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Analysis and Synthesis of Realistic Random Environments

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Outline

- Motivation
- Introduction
- Analysis and synthesis of mechanical environments
  - Stationary random vibration
    - Estimation of spectral densities (including durations)
    - Generation of stationary random process realizations
  - Oscillatory shock
    - Estimation of parameters via product model (a simple version of the Priestley random process)
    - Generation of realizations
  - Classical shock (We will not cover this, though it is included in the software.)
    - Estimation of parameters
    - Generation of realizations

Motivation

- Practically all mechanical environments are mixtures of:
  - Quasi-stationary random vibration
  - Oscillatory shocks, and
  - Classical shocks

  operating in 6 degrees-of-freedom. These are *mixed spectrum* signals.

- Because of the difficulties involved in analyzing and simulating the components simultaneously, usually we:
  - Separate environments into parts
  - Estimate parameters in models of the parts
  - Simulate the parts, separately (And, usually, include a very substantial safety factor/confidence limit/tolerance limit.)

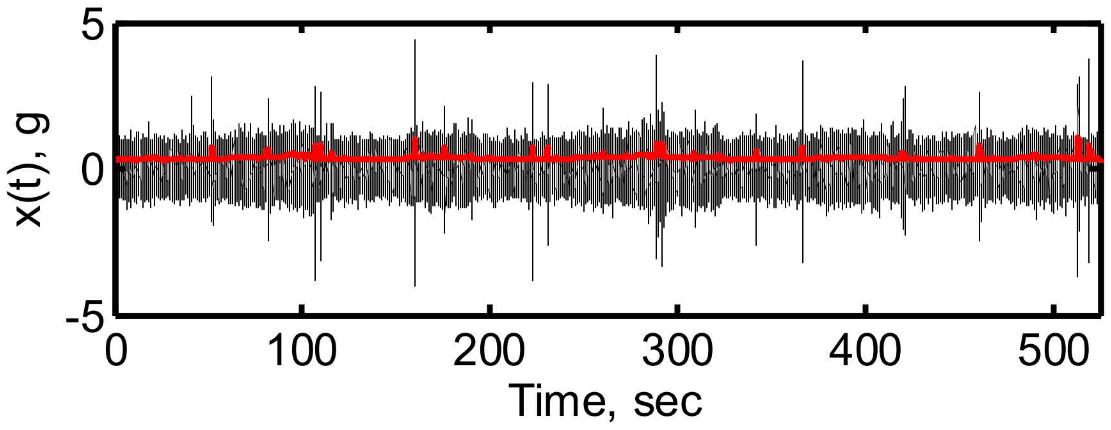
  and perform tests in one translational direction at a time.

Motivation

- Why do we wish to analyze and synthesize mixed spectrum environments?

1. So that, following statistical analysis, we can specify tests for physical systems subjected to the environments.
2. So that we can generate realizations of the stochastic environments for analysis.
3. So that we can use the results of 1. and 2. in the design and decision-making process.

Example



This is an acceleration signal that is a mixed environment. It is composed of:

- Background, stationary random and
- Shocks that are superimposed on the background.
- The signal duration is $524\ sec$. Time increment is $\Delta t = 0.001\ sec$.
- The red line is the time dependent RMS.

- There are multiple stationary random environments composing the background – various RMS levels.
- The shocks appear at random times and have different random amplitudes. They may come from different shock sources.

Are there any (low-level) shock realizations hidden in the stationary random background?

Example

- Where do mixed environments occur in structural engineering?
  - Automotive highway travel and accidental impacts
  - Commercial aircraft takeoff, cruise/maneuver, landing
  - Missile ignition, launch, stage separations
  - Reentry vehicle reentry
  - Military ground vehicle travel over rough terrain and gunfire
  - Military aircraft takeoff, cruise/maneuver, gunfire, landing
  - Bomb and small missile handling, captive carry, launch, impact
  - Military ship cruise, maneuver, gunfire

Future Goal

- In the near future, we intend to:
  - Explore the representation of shocks using Wavelet Transforms, and
  - Develop a method for separating shocks from background random vibration using Wavelet Transforms.

- What are wavelets? Where are they used? How do they work?

Wavelet Applications

- Developed in the 1980s, largely due to advances in computing
- In vogue by the 1990s
- Ubiquitous today for any and all data exhibiting time-dependent behavior

molecular dynamics chaos theory astrophysics gravitational waves transient data analysis seismology optics turbulence quantum mechanics pattern recognition image processing medical research DNA analysis finance protein analysis climatology general signal processing speech recognition acoustics vibration signals computer graphics multifractal analysis computer vision Compression image processing Signal denoising

Wavelets

- Class of functions with the following properties:
    - Satisfy an admissibility condition (sum to zero in time)-- **must be wave (oscillatory)**
    - Satisfy regularity conditions (finite approximation)– must have **a fast decay**
        - The number of vanishing moments determines how fast a particular wavelet decays—how many vanishing moments you need depends on your application.

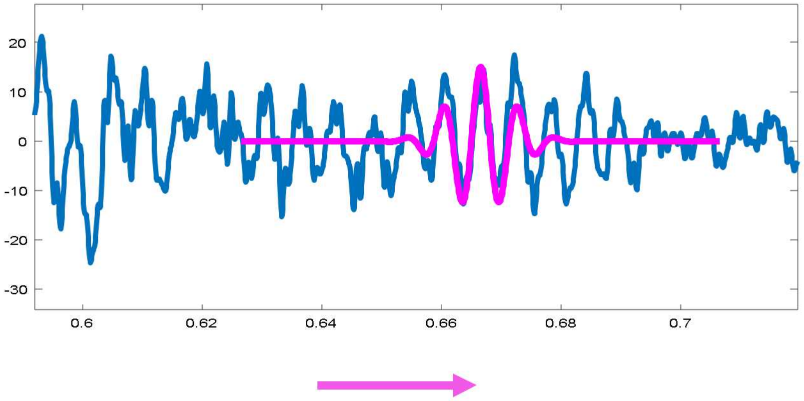
Wavelets

- Act as filters:
  - Fully scalable, modulated window
- Subtopics for better understanding
  - Fourier Analysis, Filter Banks, Sub-band coding (breaking a signal into different frequency bands)

Continuous Wavelet Transform (CWT)

$$\boldsymbol{CWT(s,\tau)} = \boldsymbol{\Upsilon(s,\tau)} = \int \boldsymbol{f(t)\Psi^{*}_{s,\tau}(t)dt}$$

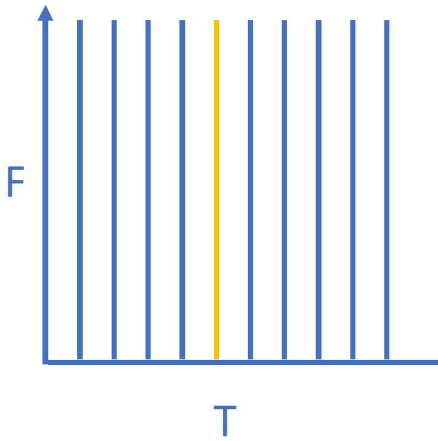
$$\boldsymbol{\Psi_{s,\tau}} = \frac{\mathbf{1}}{\sqrt{\boldsymbol{s}}}\boldsymbol{\Psi}\left(\frac{\boldsymbol{t - \tau}}{\boldsymbol{s}}\right)$$



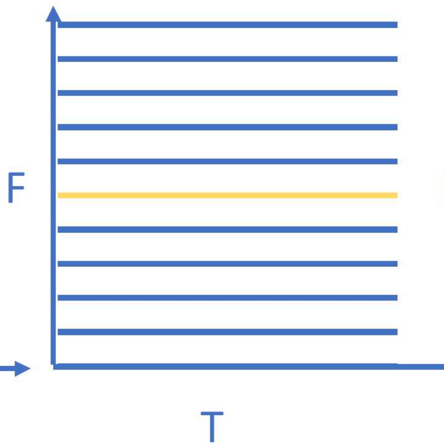
- Wavelets are scaled ($\boldsymbol{s}$) and translated ($\boldsymbol{\tau}$) along a time history.
  - The CWT is the correlation between $\boldsymbol{f(t)}$ and the wavelet $\boldsymbol{\Psi^{*}_{s,\tau}(t)}$.
- Note the CWT is two dimensional for a 1D signal
- This form is highly redundant and computationally intensive
  - In practice, the scaling function is discretized into steps and the transform is applied like a filter bank to reduce redundancy.

Resolution

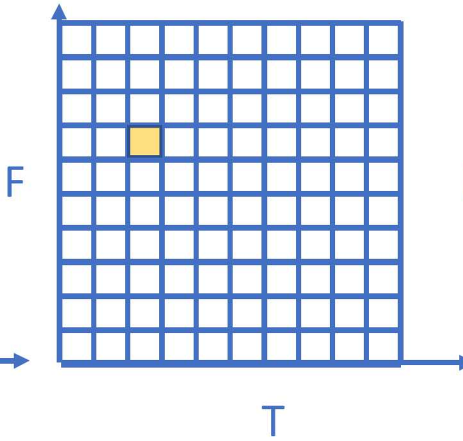
- Wavelet analysis is **Multi-resolution** analysis
- **Time-scale** instead of time-frequency (scale is converted to frequency)



