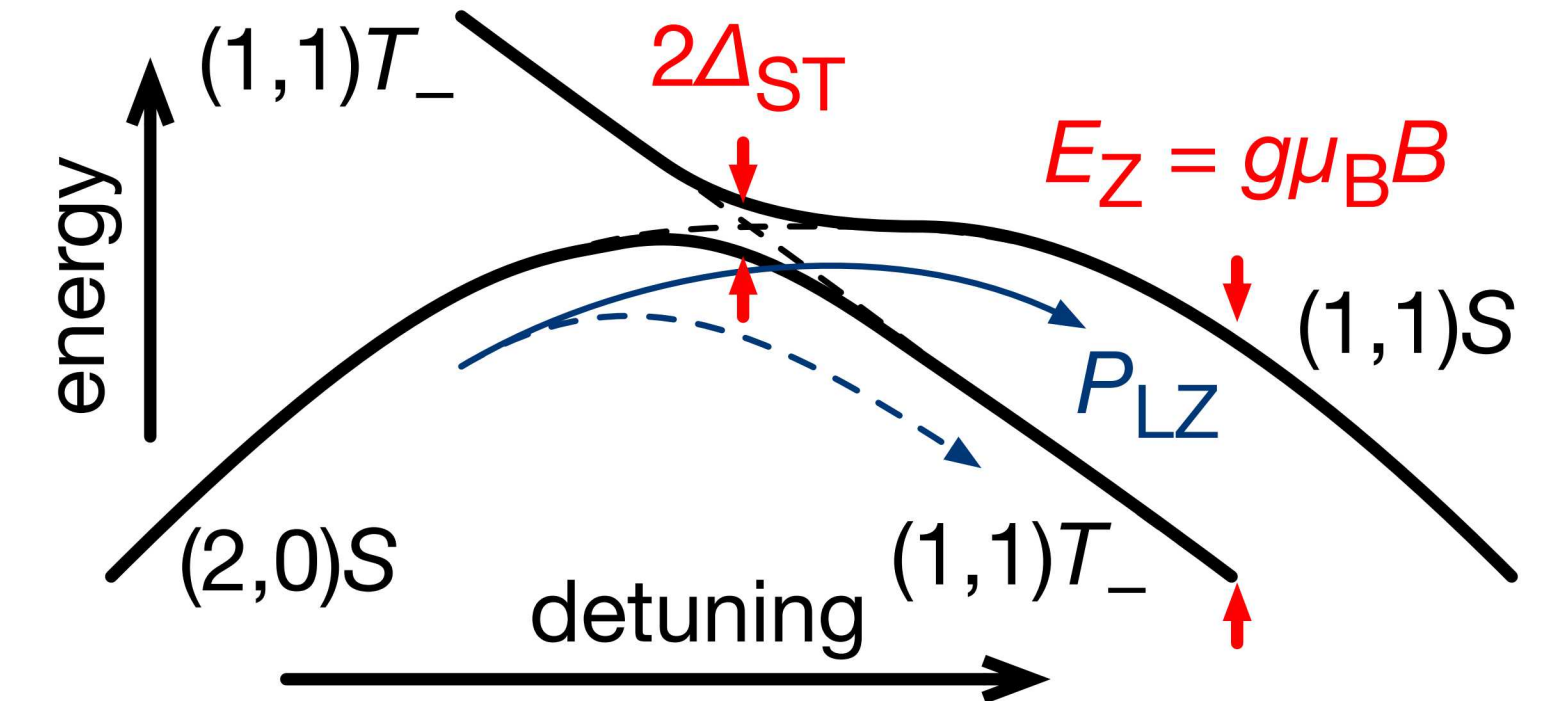
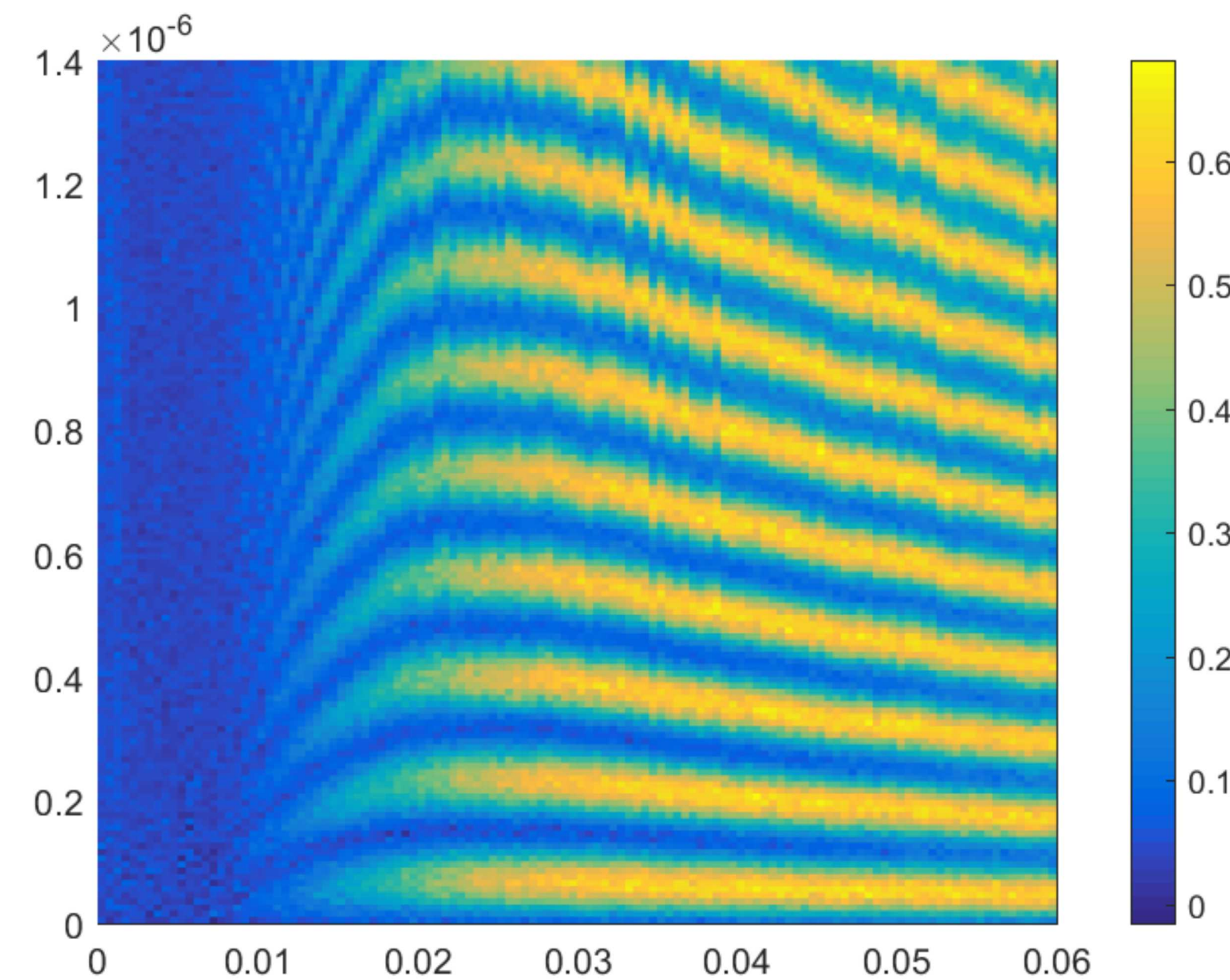
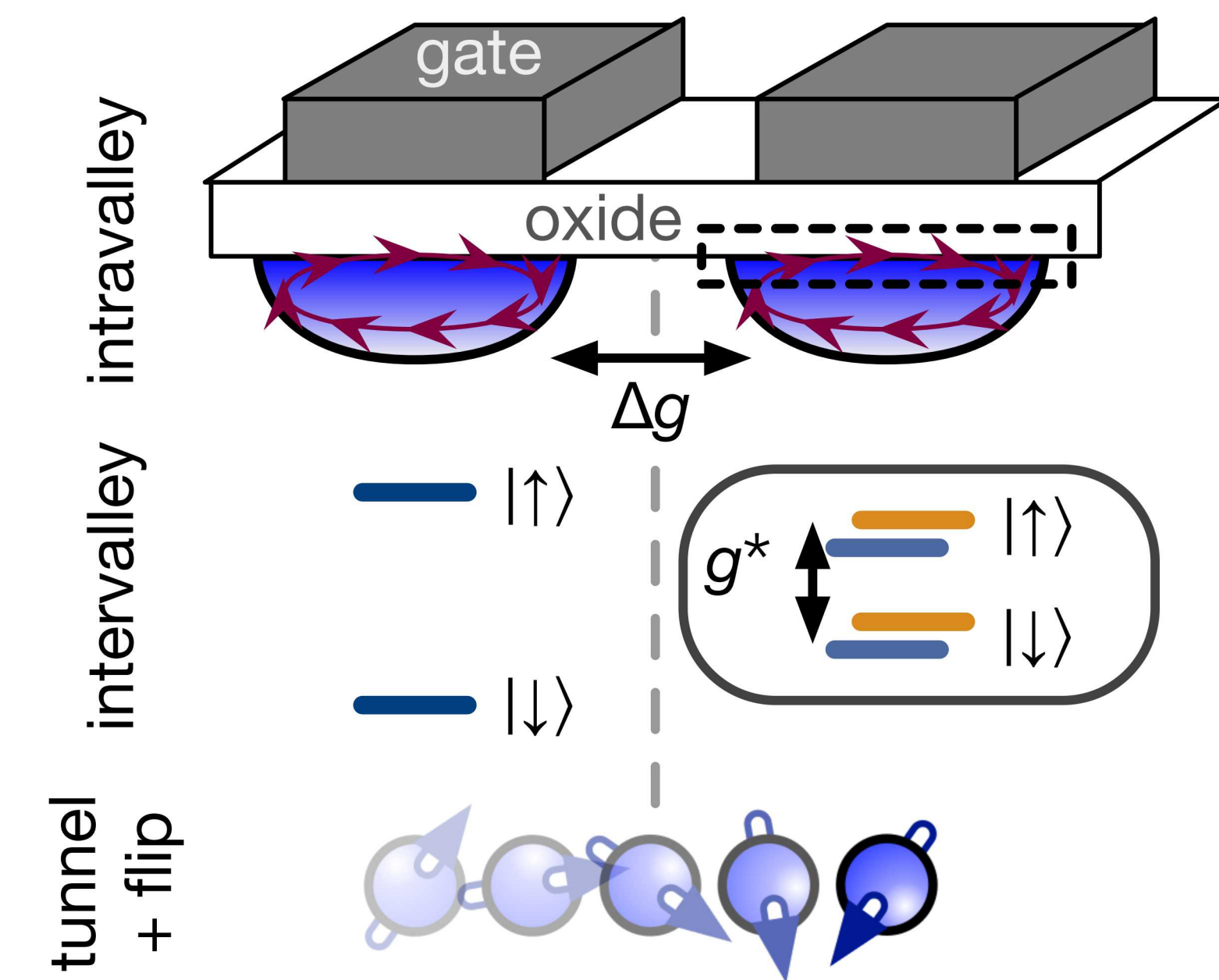


For better or for worse: spin-orbit coupling and its physical manifestations in Si quantum dot qubits



Toby Jacobson
 Sandia National Laboratories
 Albuquerque, New Mexico

Sandia National Laboratories is a multi-mission laboratory managed and operated by National Technology and Engineering Solutions of Sandia LLC, a wholly owned subsidiary of Honeywell International Inc. for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

“Spin-orbit coupling is *weak* in Si”

Historical assumptions for electron quantum dots in silicon:

- Needn't worry much about spin-orbit coupling. If we do, then Rashba but **no** Dresselhaus
- g-factor should be something close to 1.998, with slight valley-dependent anisotropy as a function of B-field orientation relative to crystallographic directions [Roth, Phys. Rev. **118**, 1534 (1960)]
- To realize an effective B-field gradient, must use something like μ -magnets (e.g. [Kawakami, et al. Nat. Nano. (2014)], [Takeda, et al. Sci. Adv. 2, e1600694 (2016)])

“Spin-orbit coupling is *non-negligible* in Si”

SOC in Si due to interface effects [Golub & Ivchenko, PRB (2004)]

- Significant Dresselhaus-like SOC (forbidden in the bulk due to symmetry)
- Can drive S/T_0 qubit rotations nicely using only intrinsic SOC
- Lots of measurements and supporting theory from community over past few years

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In this talk:

- Summary of SOC mechanisms for electrons in silicon quantum dots
- Physics behind interesting effects that one may observe in S/T₀ experiments
- Examples of our experiences with S/T₀ spin-orbit qubits at Sandia
- Challenges and prospects

“Spin-orbit coupling is *non-negligible* in Si”

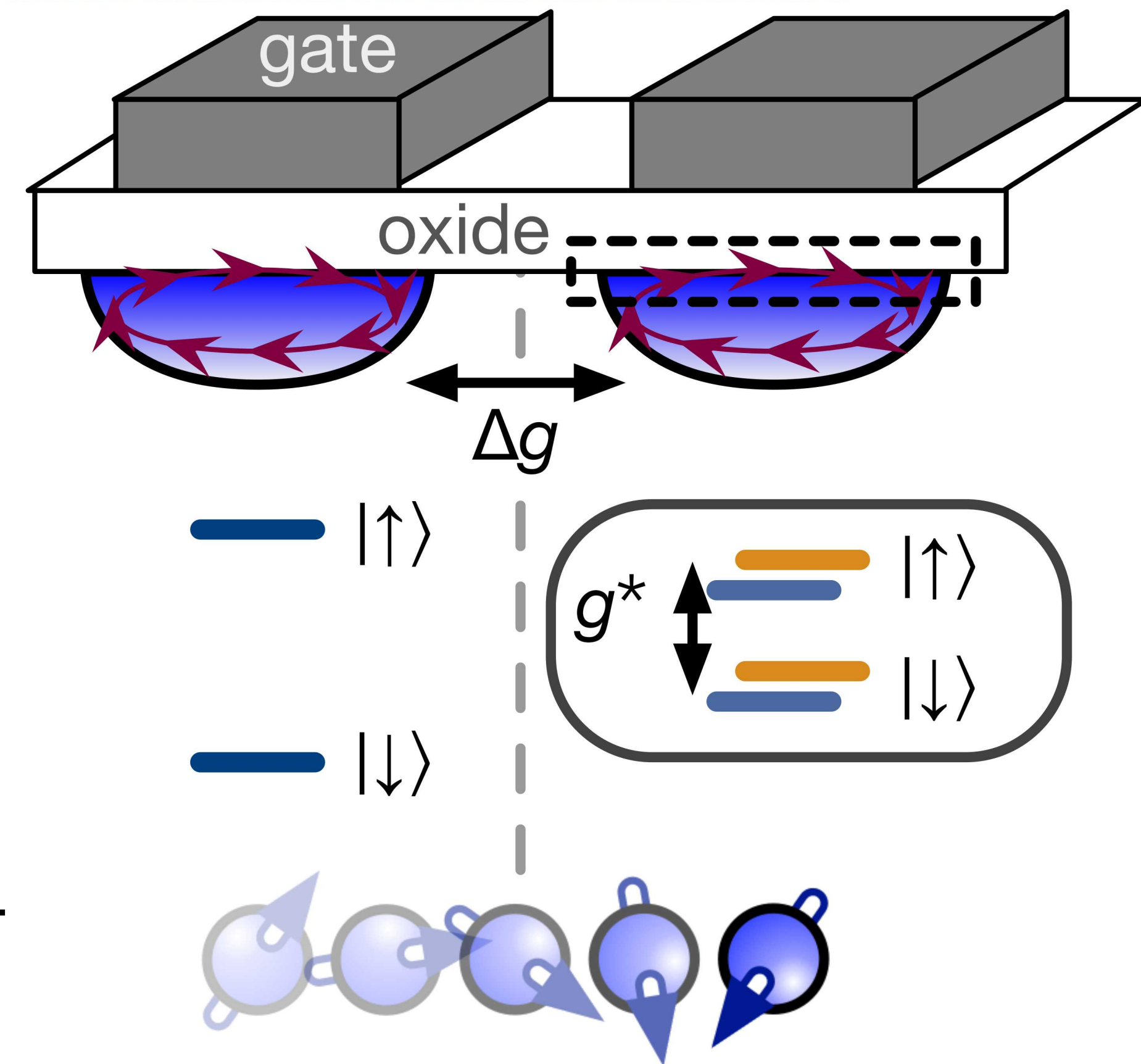
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- Can drive S/T₀ qubit rotations nicely using only intrinsic SOC
- Lots of measurements and supporting theory from community over past few years

Three mechanisms (at least) in S/T₀ qubits:

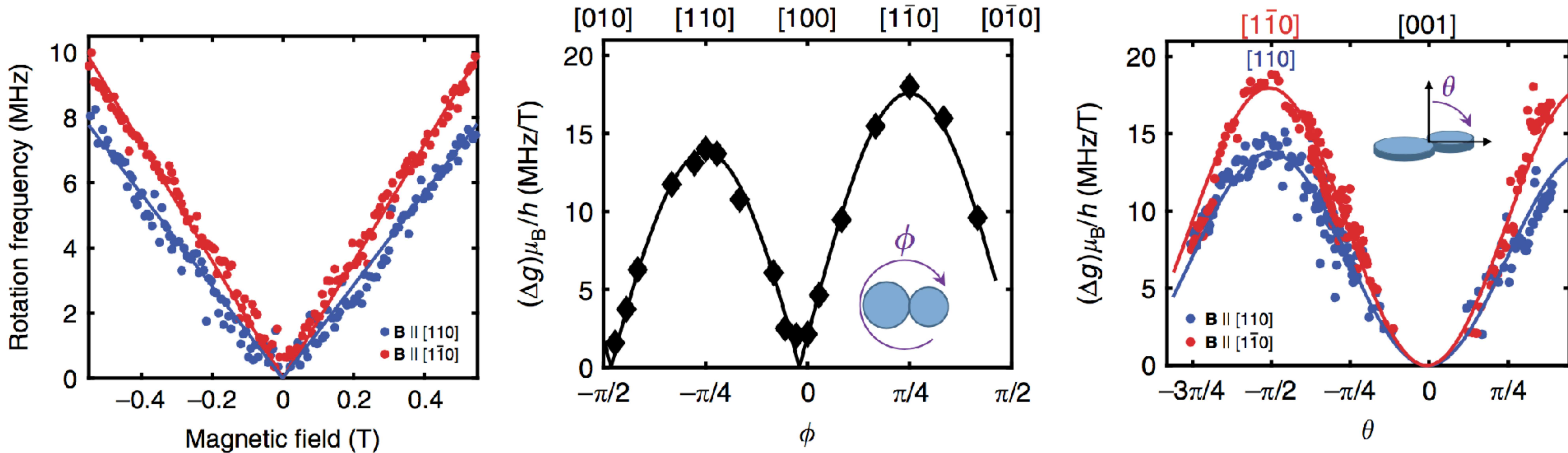
- Δg -factor between dots: Effective B-field gradient proportional to applied B
- *Inter-valley SOC*: Effective B-field gradient that is a non-linear function of B
- *Inter-dot tunneling + spin flip*: Leakage when passing through S/T anti-crossing

intravalley
intervalley
tunnel + flip



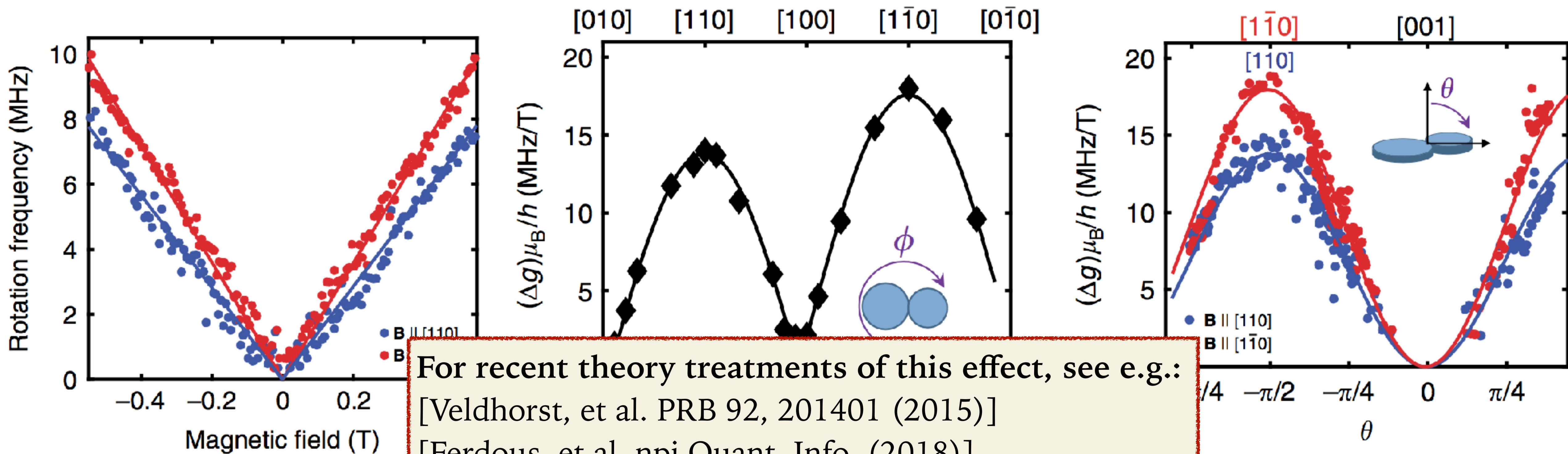
All three effects measured [Harvey-Collard, et al. arXiv:1808.07378]

A S/T₀ qubit in a poly-silicon MOS double dot, ~500 ppm ²⁹Si, no μ -magnet, and no ESR



Interpretation: Linear variation of S/T₀ rotation frequency due to *distinct g-factors* in each dot

A S/T_0 qubit in a poly-silicon MOS double dot, ~ 500 ppm ^{29}Si , no μ -magnet, and no ESR



For recent theory treatments of this effect, see e.g.:

[Veldhorst, et al. PRB 92, 201401 (2015)]

[Ferdous, et al. npj Quant. Info. (2018)]

[Ferdous, et al. PRB 97, 241401 (2018)]

[Jock, et al. Nat. Comm. 9, 1768 (2018)]

