SAND2019-0351C

A demonstration of force estimation and regularization methods for multi-shaker testing

PRESENTED BY

Ryan Schultz

30 January 2019

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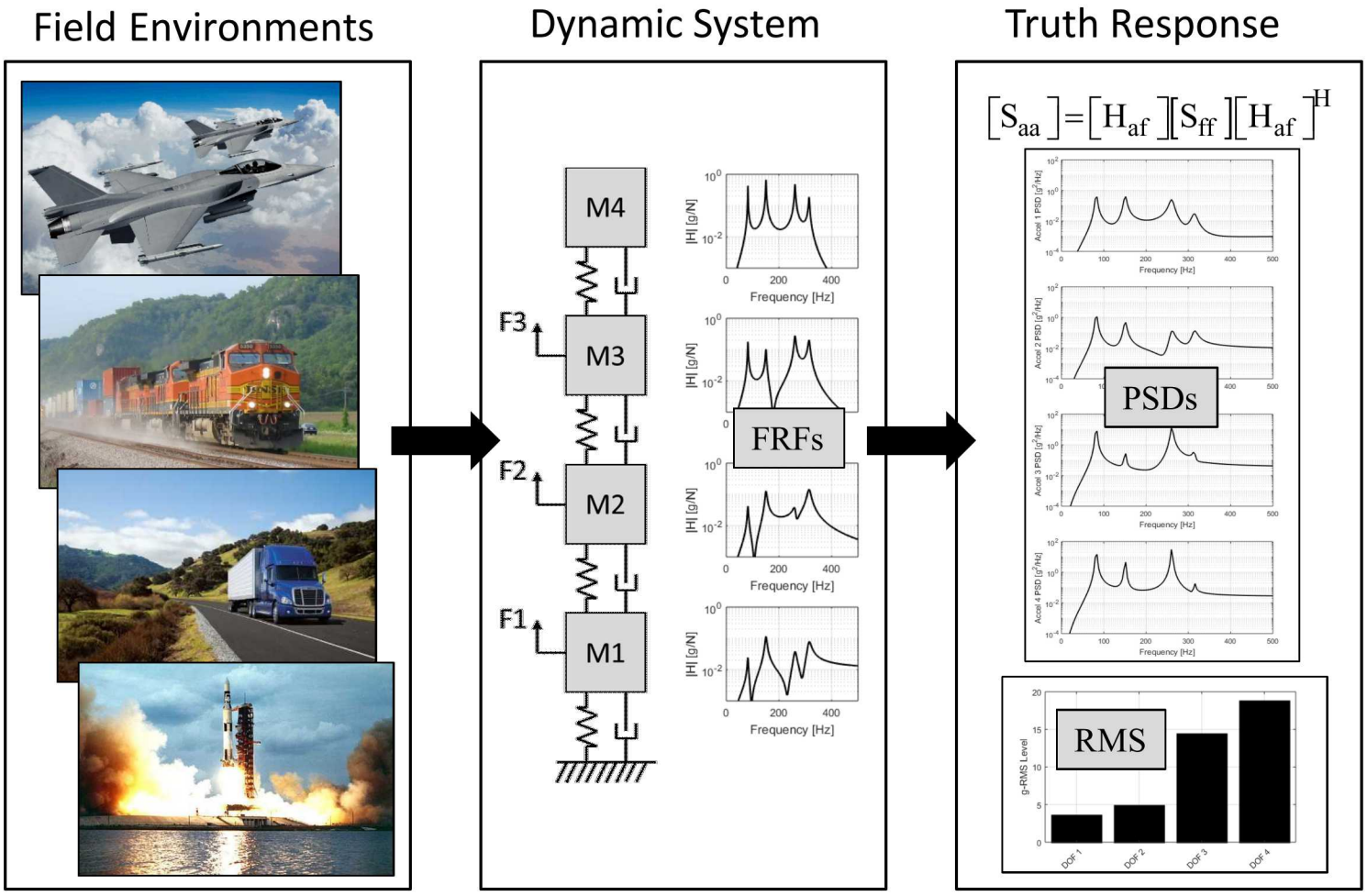
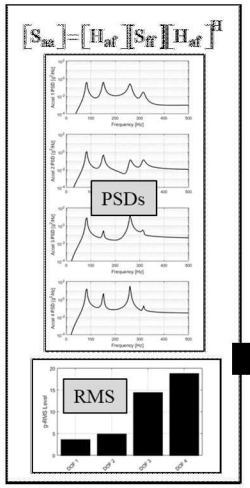
Multiple Input Vibration Testing – Just an Inverse Problem

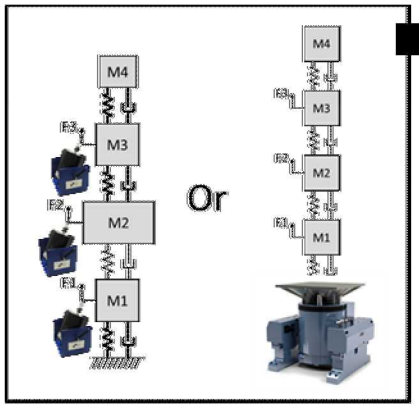


Multiple Input Vibration Testing – Just an Inverse Problem

Truth Response



Lab Test



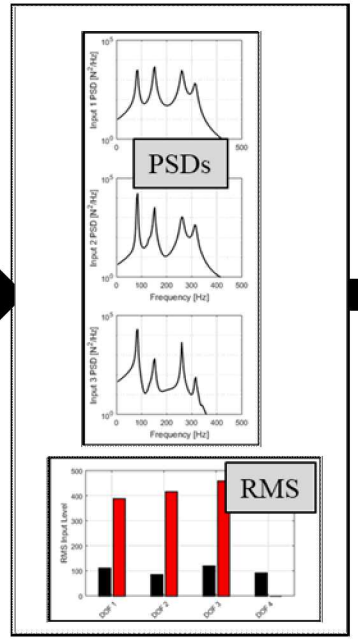
Control Problem: Estimate Inputs to Achieve Response

$$[S_{aa}] = [H_{af}][S_{ff}][H_{af}]^H$$

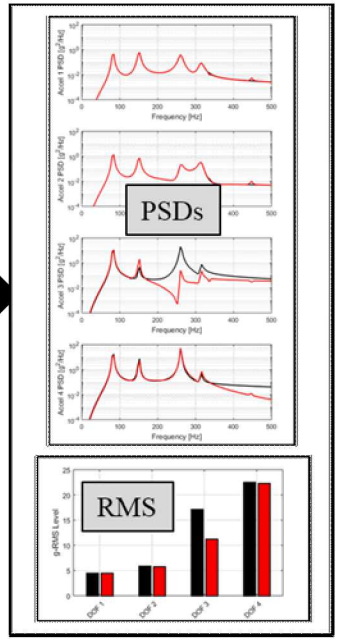
$$[S_{ff}] = [H_{af}^+][S_{aa}][H_{af}^+]^H$$

$$[H_{af}^+]$$

System FRF ID
Invert FRF



Estimated Inputs



Lab Response

Multiple Input Vibration Testing – Just an Inverse Problem

Truth Response

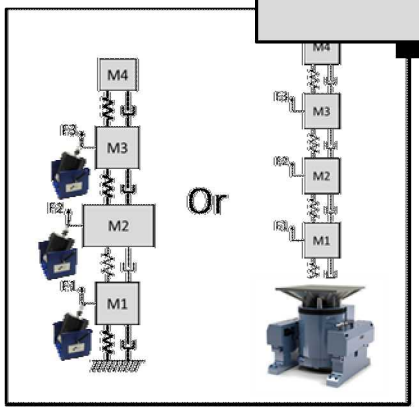
Control Problem: Estimate Inputs to Achieve Response

