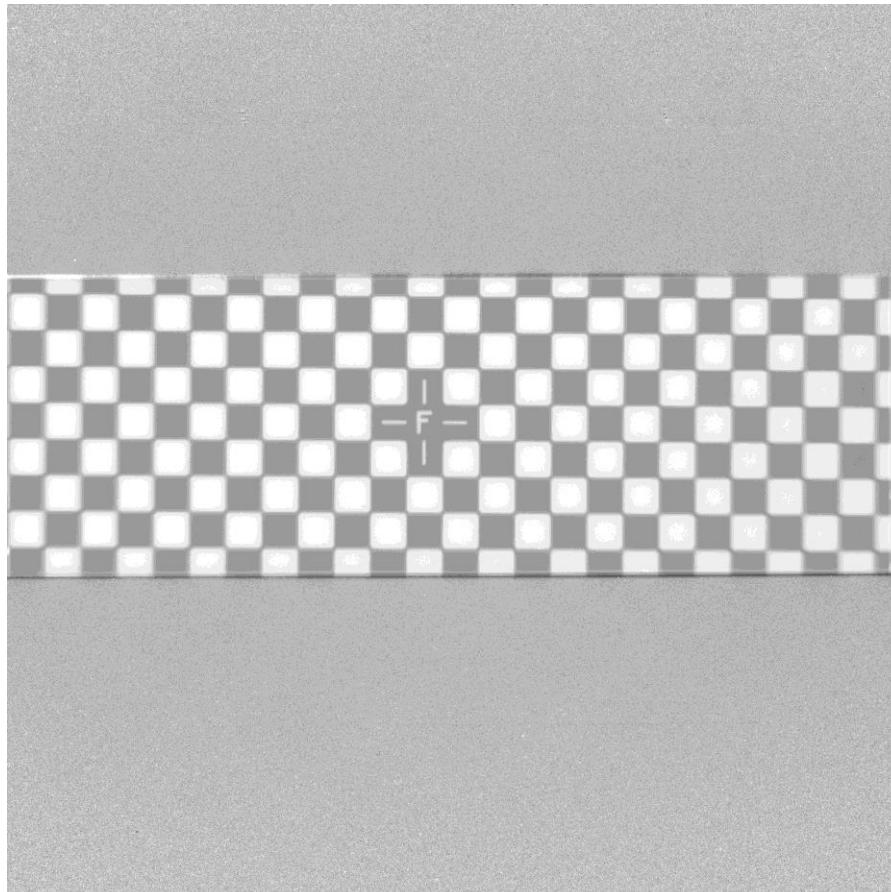


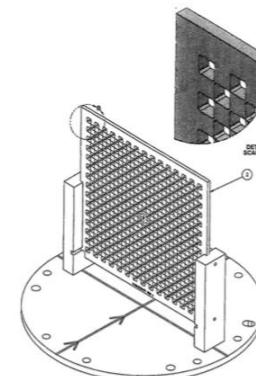
Advanced Characterization of Spatial Aspects of Image System Blur

Experiments in Spatially Varying Deconvolution

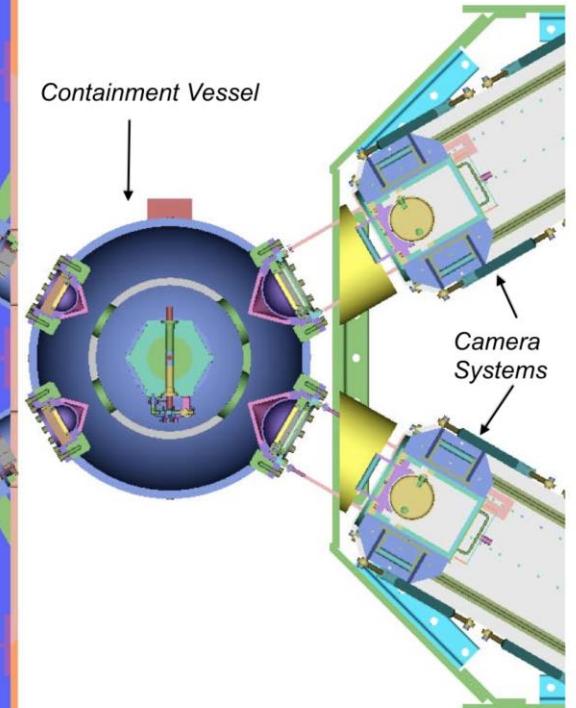
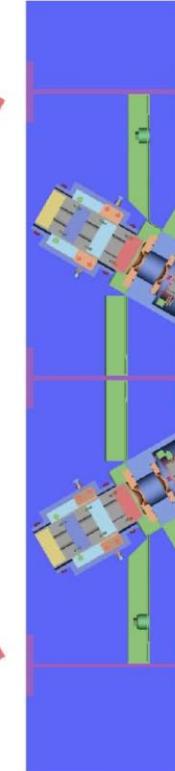
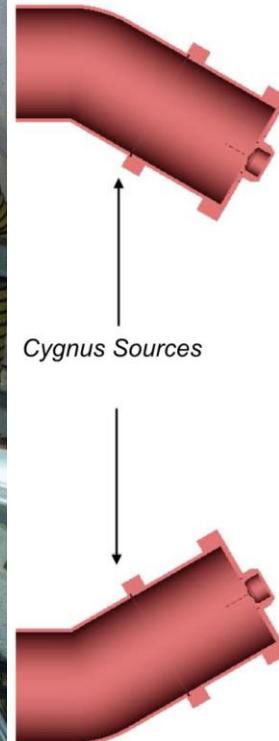