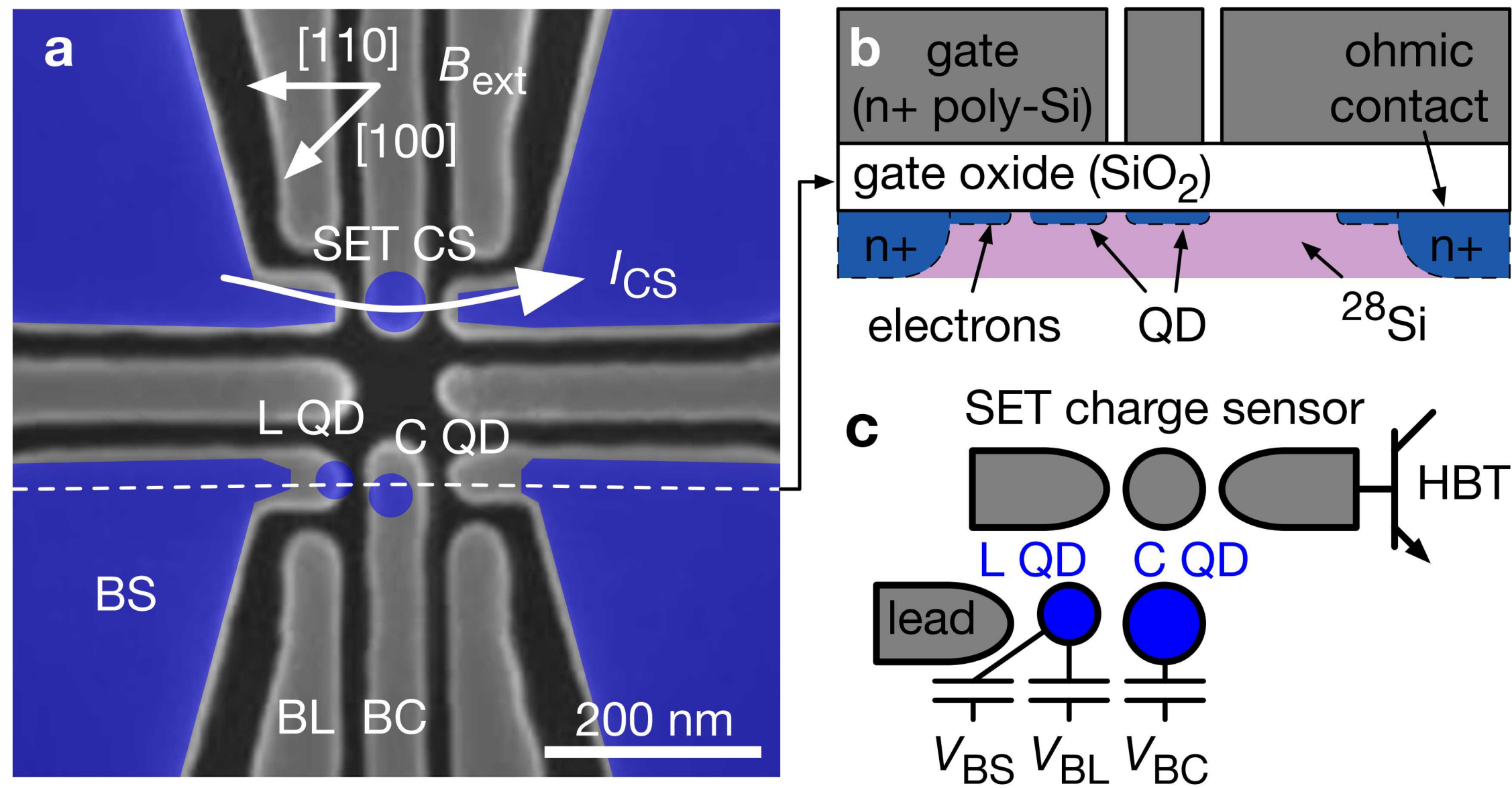
[Ruskov, et al. arXiv:1708.04555]

[Harvey-Collard, et al. arXiv:1808.07378]

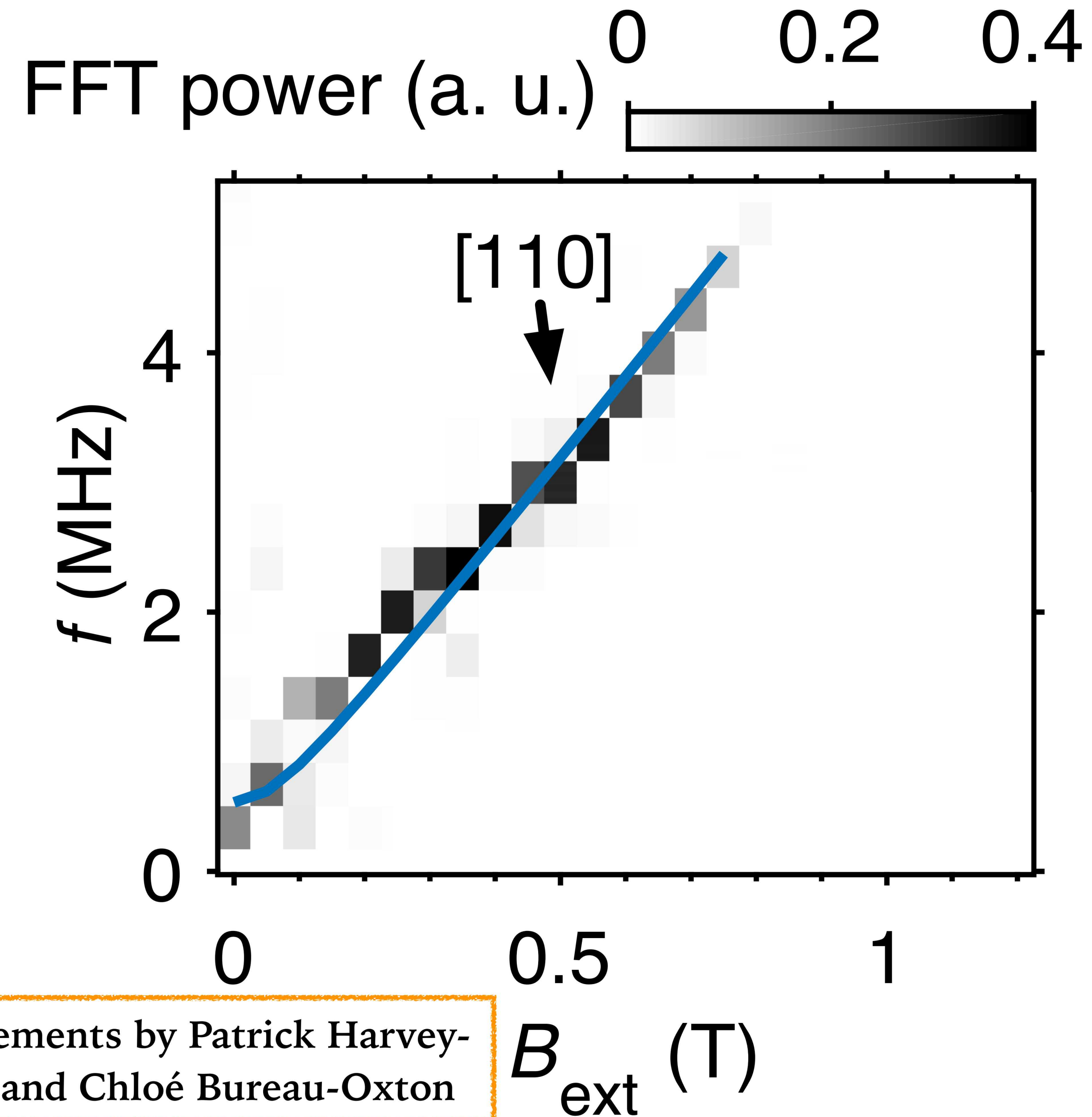
Interpretation: Linear

ectors in each dot

Consider a different device (same design)

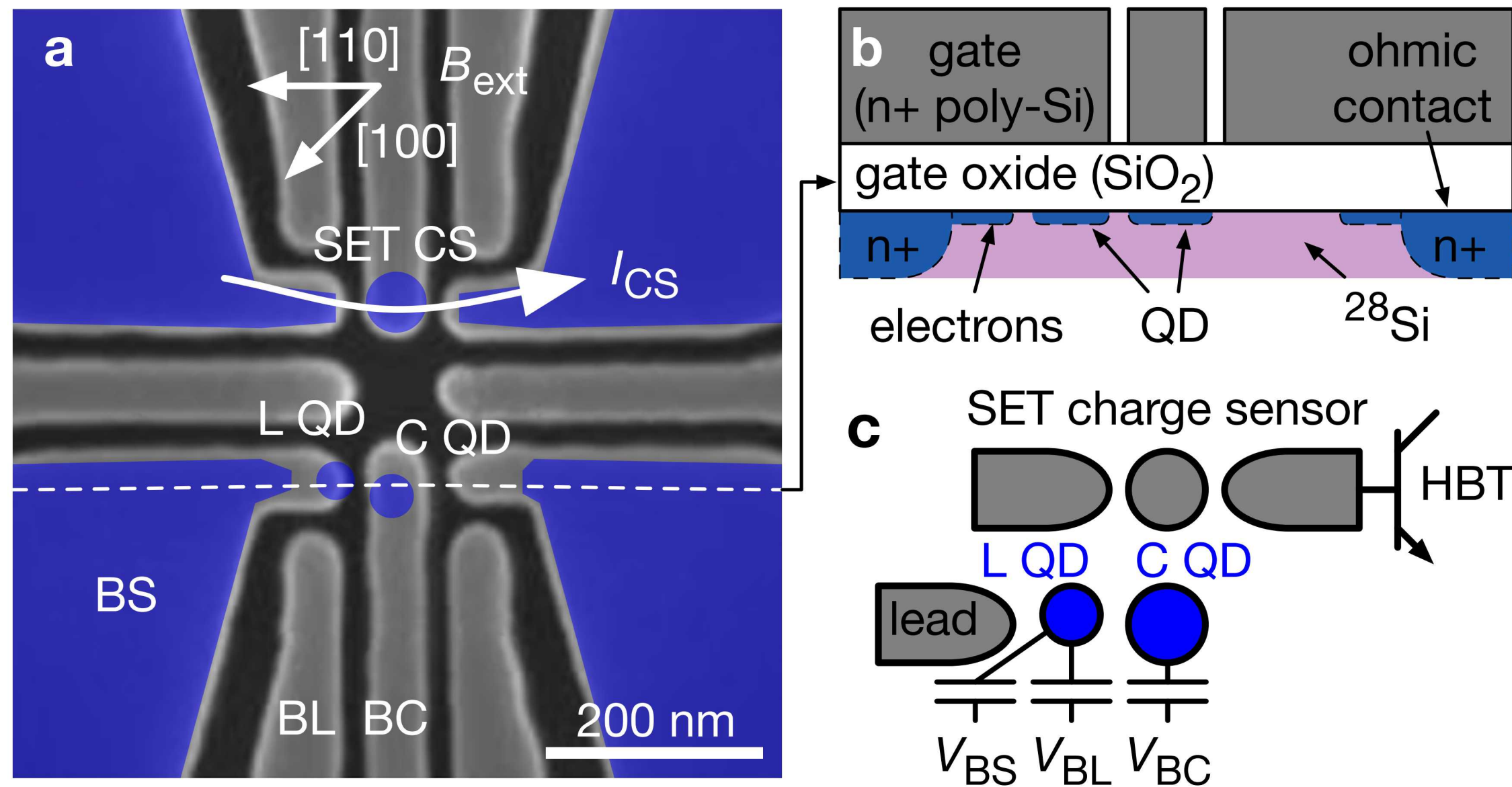


In this device, choosing a $[110]$ orientation for the B-field looks like a simple inter-dot g-factor difference.



Measurements by Patrick Harvey-Collard and Chloé Bureau-Oxton

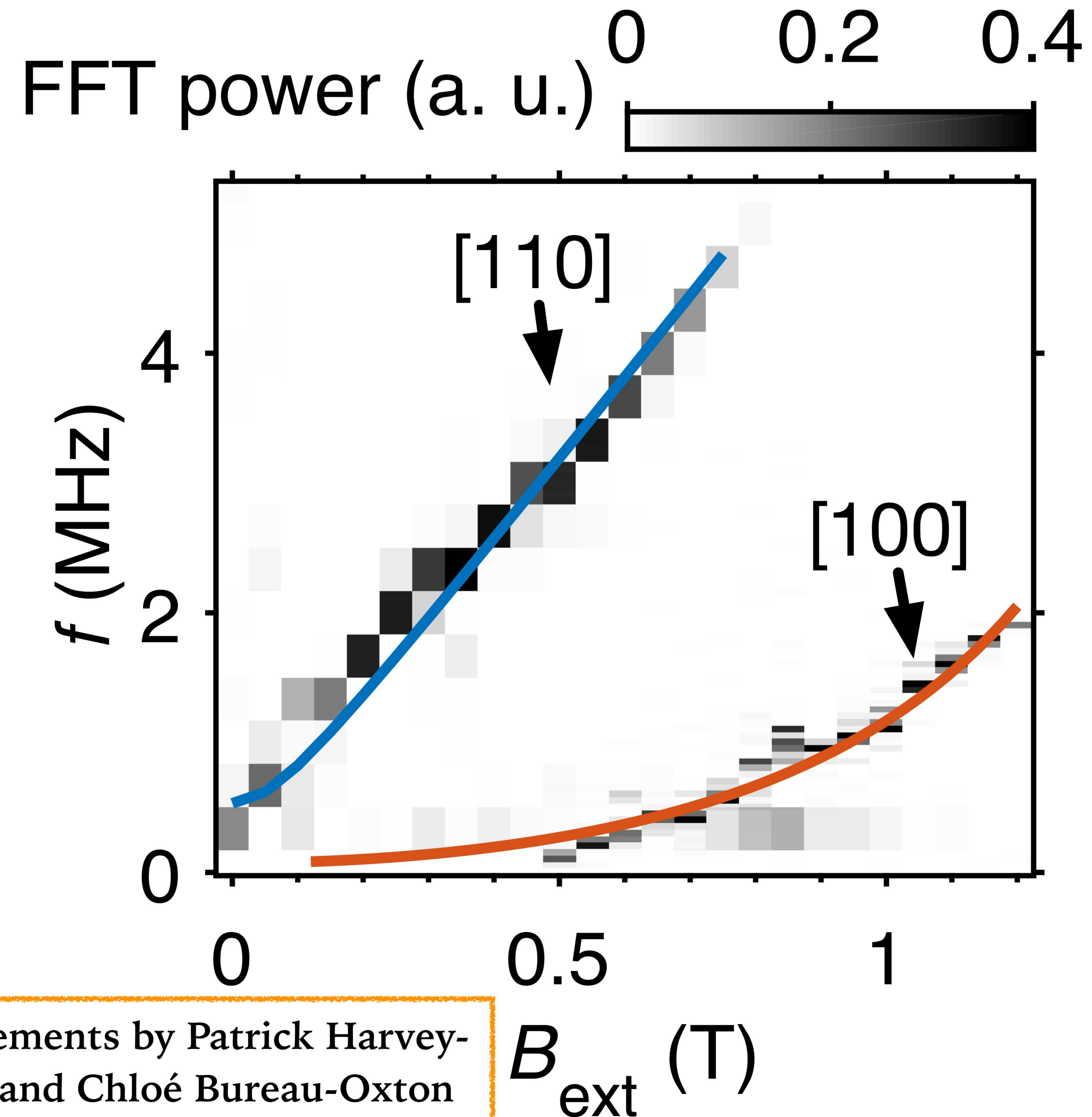
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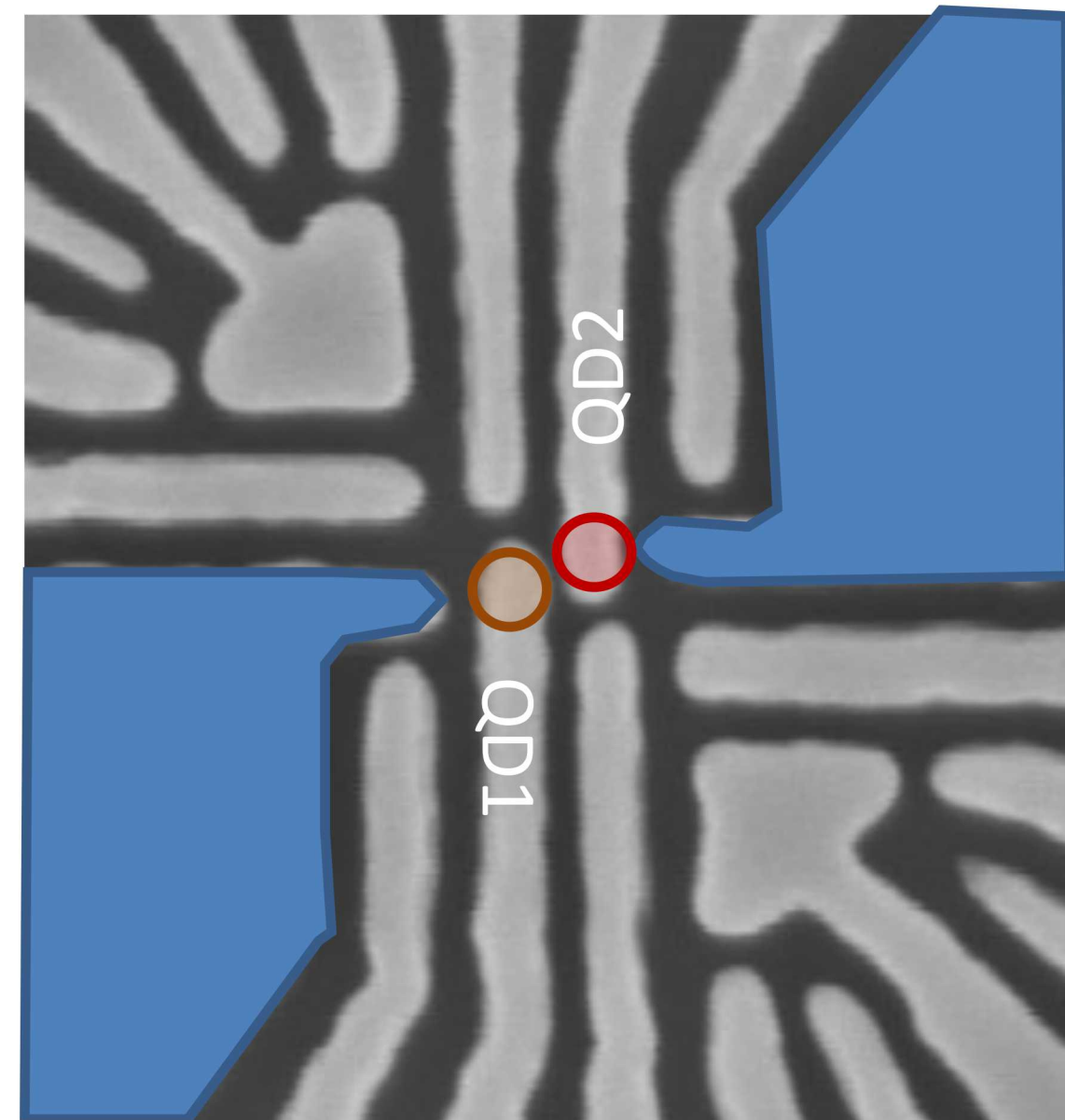


In this device, choosing a $[110]$ orientation for the B-field looks like a simple inter-dot g-factor difference.

Now, rotate the magnetic field to $[100]$.
What's going on? Clearly **not** $\propto B$

Measurements by Patrick Harvey-Collard and Chloé Bureau-Oxton

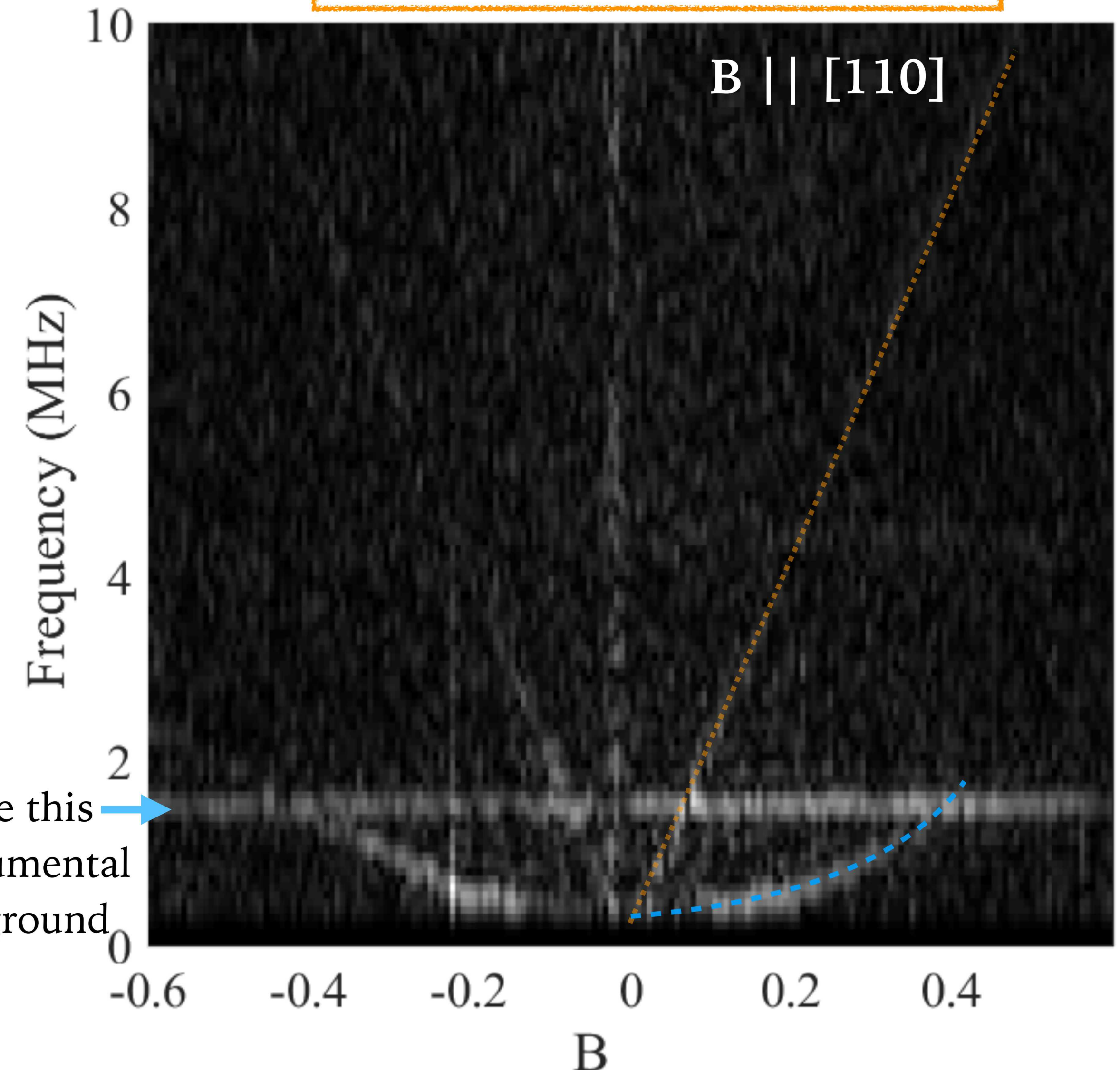




In the same measurement, we observe two frequencies with different functional forms.

