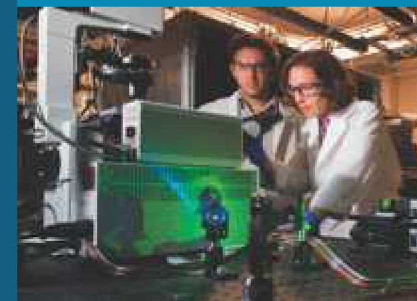
SAND2019-0352C

Input signal synthesis for open-loop multiple-input/multiple-output testing



PRESENTED BY

Ryan Schultz & Garrett Nelson

30 January 2019

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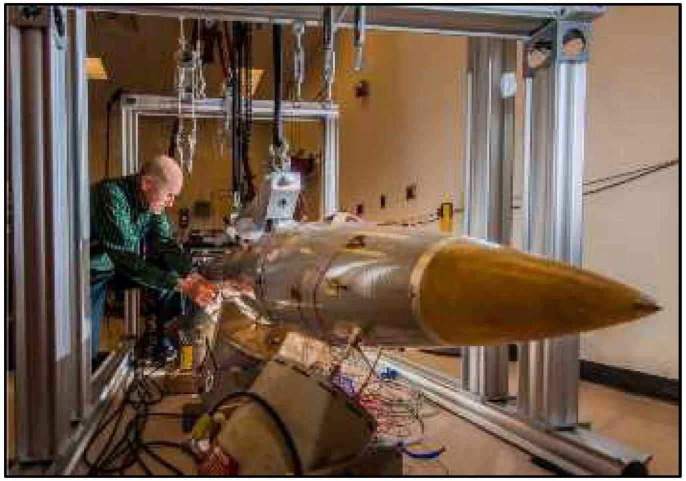
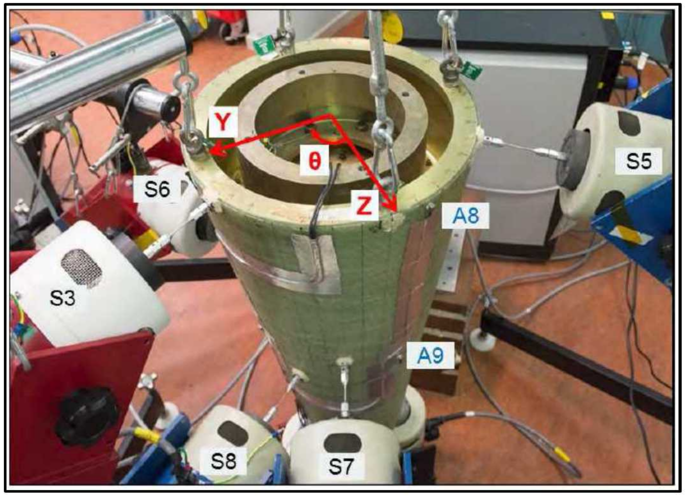
Modifying Control Algorithms Can Improve MIMO, Multi-Shaker Testing

Multi-Shaker Vibration Tests
Multiple, small shakers & MIMO control



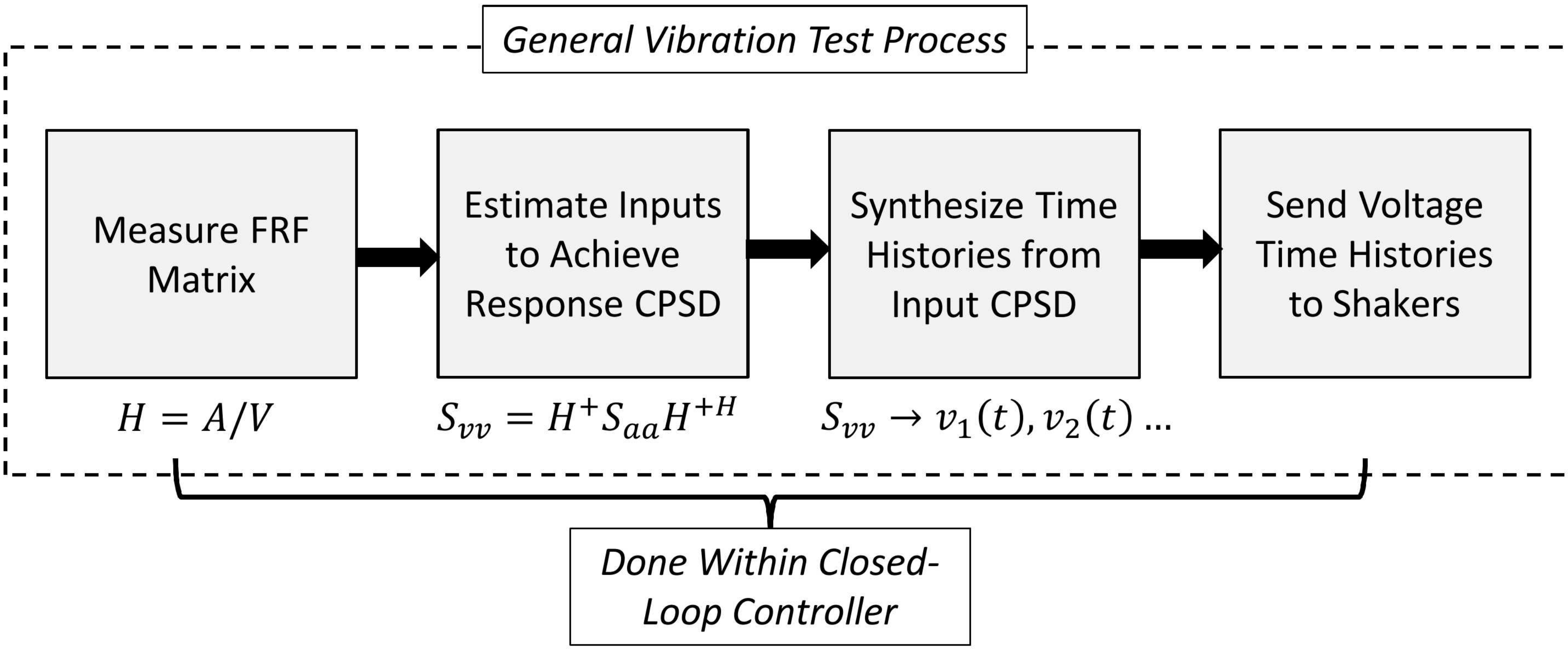
Mayes & Rohe, 2016



Daborn, 2017

- *They observed that how you control, estimate inputs matters quite a bit to accuracy & input levels*

To Implement New Control Techniques in the Lab, Need to Generate Voltage Time Signals



