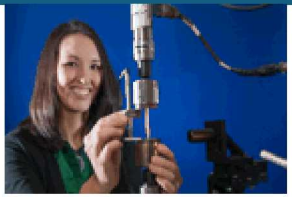
SAND2019-0501C

Investigating Nonlinearity in a Bolted Structure Using Force Appropriation Techniques



PRESENTED BY

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Structural Dynamics

Sandia National Laboratories

IMAC XXXVII 2019

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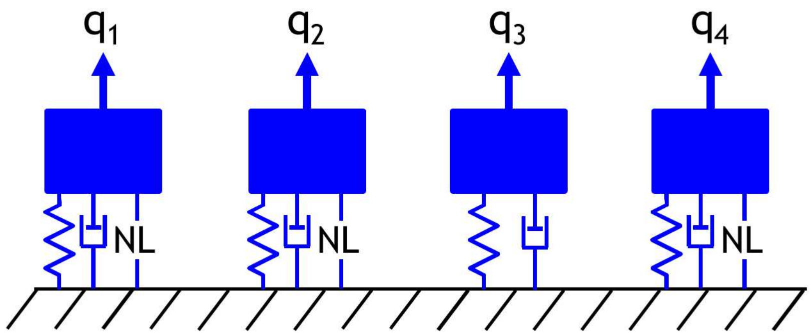
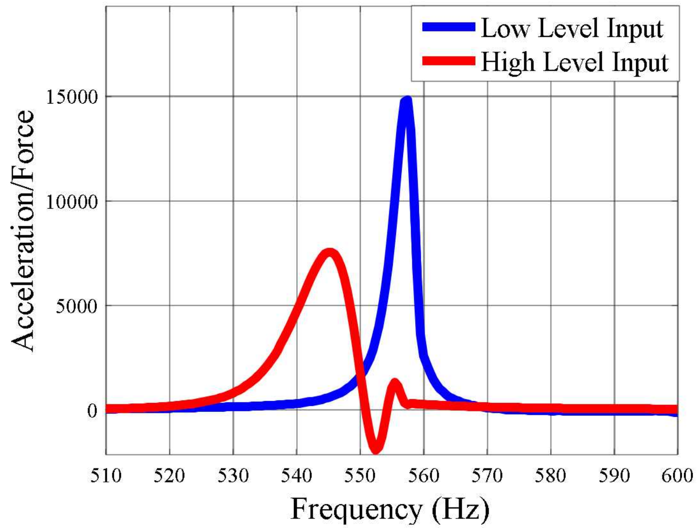
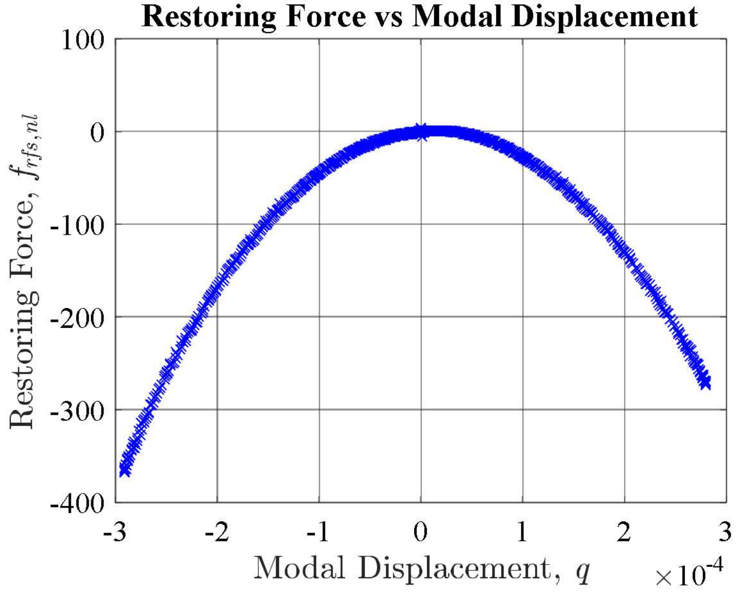
Background and Motivation

- Some structures are sufficiently nonlinear that linear models are unable to adequately capture important dynamics.
- Previous works have shown the ability of pseudo-modal modeling approaches to accurately capture the nonlinear characteristics of a structure
  - Nonlinear identification is achieved on a mode-by-mode basis
  - Excitation techniques have included hammer impacts and windowed sinusoids



- Extrapolation with nonlinear models is inadvisable, so higher responses must be achieved in testing
- Exciting a structure by dwelling at a resonant frequency of a targeted mode produces large responses and is closely aligned with Nonlinear Normal Mode (NNM) testing techniques
- **Therefore this work presents a preliminary exploration which uses a hybrid of NNM and pseudo-modal modeling testing and analysis techniques to investigate the nonlinear characteristics of a test article**





Brief Overview of Nonlinear Normal Mode Theory

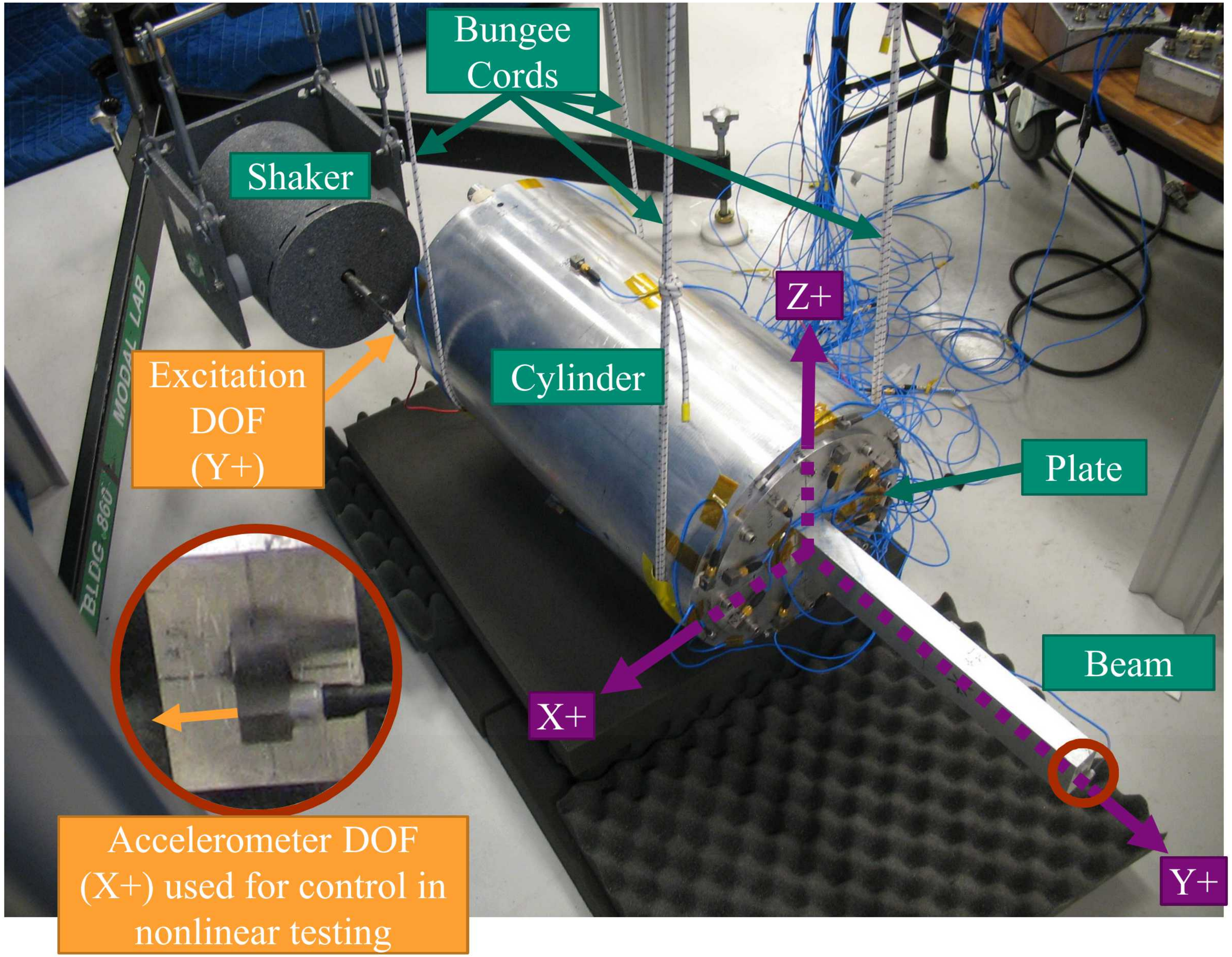
- Equation of motion of a nonlinear dynamic system
  - $\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) + \mathbf{u}_{nl,s}(\mathbf{x}(t)) + \mathbf{u}_{nl,d}(\dot{\mathbf{x}}(t)) = \mathbf{u}_e(t)$

