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Modal Projection Matching

Brandon Zwink, Peter Avitabile
Structural Dynamics and Acoustic Systems Laboratory
University of Massachusetts Lowell
One University Avenue
Lowell, Massachusetts 01854

D. Greg Tipton
Sandia National Laboratories
Engineering Sciences Center
Albuquerque, New Mexico, USA

ABSTRACT
Components of systems are excited in laboratory tests with the goal of replicating the response of the component in its field environment. A method was developed, based on projecting mode shapes of one system onto mode shapes of another system, that can describe how to excite a laboratory test system such that it replicates the test component response experienced in the field environment. This method is different from classical vibration test methods because it accounts for the dynamic differences between the boundary conditions of the two configurations. The theory is presented along with a test case to simulate the predicted response of a component under two different mounting conditions.

Keywords: Modal Projection, Boundary Condition, Fixture Neutralization

INTRODUCTION
The ability to excite a component in a laboratory environment in the same way that it was excited in its field environment is an important development tool for aerospace designs. Complications in accurately reproducing component dynamics in a laboratory often stem from limited instrumentation and the fact that the boundary conditions of the component are often different in the laboratory than they are in the field environment. Replicating the component dynamics in the laboratory can provide an opportunity to collect additional measurements, test component reliability, ensure the functionality, or test new designs in a controlled setting.

Many of the currently employed methods for testing components in the laboratory incorporate assumptions about the boundary conditions of the component and test fixturing that are known to be inaccurate for many test configurations [1]. One common assumption in classical laboratory vibration testing is that the test fixture used to hold the component in the laboratory is rigid.

Several methods have been introduced that compensate for the dynamic properties of component boundary conditions to replicate component dynamics in a laboratory setting. One approach is to utilize modal substructring techniques and component mode synthesis focusing on the equivalent “fixed base” response of the boundary conditions in both mounting configurations [2] [3] [4] [5] [6]. A second approach, the Impedance Matched Multi-Axis Test (IMMAT), focuses on matching the impedance at boundary connections and utilizing multiple shakers to match component dynamics experimentally [1]. A third method uses frequency based substructuing with a focus on matching the dynamics at connection degrees of freedom in both mounting configurations [7].

The method discussed in this paper is different from the three previously mentioned methods in that it focuses on the mode shapes of the component in both mounting configurations. The proposed method does not utilize substructing techniques but instead uses the mode shapes of the full system at only the component-degrees-of freedom to define a relationship between the laboratory and field environment dynamics.

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NOMENCLATURE

D	Device under test
L_1	Portion of field environment system model that is not the device under test
L_2	Portion of field environment system model that is not the device under test
d	Degrees of freedom on the device under test utilized for the transformation
X	Displacement in frequency domain
P	Modal displacement in frequency domain
U	Mode shapes
g	Generalized inverse

Nomenclature Examples

$X^{(DL_1)}$	Displacement in frequency domain for system comprised of D and L_1
$U_d^{(DL_1)}$	Mode shapes for system comprised of D and L_1 at the "d" degrees of freedom only

THEORTICAL BACKGROUND

The basis of the approach used herein relies on the basic structural dynamic modification and system modeling theory that has been used for many years [8]. The basis used is that of the decomposition of a structural system into its modal contributions and the fact that in component mode synthesis (system modeling), the final modified system modes can be comprised of the original unconnected component modes from the general relationship

$$U_2 = U_1 U_{12} \quad (1)$$

Where U_2 represents the mode shapes of the mode shapes of the final system, U_1 represents the mode shapes of the original system and U_{12} represents the transformation matrix between the two systems. Because the response of the laboratory test model is described by a sum of the mode shapes of the laboratory test model at any particular point in time (modal superposition), this method focuses on defining a transformation from the modal responses of the field environment system to target modal responses of the laboratory test system through a single transformation matrix. Because the goal of this method is to focus on faithfully re-creating the dynamics of the device under test, the derivation of the transformation utilizes only the degrees of freedom that are located on the device under test. These degrees of freedom are designated as "d" degrees of freedom.

The modal transformations for the field environment system and laboratory test system are given in the equations

$$X^{(DL_1)} = U^{(DL_1)} P^{(DL_1)} \quad (2)$$

and

$$X^{(DL_2)} = U^{(DL_2)} P^{(DL_2)} \quad (3)$$

The physical displacements of the device under test (component D) are then set to be equal for both systems as shown in the equation

$$X_d^{(DL_1)} = X_d^{(DL_2)} \quad (4)$$

Combining equations 2, 3, and 4 and solving for the modal response of the laboratory test system yields the following relationship

$$P^{(DL_2)} = U_d^{(DL_2)^g} U_d^{(DL_1)} P^{(DL_1)} \quad (5)$$

The two mode shapes in equation 5 can be combined into a single matrix that describes the transformation between the modal responses of the field environment system and the laboratory test system as follows

$$\mathrm{P}^{(\mathrm{DL}_2)} = \mathrm{U}_{21}\mathrm{P}^{(\mathrm{DL}_1)} \quad 6$$

The U_{21} matrix is incredibly important to look at when performing this method because it describes how the modes of the laboratory test system are created using linear combinations of the field environment mode shapes. It should be noted that this process is a modal projection, so if the environmental model mode shapes do not span the laboratory test model mode shapes, the solution will be the best fit possible in a least squares sense.