Dot occupancy: (4,0)-(3,1)
(filled lower valley in left dot)

Measurements by Ryan Jock



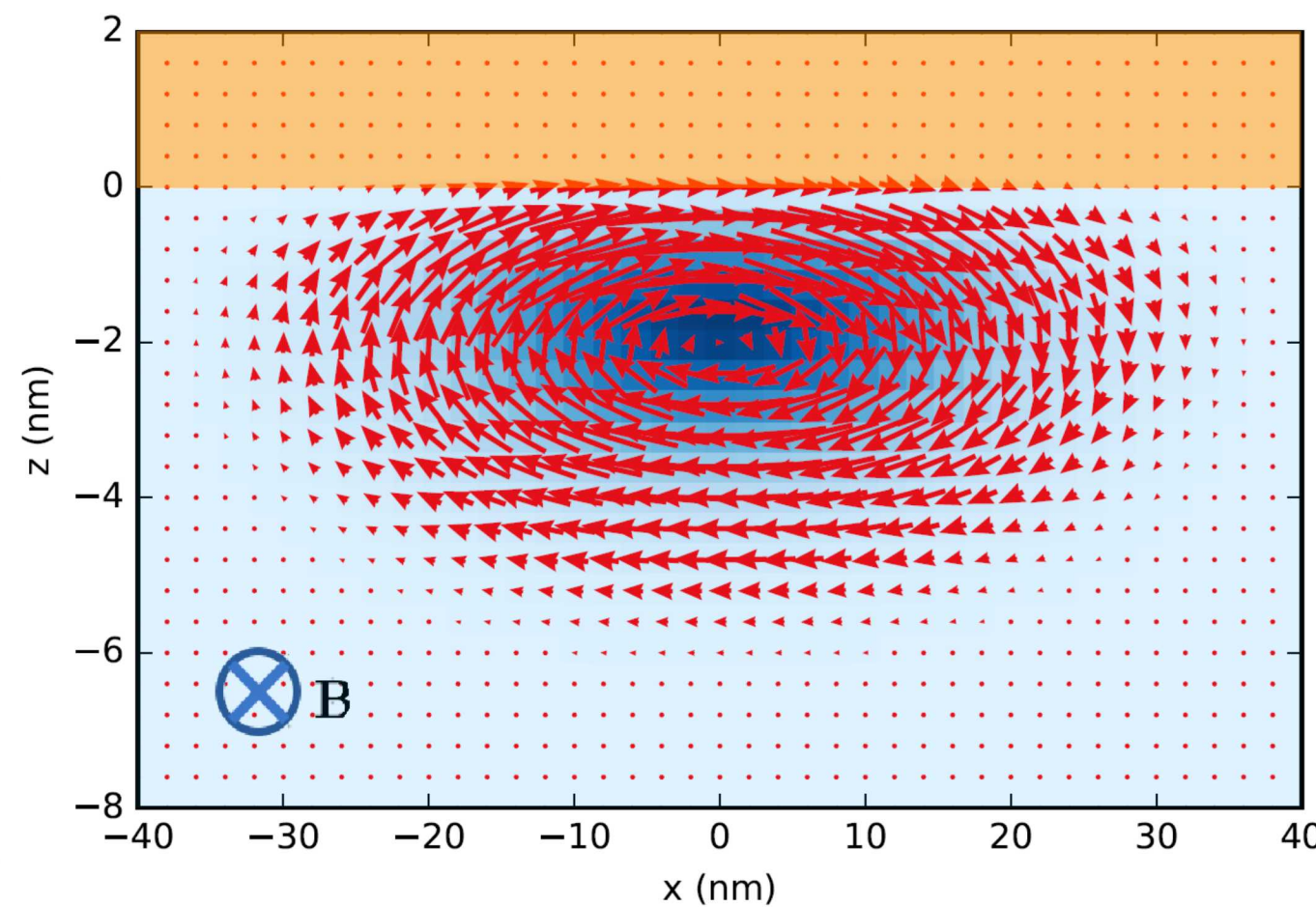
Note: Two simultaneous frequency components have been observed previously due to valley-dependent g-factors [Kawakami, et al. Nat. Nano. (2014)].
(see also [Veldhorst, et al. PRB (2015)])

Why the distinct functional forms?

A menagerie of SOC effects

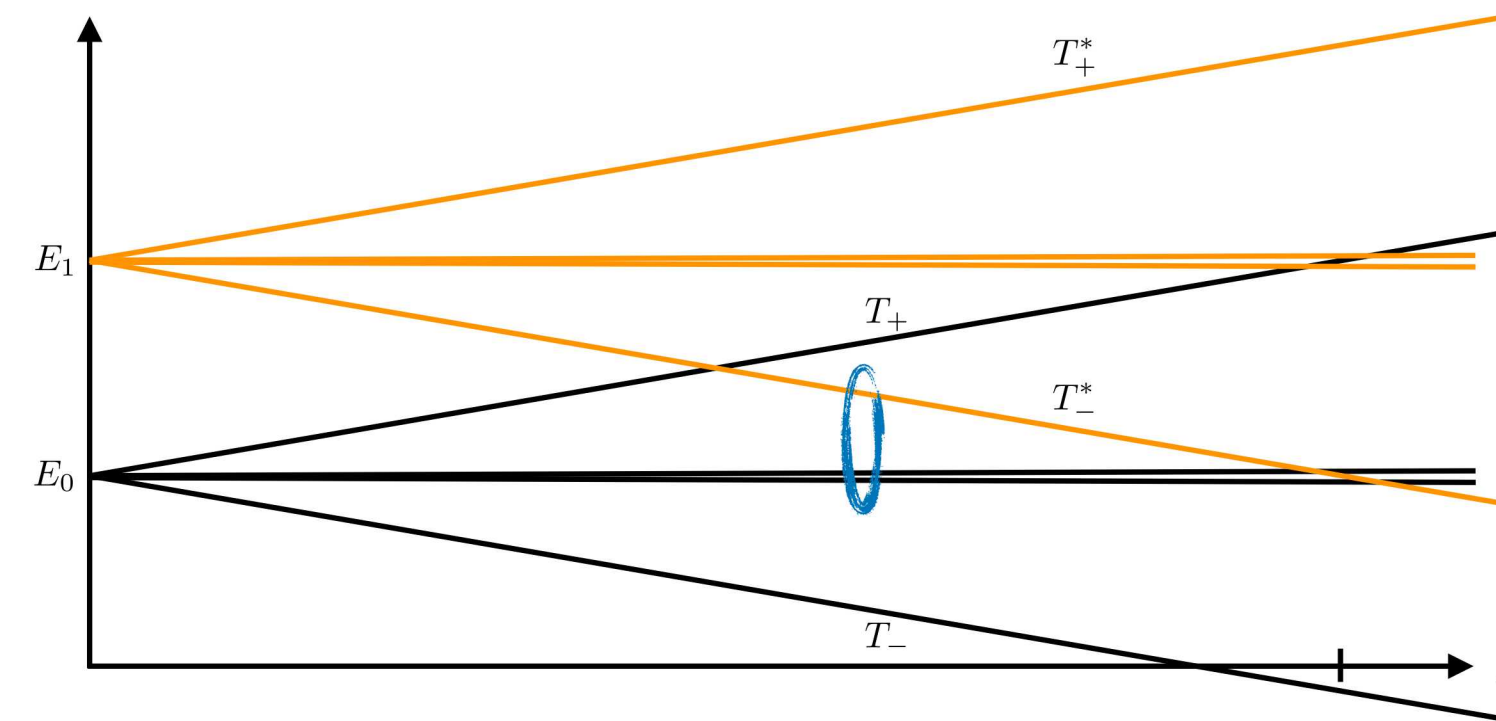
The underlying microscopic SOC physics can manifest in a variety of ways:

(1) Intra-valley (*g-tensor shift*)



$$\text{e.g. } \langle R \uparrow | H_{\text{SO}} | R \downarrow \rangle$$

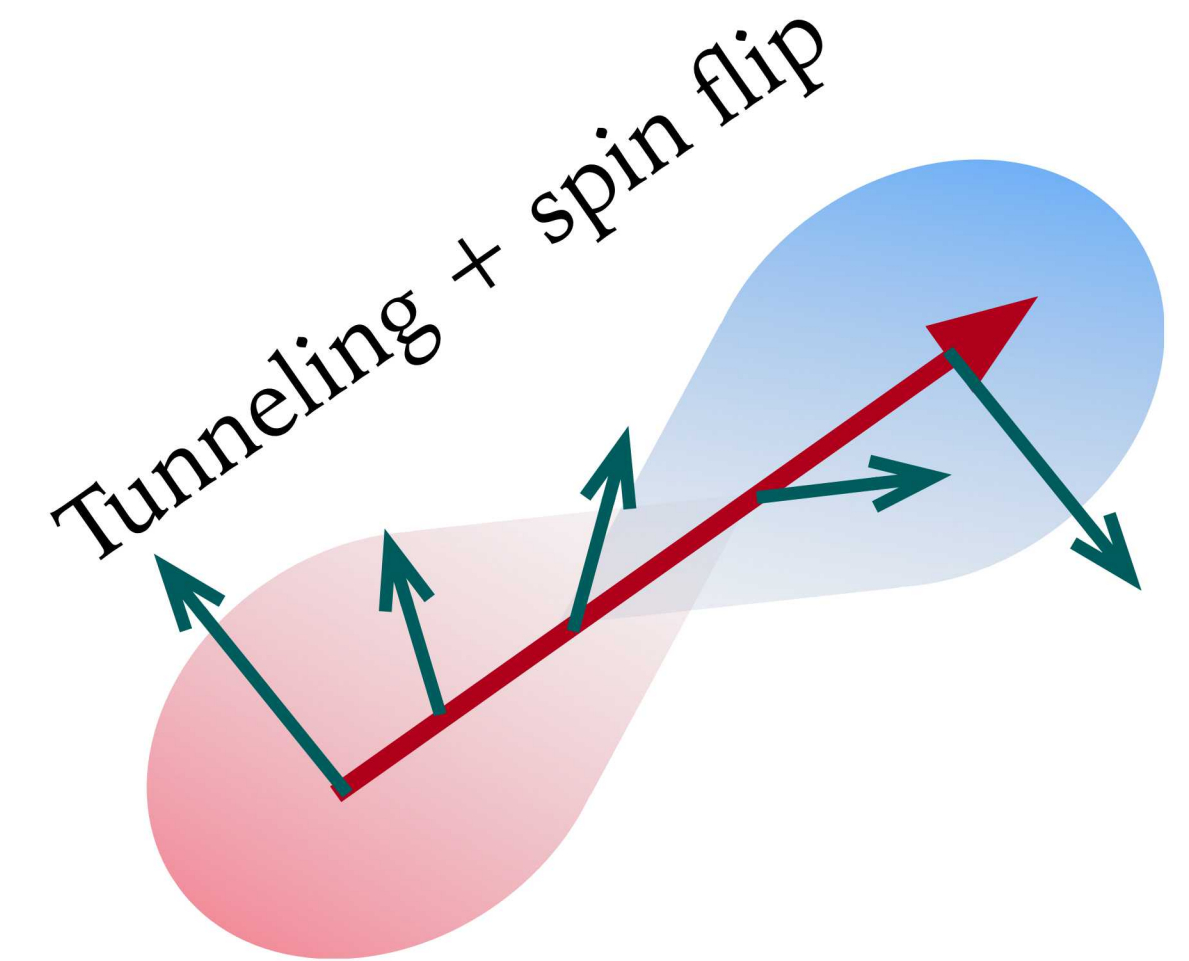
(2) Inter-valley (*nonlinear effects*)



Related to T_1 hot-spots
[Yang, et al. Nat. Comm. (2013)]
(UNSW + LPS + Purdue)

$$\text{e.g. } \langle R^* \uparrow | H_{\text{SO}} | R \downarrow \rangle$$

(3) Inter-dot $\rightarrow$ S/T anti-crossing (*leakage, SPAM error*)

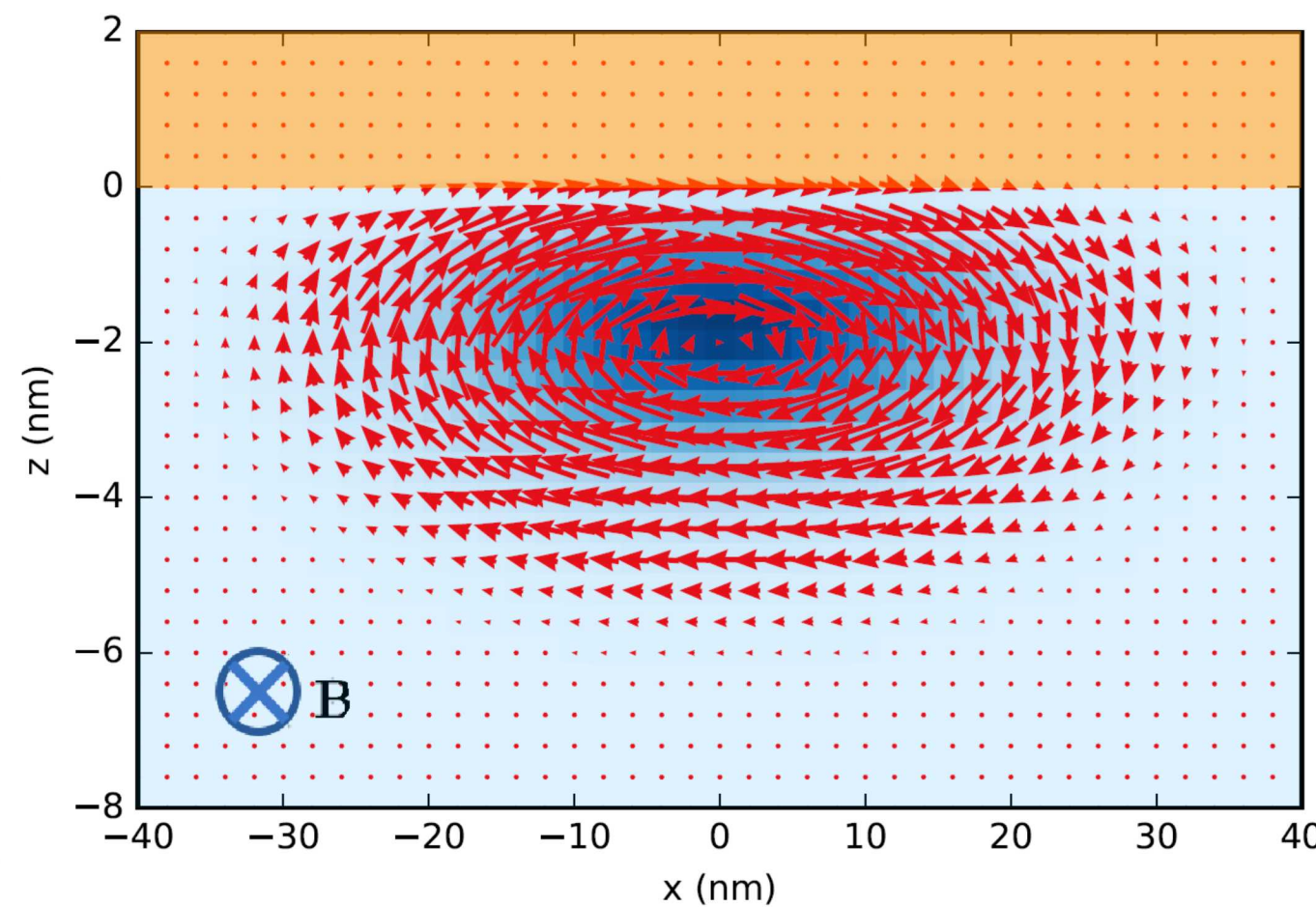


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A menagerie of SOC effects

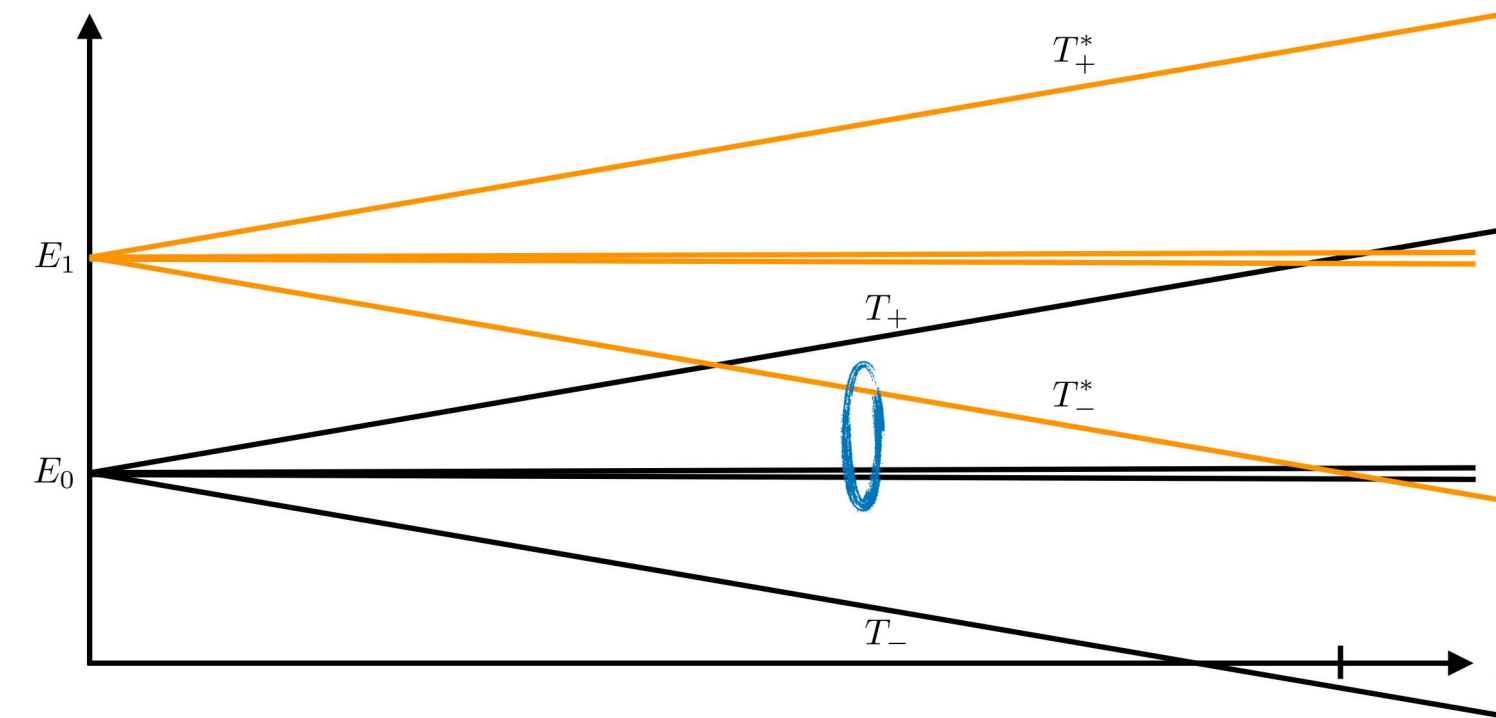
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e.g. $\langle R \uparrow | H_{\text{SO}} | R \downarrow \rangle$

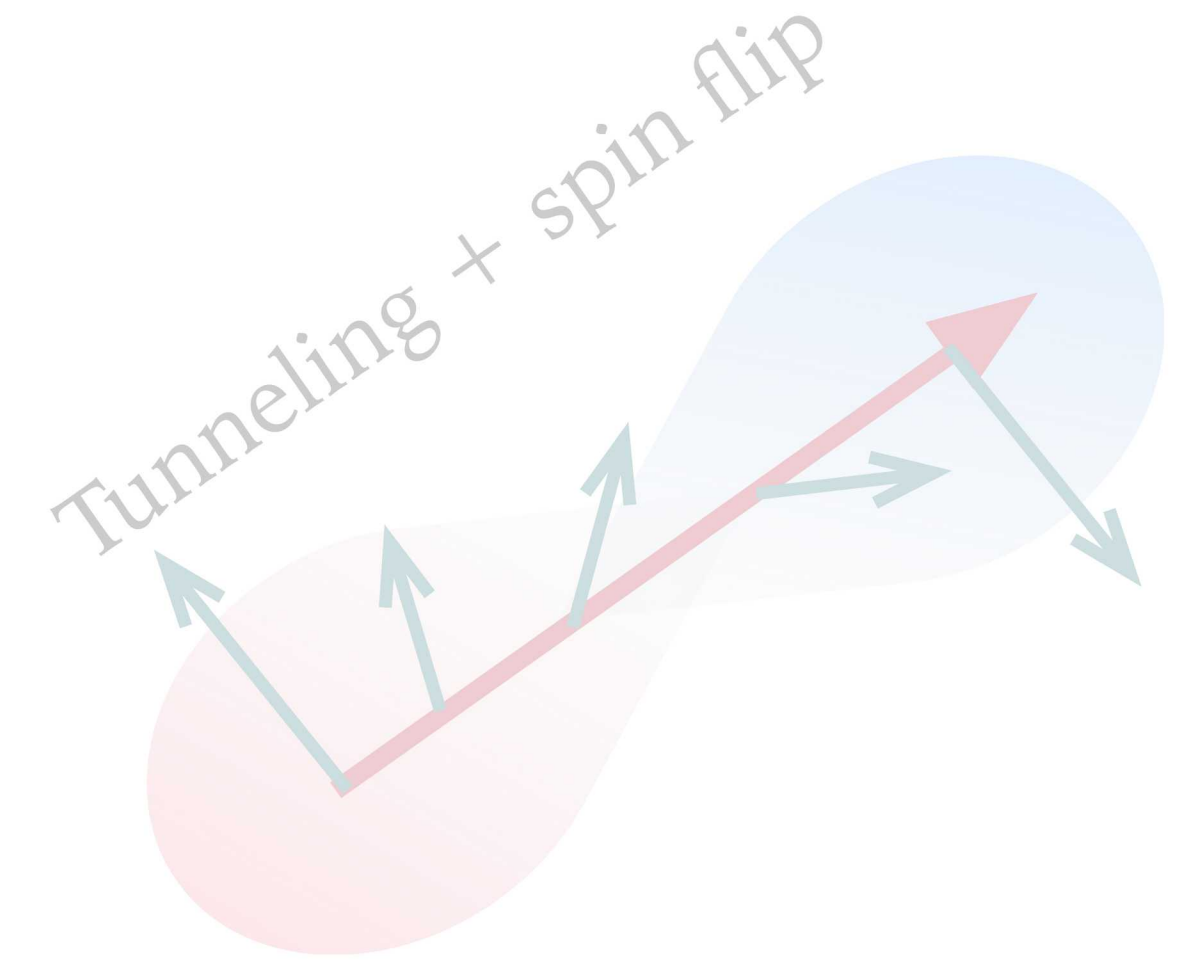
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Related to T₁ hot-spots
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e.g. $\langle R^* \uparrow | H_{\text{SO}} | R \downarrow \rangle$

(3) Inter-dot $\rightarrow$ S/T anti-crossing (leakage, SPAM error)



e.g. $\langle L \uparrow | H_{\text{SO}} | R \downarrow \rangle$

Now for some theory on intra- and inter-valley SOC mechanisms...