Daniel Frayer (PI), Marylesa Howard,
Jessica Pillow, Jesse Adams

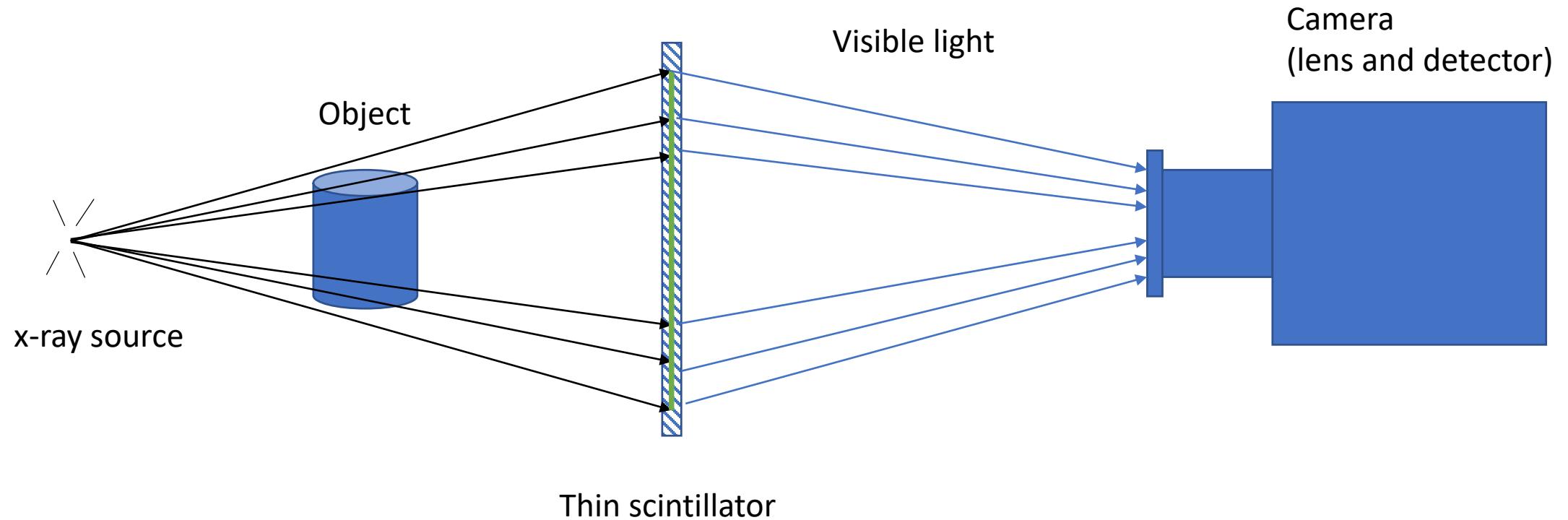
With contributions from
Matthias Morzfeld (U of Arizona),
Jessica Pillow (U of Arizona),
Kevin Joyce (LLNL),
Aaron Luttman (PNNL)