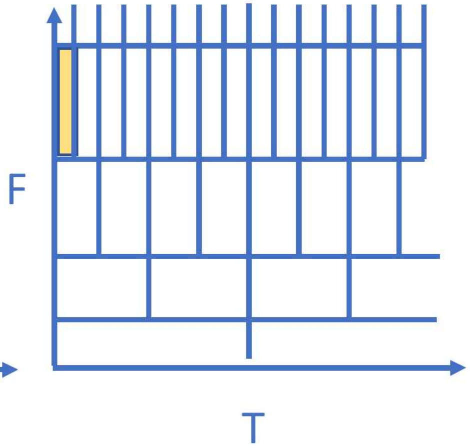
Time Domain



Frequency Domain

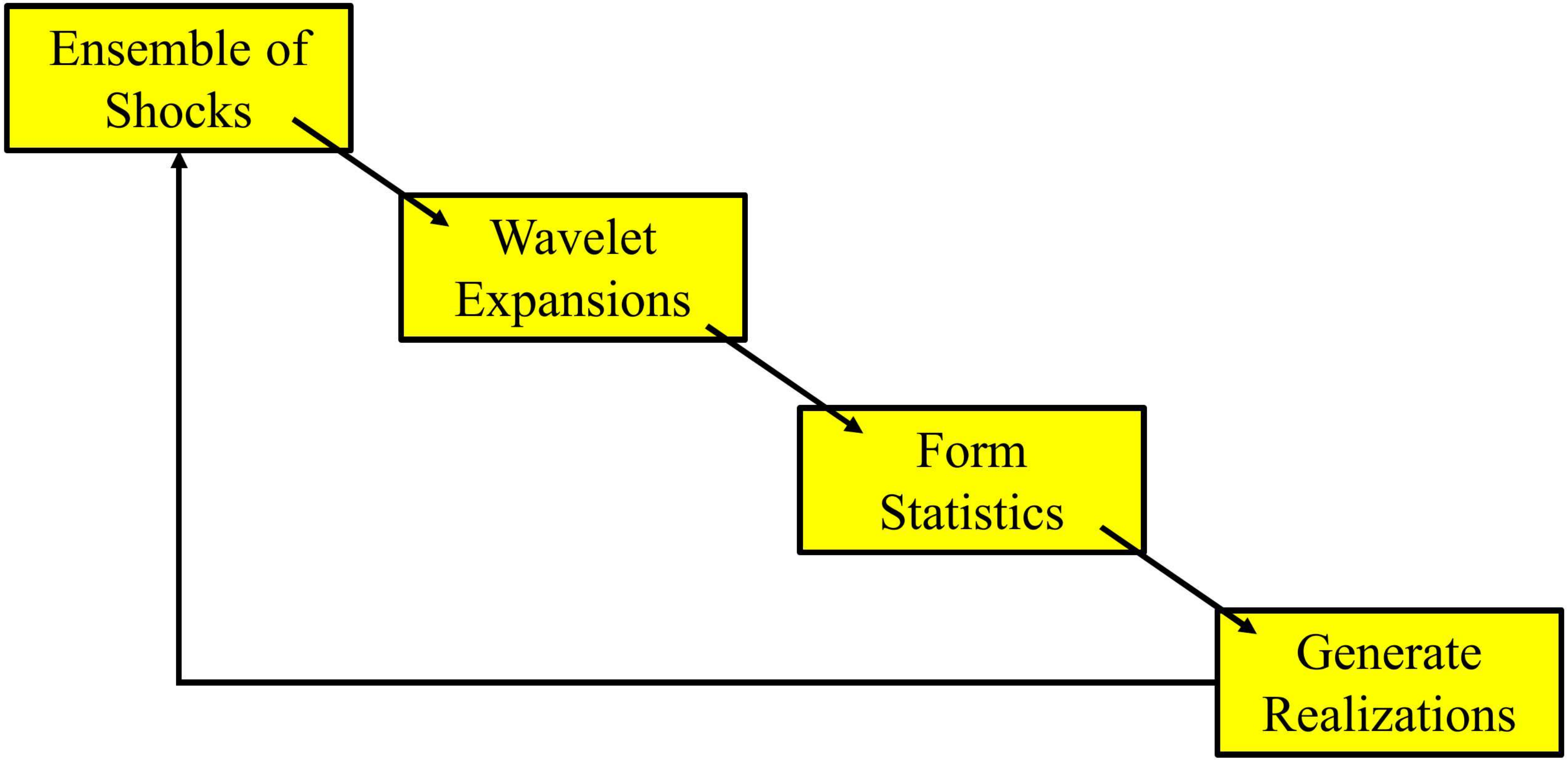


Time-Frequency Domain via **STFT**



Time-Frequency Domain via **Wavelet**

Part of Our Intended Procedure



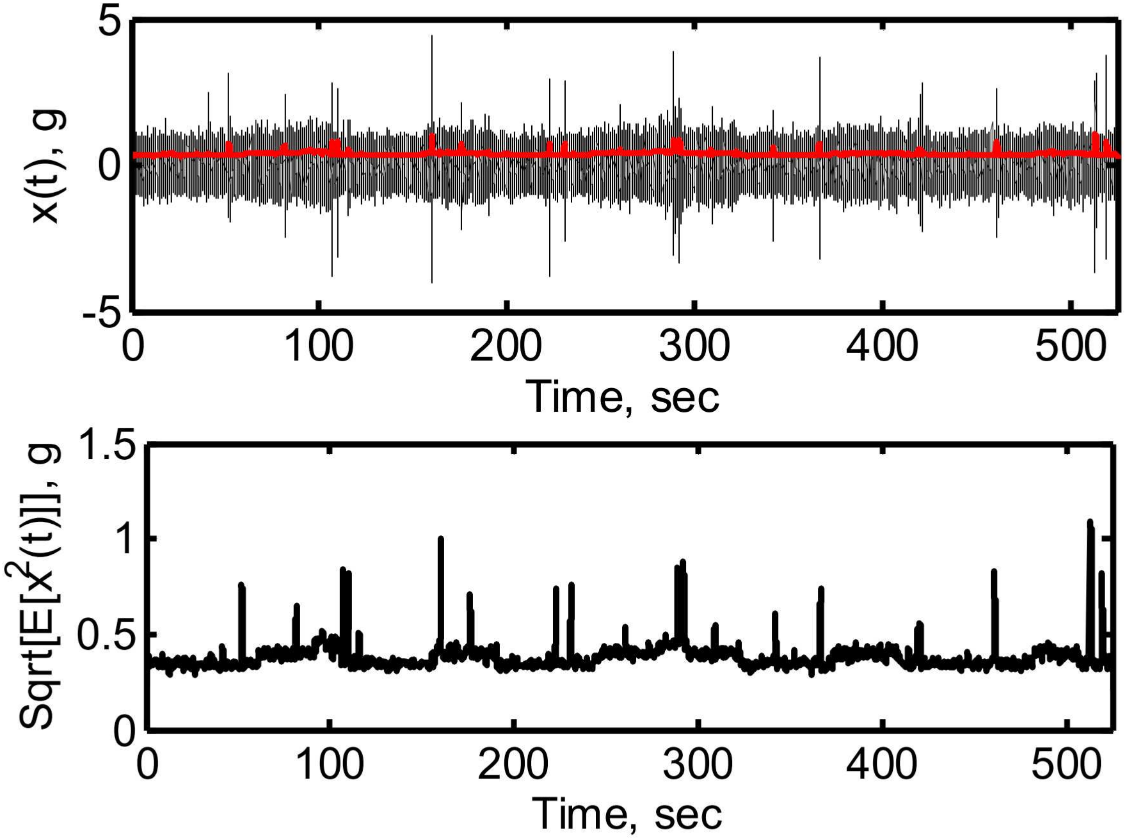
Ensemble of Shocks

Wavelet Expansions

Form Statistics

Generate Realizations

Example



- The top figure is a re-plot of the environment shown on slide 5.
- The lower figure re-plots the time dependent RMS, by itself.
- This function (and, perhaps, others) may be useful in establishing where most of the shocks in the signal occur.
- The function also indicates different mean squares in the signal segments.

Current Approach to Modeling the Parts of a Complex Environment

- Our approach to analysis of mixed spectrum signals will include the following:
  1. Divide the mixed spectrum signal into (background) stationary segments with different RMS levels (and different frequency content), and with superposed shocks.
  2. For each stationary segment, eliminate the shocks and estimate the spectral density of what remains.
  3. Model the stationary background using the spectral density of separate segments.
  4. Filter each isolated shock into components.
  5. Estimate the time-dependent mean square (MS) of all components of each isolated shock.
  6. Use knowledge of the spectral density of the stationary signal to subtract stationary random effects from the shock. What is left is the MS of the shock.

Current Approach to Modeling the Parts of a Complex Environment

- Our approach to analysis of mixed spectrum signals will include the following:
  7. Divide the shocks and their components into groups according to amplitude level.
  8. Compute the sample means of shock component RMSs within each group.
  9. Fit a parametric model to the component RMS shapes for shocks in each group.
  10. Perform statistical analysis of shock amplitudes.
  11. Use the spectral density information to generate random segments of stationary background. Use the shape information and the amplitude information on shocks to generate shock realizations; Superpose shocks on the stationary background.

Current Approach to Modeling the Parts of a Complex Environment

- Suppose we had some method for separating a complex environment into:
  - Stationary random environments – possibly, at multiple levels and each with its own frequency content.
  - Oscillatory nonstationary random environments. (Separated from the background, stationary random environment)
  - Random classical pulses. (Separated from the background , stationary random environment)
- How would we describe each component?
- When we synthesize the components, will the result resemble the original signal in some sense?
- (We will try to do this for a scalar signal.)

Stationary Random Environments

- We will model the background environment as a stationary random process.
- The *random process* is denoted $\{X(t_j), t_j \in T\}$. The random process is discrete-parametered (time), and continuous–valued.
- The random process is a sequence of *random variables*, $X(t_j), j = 0, \ldots, n-1$, defined at times $t_j = j\Delta t, j = 0, \ldots, n-1$.
- Each random variable in the random process has its own *probability distribution* (*probability density function* (PDF)).

Stationary Random Environments

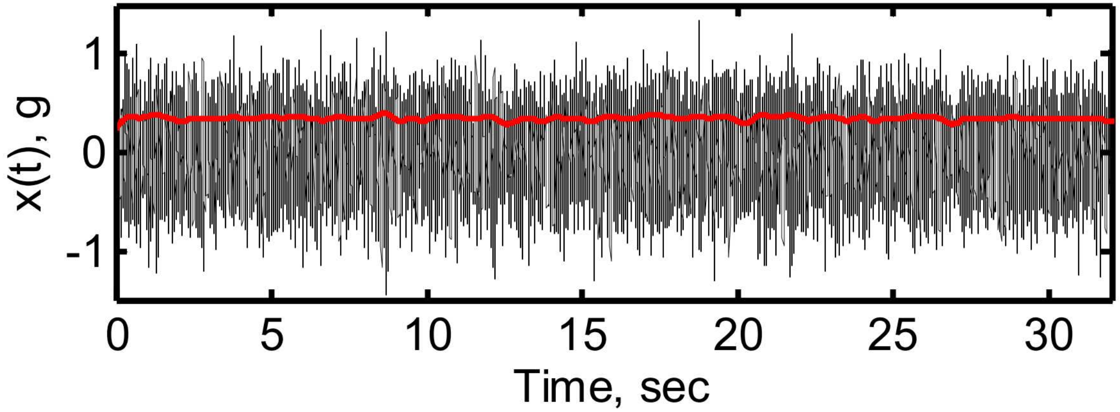
- Each pair of random variables in the random process has its own *joint PDF*. Etc.
- A *stationary random process* is one that is in a temporal steady-state at all times. (In a practical sense, this means that we treat random processes that are in a steady-state over a "long" period of time as though they are stationary.)
- A *nonstationary random process* is one that is not in a steady-state over time. Either,
  - Its amplitude content varies with time, or
  - Its frequency content varies with time, or
  - Both.

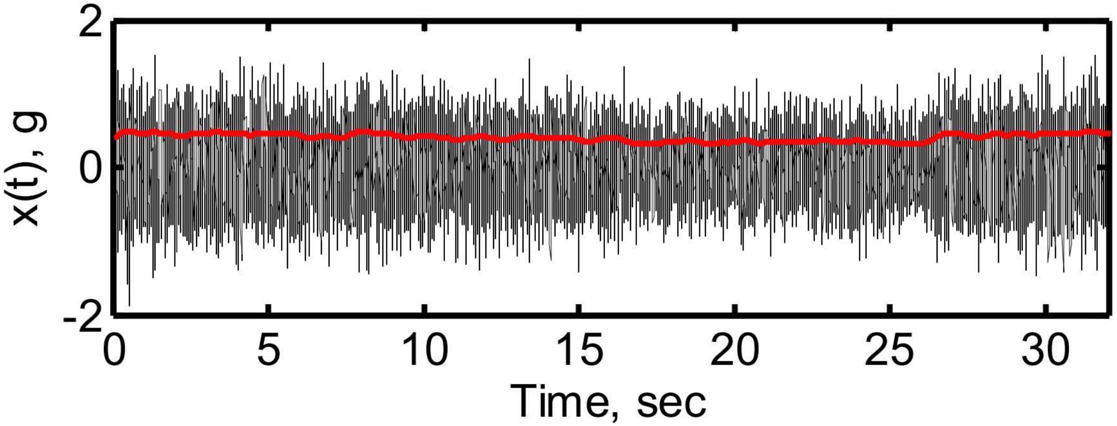
Stationary Random Environments

- For present purposes:
  - We will treat the background as a sequence of stationary random processes. (The number of different stationary environments must be established automatically, or "by hand.")
  - We will treat the shocks, both oscillatory and classical, as realizations of a nonstationary random processes. To keep the analysis simple, we will use a simple, generic nonstationary random process model.

Stationary Random Environments - Example

- What do we mean by sequence of stationary random processes?



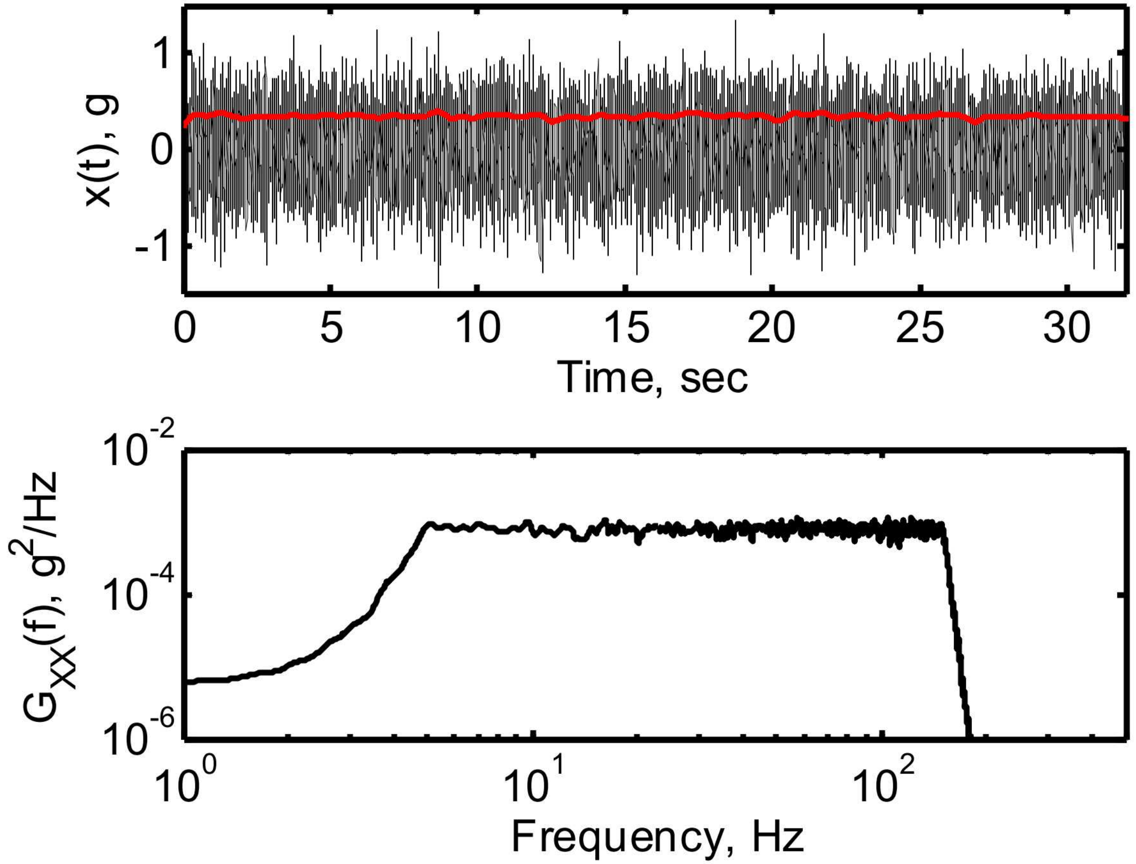


- The top signal is a realization from a single stationary random process. $t_{max} = 32.8\ sec, \Delta t = 0.001\ sec, \sigma_X = 0.35\ g.$
- The signal shown below is a realization that comes from two or more sources, each with its own RMS. $t_{max} = 32.8\ sec, \Delta t = 0.001\ sec.$

Stationary Random Environments

- Every stationary random process has a spectral density. (This can be developed from fundamentals, but we will not do that, here.)
- The *spectral density* is a nonnegative function defined as a function of frequency. (It is the Fourier transform of an autocorrelation function.)
  - The area under a spectral density function is the mean square of its stationary random process.
  - Its amplitudes reflect the relative, mean square signal content at various frequencies in a random source.