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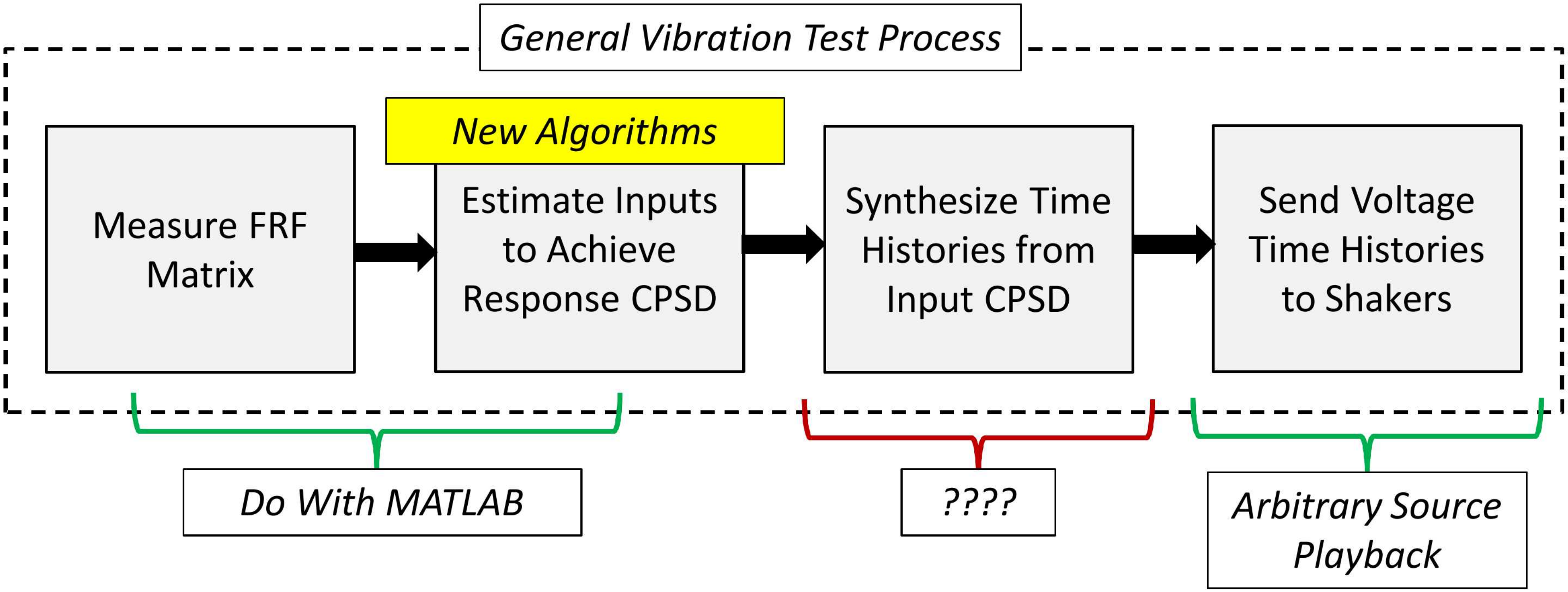

Note: These techniques are not new – Smallwood presented them in the late 1970's
This is an exploration in what affects the resulting signals

Synthesizing Time Histories for Single & Multiply-Correlated Inputs

1. Synthesis of Single Time Signal from APSD
   - Random processes & realizations
   - Tone and broadband signals

2. Generating Smooth, Long-Duration Signals
   - Constant-Overlap and Add
   - Effect of windowing on variance
   - Window corrections

3. Synthesis of Multiply-Correlated Signals
   - Matrix decomposition techniques
   - Comparison of decomposition & random process techniques

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Synthesis of Single Time History from an APSD

- Premise:
  - Have some desired signal auto-power spectral density (APSD)
  - Want to generate a time history which has that desired APSD
  - Can do in the time domain, but more efficient in frequency domain

$$G_{vv} \rightarrow v(t)$$

- What's the challenge?
  - APSDs do not have phase, so cannot simply use IDFT

$$v(t) = \mathcal{F}^{-1}(X_v)$$
$$X_v = ?$$

- Approach:
  1. Convert the APSD to linear spectrum magnitude
  2. Pick a random phase to generate a realization of a complex linear spectrum
  3. Take the IDFT to generate the time history

1. Linear Spectrum Magnitude	2. Random Phase	3. Inverse Fourier Transform
$\lvert X_v \rvert = \sqrt{G_{vv}/df}$	$\psi = U(0,2\pi)$ $X_v = \lvert X_v \rvert e^{j\psi}$	$v(t) = \mathcal{F}^{-1}(X_v)$

Synthesis of Single Time History from an APSD

- 2 Methods for Generating Random Phase for the Linear Spectrum:
  - Method 1: Generate 2 Gaussian random variables, one for the real part, A, & one for the imaginary part, B
  - Method 2: Generate 1 Uniformly distributed random variable, ψ, for the phase
- Do at each frequency line to generate the broadband linear spectrum
  - New random variables for each frequency line

Method 1:

$$X_k = \alpha_k \frac{1}{\sqrt{2}} (A_k + jB_k)$$

Method 2:

$$X_k = \alpha_k e^{j\psi_k}$$

Comparison of Random Process Methods

- Generate multiple pure-tone signals (realizations) with Method 1 and Method 2

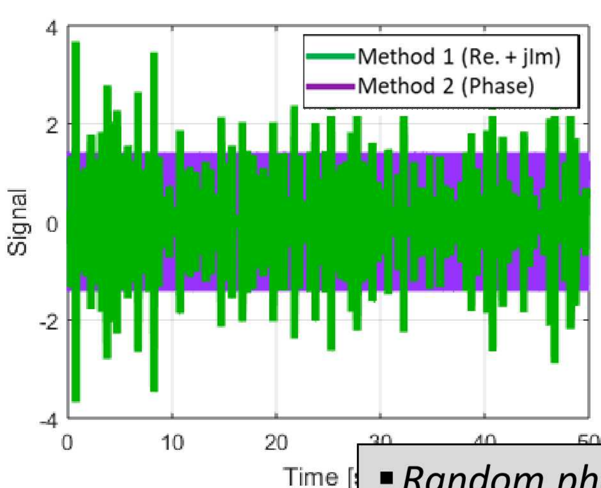
Method 1:

$$X_k = \alpha_k \frac{1}{\sqrt{2}} (A_k + jB_k)$$

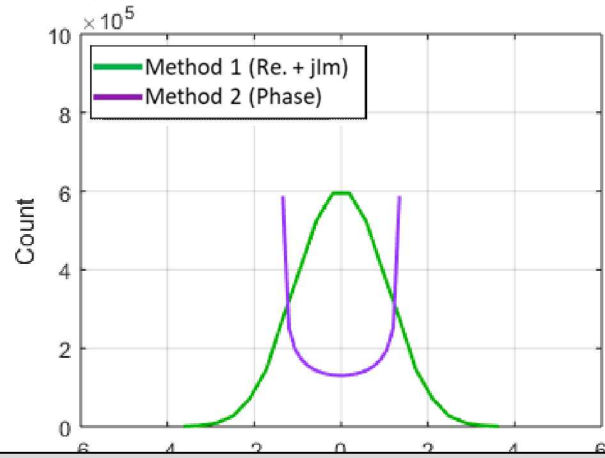
Method 2:

$$X_k = \alpha_k e^{j\psi_k}$$

Time Histories (100 Realizations)



