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Chance-constrained optimization: an approximation and an application

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Chance Constraint Setting

This is a linear Joint Chance Constraint:

$$P(x_t \leq y_t^\omega + w_t^\omega, \forall t \in T) \geq 1 - \varepsilon$$

Background:

- Two-stage stochastic program with recourse
- First stage decision, x_t, second-stage decision, y_t^ω
- Possibly integer restrictions on x and/or y
- i.i.d. samples of uncertainty w_t^ω

Challenges

- CC models are computationally intractable
- A known NP-hard problem
- Existing algorithms not scalable to practical sized problems
- Feasible region is non-convex

Generic CC model

$$\max_{x,y} \quad \sum_{t\in T} (R_t x_t - \mathbb{E}[B_t y_t^\omega]) \tag{1a}$$

$$\text{s.t.} \quad \mathbb{P}(y_t^\omega + w_t^\omega \geq x_t, \forall t \in T) \geq 1 - \varepsilon \tag{1b}$$

$$x \in X \tag{1c}$$

$$y \in Y. \tag{1d}$$

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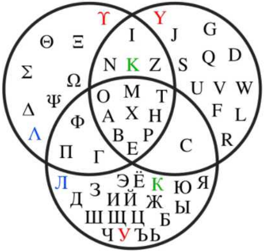
Approximations with classical probability bounds

Satisfying a JCC is an intersection of events. Failing a JCC is a union of events.



We can rewrite the JCC as follows:

$$\mathbb{P}\left(\bigcup_{t \in T} F_t\right) \leq \varepsilon$$

where $F_t = \{\omega : x_t > y_t^{\omega} + w_t^{\omega}\}$.

Approximations with classical probability bounds

$$\mathbb{P}(\bigcup_{t \in T} F_t) \leq \varepsilon$$

Consider an optimization model with a JCC with a maximization objective (such as model (1)).

- Lower Bound (LB): Approximate the LHS using a quantity **larger** than $\mathbb{P}(\bigcup_{t \in T} F_t)$. Feasible region is **restricted**.
- Upper Bound (UB): Approximate the LHS using a quantity **smaller** than $\mathbb{P}(\bigcup_{t \in T} F_t)$. Feasible region is **enlarged**.

Approximations with classical probability bounds

$\mathbb{P}(\bigcup_{t\in T} F_t) = S_1 - S_2 + \cdots (-1)^{|T|-1} S_T$
where, $S_k = \mathbb{P}(\sum_{1\leq i_1<\cdots<i_k\leq |T|} F_{i_1} \cap \cdots \cap F_{i_k})$.

Bonferroni bounds:

$$\mathbb{P}(\bigcup_{t\in T} F_t) \leq \quad S_1 \leftarrow LB \tag{2a}$$

$$\mathbb{P}(\bigcup_{t\in T} F_t) \geq \quad S_1 - S_2 \leftarrow UB. \tag{2b}$$

Approximations with classical probability bounds

$\mathbb{P}(\bigcup_{t\in T} F_t) = S_1 - S_2 + \cdots (-1)^{|T|-1} S_T$
where, $S_k = \mathbb{P}(\sum_{1\leq i_1<\cdots<i_k\leq|T|} F_{i_1} \cap \cdots \cap F_{i_k})$.

Tighter bounds from Sathe et al. [1980]:

$$\mathbb{P}(\bigcup_{t\in T} F_t) \leq \quad S_1 - \tfrac{2}{T} S_2 \leftarrow LB \tag{3a}$$

$$\mathbb{P}(\bigcup_{t\in T} F_t) \geq \quad \frac{S_1 + 2S_2}{T^2} \leftarrow UB. \tag{3b}$$

Approximations with classical probability bounds

$\mathbb{P}(\bigcup_{t\in T} F_t) = S_1 - S_2 + \cdots (-1)^{|T|-1} S_T$
where, $S_k = \mathbb{P}(\sum_{1\leq i_1<\cdots<i_k\leq|T|} F_{i_1} \cap \cdots \cap F_{i_k})$.