MODEL DESCRIPTION

A two-beam finite element model was utilized to demonstrate the application of this method. The two-beam model was developed for studying modal dynamics of a two-beam system [9]. The system is comprised of a lower beam which represents the dynamics of the field environment boundary conditions as well as an upper beam which represents the component whose dynamics need to be replicated in different boundary conditions. The upper beam will be referred to as the "device under test".

The upper beam, designated "D", is made using 14 planar beam elements with 15 nodes and two-degrees of freedom at each node (translation and rotation). The lower beam, designated "L_1", was made using 28 planar beam elements with 29 nodes and two-degrees of freedom at each node (translation and rotation).

A second system model was created to represent the laboratory test configuration by modifying the lower beam to be much stiffer than the lower beam for the field environment configuration. The stiffer lower beam was designated "L_2". A depiction of both models is shown in Figure 1.

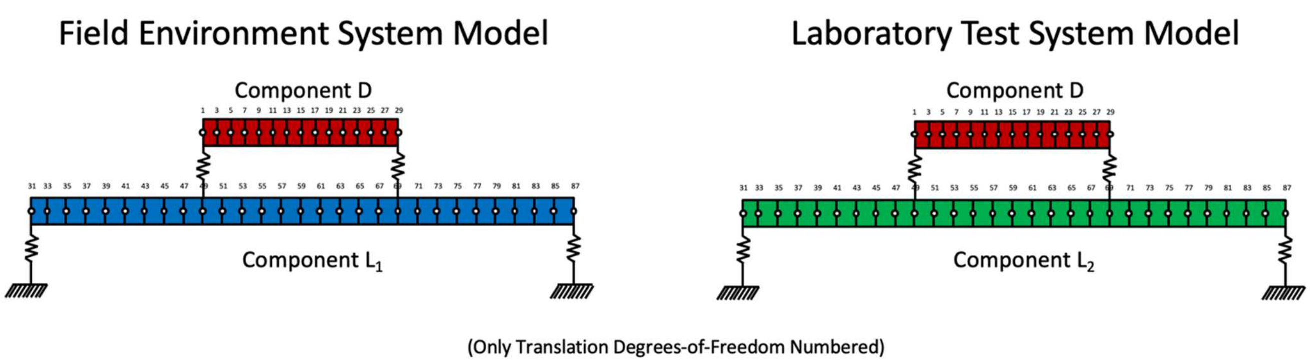


Figure 1: Finite Element Model Depictions

The upper beam and lower beams were tied together using two translational springs and two translational springs were also used to tie the lower beams to ground. The Eigen solution was computed for both models and the resulting frequencies and mode shapes are shown in Table 1 and Table 2 for the first 12 modes of each system.

