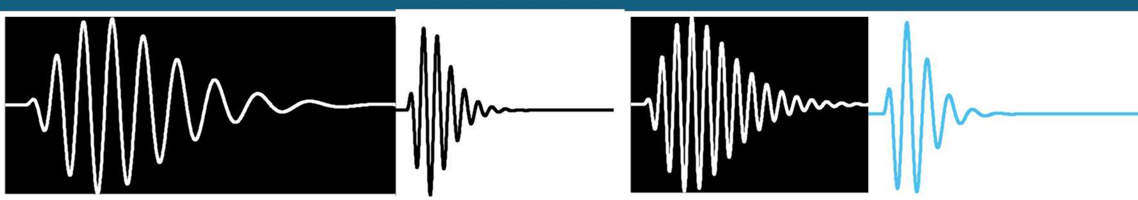
SAND2018-12116C

# Classification of transient behavior in time history data



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SCHOOL OF ENGINEERING

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# Outline

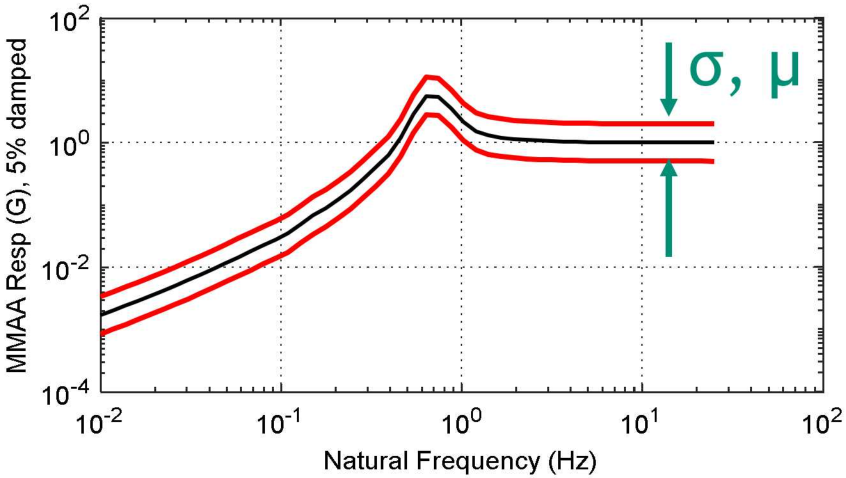
**This talk represents some early work as part of a project to develop a specification framework for mechanical shock.**

1. Motivation
2. Tools
3. The Method
4. Numerical Example
5. Concluding Remarks & Discussion

# The Current Framework is One-Dimensional

Currently, we specify shocks with the shock response spectrum (SRS).

- Mean, variance, and tolerance limits are defined (per frequency) as a function of amplitude.




The shock response spectrum is ill-equipped to describe transient motion.

- It contains no information about the duration and little information on the time-dependent character of the event.
- More information is always required to specify a shock for laboratory testing.