And more from Dawson and Sankoff [1967]:

$$\mathbb{P}(\bigcup_{t\in T} F_t) \geq \frac{{S_1}^2}{S_1 + 2S_2}. \leftarrow UB \quad (4a)$$

can be linearized for JCC $\leq \varepsilon$:

$$2\varepsilon S_2 \geq \alpha_n S_1 + \beta_n, n = 0, 1, \ldots |N| - 1, \leftarrow UB \quad (4b)$$

Optimizing over JCCs

$u_t^\omega = 1$: failure at t in scenario ω
$v_{tt'}^\omega = 1$: failure at t and t' in scenario ω

$$x_t - y_t^\omega - w_t^\omega \leq M_t^\omega u_t^\omega, \forall t \in T, \omega \in \Omega$$

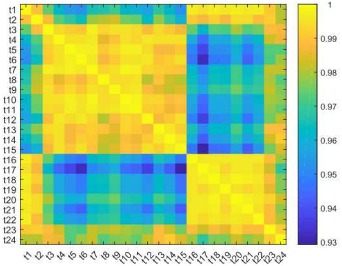
$$\text{McCormick envelope} \begin{cases} v_{t,t'}^\omega \leq u_t^\omega, (t,t') \in T, t < t', \omega \in \Omega \\ v_{t,t'}^\omega \leq u_{t'}^\omega, \forall (t,t') \in T, t < t', \omega \in \Omega \\ v_{t,t'}^\omega \geq u_t^\omega + u_{t'}^\omega - 1, \forall (t,t') \in T, t < t', \omega \in \Omega \end{cases}$$

$$u_t^\omega = \{0,1\}, \forall t \in T, v_{t,t'}^\omega = \{0,1\}, \forall (t,t') \in, \omega \in \Omega$$

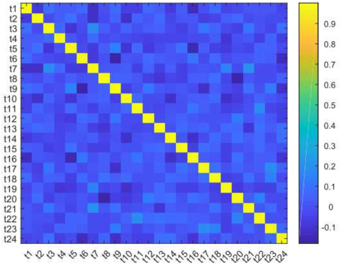
A case study

$$
\begin{aligned}
\max_{x,y} \quad & \sum_{t \in T} (R_t x_t - \mathbb{E}[B_t y_t^\omega]) && \text{(5a)} \\
\text{s.t.} \quad & \mathbb{P}(y_t^\omega + w_t^\omega \geq x_t, \forall t \in T) \geq 1 - \varepsilon && \text{(5b)} \\
& 0 \leq y_t^\omega \leq \Delta, \forall t \in T, \omega \in \Omega && \text{(5c)} \\
& x_t \geq 0, \forall t \in T. && \text{(5d)}
\end{aligned}
$$

We compare two sampling procedures: (a) ARMA(2,2) process, and (b) normal random variables. Both samples have the same hourly means and variances.



(a)



(b)

Figure: Correlation structure of w_t

Computational results: ARMA (large correlation)

ε	Bounding constraint	Optimal objective value Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(2a)	**8,351.3**	8,351.3	0%	2	3.3%
	(2b)	21,282.8	**21,282.8**	0%	12	59.4%
	(3a)	**8,351.3**	8,365 .8	0.1%	2100	3.3%
	(3b)	8,339.6	**10,682.1**	21.9%	2100	19.2%
	(4a)	8,339.7	**8,726.7**	4.5%	2100	1.1%
	(4b)	8,688.9	**8,702.1**	0.2%	2100	0.8%
0.03	(2a)	**8,374.6**	8,374.6	0%	2	8.5%
	(2b)	22,353.2	**22,353.2**	0%	14	59.0%
	(3a)	**8,339.6**	8,755.4	4.7%	2100	8.9%
	(3b)	8,339.6	**13,321.2**	37.4%	2100	31.3%
	(4a)	9,137.3	**9,311.4**	1.9%	2100	1.7 %
	(4b)	9,074.4	**9,252.2**	1.9%	2100	1.1%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 8,351.3 and 8,702.1; true optimal value is 8,634.1
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 8,374.6 and 9,252.2; true optimal value is 9,154.9

Computational results: Gaussian (weak correlation)