Table 1: Field Environment Model Mode Shapes

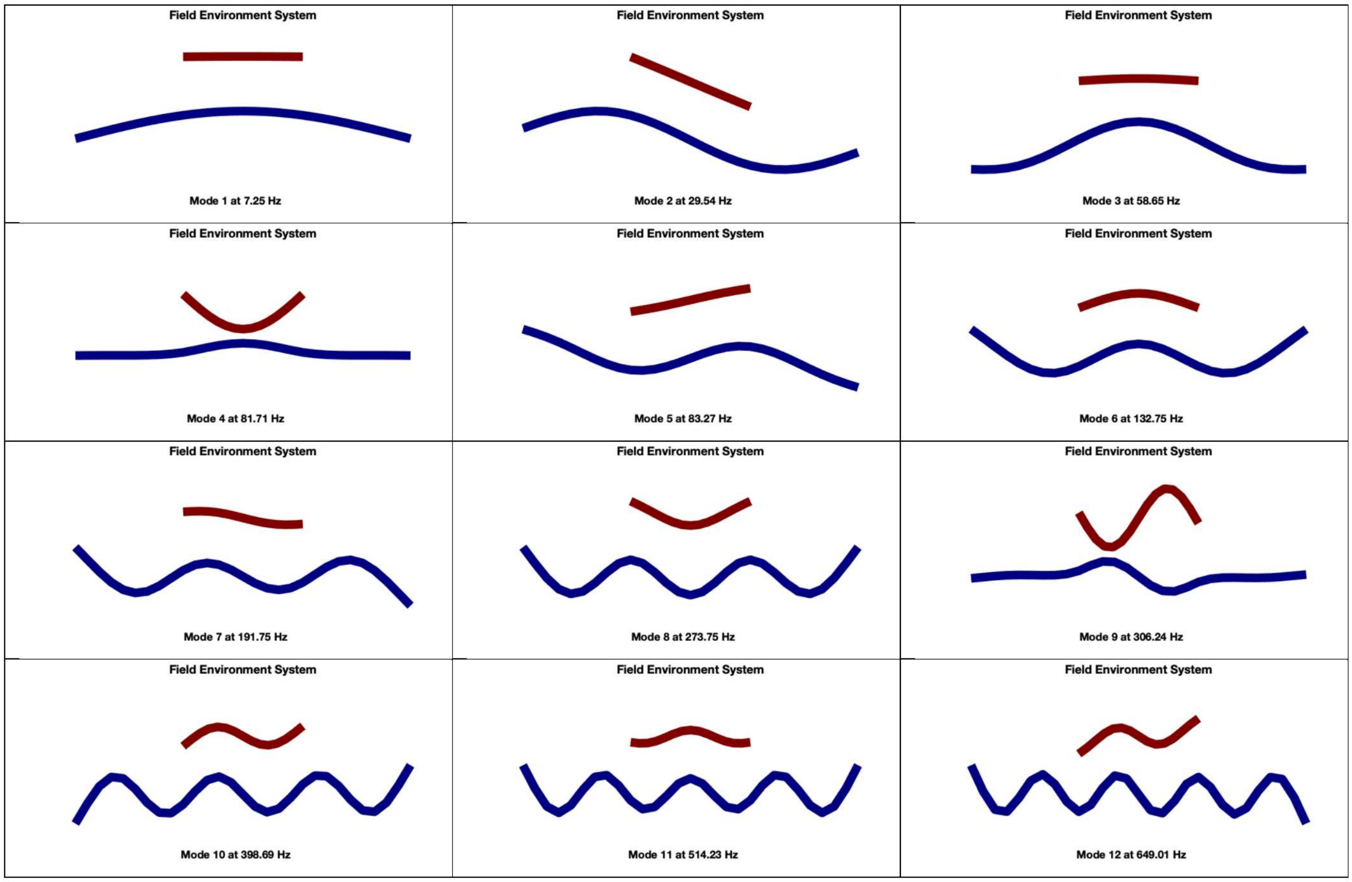
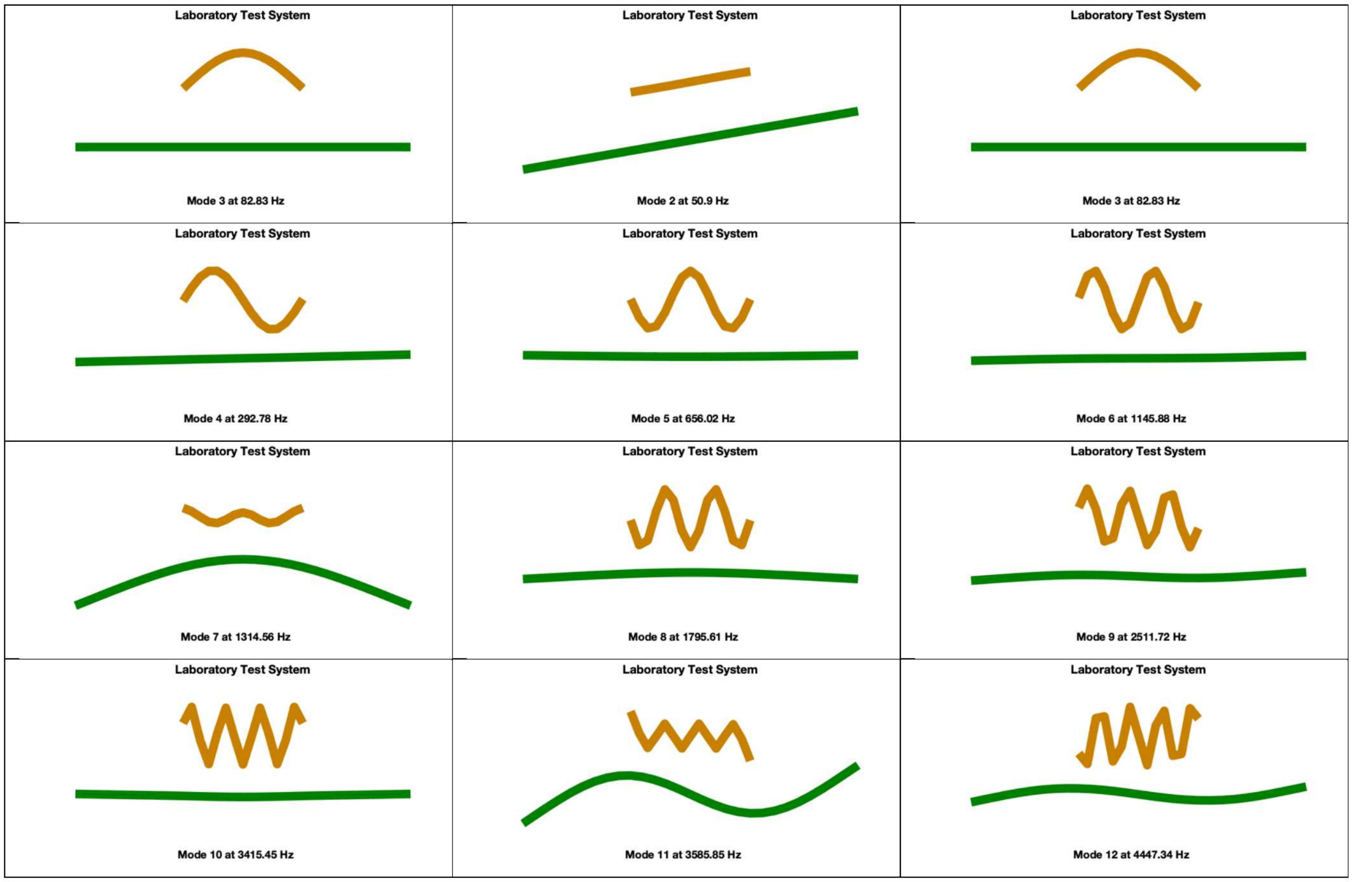