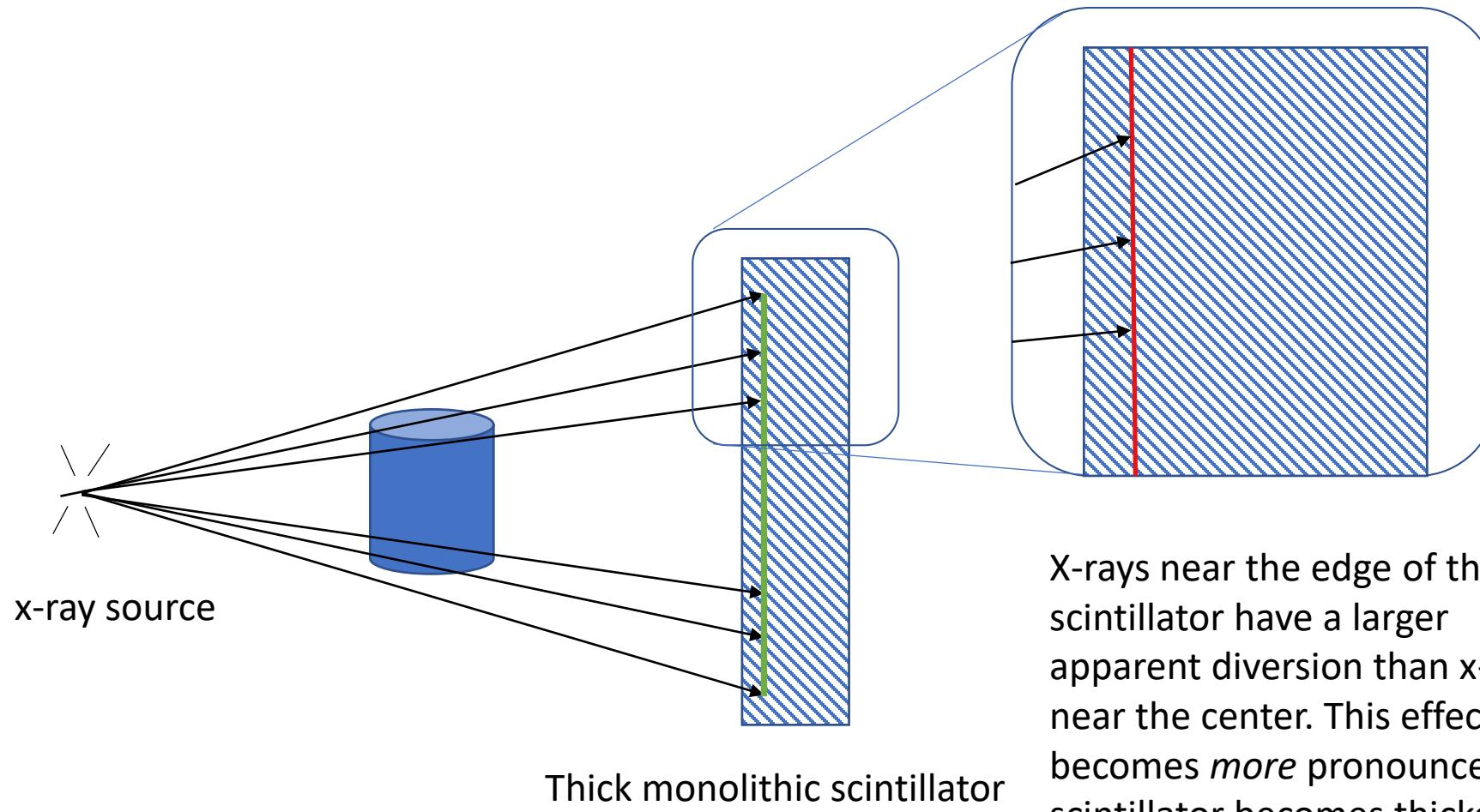
Radiography at the NNSS



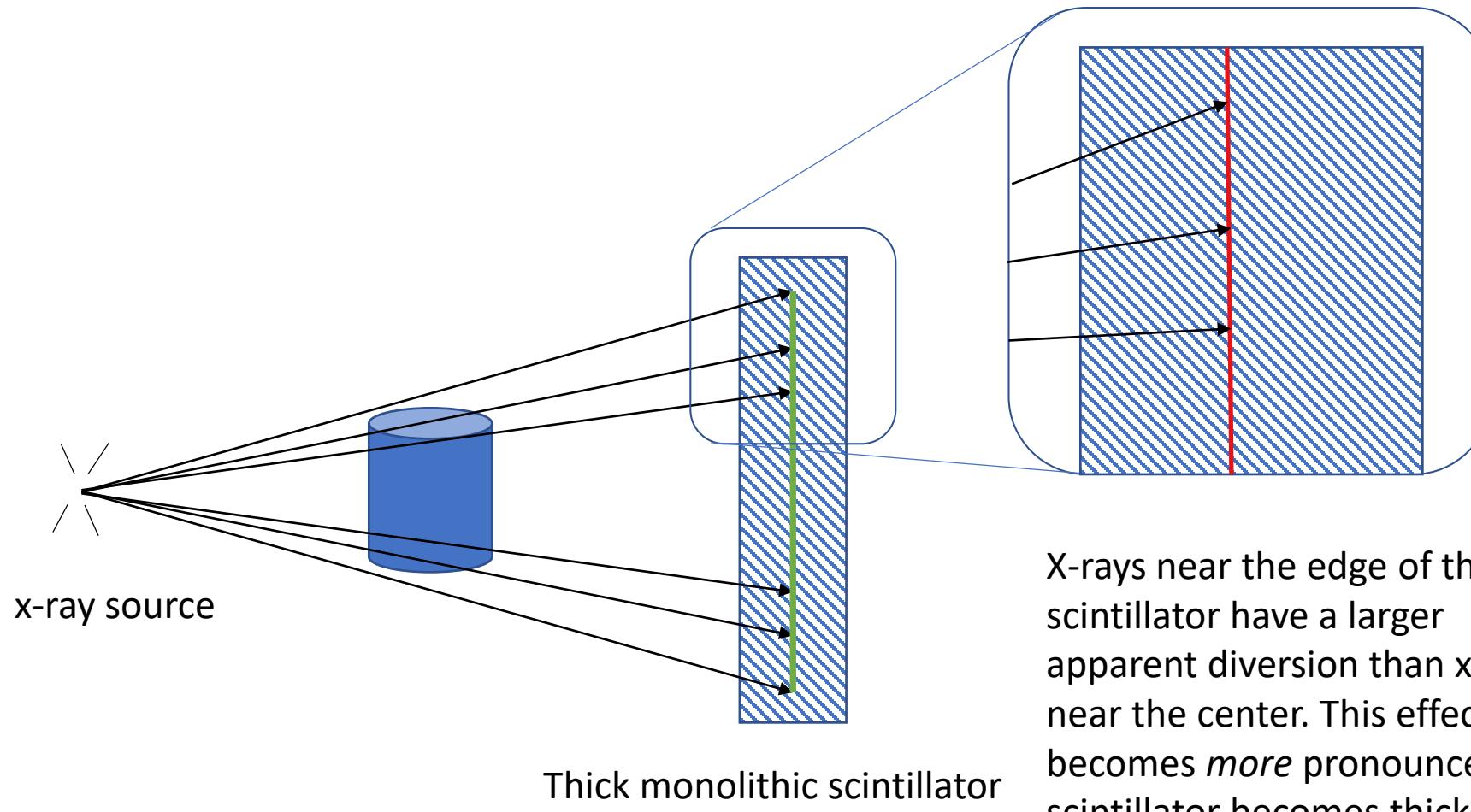
Blur in x-ray imaging