ε	Bounding constraint	Optimal objective value Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(2a)	**9,100.8**	9,100.8	0%	1	2.7%
	(2b)	21,606.6	**21,606.6**	0%	18	56.7%
	(3a)	**9,102.0**8	9,113.3	0.1%	2100	2.7%
	(3b)	9092.3	**11,365.5**	20%	2100	17.7%
	(4a)	9,434.3	**9,486.3**	0.5%	2100	1.4%
	(4b)	9,421.5	**9,452.3**	0.3%	2100	1.1%
0.03	(2a)	**9,124.3**	9,124.3	0%	2	7.7%
	(2b)	22,762.1	**22,762.1**	0%	21	56.6%
	(3a)	**9,124.8**	9,198.4	0.8%	2100	7.7%
	(3b)	9,092.3	**13,907.6**	34.9%	2100	28.9%
	(4a)	9,092.3	**10,062.6**	9.6%	2100	1.8%
	(4b)	9,092.3	**10,004.8**	9.1%	2100	1.2%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 9,100.8 and 9,449.9; true optimal value is 9,353.2
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 9,124.3 and 10,004.8; true optimal value is 9,884.0

Computational results

- Bonferroni lower bound and Dawson & Sankoff upper bound consistently perform better than others
- Weaker correlation in uncertainty leads to easier-to-solve models
- MIQCP formulation of Dawson & Sankoff bound is challenging

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Stochastic unit commitment

Standard unit commitment (UC) problem: which thermal generators should be scheduled to meet power demand, while ensuring feasible operations?

Stochastic unit commitment

Standard unit commitment (UC) problem: which thermal generators should be scheduled to meet power demand, while ensuring feasible operations?

- Thermal generator operational limits are based on engineering judgments
- Can be exceeded in practice, for short periods
- System operators can and do run thermal generators beyond these limits

Stochastic unit commitment

Proposed model

- Allow thermal generators to "occasionally" violate operational limits
- Violations should be few (else, increased maintenance costs)
- Violations should not be large (there are absolute ratings of generators)
- 1% savings in energy production is worth $\approx$ \$1 billion per year in the U.S. alone

Stochastic unit commitment

Proposed model

- Let $y_t^{g,\omega}$ denote a "non-nominal" operation in hour t for generator g in scenario ω
- During non-nominal operations, generator's operating region expands from $[\underline{P}^g, \overline{P}^g]$ to $[\underline{\underline{P}}^g, \overline{\overline{P}}^g]$
- Non-nominal mode of generation is more expensive
- Number of non-nominalities is few: $\frac{1}{|\Omega||\mathcal{T}||\mathcal{G}|} \sum_{g \in \mathcal{G}} \sum_{t \in T} \sum_{\omega \in \Omega} y_t^{g,\omega} \leq \varepsilon \leftarrow$ almost a chance-constraint!

Setup

We use:

$$
\begin{aligned}
\overline{\overline{P}}^{g} &= (1+\beta)\overline{P}^{g} \\
\underline{\underline{P}}^{g} &= (1-\beta)\underline{P}^{g} \\
\overline{C}^{g} &= (1+\gamma)C^{L^{g},g} \\
\underline{C}^{g} &= (1+\gamma)C^{L^{g},g}.
\end{aligned}
$$

WECC240++ system with 85 thermal generators, 50 scenarios and RTS-GMLC system with 73 thermal generators, 16 scenarios

Computational results for the WECC240++ 50 scenario test case for 11 May 2013.

Table: MIP gap = 0.1%

ε	β	γ	Cost (K$)	Savings (%)	Time (sec)	MIP gap (%)
0			64.41	0.00%	183	-
0.01	0.05	0.1	64.20	0.33%	275	-
		0.2	64.21	0.31%	242	-
	0.1	0.1	64.03	0.59%	258	-
		0.2	64.04	0.58%	317	-
0.05	0.05	0.1	63.86	0.85%	275	-
		0.2	63.90	0.80%	343	-
	0.1	0.1	63.35	1.64%	378	-
		0.2	63.42	1.55%	371	-

- Increase $\varepsilon \Rightarrow$ increase savings
- Increase $\beta \Rightarrow$ increase savings
- Increase $\gamma \Rightarrow$ decrease savings

Computational results for the RTS-GMLC 16 scenario case for 10 July 2020.

