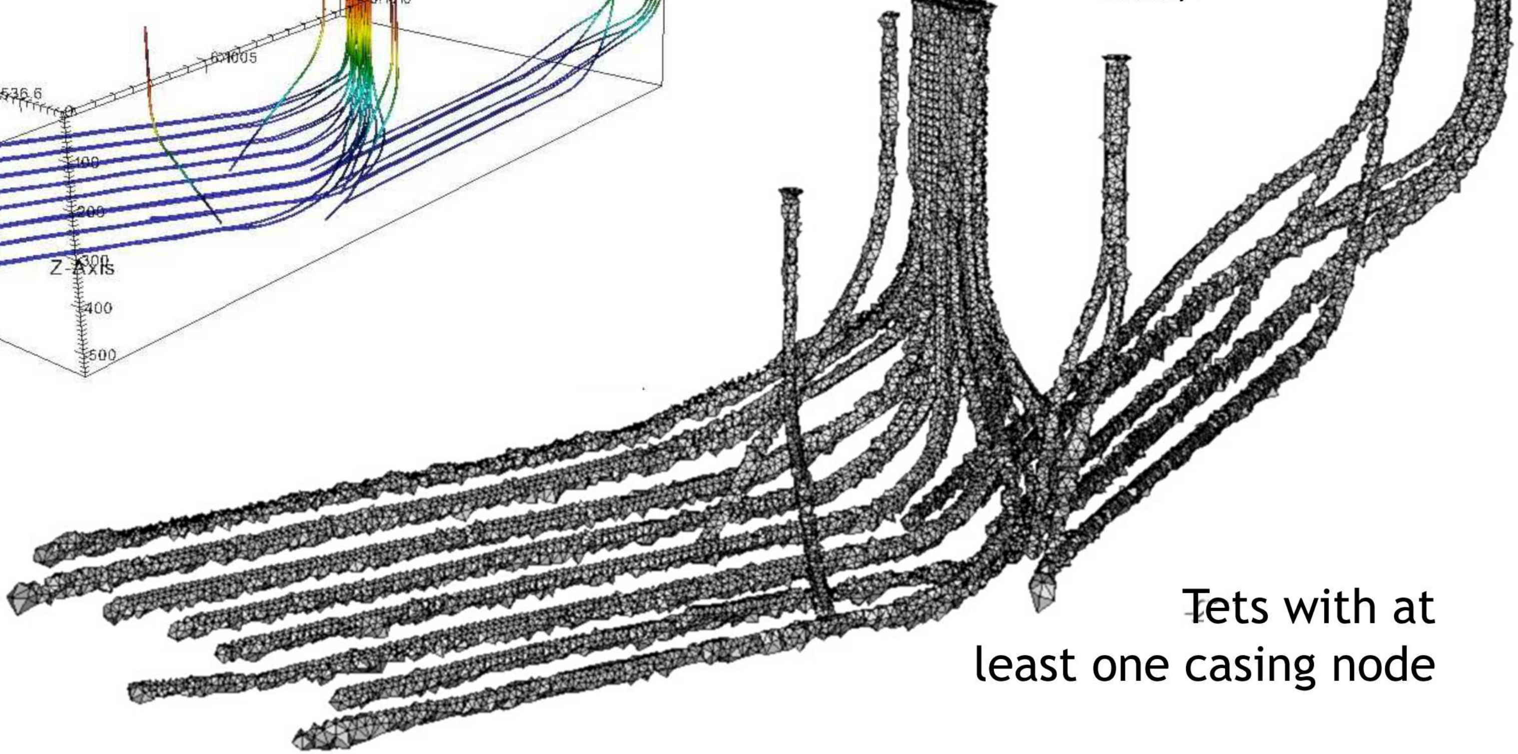
What does the time-lapse surface electric field look like for different casing configurations? Better get all the steel in your model!

EXAMPLE: SAGD MULTILAERAL

20 m node spacing, 45 km of casing, 31 wells: 2313 edges
50 m node spacing on air/earth interface over oilfield
332k tets, 60k nodes, 10 x 10 x 5 km domain