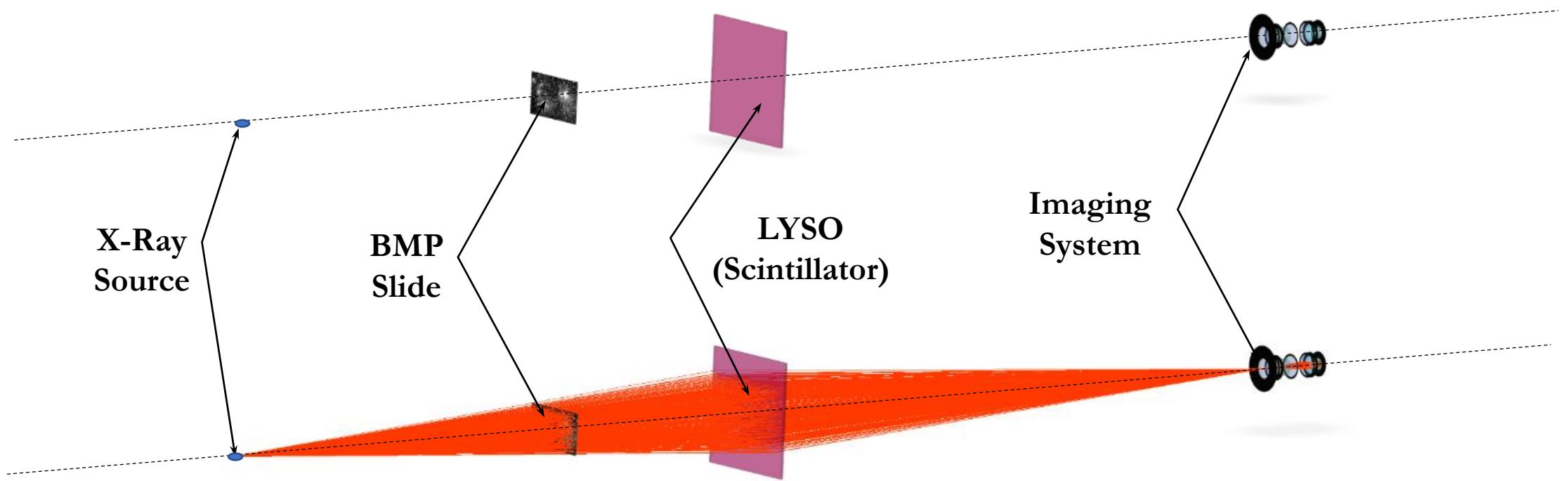
Spatially varying blur in x-ray imaging with thick scintillators and cone beams