Stationary Random Environments - Example



- The signal shown earlier (single source) is repeated at top.
- Its estimated spectral density is shown below.
- For later reference, Parzen’s method was used. Triangular lag weighting in time domain. Time domain lag weighting width, $T = 1\ sec$.

Stationary Random Environments

- There are, at least, five good methods to estimate spectral density.
    - Assessment of mean square – Parseval's rule.
    - Welch's method.
    - Parzen's method.
    - Karhunen-Loeve expansion.
    - ARMA modeling.
- We use Parzen's method.

Stationary Random Environments – Parzen's Estimate of Spectral Density

- Parzen's estimate of spectral density starts with:

$$\sigma_X^2 = \int_{-\infty}^{\infty} S_{XX}(\omega)d\omega$$

where σ_X^2 is the *mean square* of a random process (We assume the random process is zero-mean.) and $S_{XX}(\omega), -\infty < \omega < \infty$ is the two-sided spectral density of the random process.

- Given a measured signal, $x_j, j = 0, \dots, n-1$, from a stationary source, the mean square is estimated with the formula:

$$s_X^2 = \frac{1}{n}\sum_{j=0}^{n-1} x_j^2$$

Stationary Random Environments – Parzen's Estimate of Spectral Density

- There is an expression known as Parseval's relation that expresses the mean square in the frequency domain.
- Let $\xi_k, k = 0, \dots, n-1$ denote the discrete Fourier transform of the time domain signal. It is defined:

$$\xi_k = \sum_{j=0}^{n-1} x_j e^{-i2\pi jk/n} \qquad k = 0, \dots, n-1$$

- Parseval's relation is:

$$\sum_{k=0}^{n-1} |\xi_k|^2 = n \sum_{j=0}^{n-1} x_j^2$$

Stationary Random Environments – Parzen's Estimate of Spectral Density

- Parseval's relation implies:

$$s_X^2 = \frac{1}{n^2}\sum_{k=0}^{n-1}|\xi_k|^2$$

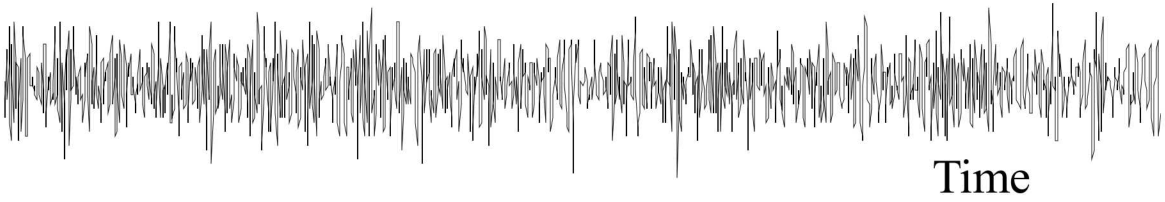
- And this implies that $(1/n)|\xi_k|^2, k = 0, \dots, n-1$ is an (unsmoothed) estimator for the spectral density. It is known as the *periodogram*.
- Smoothing the periodogram results in Parzen's estimator for the spectral density. In one-sided form, this is:

$$\hat{g}_{XX}(f_k) = \frac{2}{n\Delta t}\sum_m W(k-m)|\xi_m|^2 \qquad k = 0, \dots, n/2$$

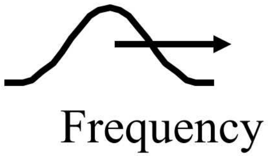
where $W(k), -n_W \leq k \leq n_W$, is a finite, symmetric, window function.

Stationary Random Environments – Parzen's Estimate of Spectral Density - Graphically

Measure realization, x_j, from stationary random source



Specify window, W_k, type and width for use in spectral density estimation



Compute modulus squared, $|\xi_k|^2$, of DFT of signal

Convolve window with modulus squared of DFT and scale to obtain spectral density estimate

Stationary Random Environments

- We are not yet ready to exercise the capability, but eventually, we will need to generate realizations of stationary random processes. This is how we do it.
- Let $\{X(t), -\infty < t < \infty\}$ be a weakly stationary random process with two-sided spectral density $S_{XX}(f), -1/(2\Delta t) \leq f \leq 1/(2\Delta t)$.
- To generate a realization from the random process containing n discrete points at a time increment of Δt, compute:

$$x_j = \frac{1}{n}\sum_{k=0}^{n-1} \xi_k e^{i2\pi jk/n} \qquad j = 0, \ldots, n-1$$

Stationary Random Environments

where

$$\xi_k = |\xi_k| e^{i\varphi_k} \qquad k = 0, \ldots, n-1$$
$$\xi_{n/2+k} = \xi_{n/2-k}^* \qquad k = 1, \ldots, n/2-1$$

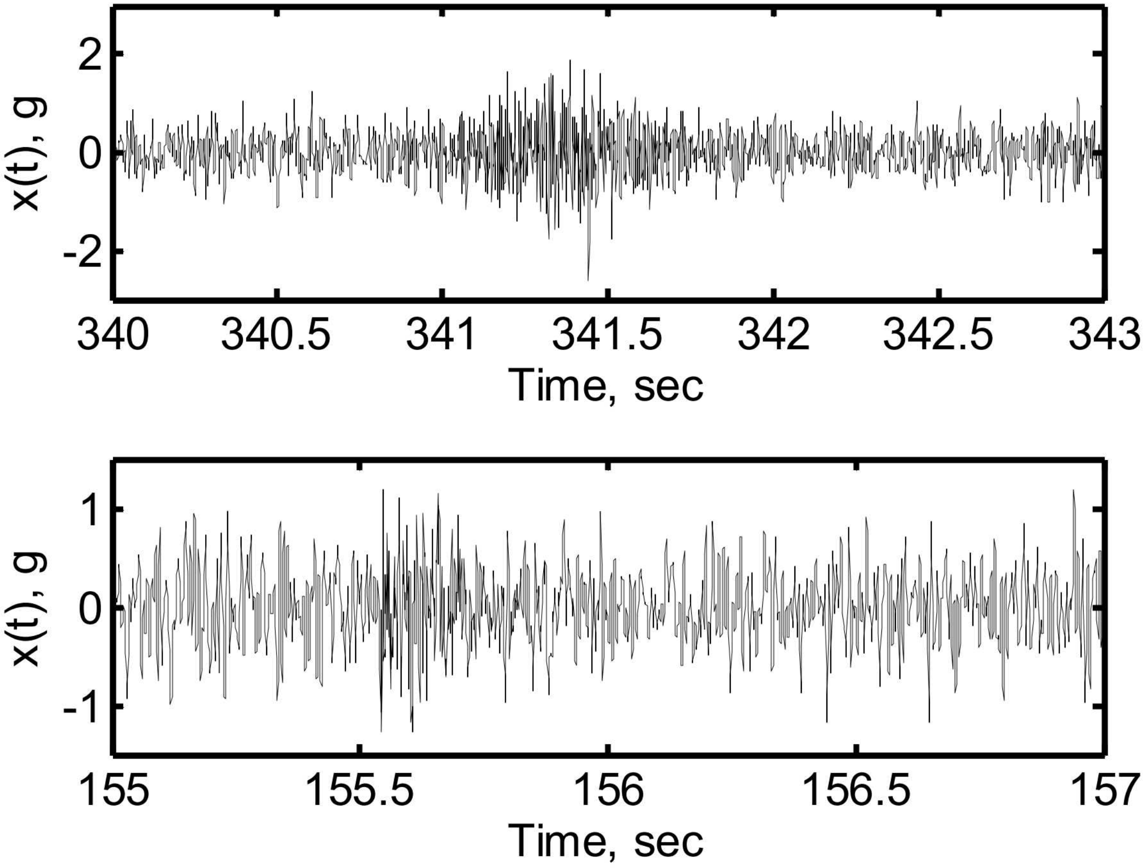
$$|\xi_k| = \sqrt{\frac{n}{\Delta t} S_{XX}(f_k)} \qquad f_k = k/(n\Delta t) \qquad k = 0, \ldots, n/2$$

The $\varphi_k, k = 0, \ldots, n/2$ are uniform random variable realizations that come from the source, $U(-\pi, \pi)$. The sources of the φ_k are independent, and $Im[\xi_0] = Im[\xi_{n/2}] = 0$.

Stationary Random Environments

- We can estimate the spectral density when we are confident that we possess a sample from a single, stationary random source.
- But we said earlier that we assume our signal of interest is from a mixed spectrum source. There may be multiple stationary random process measurements in a single measurement.
- Further, there may be shocks in the measurement.
- Our next objective is to identify shocks and separate stationary environments in a mixed spectrum signal.

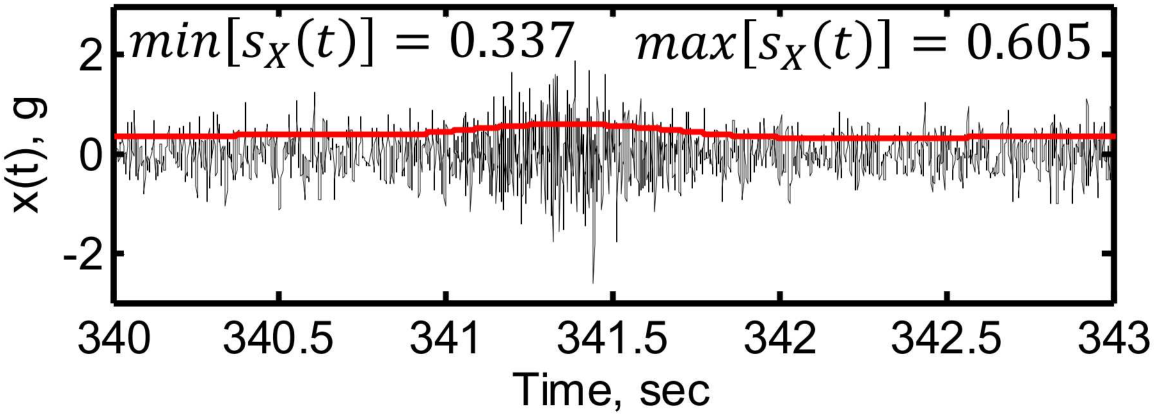
Separation of Shocks from Stationary Random Environments - Example

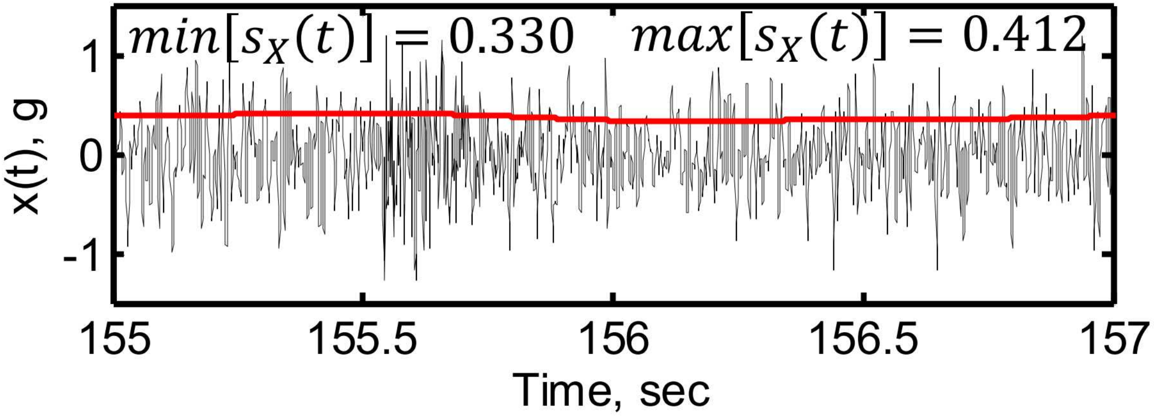


These are re-plots of short segments of the original, 524 *sec* signal on *slide 5*.

- The top slide shows that it may be relatively easy to observe a shock superimposed on the background random process realization.
- The lower slide shows that it may not be easy to observe a shock "hidden" in the stationary background.

Separation of Shocks from Stationary Random Environments - Example





- The time dependent RMS may help to establish the presence of a shock (upper figure),.
- Or it may not (lower figure).

Separation of Shocks from Stationary Random Environments

- Computation of the time dependent MS. Let $x_j, j = 0, \ldots, n-1$ be a (potentially) mixed spectrum signal.
- Its time dependent mean square can be computed as the convolution between a lag weighting and the square of the signal:

$$s_X^2(t_j) = \sum_r w(j-r)x_r^2 \qquad j = 0, \ldots, n-1$$

  where $w(j), -j_w \leq j \leq j_w$ is a symmetric, finite, positive function whose elements sum to one.
- The time dependent RMS, $s_X(t_j), j = 0, \ldots, n-1$, is the square root of the MS, above.

Separation of Shocks from Stationary Random Environments

- There are other frequency-based means for sensing the presence of a shock within a stationary background signal. A few are:
  - Effective bandwidth.
  - Centroid and moments about centroid of time dependent spectral density. (We pursue this one, here.)
- The spectral density may be estimated as outlined, above (Parzen’s method).
- We may repeat the estimate on relatively short segments of a (potentially) mixed spectrum signal.