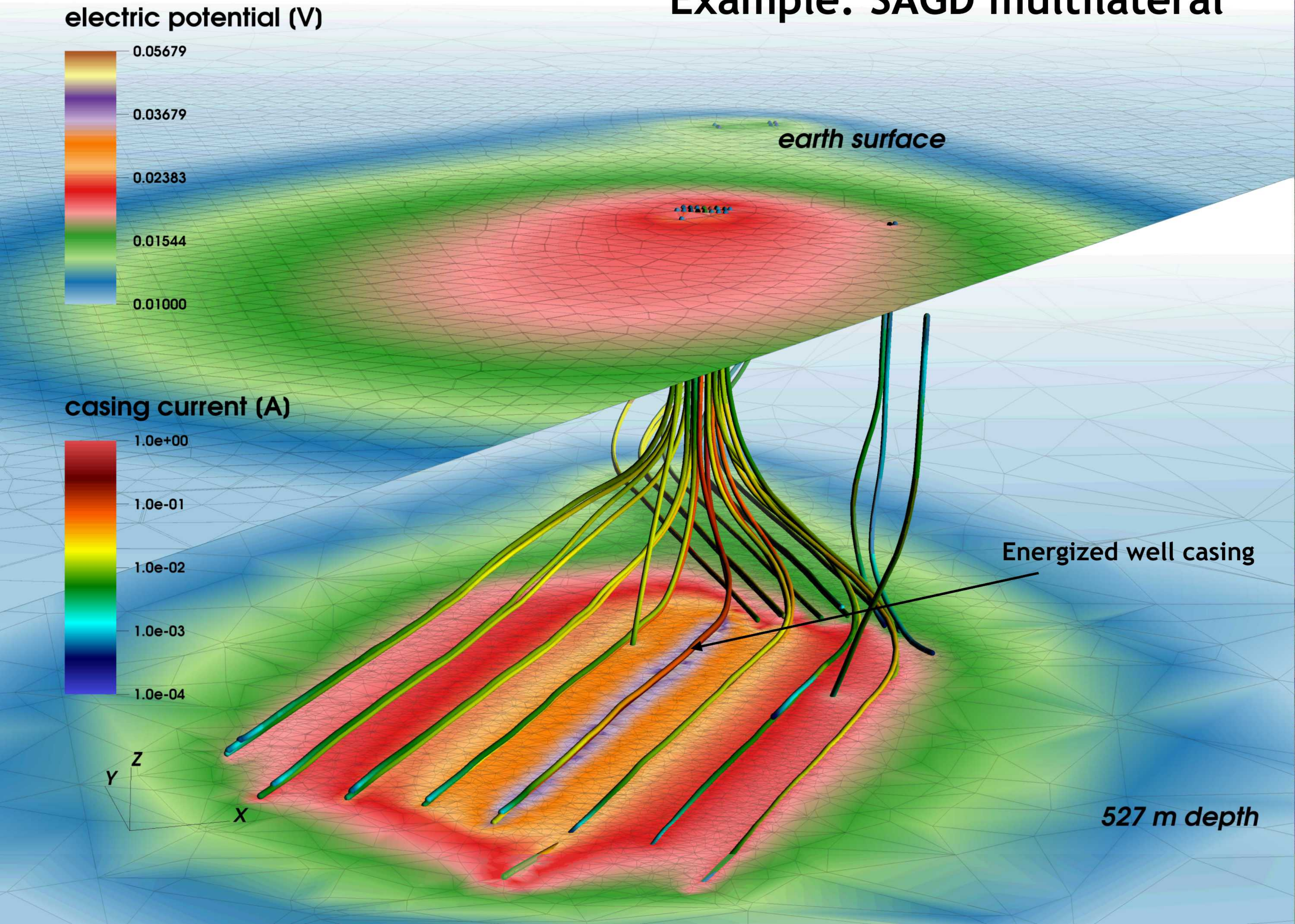




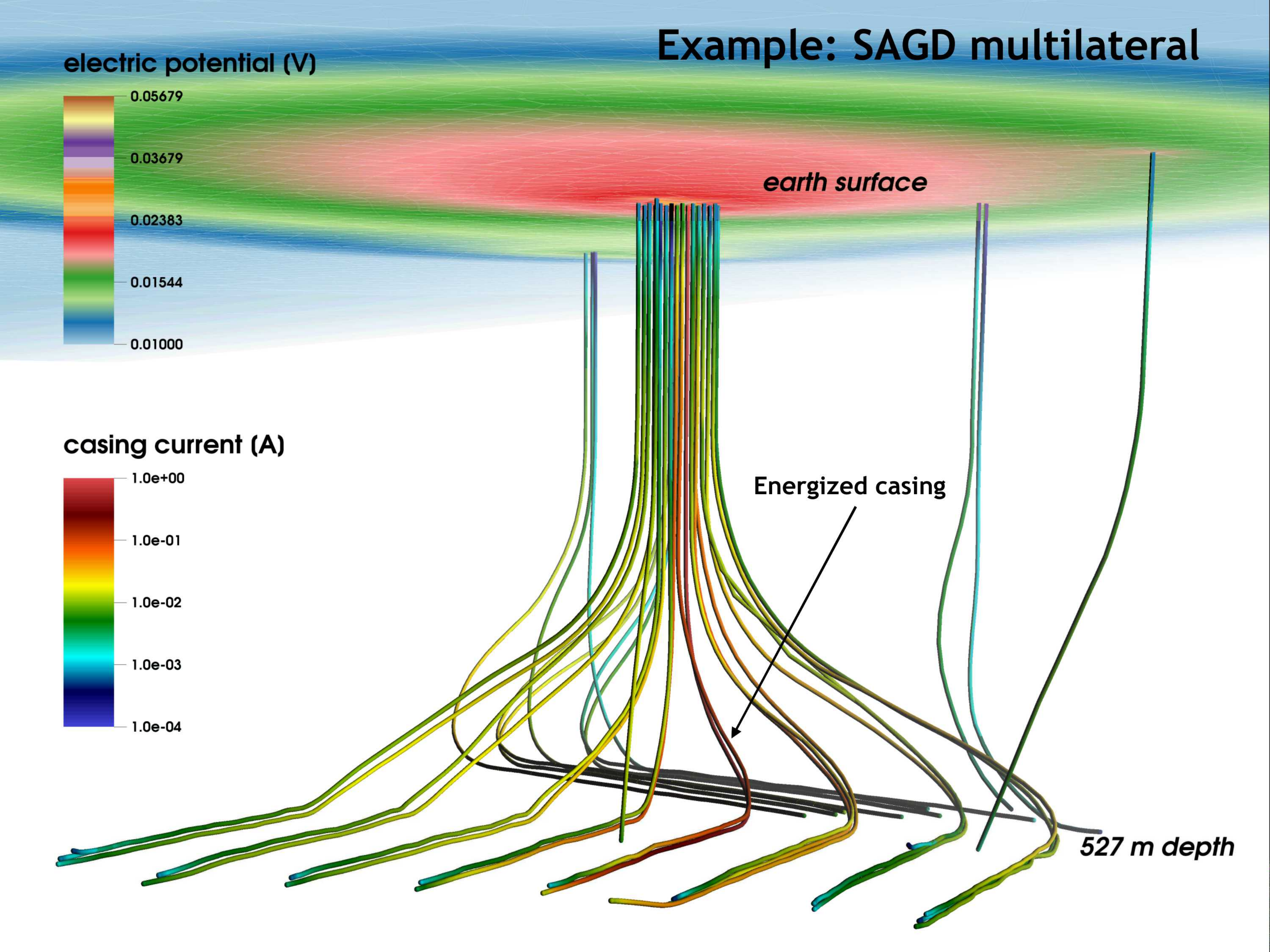


Tets with at least one casing node

Example: SAGD multilateral



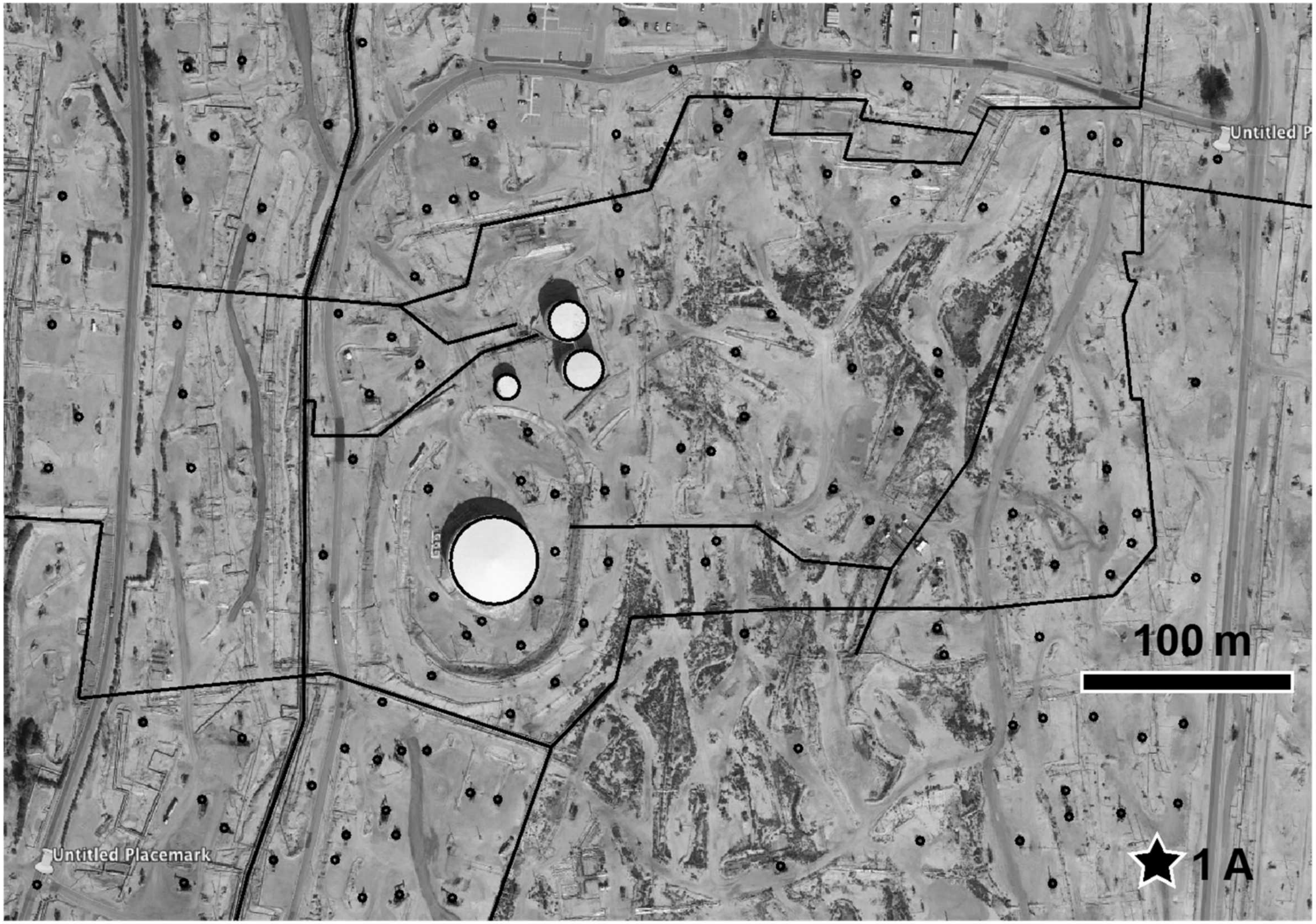
Example: SAGD multilateral



EXAMPLE: CASING AND SURFACE INFRASTRUCTURE

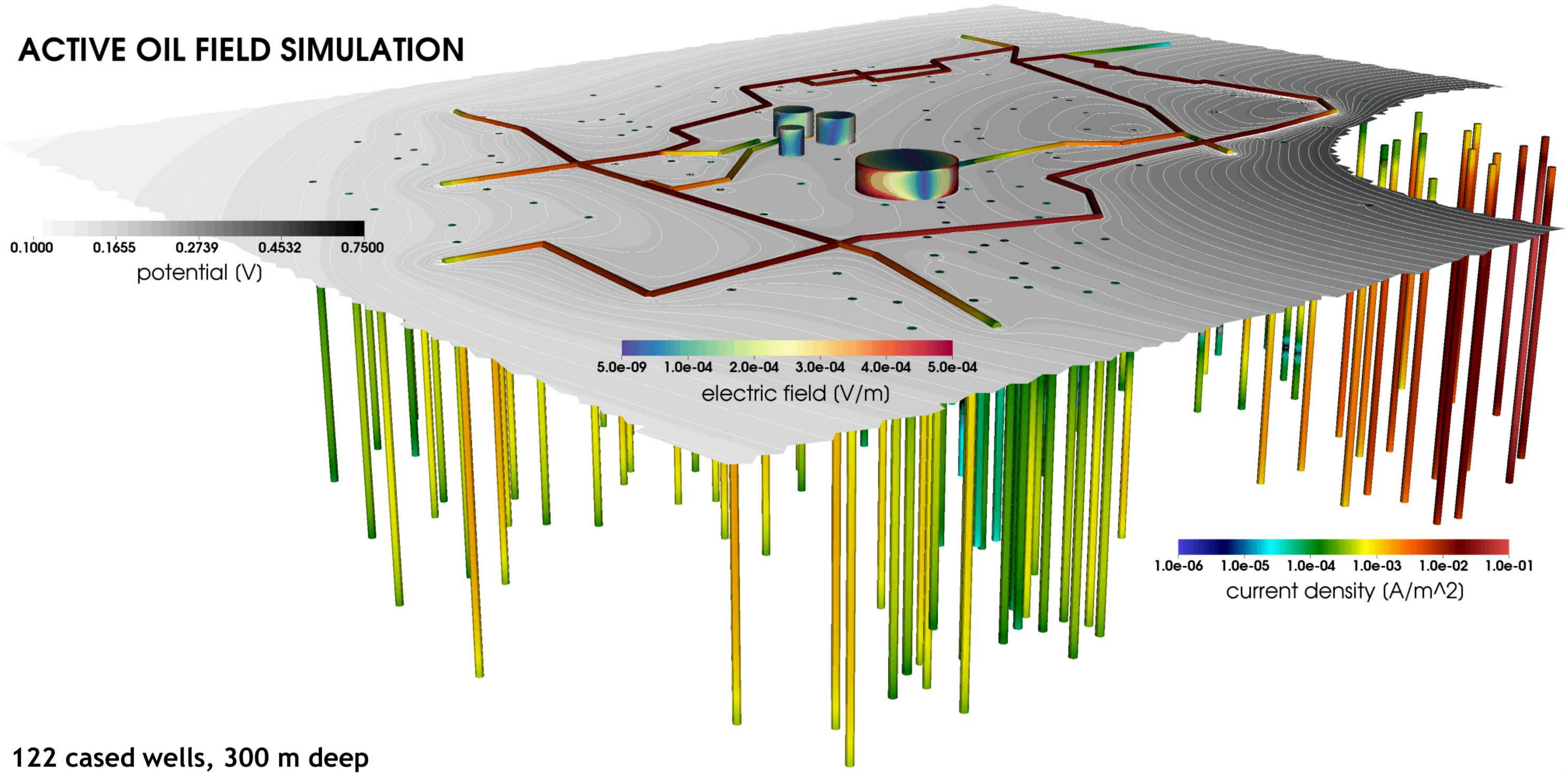
Kern River Formation Site

0.7 km^2 area + 122 wells + ~2 km surface pipes



EXAMPLE: CASING AND SURFACE INFRASTRUCTURE