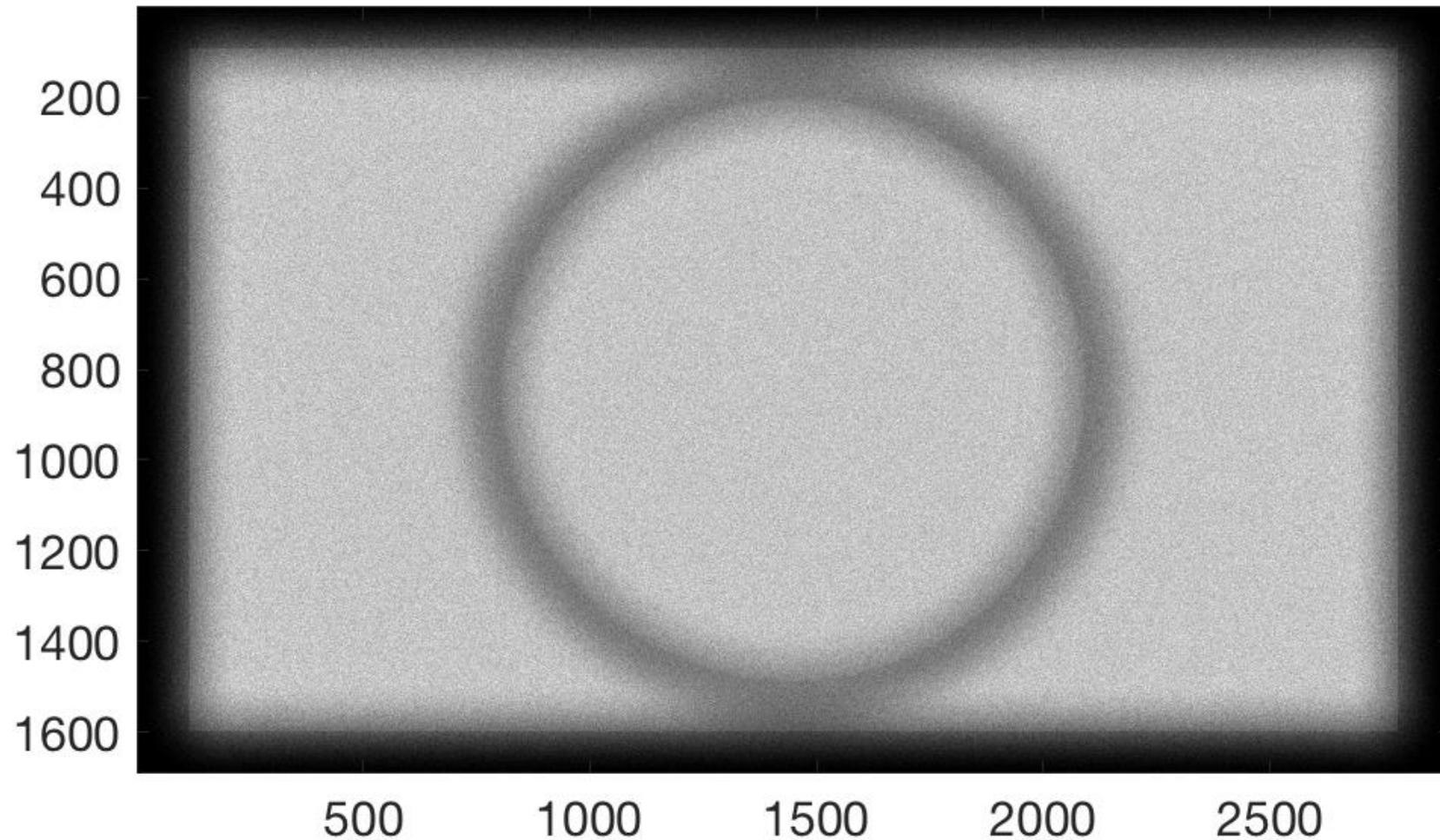
Spatially varying blur in x-ray imaging with thick scintillators and cone beams