Interface SOC: [Golub & Ivchenko, PRB 69, 115333 (2004)]

$$H_{\text{SO}} = H_R + H_D$$

$$= \delta(z) [\gamma_R (P_y \sigma_x - P_x \sigma_y) + \gamma_D (P_x \sigma_x - P_y \sigma_y)]$$

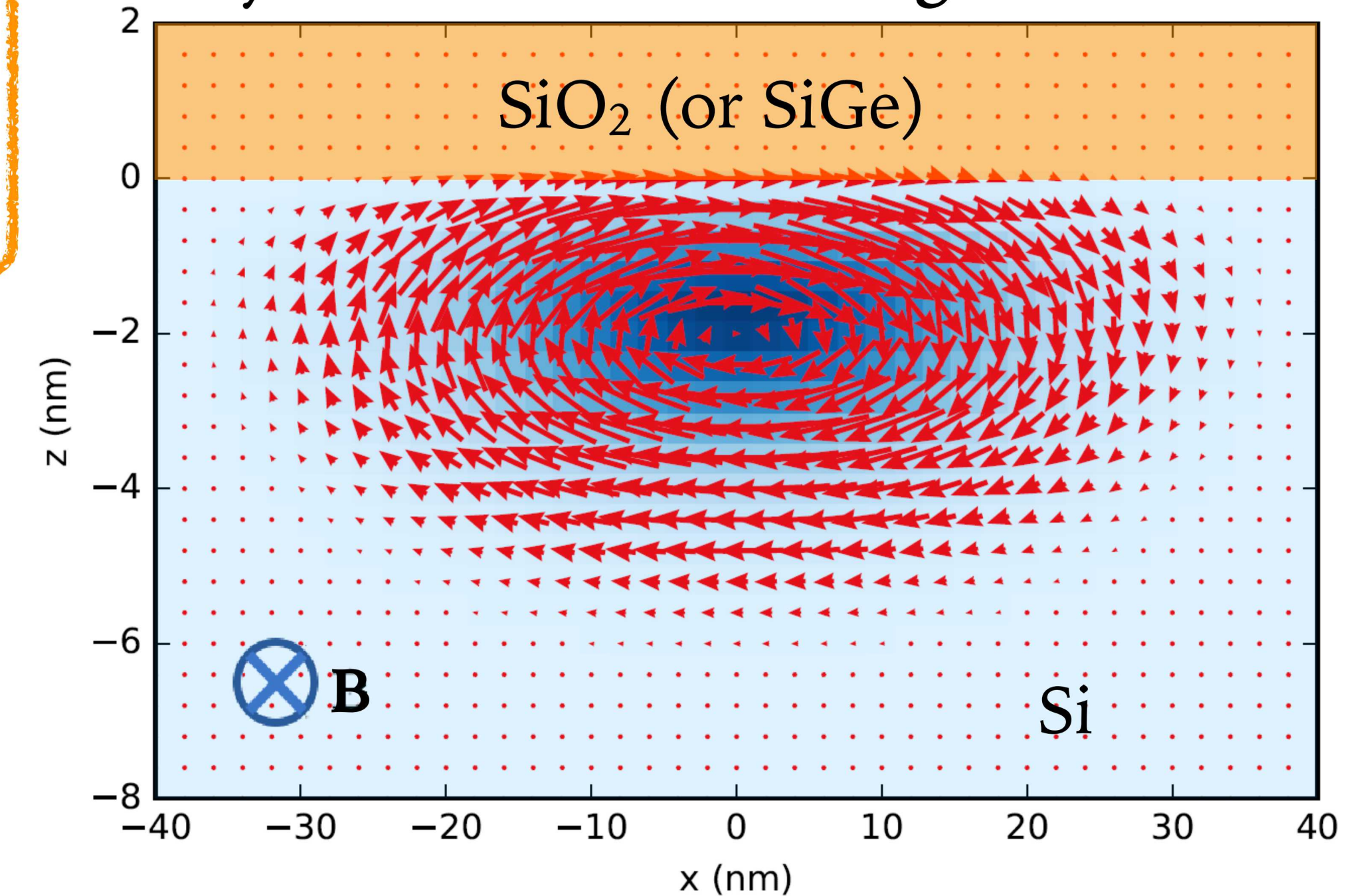
Valley eigenstates for L and R dots:

$$\langle \mathbf{r} | \phi_R \rangle = \frac{1}{\sqrt{2}} (e^{ik_0 z} + e^{i\varphi_{v,R}} e^{-ik_0 z}) \psi_R(\mathbf{r})$$

$$\langle \mathbf{r} | \phi_{R^*} \rangle = \frac{1}{\sqrt{2}} (e^{ik_0 z} - e^{i\varphi_{v,R}} e^{-ik_0 z}) \psi_R(\mathbf{r})$$

$$\langle \mathbf{r} | \phi_L \rangle = \frac{1}{\sqrt{2}} (e^{ik_0 z} + e^{i\varphi_{v,L}} e^{-ik_0 z}) \psi_L(\mathbf{r})$$

Cyclotron motion along interface



Interface SOC: [Golub & Ivchenko, PRB 69, 115333 (2004)]

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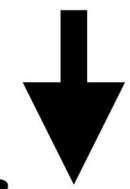
$$= \delta(z) [\gamma_R (P_y \sigma_x - P_x \sigma_y) + \gamma_D (P_x \sigma_x - P_y \sigma_y)]$$

Valley eigenstates for L and R dots:

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Dependence of momentum matrix elements on valley phase:

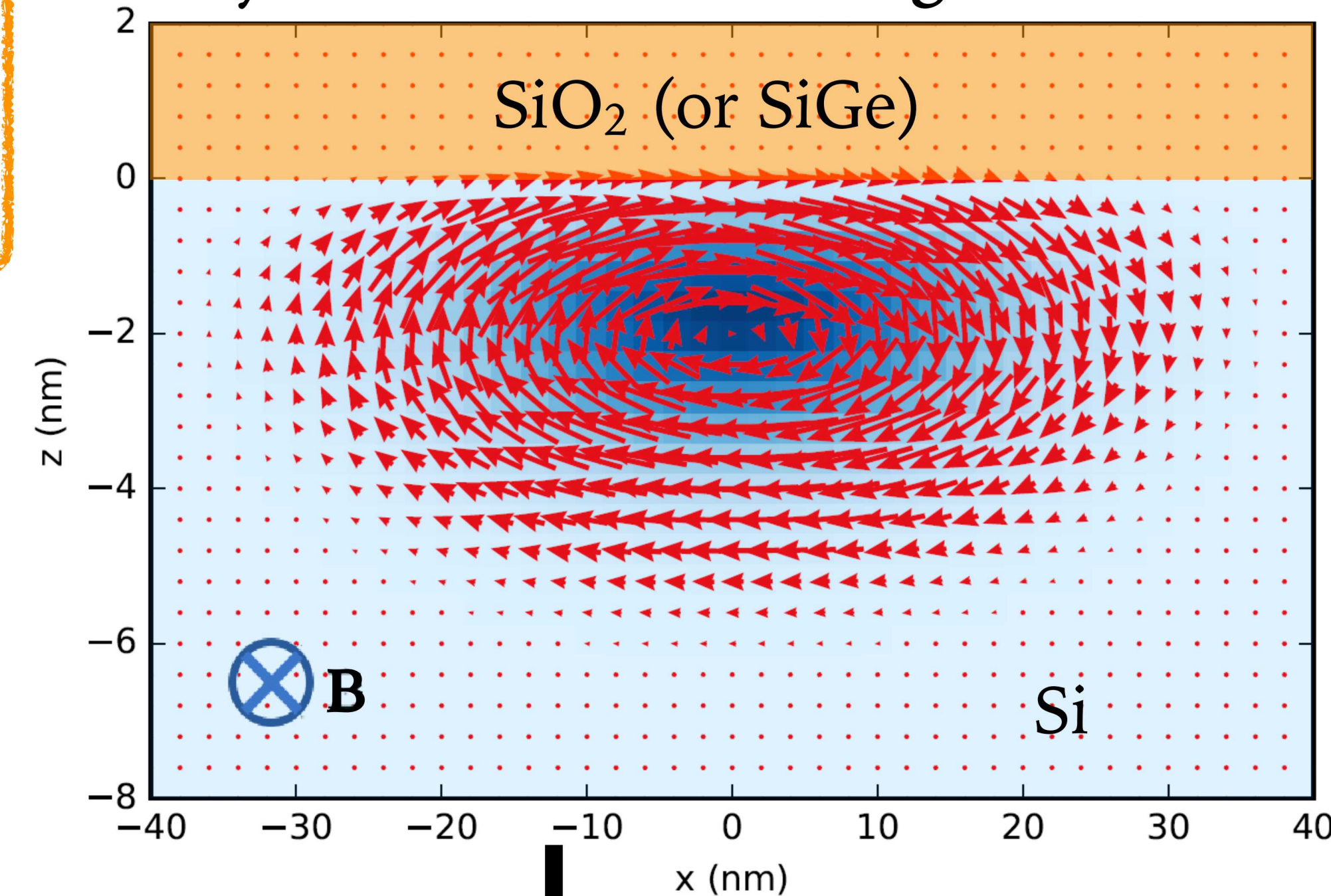
$$\langle \phi_L | \delta(z) P_k | \phi_L \rangle = (1 + \cos(\varphi_{v,L})) \langle \psi_L | \delta(z) P_k | \psi_L \rangle$$

$$\langle \phi_R | \delta(z) P_k | \phi_R \rangle = (1 + \cos(\varphi_{v,R})) \langle \psi_R | \delta(z) P_k | \psi_R \rangle$$

$$\langle \phi_R | \delta(z) P_k | \phi_{R^*} \rangle = -i \sin(\varphi_{v,R}) \langle \psi_R | \delta(z) P_k | \psi_R \rangle,$$

Important: Relative strength of intra- and inter-valley SOC modulated by valley phase

Cyclotron motion along interface



Dependence of momenta on B-field:

$$\langle \psi_L | \delta(z) P_x | \psi_L \rangle = \lambda_L^x B_y$$

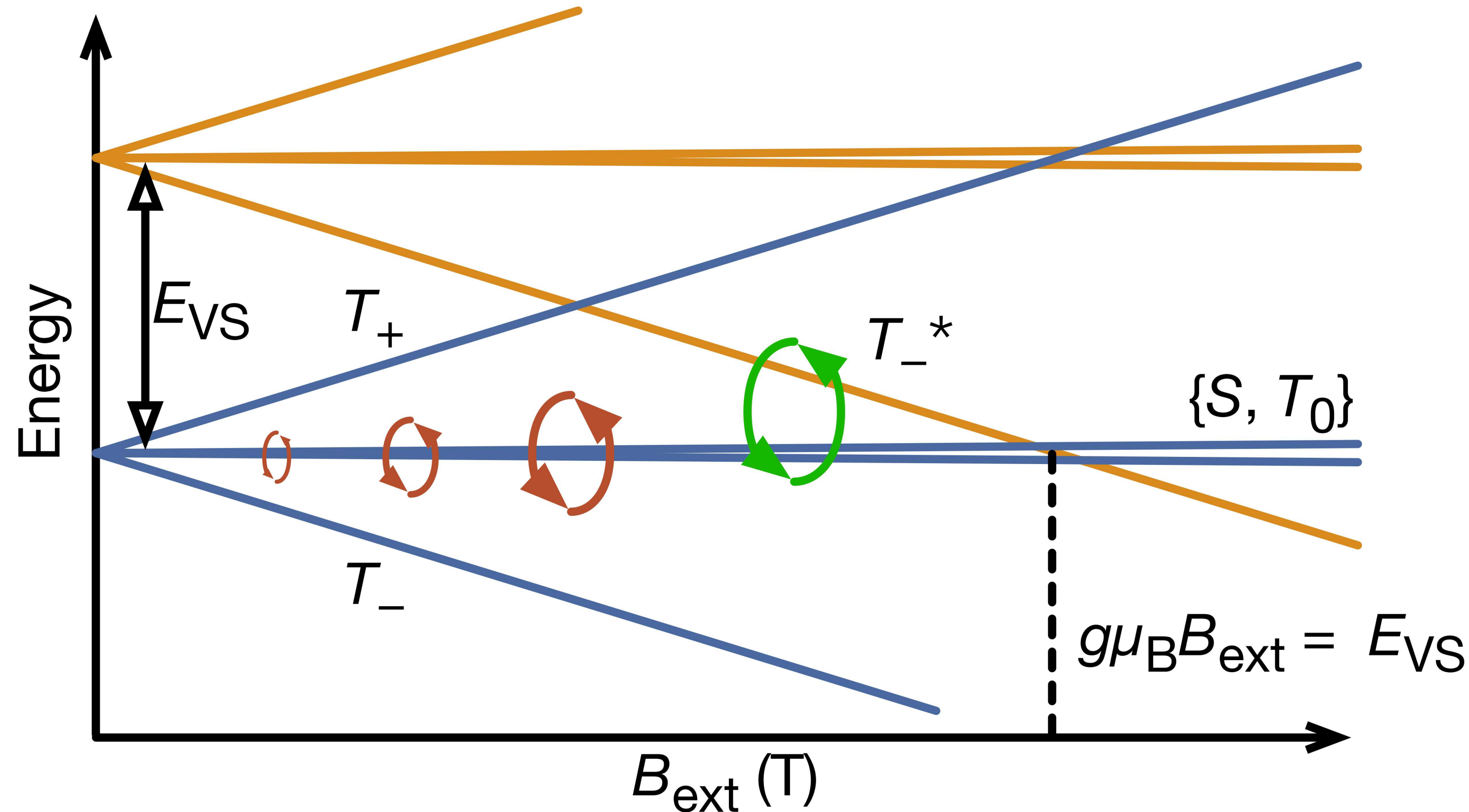
$$\langle \psi_L | \delta(z) P_y | \psi_L \rangle = -\lambda_L^y B_x$$

$$\langle \psi_R | \delta(z) P_x | \psi_R \rangle = \lambda_R^x B_y$$

$$\langle \psi_R | \delta(z) P_y | \psi_R \rangle = -\lambda_R^y B_x$$

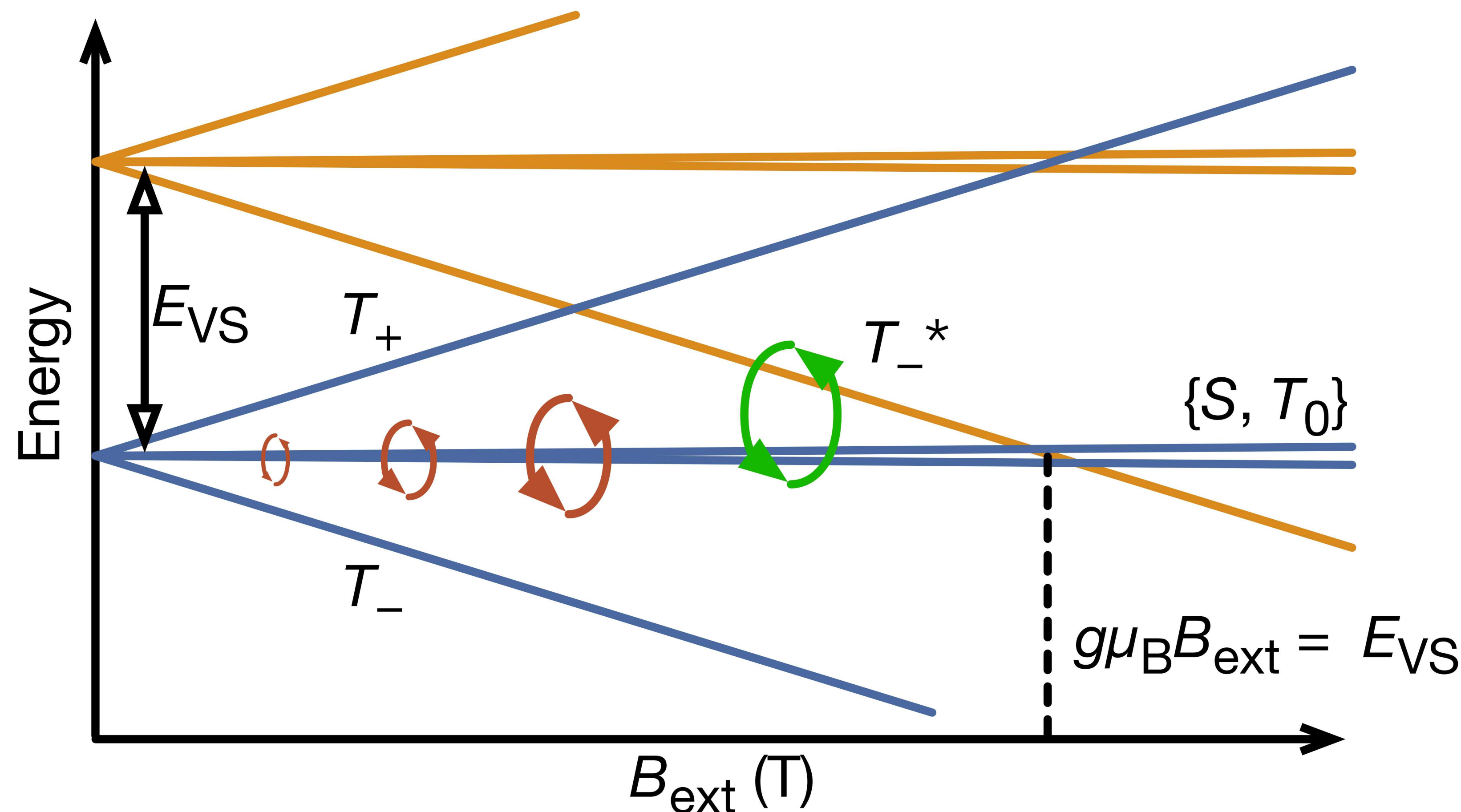
Putting this all together:

- Direct S/ T_0 coupling driven by *intra-valley* SOC
- Excited T_-^* branch enters into the mix through *inter-valley* SOC
- Both *intra-* and *inter-valley* SOC matrix elements *proportional to B*
- Relative weights of intra- and inter-valley contributions *governed by valley phases in dots*



Putting this all together:

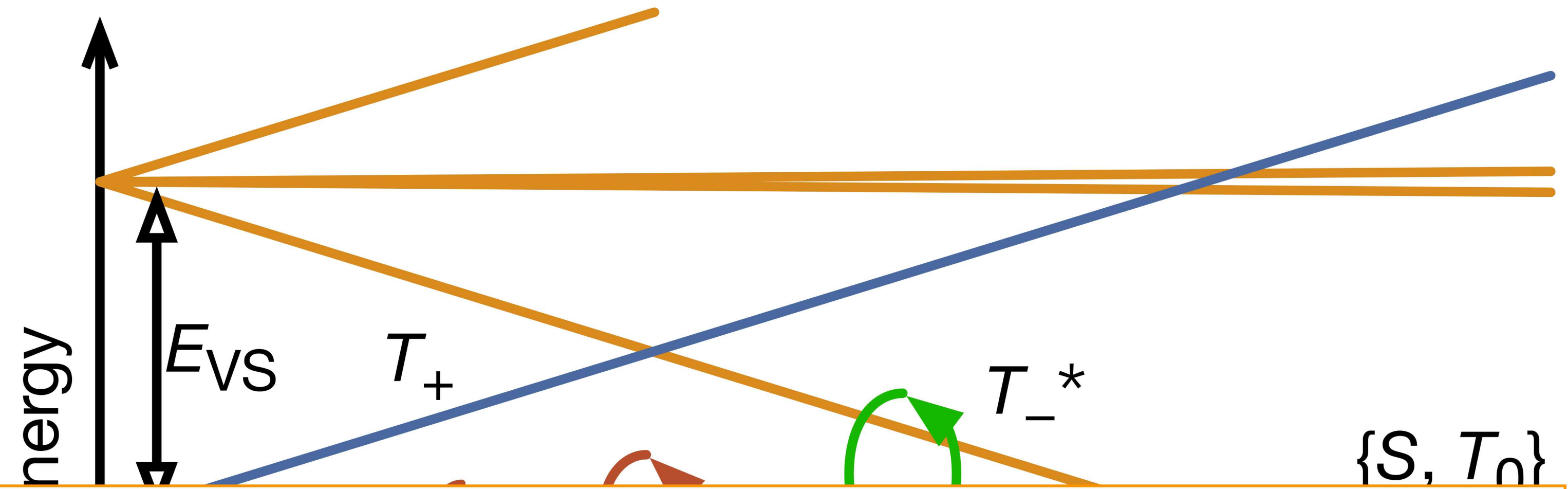
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- Relative weights of intra- and inter-valley contributions *governed by valley phases in dots*



Upshot: Depending on the valley phase (which may change somewhat between tune-ups) and B-field orientation, intra- or inter-valley mechanisms may dominate. If excited valleys in both dots relevant, more complicated behavior can emerge.