Table 2: Laboratory Test Model Mode Shapes



FIELD ENVIRONMENT DEFINITION

A response of the field environment model is needed in order to attempt replicating that response in the laboratory test model. The response is arbitrary other than that it needs to exercise the dynamics of the system in order to be a relevant environment to study. Therefore, an impulse excitation force was defined and applied to the field environment model at translational degree of freedom (DOF) 43 as depicted in Figure 2. The haversine impulse force that was generated and applied to the field environment model is shown in Figure 3.

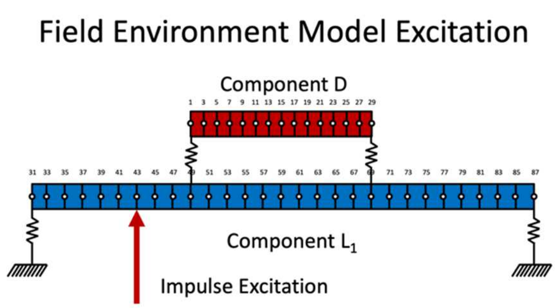


Figure 2: Field Environment Model Excitation Location

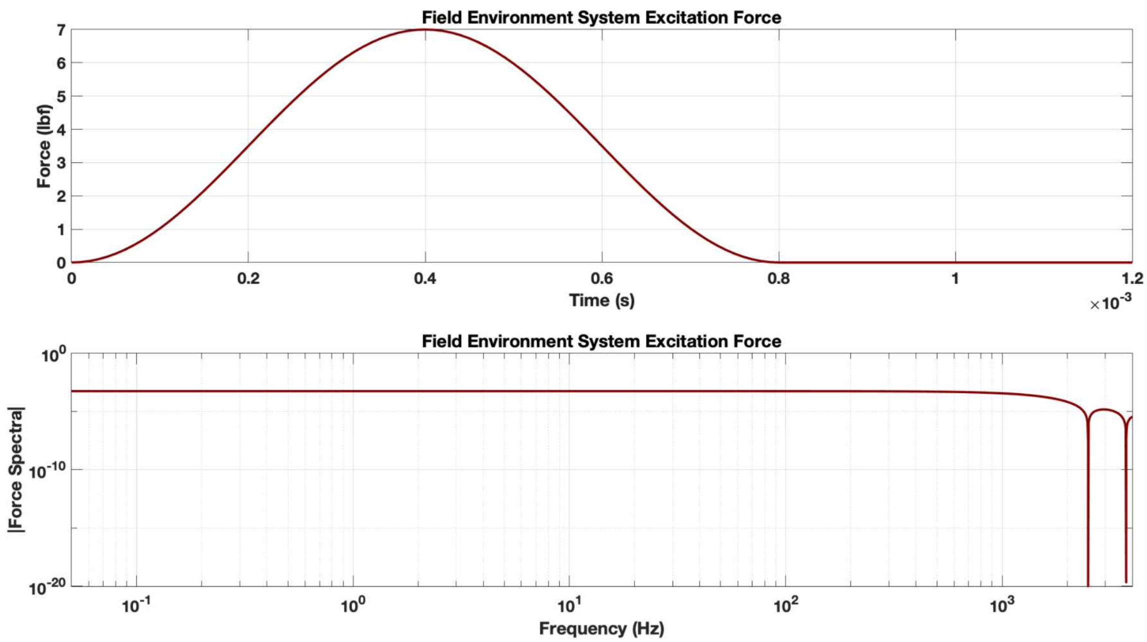


Figure 3: Impulse Excitation Force Applied to Field Environment Model at DOF 43

The resulting modal responses for the field environment model are shown in Figure 4 for the first 12 modes of the system.