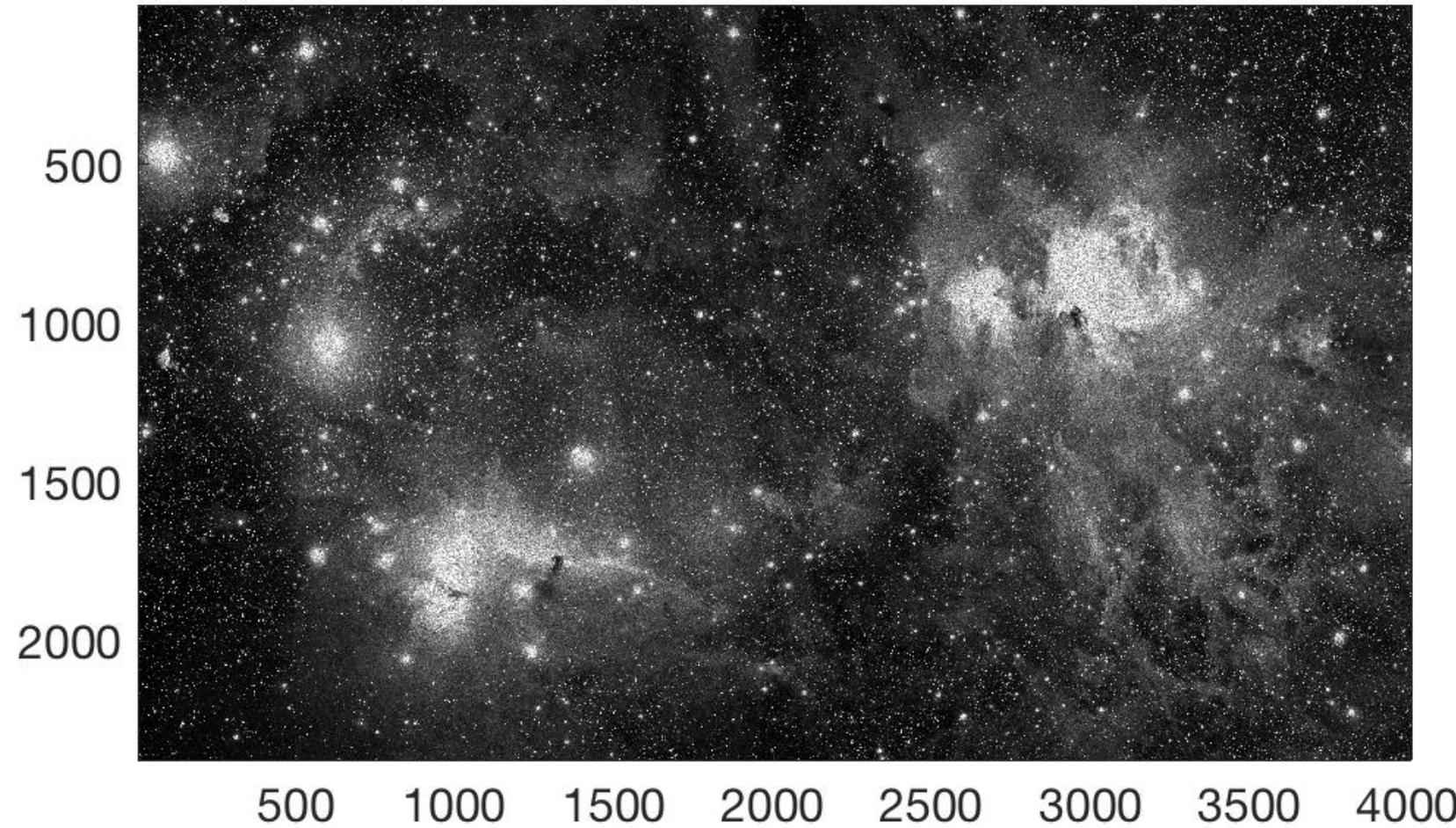
Synthetic data from optical software



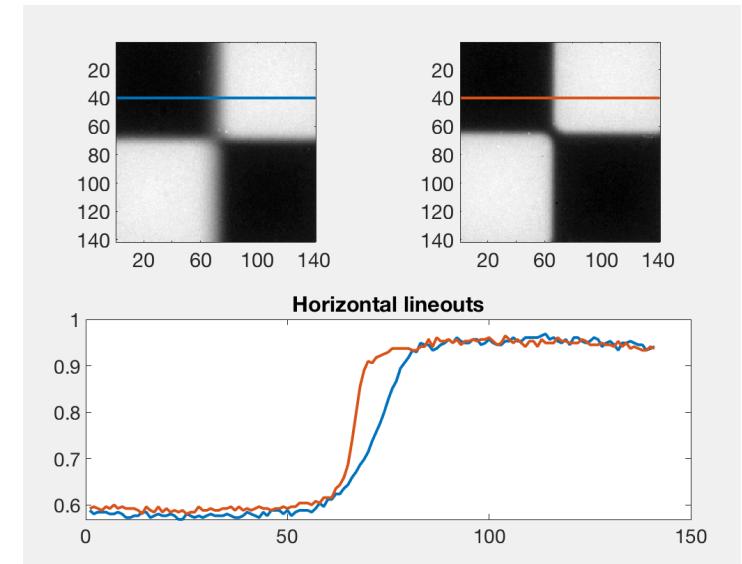
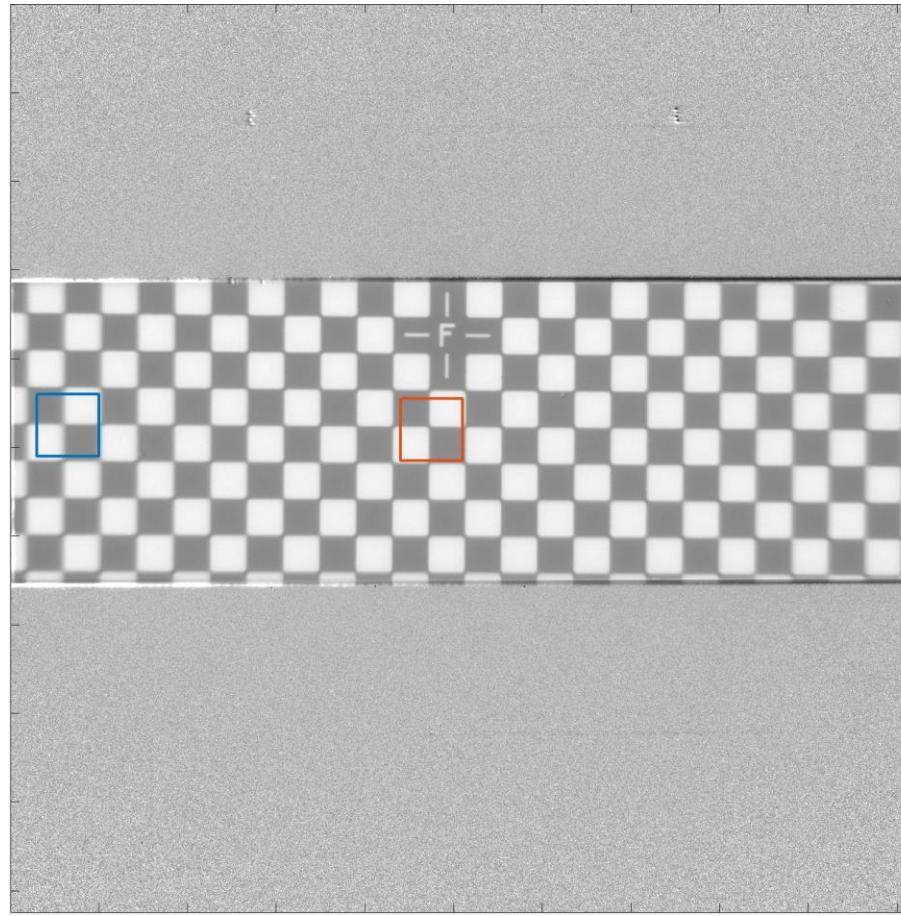
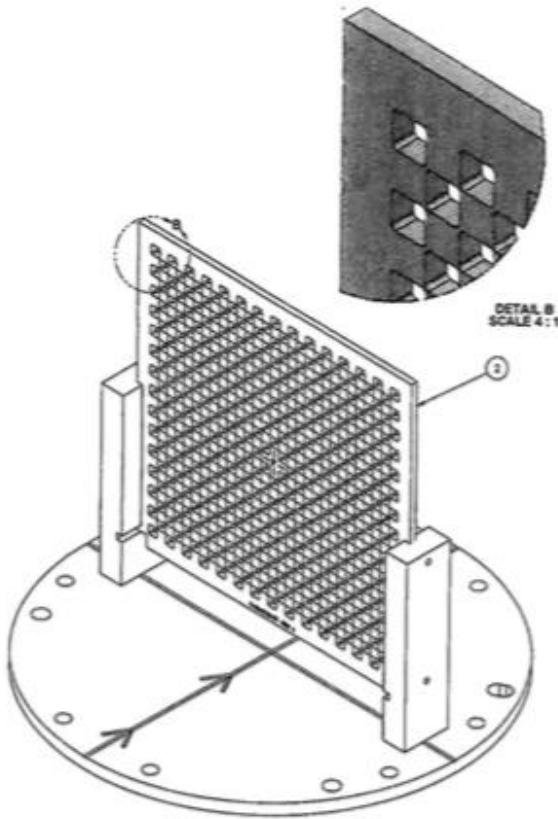
Synthetic data example 1: radially symmetric object