Histogram of Signal Values



Variance & Peak Values

Method	Variance	Peak
Method 1	1.100	3.67
Method 2	1.000	1.41

- *Random phase gives constant amplitude*
- *Random real & imaginary gives variations in the amplitudes, normal distribution of signal values*
- *With sufficient averages, both provide desired variance*

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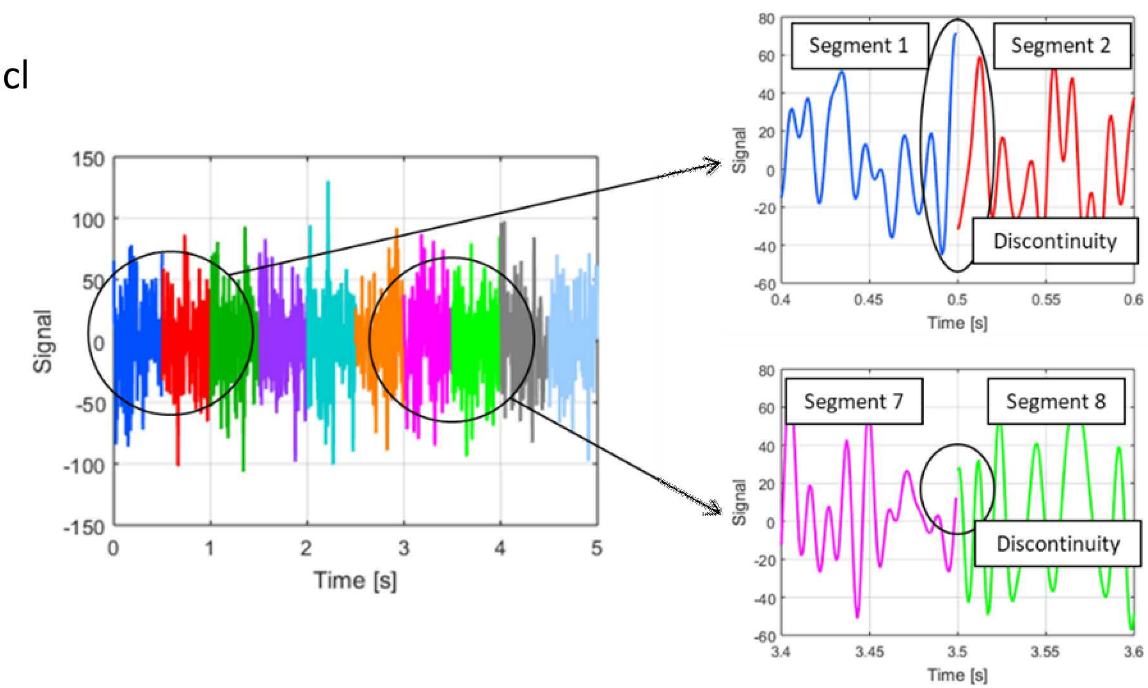
Generating Smooth, Long-Duration Signals

- Premise:
  - Generally, sampling parameters result in a time history that is too short to be used for an entire vibration test
  - Need to generate multiple signals & put them together

$$T_p = \frac{1}{df}$$

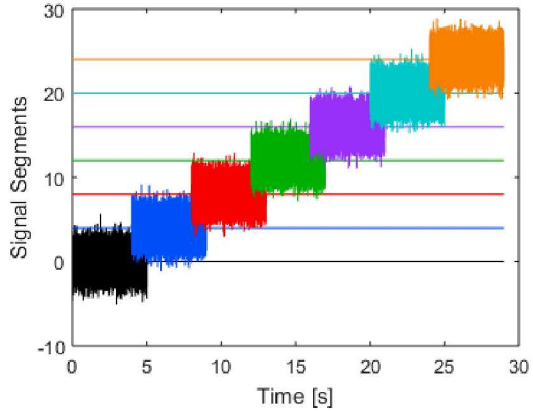
- What's the challenge?
  - Each signal does not start & end at the same point – get a jump discontinuity
- Approach:
  - Apply a window to smooth eacl
  - Overlap & Add



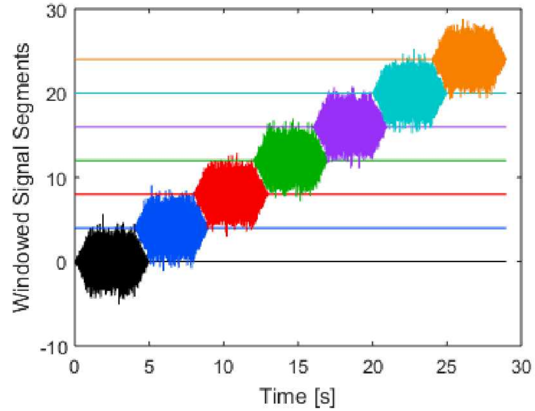
Generating Smooth, Long-Duration Signals

- Constant-Overlap & Add (COLA)

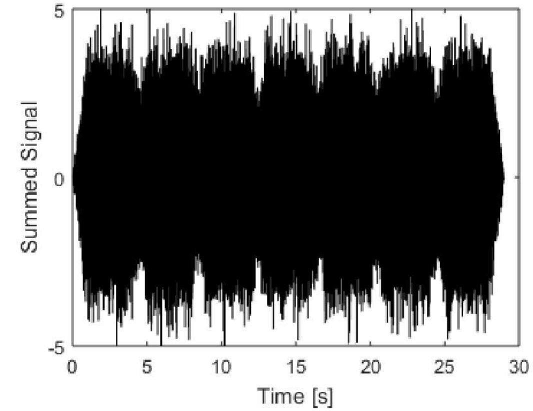
1. Generate Multiple Signals (Realizations) & Overlap in Time



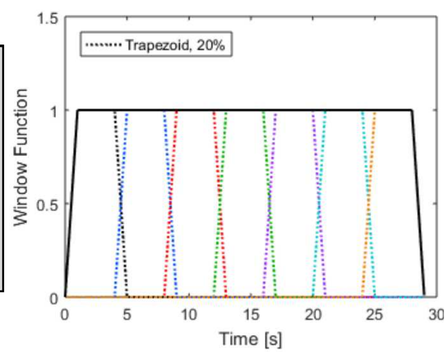
2. Window Each Signal to Taper the Ends to Zero

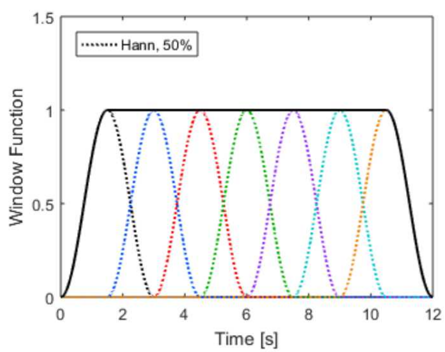


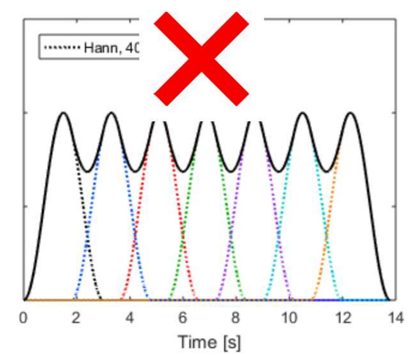
3. Add All Signals Together to Form a Long, Single Signal