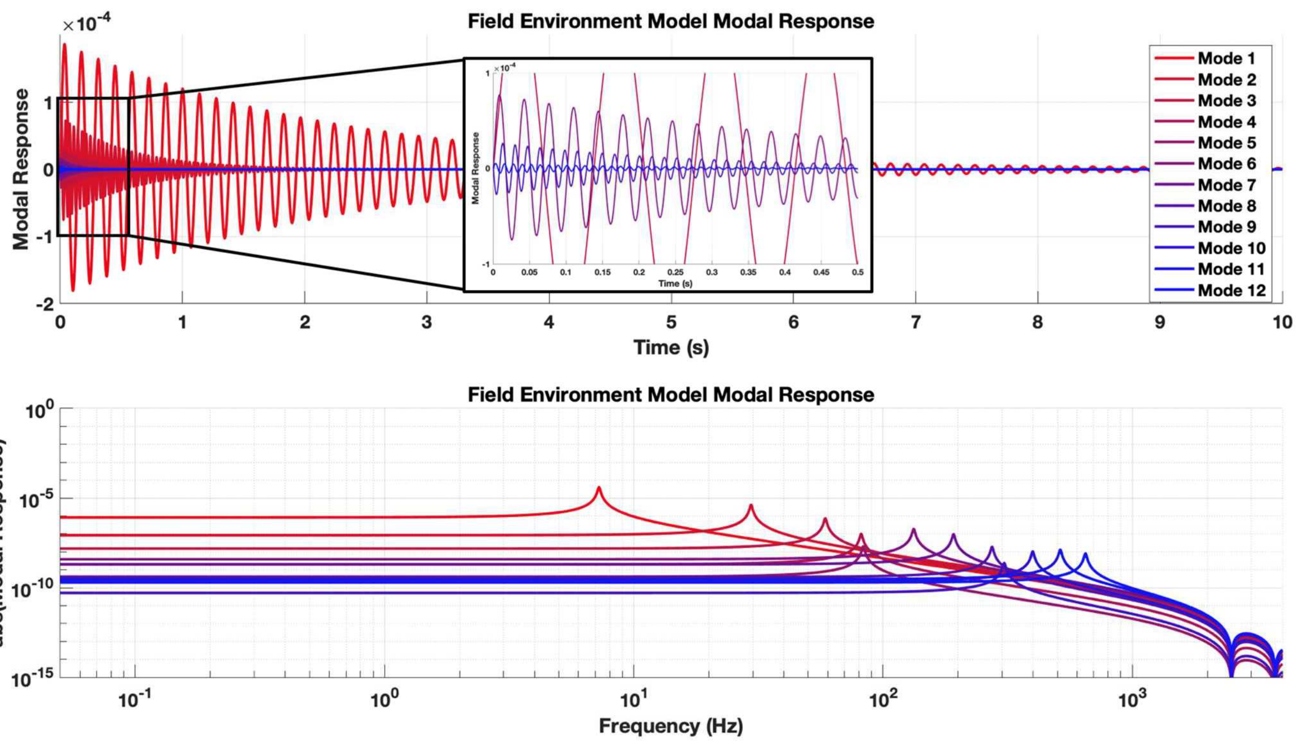


Figure 4: Modal Response of Field Environment Model

TEST CASES STUDIED

A set of degrees of freedom was selected on the device under test that would span the space of the first 12 modes of the field environment model. It was also important to include more degrees of freedom than mode shapes included. For that reason, 13 translation degrees of freedom were selected as the “d” degrees of freedom and are shown in Figure 5.