Truth Response to Field Environments: $\boldsymbol{Y_0 = H_0 F_0}$

Invert FRF to Estimate Lab Inputs: $\boldsymbol{F_1 = H_1^{+} Y_0}$

Run the Test: $\boldsymbol{Y_1 = H_1 F_1}$

Lab Tes



System FRF ID
Invert FRF

Estimated Inputs

Lab Response

Multiple Input Vibration Testing – Just an Inverse Problem

Truth Response

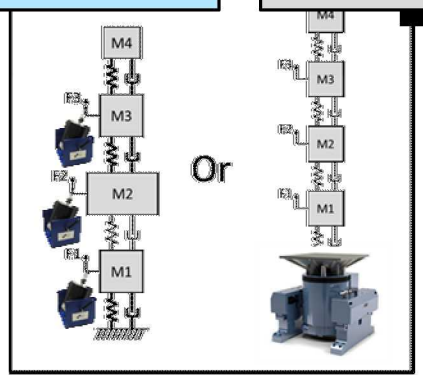
Control Problem: Estimate Inputs to Achieve Response

Multiple Ways to Estimate Inputs & Reduce Numerical Errors

Truth Response to Field Environments: $\boldsymbol{Y_0 = H_0 F_0}$

Invert FRF to Estimate Lab Inputs: $\boldsymbol{F_1 = H_1^{+} Y_0}$

Run the Test: $\boldsymbol{Y_1 = H_1 F_1}$



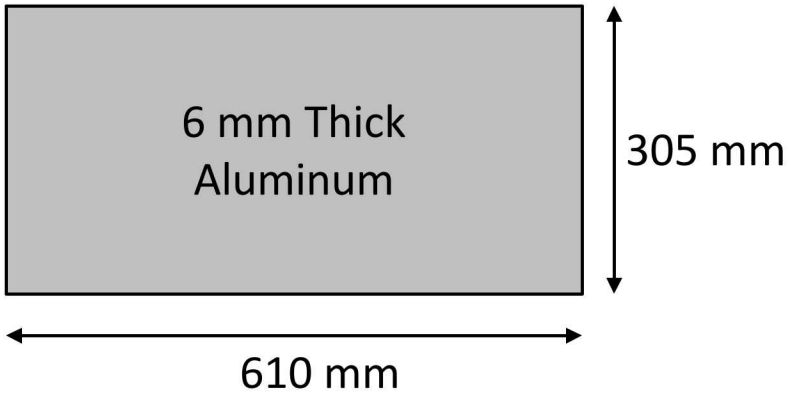
System FRF ID
Invert FRF

Estimated Inputs

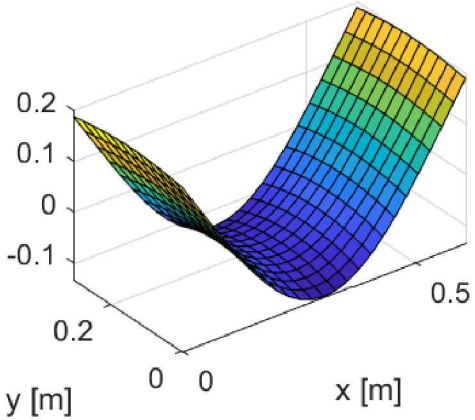
Lab Response

Techniques Demonstrated with a Model of a Dynamic System

Free-Free Aluminum Plate

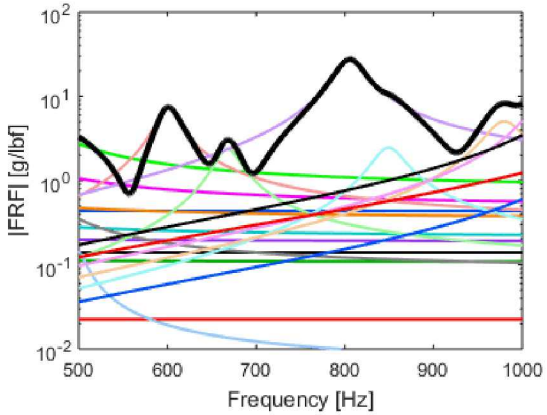


Modes from FE Moel



FRFs Synthesized from Modes

$$H_{af}(\omega) = U_{output}\overline{H}(\omega)U_{input}^{T}$$



Techniques Demonstrated with a Model of a Dynamic System

1. Condition Number of the FRF Matrix
   - What affects condition number?
   - How does condition number affect estimated inputs & responses?

2. Fixing Poorly Conditioned Systems – Regularization
   - Tikhonov & Singular Value methods
   - How can regularization can help or hurt?

3. Force Estimation Techniques
   - What methods are available?
   - How do they differ in terms of response and inputs?

Objective: Learn different MIMO techniques, apply them to a dynamic system & understand how the different techniques affect results

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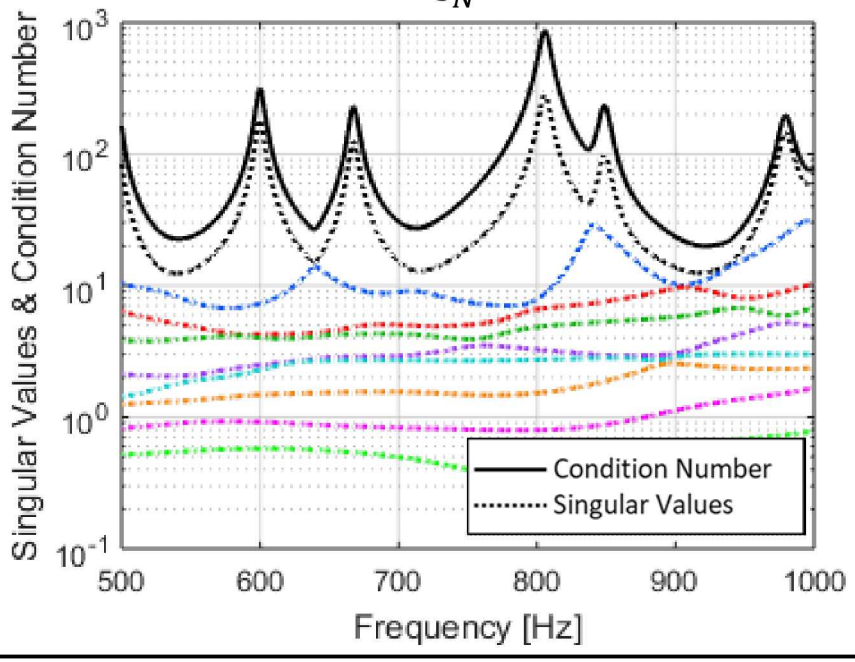
Condition Number of the FRF Matrix

- What affects condition number?
- How does condition number affect estimated inputs & responses?