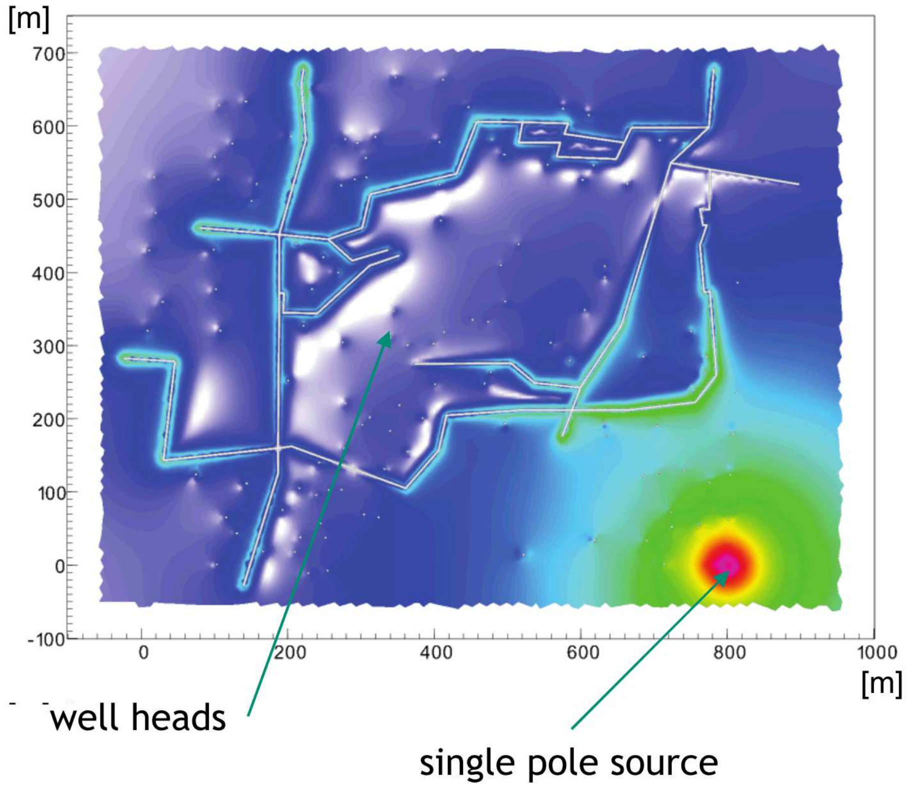
ACTIVE OIL FIELD SIMULATION



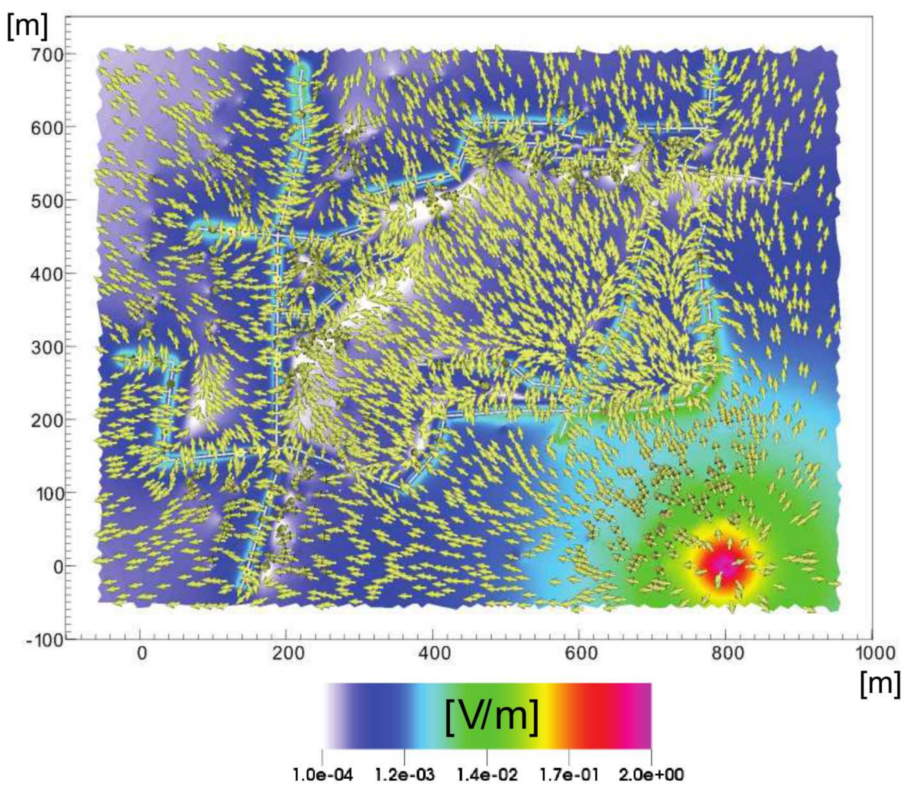
122 cased wells, 300 m deep
5 km surface pipes
~35 km pipeline/casing modeled at 10 m grid spacing: 3500 elements
Traditional FEM requires ~7e6 elements per km of pipline/casing.
HFEM decreases computational burden by ~4 orders of magnitude in this example (10 min vs 2 mo, estimated runtime)