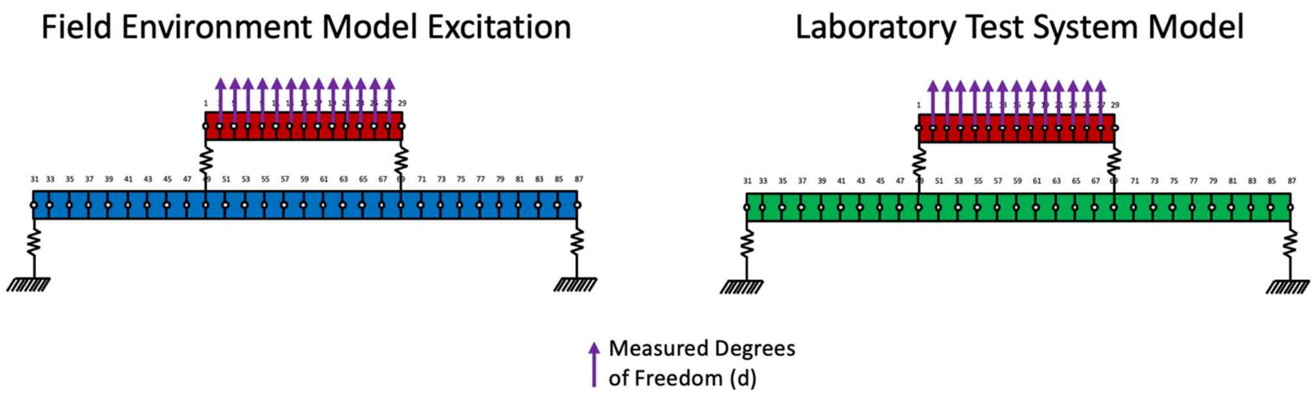


Figure 5: 13 "d" Measured Degrees of Freedom

Utilizing the measurement degrees of freedom depicted in Figure 5 and the first 12 mode shapes of both systems, the resulting U_{21} matrix values are shown in Figure 6. Each column of the U_{21} matrix describes how a particular field environment mode is created using linear combinations of laboratory test model modes.

When considering only the dynamics of the device under test, the U_{21} matrix demonstrates that the first 12 mode shapes of the field environment model have been described (mostly) using the first four mode shapes of the laboratory test model. This same realization can be made by closely investigating the mode shapes of both systems shown in Table 1 and Table 2.

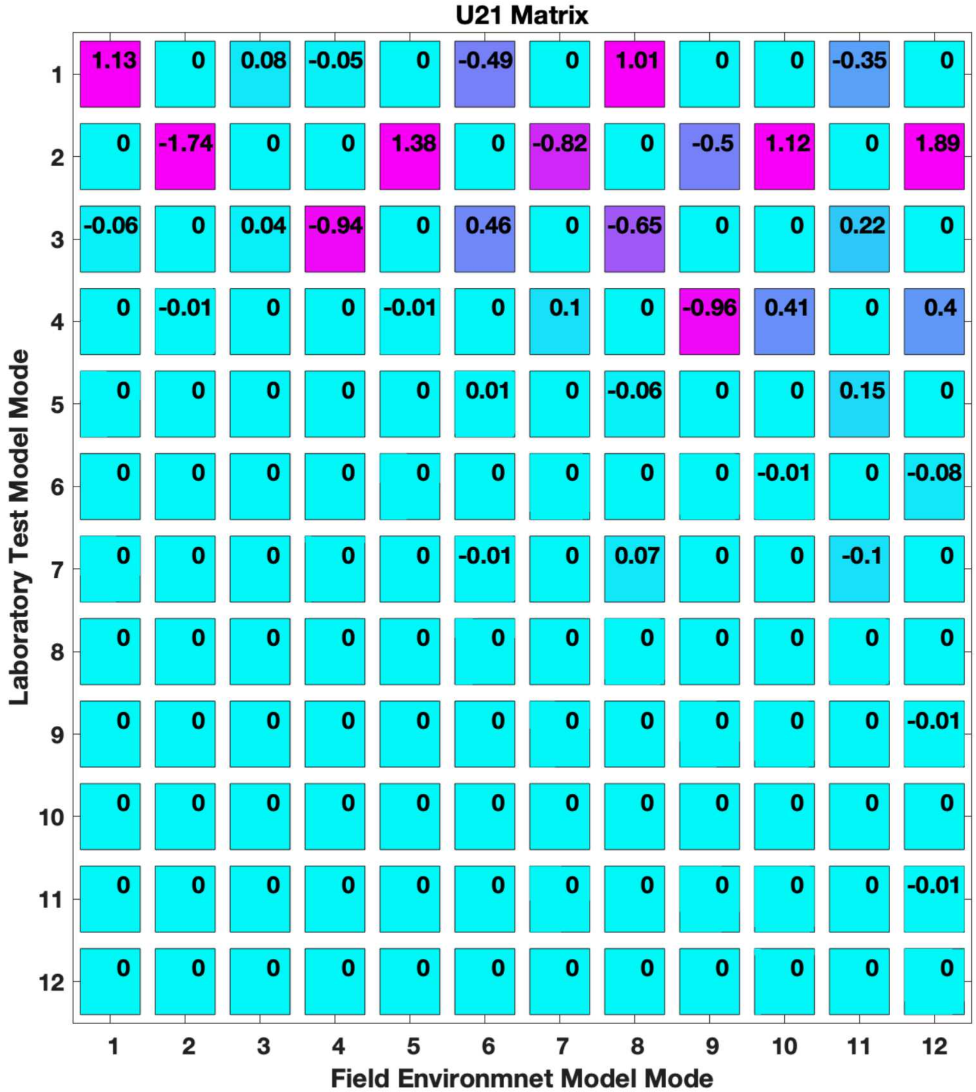


Figure 6: U_{21} Matrix (12 Modes of Field Environment Model, 12 Modes of Test Lab Model)

In order to demonstrate how this method can be applied when the U_{21} matrix is not square, the example problem will be computed using the first 12 modes of the field environment model and only the first 4 modes of the test laboratory system. The resulting U_{21} matrix for this test case is shown in Figure 7.