Condition Number:
Ratio of Largest/Smallest Singular Value

$$\kappa = \frac{s_1}{s_N}$$



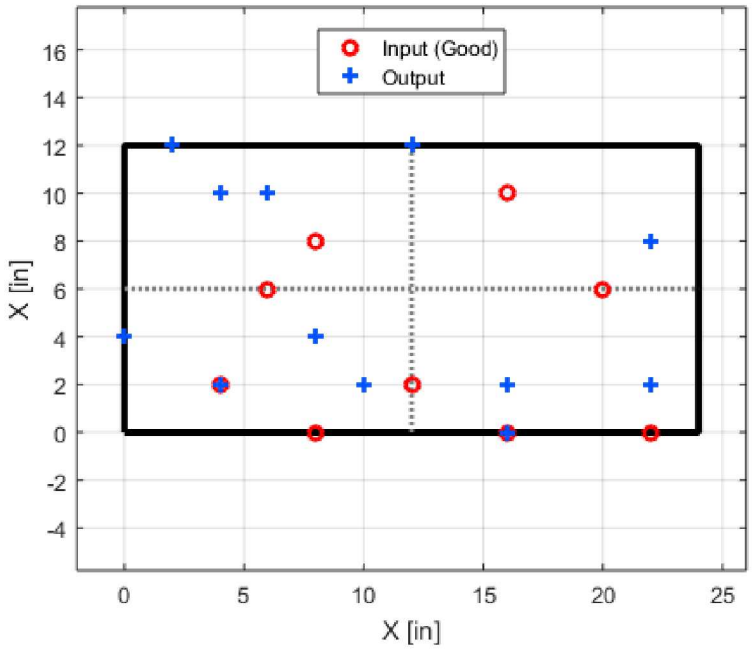
$$H = USV^H$$

$$S = \begin{bmatrix} s_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_N \end{bmatrix}$$

- Singular Values indicate the number of strong, unique contributors to the matrix
- If two inputs or outputs are very similar (redundant), the Singular Values will reflect this

Condition Number of the FRF Matrix

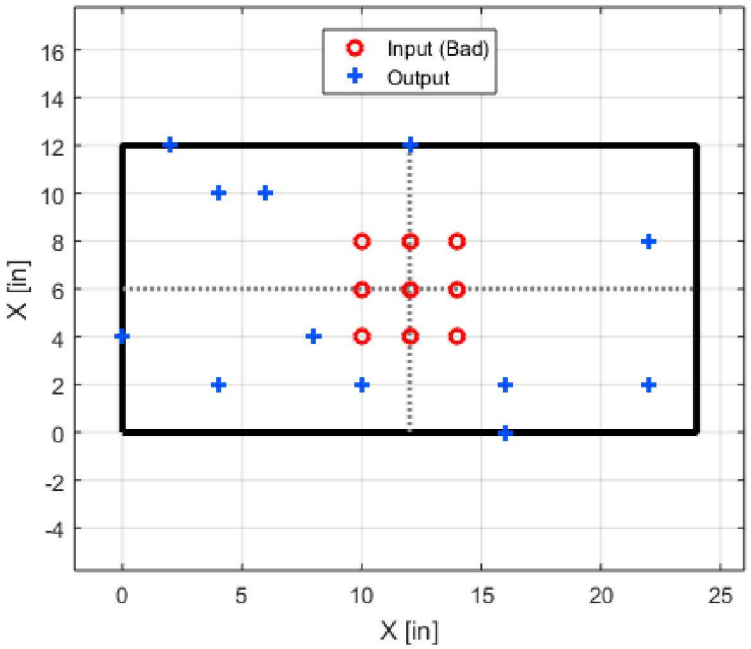
- What affects condition number?
  - Location of Inputs & Outputs (other cases in the paper)



Good

Highly Unique Inputs & Outputs

(Chosen with C.N.M.)

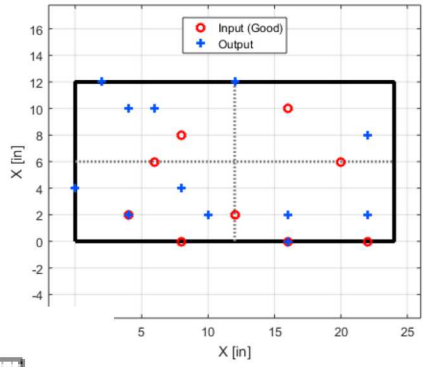


Bad

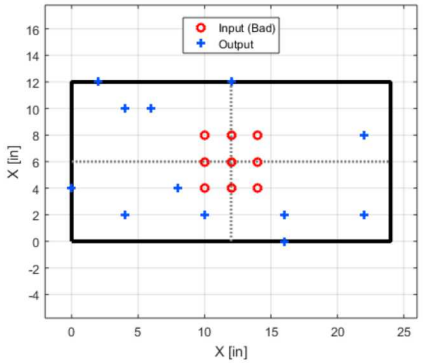
Many Similar Inputs

Condition Number of the FRF Matrix

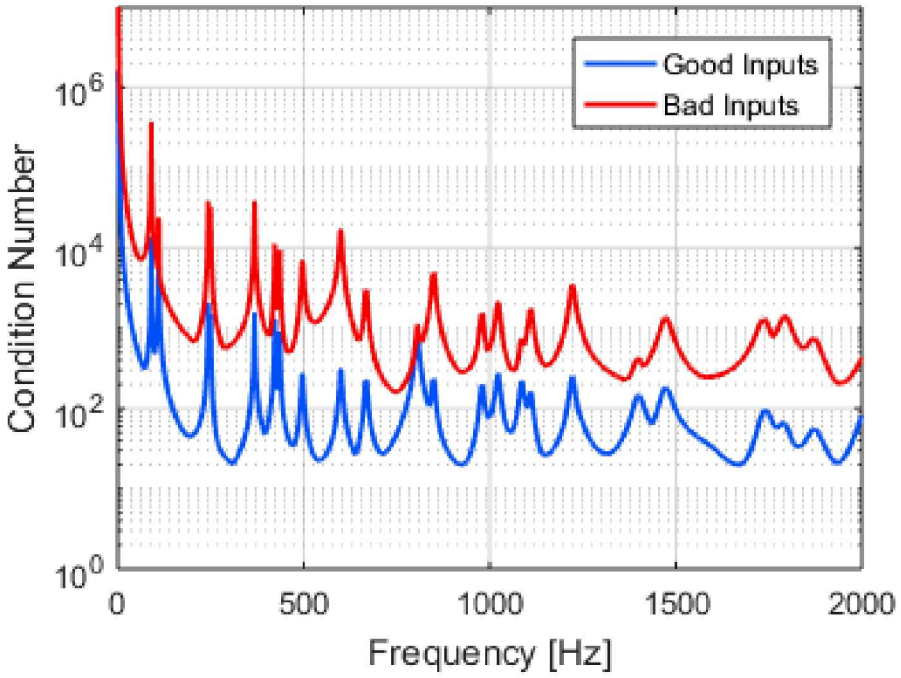
- What affects condition number?
  - Location of Inputs & Outputs



Good Inputs



Bad Inputs



- Location of Inputs has very strong effect on condition number
- Redundant information = small singular values
- Each input should excite modes in a strong, different way
- Generally, fewer inputs/outputs results in lower condition number, but not universally true

How does condition number affect estimated inputs & responses?