Separation of Shocks from Stationary Random Environments

- Such computations yield an estimate of the time dependent spectral density of a random process, denoted:

$$G_{XX}(f_k, t_j) \qquad k = 0, \ldots, n_f - 1, j = 0, \ldots n - 1$$

- The spectral density estimate at each time can be normalized to unit area:

$$G_{norm}(f_k, t_j) = G_{XX}(f_k, t_j) \Big/ \left(\Delta f \sum G_{XX}(f_k, t_j) \right)$$

$$k = 0, \ldots, n_f - 1, j = 0, \ldots n - 1$$

Separation of Shocks from Stationary Random Environments

- $G_{norm}(f_k, t_j), k = 0, \ldots, n_f - 1, j = 0, \ldots n - 1$, has unit area. Treat it like a PDF – find its expected value and variance (These will have units of frequency and frequency squared):

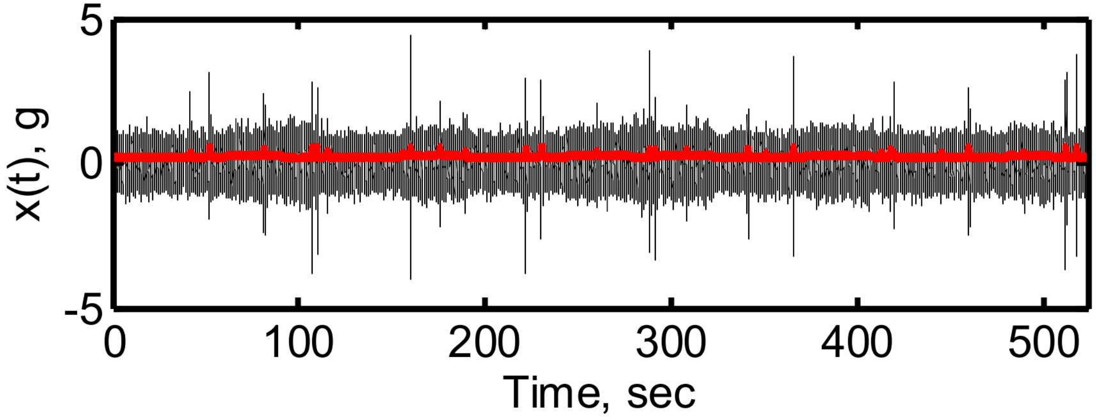
$$Freq_{G_{norm}}(t_j) = \Delta f \sum f_k G_{XX}(f_k, t_j)$$

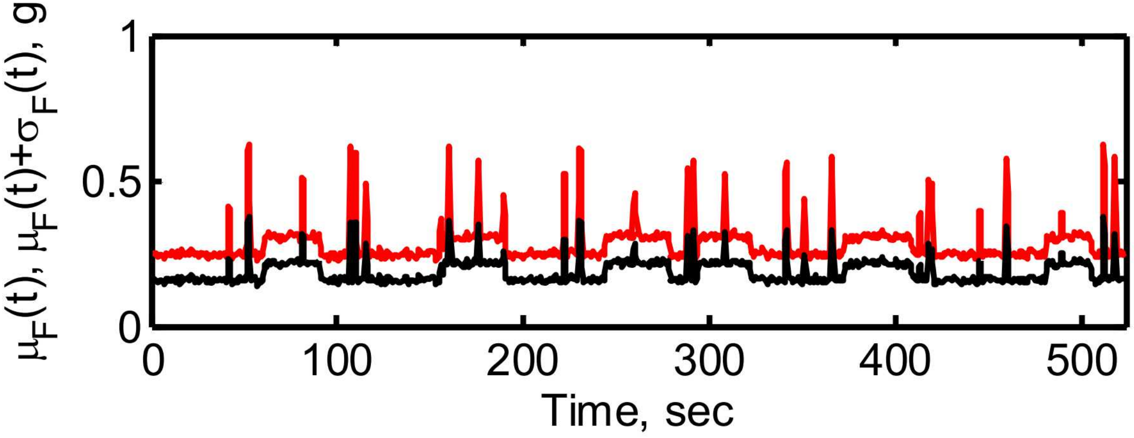
$$s^2_{Freq_G}(t_j) = \Delta f \sum \left[f_k - Freq_{G_{norm}}(t_j)\right]^2 G_{XX}(f_k, t_j)$$

$$j = 0, \ldots n - 1$$

- If the centroid and second moment about the centroid of spectral density of a mixed spectral signal change over time, these measures may detect that fact.
- These are time-varying spectral moments.

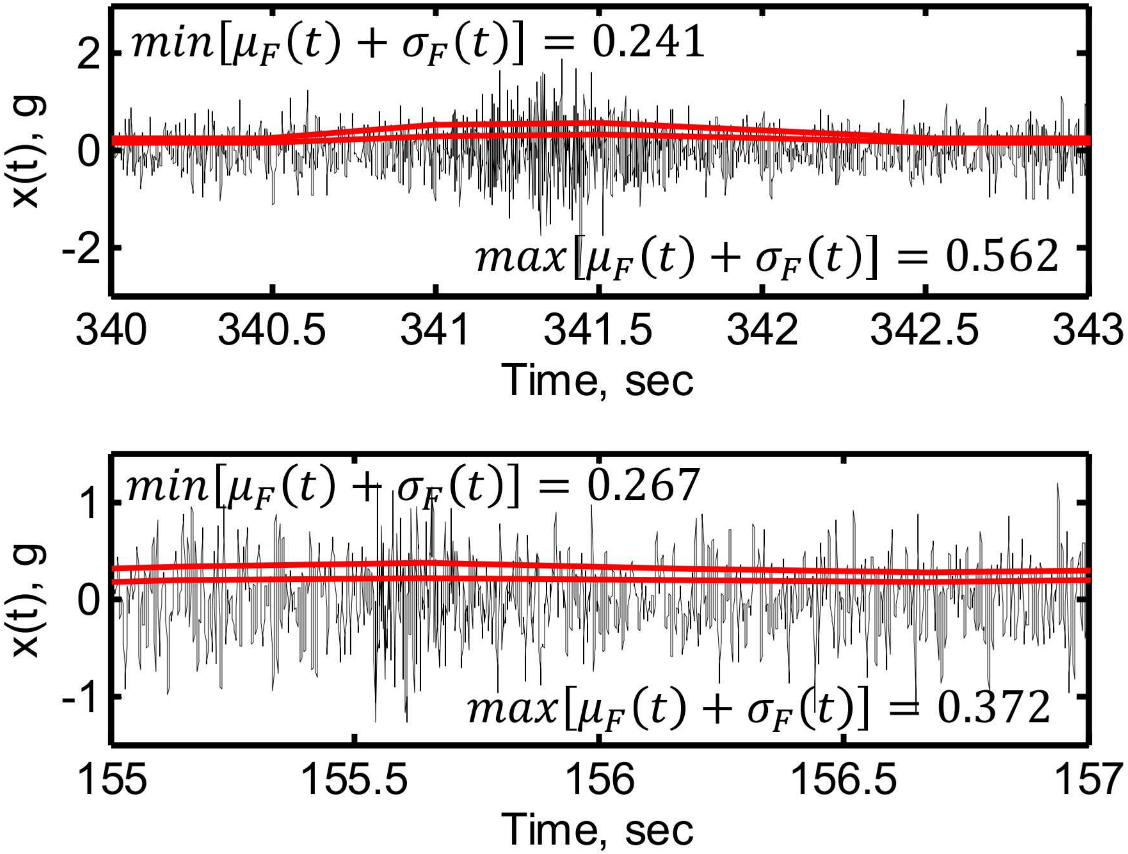
Separation of Shocks from Stationary Random Environments - Example





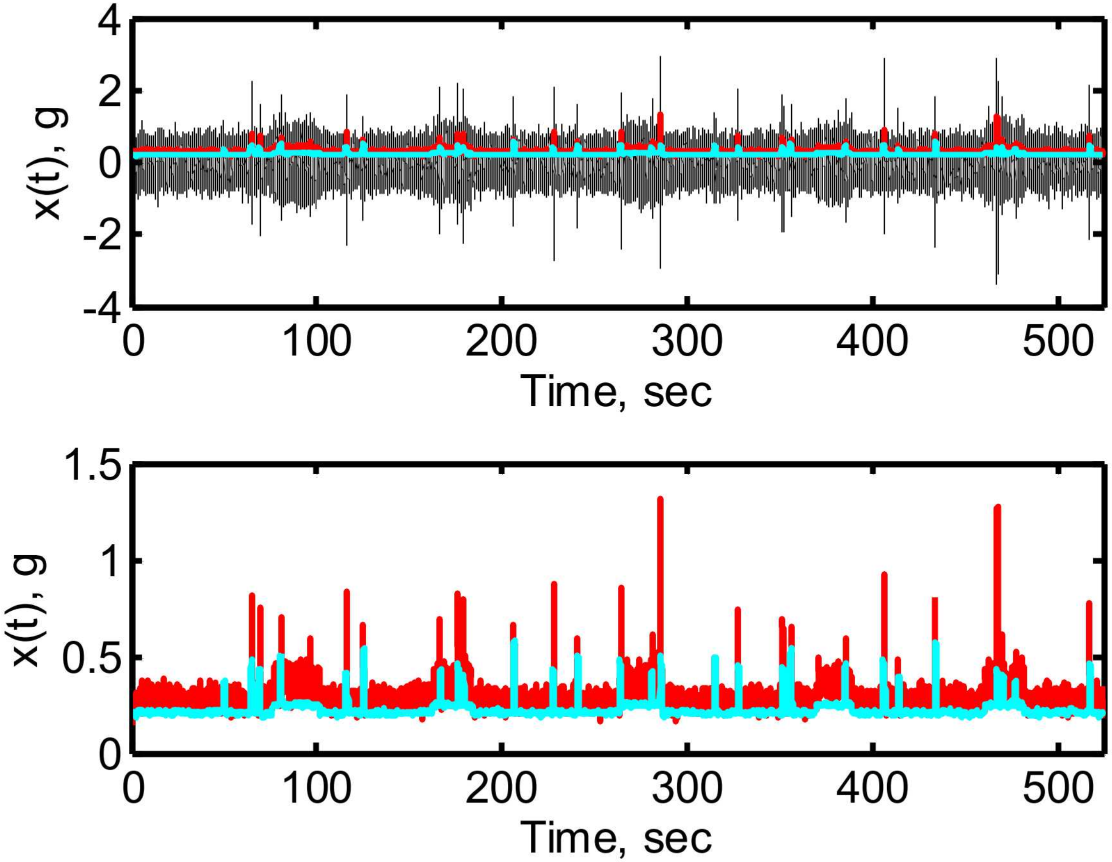
- This is a re-plot of the mixed spectrum signal from slide 4 (top) with the centroid and centroid plus square root of the second moment about the centroid of the spectral density. (Normalized by f_{max})
- The lower plot is the latter two curves, only.

Separation of Shocks from Stationary Random Environments - Example



- This re-plots the signal segments shown before (slide 32) with the centroid and centroid plus one “standard deviation.”
- These measures may or may not help reveal the presence of shocks.
- These measures of signal behavior are not superior to time dependent mean square; they are simply alternatives.

Example



- This re-plots the overall signal from slide 5.
- The time-dependent RMS is red.
- The time-dependent spectral density centroid plus one standard deviation is cyan.
- When a shock superimposed on the background is small, the RMS may miss it while the spectral measure may find it (in this case).

- Also, the spectral centroid plus one standard deviation indicate, clearly, when the background spectral density changes.

Separation of Shocks from Stationary Random Environments

- The measures shown on the previous slides – time-varying RMS and time-varying spectral moments – indicate:
  - The times when the stationary, background random vibration changes from one level to another, and
  - The times of occurrences and durations of shocks.
- We can use the time-varying RMS and time-varying spectral moments to separate the overall signal into separate stationary segments, but we would like to develop a method to separate the shocks from their background.
- One shock model is conducive to accomplishing that goal.

Nonstationary Random Environments

- There are several frameworks for modeling nonstationary random environments, and we intend to develop the Wavelet transform as a framework for modeling nonstationary random environments.
- The fundamental framework used by mathematicians is the Priestley model:

$$X(t) = \int_{-\infty}^{\infty} A(\omega, t) e^{i\omega t} dZ(w) \qquad t \in T$$

where ….

- Its use is very complicated!

Nonstationary Random Environments

- An alternate model that is a special case of the Priestley model is the product model:

$$X(t) = \sum_{k=1}^{n_c} A_k(t) W_k(t) \qquad t \in T$$

where:

n_c is the number of frequency components in the model. (Usually, this is a small number, say 4.)

$A_k(t), t \in T$ is a slowly-varying, modulating function that defines the time variation of the k^{th} component.

$W_k(t), t \in T$ is a band-limited white noise random process.