**One-dimension may not be sufficient**

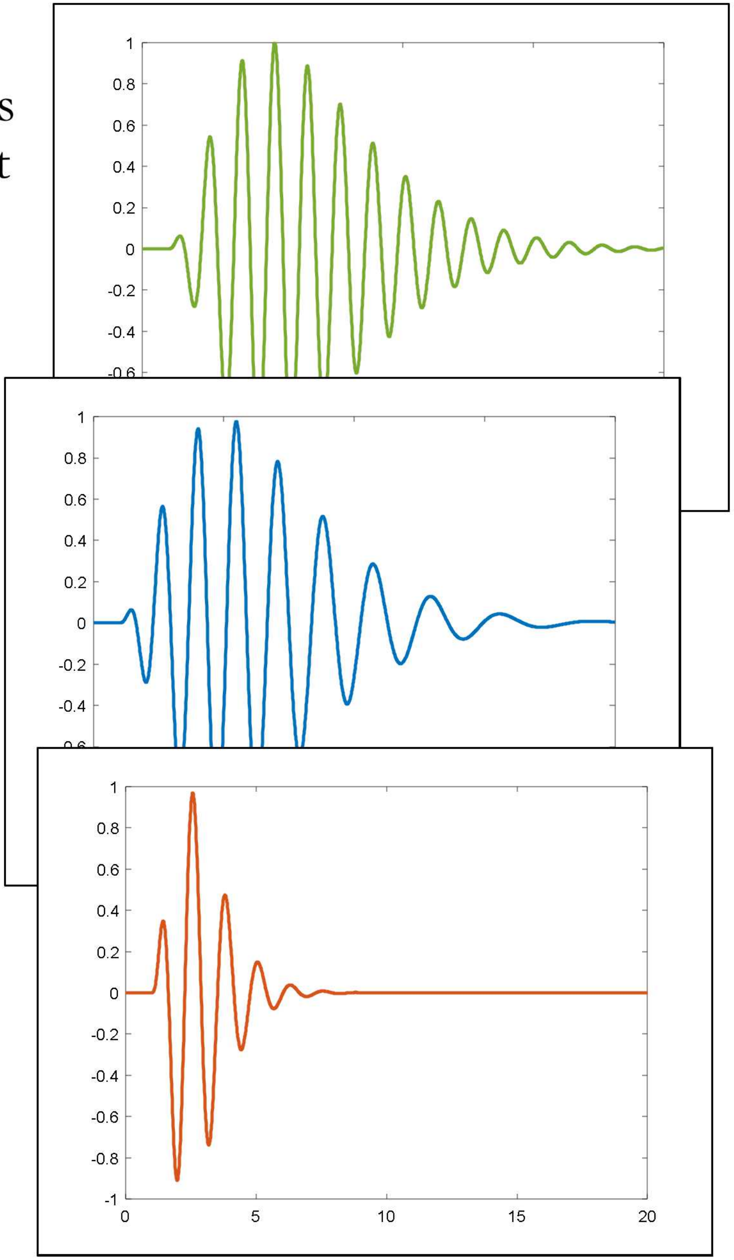
# Comparing Events

In order to define a mean, variance, or tolerance limits on an ensemble of events -- we need to define what it means for **one event to be different from another.**

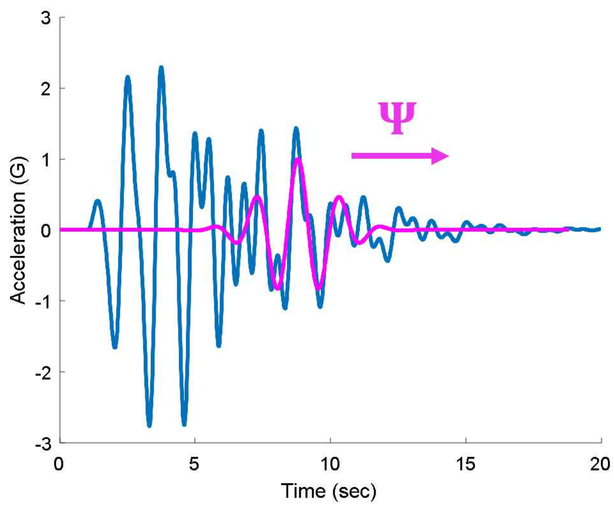
A **comparison method** is proposed that can handle differences across time and frequency.

It may be used to help define an **appropriate ensemble** for statistical analysis.

The tools used to define that space include the continuous wavelet transform and singular value decomposition.



# Continuous Wavelet Transform




The transform:

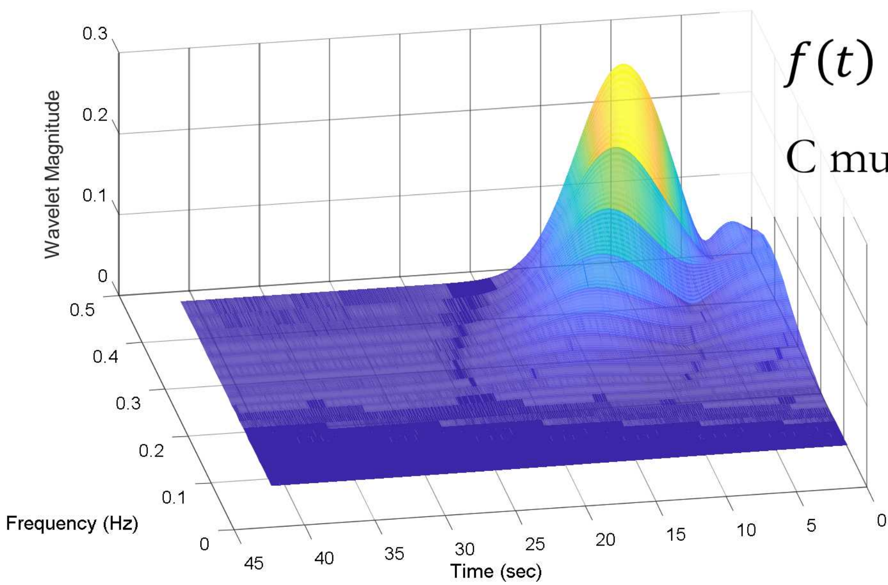
$$\Upsilon(s,\tau) \equiv \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{s}}\Psi^*\left(\frac{t-\tau}{s}\right)dt$$

The inverse (with admissibility criterion, C ):

$$C \equiv \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|}d\omega, \quad 0 \leq C \leq \infty$$