- Inputs are estimated as:

$$Ax = b \rightarrow x = A^{+}b$$

- For MIMO vibration:

$$S_{aa} = HS_{ff}H^{H} \rightarrow S_{ff} = H^{+}S_{aa}\,H^{+H}$$

- Errors in the estimates can come from noise or uncertainty on the target response:

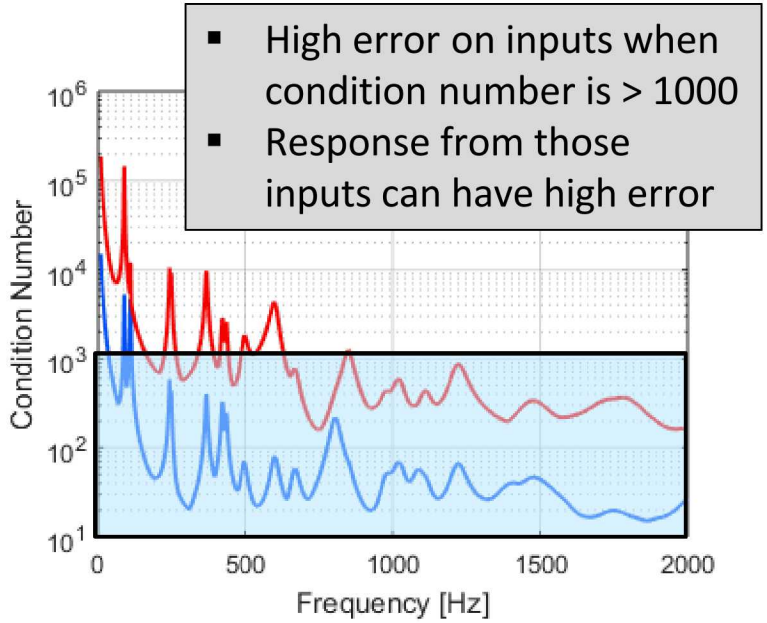
$$Ax = b + e$$

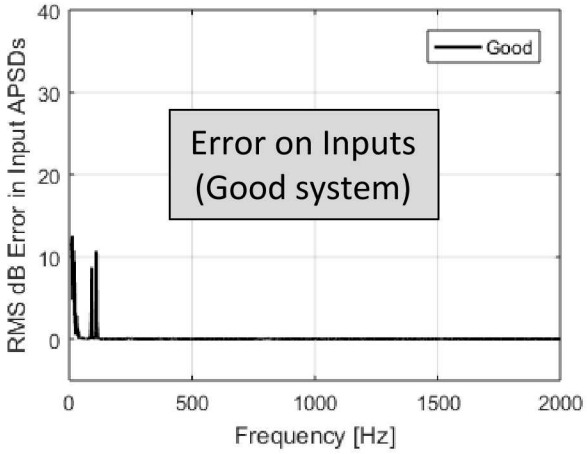
- This noise can be amplified as it is propagated through the inverse of the FRF matrix:

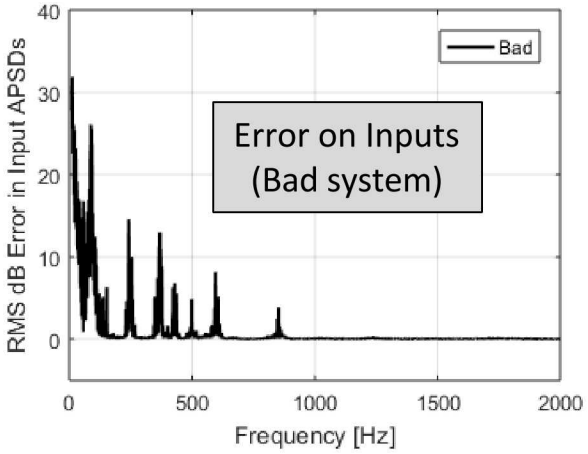
$$x = A^{+}(b + e)$$

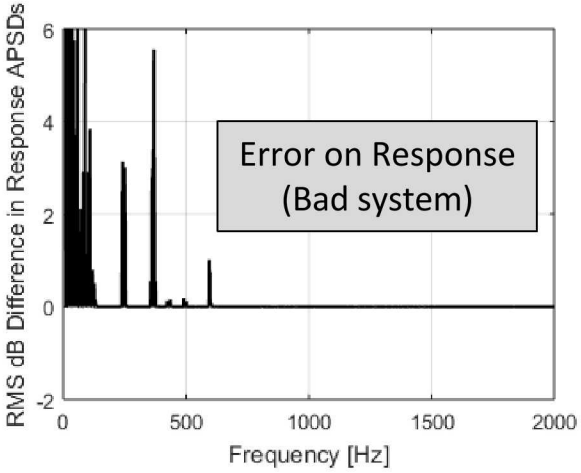
How does condition number affect estimated inputs & responses?

- Example: Unit input loads, apply noise to Truth response
- Estimate inputs with noisy response
  - $dB_{input} = 10\log_{10}\left(\frac{G_{ff,1}}{G_{ff,0}}\right)$
  - $dB_{response} = 10\log_{10}\left(\frac{G_{aa,1}}{G_{aa,0}}\right)$









Techniques Demonstrated with a Model of a Dynamic System

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2. Fixing Poorly Conditioned Systems – Regularization
   - Tikhonov & Singular Value methods
   - How can regularization can help or hurt?

3. Force Estimation Techniques
   - What methods are available?
   - How do they differ in terms of response and inputs?

Fixing Poorly Conditioned Systems – Regularization

- Regularization is a numerical correction to the FRF matrix to improve the conditioning to reduce errors in the estimates
- Example Problem:
  - White noise inputs to plate system, bad input locations
- Methods:

Tikhonov:

- Various flavors
- One type:

$$A^+ = (A^T A)^{-1} A^T$$

$$A^+ = (A^T A + \lambda^2 I)^{-1} A^T$$

- Regularization value: λ^2
- λ Can be constant or function of frequency

Singular Value:

- Decompose FRF

$$H = USV^H$$

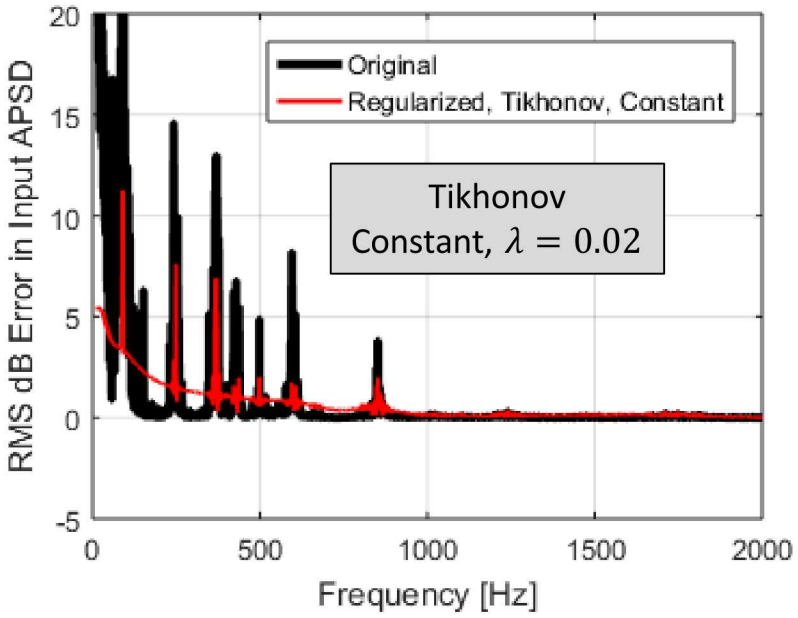
- Change the smallest singular values to be larger or set them to zero