Nonstationary Random Environments

- Here are some more details on $W_k(t), t \in T$, the band-limited white noise random process. The random process has mean zero. Its one-sided spectral density is:

  $$G_{W_k W_k}(f) = \frac{1}{\Delta F} \qquad (k-1)\Delta F \leq f \leq k\Delta F, k = 1, \ldots, n_c$$

  where ΔF is a wide frequency interval, defined:

  $$\Delta F = \frac{f_{max}}{n_c}$$

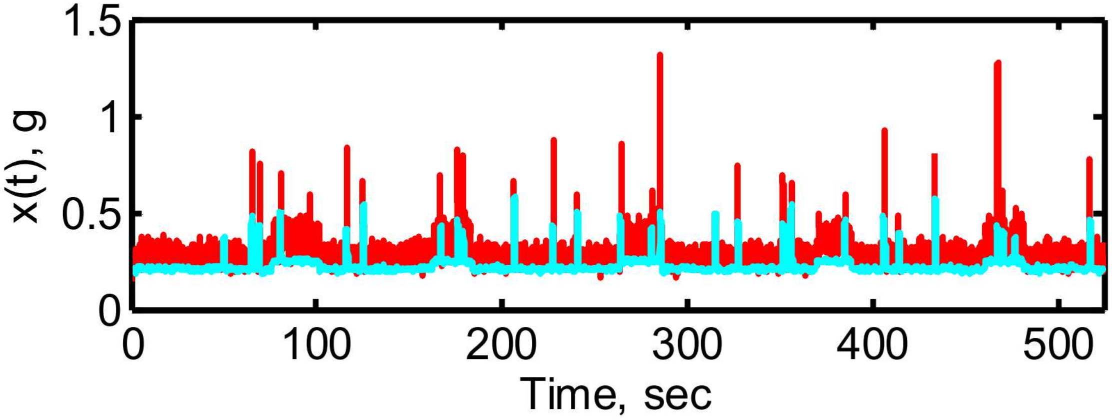
  and f_{max} is the greatest frequency to be represented in the nonstationary random process, $X(t)$.
- Note that the band-limited white noise has unit RMS. (Area under spectral density equals one.)

Nonstationary Random Environments

- If we can subtract off the background, stationary random vibration from every shock, then the form of the nonstationary random process model makes it easy to model the shock as a nonstationary random process.
- To start the process, we divide the overall signal into segments with constant spectral density and superimposed shocks. (For now, we do it “by hand.”)

Nonstationary Random Environments

Level	Start	End
1	0	75.9
2	75.9	101.6
1	101.6	162.5
2	162.5	183.6
1	183.6	262.9
2	262.9	286.8
1	286.8	369.3
2	369.3	388.5
1	388.5	460.0
2	460.0	481.2
1	481.2	523.7

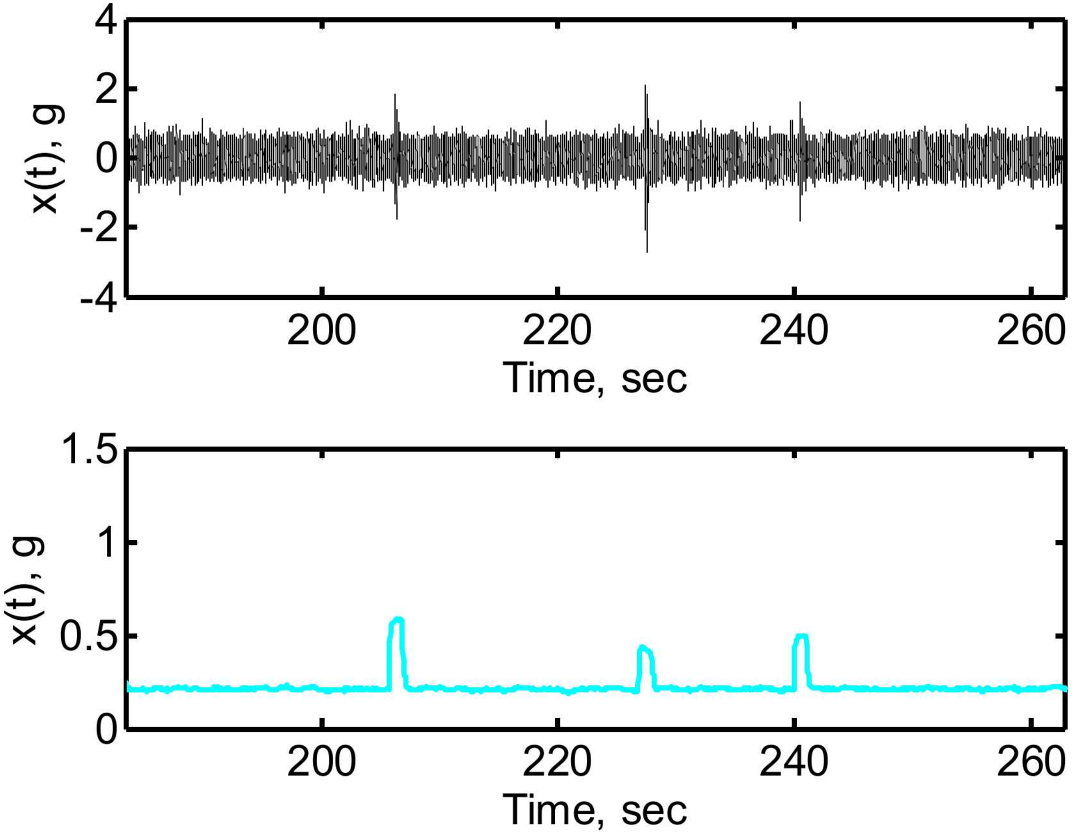


- These are the start and end-times we chose for the stationary segments.
- Choices were based on values of spectral moments. Centroid of spectral density plus one standard deviation.

Nonstationary Random Environments

- For example, consider the fifth stationary segment: $t \in [183.6, 262.9]$
- There appear to be three shocks within the stationary segment.
- Their times of occurrence can be identified:

Start	End
205.7	207.3
226.8	228.2
239.9	241.3

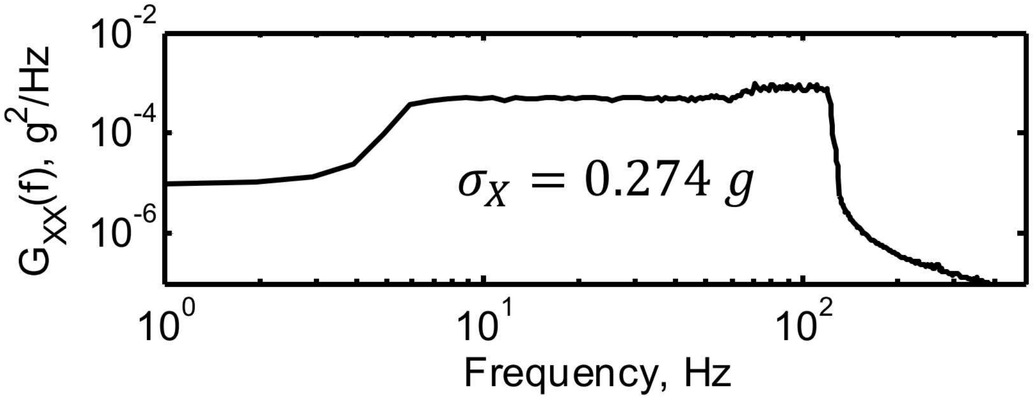


All the shocks (27) can be identified in this way. (They are, in Ex12_AnalMixedEnv.m, line 142.)

Nonstationary Random Environments

- By splicing out the shocks (and smoothing the transitions), we are left with a stationary segment of signal.
- Estimate the spectral density. Shown at right.
- The RMS values of components can be computed:

Freq band (Hz)	RMS (g)
0-128	0.273
128-256	0.019
256-384	0.004
384-512	0.003



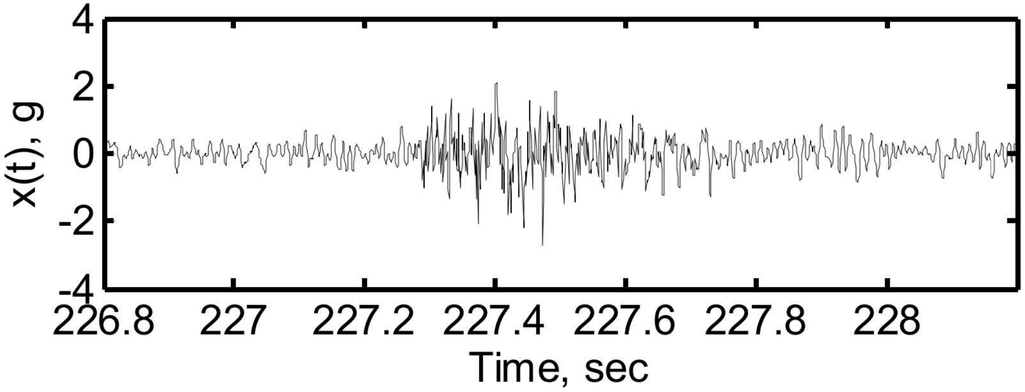
- Record the duration of this stationary segment of random vibration:

$$T = 262.9 - 183.6 = 79.3\ sec$$

- Later, the durations of stationary segments will be used to:
  - Estimate probability of a given level of stationary background, and
  - Estimate parameters of a probability law for the duration.

Nonstationary Random Environments

- Now, isolate one of the shocks shown two slides up – say, the second shock - $t \in [226.8, 228.2]$.
- Filter the shock into components. (For this application, we define four components in the frequency bands listed on the slide, above. We use four-pole, Butterworth, low-pass, band-pass, and high-pass filters.)



Nonstationary Random Environments

- The components of the shock on the previous slide are shown here.
- Their frequency bands are:

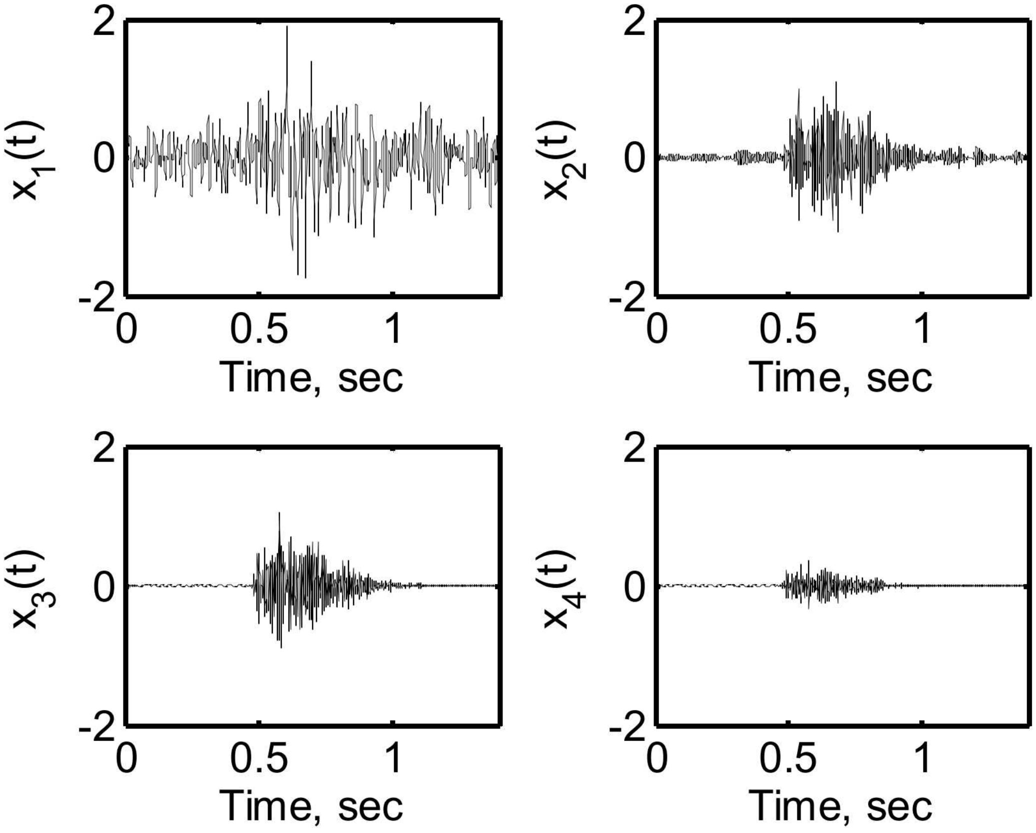
Freq band (Hz)

0-128

128-256

256-384

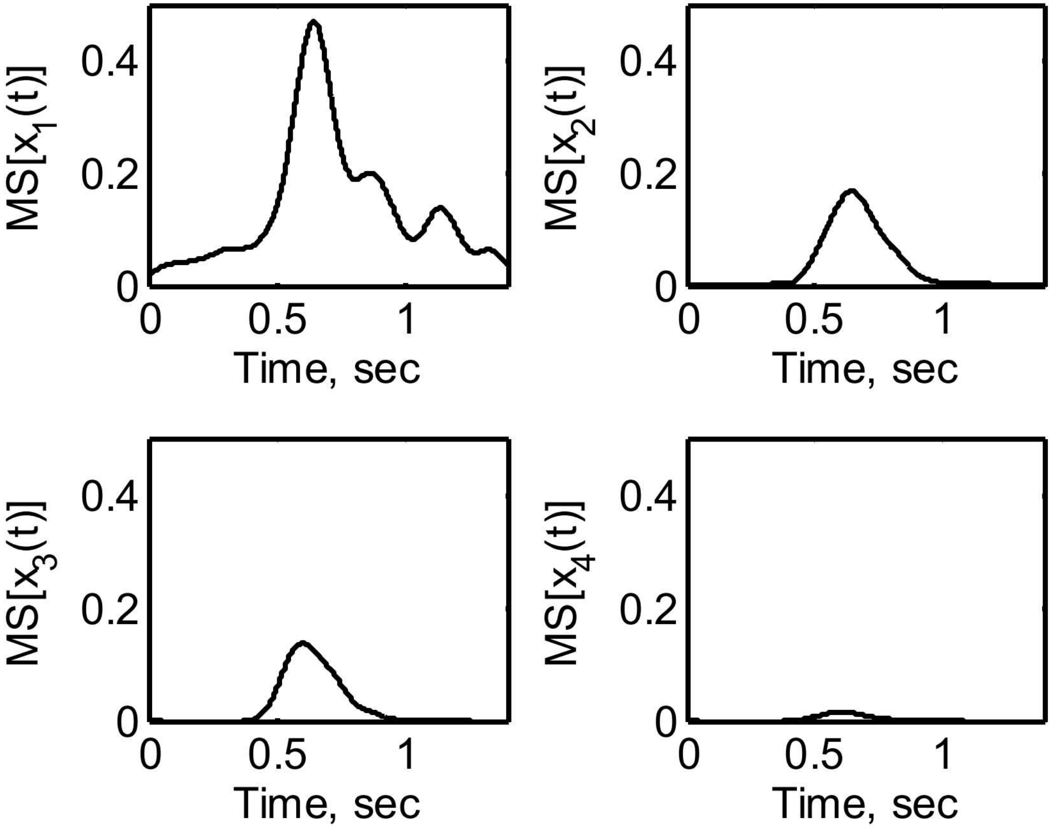
384-512



All the shocks (27) can be isolated and then filtered in this manner.