Objective of nonlinear identification is to determine these quantities

- A Nonlinear Normal Mode (NNM) is a solution to the underlying conservative system
  - $\mathbf{M}\ddot{\mathbf{x}}_{\mathbf{nnm}} + \mathbf{K}\mathbf{x}_{\mathbf{nnm}} + \mathbf{u}_{nl,s}(\mathbf{x}_{\mathbf{nnm}}) = \mathbf{0}$
  - $\mathbf{x}_{\mathbf{nnm}}$ is the NNM response
- This implies
  - $\mathbf{C}\dot{\mathbf{x}}_{\mathbf{nnm}} + \mathbf{u}_{nl,d}(\dot{\mathbf{x}}_{\mathbf{nnm}}) = \mathbf{u}_e$
- Thus, if $\mathbf{x}_{\mathbf{nnm}} = \sum_n \mathbf{a}_n \cos(n\omega t)$ then $\mathbf{u}_e = \sum_n \mathbf{b}_n \sin(n\omega t)$
  - Harmonics due to nonlinearities
  - Excitation is 90° out of phase with response (phase quadrature)
  - Analogous to force appropriation testing to extract linear frequency and damping
  - This must hold for each harmonic of each DOF
- Peeters [1,2] showed NNM isolation can be approximately achieved using single input excitation

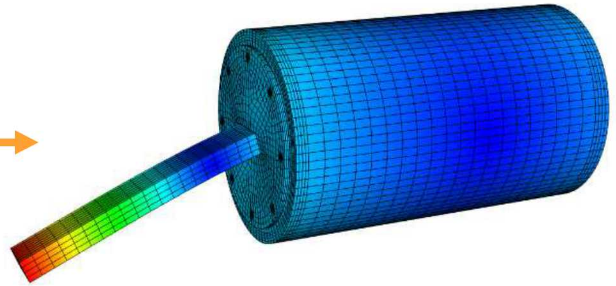
This phase quadrature criterion will be used in nonlinear testing to isolate the target mode

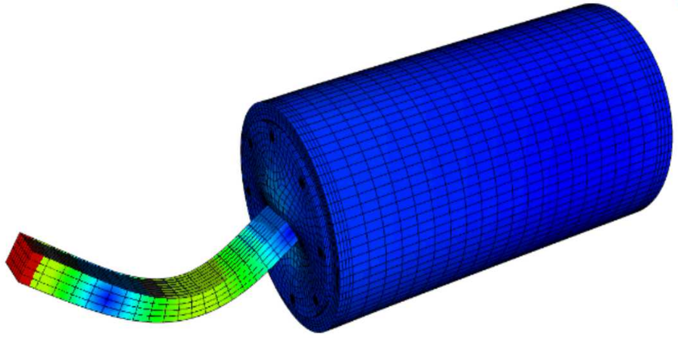
Test Set-Up and Linear Modal Analysis Results



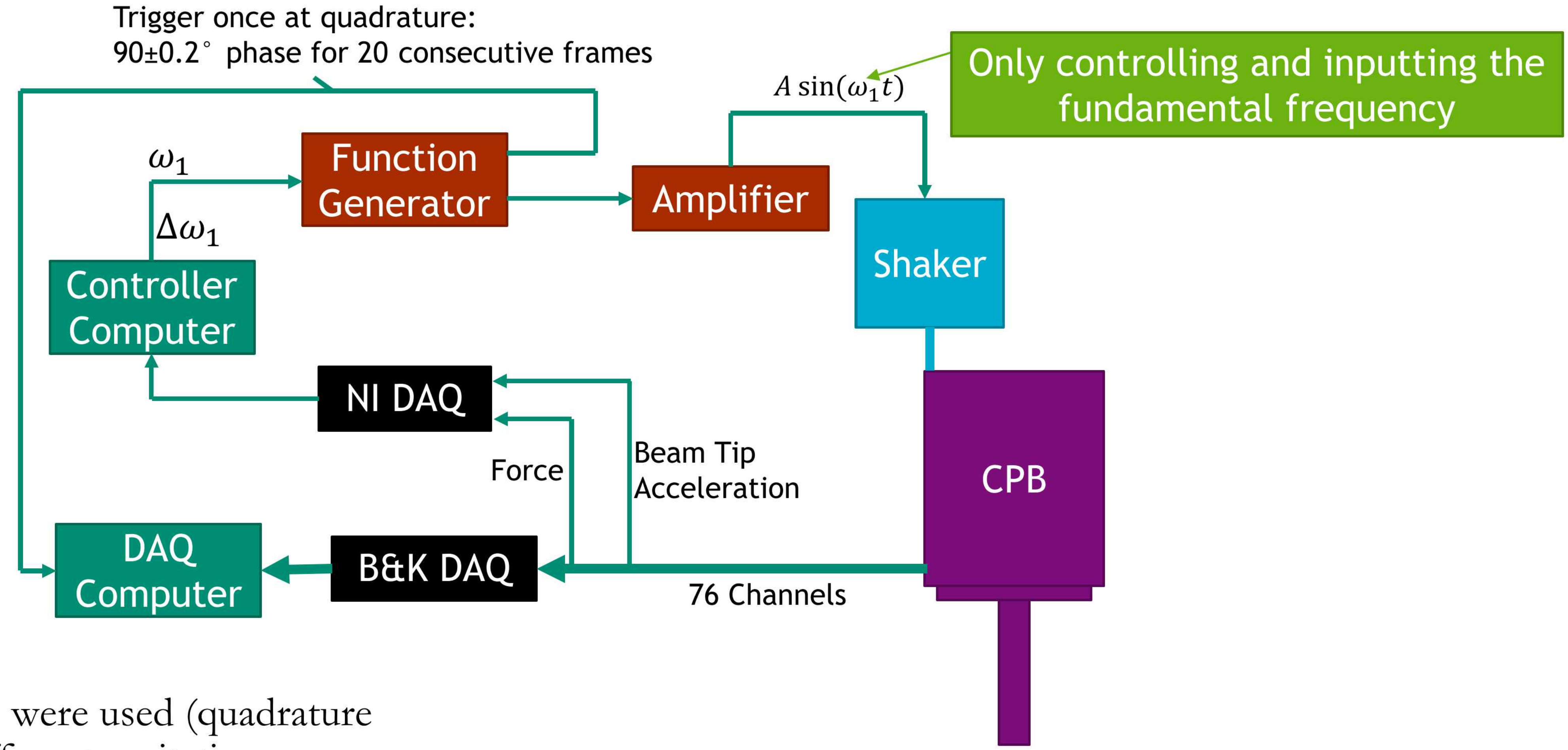
Mode*	Frequency (Hz)	Damping (%)	Description
7	129	0.61	1st bend of Beam in X-direction
8	171	0.09	1st bend of Beam in Z-direction
9	386	0.07	Ovaling of Cylinder
10	392	0.06	Ovaling of Cylinder
11	547	0.30	Axial mode
12	945	0.42	Ovaling of Cylinder
13	950	0.46	Ovaling of Cylinder
14	1025	0.08	2nd bend of Beam in X-direction
15	1224	0.42	Ovaling of Cylinder