Synthetic data example 2: Orion nebula



Real data: checkerboard object



Standard deconvolution model

- \mathbf{B} : image data
 - Blurred and noisy
 - $\mathbf{b} = \text{vec}(\mathbf{B})$: column stacked image data
- \mathbf{X} : image reconstruction
 - Unknown, deblurred
 - $\mathbf{x} = \text{vec}(\mathbf{X})$: column stacked image
- \mathbf{A} : Blurring matrix
 - Based on a known blurring kernel

$$\begin{aligned}\mathbf{b} &= \mathbf{A}\mathbf{x} \\ \mathbf{x} &= (\mathbf{A}^\top \mathbf{A} + \gamma \mathbf{L})^{-1} \mathbf{A}^\top \mathbf{b}\end{aligned}$$

- γ : regularization scaling parameter
- \mathbf{L} : regularization matrix

Standard deconvolution model with UQ

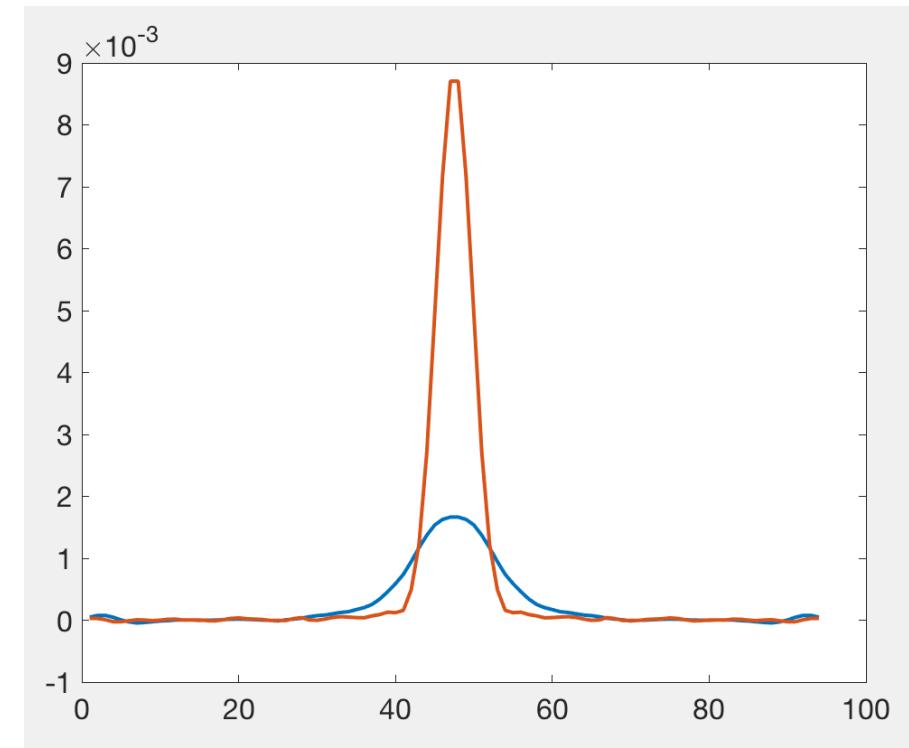
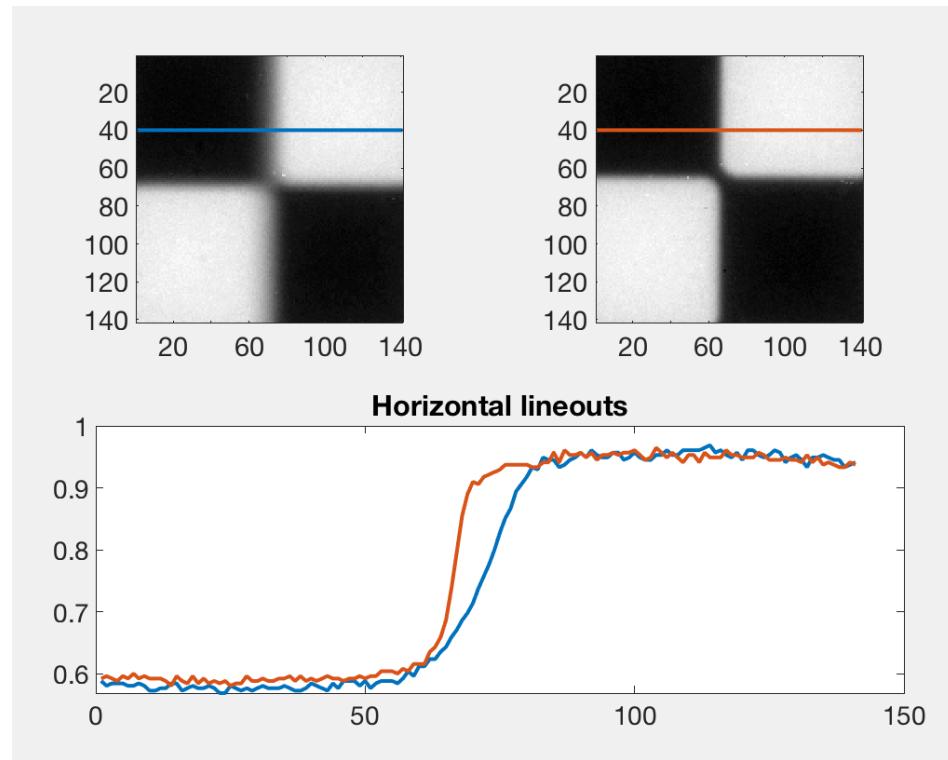
- Likelihood:
 - $\mathbf{b}|\mathbf{x}, \lambda \sim N(\mathbf{Ax}, \lambda^{-1}\mathbf{I})$
 - λ : likelihood precision
- Prior:
 - $\mathbf{x}|\delta \sim N(\mathbf{0}, (\delta\mathbf{L})^{-1})$
 - δ : prior precision
- \mathbf{A} : Blurring matrix
 - Based on a known blurring kernel
- \mathbf{L} : regularization matrix
- $\mathbf{H}_{\lambda,\delta} = \lambda\mathbf{A}^\top\mathbf{A} + \delta\mathbf{L}$

$$\mathbf{b} = \mathbf{Ax} + \epsilon$$
$$\mathbf{x}|\mathbf{b}, \lambda, \delta \sim N\left(\lambda\mathbf{H}_{\lambda,\delta}^{-1}\mathbf{A}^\top\mathbf{b}, \mathbf{H}_{\lambda,\delta}^{-1}\right)$$

- Hyperprior parameters:
 - $\lambda \sim \Gamma(\alpha_\lambda, \beta_\lambda)$
 - $\delta \sim \Gamma(\alpha_\delta, \beta_\delta)$