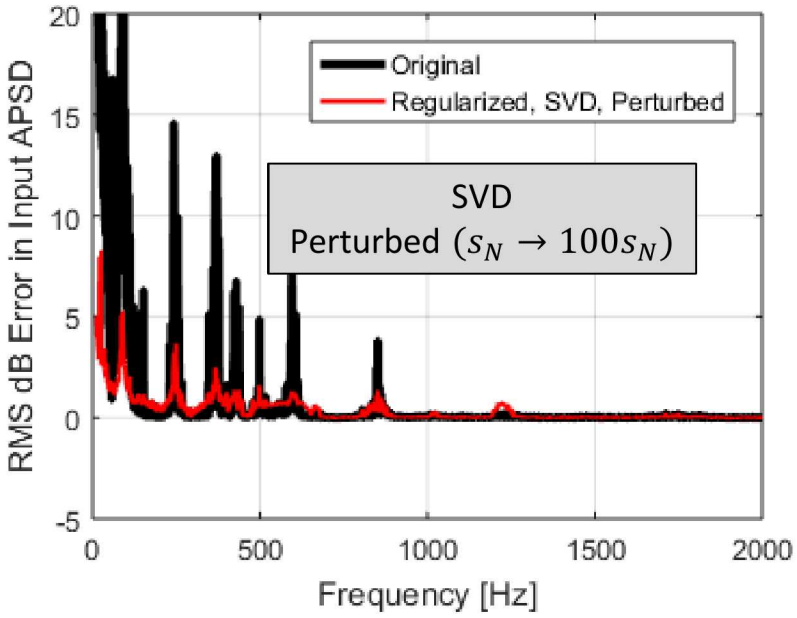
$$s_N \rightarrow 100 s_N$$

$$s_N \rightarrow 0$$

Regularization changes the FRF matrix, ideally by just enough to improve numerics without changing the system

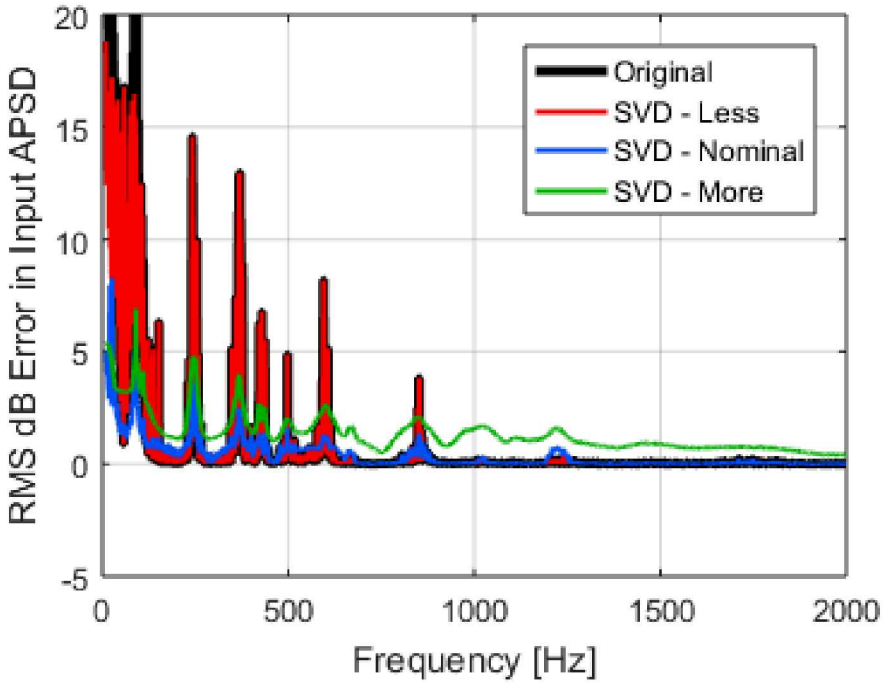
Fixing Poorly Conditioned Systems – Regularization





- Regularization can reduce errors in estimated inputs
- Tikhonov & SVD both work, just change FRF matrix differently

Fixing Poorly Conditioned Systems – Regularization



- Too little regularization does not change the matrix enough to reduce noise propagation
- Too much regularization changes the form of the matrix, introducing new errors

Techniques Demonstrated with a Model of a Dynamic System

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Force Estimation Techniques

- What techniques are available?
- How do they differ in terms of response and inputs?

Standard Method

- Inputs estimated with pseudo-inverse of the FRF matrix applied to the full target response CPSD
- $S_{ff,1} = H^{+}S_{aa,0}H^{+H}$

Independent Drives

- Estimate inputs to achieve only APSDs of target response
- $G_{ff,1} = (H \circ H^{*})^{+}G_{aa,0}$
- Can then update the target CPSD & estimate non-independent inputs

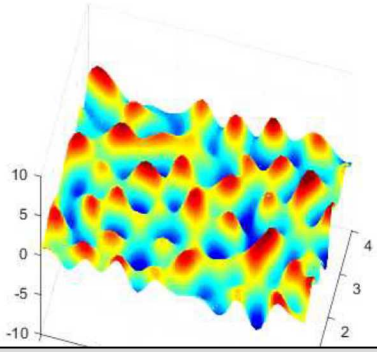
Buzz Test

- Replace the cross terms in the target response CPSD using a white noise input
- Estimate inputs with standard method, new target CPSD

Not an exhaustive list of available techniques – these are just the most prominent in MIMO shaker testing today

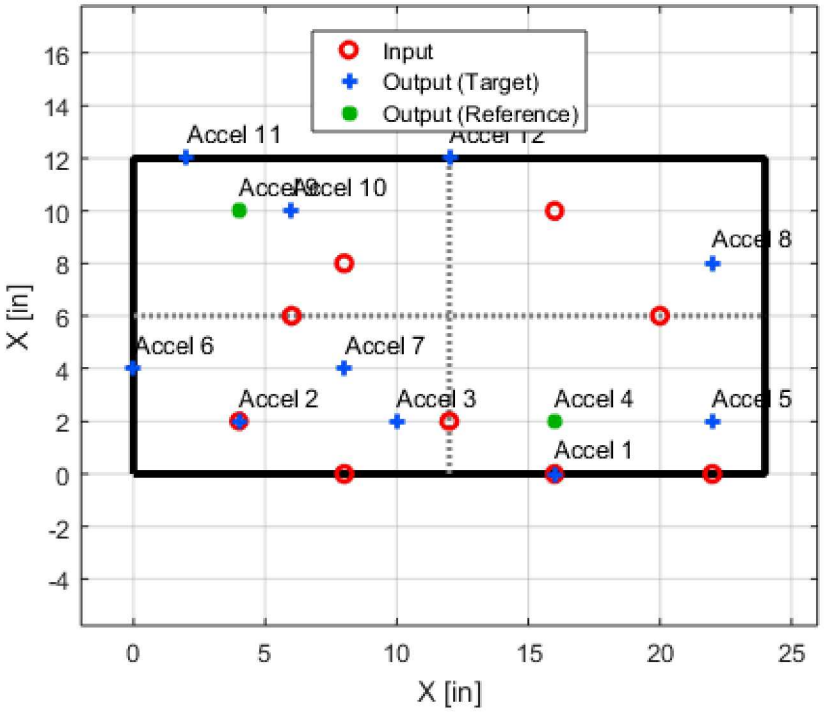
Force Estimation Techniques

- Example using different techniques
- Target response from acoustic pressure loads
  - 12 gages total
  - 10 used for control (targets)
  - 2 at non-control locations (references)
- Replicate with a lab test using 9 inputs at “good” locations