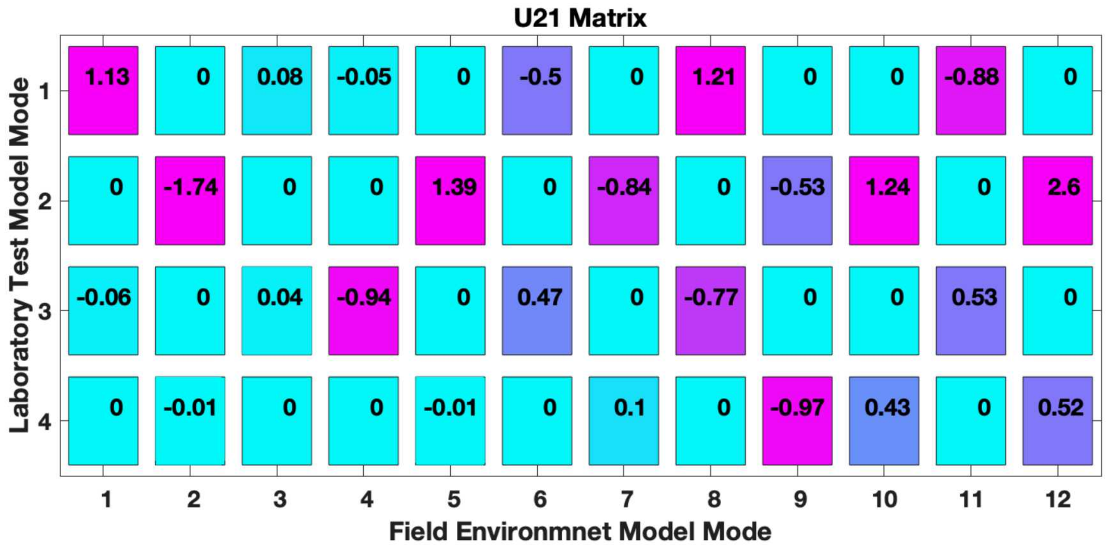


Figure 7: U_{21} Matrix (12 Modes of Field Environment Model, 4 Modes of Test Lab Model)

Utilizing the field environment model response shown in Figure 4 and the U_{21} matrix from Figure 7, the modal responses for the laboratory test system were calculated as described in equation 6. The resulting modal responses for the laboratory test system are shown in Figure 8.