*Rigid body modes not listed





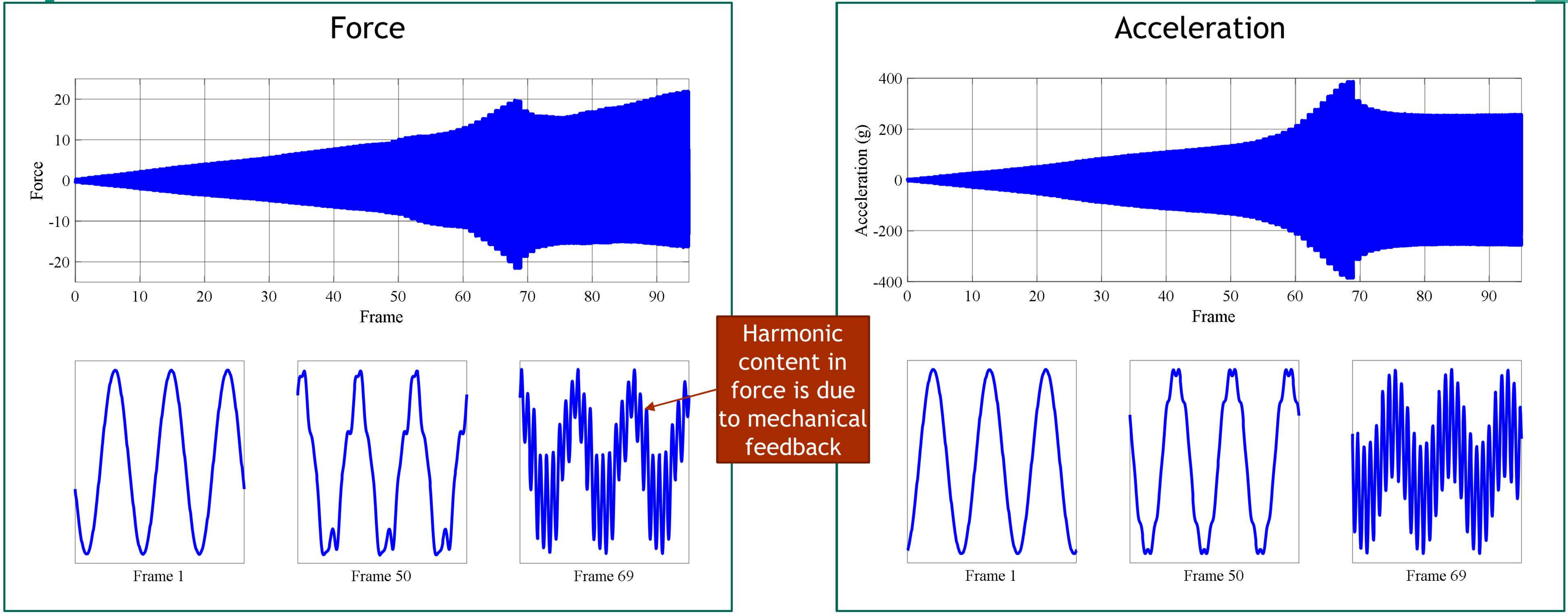
Control Scheme for Nonlinear Testing



- 95 voltage increments were used (quadrature was achieved for 95 different excitation amplitude levels)
  - Small increases in amplitude

Objective of nonlinear testing: excite mode 7 using force appropriation to characterize nonlinearity