Field (Truth) = Acoustic Loads
Lab (Replica) = 9 Shakers

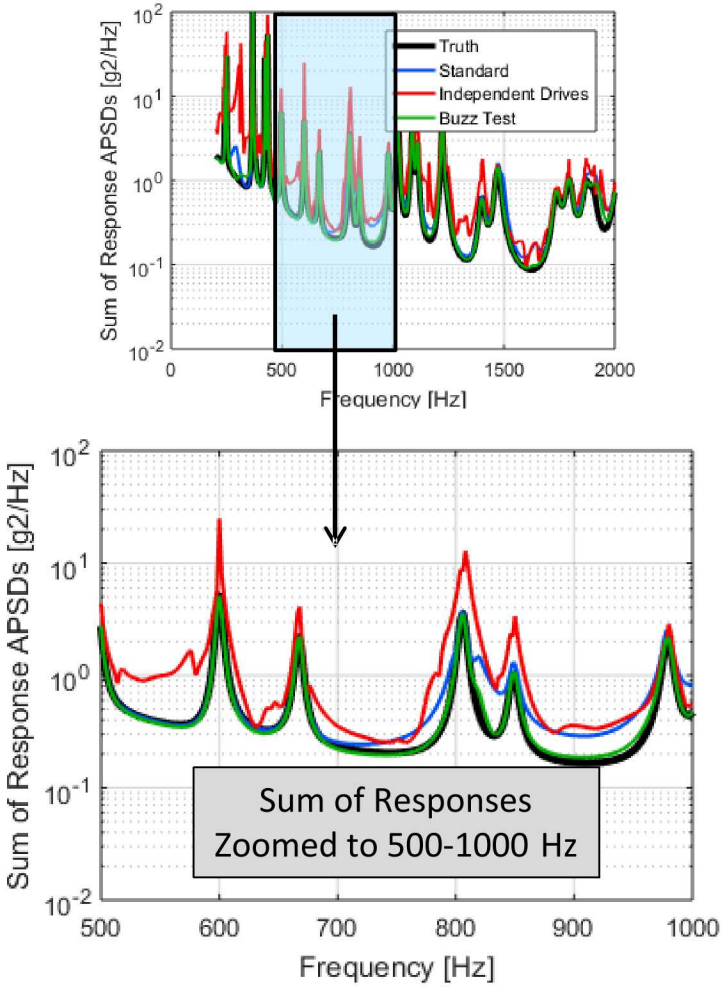
- 10 controls
- 2 references
- 9 inputs

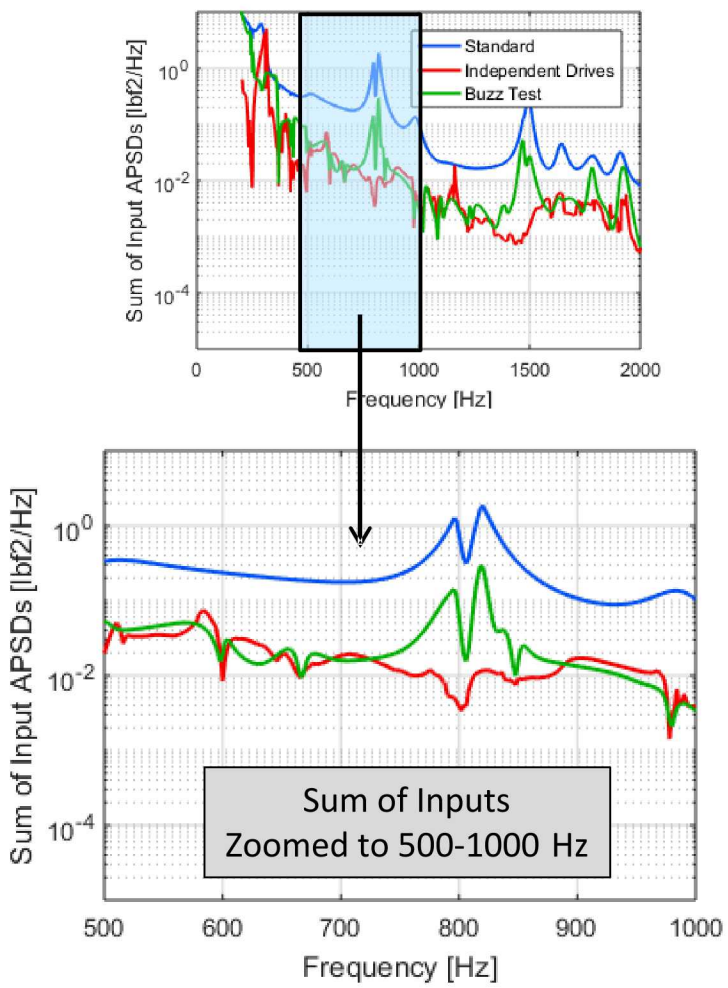


Force Estimation Techniques

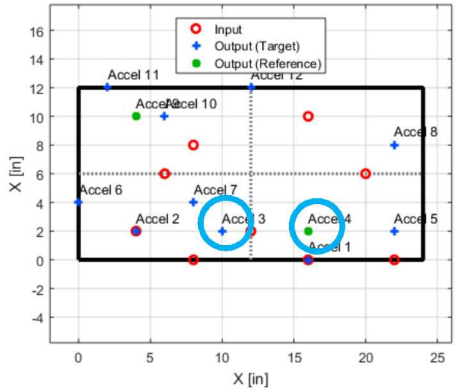
- Sum of input & output APSDs

- Forcing drives to be fully independent can reduce required inputs, but response error is high
- Standard method is accurate, but requires more input

