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BDDC in Trilinos

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Thanks

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- Jan Mandel (UC Denver)
- Olof Widlund (Courant Institute)

Outline

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- **Background:**
  - What is BDDC
  - Where does it fit in Trilinos
- **Recent Developments:**
  - Reduced size coarse spaces
  - Higher order elements
  - Inexact subdomain solvers
- **Using BDDC:**
  - Getting started
  - Features & Limitations
- **Ongoing Efforts:**
  - ExaWind project
  - Communication-friendly decompositions
  - GPU-based inexact solvers

Two Flavors of Domain Decomposition

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- **Iterative Substructuring:**
  - FETI-DP[1] and BDDC[2] fall into this class
  - Schur complement methods (if exact solvers)
  - Communication restricted to interfaces
  - Ideas tied to substructuring analysis
- **Overlapping Schwarz:**
  - FROSch[3] in Trilinos/ShyLU does this
  - Communication in overlap regions
  - Can work with fully-assembled matrices
  - Ideas date back to Hermann Schwarz, 1870.

[1] Finite Element Tearing and Interconnecting – Dual Primal
[2] Balancing Domain Decomposition by Constraints
[3] Fast and Robust Overlapping Schwarz (parallel implementation of GDSW[4])
[4] Generalized Dryja Smith Widlund

Substructuring

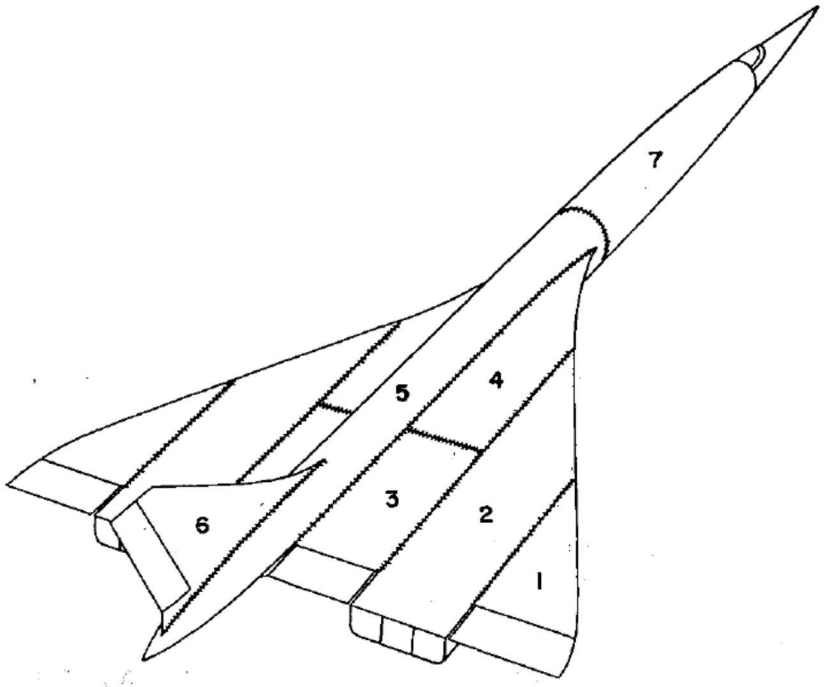


FIG. 3. Typical substructure arrangement for delta aircraft.