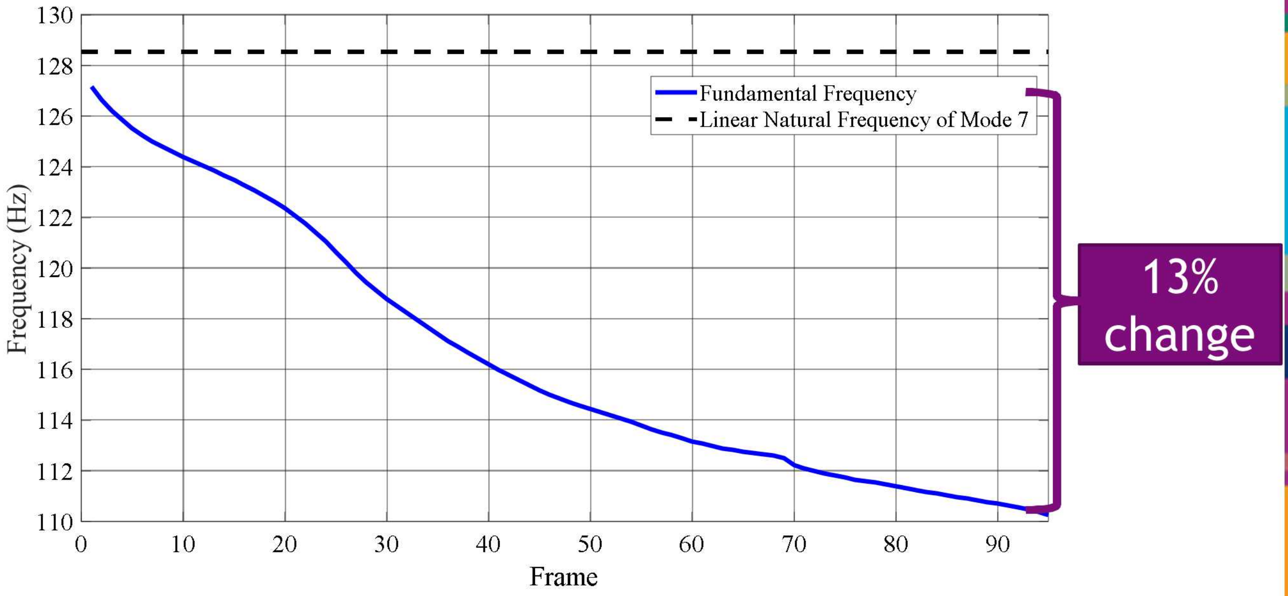
Force and Acceleration Response

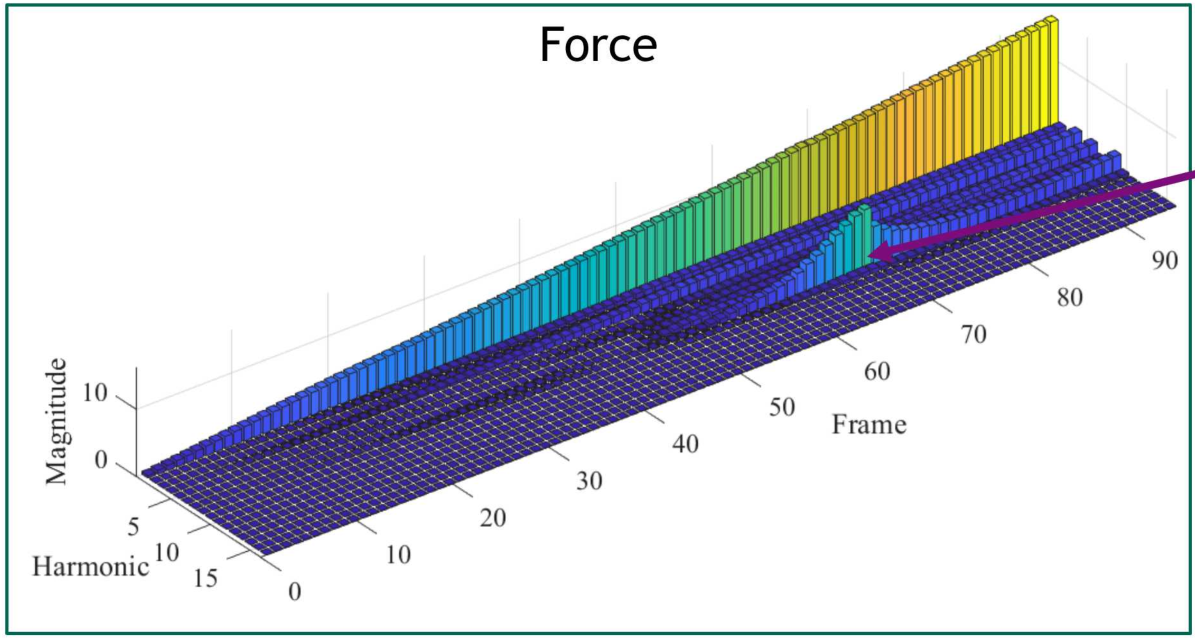


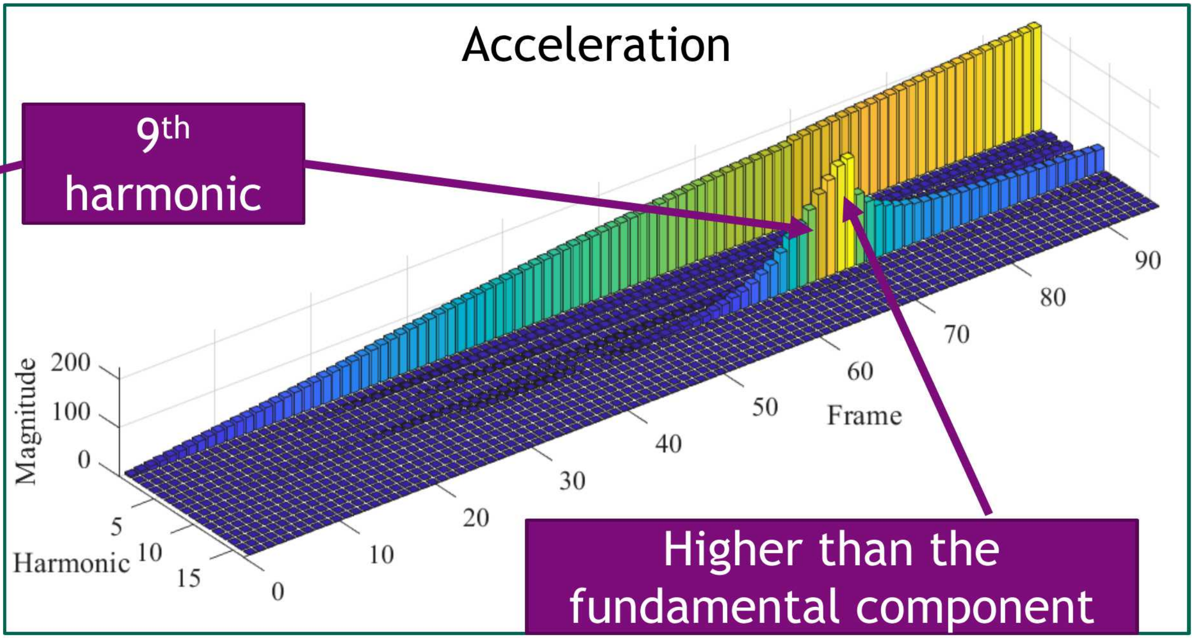
- Force appropriation testing controlled as desired and nonlinear effects evident in data
- Remainder of presentation will investigate nonlinear analogies of linear modal analysis:
  - Natural frequency, MIFs, mode shapes, and damping

Frequency Content and Tracking

- Measured signals were fit to a sum of sines and cosines for a selected number of harmonics
  - $r(t) = \sum_{k=1}^{N} A_k \sin(k\omega t) + B_k \cos(k\omega t)$
- Amplitude and phase of each harmonic was then computed
  - Plots below show amplitude of first 16 harmonics for each frame for the excitation force and control acceleration

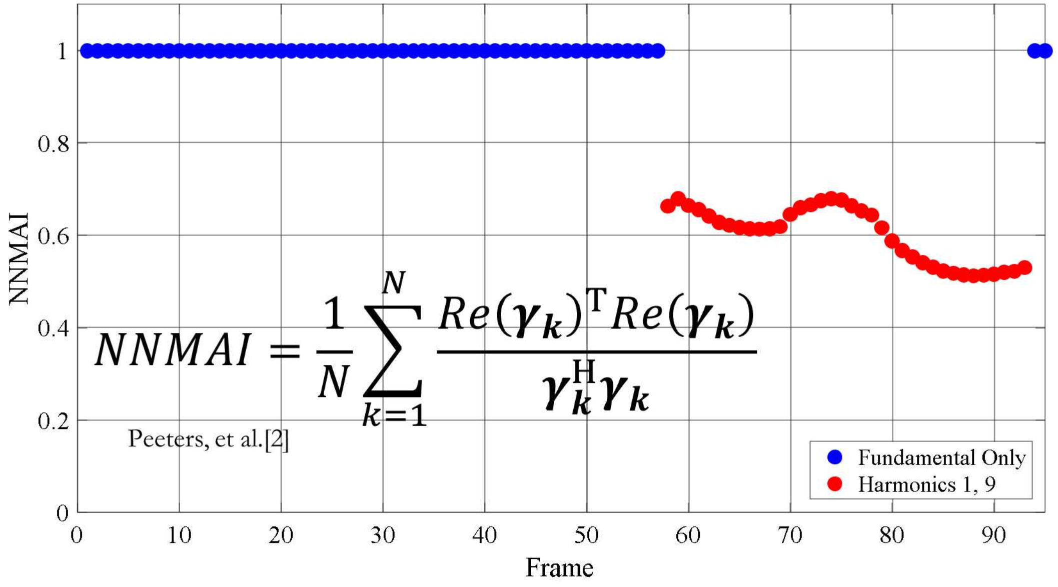






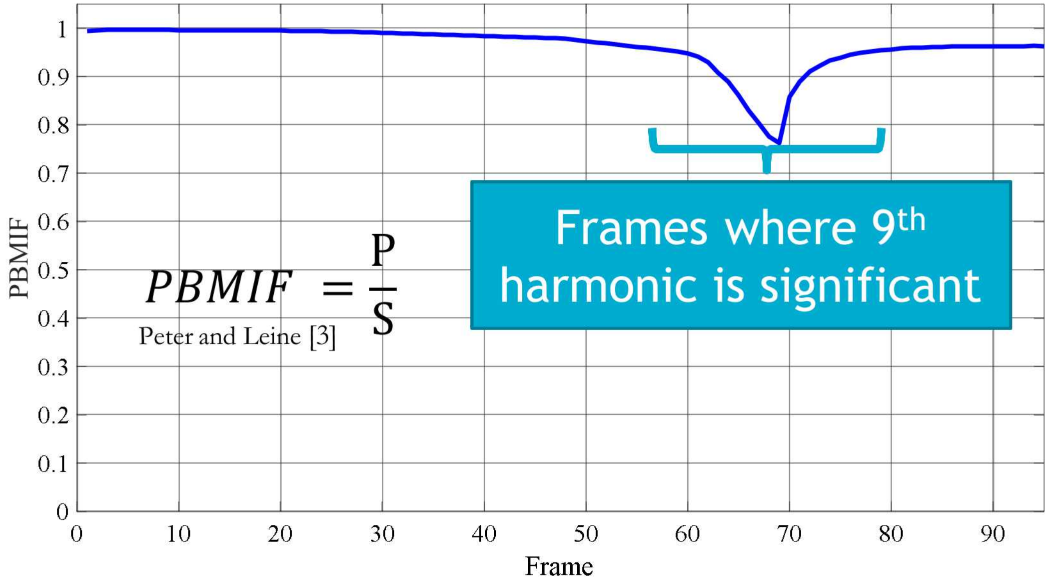
Mode Indicator Functions

Nonlinear Normal Mode Appropriation Indicator (NNMAI)