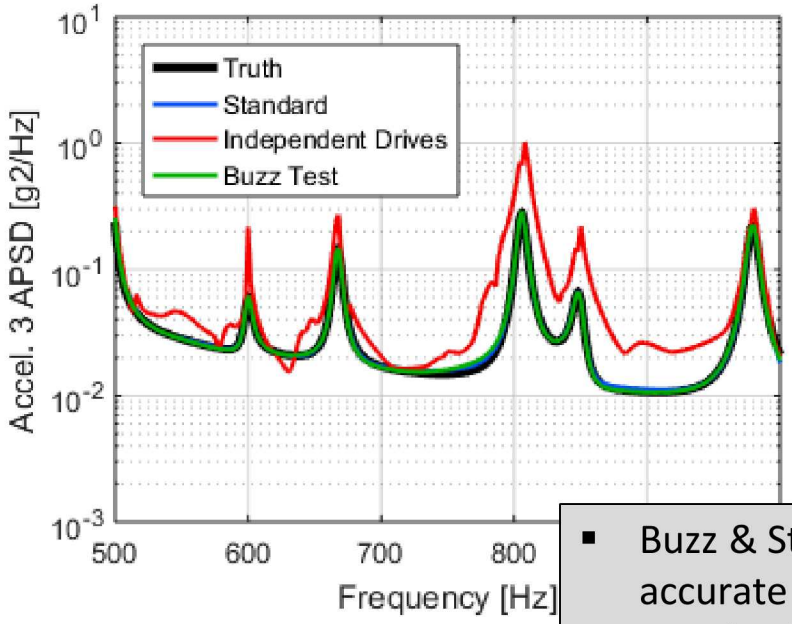




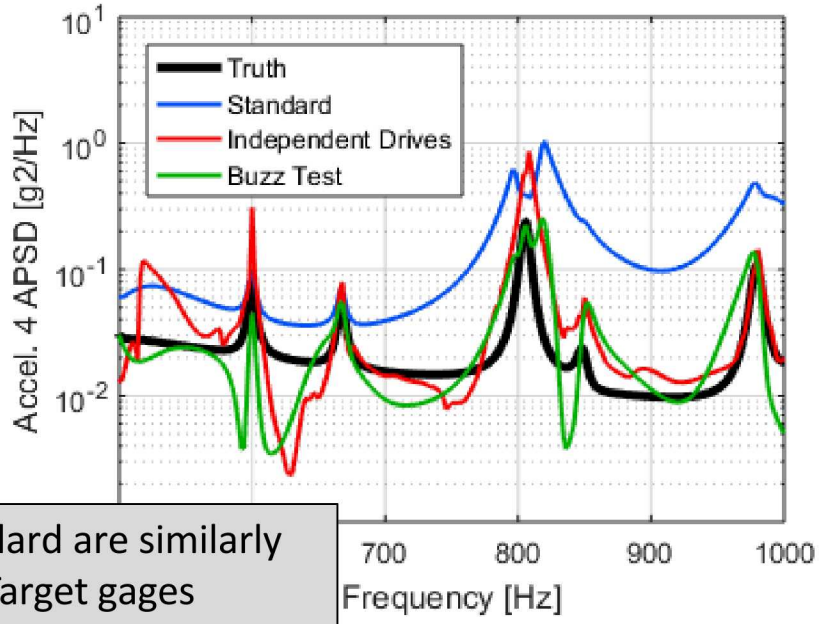
Force Estimation Techniques



Target Gage APSD

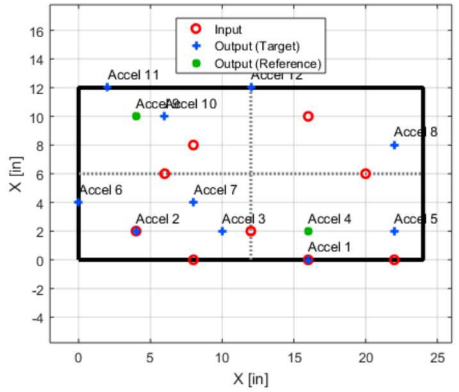


Reference Gage APSD

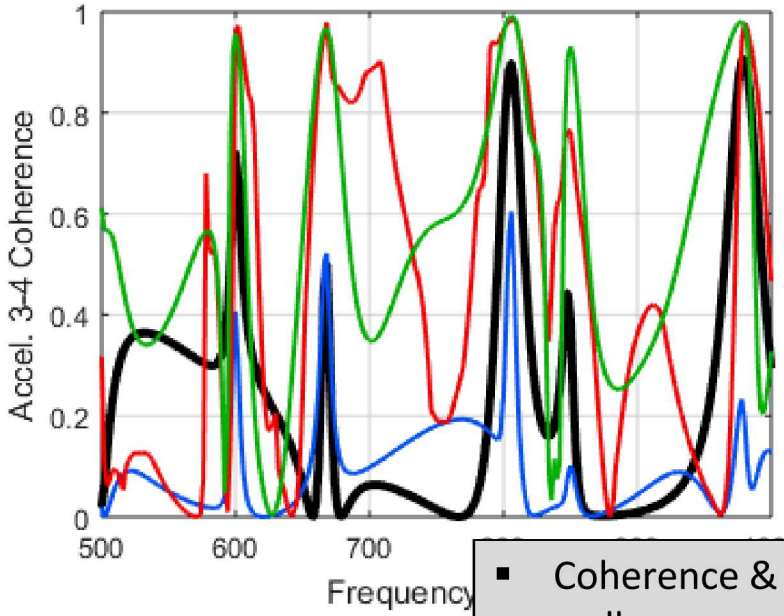


- Buzz & Standard are similarly accurate at Target gages
- Standard is not accurate at Reference gages

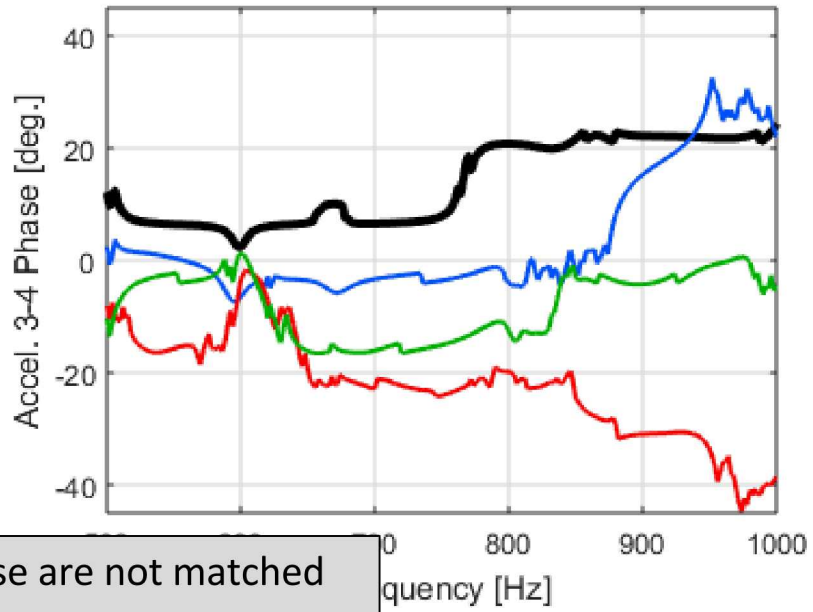
Force Estimation Techniques



Coherence



Phase



- Coherence & phase are not matched well
- Buzz & Independent make response more coherent than desired

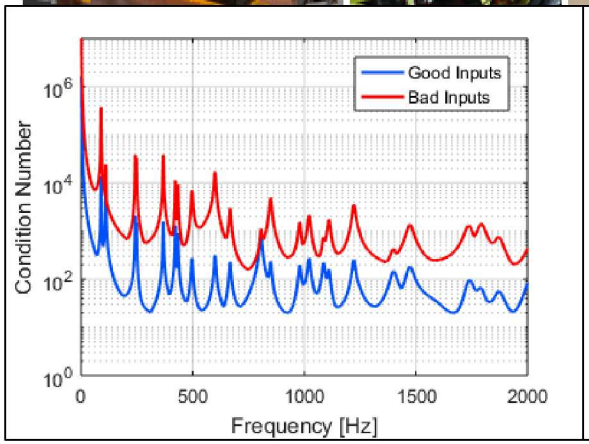
Conclusions: Regularization & Force Estimation Techniques Demonstrated with a Simple Dynamic System

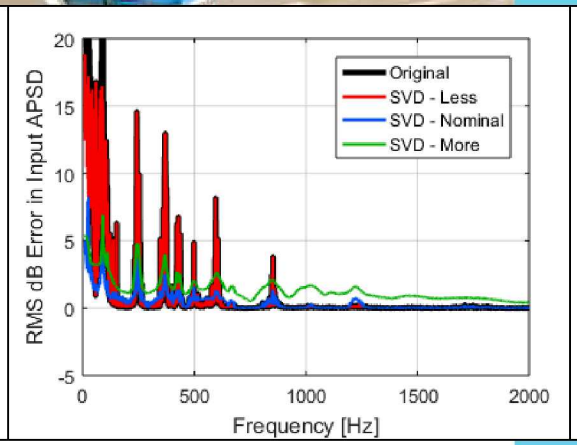
1. Condition Number of the FRF Matrix
   - Location of inputs & outputs is critical – need unique information in the FRF
   - Inverse susceptible to error propagation if condition number >1000

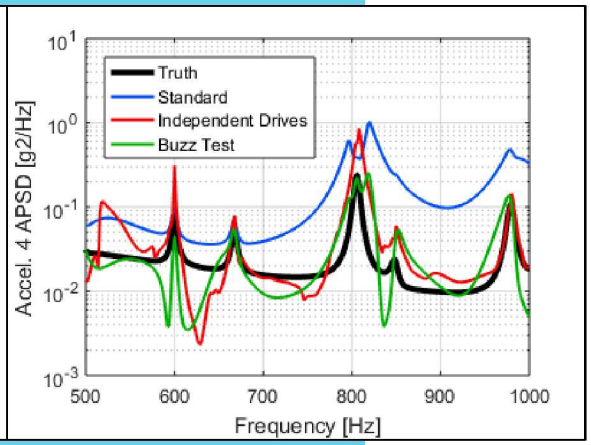
2. Fixing Poorly Conditioned Systems – Regularization
   - Numerical perturbations of the FRF matrix, multiple ways to achieve this
   - Change FRF matrix just enough to improve conditioning without changing overall form