Idea: restrict problem to interfaces

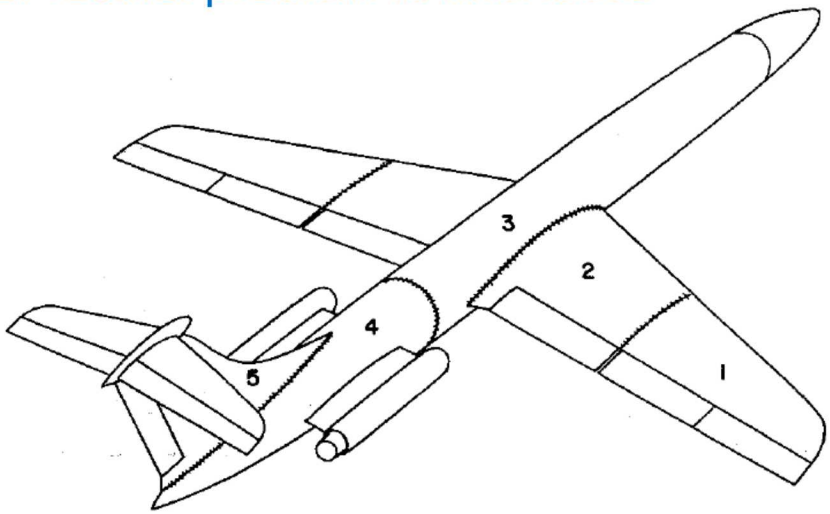
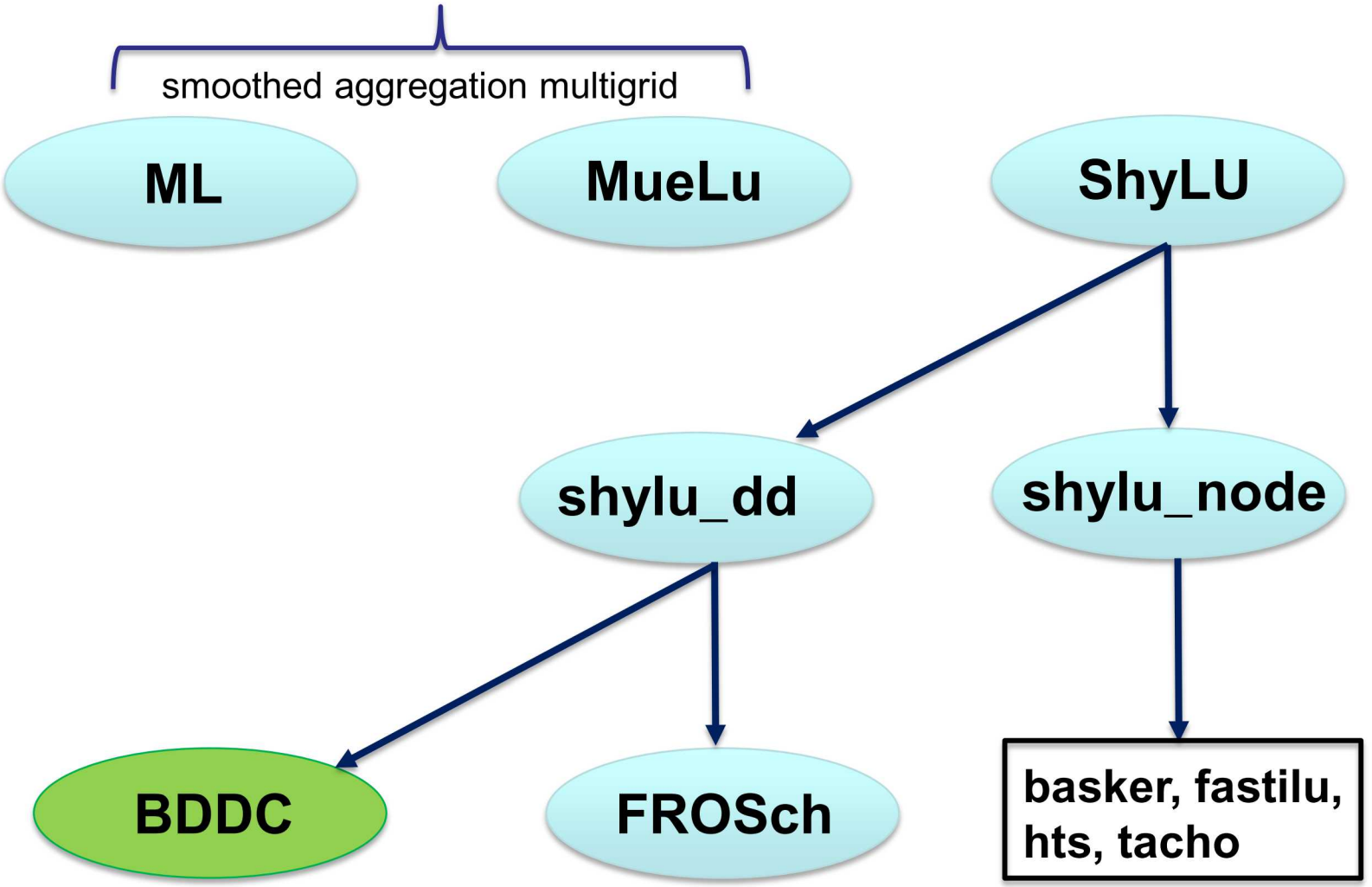


FIG. 4. Typical substructure arrangement for conventional aircraft.