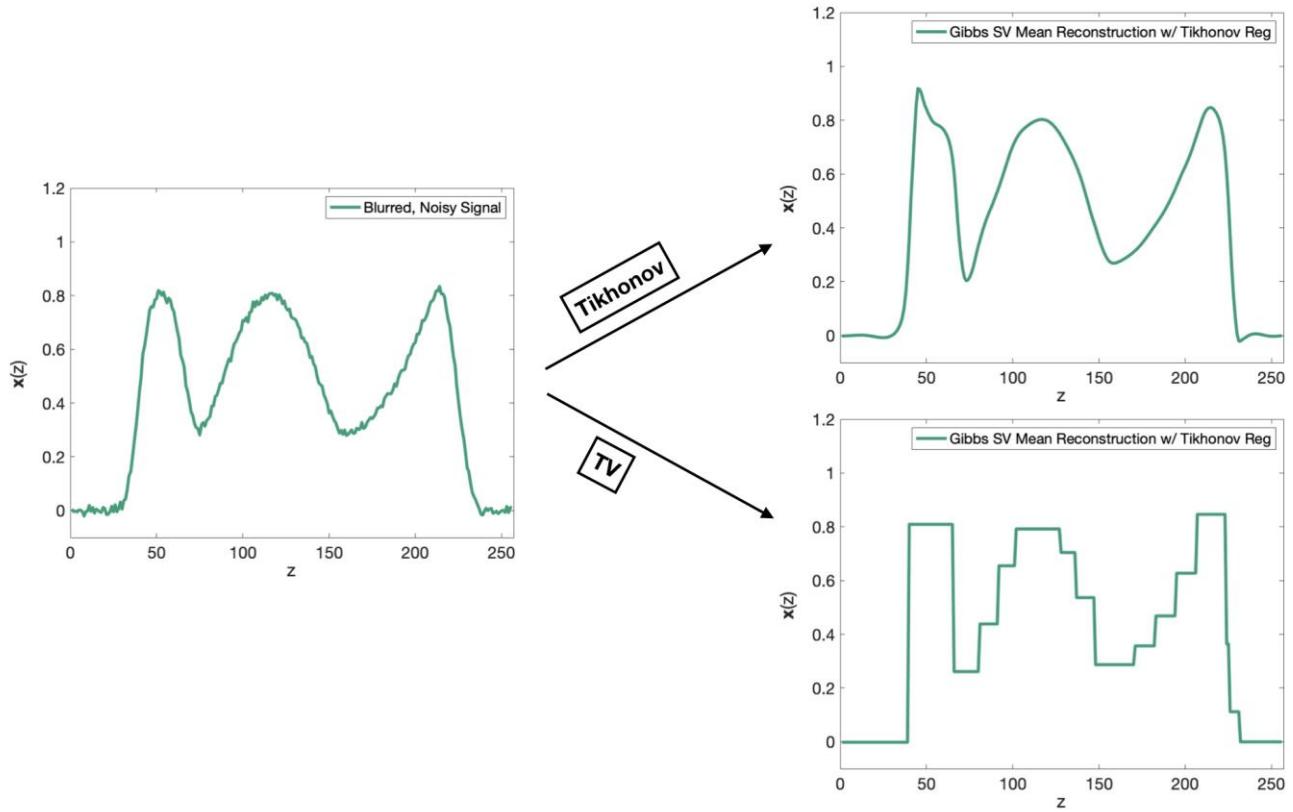
Problem with the deconvolution approach

- When blur varies spatially, assuming the same blurring kernel everywhere is incorrect and can result in poor reconstructions



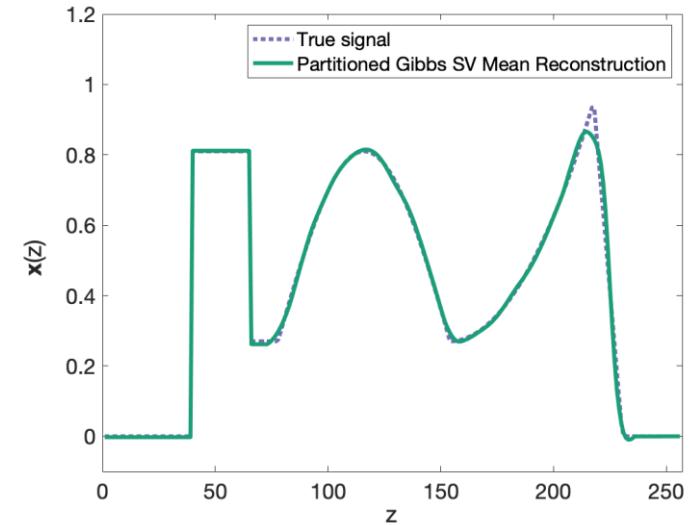
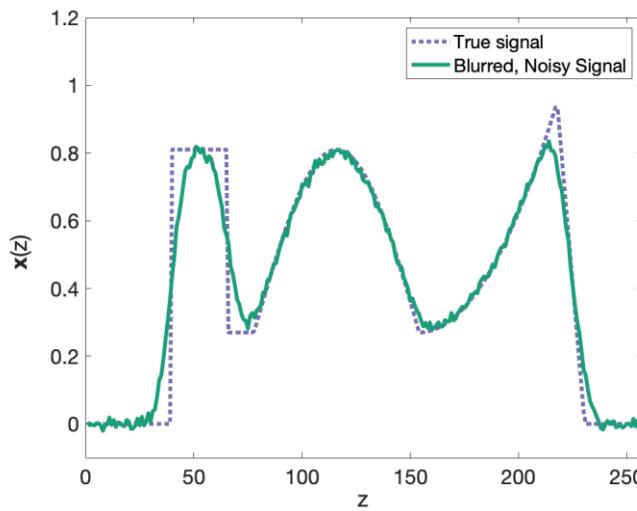
Spatially Varying Parameters and Multi-regularization

- Allow different regularization parameters at different locations in the signal
 - λ, δ are now vectors
- Allow different regularizations depending on the signal



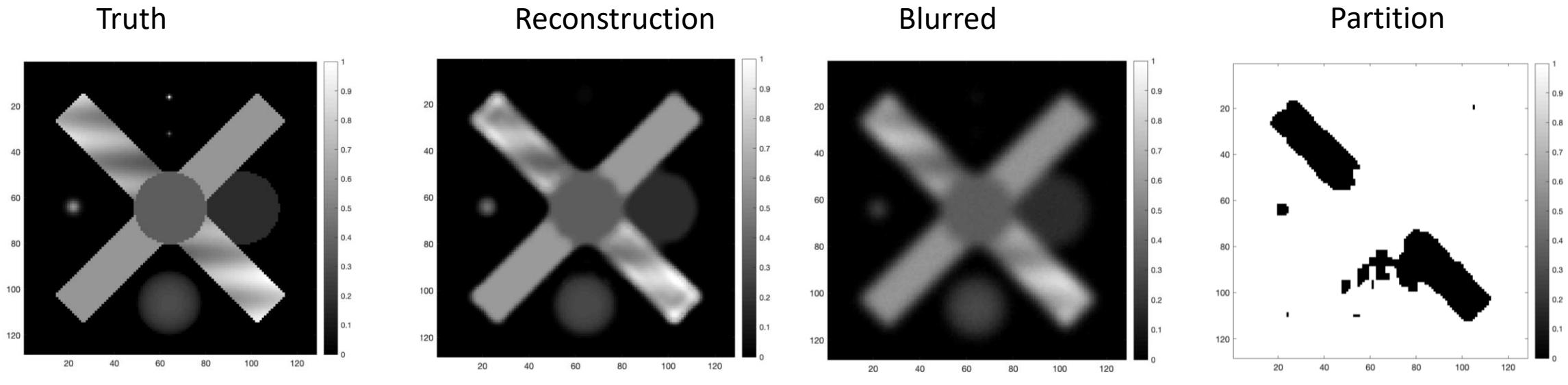
Spatially Varying Parameters and Multi-regularization

- Allow different regularization parameters at different locations in the signal
 - λ, δ are now vectors
- Allow different regularizations depending on the signal



Spatially Varying Parameters and Multi-regularization in 2D

- Need to choose how to partition the image for regularization
- Need methods that will work on large (e.g. 4k x 4k) images



Possible approaches to spatially varying deblurring

- Piecewise convolution algorithms
- Wavelet based deblur
- Machine Learning