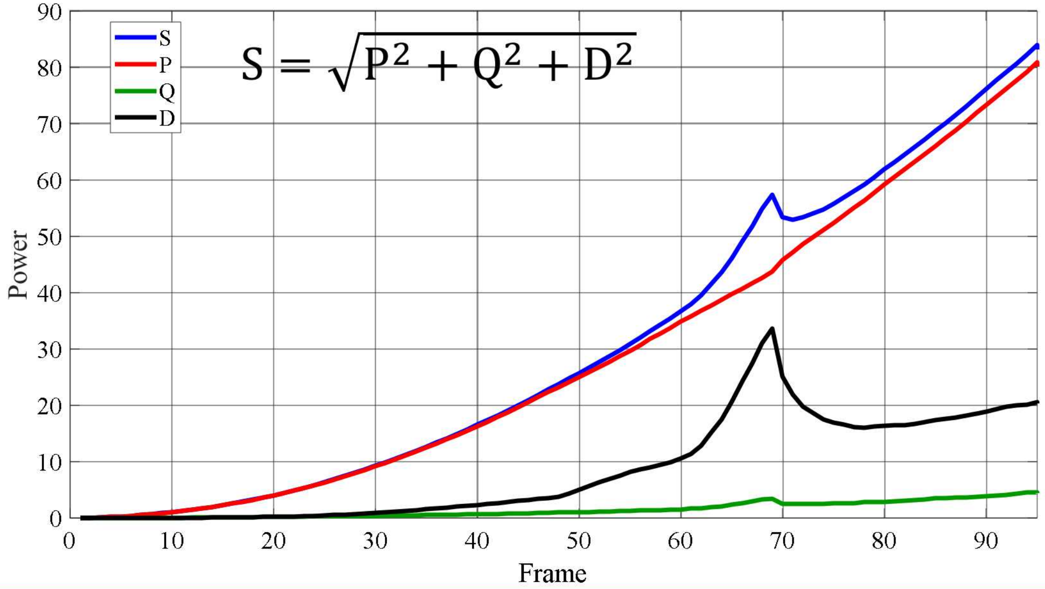


$$NNMAI = \frac{1}{N}\sum_{k=1}^{N}\frac{Re(\boldsymbol{\gamma_k})^{T}Re(\boldsymbol{\gamma_k})}{\boldsymbol{\gamma_k}^{H}\boldsymbol{\gamma_k}}$$

Power Based Mode Indicator Function (PBMIF)



$$PBMIF = \frac{P}{S}$$

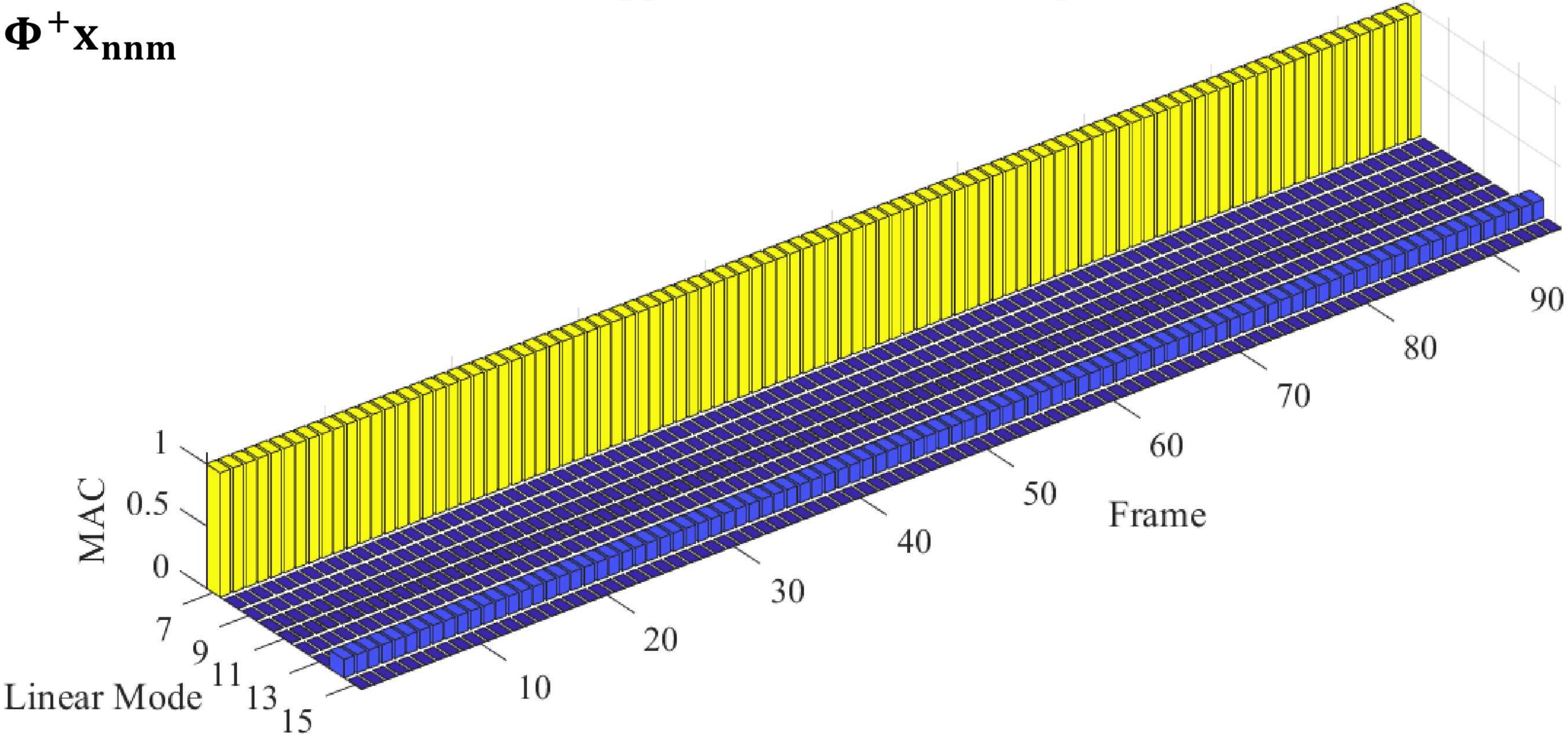


$$S = \sqrt{P^2 + Q^2 + D^2}$$

- Mode well isolated but purity is degraded when the 9th harmonic response becomes significant (≥20% of fundamental)
- This is not unexpected because the control system only operated on the fundamental frequency

Mode Shape Analysis

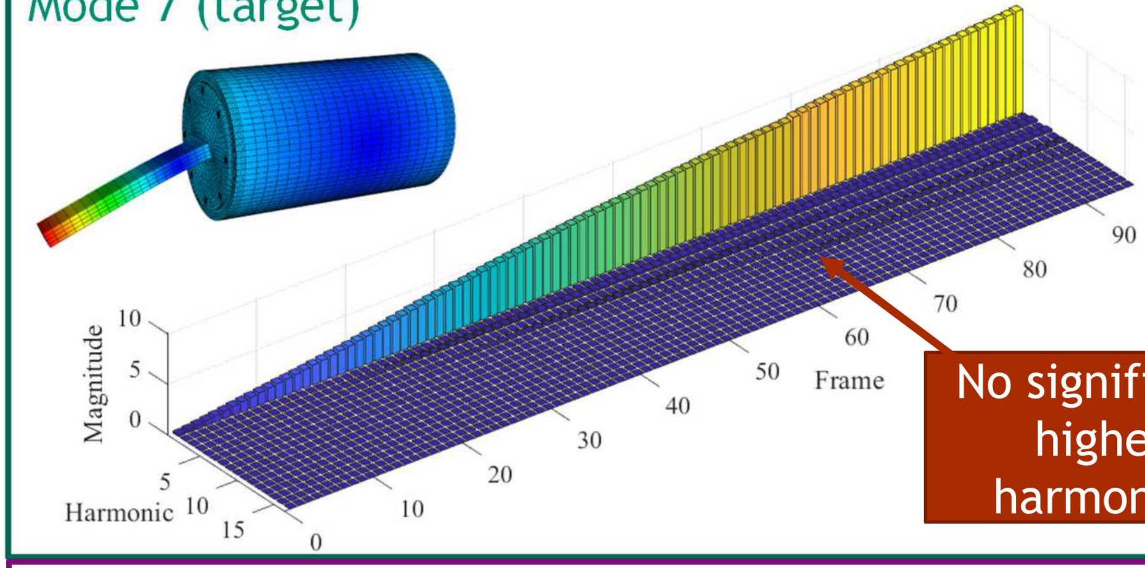
- Computed Modal Assurance Criterion (MAC) between the deflection shapes of the fundamental frequency and the linear mode shapes
- MAC value for mode 7 (target mode) does not drop below 0.999
  - Shape of target mode is not changing with amplitude
- Thus, we assume that a modal filter can be applied to isolate the responses of each mode
  - $\mathbf{q_{nnm}} = \mathbf{\Phi}^{+}\mathbf{x_{nnm}}$