Table: MIP gap = 0.1%

ε	β	γ	Cost (M$)	Savings (%)	Time (sec)	MIP gap (%)
0			3.89	0.00%	33	-
0.01	0.05	0.1	3.84	1.21%	46	-
		0.2	3.84	1.20%	48	-
	0.1	0.1	3.83	1.51%	82	-
		0.2	3.83	1.50%	106	-
0.05	0.05	0.1	3.83	1.53%	65	-
		0.2	3.83	1.45%	100	-
	0.1	0.1	3.81	2.08%	1800	0.22%
		0.2	3.82	1.82%	1800	0.15%

- Increase $\varepsilon \Rightarrow$ increase savings
- Increase $\beta \Rightarrow$ increase savings
- Increase $\gamma \Rightarrow$ decrease savings

Cost savings for the RTS-GMLC 16 scenario case for 10 July 2020.

ε	β	γ	Optimal	Limited	No nuclear
0.01	0.05	0.1	1.21%	0.71%	1.06%
		0.2	1.20%	0.69%	1.04%
	0.1	0.1	1.51%	1.14%	1.15%
		0.2	1.50%	1.10%	1.11%
0.05	0.05	0.1	1.53%	0.70%	1.22%
		0.2	1.45%	0.69%	1.15%
	0.1	0.1	2.08%	1.14%	1.41%
		0.2	1.82%	1.10%	1.28%

Limited = a generator can have at most one non-nominal operations per day
No nuclear = no non-nominal operation for the nuclear unit in this system

Contents

Computational results: ARMA (large correlation) with 500 scenarios

ε	Bounding constraint	Optimal objective value Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(2a)	**8,453.4**	8,453.4	0%	1	2.9%
	(2b)	21,582.9	**21,582.9**	0%	129	59.7%
	(3a)	**8,701.0**	8,701.0	0%	1717	0%
	(3b)	10,462.7	**11,318.4**	7.5%	2100	23.1%
	(4a)	8,348.9	**40,116.9**	79.2%	2100	78.3%
	(4b)	8,348.9	**8,772.9**	4.8%	2100	0.8%
0.03	(2a)	**8,542.5**	8,542.5	0%	3	7.3%
	(2b)	22,570.6	**22,570.6**	0%	175	59.2%
	(3a)	**8,348.9**	9,396.1	11.1%	2100	9.4%
	(3b)	8,348.9	**15,127.8**	44.8%	2100	39.1%
	(4a)	8,348.9	**41,151.4**	79.8%	2100	77.6 %
	(4b)	8,348.9	**9,352.9**	10.7%	2100	1.5%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 8,701.0 and 8,772.9; true optimal value is 8,701.0
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 8,542.5 and 9,352.9; true optimal value is 9,211.3

Computational results: Gaussian (weak correlation) with 500 scenarios

ε	Bounding constraint	Optimal objective value: Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(2a)	**9,005.1**	9,005.1	0%	1	3.7%
	(2b)	21,503.7	**21,503.7**	0%	75	56.5%
	(3a)	**8866.9**	8,889.3	1.3%	2100	5.1%
	(3b)	8,866.9	**11,071.9**	19.9%	2100	15.6%
	(4a)	8,866.9	**40,126.1**	77.9%	2100	76.7%
	(4b)	9,343.6	**9,390.3**	0.5%	2100	0.5%
0.03	(2a)	**9,148.2**	9,148.2	0%	3	7.4%
	(2b)	22,565.4	**22,565.4**	0%	46	56.2%
	(3a)	**8,866.9**	9,315.3	4.8%	2100	10.2%
	(3b)	8,866.9	**13,711.9**	35.3%	2100	27.9%
	(4a)	8,866.9	**41,187.8**	78.5%	2100	76.0 %
	(4b)	8,866.9	**9,990.9**	11.2%	2100	1.2%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 9,005.1 and 9,390.3; true optimal value is 9,346.4
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 9,148.2 and 9,990.9; true optimal value is 9,874.1

Stochastic unit commitment model

Indices and Sets:

$g \in \mathcal{G}$	Thermal generators.
$t \in \mathcal{T}$	Hourly time steps: $1, \ldots, T$; i.e., $[a, b) \in \mathcal{T} \times \mathcal{T}$ such that $b \geq a + UT^g$.
$l \in \mathcal{L}^g$	Piecewise production cost intervals for generator g: $1, \ldots, L_g$.
$s \in \mathcal{S}^g$	Start-up categories for generator g, from hottest (1) to coldest (S_g).
$\omega \in \Omega$	Scenarios: $\omega_1, \ldots, \omega_N$.