Note: Amount of Overlap Depends on the Window Type (Hann, Trap, Tukey, etc.) Objective is to Achieve Sum of Windows Equal to One

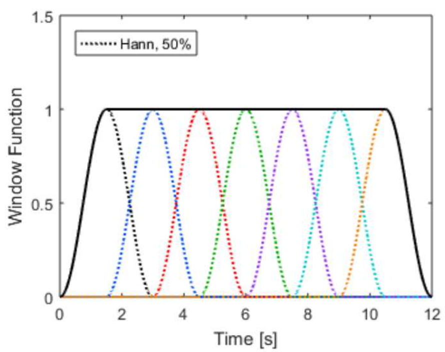




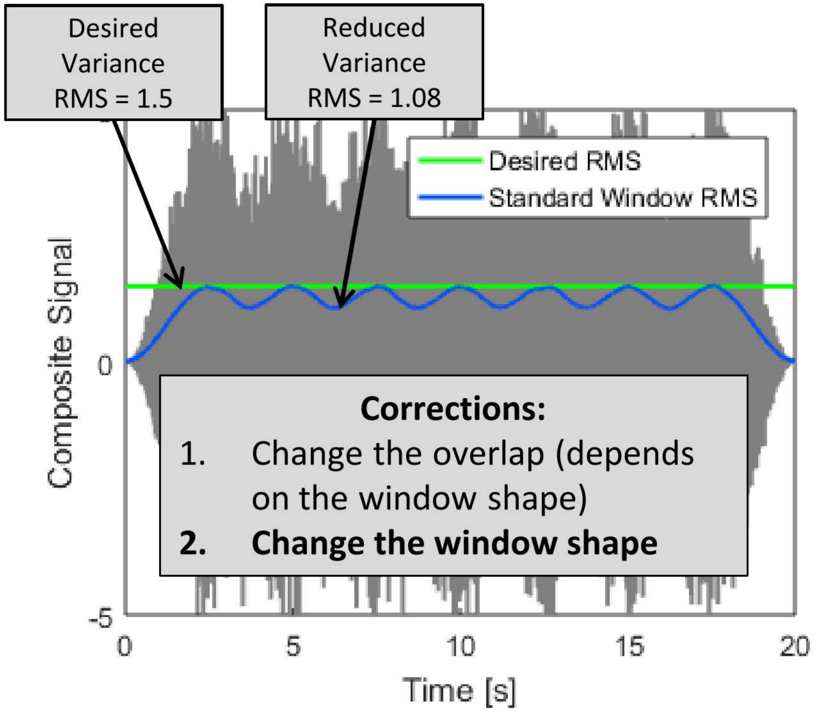


Generating Smooth, Long-Duration Signals

- Typical overlap is not sufficient to achieve constant variance with random signals
- The summed window amplitude is 1, but the composite random signal has obvious variance deviations in the overlap region

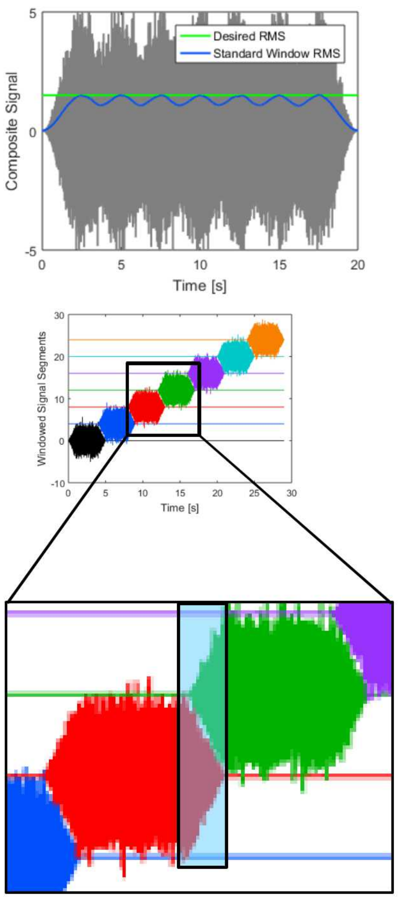


Window: Hann with 50% Overlap



Generating Smooth, Long-Duration Signals

Determine how to change the window shape by looking at the variance of the composite signal



- Total signal = sum of two windowed signals:

$$x_{total} = wx_1 + wx_2$$

- Variance of the total signal:

$$\sigma^2 = \frac{1}{N}\sum x_{total}^2 = \frac{1}{N}\sum (wx_1 + wx_2)^2 = \frac{1}{N}\sum (w^2x_1^2 + w^2x_2^2 + 2w^2x_1x_2)$$

- Variance of each individual signal is σ_0^2:

$$\sigma^2 = w^2\sigma_0^2 + w^2\sigma_0^2 + 2w^2\frac{1}{N}\sum x_1x_2 = w^2\sigma_0^2 + w^2\sigma_0^2$$

- See that the sum of the window functions squared must equal one:

$$\sigma^2 = w^2\sigma_0^2 + w^2\sigma_0^2 = \sigma_0^2(w^2 + w^2)$$

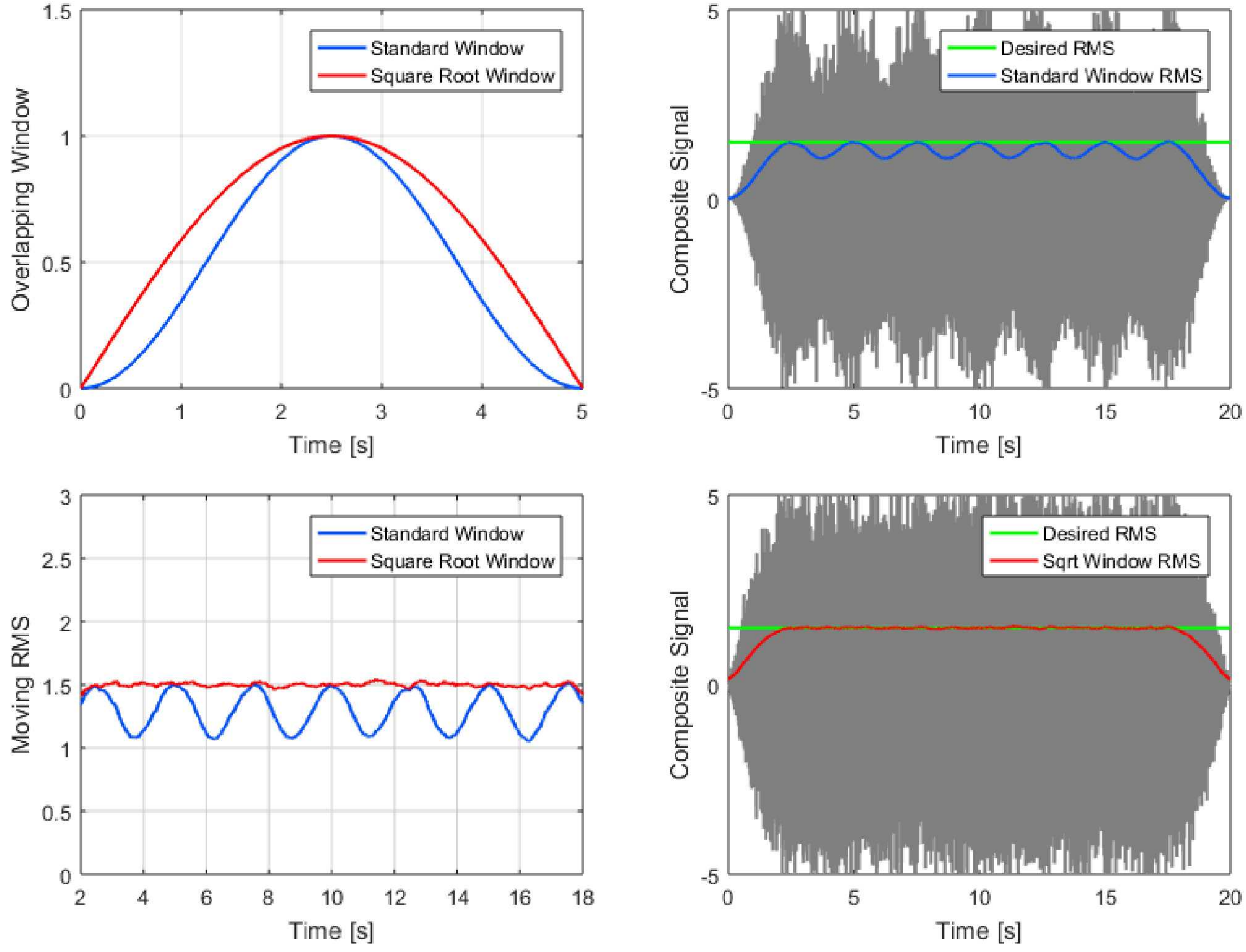
$(w^2 + w^2) = 1$ instead of $w + w = 1$

- Thus, the window functions should be the square root of the native window:

$$\hat{w} = \sqrt{w}$$
$$\hat{w}^2 + \hat{w}^2 = w + w = 1$$
$$x_{total} = \hat{w}x_1 + \hat{w}x_2$$

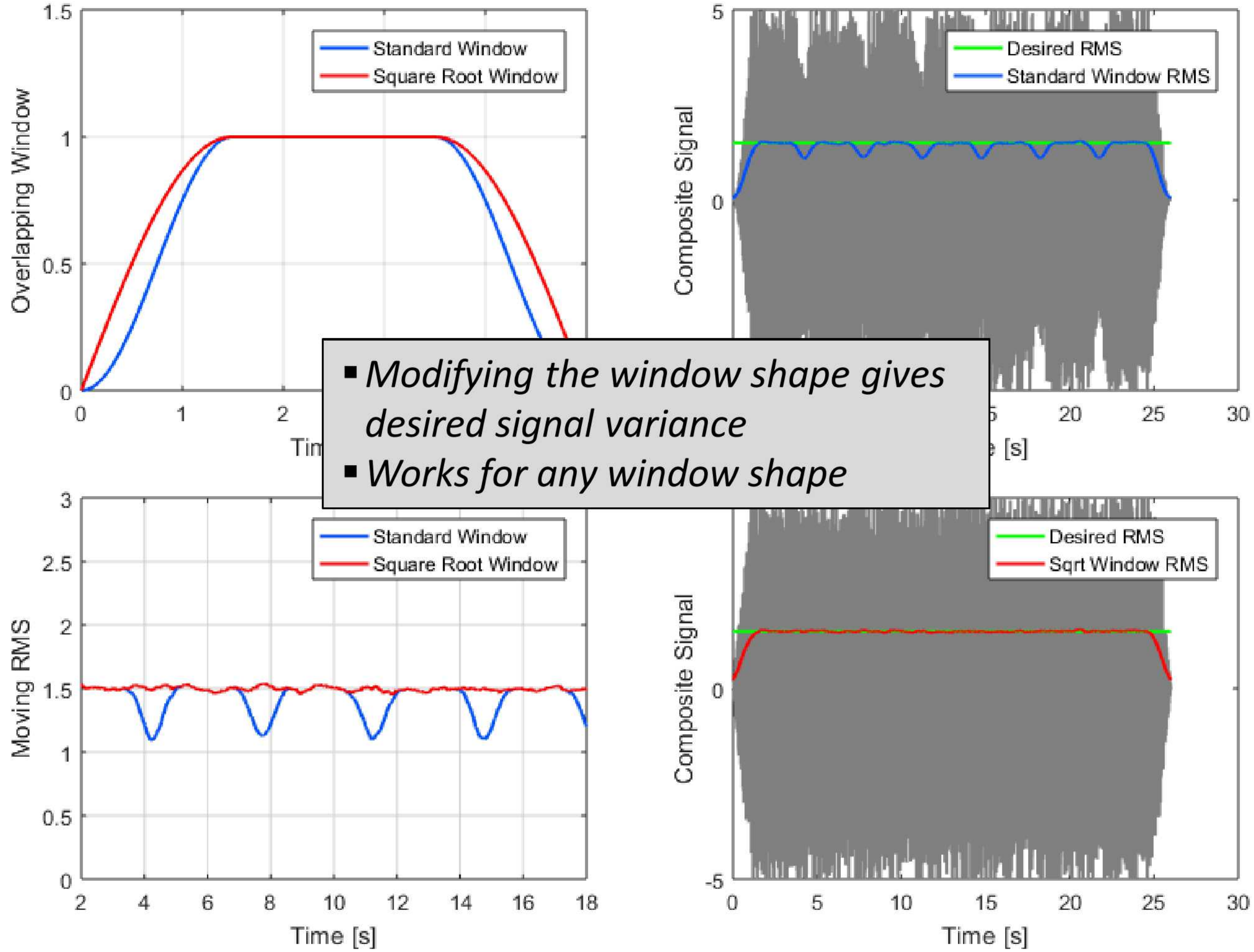
Generating Smooth, Long-Duration Signals

Example: Hann Window



Generating Smooth, Long-Duration Signals

Example: Tukey Window



- *Modifying the window shape gives desired signal variance*
- *Works for any window shape*

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Synthesis of Multiply-Correlated Signals

- Premise:
  - In MIMO problems, the inputs are often correlated
  - Synthesized time signals must reflect this desired correlation, along with the amplitude
- What's the challenge?
  - How do you enforce correlation on multiple time signals?