Putting this all together:

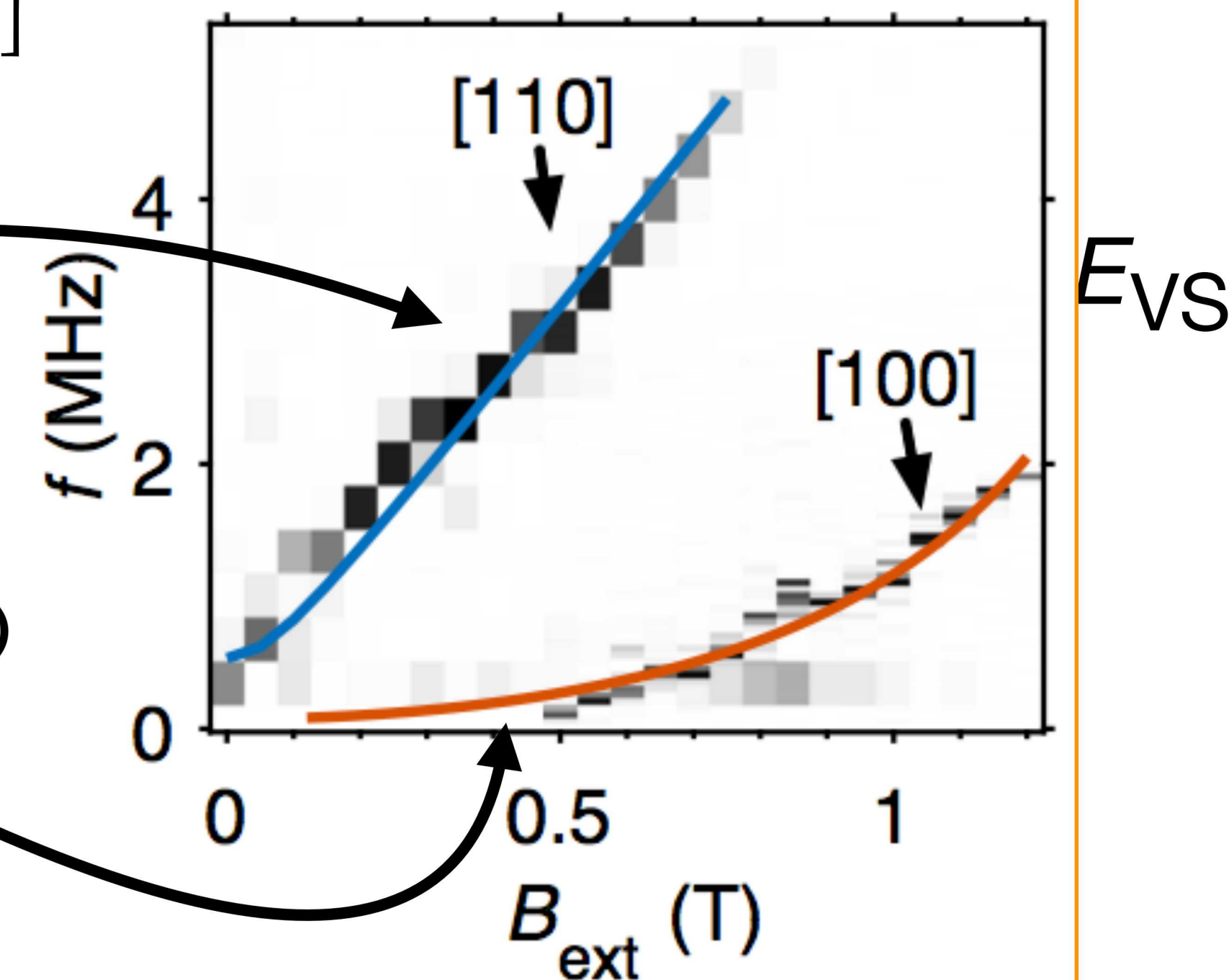
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- Excited T₋* branch enters into the mix through *inter-valley* SOC
- Both *intra-* and *inter-valley* SOC matrix elements *proportional to B*
- Relative weights valley contribution *valley phases* in d



Example: in [Harvey-Collard, et al. arXiv:1808.07378] we observed S/T₀ rotation frequencies believed to be

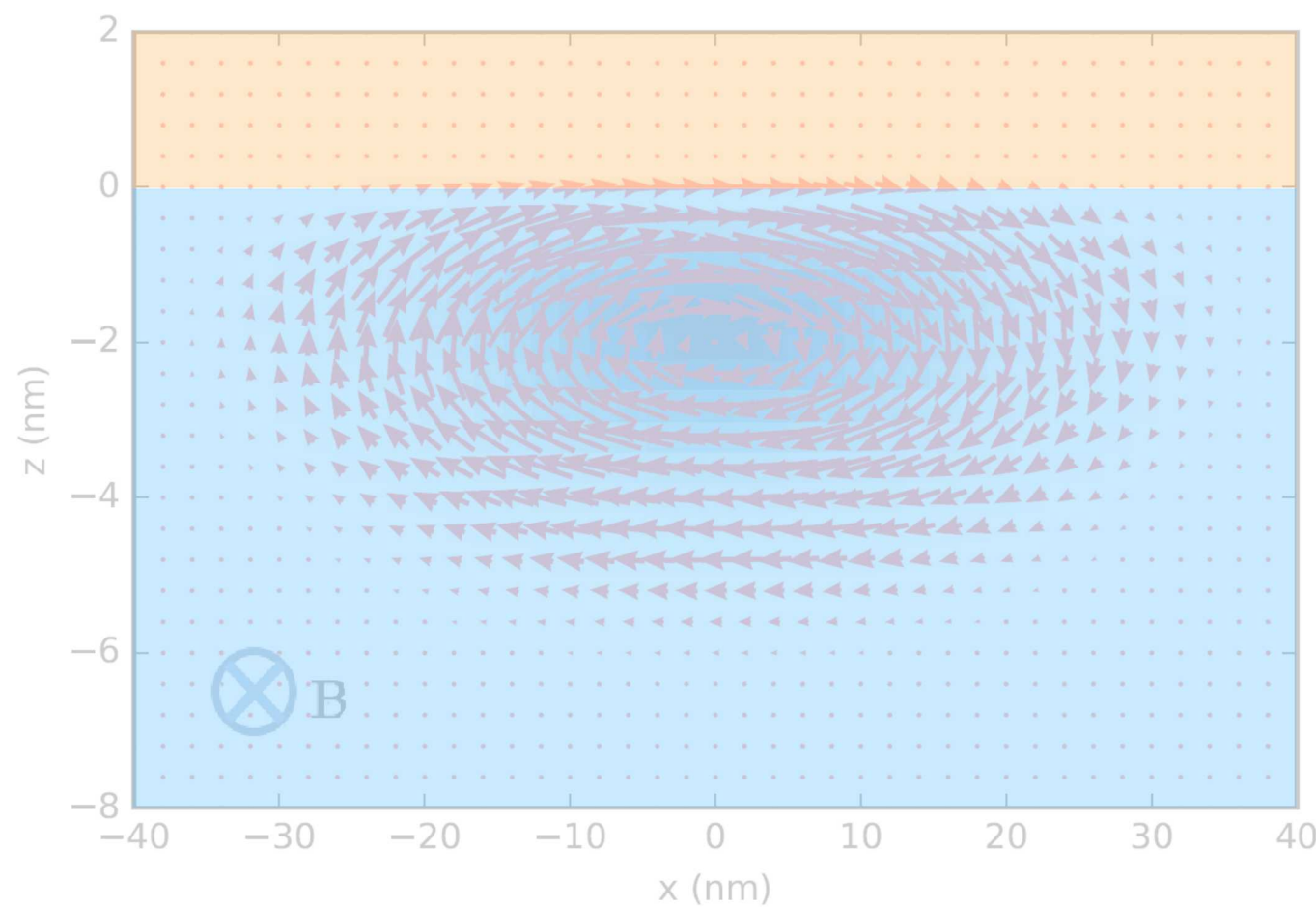
for $B \parallel [110]$: $\propto B$ (mainly *intra-valley*)

for $B \parallel [100]$: approximately $\propto B^2$ (mainly *inter-valley*)



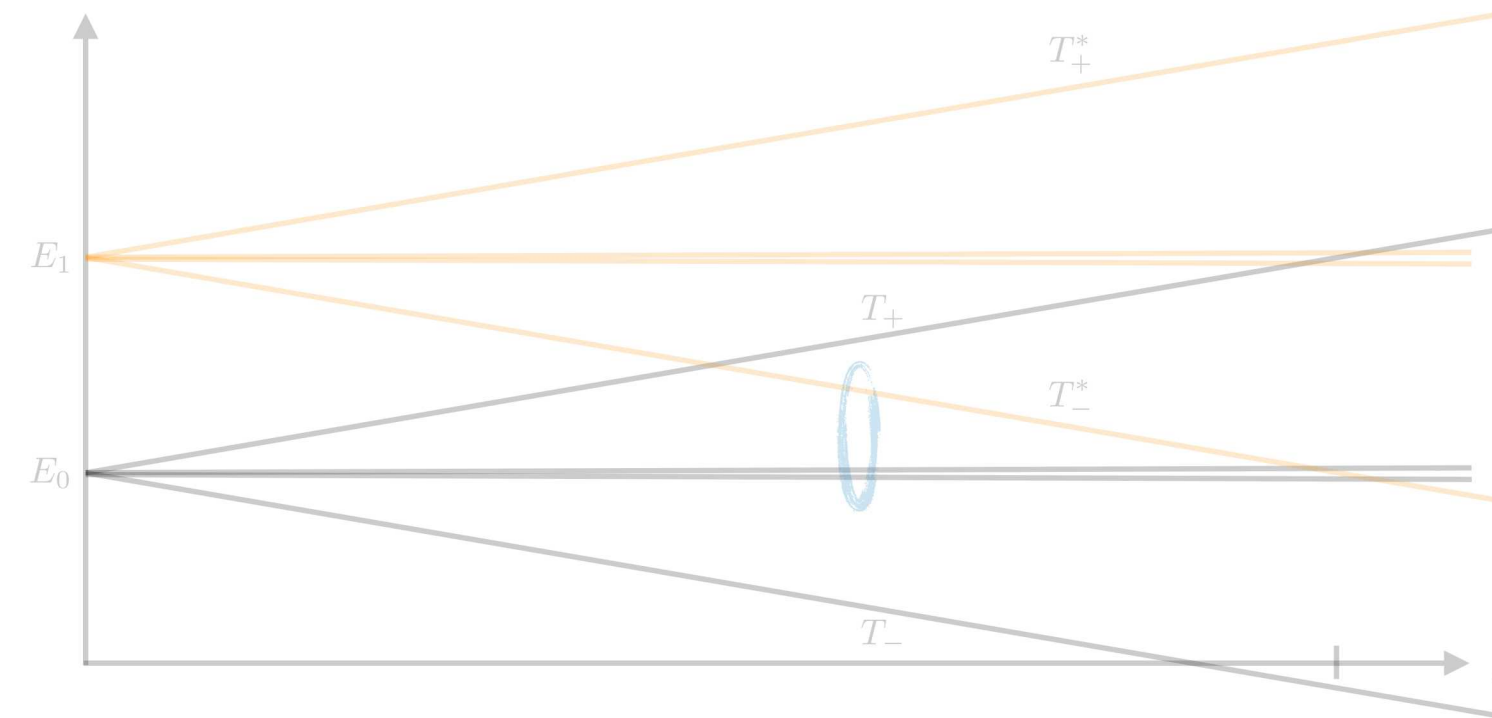
A menagerie of SOC effects

(1) Intra-valley (*g-tensor shift*)



$$\text{e.g. } \langle R \uparrow | H_{\text{SO}} | R \downarrow \rangle$$

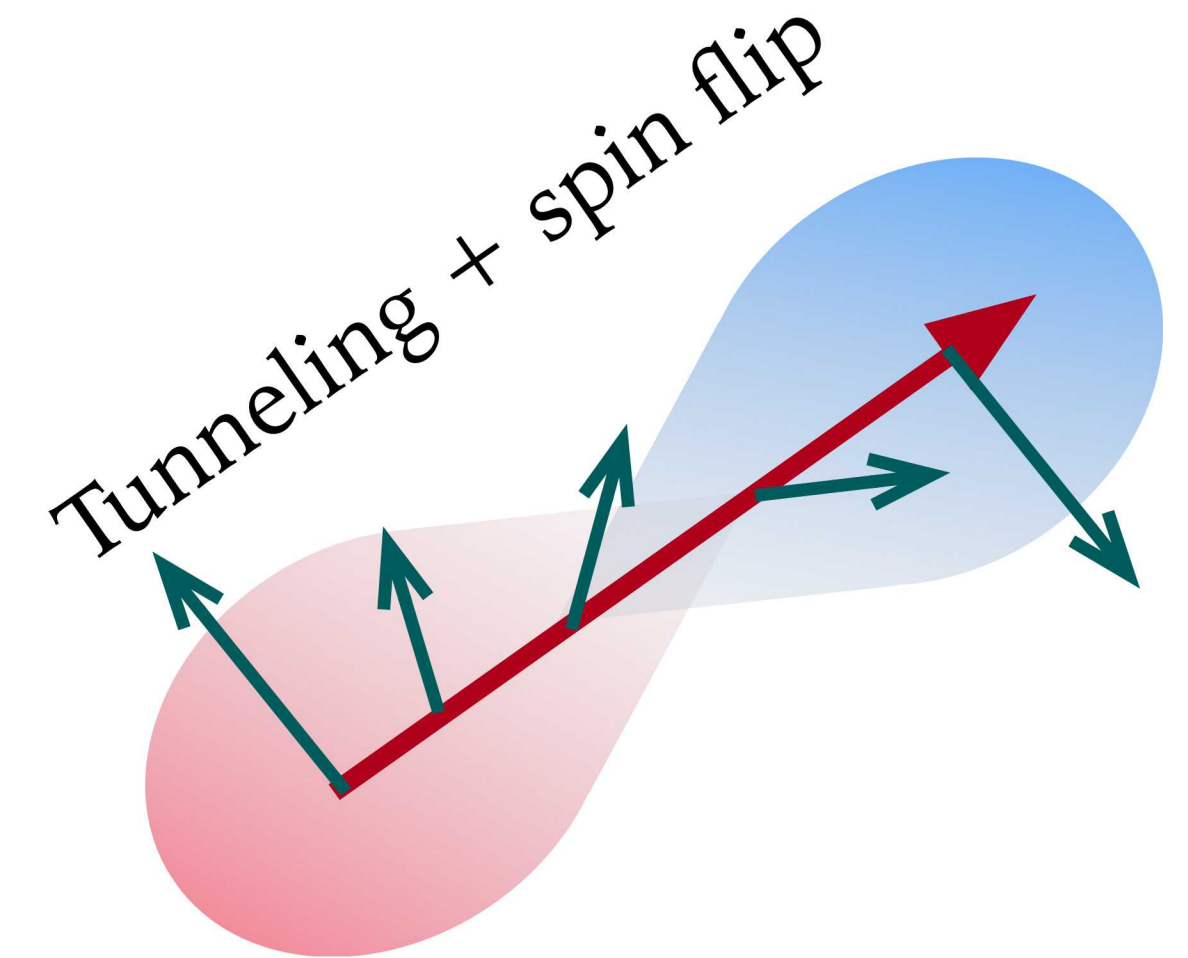
(2) Inter-valley (*nonlinear effects*)



Related to T_1 hot-spots
[Yang, et al. Nat. Comm. (2013)]
(UNSW + LPS + Purdue)

$$\text{e.g. } \langle R^* \uparrow | H_{\text{SO}} | R \downarrow \rangle$$

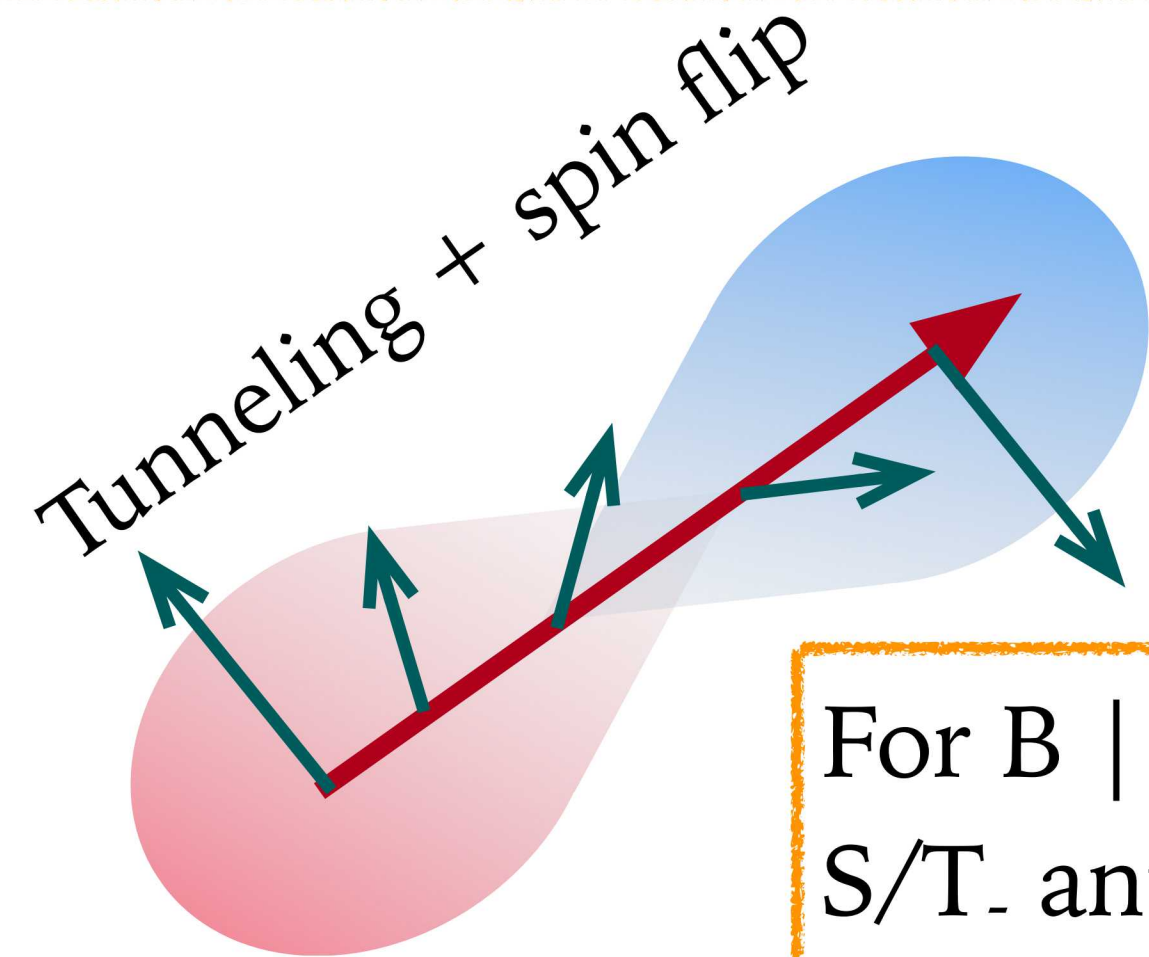
(3) Inter-dot $\rightarrow$ S/T anti-crossing (*leakage, SPAM error*)



$$\text{e.g. } \langle L \uparrow | H_{\text{SO}} | R \downarrow \rangle$$

Now, measurements of the last of these three SOC mechanisms...