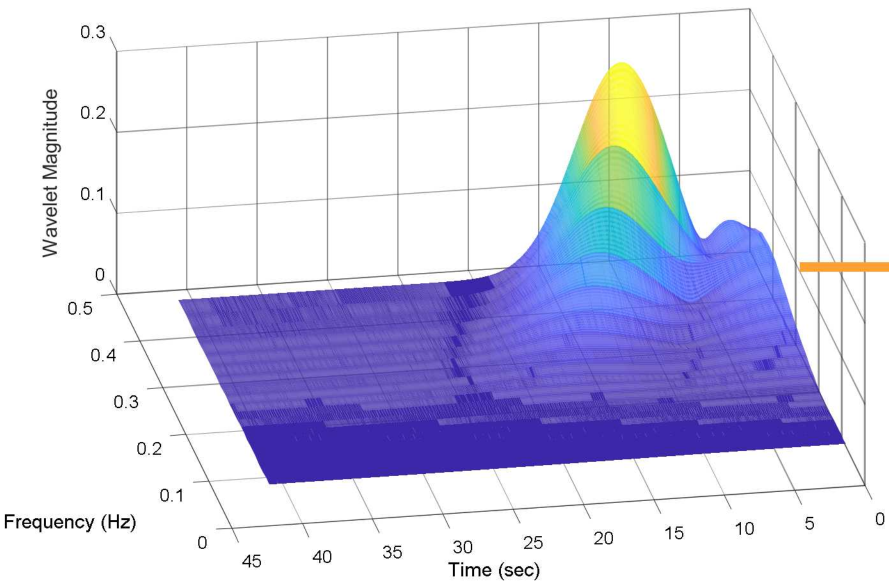
$$f(t) \equiv \frac{1}{C}\int_{s=-\infty}^{\infty}\int_{\tau=-\infty}^{\infty}\frac{1}{|s|^2}\Upsilon(s,\tau)\frac{1}{\sqrt{s}}\Psi\left(\frac{t-\tau}{s}\right)ds\,d\tau$$

C must be positive and finite for the inverse to exist.




The magnitude of the transform can be thought of as a correlation between Ψ and f.

# Continuous Wavelet Transform




$$[\Upsilon(s,\tau)]$$

$$m\ x\ n$$

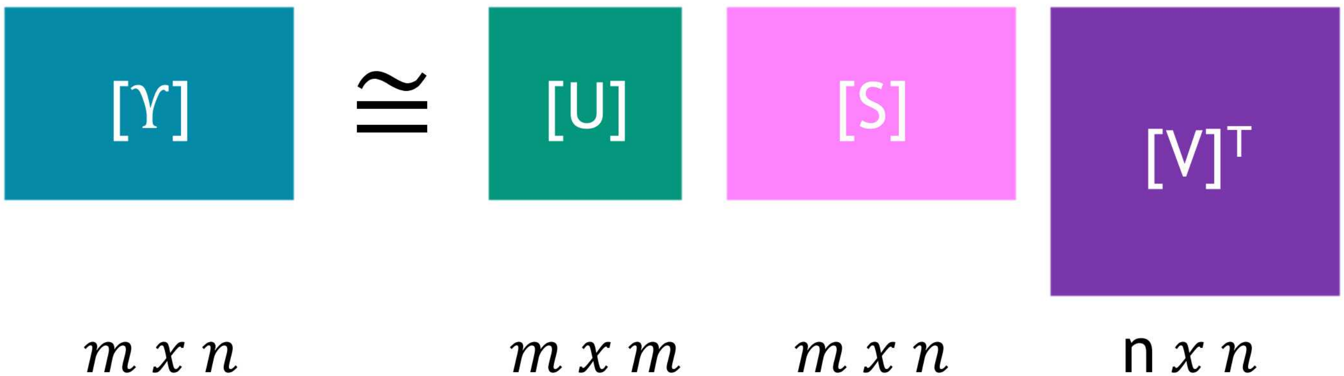
$$0 \leq s \leq m$$

$$0 \leq \tau \leq n$$

$m$ is the number of 'frequencies'
$n$ is the number of points in the time history

# Singular Value Decomposition (SVD)

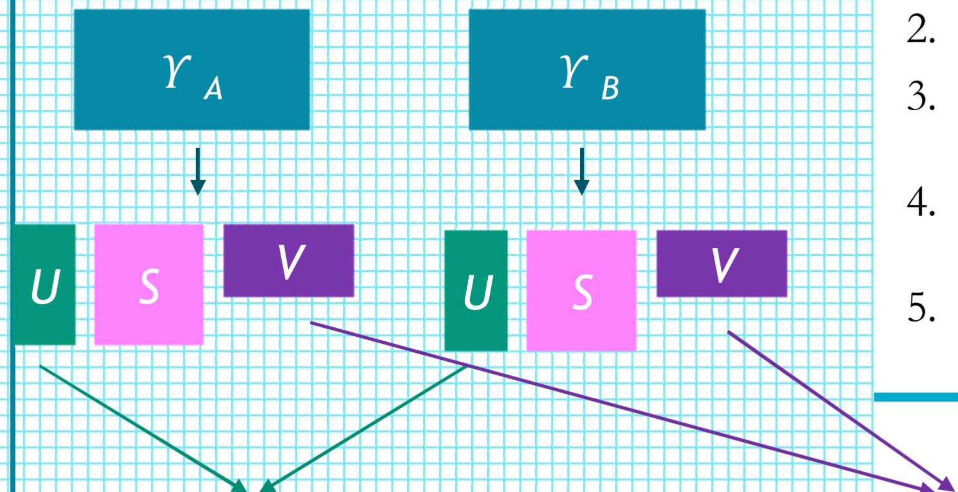
$$\Upsilon \cong USV^T$$




U contains the **left singular vectors** of A (eigenvectors of $\Upsilon\Upsilon^T$)
V contains the **right singular vectors** of A (eigenvectors of $\Upsilon^T\Upsilon$)
$S^2$ are the **eigenvalues**, in rank-order

An approximation for ϒ can be made by only retaining singular values up to a desired number ($< m$).