P63: J. S. Przemieniecki, "Matrix Structural Analysis of Substructures", AIAA Journal, Vol. 1, pp. 138-147, 1963.

Where BDDC Fits



Recent Developments

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- **Reduced size coarse spaces*:**
  - Why: Less memory, fewer computations, fewer coarse levels
  - Available in next pull request

Problem: Unit cube decomposed into smaller cubic subdomains. Results for scalar case & edge-based coarse spaces.

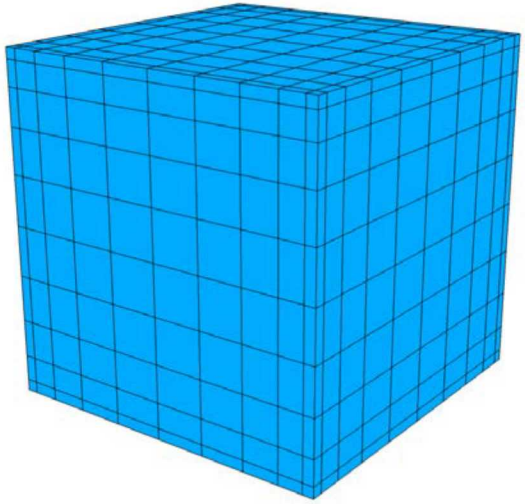
Speedups for proposed approach and ratios of coarse problem dimensions

# subdomains	Initialization	Solve	ratio n_c
13,824	1.8	1.0	0.32
21,952	2.1	1.1	0.32
32,768	2.5	1.2	0.32
46,656	3.0	1.4	0.32

* A Vertex Coarse Space for BDDC in Three Dimensions, 25th International Domain Decomposition Conference, St. John's, Newfoundland, Canada, July 23-27, 2018 (with KHP and Olof Widlund)

Recent Developments

- **Higher order elements*:**
  - Treat each higher order element as a subdomain
    - Challenging computations isolated to individual elements
  - Coarse problem involves assembly of lower order elements
    - We know how to solve this problem
  - Existing theory, promising results, available next pull request
  - Use old idea from Orszag (1980)



	K_p (dense)	K_h (sparse)
Memory	$O(p^6)$	$O(p^3)$
Factorization work	$O(p^9)$	$O(p^6)$
Factorization memory	$O(p^6)$	$O(p^{4/3})$
AMG initialization		$O(p^3)$
AMG memory		$O(p^3)$

optimized matrix-vector products require $O(p^4)$ work

* A Memory Efficient BDDC Algorithm for Higher Order Elements, 25th International Domain Decomposition Conference, St. John's, Newfoundland, Canada, July 23-27, 2018.

Recent Developments

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- **Inexact subdomain solvers:**
  - Factorization times and memory usage of sparse direct solvers can be prohibitive
  - Theory exists for inexact subdomain solvers, but some care must be taken to preserve scalability
  - Optional dependency on MueLu now available
  - Prospects for sparse direct solvers on GPUs not promising
  - Work in progress on GPU-based inexact solvers
    - Early results encouraging
    - Plan to make available next pull request