$$\boldsymbol{S_{aa} = H S_{vv} H^{H}}$$
$$\boldsymbol{S_{vv} = H^{+} S_{aa} H^{+H}}$$

$$\boldsymbol{S_{vv} = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix}}$$

Correlated Inputs

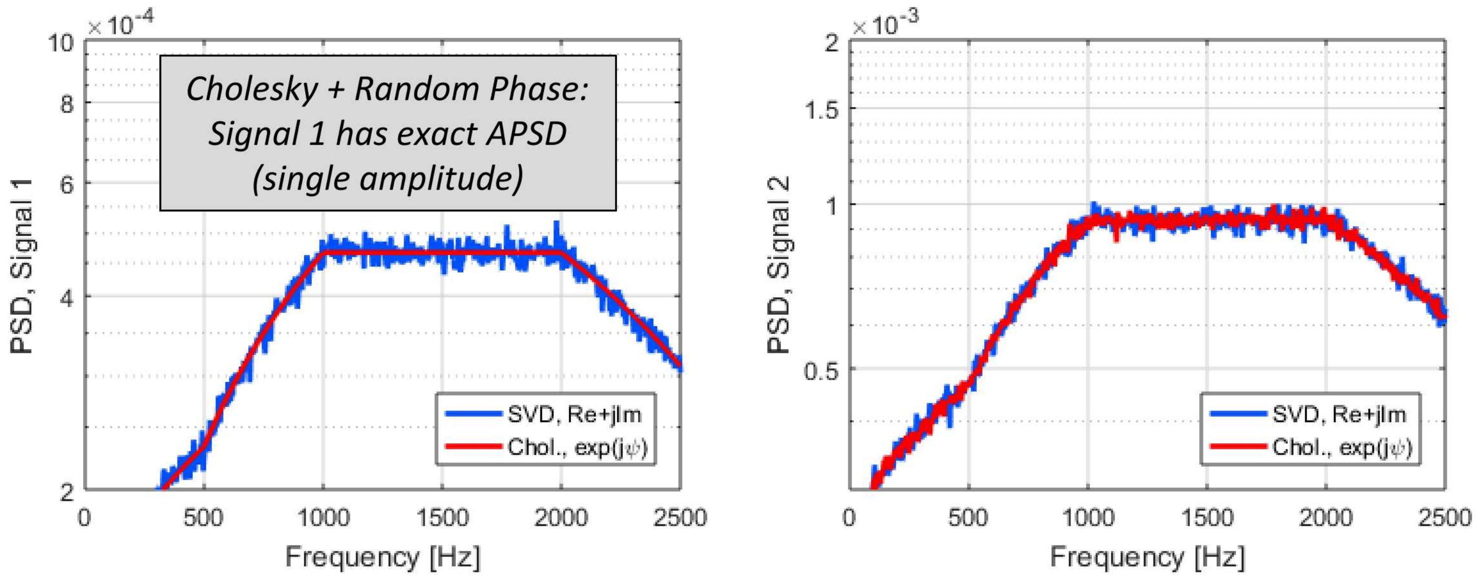
Synthesis of Multiply-Correlated Signals

$[S_{vv}]$	→	$[L]$, $[S_{vv}] = LL^H$	→	$\{\Psi\} = \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{Bmatrix}$	→	$\{X_v\} = [L]\{\Psi\}$	→	$v_i(t) = \mathcal{F}^{-1}(X_{v,i})$
CPSD Matrix		Decompose to Linear		Generate Vector Random Process		Multiply To Get Linear Spectra		IDFT To Time Histories

- Start with desired voltage CPSD: $S_{vv} = H^+ S_{aa} H^{+H}$
- Matrix Decomposition:
  - Convert CPSD from power domain to linear domain (matrix square root)
  - Cholesky Decomposition: $[S_{vv}] = [L][L^H]$ (lower triangular matrix)
  - Singular Value Decomposition: $[S_{vv}] = [U][S][V]^H \rightarrow [L] = [U][S]^{\frac{1}{2}}[V]^H$
- Vector Random Process:
  - Vector of random variables (random phase or random real & imaginary parts), one for each signal
- Multiplying the $[L]$ matrix by the vector random process generates a realization of the multiply-correlated linear spectra, $\{X_v\}$

Synthesis of Multiply-Correlated Signals

- Matrix Decomposition method & random process method both affect the resulting signals
- Example Problem:
  - 4 signals, coherence of 0.25, phase of $\pi/4$ for all frequency lines. 100 averages



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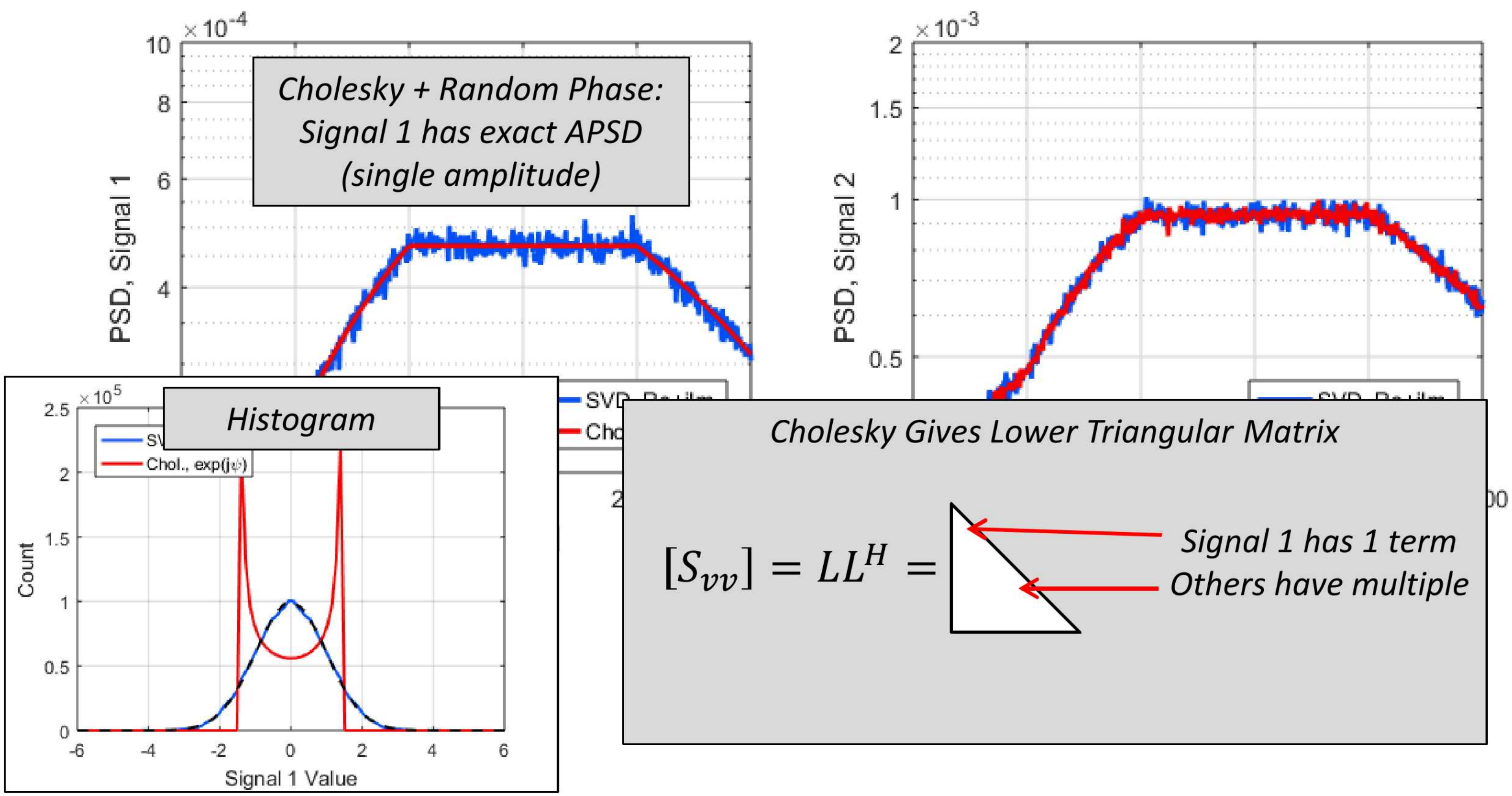