# Method




$$(cosine_{A,B})_i \frac{Real(\overrightarrow{U_{A,i}} \cdot \overrightarrow{U_{B,i}})}{\|\overrightarrow{U_{A,i}}\|\|\overrightarrow{U_{B,i}}\|}$$

$$(cosine_{A,B})_i = \frac{Real(\overrightarrow{V_{A,i}} \cdot \overrightarrow{V_{B,i}})}{\|\overrightarrow{V_{A,i}}\|\|\overrightarrow{V_{B,i}}\|}$$

$i = 1 \ to \ N$

$$du_i = 1 - \left|(cosine_{A,B})_i\right|$$

$$dv_i = 1 - \left|(cosine_{A,B})_i\right|$$

$$D_u = \sqrt{du_1^2 + du_2^2 + \cdots + du_N^2}$$

$$D_v = \sqrt{dv_1^2 + dv_2^2 + \cdots + dv_N^2}$$

*Optional Sub-step*

$$D_{all} = \sqrt{D_u^2 + D_v^2}$$

*Final Metri* T10

1. Wavelet Transform Signals A and B
2. Decompose with SVD
3. Compute Cosine Angles for N desired number of singular values
4. Subtract the absolute value from 1 to convert to a distance
5. Compute Euclidian distances between U and V vectors for an overall metric

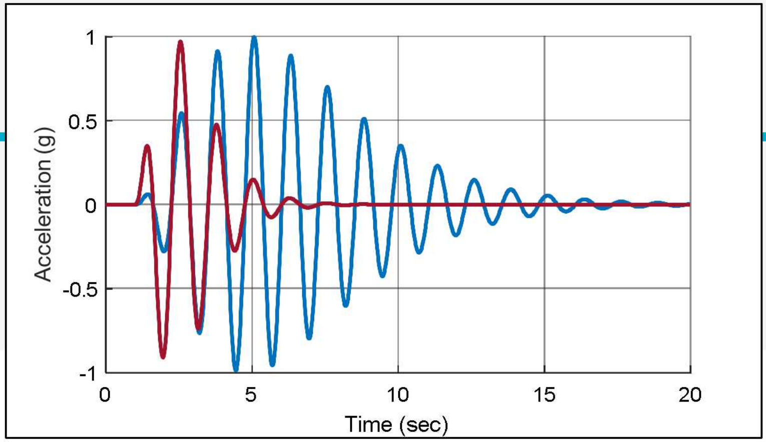
## Slide 8

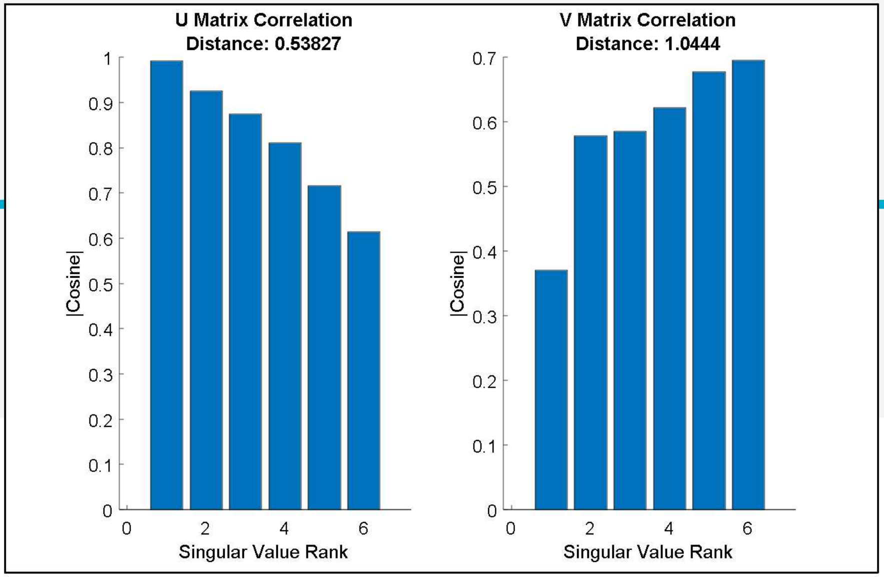
**T10** Be prepared to explain what this means, intuitively.