Mode Shape Analysis

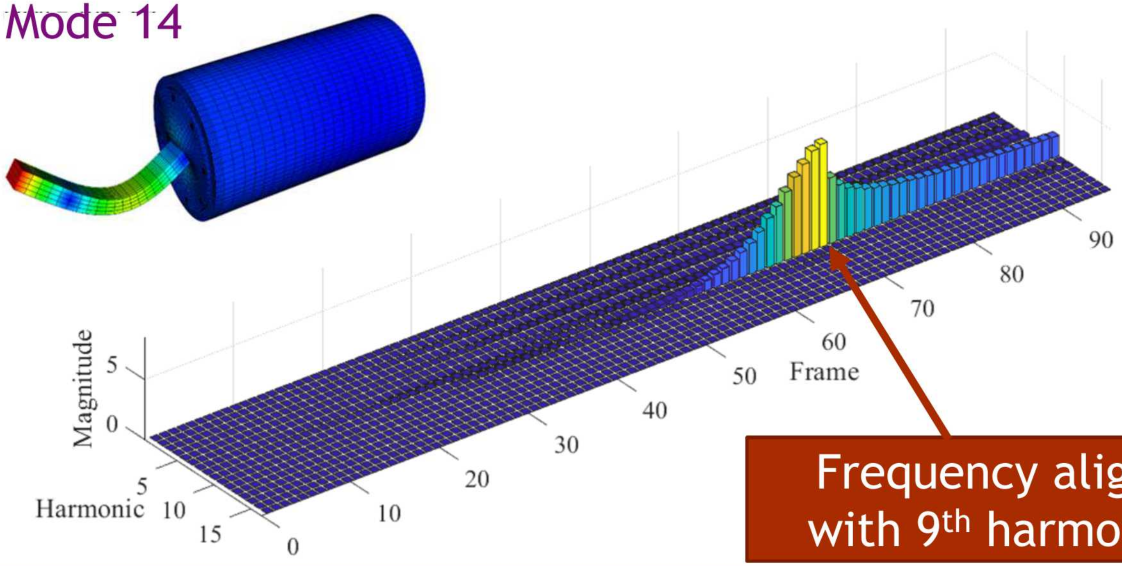
- The modal filter was applied to the response data to investigate the frequency content of each mode

Mode 7 (target)

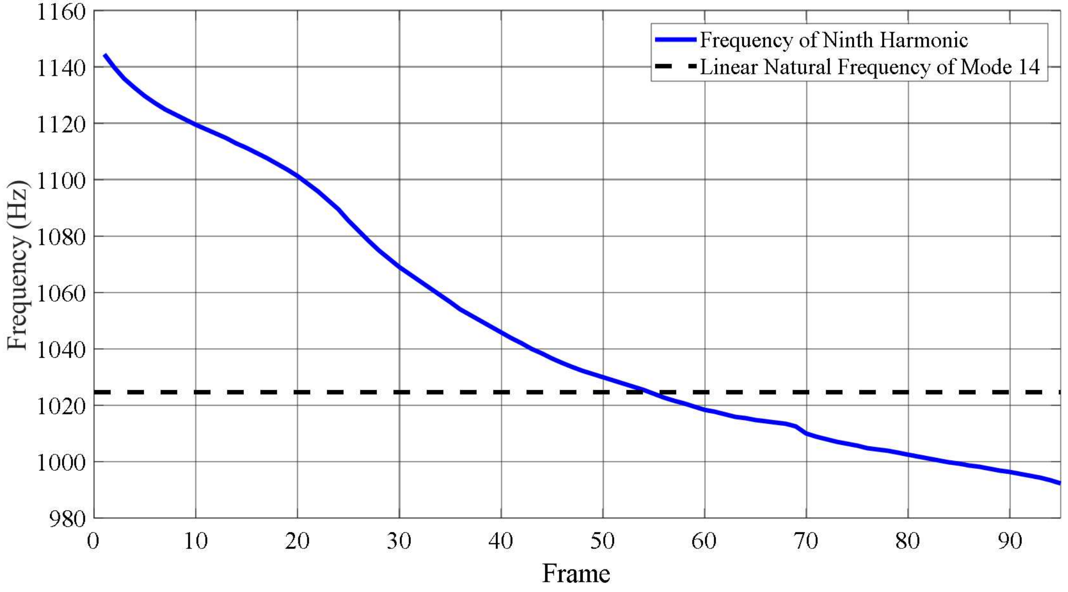


No significant higher harmonics

Mode 14



Frequency aligns with 9th harmonic



Higher harmonics seen in physical responses are due to a modal interaction between modes 7 and 14

Conclusions and Future Work

- A SISO control system was used to excite mode 7 by achieving quadrature between the excitation and control location
- Achieved maximum acceleration of 400g while being well below amplifier limits
- The response of the 9^{th} harmonic was due to an interaction between modes 7 and 14
- The uncontrolled response of the 9^{th} harmonic increased uncertainty in the purity of NNM response as indicated by MIFs
- Future work will focus on multi-harmonic control to achieve quadrature for all significant harmonics
- A framework for extracting nonlinear damping has been started, but more development is needed

[1] M. Peeters, G. Kerschen and J. Golinval, "Dynamic testing of nonlinear vibrating structures using nonlinear normal modes," *Journal of Sound and Vibration,* vol. 330, pp. 486-509, 2011.
[2] M. Peeters, G. Kerschen and J. Golinval, "Modal testing of nonlinear vibrating structures based on nonlinear normal modes: Experimental demonstration," *Mechanical Systems and Signal Processing,* vol. 25, pp. 1227-1247, 2011.
[3] S. Peter and R. I. Leine, "Excitation power quantities in phase resonance testing of nonlinear systems with phase-locked-loop excitation," *Mechanical Systems and Signal Processing,* vol. 96, pp. 139-158, 2017.