EXAMPLE: CASING AND SURFACE INFRASTRUCTURE

SURFACE ELECTRIC FIELD AMPLITUDE



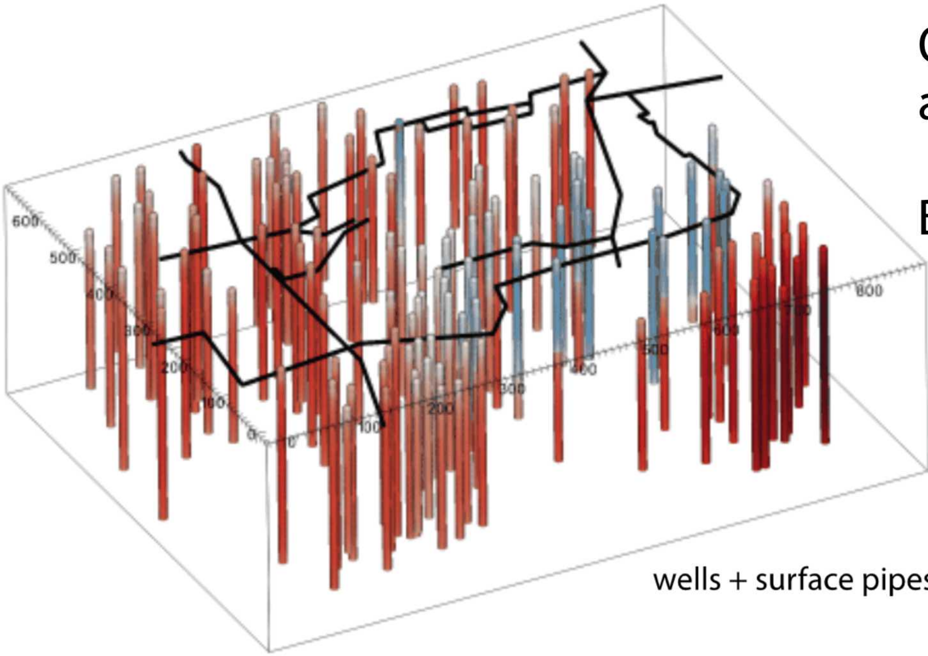
DIRECTION AND AMPLITUDE



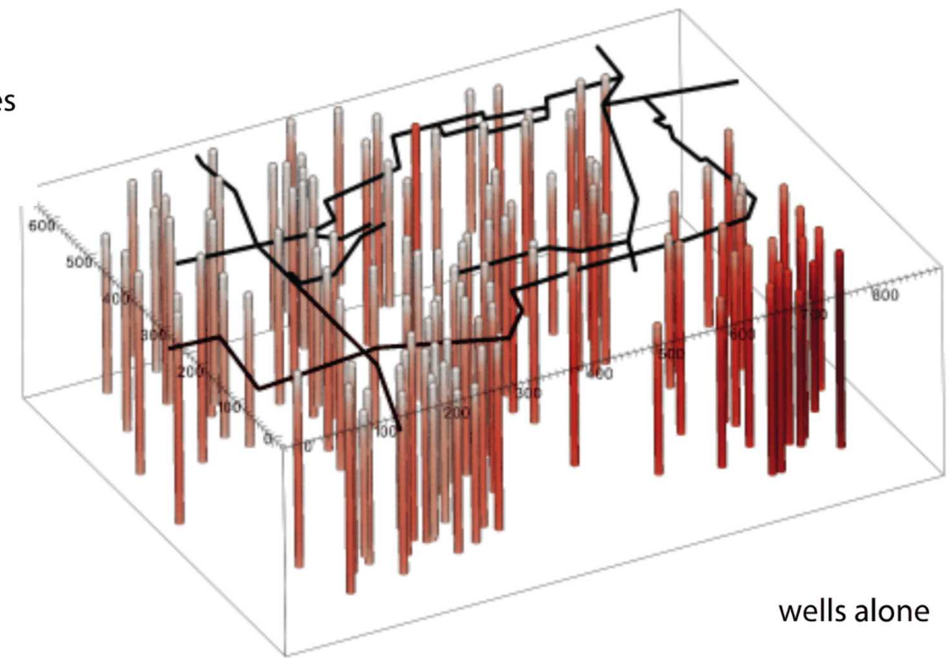
SURFACE PIPELINE COUPING WITH CASING CURRENTS

QUESTION: How does pipeline coupling affect casing current direction?

Blue, upward current; red, downward.



wells + surface pipes

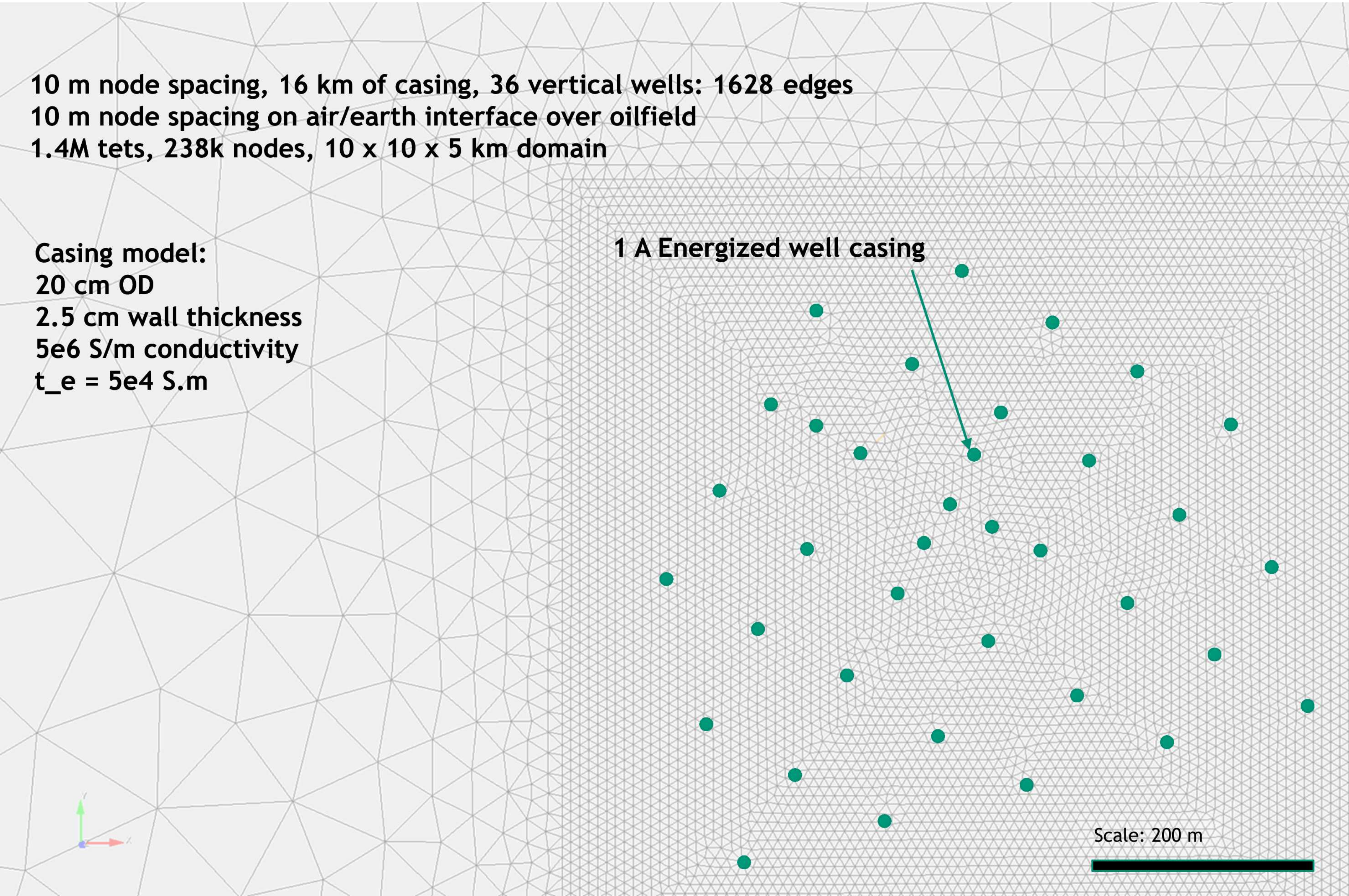


wells alone

EXAMPLE: SHALLOW HEAVY OIL RESERVOIR

10 m node spacing, 16 km of casing, 36 vertical wells: 1628 edges
10 m node spacing on air/earth interface over oilfield
1.4M tets, 238k nodes, 10 x 10 x 5 km domain