Tom, 10/16/2018

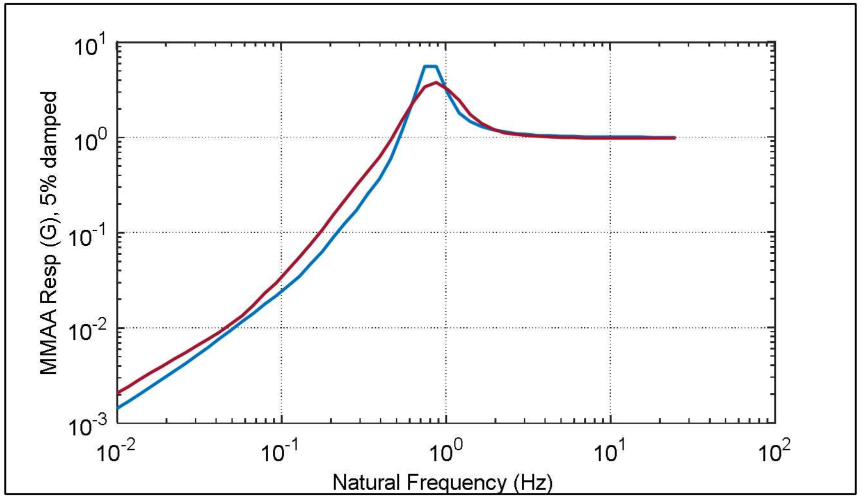
# Numerical Example – Direct Comparison

Same frequency, different envelope



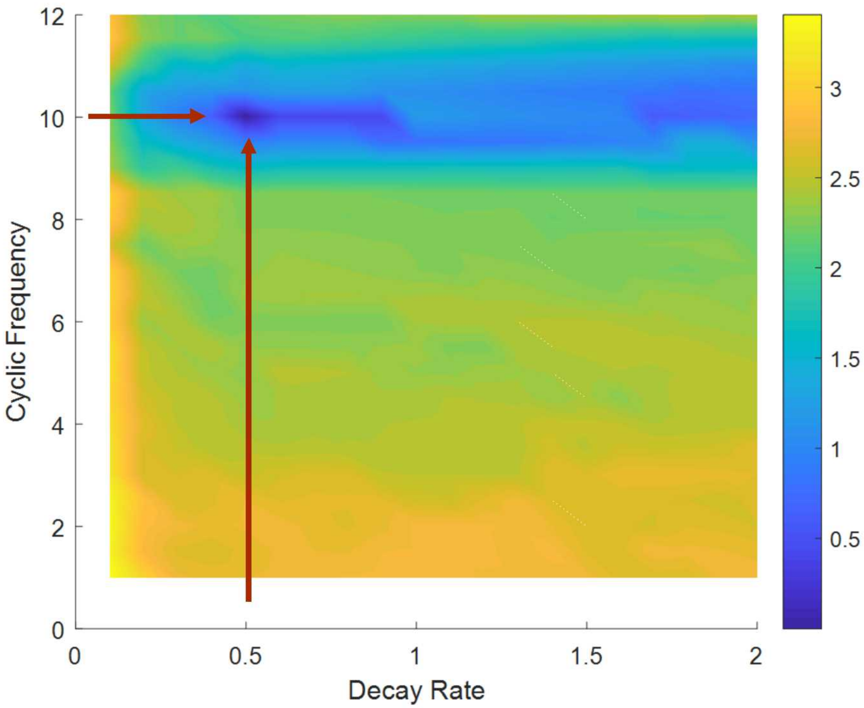




**Distance 1.05**




The SRS gives you some indication that these signals are different, but little way to gauge by ***how much***.

# Numerical Example – Model Parameters




This method might be useful for estimating parameters in a statistical model.

With a known model, permutate the comparison signal and look for low distance values.

In this case, the sample signal was given by $x_4$:

$$w(n) = 0.5\left(1 - \cos\left(2\pi \frac{n}{4095}\right)\right), 0 \leq n \leq 4095$$

$$x_4(t) = \left\| w\left(e^{-0.5t}\right) \right\| \sin(10t)$$

$$0 \leq t \leq 40.95$$

The comparison signal had the same form:

$$x_4(t) = \left\| w\left(e^{-dt}\right) \right\| \sin(ct)$$

$$0.1 \leq d \leq 2$$

$$1 \leq c \leq 12$$

The heat map at left shows that the lowest distance (dark blue) is located at **d** = 0.5 and **c** = 10.

This corresponds to high correlation for the sample signal and the comparison signal for those parameters.

# Conclusion

Wavelet decomposition with SVD has been used in multiple forms as a method of feature extraction for various types of pattern recondition analysis.

It has also been used to estimate modal parameters for aerospace applications.

This method shows that it is possible to examine time and frequency behavior in a quantifiable way.

However, a lot of work needs to be done to show that this method can be used as basis for specification development.

- Robustness to noise has not been investigated
- Sensitivity of the method to different wavelets has only been examined in a cursory way.
- Estimation methods with an approximate inverse need to be developed T14