Nonstationary Random Environments

- Estimate the time-dependent mean square (MS) of each shock component.
- Then subtract the MS of each component of the stationary background from each shock component. (In effect, this treats the background within each filter band as a band-limited white noise.)
- What remains should be the MS of one component of the shock.



We can estimate the MSs of all the components of all the shocks (27) in this manner. (Note that the MSs all appear to be pulses.)

Nonstationary Random Environments

- Let's go back to the product model:

$$X(t) = \sum_{k=1}^{n_c} A_k(t) W_k(t) \qquad t \in T$$

- We could, at this point:
  - Specify one or more parametric models for $A_k(t), t \in T$, and then
  - Perform a least squares fit of the model to the MSs (or corresponding RMSs) shown on the previous slide.
- This would create a model for each shock.
- We choose to do something a little bit different.

Nonstationary Random Environments

Group 1	Group 2	Group 3
1	10	23
22	6	2
16	18	8
26	7	5
20	4	13
12	17	11
25	3	21
14	27	24
19	9	15

- We identified 27 shocks in the original signal.
- We will group them according to their peak amplitudes. Nine with the lowest amplitudes. Nine with intermediate amplitudes. Nine with the highest amplitudes. (We could also use durations, frequency content, or all three criteria, simultaneously.)
- When grouping is done as described, the groups are the shocks with indices shown at left.

Nonstationary Random Environments

Group 1	 2	 3
1	10	23
22	6	2
16	18	8
26	7	5
20	4	13
12	17	11
25	3	21
14	27	24
19	9	15

- We will take the RMSs associated with the four components of each group of shocks and average them.
- Then, we will use least squares to identify the parameters of each component at each level – 1, 2, or 3.

Nonstationary Random Environments

- There is no limit to the parametric models we might use. Here are some models:

1. $A(t) = A_0 = constant \qquad 0 \le t \le T$
2. $A(t) = A_0 e^{-\alpha t} \qquad 0 \le t \le T$
3. $A(t) = A_0 (t/T)^C \qquad 0 \le t \le T$
4. $\mathrm{A}(t) = A_0 (1 - t/T)^C \qquad 0 \le t \le T$
5. $\mathrm{A}(t) = A_0 sin(\pi t/T) \qquad 0 \le t \le T$
6. $\mathrm{A}(t) = \frac{1}{2} A_0 \left(1 - cos(2\pi t/T)\right) \qquad 0 \le t \le T$
7. $\mathrm{A}(t) = A_0 t e^{-\alpha t} \qquad 0 \le t \le T$
8. $\mathrm{A}(t) = A_0 t^n e^{-\alpha t} \qquad 0 \le t \le T$

These parametric models are included in the function nsrp11(), distributed here.

Nonstationary Random Process Model

- We choose to use parametric model number 8.

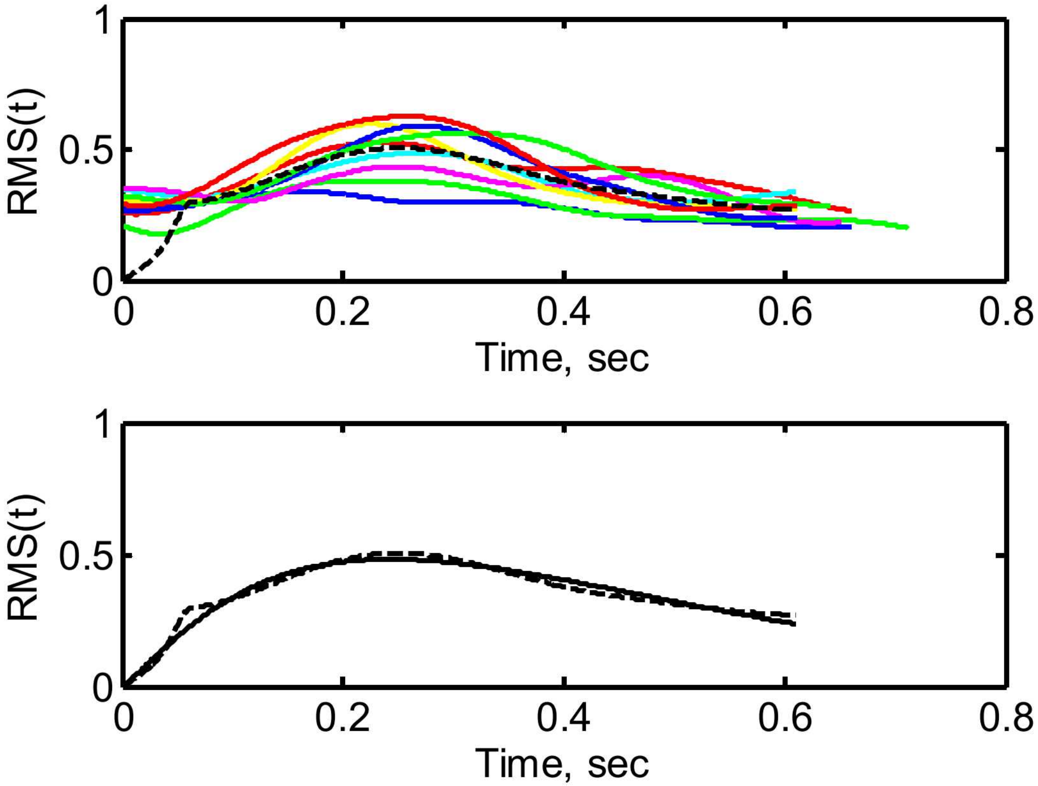
$$\mathrm{A}(t) = A_0 t^n e^{-\alpha t} \qquad\qquad 0 \le t \le T$$

- Given an experimental RMS, $s_X\left(t_j\right), j = 0, \dots, n-1$, we solve the least squares problem:

$$\begin{bmatrix} ln\left(s_X(t_0)\right) \\ ln\left(s_X(t_1)\right) \\ \vdots \\ ln\left(s_X(t_{n-1})\right) \end{bmatrix} = \begin{bmatrix} 1 & ln(t_0) & t_0 \\ 1 & ln(t_1) & t_1 \\ & \vdots & \\ 1 & ln(t_{n-1}) & t_{n-1} \end{bmatrix} \begin{bmatrix} ln(A_0) \\ n \\ -\alpha \end{bmatrix}$$

via pseudo-inverse of the coefficient matrix on the right side.

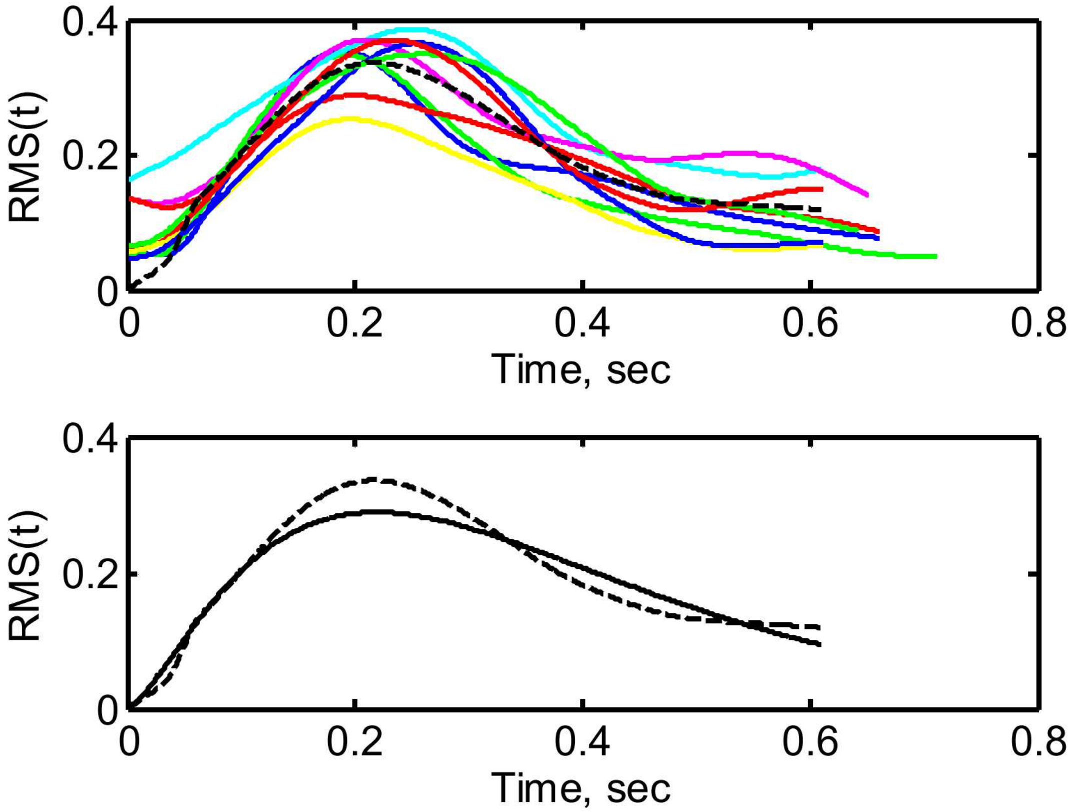
Nonstationary Random Process Model



- Here are some examples of the results we obtain when we apply the technique of the previous slide to the group 2 shocks and use parametric model 8.
- Component 1.
- RMSs of shocks 1-9 and average (dashed). Top.
- Average of RMSs (dashed) and model from previous slide. Bottom

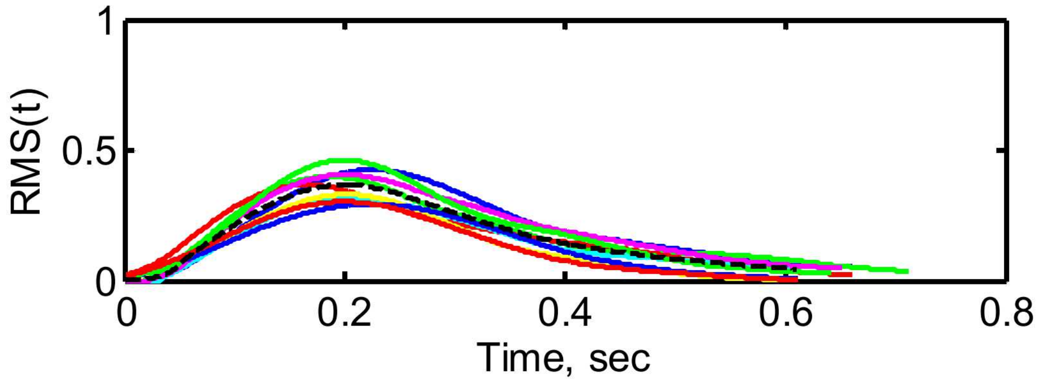
Note that the average is tapered to zero at the start to assure a good fit of the parametric model.

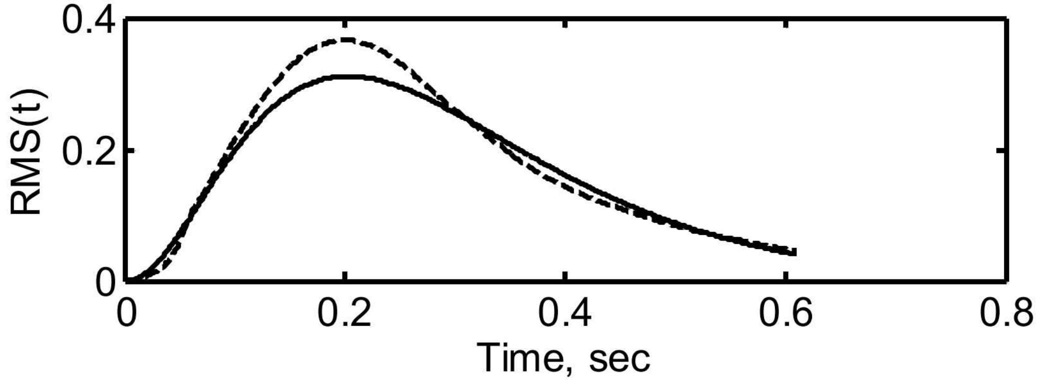
Nonstationary Random Process Model



- Group 2. Component 2.
- RMSs of shocks 1-9 and average (dashed). Top.
- Average of RMSs (dashed) and model. Bottom.
- The parametric model underestimates the highest experimental peak.