Casing model:
20 cm OD
2.5 cm wall thickness
5e6 S/m conductivity
t_e = 5e4 S.m



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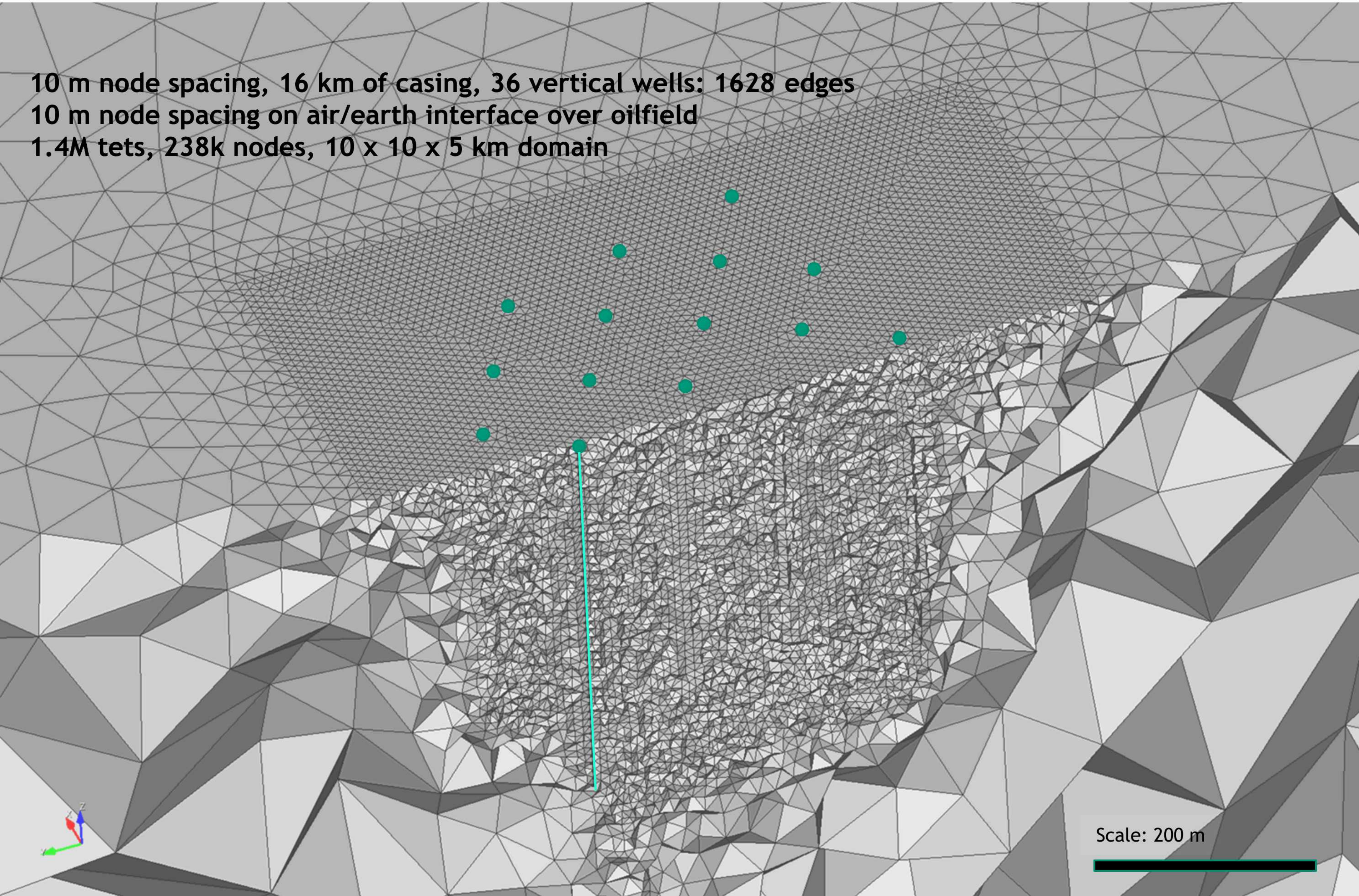

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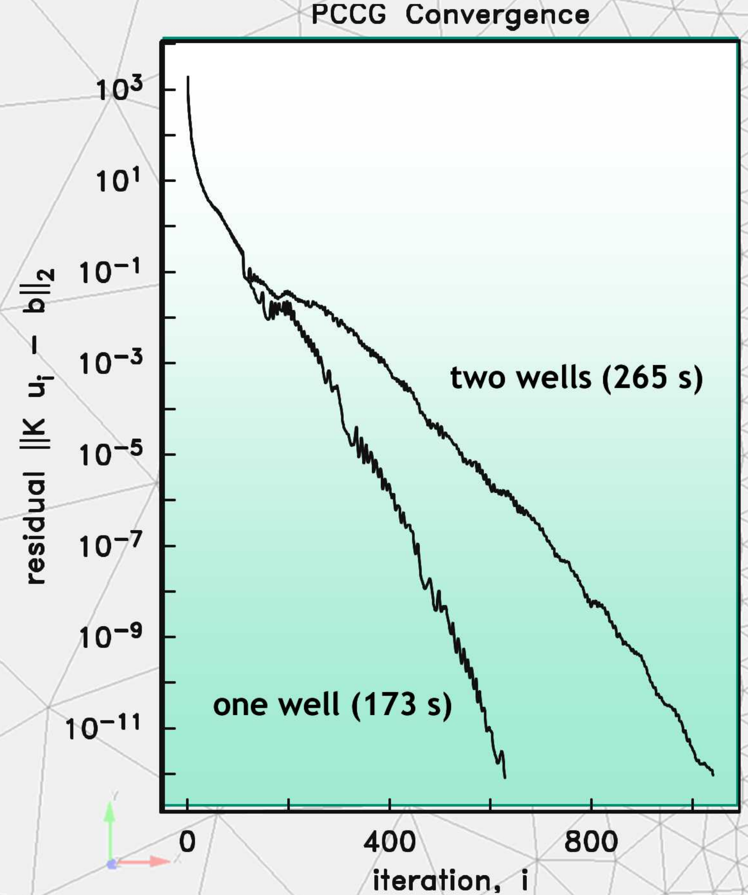
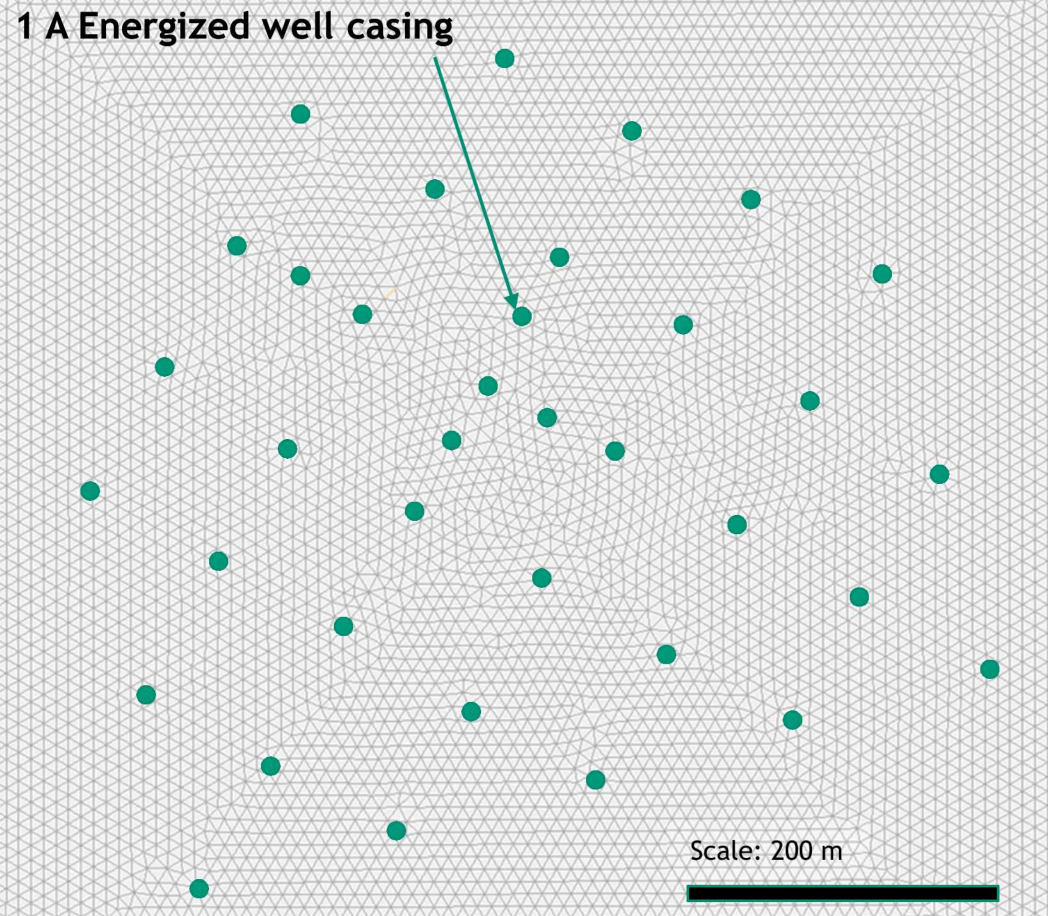
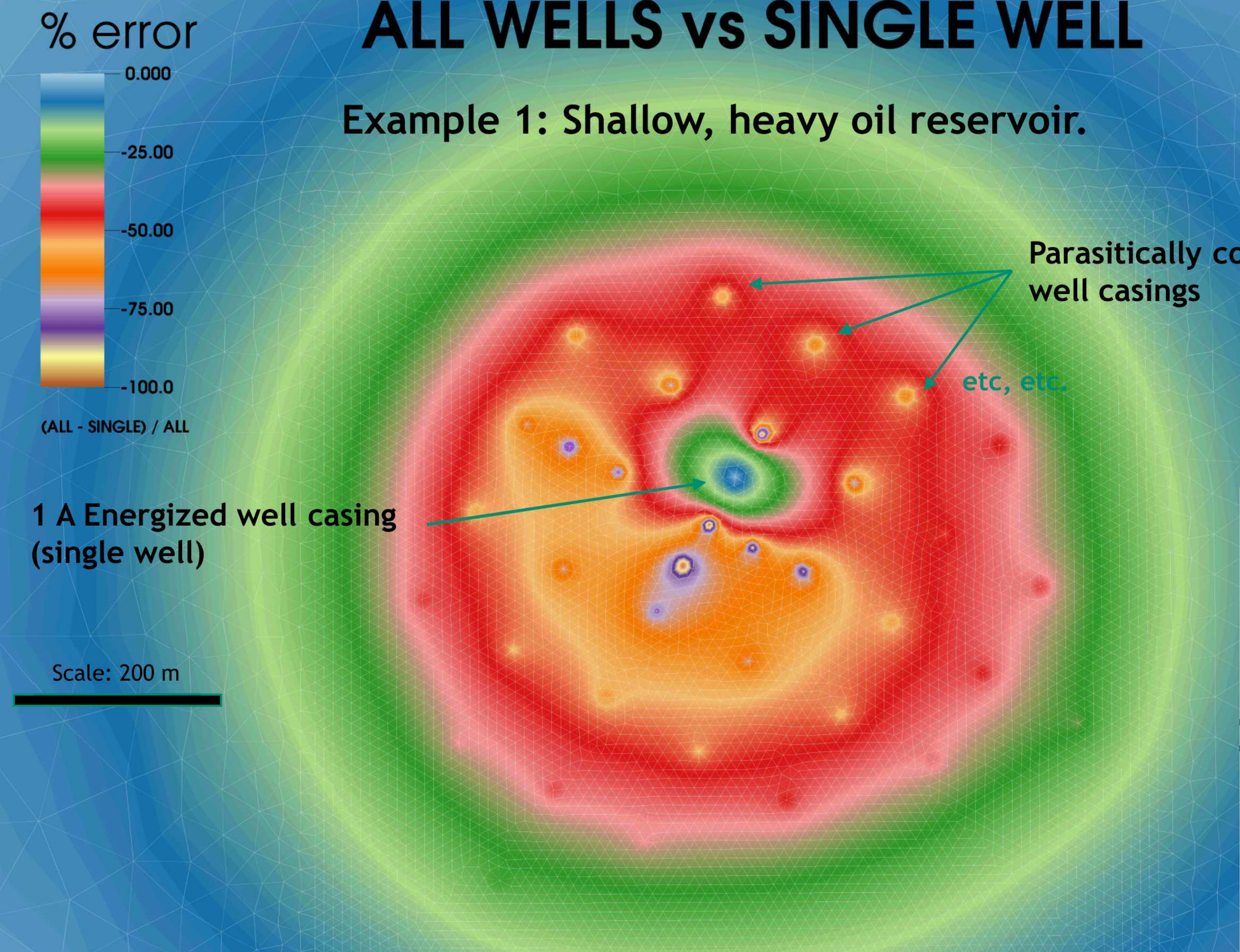
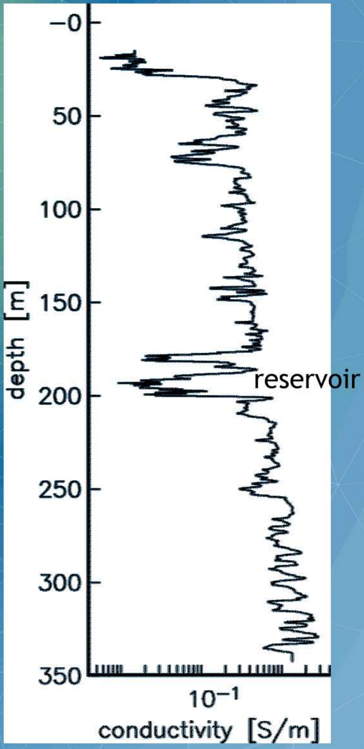




ALL WELLS vs SINGLE WELL

Example 1: Shallow, heavy oil reservoir.