## Slide 11

**T14** Do you want to add a comment on random processes? Do so only if you think it is appropriate.
Tom, 10/16/2018

# References

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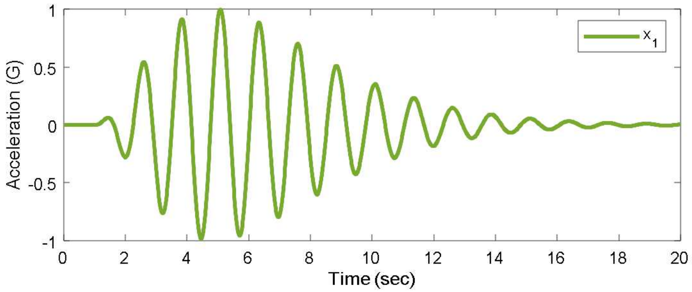
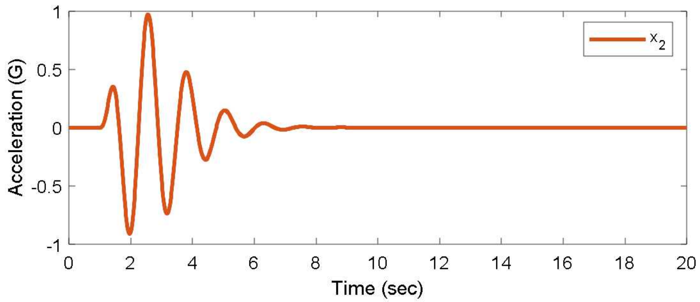
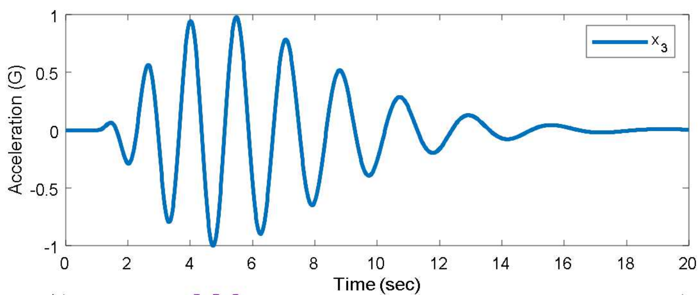
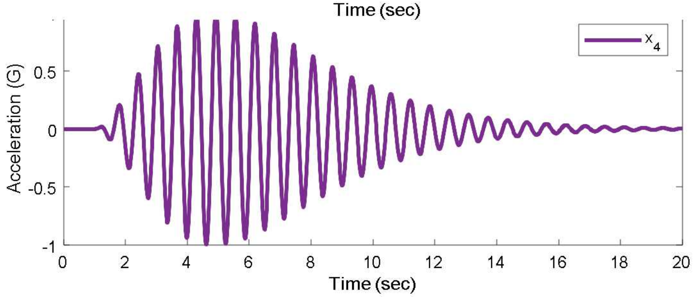
Hassanpour, Hamid, Mostefa Mesbah, and Boualem Boashash. "Time-frequency feature extraction of newborn EEG seizure using SVD-based techniques." *EURASIP Journal on Applied Signal Processing* 2004 (2004): 2544-2554..

# Backup- Sample Data




Hann window :

$$w(n) = 0.5\left(1 - \cos\left(2\pi \frac{n}{4095}\right)\right), 0 \leq n \leq 4095$$

**Baseline:**

$$x_1(t) = \left\| w\ (e^{-0.5\ t}) \right\| \sin(5t)\ , 0 \leq t \leq 40.95$$

**Different Envelope**

$$x_2(t) = \| w\ (e^{-0.1t}) \| \sin(5t)$$

**Time dependent frequency**

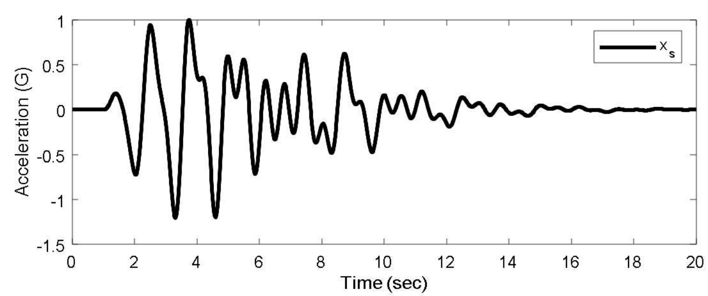
$$x_3(t) = \left\| w\ (e^{-0.5t}) \right\| \sin(f(t)\ t), f(t) = 5 - 0.1t$$

**Different frequency**

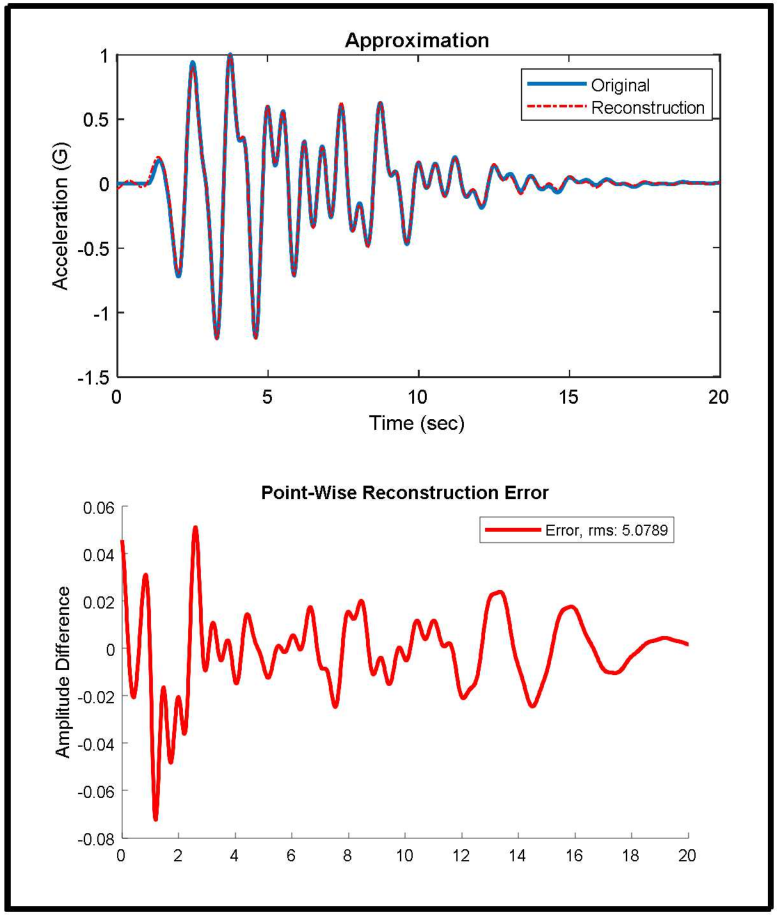
$$x_4(t) = \left\| w\ (e^{-0.5t}) \right\| \sin(10t)$$