Parameters: First Stage

$C^{l,g}$	Marginal cost for piecewise segment l for generator g (\$/MWh).
$\overline{C}^g$	Marginal cost for production above $\overline{P}^g$ (\$/MWh).
$\underline{C}^g$	Marginal cost for production below $\underline{P}^g$ (\$/MWh).
$C^{R,g}$	Cost of generator g running and operating at minimum production $\underline{P}_g$ (\$/h).
$C^{s,g}$	Start-up cost of category s for generator g (\$).
DT^g	Minimum down time for generator g (h).
$\overline{P}^g$	Maximum power output for generator g under normal operations (MW).
$\overline{\overline{P}}^g$	Maximum power output for generator g under non-nominal operations (MW).
$\underline{P}^g$	Minimum power output for generator g under normal operations (MW).
$\underline{P}^g$	Minimum power output for generator g under non-nominal operations (MW).
$\overline{\overline{\overline{P}}}^{l,g}$	Maximum power available for piecewise segment l for generator g (MW) (with $\overline{P}^{0,g} = \underline{P}^g$).
RD^g	Ramp-down rate for generator g (MW/h).
RU^g	Ramp-up rate for generator g (MW/h).
SD^g	Shutdown ramp rate for generator g (MW/h).
SU^g	Start-up ramp rate for generator g (MW/h).
TC^g	Time down after which generator g goes cold (h).
$\underline{T}^{s,g}$	Time offline after which the start-up category s is available (h) (with $\underline{T}^{1,g} = DT^g$, $\underline{T}^{S^g,g} = TC^g$)
UT^g	Minimum up time for generator g (h).

Stochastic unit commitment model

Parameters: Second Stage

D_t^{ω}	Load (demand) at time t in scenario ω (MW).
$\overline{W}_t^{\omega}$	Maximum power from renewables at time t in scenario ω (MW).
$\underline{W}_t^{\omega}$	Minimum power from renewables at time t in scenario ω (MW).

Variables: First Stage

u_t^g	Commitment status of generator g at time t, $\in \{0,1\}$.
v_t^g	Start-up status of generator g at time t, $\in \{0,1\}$.
w_t^g	Shutdown status of generator g at time t, $\in \{0,1\}$.
$x_{[t,t')}^g$	Indicator arc for shutdown at time t, start-up at time t', uncommitted for $i \in [t,t')$, for generator g, $\in \{0,1\}$, $[t,t')$ such that $t + DT^g \leq t' \leq t + TC^g - 1$.

Variables: Second Stage

$p_t^{g,\omega}$	Power above minimum from generator g at time t in scenario ω (MW).
$\overline{P}_t^{g,\omega}$	Power above maximum from generator g at time t in scenario ω (MW).
$\underline{P}_t^{g,\omega}$	Power below minimum from generator g at time t in scenario ω (MW).
$p_t^{l,g,\omega}$	Power from piecewise interval l for generator g at time t in scenario ω (MW).
$r_t^{h,\omega}$	Power from renewables at time t in scenario ω (MW).
$y_t^{g,\omega}$	Non-nominal operation status of generator g at time t in scenario (MW).

Stochastic unit commitment

$$\min \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \left(\sum_{l \in \mathcal{L}^g} \mathbb{E}[C^{l,g}\, p_t^{l,g,\omega} + \overline{C}^g\, \overline{P}_t^{g,\omega} + \underline{C}^g\, \underline{P}^{g,\omega}] + C^{R,g}\, u_t^g + c_t^{SU,g} \right) \tag{6}$$

subject to:

$$u_t^g - u_{t-1}^g = v_t^g - w_t^g \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{7a}$$

$$\sum_{i=t-UT^g+1}^{t} v_i^g \le u_t^g \qquad \forall t \in [UT^g, T],\ \forall g \in \mathcal{G} \tag{7b}$$

$$\sum_{i=t-DT^g+1}^{t} w_i^g \le 1 - u_t^g \qquad \forall t \in [DT^g, T],\ \forall g \in \mathcal{G} \tag{7c}$$

$$\sum_{t'=t-TC^g+1}^{t-DT^g} x_{[t',t)}^g \le v_t^g \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{7d}$$

$$\sum_{t'=t+DT^g}^{t+TC^g-1} x_{[t,t')}^g \le w_t^g \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{7e}$$

$$c_t^{SU,g} = C^{S,g} v_t^g + \sum_{s=1}^{S^g-1} (C^{s,g} - C^{S,g}) \left(\sum_{t'=t-\underline{T}^{s+1,g}+1}^{t-\underline{T}^{s,g}} x_{[t',t)}^g \right) \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{7f}$$