What is the relative effect on electrostatic potential when ignoring infrastructure?

ALL WELLS vs SINGLE WELL

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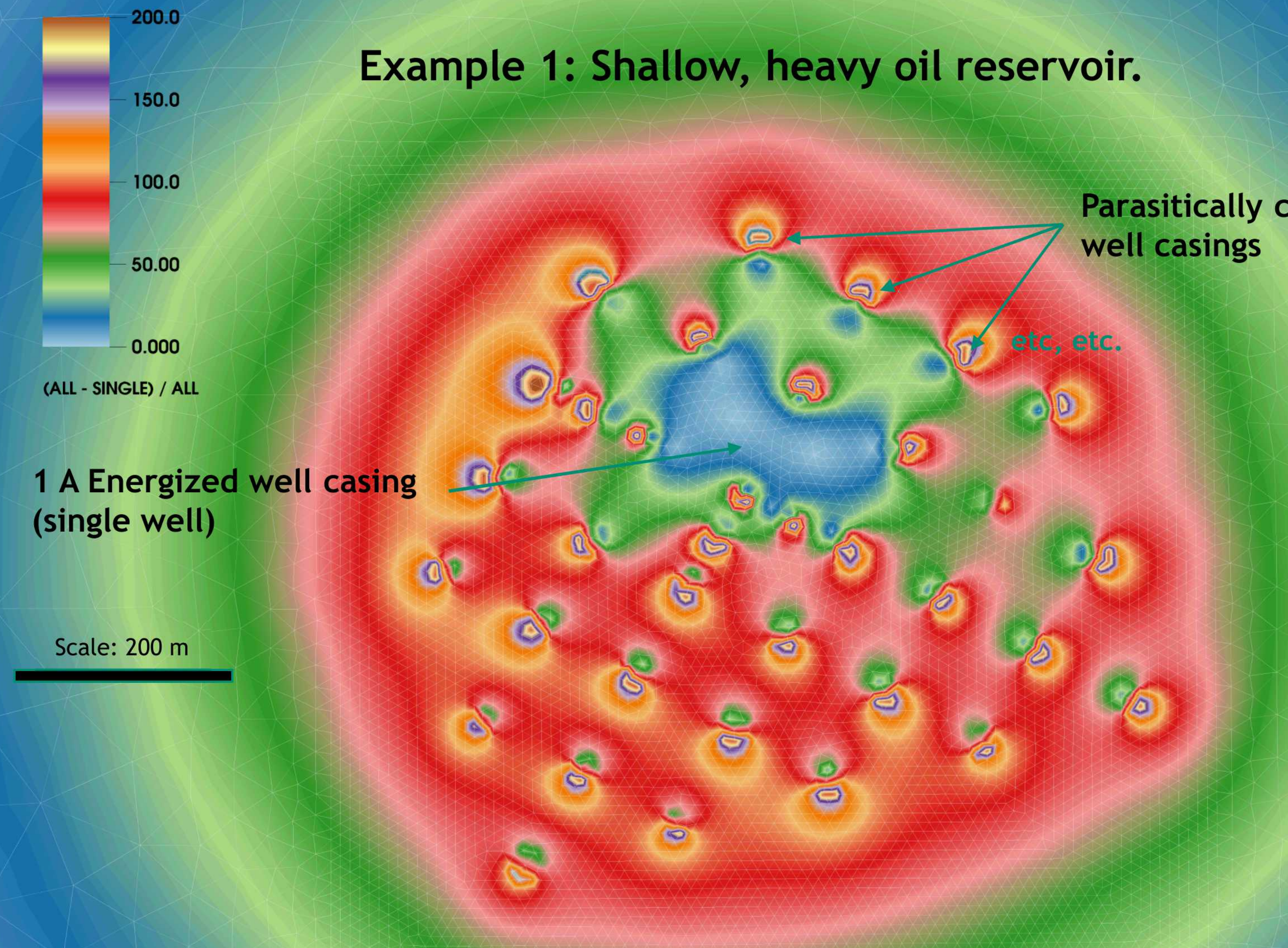
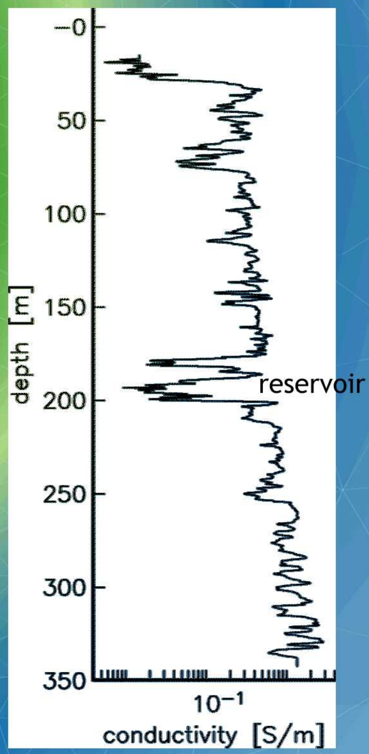




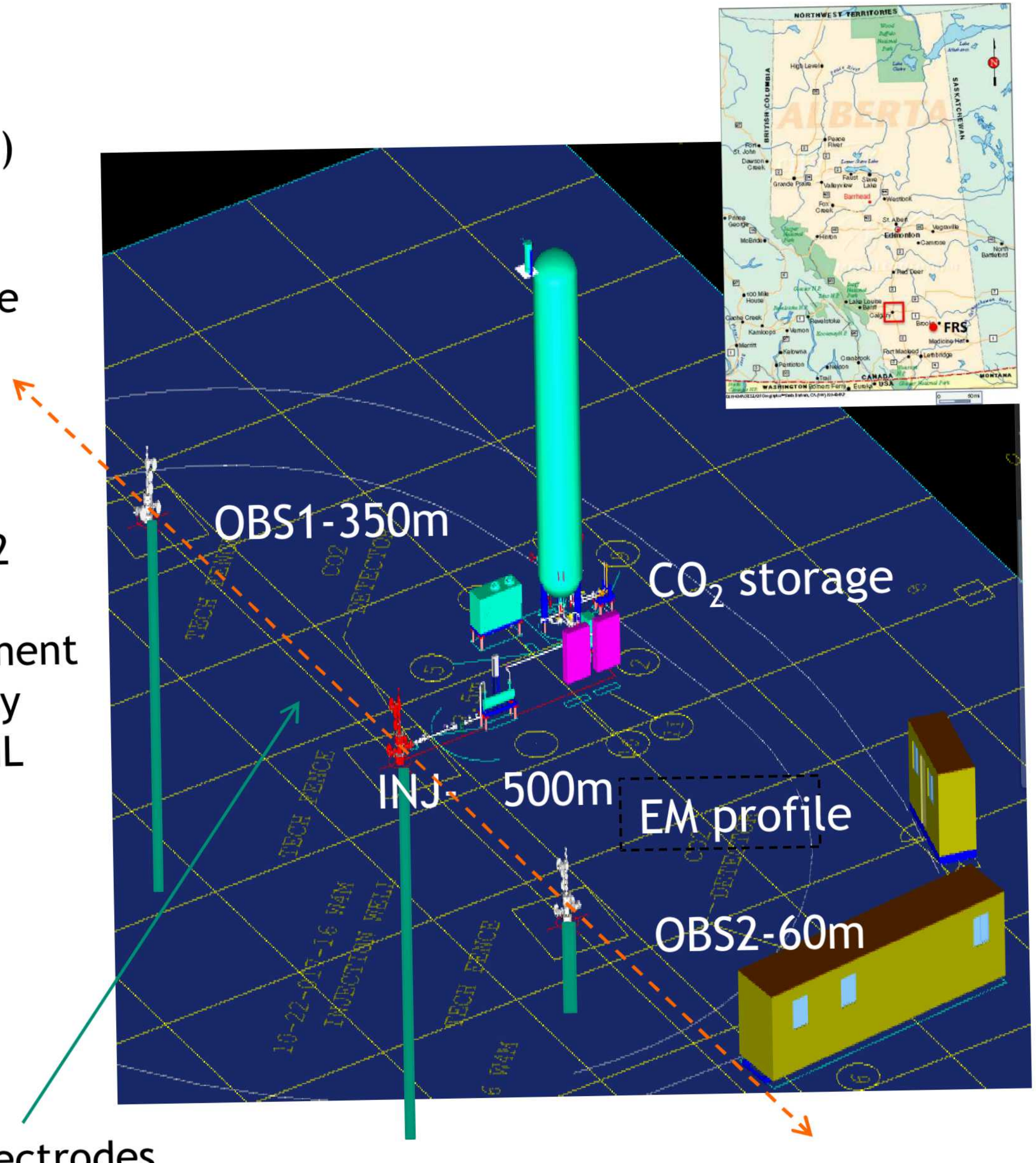
What is the relative effect on electric field when ignoring infrastructure?

FIELD EXPERIMENT: CAMI CO_2 SEQUESTRATION SITE

- Current on well head
  - Used OBS2 (60m) and OBS1 (350m) casing length
  - 5 Hz signal used
  - Trench electrodes used for voltage measurements
    - Trench Offset 5m
    - 10m spacing +/- 500m from wellheads
  - Return electrode 500m from OBS2

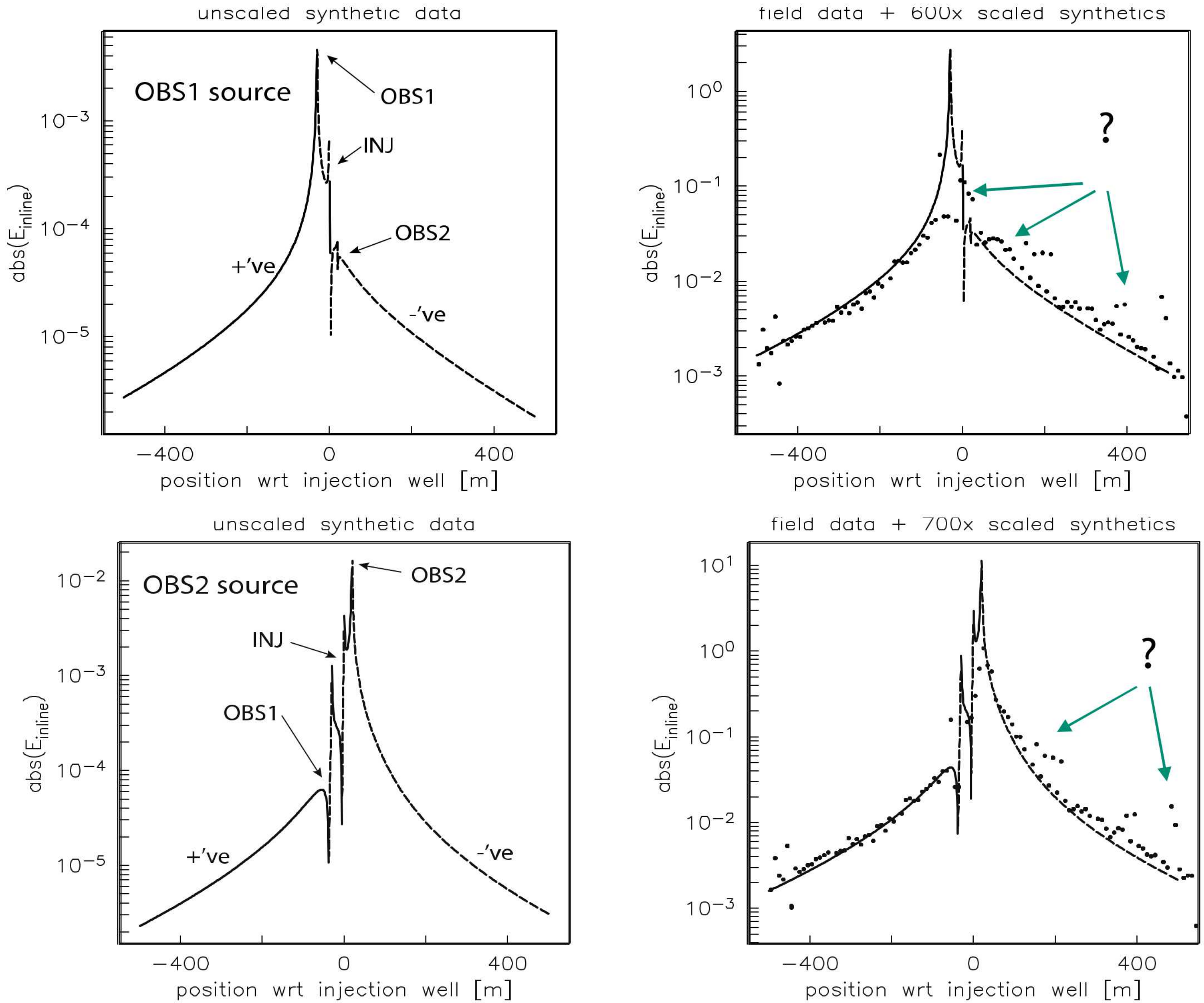
One component of a multi-physics experiment for monitoring/assessing wellbore integrity (DOE/EERE/GTO project joint between SNL and LBNL)

Site is characterized by a operational infrastructure with potentially negative impacts on data quality.