**Sum**

$$x_S(t) = x_1 + x_2 + x_3 + x_4$$

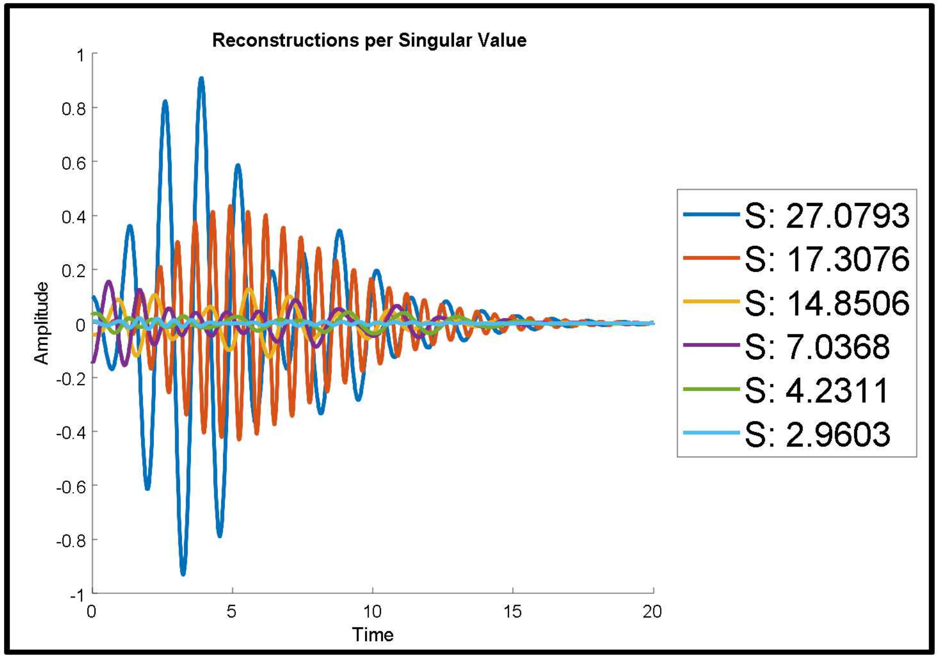


# Example Decomposition



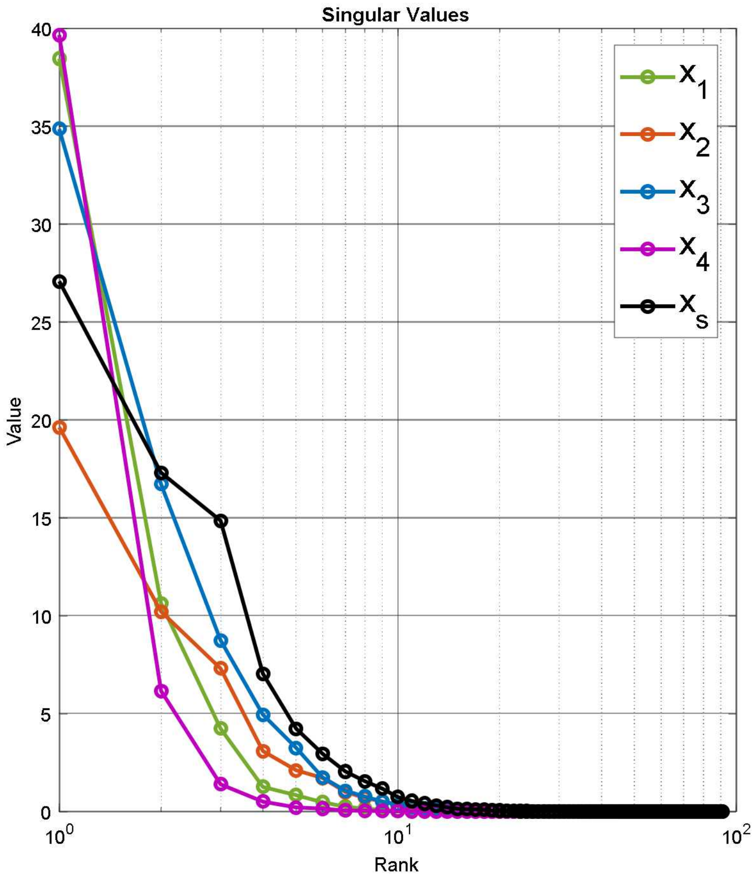

In this case, 6 singular values account for 90% of the behavior.