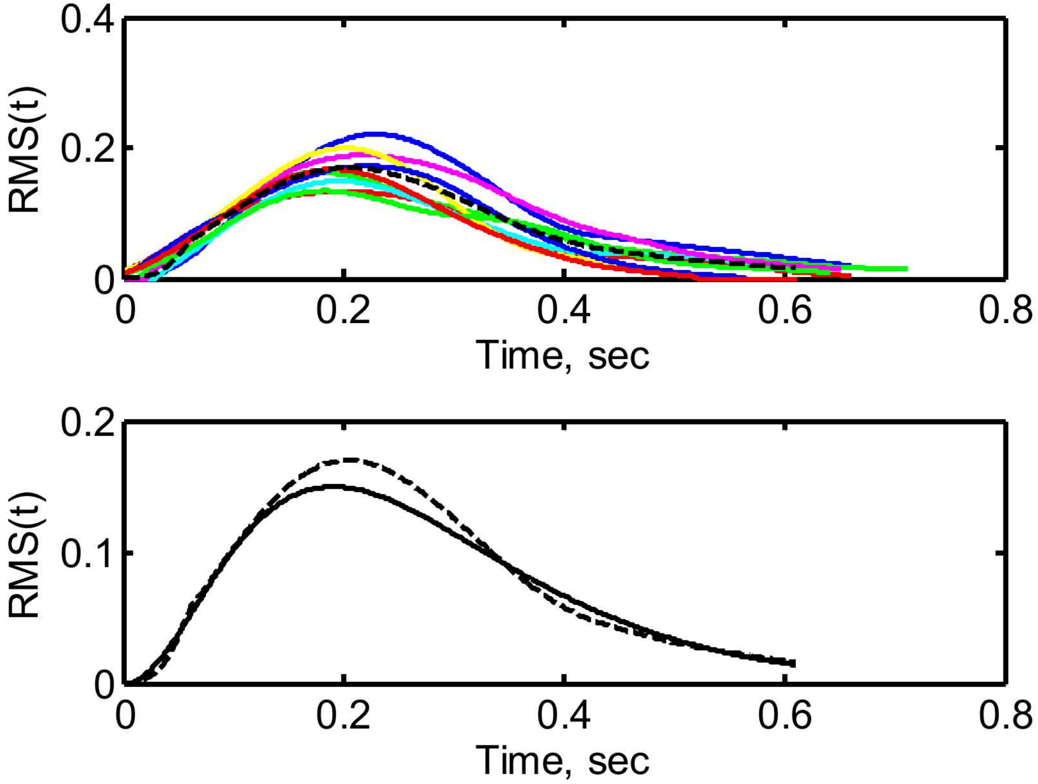
Nonstationary Random Process Model





- Group 2. Component 3.
- RMSs of shocks 1-9 and average (dashed). Top.
- Average of RMSs (dashed) and model. Bottom.

Nonstationary Random Process Model



- Group 2. Component 4.
- RMSs of shocks 1-9 and average (dashed). Top.
- Average of RMSs (dashed) and model. Bottom.

Nonstationary Random Process Model

- It now remains to establish a model for the randomness of the amplitudes of the shocks in Groups 1, 2, and 3.
- The approach we choose to accomplish this uses the peak value of the overall shock – not its components.
- Denote the peak values of the four component RMSs $Z_k, k = 1, \ldots, 4$. Then we estimate the peak of the overall signal to be:

$$Z^{(j)} = \sqrt{\frac{\pi}{2}} \sqrt{Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2} \qquad j = 1, \ldots, 9$$

(The $\sqrt{\pi/2}$ factor comes from analogy with the Rayleigh distribution. Index $j = 1, \ldots, 9$ refers to the 9 shocks in each group.)

Nonstationary Random Process Model

- The means and standard deviations of peaks for the three groups of shocks are:

	Group		
	1	2	3
Mean	1.327	1.837	2.219
Standard Dev	0.411	0.388	0.512

Nonstationary Random Process Model

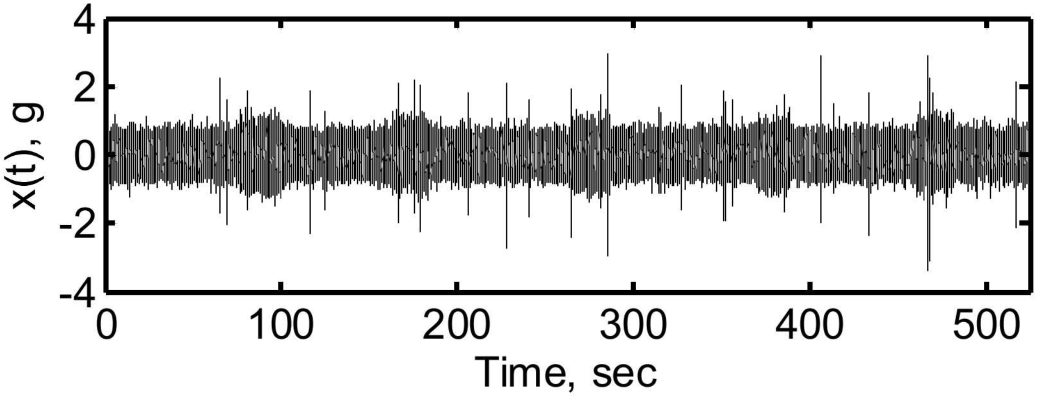
- When the components of a shock have the parameters, $[A_k, n_k, \alpha_k], k = 1, \ldots, 4$, then we can generate a realization of the shock using the formulas:

$$g(t) = \sum_{k=1}^{4} A_k t^{n_k} e^{-\alpha_k t} w_k(t) \qquad 0 \le t \le T$$

$$x(t) = z \left[\sum_{k=1}^{4} A_k t^{n_k} e^{-\alpha_k t} w_k(t) \right] \Big/ \max_t [g(t)]$$

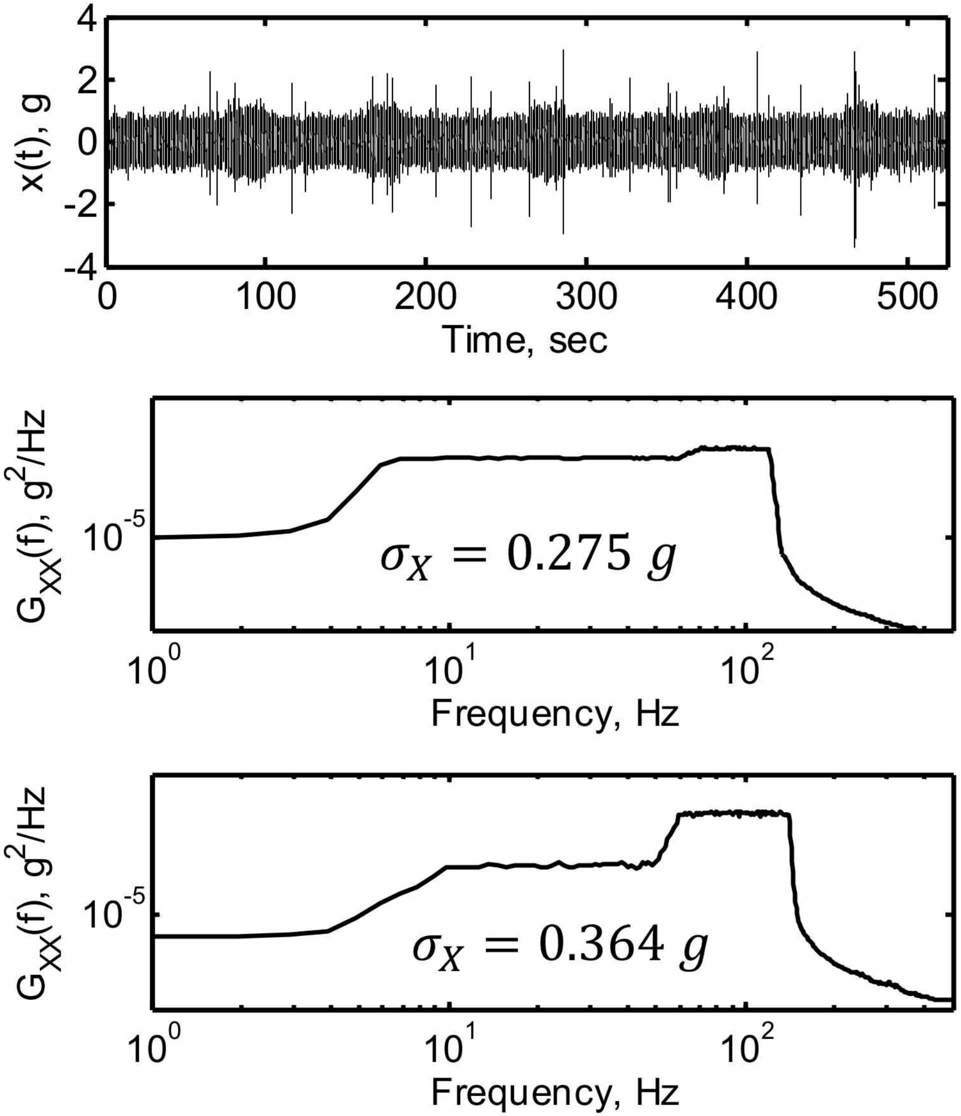
where z is a realization of a *lognormal* random variable with mean and standard deviation from one column on the previous slide, and the $w_k(t), k = 1, \ldots, 4, 0 \le t \le T$ are band-limited white noise random process realizations, as defined earlier.

Identified Nonstationary Random Environment



The signal shown at left (the same one we have considered throughout this sequence of slides) was analyzed, and the following results were obtained.

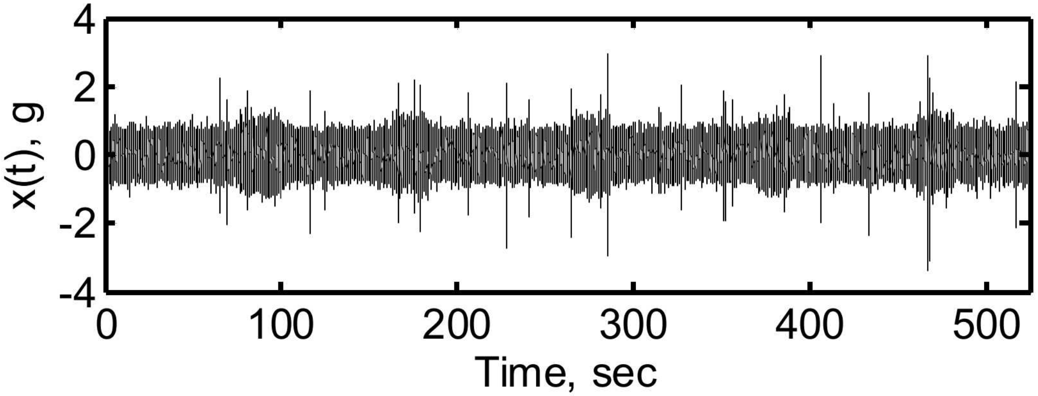
Identified Nonstationary Random Environment



- We chose to specify two, stationary random vibration, background environments. Their spectral densities are shown at left.
- Their probabilities of occurrence are: 0.756, 0.244
- The mean and standard deviation of environment durations (in seconds) are:

Index	Mean	Std dev
1	68.77	14.90
2	22.22	2.57

Identified Nonstationary Random Environment

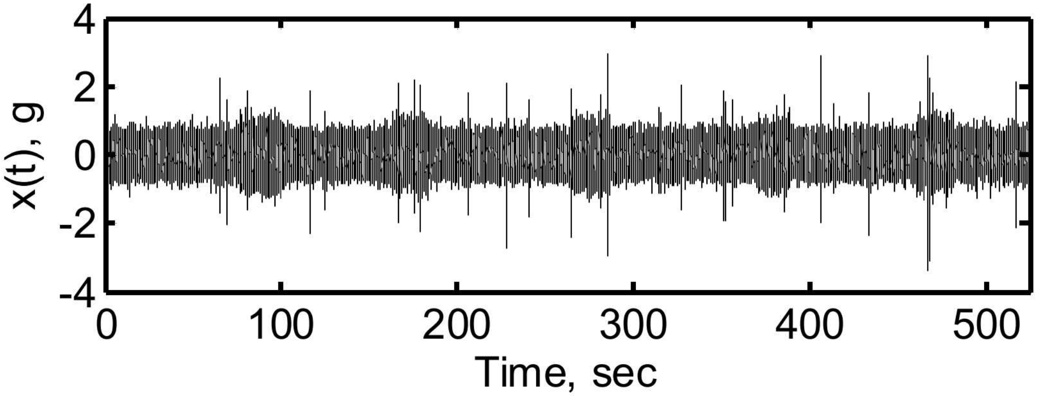


As stated earlier, 27 shocks were identified, and separated into three groups of nine each. The component shape parameters are listed below:

	Group 1			
	Component			
	1	2	3	4
A	3.66	12.01	704.7	223.3
n	0.98	1.56	3.01	2.79
α	3.78	6.81	14.56	14.46

Mean and standard deviation of wait times between shocks (seconds):
52.9   53.5

Identified Nonstationary Random Environment

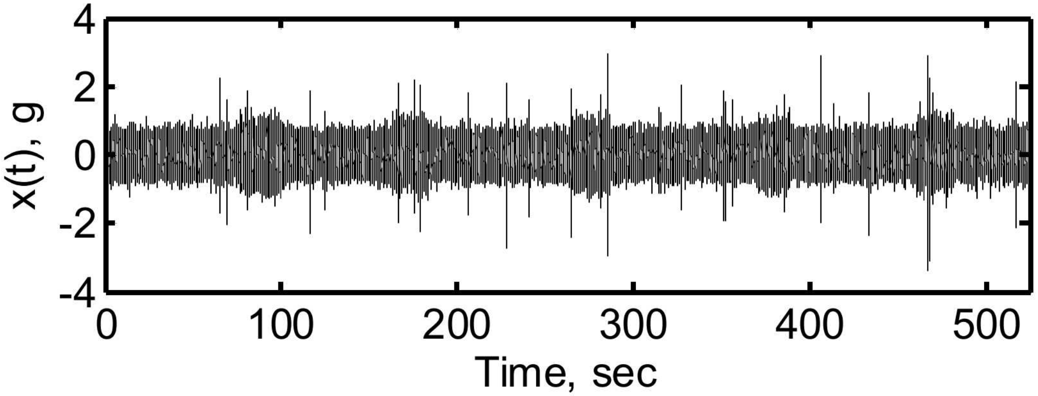


As stated earlier, 27 shocks were identified, and separated into three groups of nine each. The component shape parameters are listed below:

	Group 2			
	Component			
	1	2	3	4
A	8.81	11.82	105.22	61.79
n	1.20	1.47	2.24	2.27
α	4.95	6.73	11.06	11.89

Mean and standard deviation of wait times between shocks (seconds):
57.4 52.9

Identified Nonstationary Random Environment

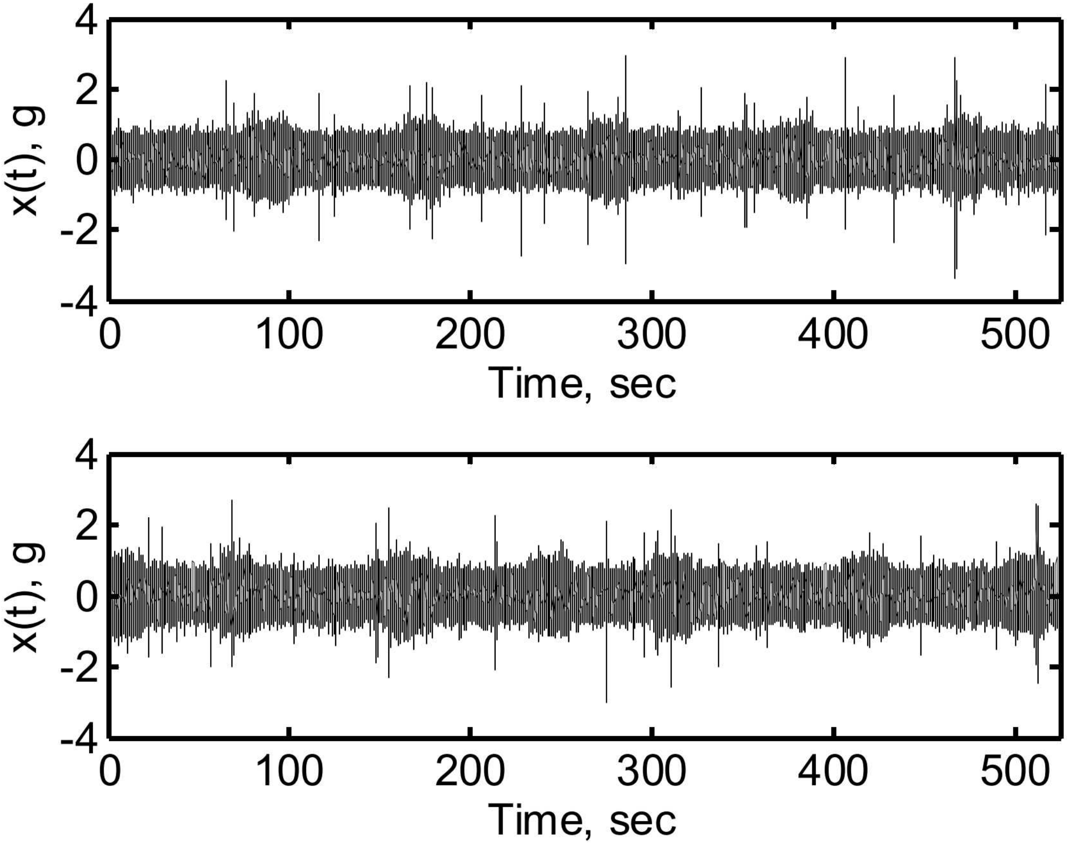


As stated earlier, 27 shocks were identified, and separated into three groups of nine each. The component shape parameters are listed below:

	Group 3			
	Component			
	1	2	3	4
A	14.28	14.80	296.0	89.01
n	1.30	1.51	2.55	2.36
α	4.97	6.19	11.98	12.15

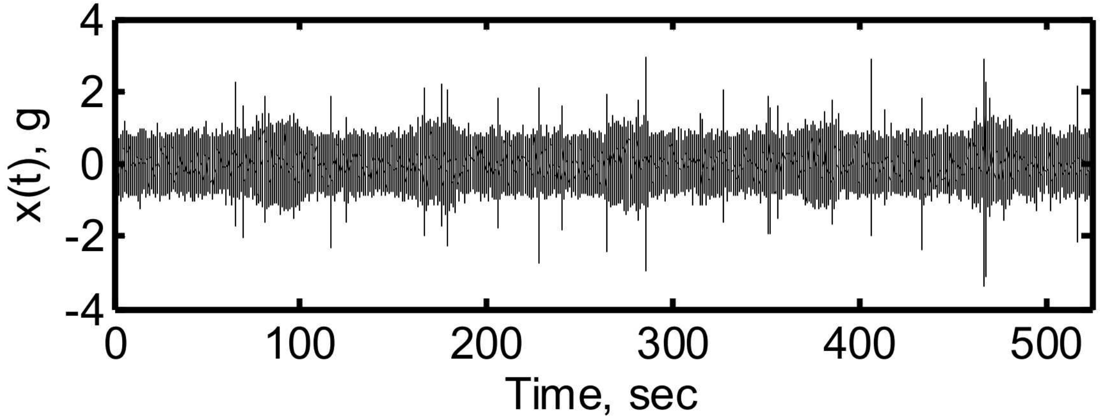
Mean and standard deviation of wait times between shocks (seconds):
51.9 29.8

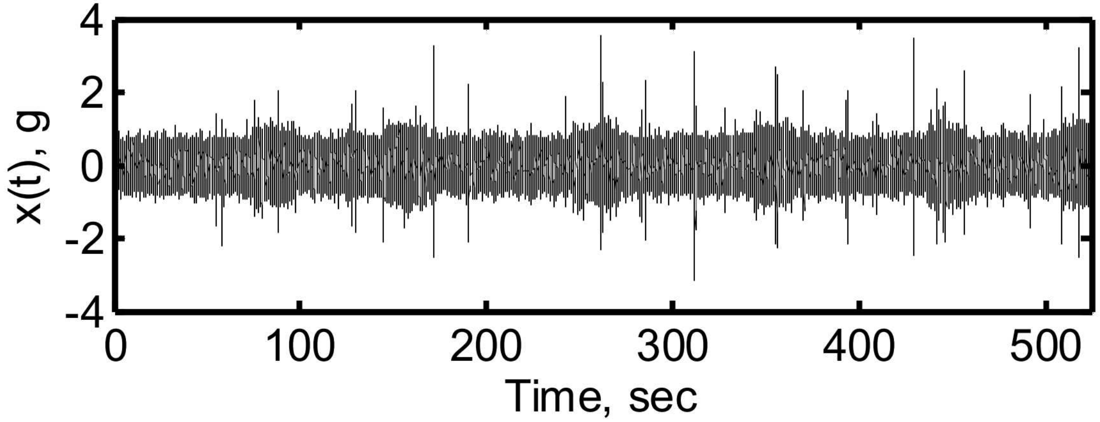
Generated Nonstationary Random Environments
Example 1



- The statistics can be used to generate mixed spectrum environments.
- The original environment is shown at top.
- A generated mixed spectrum environment is shown below.

Generated Nonstationary Random Environments
Example 2





- The statistics can be used to generate mixed spectrum environments.
- The original environment is shown at top.
- A different generated mixed spectrum environment is shown below.

Summary

- One method for analyzing and synthesizing mixed spectrum environments has been developed and demonstrated.
- The method assumes that mixed spectrum signals are composed of:
  - Segments of stationary, background random vibration, with
  - Superimposed oscillatory shocks, and perhaps
  - Superimposed classical shocks (These were not included in the discussion.)
- The computer code Gen_Mixed_Env12() was used to generate the environment used as the basis for this discussion.
- The computer code Ex12_AnalMixedEnv() was used to analyze the signal.

Summary

- Many modifications could be made to both codes (and their functions). For example:
    - The Wavelet transform might be used for modeling of the shocks and for separation of the shocks from the stationary background.
    - Correlations among random vibration durations and times of occurrence of the shocks may be considered and modeled.
    - Deterministic times of occurrence of the shocks might be modeled. (As appropriate for missile launch environments.)

Analysis of Mixed-Spectrum Environments

- Operation of the code Gen_Mixed_Env12().
- The call to the function takes the form:

```
[time,x,xRV,xOS,xCS,OSdata_Real,CSdata_Real] =
    Gen_Mixed_Env12(dt,nt,RVdata,OSdata,CSdata)
```

- These are the interpretations of the inputs:
  - dt is the time increment of the signal to be generated
  - nt is the number of samples in the signal to be generated.

Analysis of Mixed-Spectrum Environments

- These are the interpretations of the inputs:
  - RVdata is a (1)x(1) cell structure containing the following information:
    - .nRVenv The number of random vibration environments to be specified in terms of their spectral densities. Dimension: Scalar
    - .probRVenv Probability that, at a given time, a particular random vibration environment will be in effect. These values must sum to one. Dimension: (nRVenv)x(1)

Analysis of Mixed-Spectrum Environments

- These are the interpretations of the inputs:
  - RVdata is a (1)x(1) cell structure containing the following information:
    - .EdurRVenv Average duration of the random vibration environments. Durations will be generated at random, according to the lognormal law. Dimension: (nRVenv)x(1)
    - .StDdurRVenv Std dev of duration of the random vibration environments. Durations will be generated at random, according to the lognormal law. Dimension: (nRVenv)x(1)

Analysis of Mixed-Spectrum Environments

- These are the interpretations of the inputs:
  - RVdata is a (1)x(1) cell structure containing the following information:
    - .SDfreq Cell structure containing frequency break points of spectral density environments. Dimension: (nRVenv)x(1) cell structures Dimension: (nf(nRVenv))x(1)
    - .SDord Cell structure containing amplitude break points of spectral density environments. Dimension: (nRVenv)x(1) cell structures Dimension: (nf(nRVenv))x(1)

Analysis of Mixed-Spectrum Environments

- These are the interpretations of the inputs:
  - OSdata is a (1)x(1) cell structure containing the following information:
    - .nOSsrc The number of oscillatory, random shock sources to be specified in terms of the nsrp10() framework. Dimension: Scalar
    - .ntOSsrc Number of time points in an oscillatory shock source. Positive integers. Dimension: (nOSsrc)x(1)

Analysis of Mixed-Spectrum Environments

- These are the interpretations of the inputs:
  - OSdata is a (1)x(1) cell structure containing the following information:
    - .mnWtOSsrc Occurrence times for oscillatory shocks will be defined as lognormal random realizations. This is the mean waiting time between occurrences of the oscillatory sources. Dimension: (nOSsrc)x(1)
    - .sdWtOSsrc Standard deviation of waiting time between occurrences of the oscillatory sources. Dimension: (nOSsrc)x(1)

Analysis of Mixed-Spectrum Environments

- These are the interpretations of the inputs:
  - OSdata is a (1)x(1) cell structure containing the following information:
    - .mnA_OSsrc Mean values of the amplitudes of the oscillatory sources. (Lognormal source) Dimension: (nOSsrc)x(1)
    - .sdA_OSsrc Std devs of the amplitudes of the oscillatory sources. (Lognormal source) Dimension: (nOSsrc)x(1)

Analysis of Mixed-Spectrum Environments

- These are the interpretations of the inputs:
  - OSdata is a (1)x(1) cell structure containing the following information:
    - .a_OSsrc   Cell structure containing shock information described in nsrp11(). Cell dimension: (nOSsrc)x(1)

      .a Data in nsrp11().

      Dimension: (nc)x(4)

Analysis of Mixed-Spectrum Environments

- These are the interpretations of the inputs:
  - CSdata is a (1)x(1) cell structure containing the following information:
    - .On_Off Shall the option to generate classical shock pulses be turned off (0) or on (1)? Dimension: (nOSsrc)x(1)
    - .multA_CSh The classical pulse, if it is present, will have an amplitude that is (Rval)x(multA_CSh)x(A_OSsrc). These are the multiplicative factors. This can be entered as empty, [], if the classical pulse is turned off. Dimension: (nOSsrc)x(1)

Analysis of Mixed-Spectrum Environments

- These are the interpretations of the inputs:
  - CSdata is a (1)x(1) cell structure containing the following information:
    - .sdA_CSh The Rval in the previous description is a lognormal random variable realization. Its mean is one. This is its standard deviation. This can be entered as empty, [], if the classical pulse is turned off. Dimension: (nOSsrc)x(1)
    - .Comp_CSh Shall the option to include a compensating pulse be turned off (0) or on (1)? Dimension: (nOSsrc)x(1)

Analysis of Mixed-Spectrum Environments

- These are the interpretations of the inputs:
  - CSdata is a (1)x(1) cell structure containing the following information:
    - .MDur_Comp_CSh Integer multiples of durations of the classical pulses that define length of the compensating shock pulses. This can be entered as empty, [], if the classical pulse is turned off. Dimension: (nOSsrc)x(1) Positive Integer