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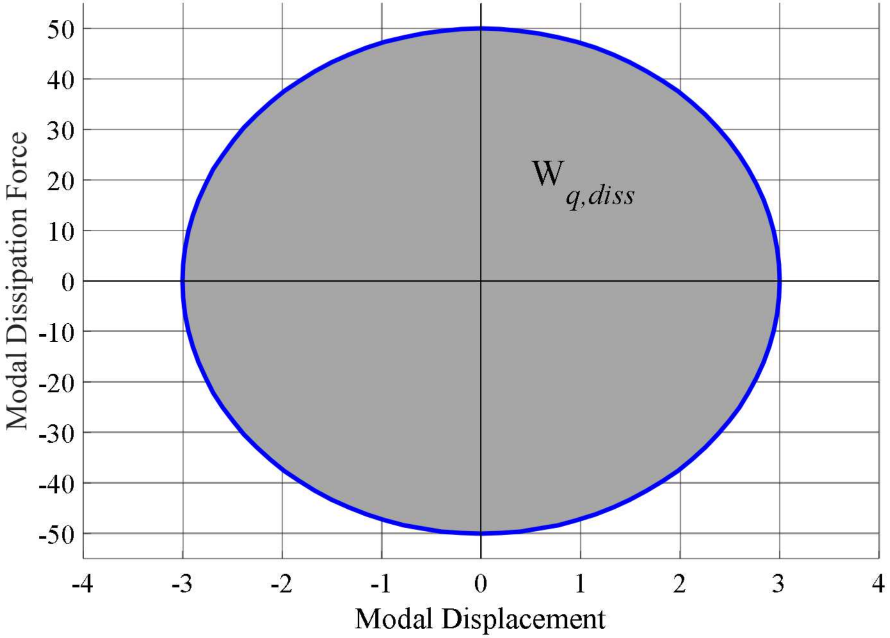
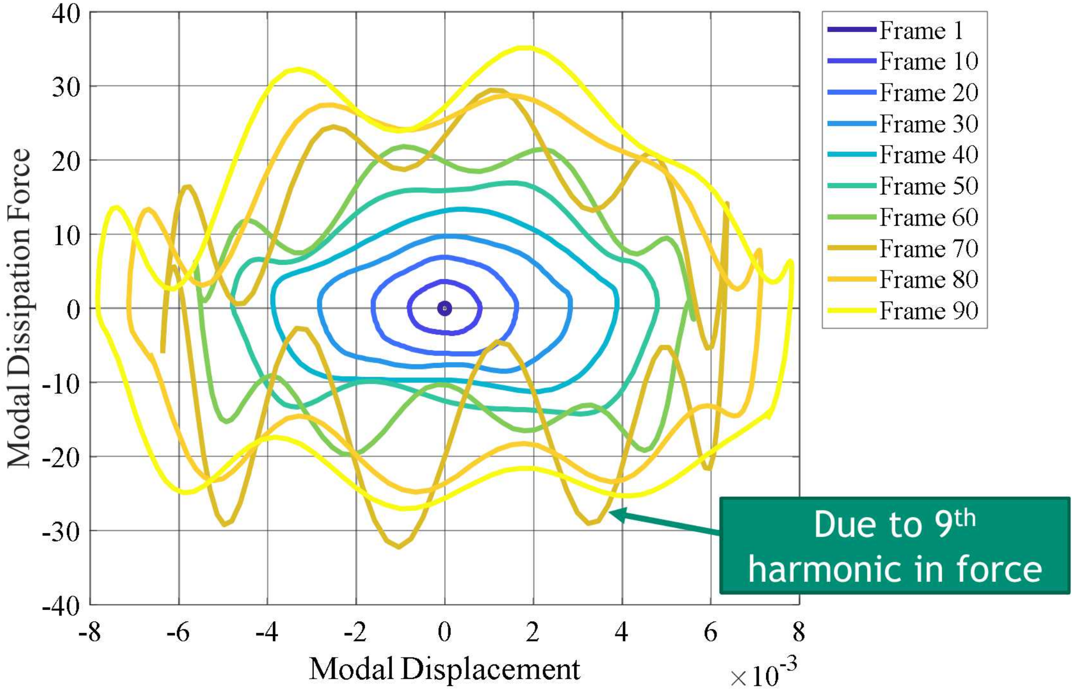
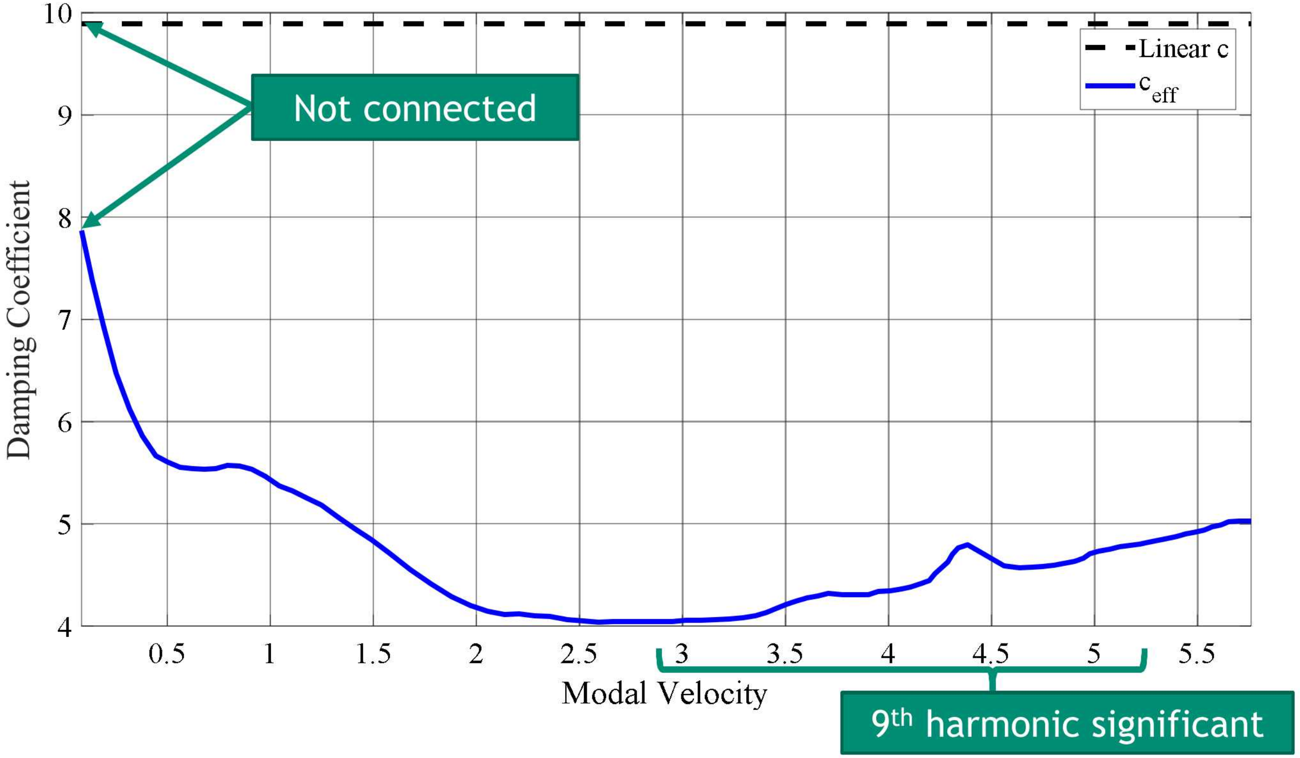
Nonlinear Damping Estimation

- Nonlinear damping is computed using the energy dissipated per cycle of oscillation of the target mode
- The work done by the modal dissipation force, $W_{q,diss}$, is the integral of the modal dissipation force, $u_{q,d}$, times the modal displacement, q
  - $W_{q,diss} = \oint u_{q,d} dq$
- If the NNM is appropriately isolated, the modal dissipation force is equal to the modal excitation force, $u_{q,e}$. Thus
  - $W_{q,diss} = \oint u_{q,e} dq = \oint u_{q,e} \dot{q} dt = \oint c_{eff} \dot{q}^2 dt$
  - $u_{q,e} = u_{q,d} = c_{eff} \dot{q}$
  - c_{eff} is analogous to $2\zeta\omega_n$ and is assumed constant for each excitation level
- c_{eff} can be tracked to determine its dependency on response amplitude