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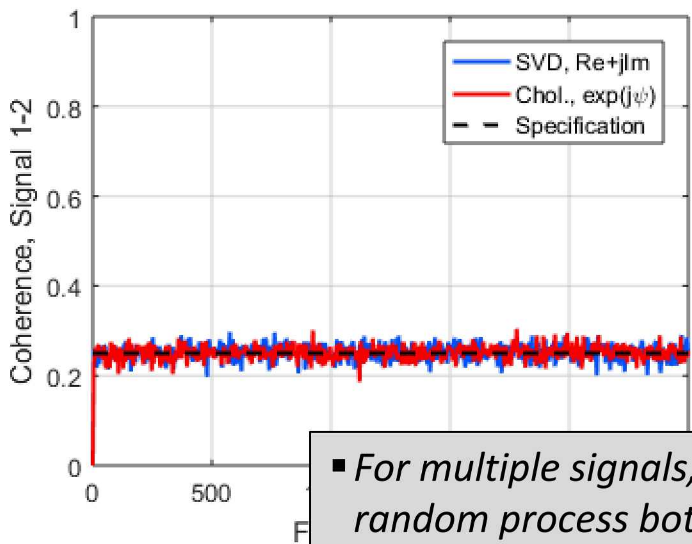
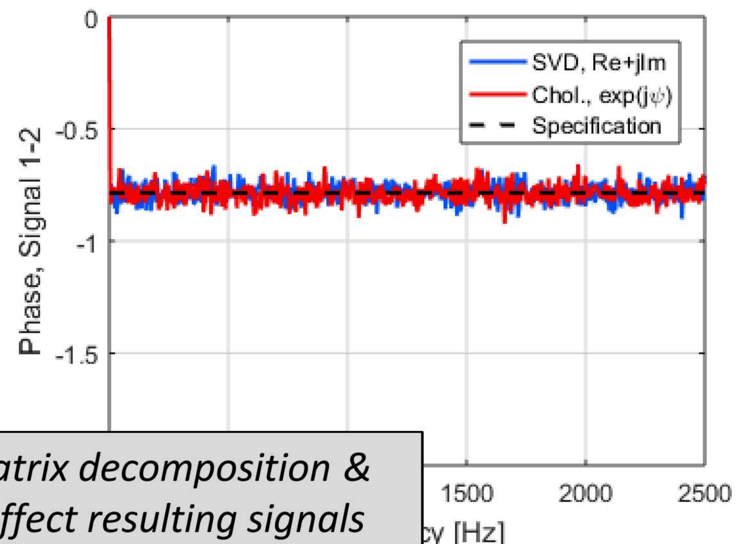




- *For multiple signals, matrix decomposition & random process both affect resulting signals*
- *On average, signals match the desired APSD, coherence and phase*

Conclusions: Generating multiply-correlated signals for MIMO testing

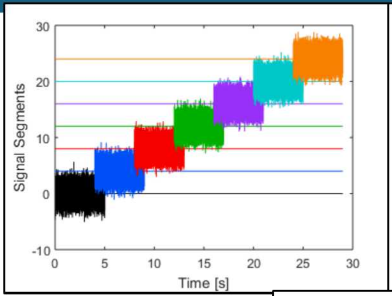
Goal: Enable the Use of New MIMO Control Algorithms for Multi-Shaker Vibration Testing

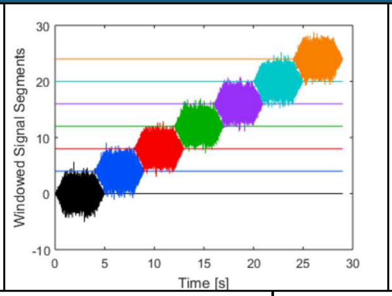
1. Synthesis of Single Time Signal from APSD
   - Not deterministic – use random sampling
   - One average, signals represent desired APSD
   - Random process matters – real & imaginary vs. phase

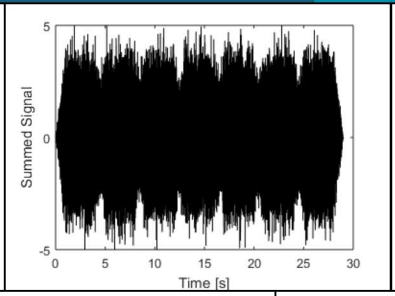
2. Generating Smooth, Long-Duration Signals
   - Used to concatenate multiple, short signals into longer signal for a test
   - COLA smooths the transitions
   - Changing to a square-root window function preserves variance

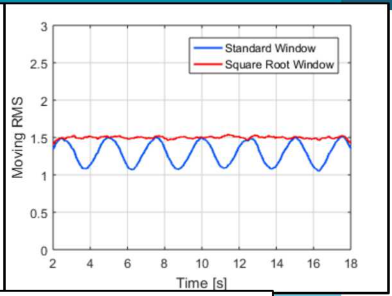
3. Synthesis of Multiply-Correlated Signals
   - Procedure is similar to single signals
   - Convert from power to linear space with a matrix decomposition (Cholesky or SVD)
   - Random process is now a vector with Nsignals terms
   - Type of decomposition and random process affects the resultant signals

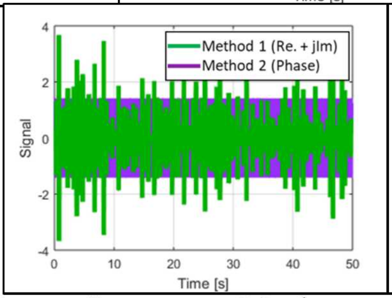
Input signal synthesis for open-loop multiple-input/multiple-output testing