3. Force Estimation Techniques
   - Multiple ways to solve the problem, each with strengths and weaknesses
   - Standard method can be accurate at targets, but not references
   - Standard method generally requires higher input forces
   - Strictly requiring independent inputs is not ideal – use Smallwood's method
   - Buzz test method provides nice balance of accuracy and inputs

A demonstration of force estimation and regularization methods for multi-shaker testing







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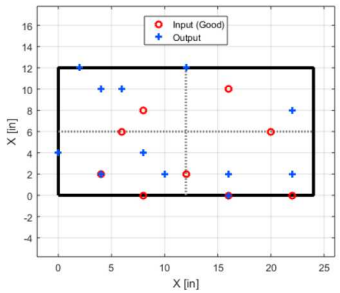
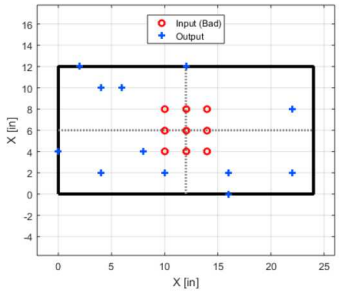
Condition Number of the FRF Matrix

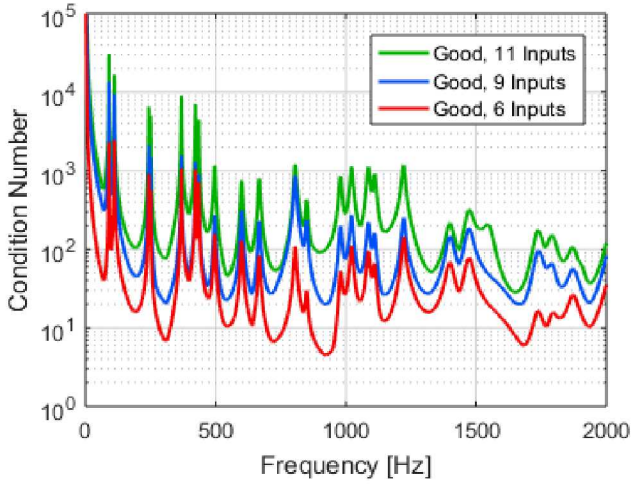
- What affects condition number?
  - Number of Inputs & Outputs



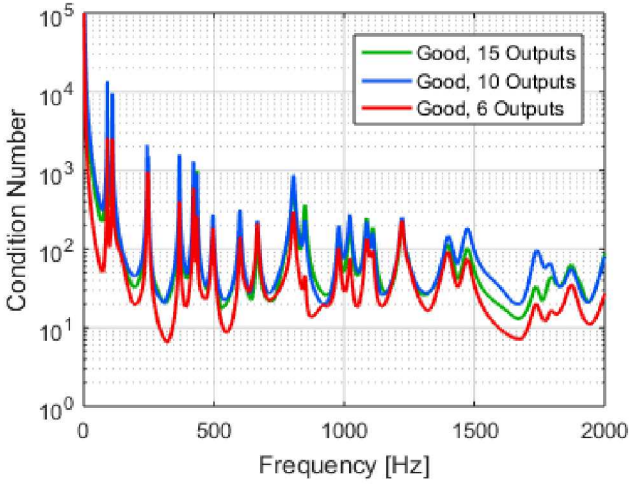
Good Inputs



Bad Inputs



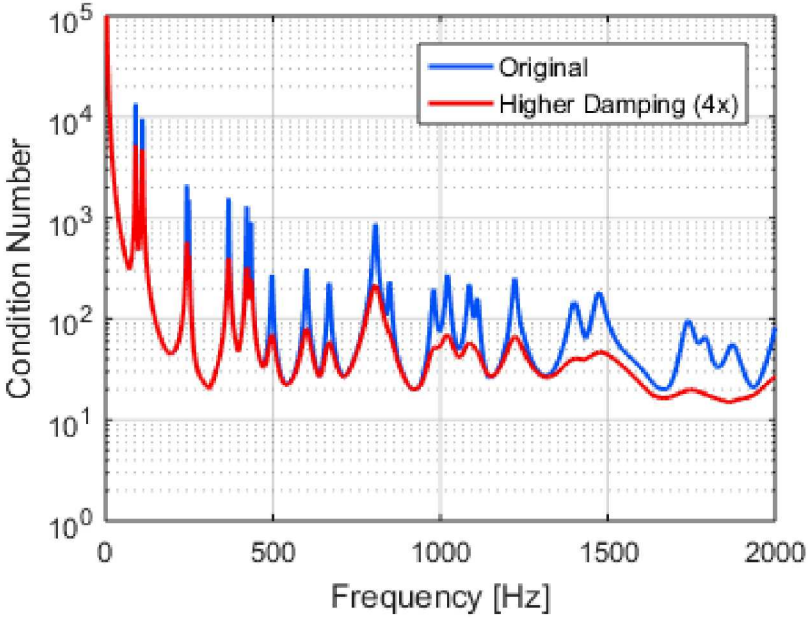
- More inputs increases condition number



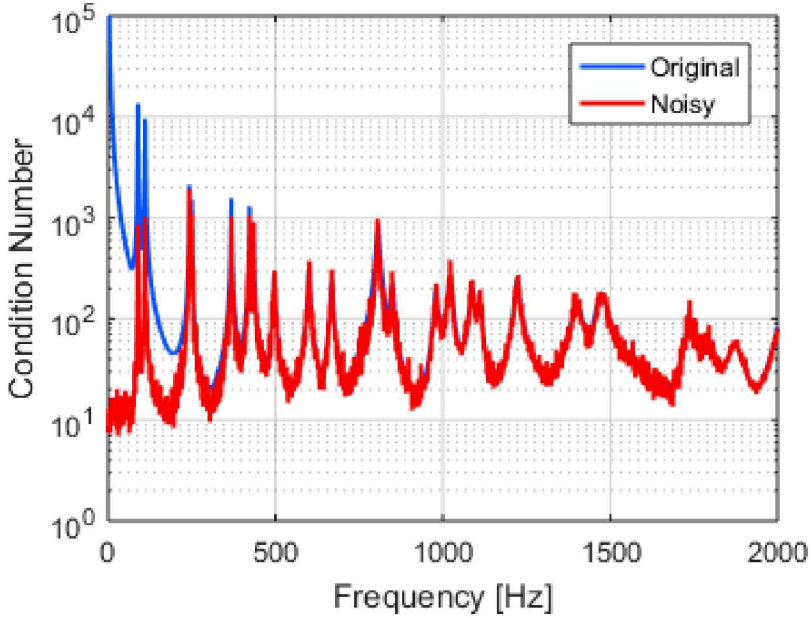
- More outputs does not trend the same way

Condition Number of the FRF Matrix

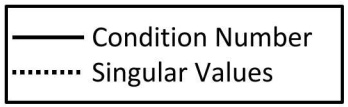
- What affects condition number?
  - Location and number of Inputs & Outputs
  - Damping
  - Noise



- Increased damping reduces the condition number somewhat



- Uncorrelated noise does not universally affect condition number


Condition Number
Singular Values