Nonlinear Damping Estimation



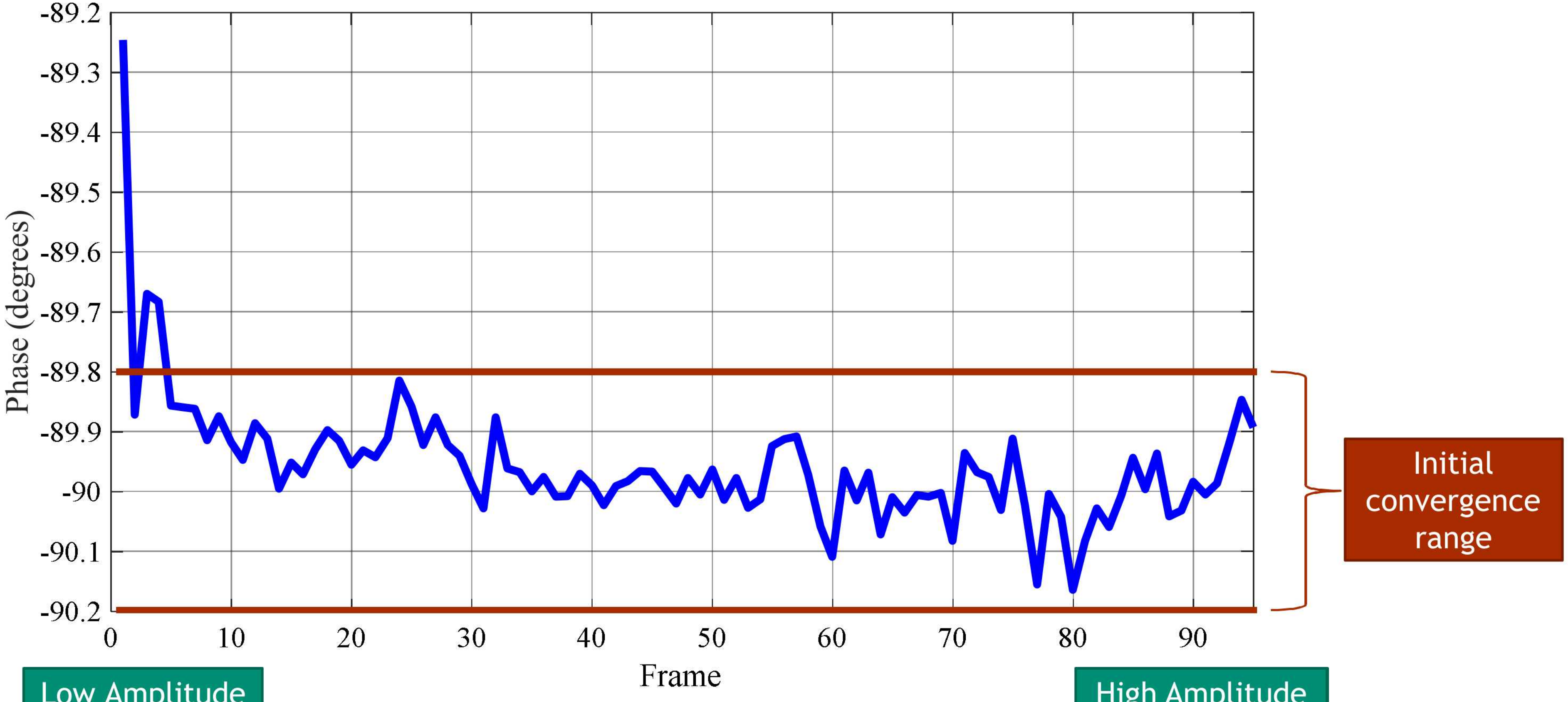


- Oscillations in modal dissipation force vs displacement due to 9th harmonic in excitation force
- Low level c_{eff} does not connect with linear value
  - Not achieving low enough amplitude in nonlinear testing?
  - Throughout the many tests (low and high level) on CPB, the damping varied
  - Epoxy between plate and beam deteriorated (eventually failed altogether)
- There is higher uncertainty in damping where 9th harmonic is significant

Controller Performance

- If quadrature was not achieved after 50 tries, the convergence tolerances were doubled
  - Convergence criteria reset after each frame

- Due to noise, the low level data needed more control iterations to converge, thus the final phase error is beyond the original 0.2°



Mode Indicator Functions

- Two MIFs are shown here to judge purity of NNM isolation

Nonlinear Normal Mode Appropriation Indicator (NNMAI)

- Developed by Peeters, et al.[2]
- $NNMAI = \frac{1}{N}\sum_{k=1}^{N} \frac{Re(\boldsymbol{\gamma_k})^{\mathrm{T}} Re(\boldsymbol{\gamma_k})}{\boldsymbol{\gamma_k}^{\mathrm{H}}\boldsymbol{\gamma_k}}$

Number of included harmonics (N)

Vector of complex Fourier coefficients of acceleration response ($\boldsymbol{\gamma_k}$)

- Assumes a purely sinusoidal input (purely imaginary in the complex plane)
- NNMAI is a measure of how purely cosinusoidal the response of each harmonic is (purely real in complex plane)
- Equals 1 when NNM is perfectly isolated

Power Based Mode Indicator Function (PBMIF)

- Developed by Peter [3]
- Derived from electrical engineering concepts
- Uses only the drive point force and response
- $PBMIF = \frac{\mathrm{P}}{\mathrm{S}}$
  - $\mathrm{S} = \sqrt{\mathrm{P}^2 + \mathrm{Q}^2 + \mathrm{D}^2}$

Apparent Power (S): RMS measure of total power

Active Power (P): Power associated with response in quadrature

Reactive Power (Q): Power associated with response 90° out of quadrature

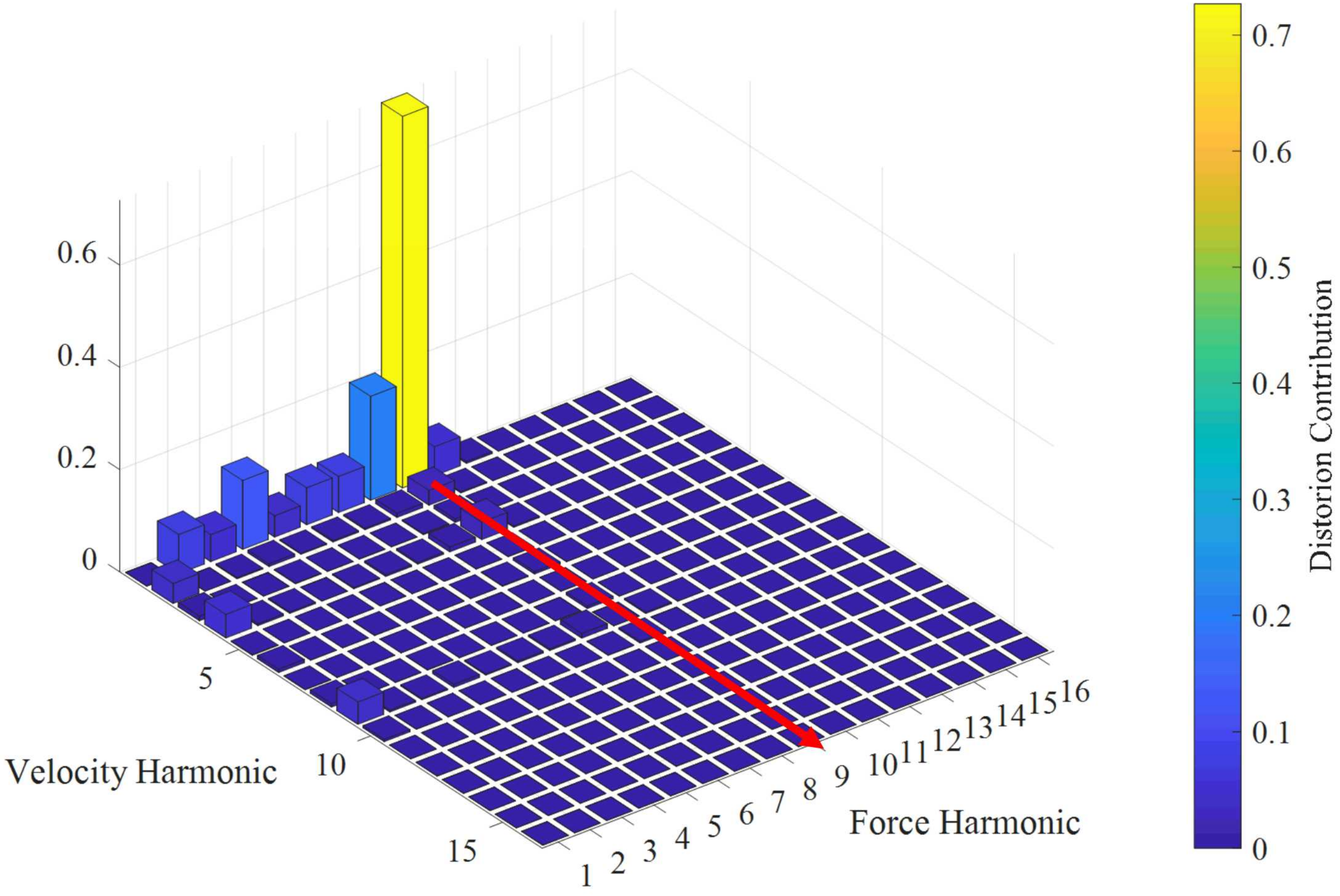
Distortion Power (D): Power exchanged between different harmonics in force and velocity

  - NNM is perfectly isolated when Q = D = 0, resulting in PBMIF = 1

[2] M. Peeters, G. Kerschen and J. Golinval, "Modal testing of nonlinear vibrating structures based on nonlinear normal modes: Experimental demonstration," *Mechanical Systems and Signal Processing*, vol. 25, pp. 1227-1247, 2011.

[3] S. Peter and R. I. Leine, "Excitation power quantities in phase resonance testing of nonlinear systems with phase-locked-loop excitation," *Mechanical Systems and Signal Processing*, vol. 96, pp. 139-158, 2017.

Distortion Power Terms at Frame 69



Mode Shape Analysis—Other Potential Interactions

- Plotting the evolution of all harmonics with the linear natural frequencies will show potential modal interactions with the target mode
- There are many instances of harmonics crossing linear natural frequencies of higher modes, but only mode 14 was shown to significantly respond in the measured data
- Potentially indicates that there must be shape compatibility in addition to frequency/harmonic alignment for modes to interact