Measurements by Patrick Harvey-Collard

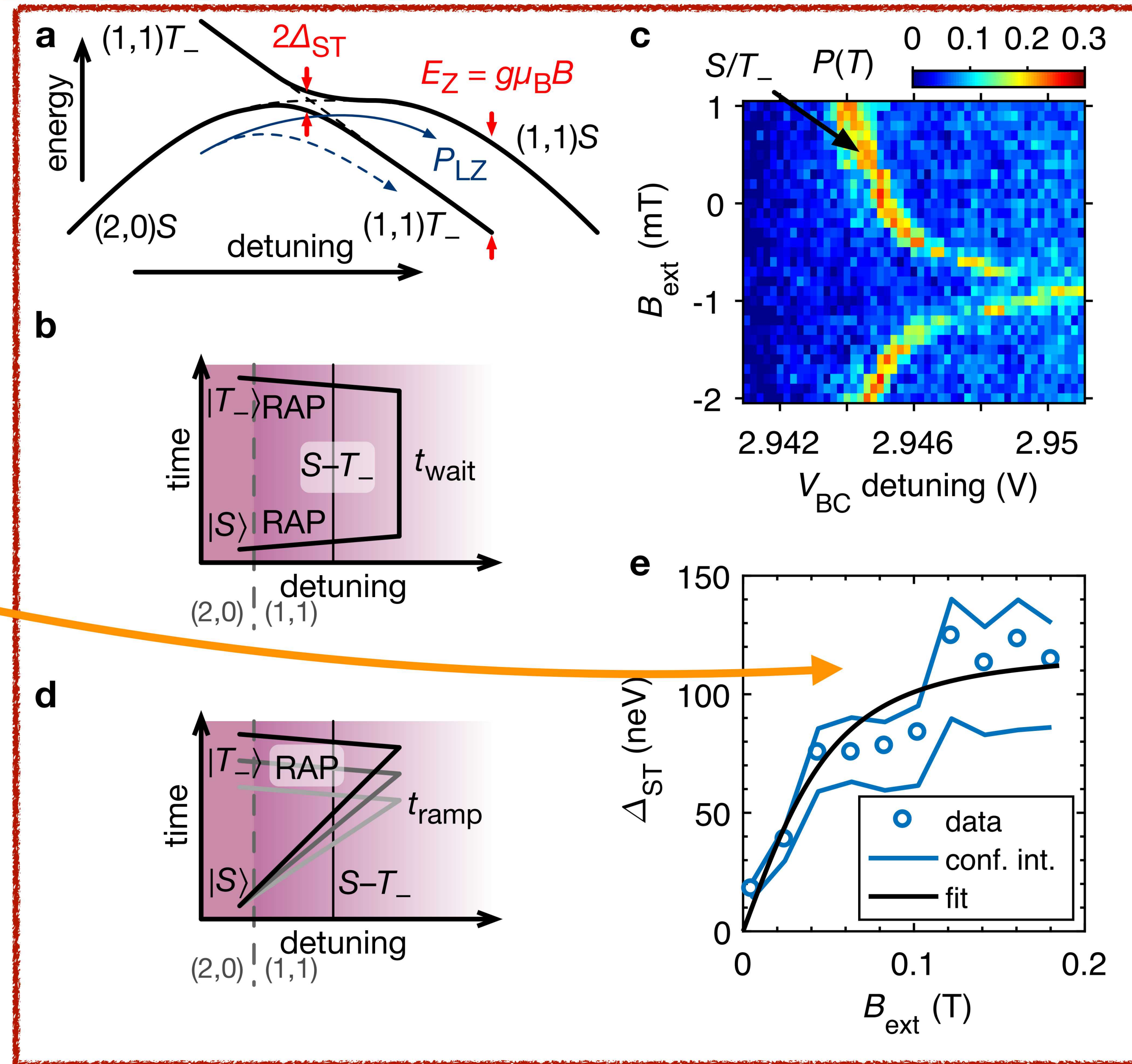


e.g. $\langle L \uparrow | H_{\text{SO}} | R \downarrow \rangle$

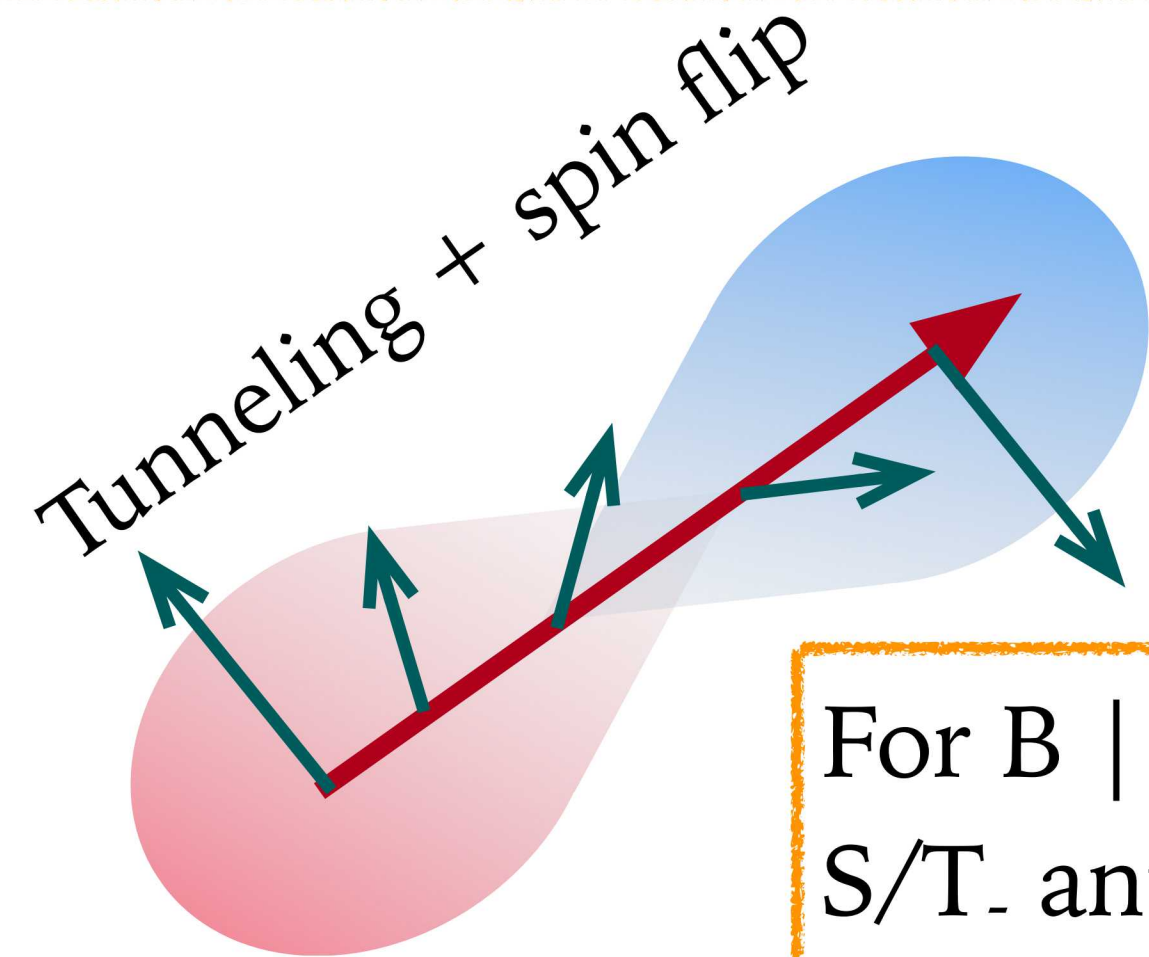
For $B \parallel [100]$,
S/T. anti-crossing half-gap
between S(2,0) and T.(1,1)
 $\Delta_{\text{ST}} \sim 110 \text{ neV}$

S/T. gap characterization technique:

[Nichol, et al. Nat. Comm. (2015)] (Yacoby group)



Measurements by Patrick Harvey-Collard



e.g. $\langle L \uparrow | H_{\text{SO}} | R \downarrow \rangle$

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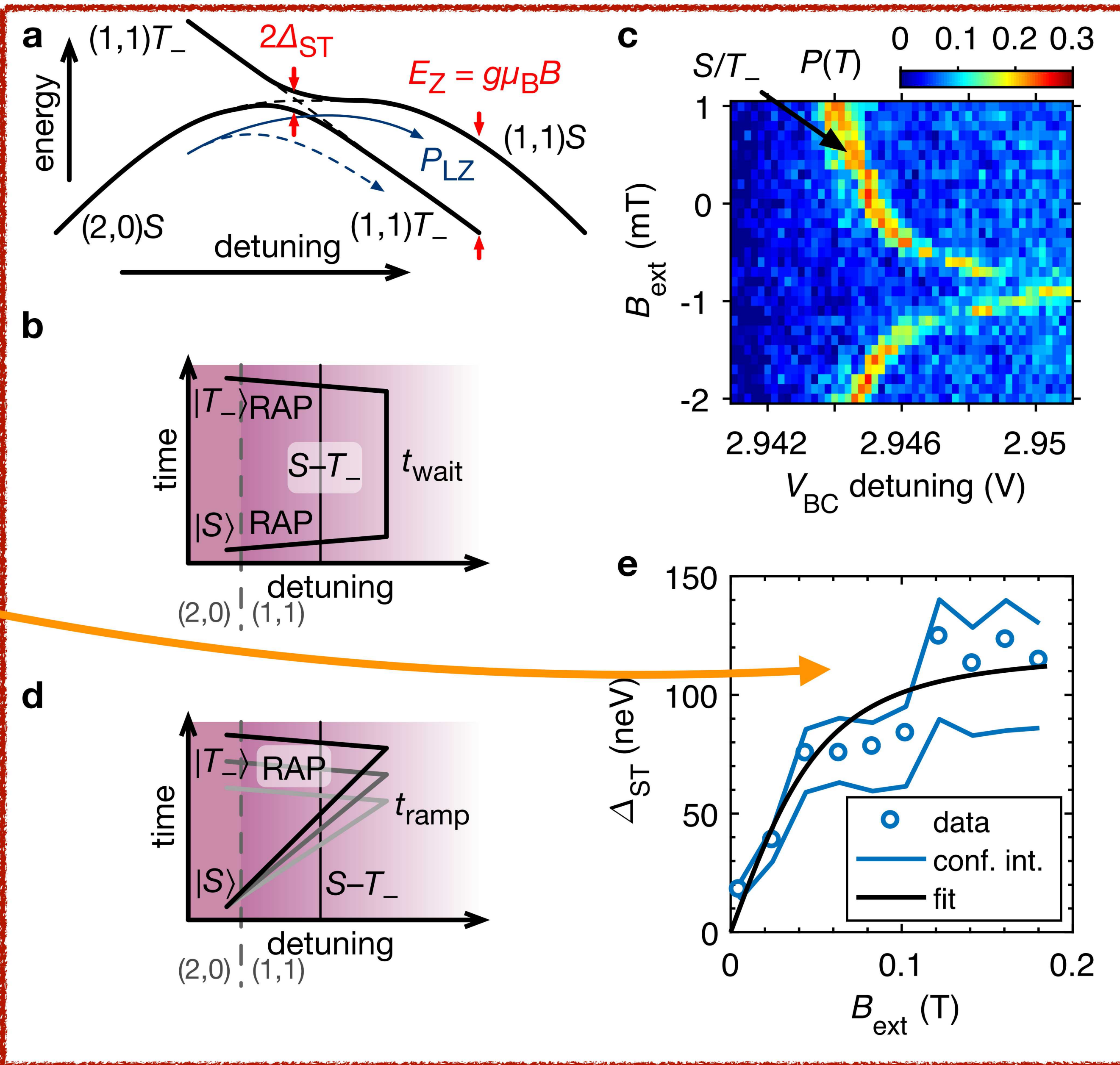
S/T. gap characterization technique:

[Nichol, et al. Nat. Comm. (2015)] (Yacoby group)

Similar magnitude in recent UNSW measurements:

[Fogarty, et al. arXiv:1708.03445]: $\Delta_{\text{ST}} \sim 70 \text{ neV}$

[Tantt, et al. arXiv:1807.10415]: $\Delta_{\text{ST}} \sim 15\text{-}230 \text{ neV}$

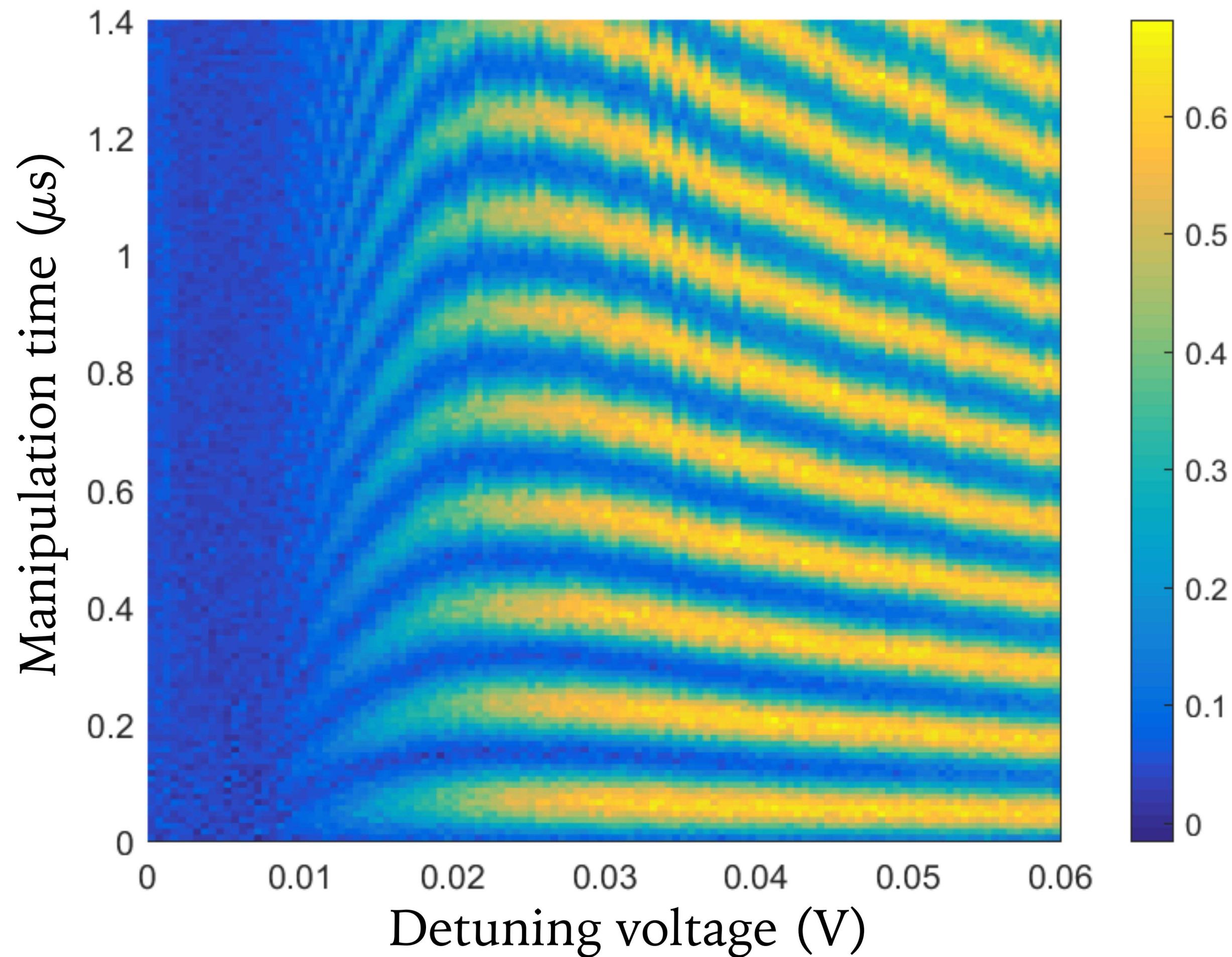


We regularly produce SOC qubits in our polysilicon MOS devices

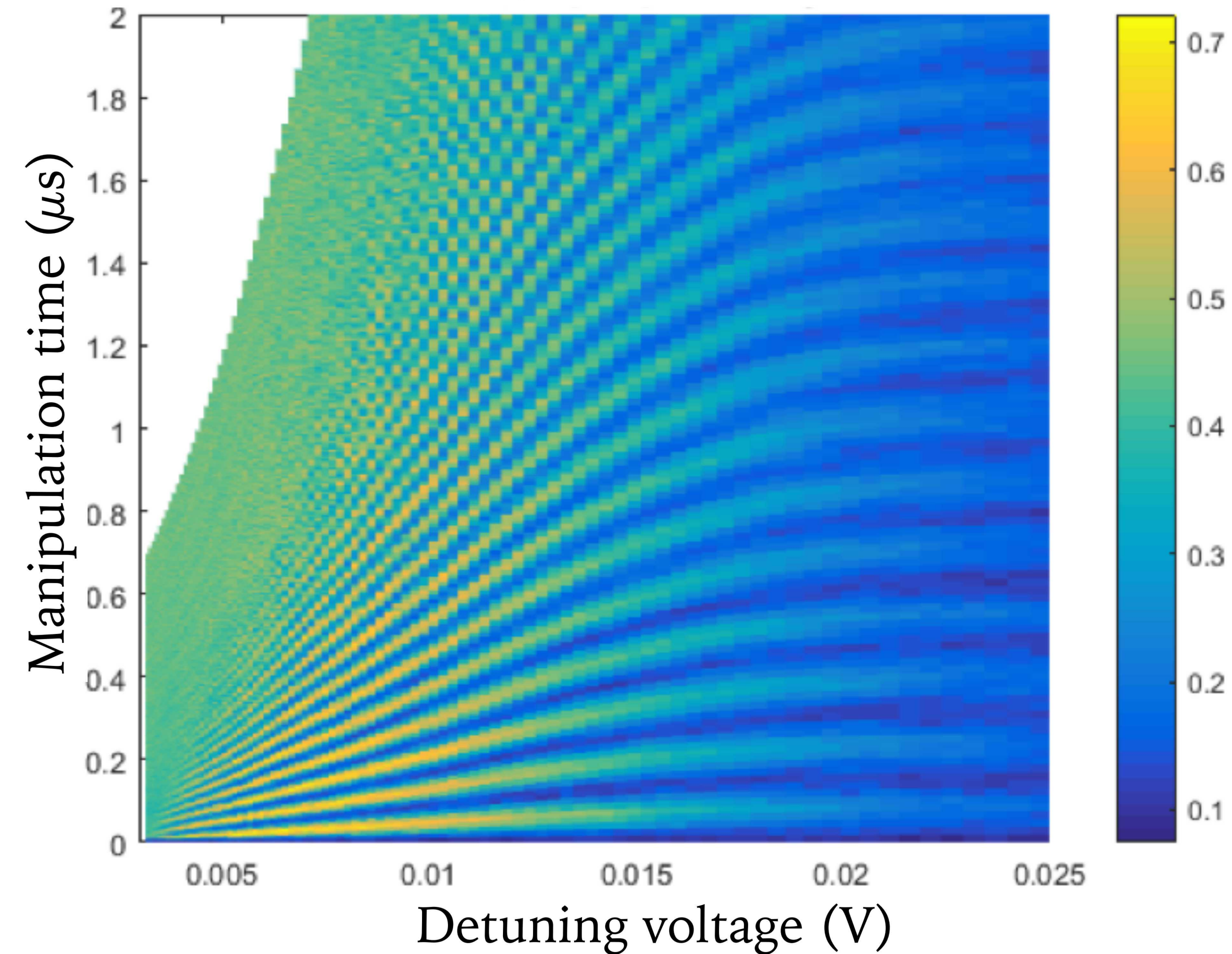
Here are some examples...

Measurements by Chloé Bureau-Oxton

X rotations



J rotations



Gates characterized with Gate Set Tomography (pyGSTi)

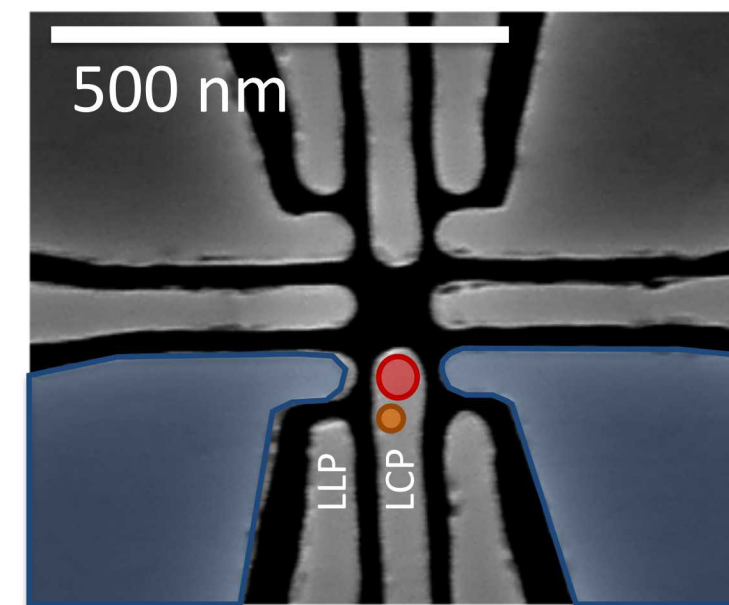
Quality of J rotations (T_2^*/T_{gate}) ~ 30

Double dot

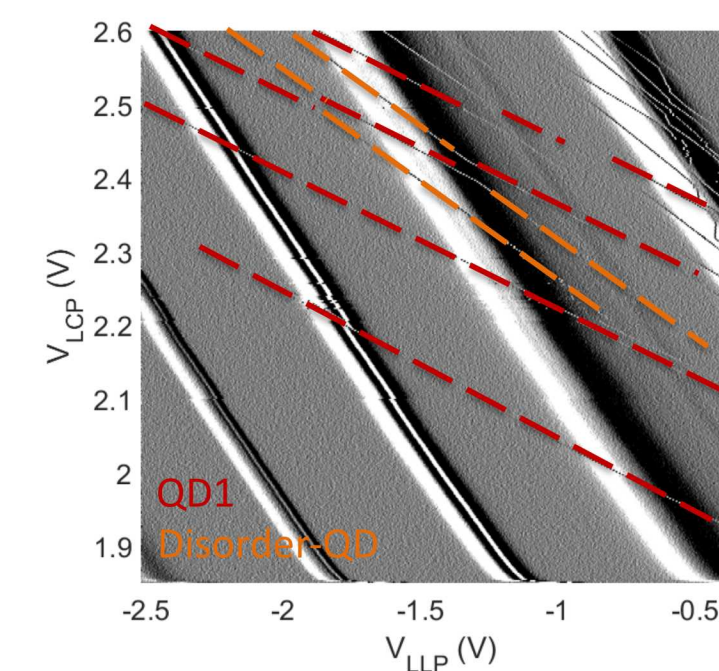
Device 1

- 1QD layout with disorder QD
- 35nm gate oxide; Isotopically enriched epi-layer with 500ppm residual ²⁹Si.
- Qubit Pair: (0,2)—(1,1) charge occupation on **disorder-QD** and **QD1**