Stochastic unit commitment

$$p_t^{g,\omega} \leq (\overline{P}^g - \underline{P}^g)u_t^g - (\overline{P}^g - SU^g)v_t^g - (\overline{P}^g - SD^g)w_{t+1}^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{>1}, \forall \omega \in \Omega \qquad (8a)$$

$$p_t^{g,\omega} \leq (\overline{P}^g - \underline{P}^g)u_t^g - (\overline{P}^g - SU^g)v_t^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \qquad (8b)$$

$$p_t^{g,\omega} \leq (\overline{P}^g - \underline{P}^g)u_t^g - (\overline{P}^g - SD^g)w_{t+1}^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^1, \forall \omega \in \Omega \qquad (8c)$$

$$p_t^{g,\omega} - p_{t-1}^{g,\omega} \leq (SU^g - RU^g - \underline{P}^g)v_t^g + RU^g u_t^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \qquad (8d)$$

$$p_{t-1}^{g,\omega} - p_t^{g,\omega} \leq (SD^g - RD^g - \underline{P}^g)w_t^g + RD^g u_{t-1}^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \qquad (8e)$$

$$p_t^{g,\omega} = \sum_{l \in \mathcal{L}^g} p_t^{l,g,\omega} \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega \qquad (8f)$$

$$p_t^{l,g,\omega} \leq (\overline{P}^{l,g} - \overline{P}^{l-1,g})u_t^g \qquad \forall t \in \mathcal{T}, \forall l \in \mathcal{L}^g, \forall g \in \mathcal{G}, \forall \omega \in \Omega \qquad (8g)$$

Stochastic unit commitment

$$y_t^{g,\omega} \le u_t^g - v_t^g - w_{t+1}^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{>1}, \forall \omega \in \Omega \tag{9a}$$

$$y_t^{g,\omega} \le u_t^g - v_t^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{1}, \forall \omega \in \Omega \tag{9b}$$

$$y_t^{g,\omega} \le u_t^g - w_{t+1}^g \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{1}, \forall \omega \in \Omega \tag{9c}$$

$$\overline{P}_t^{g,\omega} \le \left(\overline{\overline{P}} - \overline{P}\right) y_t^{g,\omega} \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{9d}$$

$$\underline{P}_t^{g,\omega} \le \left(\underline{P} - \underline{\underline{P}}\right) y_t^{g,\omega} \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{9e}$$

$$\sum_{g \in \mathcal{G}} \left(p_t^{g,\omega} + \overline{P}_t^{g,\omega} - \underline{P}_t^{g,\omega} + \underline{P}^g u_t^g\right) + r_t^\omega = D_t^\omega \forall t \in \mathcal{T},\ \forall \omega \in \Omega \tag{10}$$

$$\frac{1}{|\mathcal{G}||\mathcal{T}||\Omega|} \sum_{\omega \in \Omega} \sum_{g \in \mathcal{G}} \sum_{t \in T} y_t^{g,\omega} \le \varepsilon \tag{11}$$

$$p_t^{l,g,\omega} \in \mathbb{R}_+ \qquad \forall t \in \mathcal{T},\ \forall l \in \mathcal{L}^g,\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{12a}$$

$$p_t^{g,\omega}, \overline{P}_t^{g,\omega}, \underline{P}_t^{g,\omega} \in \mathbb{R}_+ \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G},\ \forall \omega \in \Omega \tag{12b}$$

$$r_t^{n,\omega} \in \left[\underline{W}_t^{n,\omega}, \overline{W}_t^{n,\omega}\right] \qquad \forall t \in \mathcal{T}, \forall n \in \mathcal{N}, \forall \omega \in \Omega \tag{12c}$$

$$u_t^g, v_t^g, w_t^g \in \{0,1\} \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{12d}$$

$$x_{[t,t')}^g \in \{0,1\} \qquad \forall [t,t') \in \mathcal{X}^g,\ \forall g \in \mathcal{G} \tag{12e}$$

$$y_t^{g,\omega} \in \{0,1\} \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}, \forall \omega \in \Omega. \tag{12f}$$

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