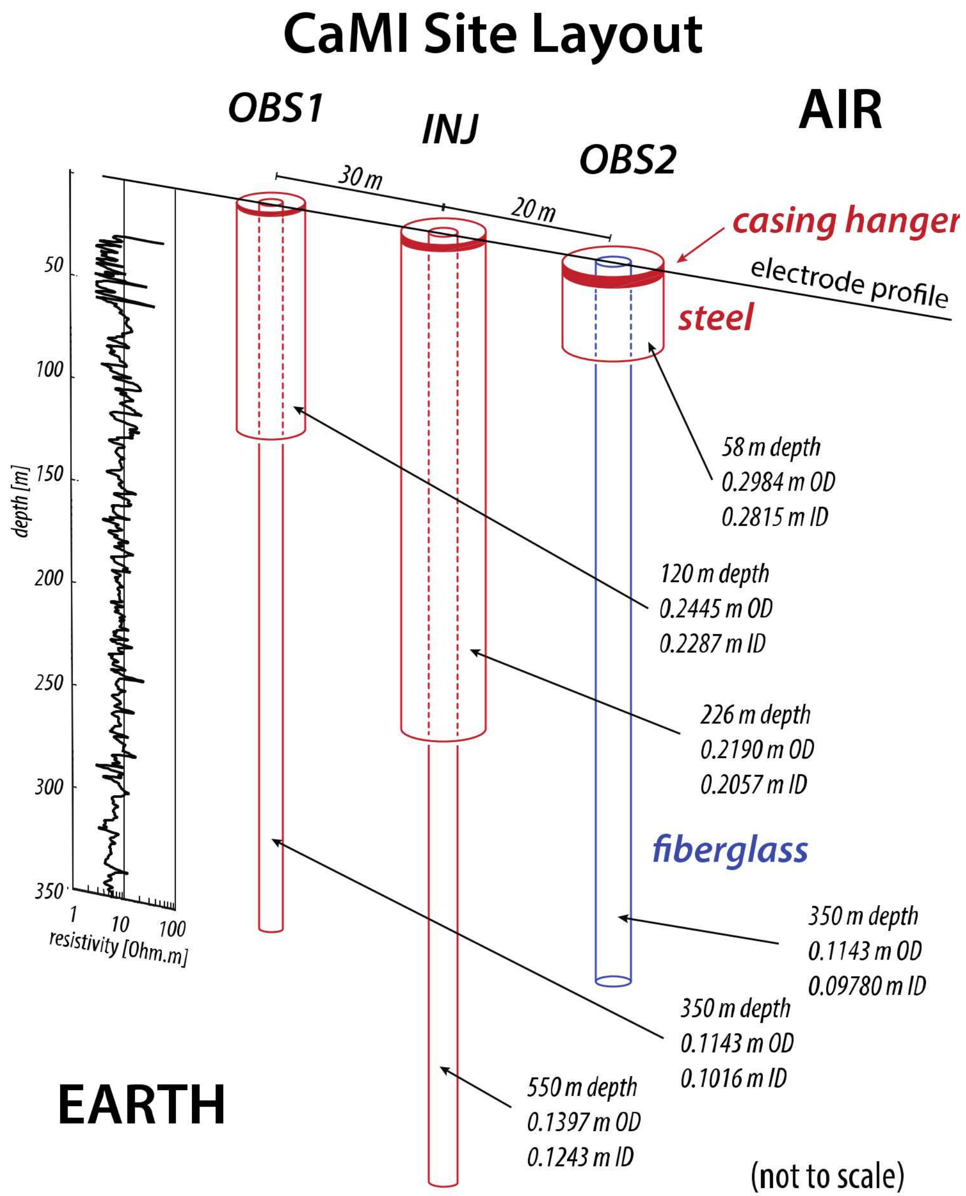


FITTING THE 5Hz DATA WITH DC RESPONSE

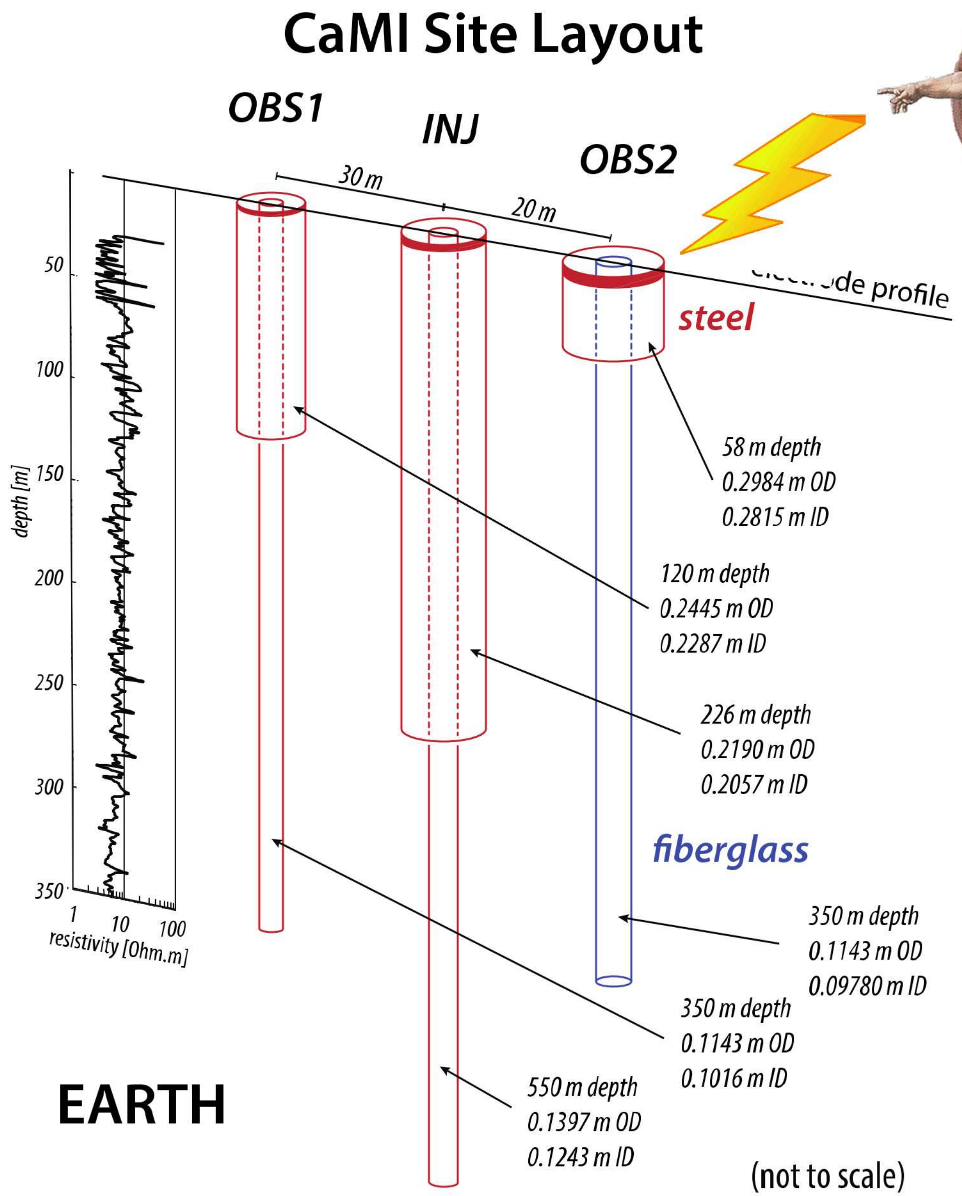
Simple, one-conductor borehole casing

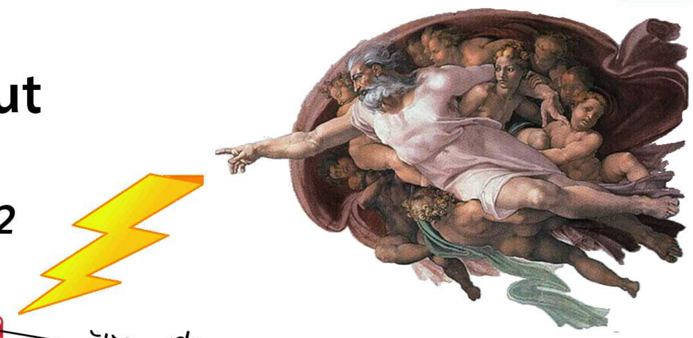


A MORE REALISTIC COMPLETION MODEL



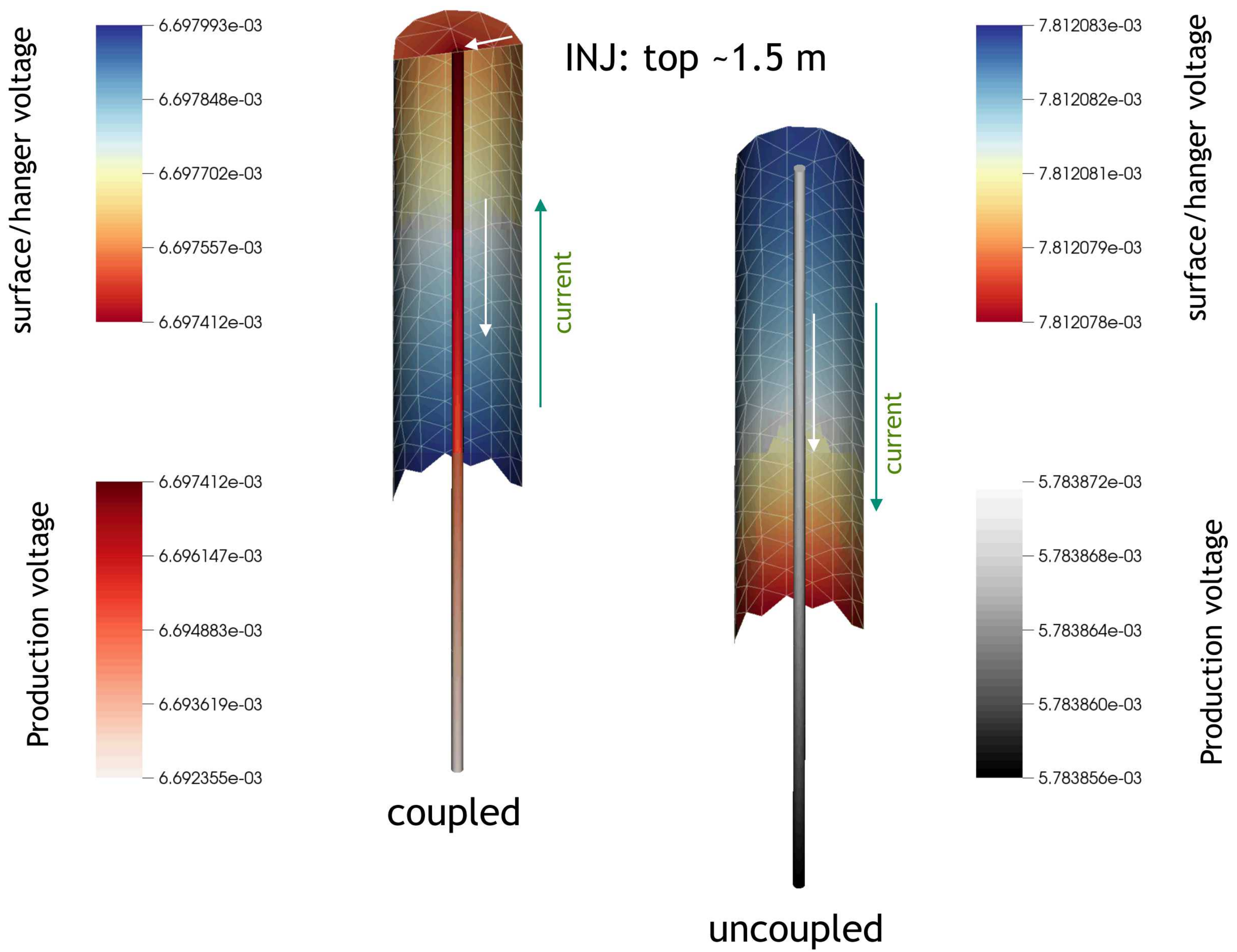
ENERGIZE OBS2 WELL AS SOURCE



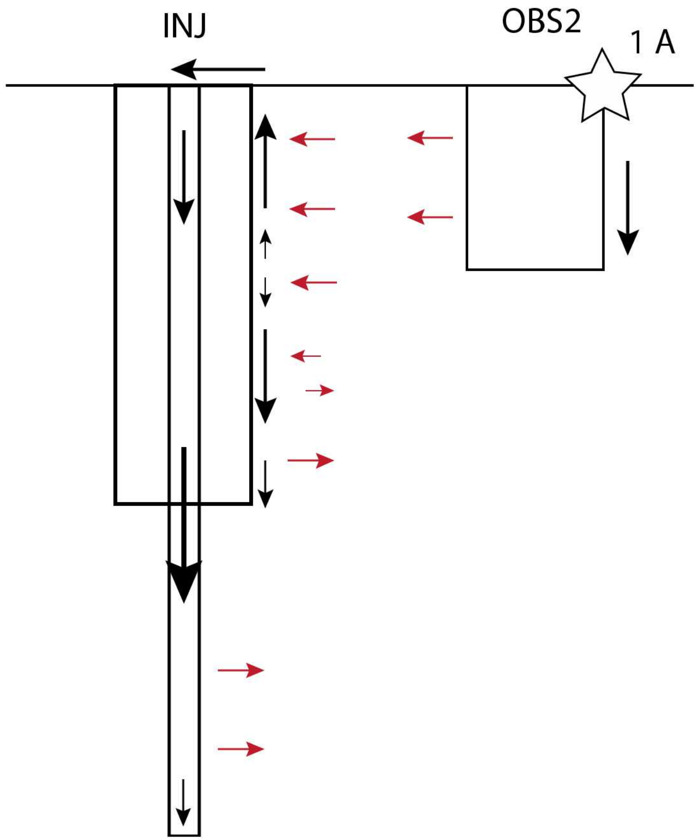


Michelangelo, detail from "The creation of Adam", Sistine Chapel Ceiling, 1508-1512, Rome

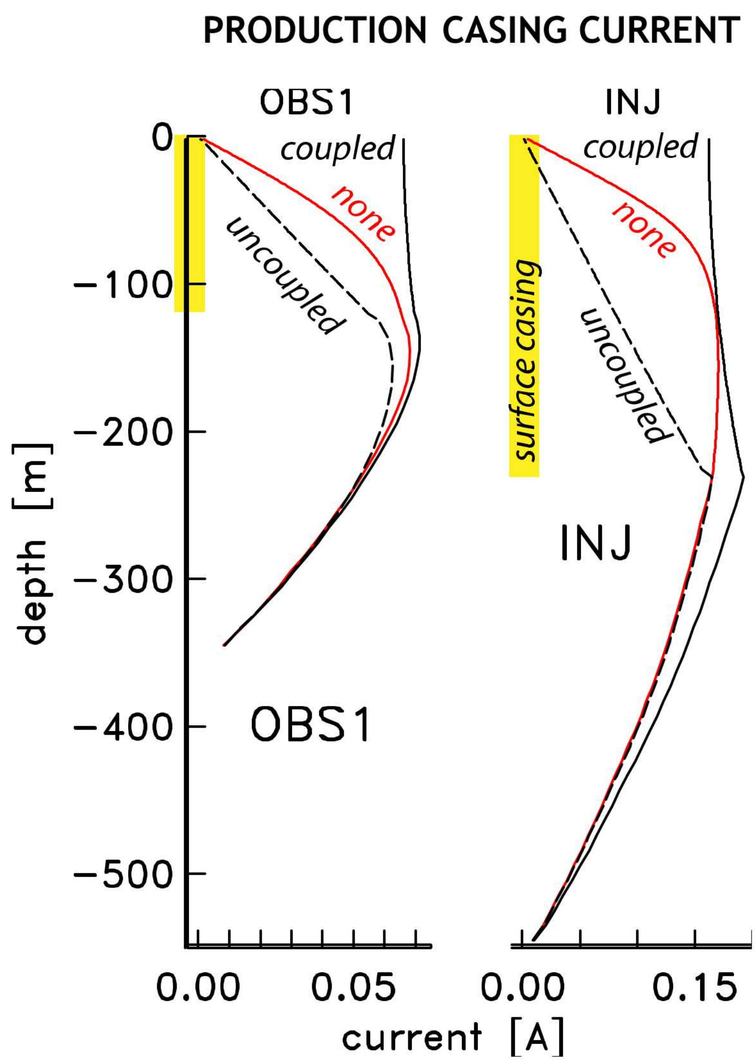
EFFECT OF AN ELECTRICALLY CONDUCTIVE HANGER



CURRENT PATHWAYS WITH COUPLED CASING

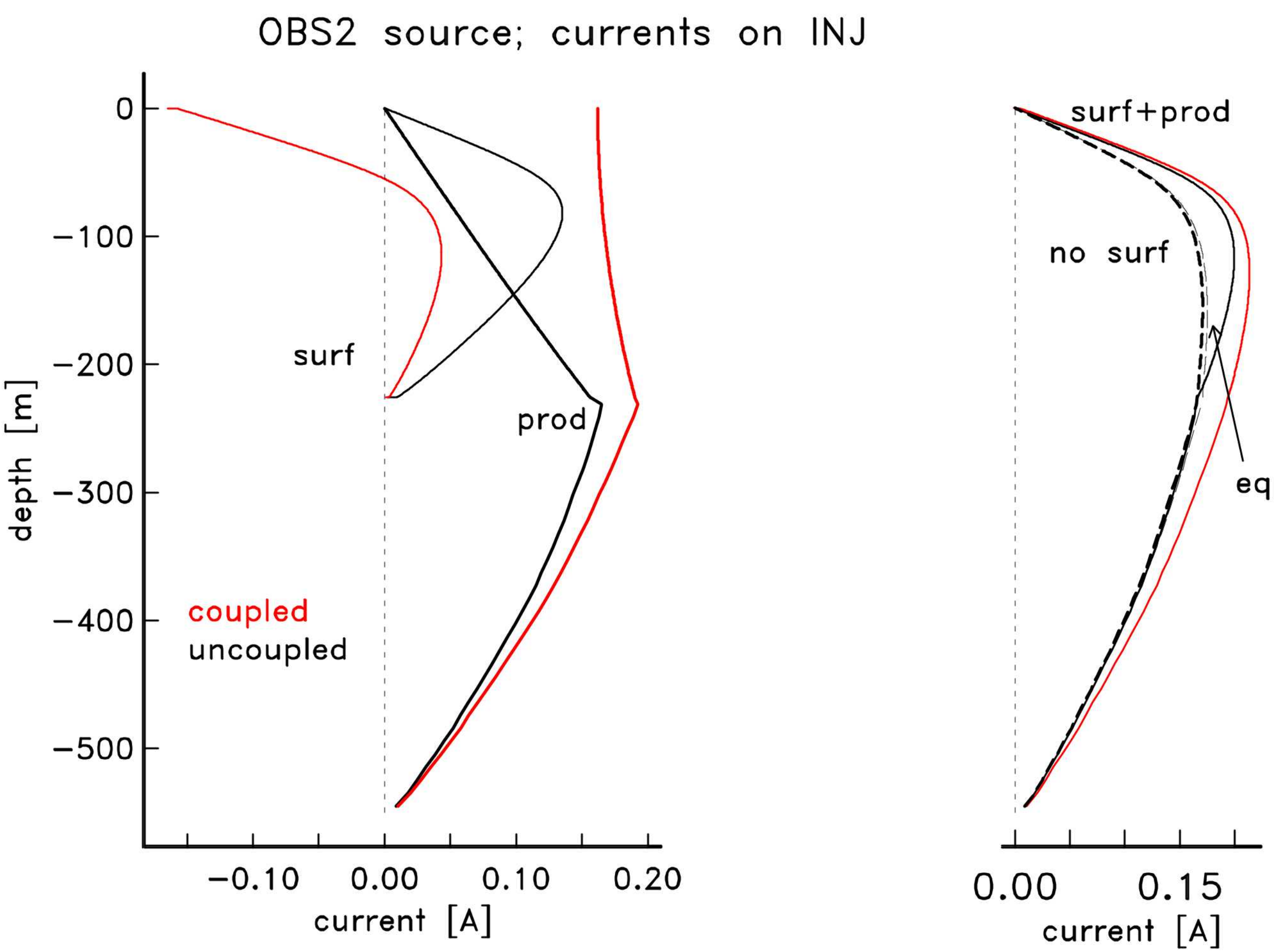


Overall current system in when production and surface casings are coupled.