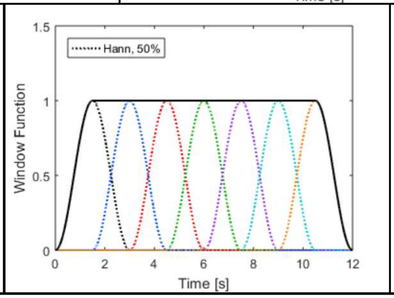


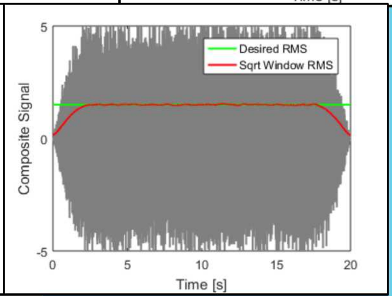












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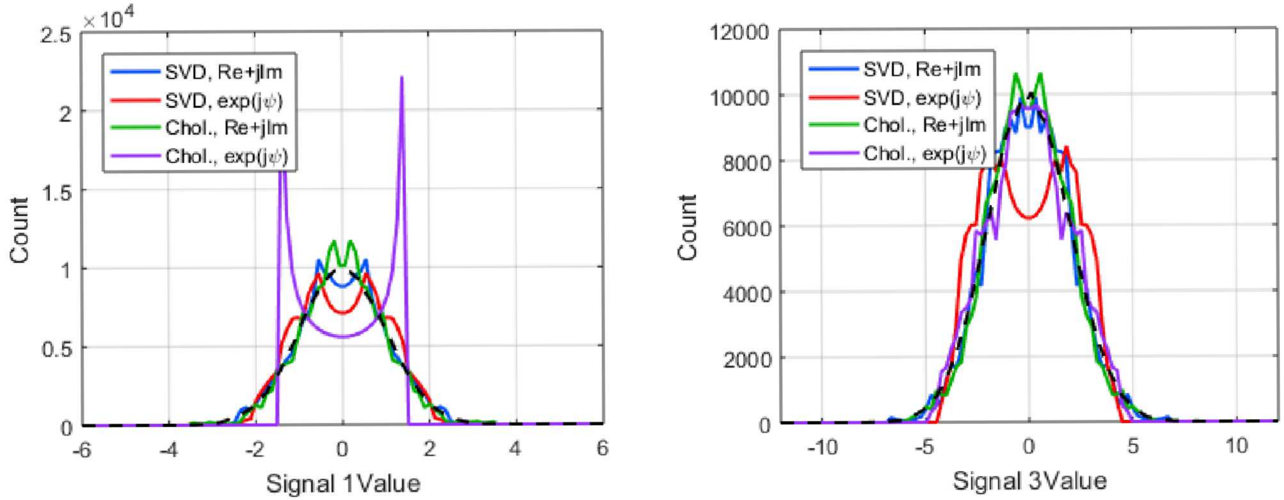
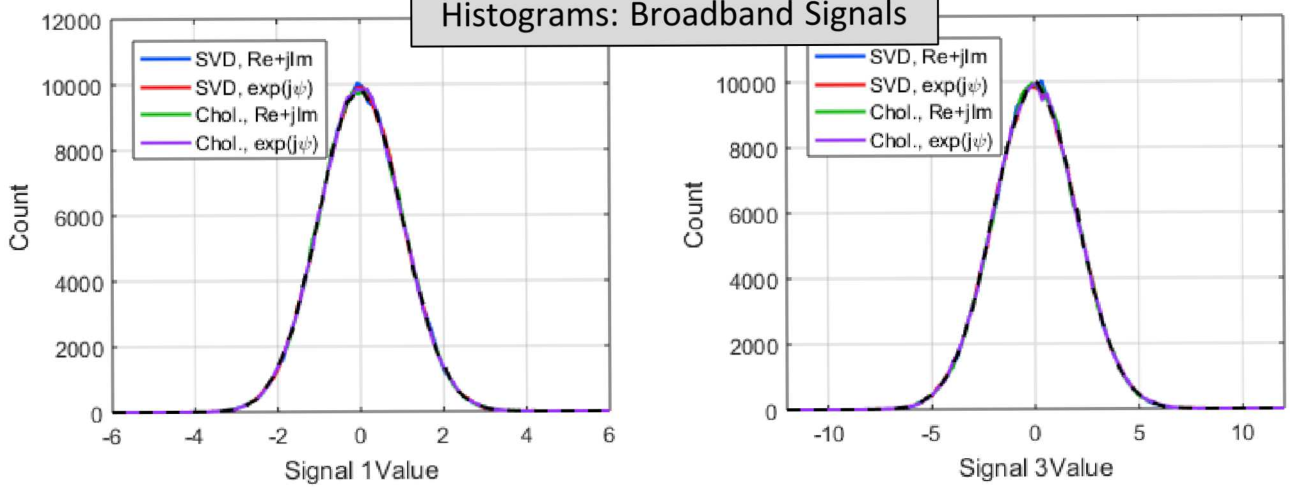
30 January 2019

Backups: Synthesis of Multiply-Correlated Signals

Histograms: Tone Signals



Histograms: Broadband Signals



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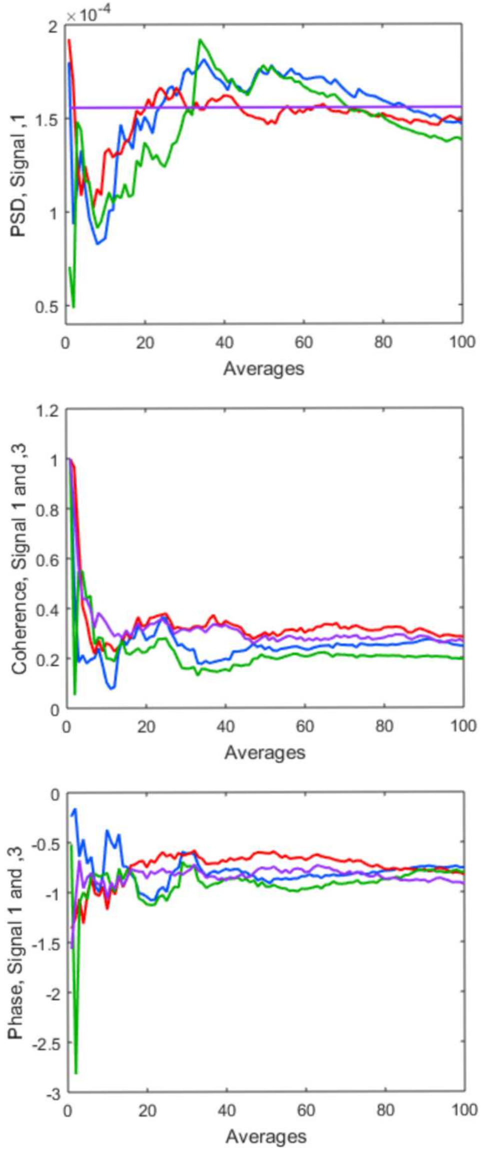

Broadband Signals