Using BDDC

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```
-D Trilinos_ENABLE_ShyLU:BOOL=OFF \
-D Trilinos_ENABLE_ShyLU_DDBDDC:BOOL=ON \
-D ShyLU_DDBDDC_ENABLE_TESTS:BOOL=ON \
```

- **Getting Started:**
  - **See SolverBDDCTest.C or SolverBDDCTestSimple.C**
  - **ProblemMakerBDDC.h generates problems on the fly**
    - **Currently just Poisson or elasticity problems**
    - **Provide problem specifics in problemMaker.xml**
  - **Run Shylu_DDBDDC_shylubddc_unit_tests.exe with gtest_filter=SolverBDDC.Test1 to try it out**

Using BDDC

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- **Features:**
  - Multiple subdomains per MPI rank
  - Tpetra for parallel communications, but no TpetraCsr
  - Matrix-vector products on each MPI rank and then reduced
  - Concurrent execution across multilevel hierarchy
- **Limitations:**
  - Only scalar elliptic & elasticity problems currently supported
  - Subdomain matrices prior to global assembly needed
    - Won't work with fully assembled matrices

Ongoing Efforts

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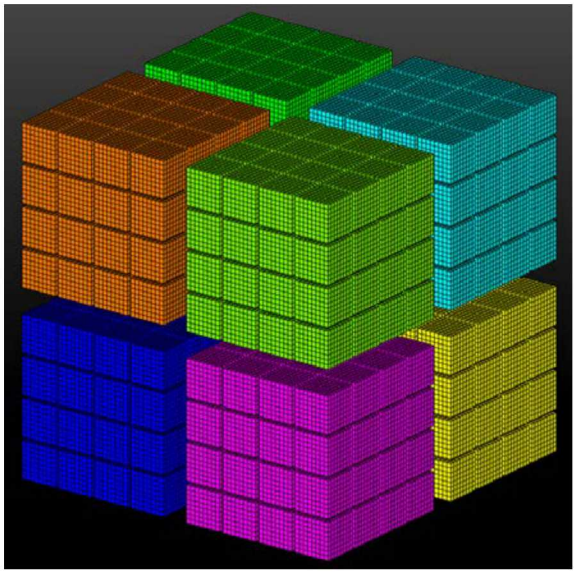
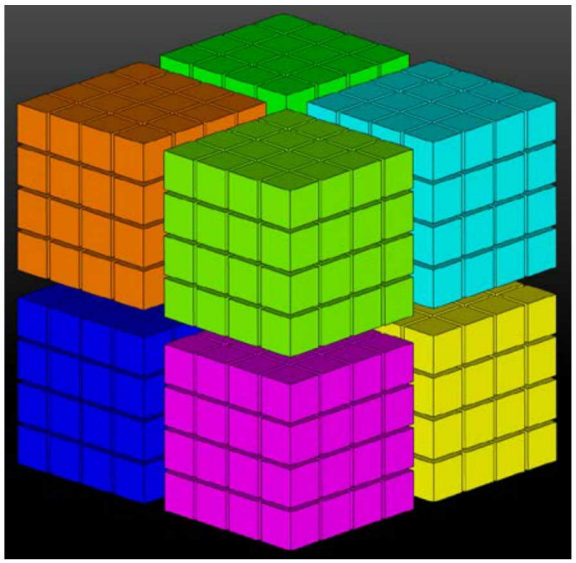
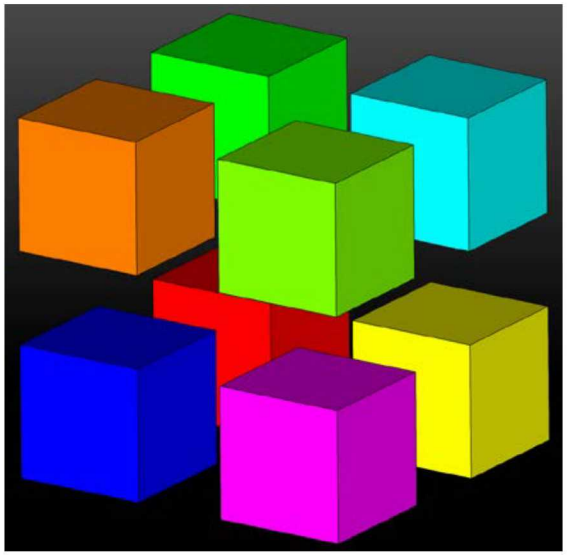
- **ExaWind Project:**
  - **Solver approaches for overset methods**
  - **Running effectively on GPUs**
- **Communication-friendly decompositions:**
  - **Reduce communication across compute nodes**
  - **Preserve favorable sparsity across all levels**



decomposition for 8 of 8000 nodes



first level coarsening, no comm



second level coarsening

Ongoing Activities

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- **GPU-based Inexact Solvers**
  - **Variety of approaches being developed/evaluated**
    - **Avoid downsides of standard coarsening**
    - **Domain decomposition like smoothers**
    - **More classical AMG-like methods w/ some twists**

Thank You!

Extra Slides

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Selected References

- BDDC
  - Clark R. Dohrmann, “A preconditioner for substructuring based on constrained energy minimization”, *SIAM J. Sci. Comput.*, 25:246-258, 2003.
  - Jan Mandel, Clark R. Dohrmann, and Radek Tezaur, “An algebraic theory for primal and dual substructuring methods by constraints”, *Appl. Numer. Math.*, 54:167-193, 2005.
  - Santiago Badia, Alberto F. Martin, and Javier Principe, “Multilevel balancing domain decomposition at extreme scales”, *SIAM J. Sci. Comput.*, 38:C22-C52, 2016.