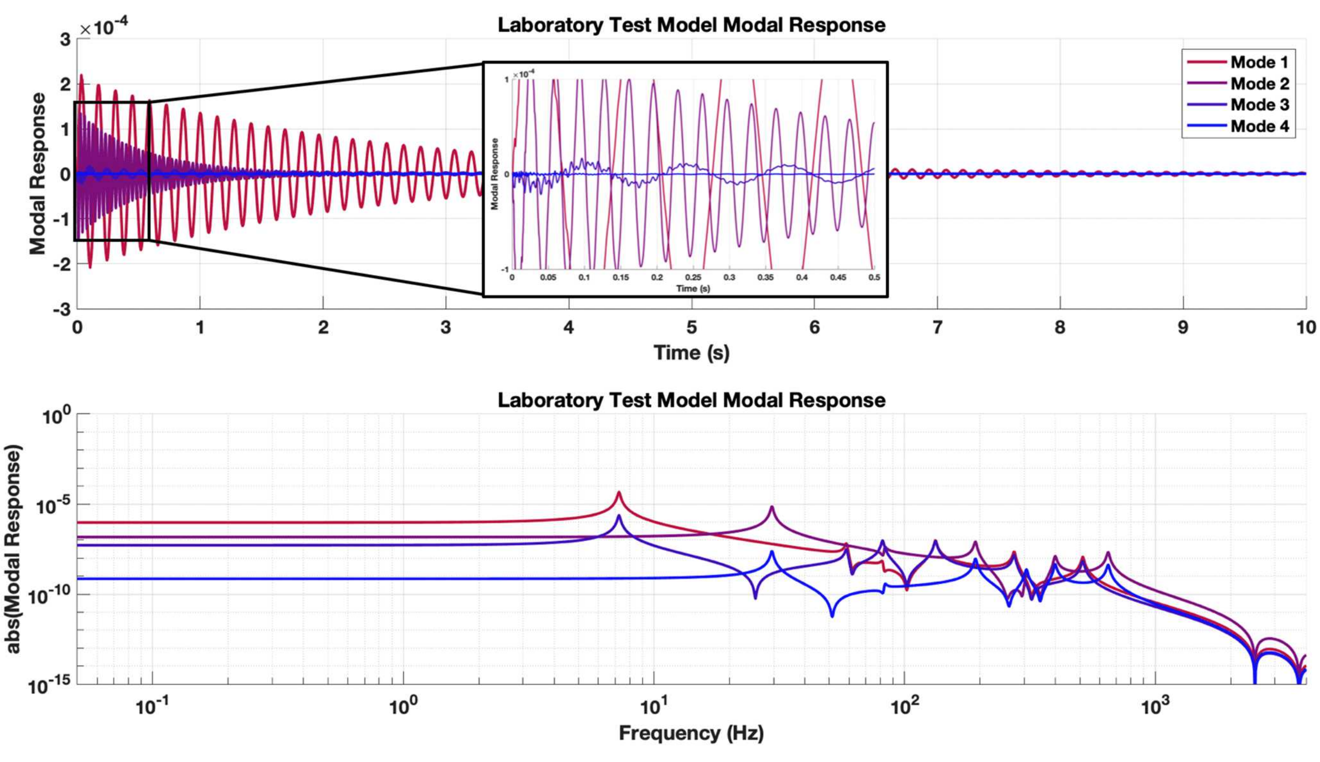


Figure 8: Laboratory Test Model Desired Modal Response

The modal responses shown in Figure 8 describe how the modes of the field test model need to be excited, and in what ratios at each frequency line, in order to match the dynamics of the device under test in the test lab. To demonstrate that the dynamics of the device under test would indeed match the field environment, the displacement at an arbitrarily chosen degree of freedom (DOF 7) is shown for both systems in Figure 9. It can be seen in Figure 9 that the response of the device under test in the test lab model matches the dynamics of the field system extremely well as intended. The other degrees of freedom on the device under test demonstrate the same results.

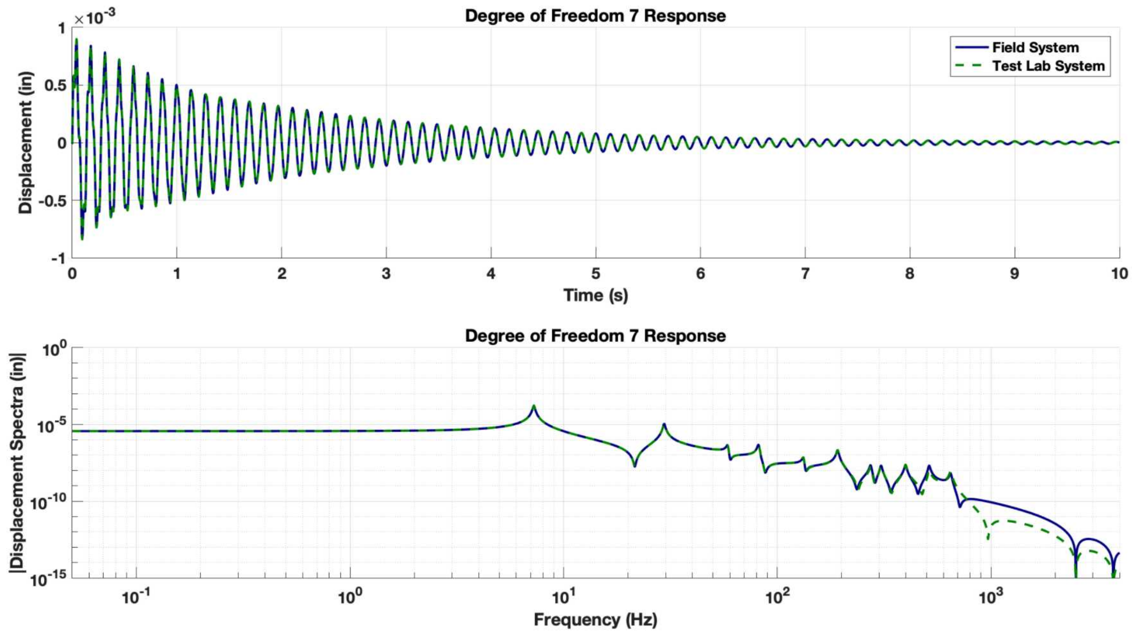


Figure 9: Response at DOF 7 for Both Models

CONCLUSION

The work presented here demonstrates an approach using modal vectors to account for boundary condition differences between two configurations – one being a field environment configuration and one being a laboratory test configuration. Using a transformation based on modal projections, the dynamics for a component in one configuration can be replicated in a different configuration; this could be two different laboratory test fixtures or two different configurations such as an operating configuration that needs to be replicated in a laboratory environment. The simulations performed demonstrated the ability to achieve the same device under test response in two different mounting configurations.

ACKNOWLEDGEMENTS

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