None: no surface casing

Coupled: hanger present; Uncoupled: hanger absent

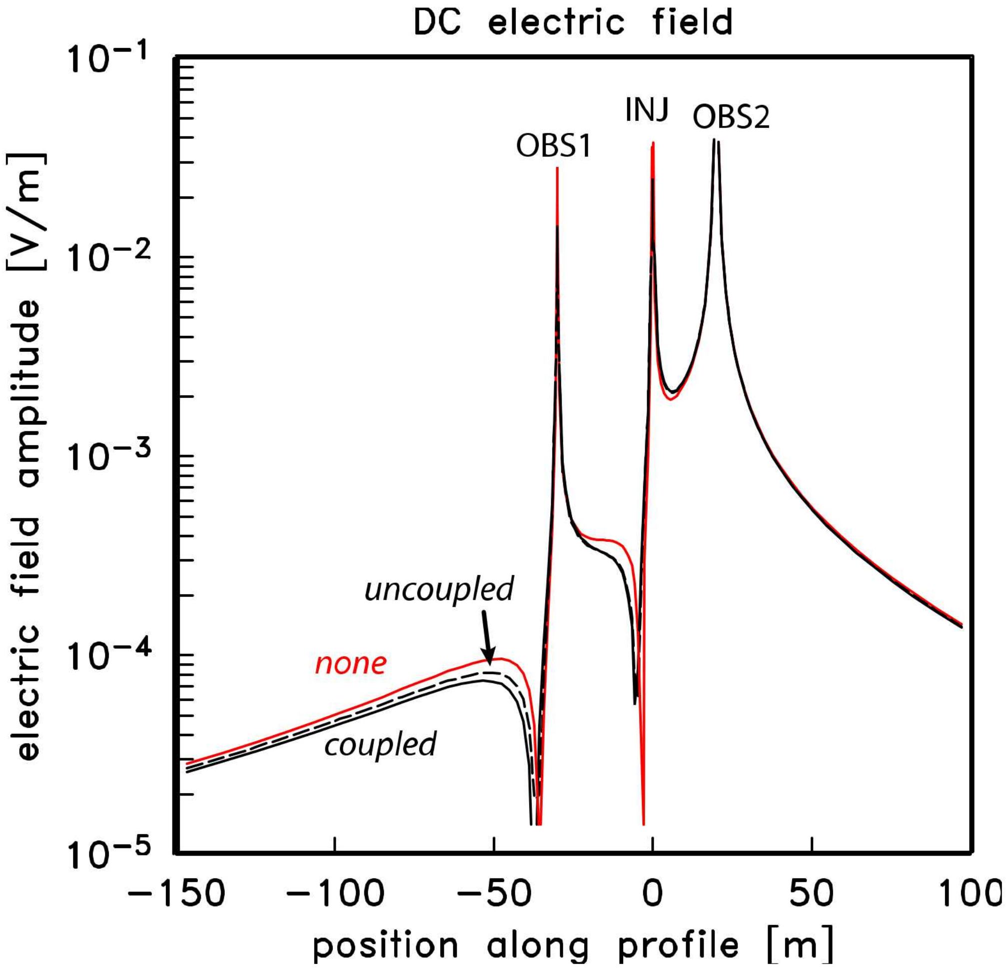


No surf: no surface casing

Eq: equivalent conductance model for surf+prod

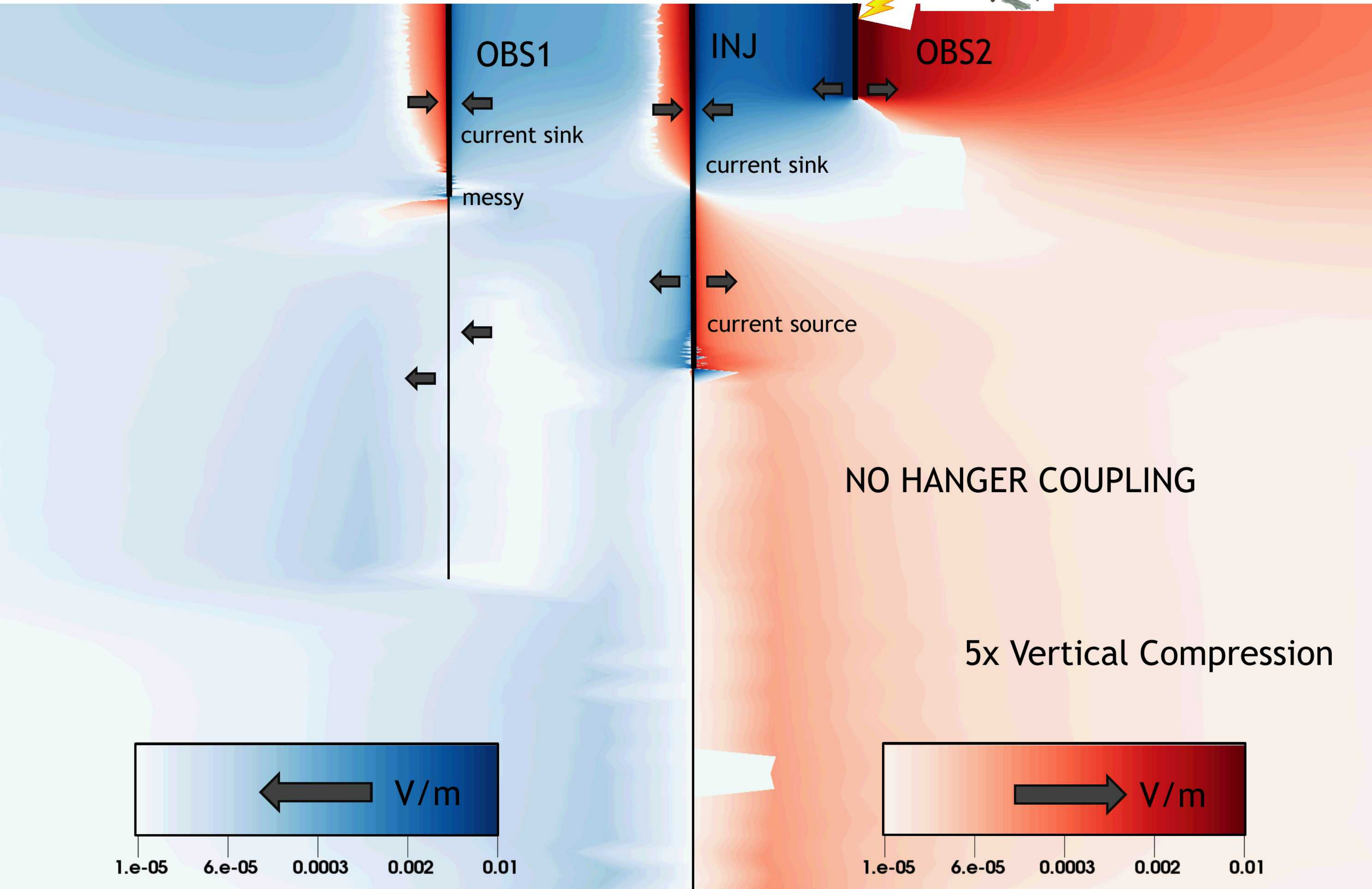
Single-conductor models (dashed) poorly approximate the current system of the dual-conductor (solid, surf + prod) models.

THE SURFACE PROFILE RESPONSE



Local errors from ignoring full completion model may be as high as a few 10s of percent.

CROSS-SECTION OF HORIZONTAL E-FIELD



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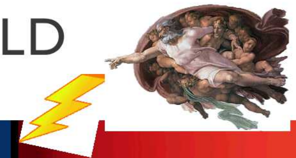

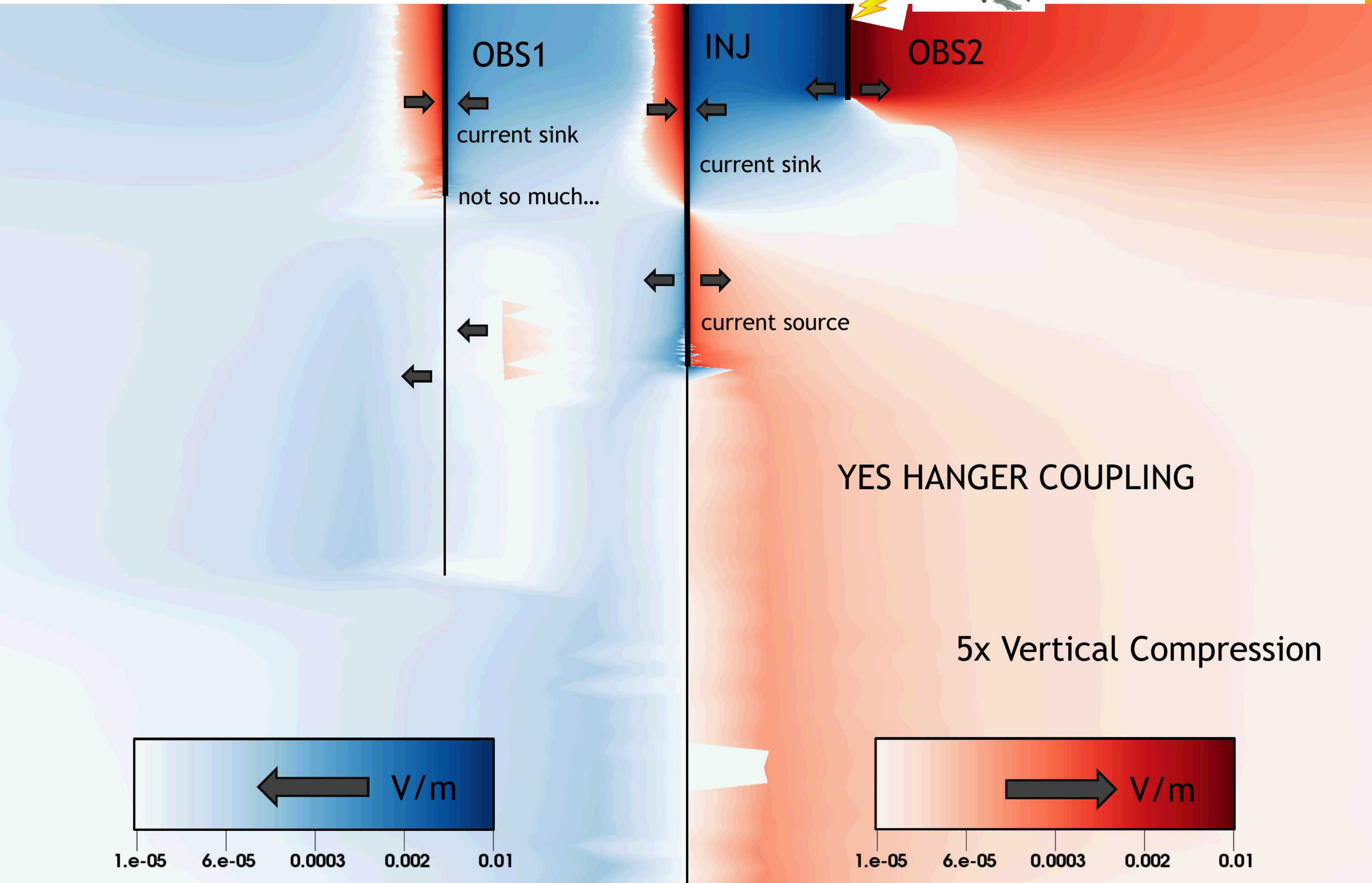

CONCLUSIONS AND PATH FORWARD

To avoid the high cost of volumetric discretization, the hierarchical FE method (Weiss, 2017) was used to simulate the borehole scattering potentials from complex, concentrically nested casing models based on the CaMI CO2 sequestration site.

The FE algorithm approximates the surface casing as a thin shell (true dimensions) surrounding a production casing approximated by a filamentary line, each endowed with conductivity of steel.

The FE algorithm performs as designed in benchmark studies and obeys the familiar convergence properties of standard FE.

Presence of these concentrically nested conductors and casing hanger, coupling the surface and production casings, significantly alters the current pathways in the casing circuit in ways that are not obviously approximated by a single, one-component casing system.

At the CaMI site, the effects of completion design on surface E-field measurements are on the order of 10s of percent, near or above the error estimates for collected field data.

MORE CONCLUSIONS AND OBSERVATIONS

Hierarchical material properties in finite element analysis offers a computationally economical way for modeling sharp, volumetrically insignificant regions, with elevated material property values (e.g. conductivity in electrostatics)

The reduction is computational burden over volumetric discretization can reach several orders of magnitude, thus leading to “real time” solutions and evaluation of problems previously believed intractable.

For the electrostatic problem, hierarchical FE solutions compare favorably with independent analytic solutions and are internally consistent with solutions from volume discretizations.

Hierarchical FE method has been applied to various “real world” oilfield examples with complex infrastructure. Although solution times are fast (10s of seconds to a minute or so), mesh generation and metadata management issues are more acute.