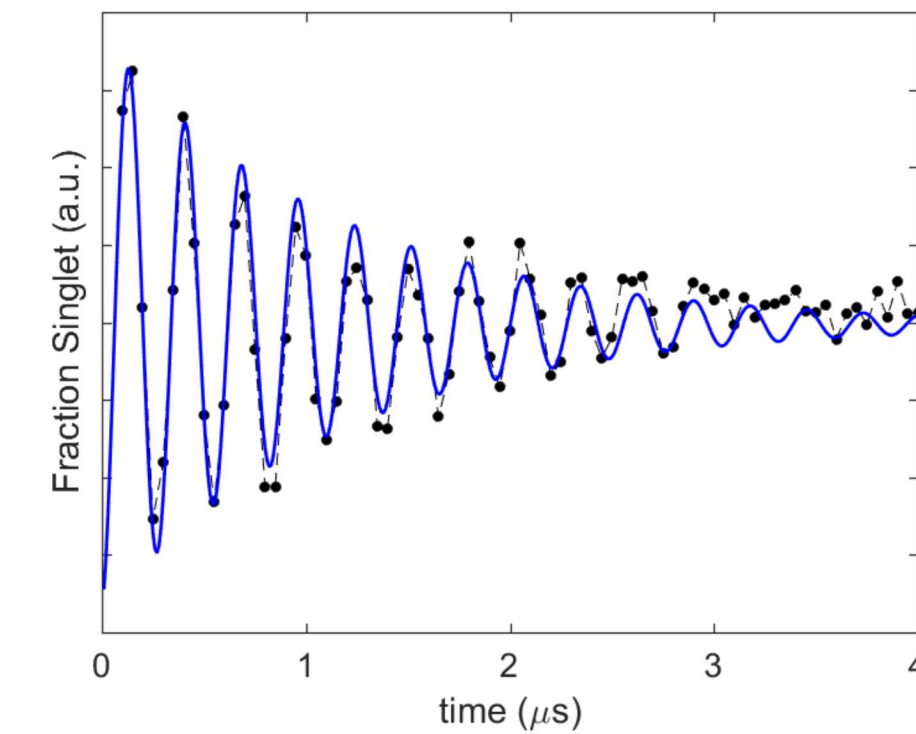
Device Layout



Charge Stability Diagram



Singlet-Triplet FID

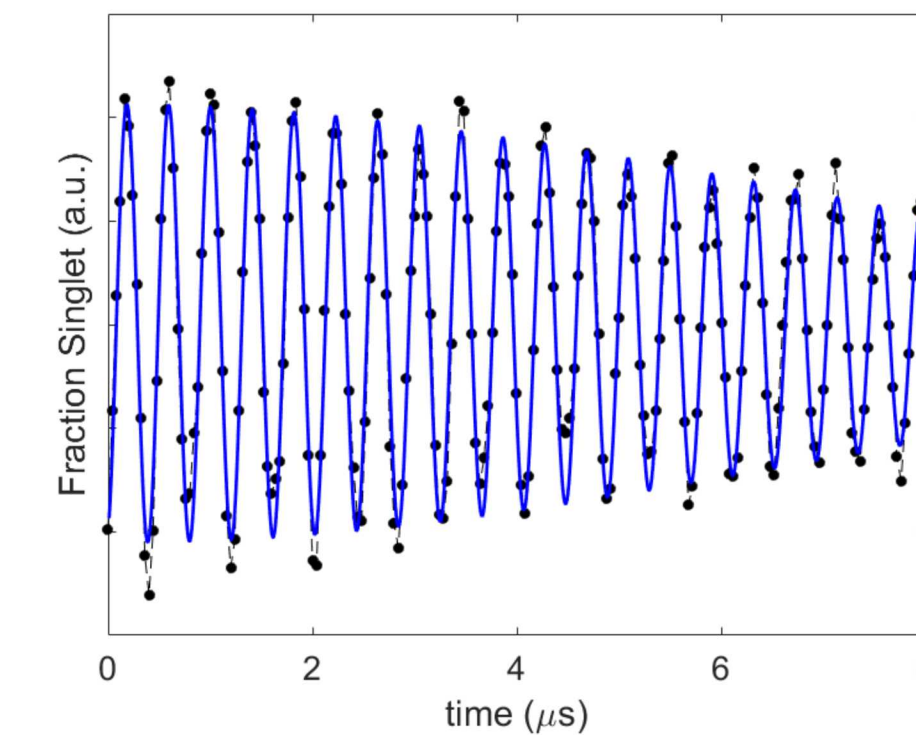
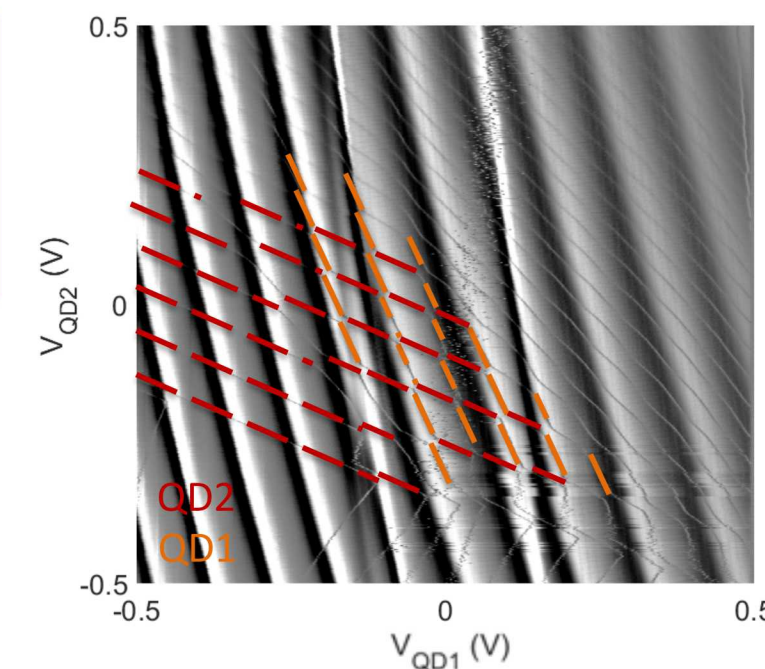
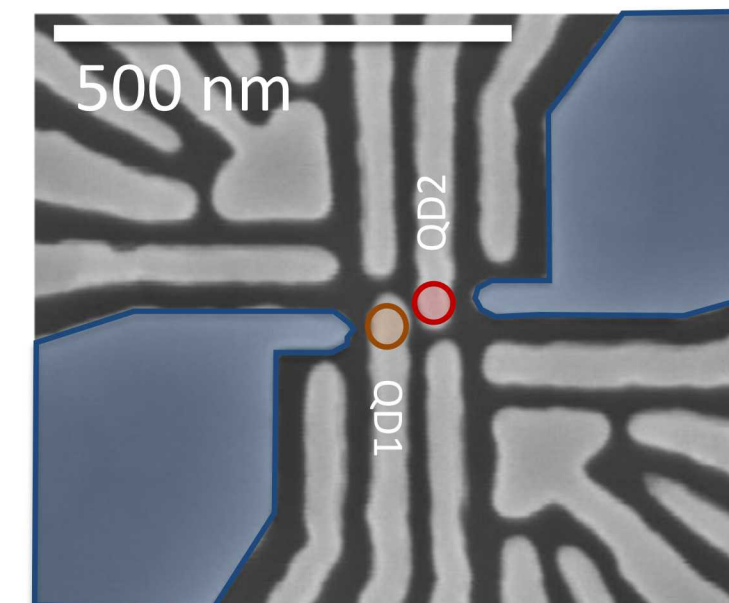


[Ryan Jock]

Double dot

Device 2

- DQD layout
- 50nm gate oxide; Isotopically enriched epi-layer with 500ppm residual ²⁹Si.
- Qubit Pair: (0,4)—(1,3) and (2,4)—(3,3) charge occupations on **QD1** and **QD2**

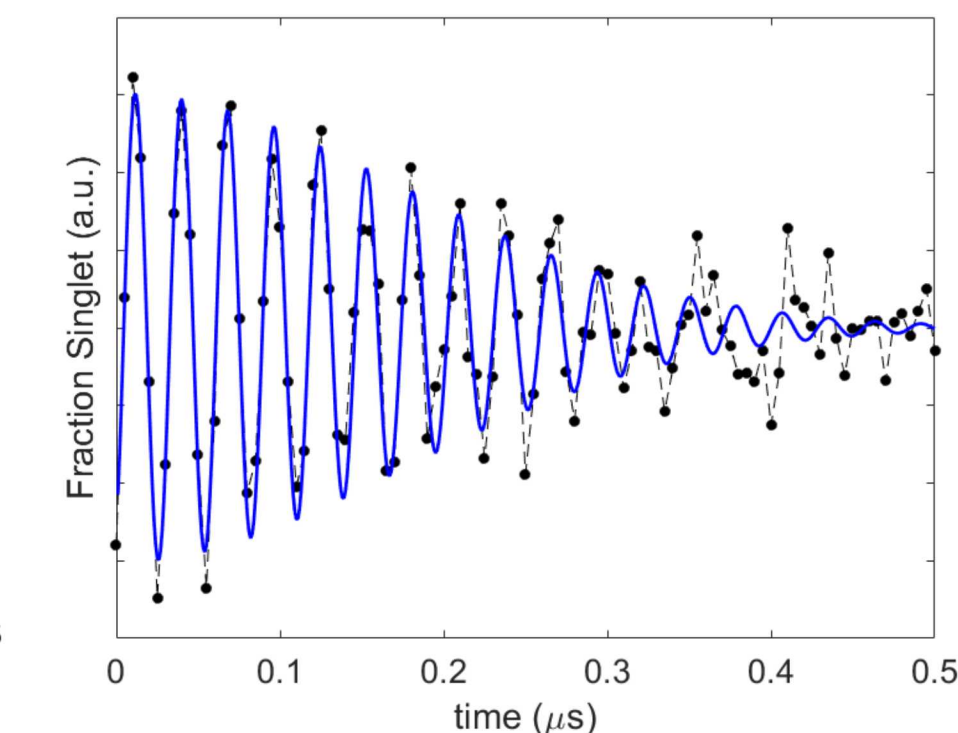
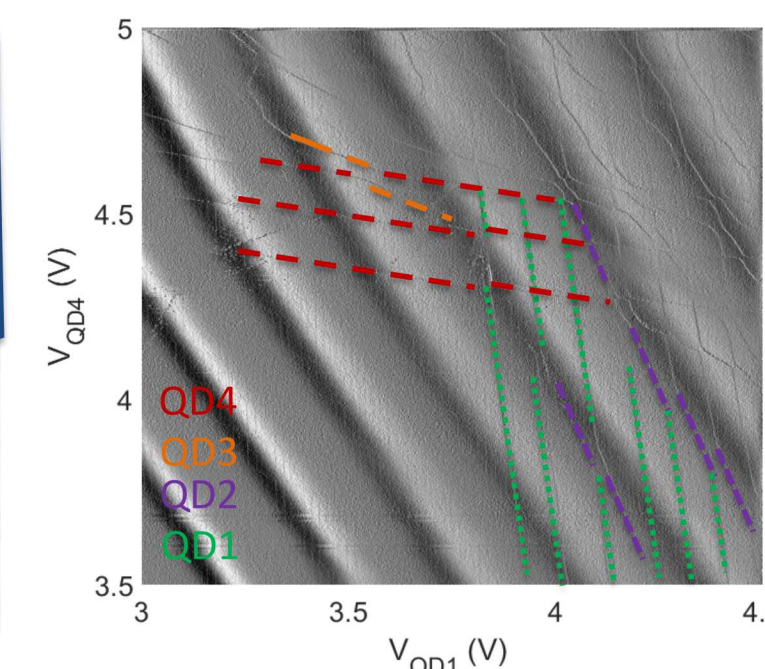
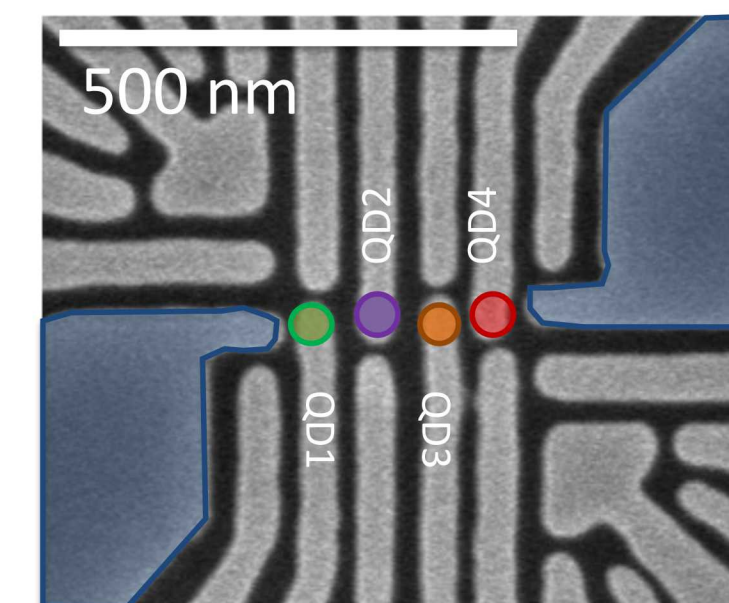


[Ryan Jock]

Quad dot

Device 3

- 4-QD layout
- 50nm gate oxide; Natural Silicon
- Qubit Pair: (0,2)—(1,2) charge occupations on **QD3** and **QD4**



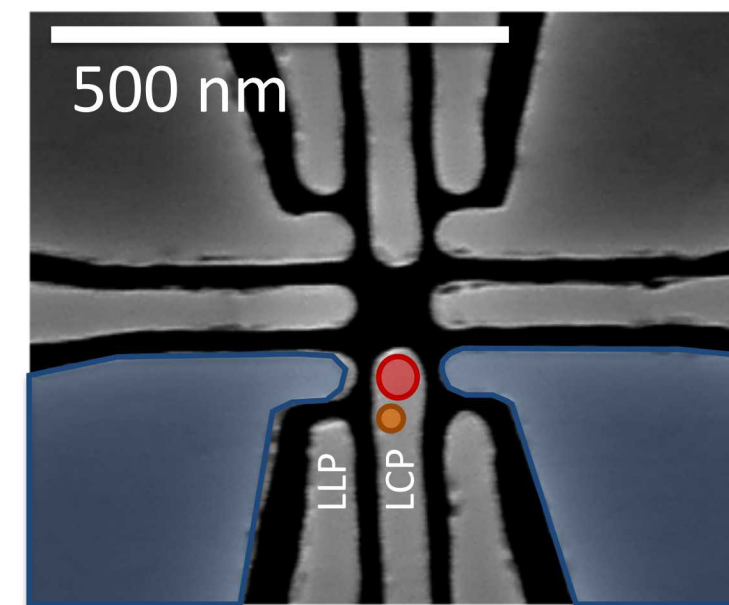
[Martin Rudolph]

Double dot

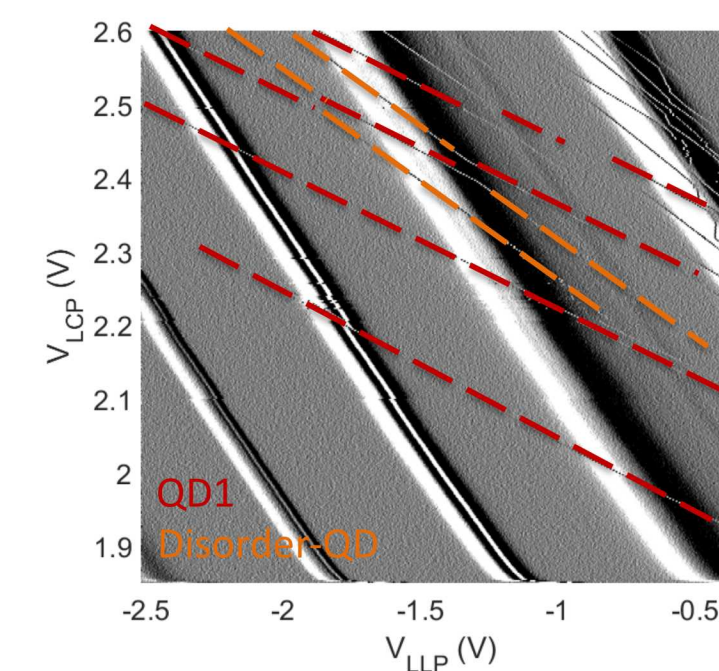
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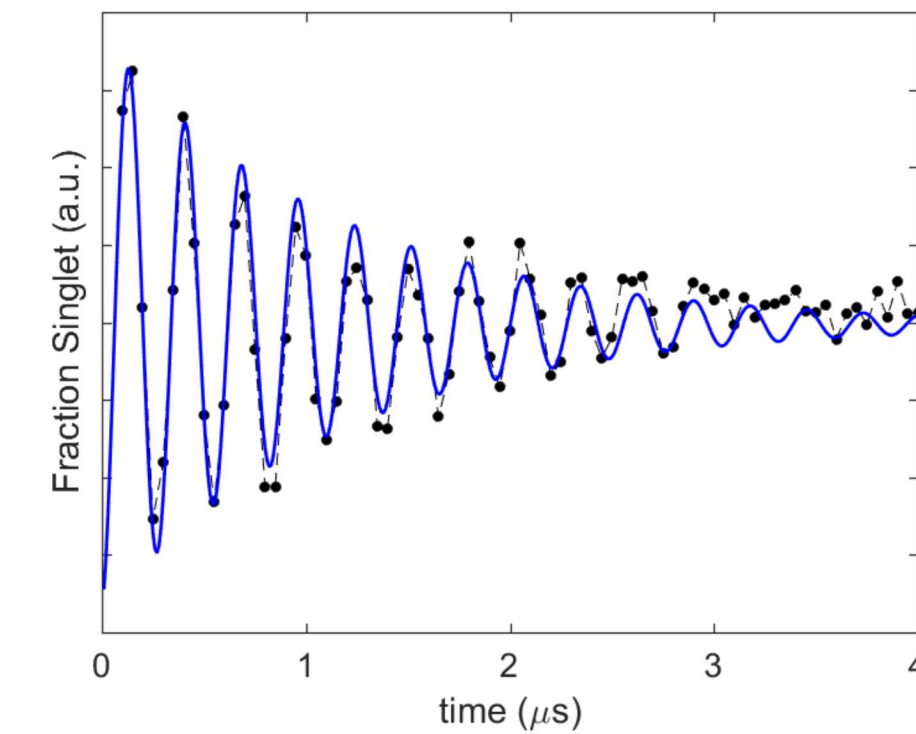
Device Layout



Charge Stability Diagram



Singlet-Triplet FID

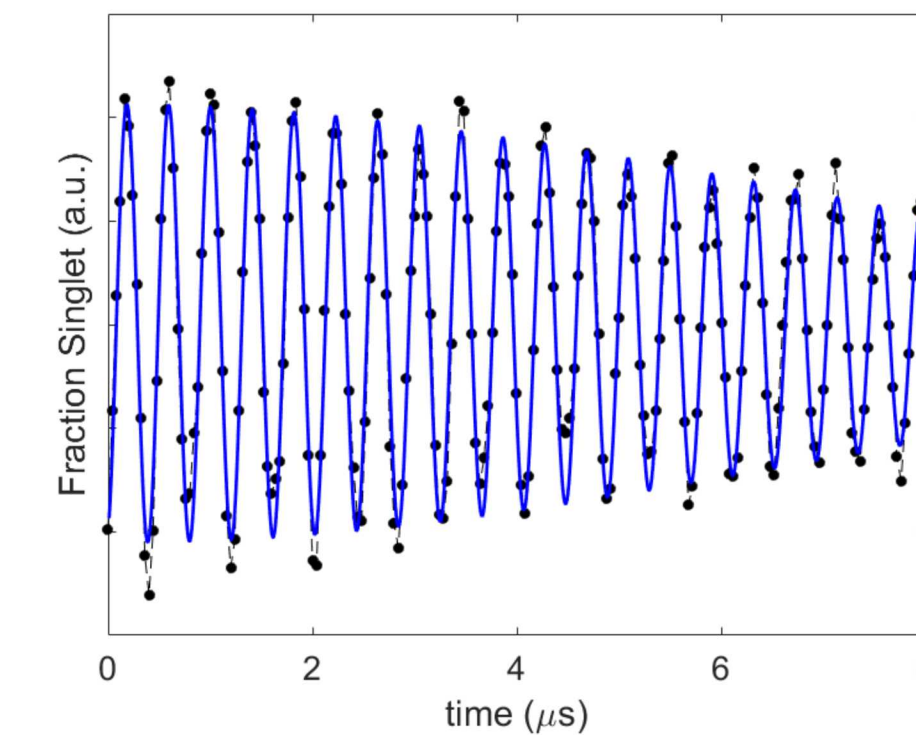
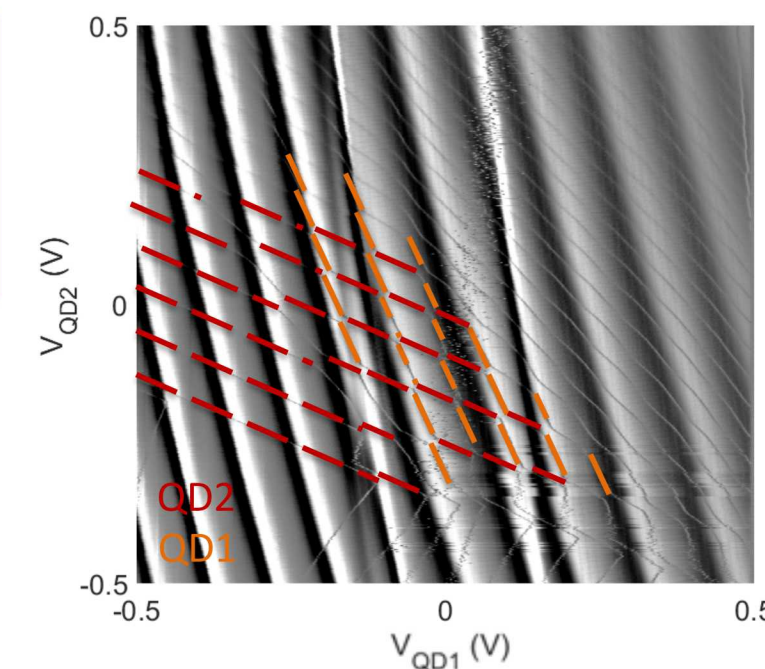
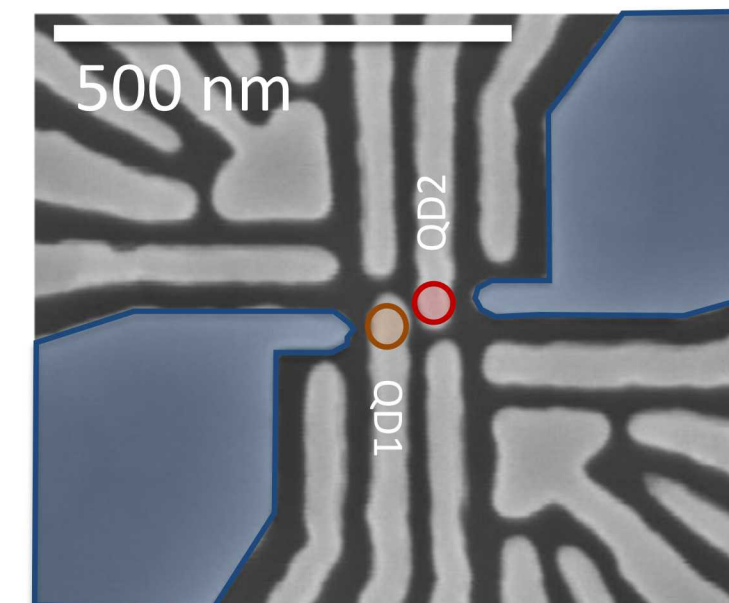


[Ryan Jock]

Double dot

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- 50nm gate oxide; Isotopically enriched epi-layer with 500ppm residual ²⁹Si.
- Qubit Pair: (0,4)—(1,3) and (2,4)—(3,3) charge occupations on **QD1** and **QD2**