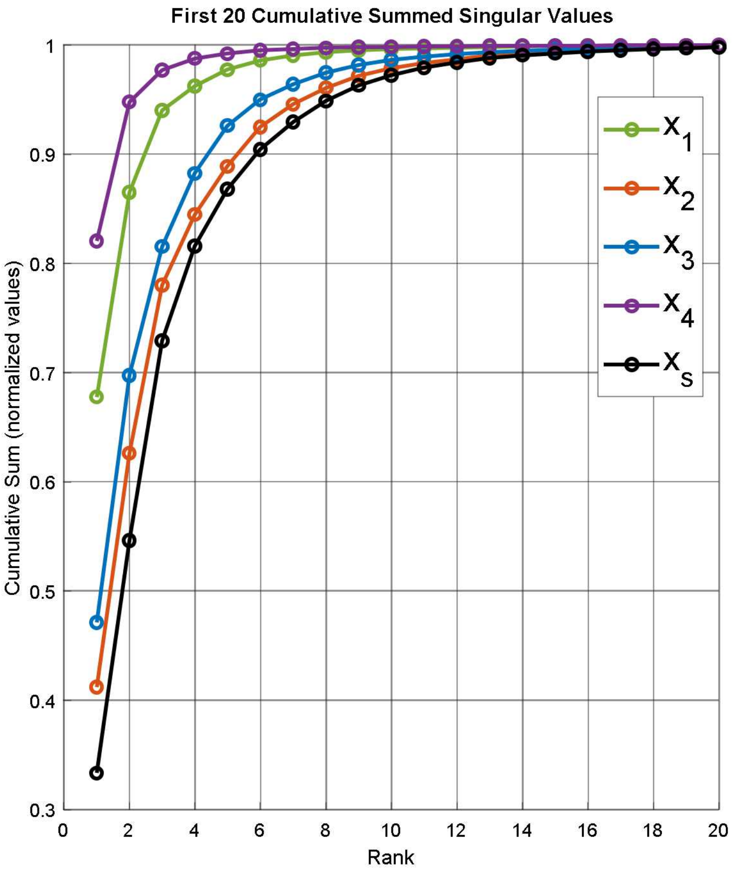
The resulting approximation has a point-wise error that peaks at 6%.




Only a few singular values are needed for a reasonable approximation

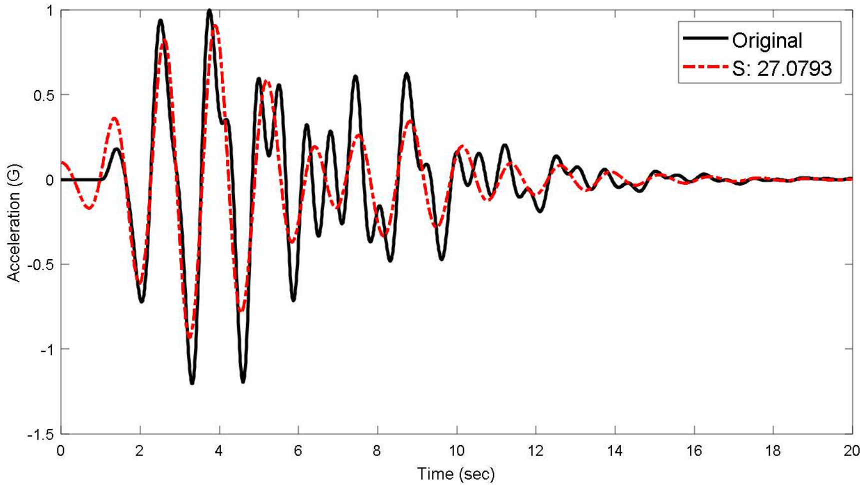
# Backup- Singular Values

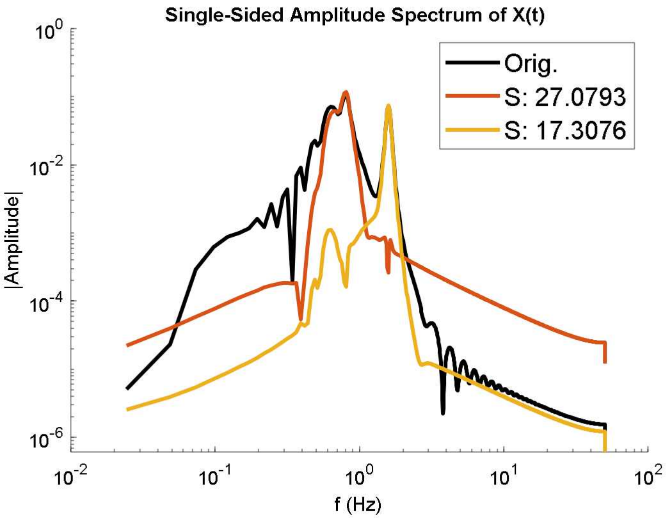



The number of singular values (consequently, the size of the feature space) will depend on the relative complexity of the signal.

# Backup – Capturing Behavior




The reconstruction (red) for the first singular value shows that it captures most of the behavior of the original time history.

As opposed to a Fourier decomposition, it contains multiple frequencies.