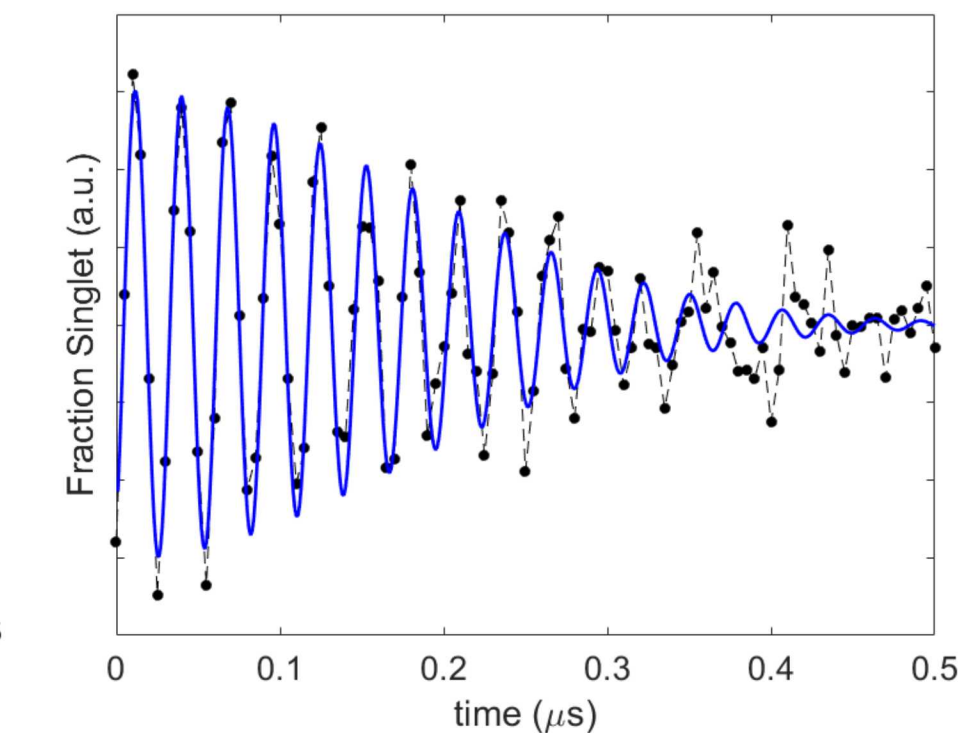
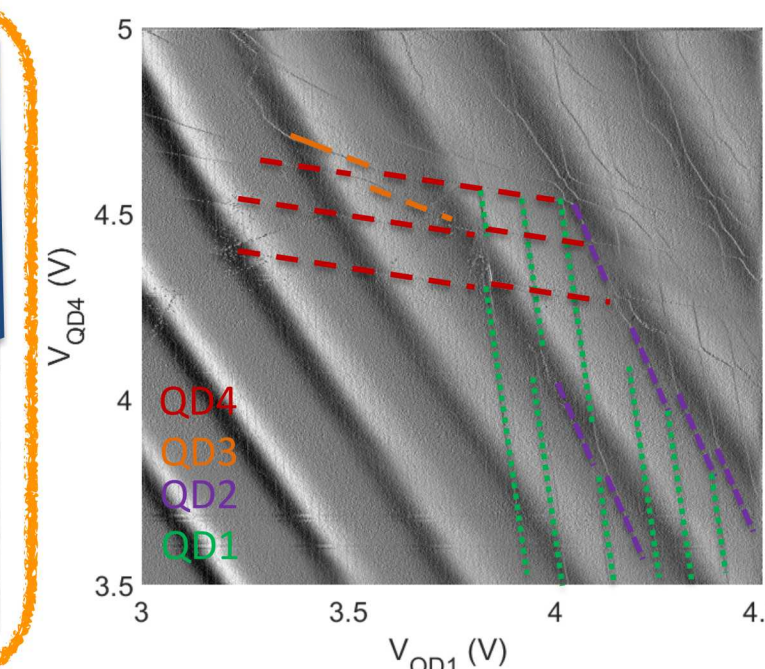
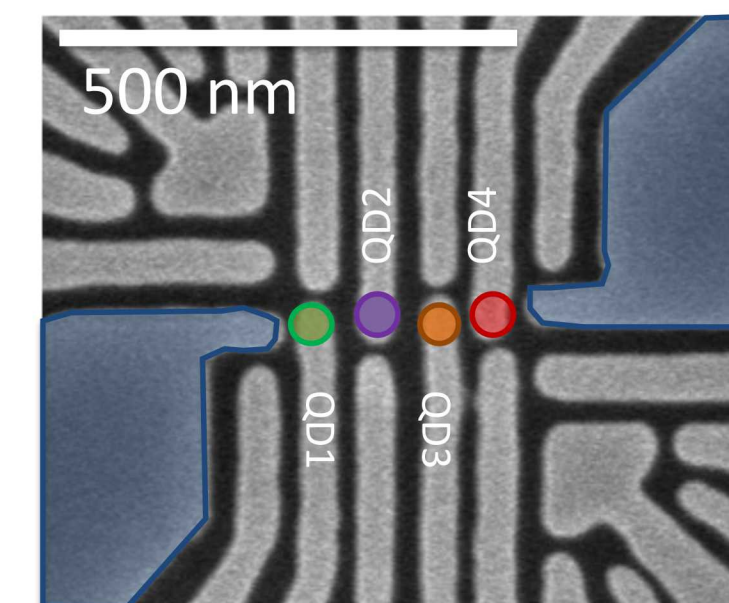


[Ryan Jock]

Quad dot

Device 3

- 4-QD layout
- 50nm gate oxide; Natural Silicon
- Qubit Pair: (0,2)—(1,2) charge occupations on **QD3** and **QD4**



[Martin Rudolph]

Survey of reported SOC strengths in silicon quantum dots

1: [Harvey-Collard, et al. arXiv:1808.07378]
2: [Jock, et al. Nat. Comm. 9, 1768 (2018)]
3: [Hao & Ruskov, et al. Nat. Comm. 5, 3860 (2014)]
4: [Hwang, et al. PRB 96, 045302 (2017)]
5: [Kawakami, et al. Nat. Nano. 9, 666 (2014)]
6: [Yang, et al. Nat. Comm. 4, 2069 (2013)]
7: [Fogarty, et al. arXiv:1708.03445]
8: [Tanttu, et al. arXiv:1807.10415]
9: [Corna, et al. npj Quant. Info. 4, 6 (2018)]
10: [Ferdous, et al. PRB 97, 241401(R) (2018)]
11: [Veldhorst, et al. PRB 92, 201401(R) (2015)]
12: [Ferdous, et al. npj Quant. Inf. 4, 26 (2018)]
13: [Scarlino, et al. PRB 95, 165429 (2017)]
14: [Eng, et al. Sci. Adv. 1, e1500214 (2015)]
15: [Jock, Rudolph, unpublished (2018)]
16: [Nichol, et al. Nat. Comm. 6, 7682 (2015)]

Broad observation:
With a few exceptions, generally consistent magnitudes of SOC amongst MOS systems and between Si/SiO₂ and Si/SiGe implementations

Rashba $\Delta\alpha$	2 MHz/T (MOS) Sandia [2] 2-2.4 MHz/T (MOS) UNSW [8] [0,7] MHz/T (MOS) Sandia [15] 1 MHz/T (Si/SiGe) Purdue+Delft+Wisconsin [12]	$\sim 0-10$ MHz/T
Dresselhaus $\Delta\beta$	16 MHz/T (MOS) Sandia [2] 10-25 MHz/T (MOS) UNSW [8] 8-25 MHz/T (MOS) Sandia [15] 5 MHz/T (Si/SiGe) Purdue+Delft+Wisconsin [12]	$\sim 5-25$ MHz/T
g or Δg between dots or eigenvalleys	$\Delta g=4.3\times 10^{-4}$ (B [110]) (MOS) UNSW [7] $\Delta g=4.3\times 10^{-4}$ (B [110]) (MOS) Sandia [1] $\Delta g=6.4\times 10^{-4}$ (B $\perp$ [001]) (Si/SiGe) HRL [14] $\Delta g=63\times 10^{-4}$ (between valleys, 1e-/3e-) (MOS) UNSW [11] $\Delta g=3\times 10^{-4}$ (between valleys) (Si/SiGe) Delft+Wisconsin [5] (2-g)$\sim [2,10]\times 10^{-3}$ (triple dot, B [110]) (MOS) Purdue+UNSW+Delft [10]	$\sim \text{few-tens} \times 10^{-4}$
Inter-valley coupling (e.g. half gap at S/T-anticrossing)	0.12 μeV @ 0.746 T (0.16 $\mu\text{eV/T}$) (MOS) UCLA+LPS [3] 0.18 μeV @ 1.45 T (0.12 $\mu\text{eV/T}$) (MOS) UNSW [4] 0.7 $\mu\text{eV/T}$ (MOS) Sandia [1] ~ 0.1 μeV @ 5 T (0.02 $\mu\text{eV/T}$) (MOS) UNSW [6] ~ 0.4 μeV @ 0.4-1 T ($\sim 0.4-1$ $\mu\text{eV/T}$) (SiGe) Delft+Wisconsin [13] 1.8 μeV @ 0.3 T (6 $\mu\text{eV/T}$) (SOI nanowire) CEA INAC/LETI [9]	$\sim 0.1-1$ μeV
Tunneling + spin flip (e.g. half gap at S/T-anticrossing)	0.1 μeV ($t_c\sim 5$ μeV) (B [100]) (MOS) Sandia [1] 0.07 μeV ($t_c\sim 8$ μeV) (B [110]) (MOS) UNSW [7] [0.015,0.23] μeV ($t_c\sim 25$ μeV, B$\perp$[001]) (MOS) UNSW [8] <i>For reference: In GaAs, 0.46 μeV measured (Harvard [16])</i>	~ 0.1 μeV

For better:

- Single-electron addressability [Veldhorst, et al. Nature (2015)]
- Drive mechanism for S/T₀ qubits (sans μ -magnets)

Challenges/upside of SOC in Si

For better:

- Single-electron addressability [Veldhorst, et al. Nature (2015)]
- Drive mechanism for S/T₀ qubits (sans μ -magnets)

For worse:

- Intrinsic dot-to-dot variability (even between cool-downs)
- Error contributions
 - ♦ Leakage (SPAM)
 - ♦ More coupling between charge noise and spin
 - ♦ Effective B-fields that depend on valley occupation (especially bad if $\Delta_{VS} \sim k_B T$)

Strategies for mitigating SOC effects in Si

To reduce Δg :

- Orient $\mathbf{B}$ along a “magic angle” where Rashba & Dresselhaus effects cancel [1-5] or “sweet spot” with low E-field sensitivity [4]
 - ♦ Downsides: *Magic angle varies from dot-to-dot; S/T. mechanism not suppressed*
- Orient $\mathbf{B} \perp$ to the Si/SiO₂ or Si/SiGe interface [3-5]
 - ♦ Downside: *S/T. mechanism not suppressed*

To reduce S/T- coupling:

- Orient $\mathbf{B}$ in-plane relative to inter-dot axis such that $\mathbf{B}_{\text{SO}} \parallel \mathbf{B}$ [5]
 - ♦ Downsides: *Won't suppress Δg mechanism; inter-dot axes likely will vary within device*

Failing that:

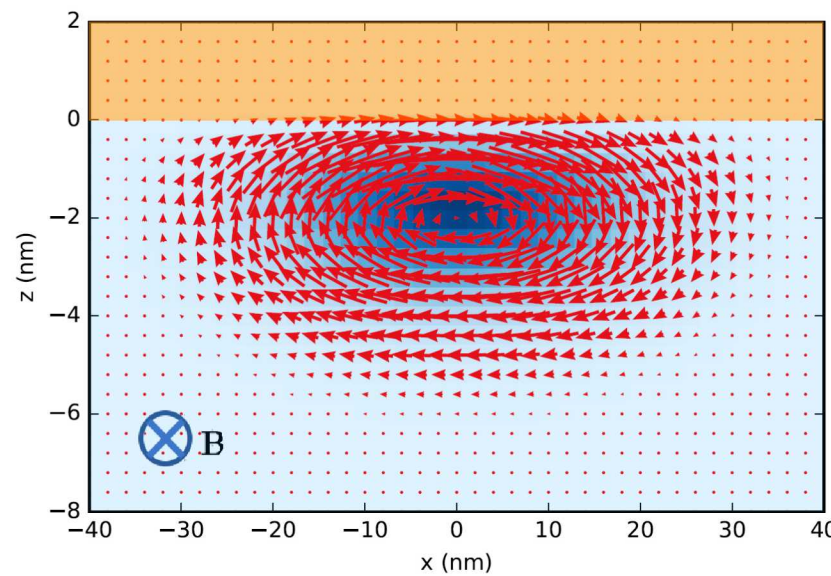
- Dynamical decoupling (for systematic or slow errors)
- Design to just be robust to these effects

References:

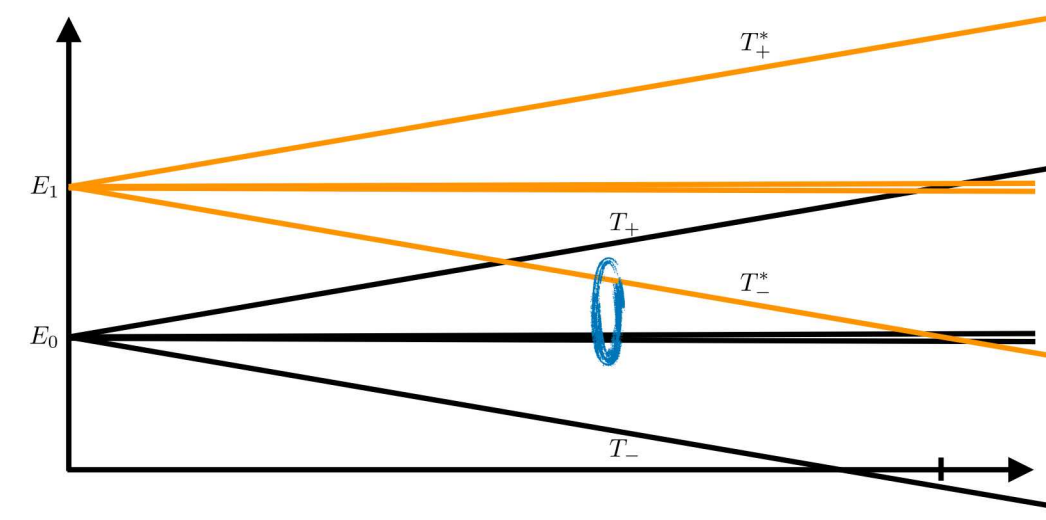
- 1: [Ferdous, et al. npj Quant. Info. 4, 26 (2018)]
- 2: [Ferdous, et al. PRB 97, 241401 (2018)]
- 3: [Jock, et al. 9, 1768 Nat. Comm. (2018)]
- 4: [Ruskov, et al. arXiv:1708.04555]
- 5: [Tantt, et al. arXiv:1807.10415]

Prospects

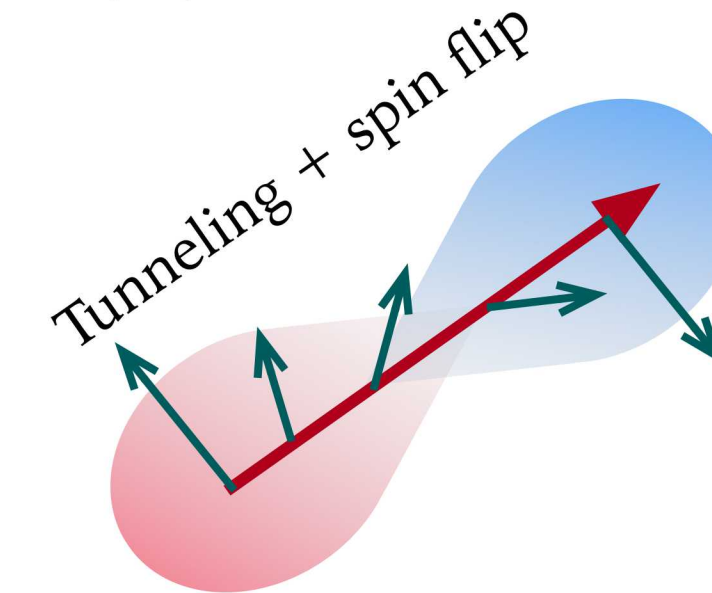
(1) Intra-valley



(2) Inter-valley



(3) Inter-dot



Upshot: We now have a reasonably solid understanding of SOC effects in Si quantum dots

Theory challenges:

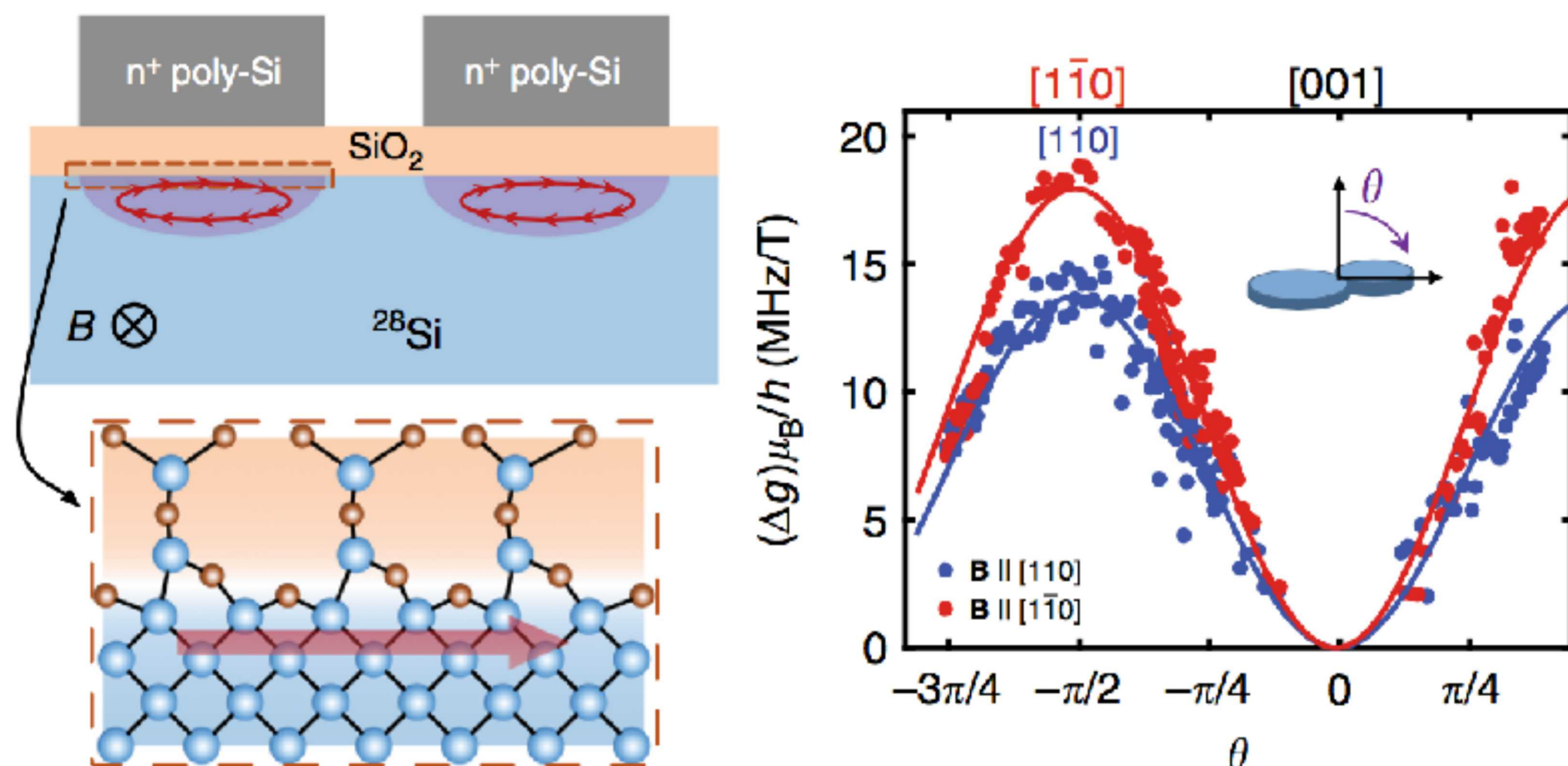
- Better microscopic and/or statistical models for interface SOC variation needed (for both MOS and SiGe)
- Intertwining of SOC and valley effects necessitate reliable quantitative models for valley splitting and valley phase

Architectural challenges:

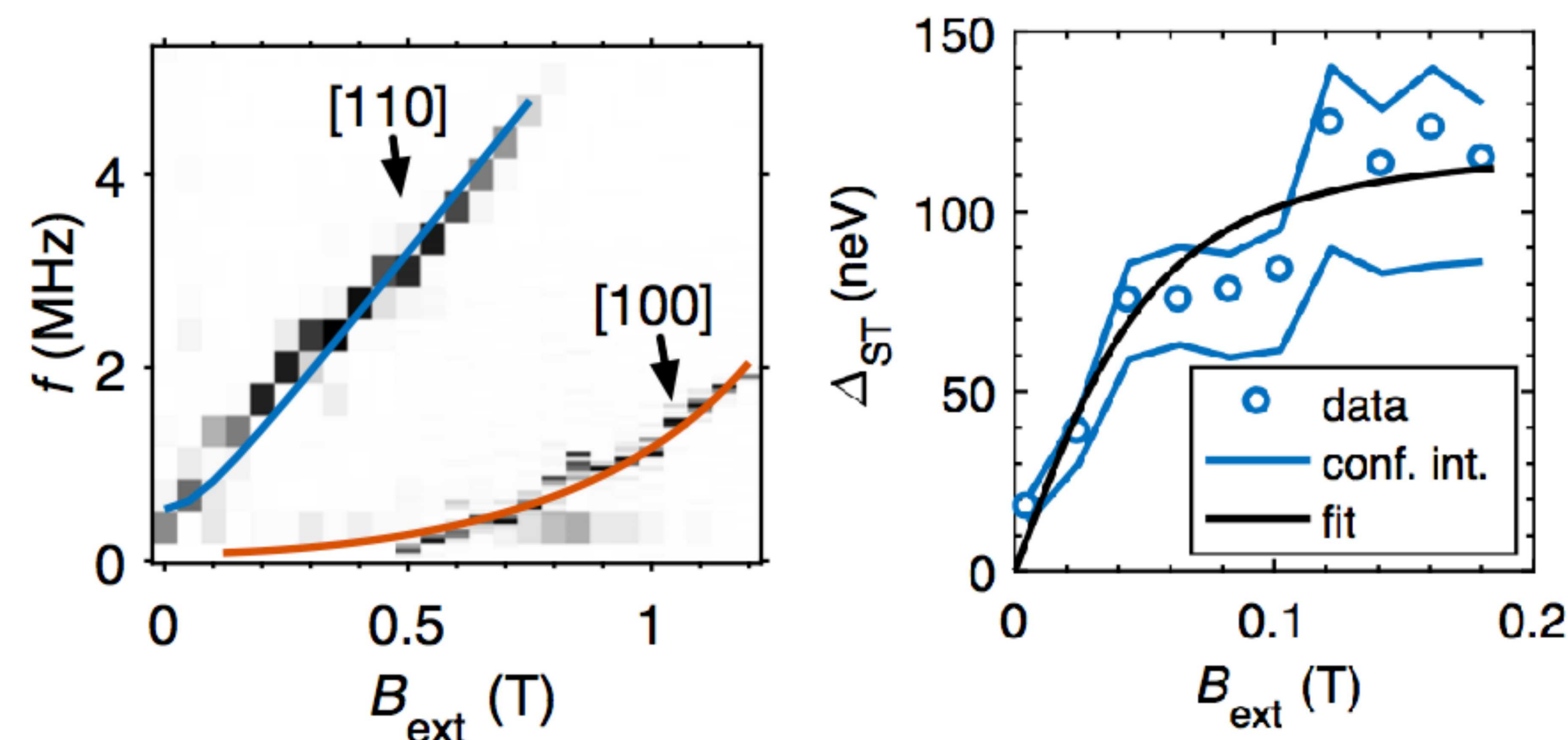
- We need to:
 1. Allow for intrinsic dot-to-dot variation (possibly complicating device tune-up) or
 2. Mitigate it (thoughtful B-field orientation, better understanding of fab dependencies)
- Effects on error channels must be incorporated

Acknowledgements

[Jock, et al. Nat. Comm. 9, 1768 (2018)]



[Harvey-Collard, et al. arXiv:1808.07378]



Collaborators at Sandia:

Experiment:

- Chloé Bureau-Oxton
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- Andy Mounce
- Martin Rudolph
- Dan Ward
- Group lead: Malcolm Carroll

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- Kenny Rudinger
- Vanita Srinivasa
- Wayne Witzel

* now a postdoc in Vandersypen group at QuTech

Listen to talks by:

Chloé Bureau-Oxton

Andy Mounce

Visit posters by:

Ryan Jock

Andrew Baczewski