

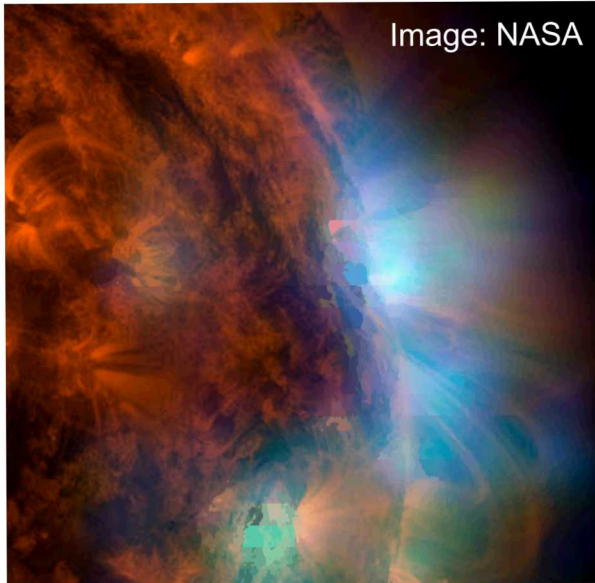
Developing Consistent Models for Matter in Extreme Conditions

S. Hansen, T. Nagayama, T. Gomez, A. Baczewski, and A. Cangi

Sandia National Laboratories

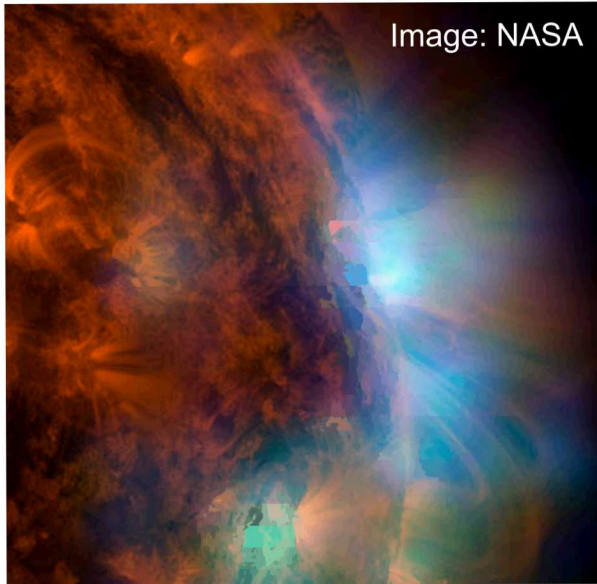
New Research Ideas Forum
Sandia National Laboratories (NM)
September 4, 2018

Context: we are interested in matter at extreme conditions relevant to fusion and astrophysical plasmas

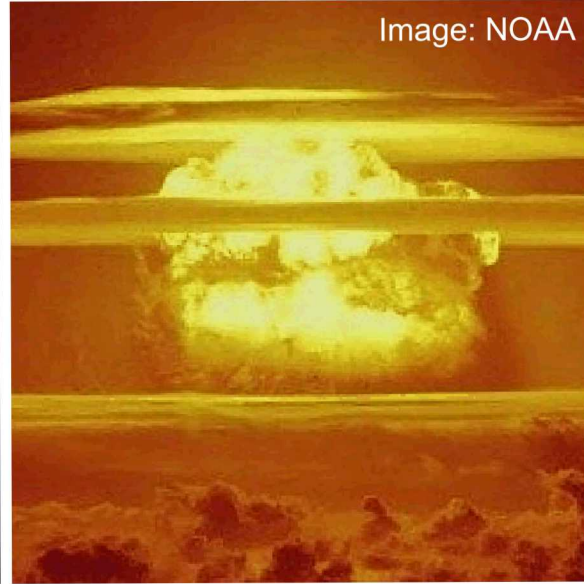


10^{+9} meters
 10^{+17} seconds

Context: we are interested in matter at extreme conditions relevant to fusion and astrophysical plasmas



10^{+9} meters
 10^{+17} seconds



10^{+3} meters
 10^{+2} seconds

Context: we are interested in matter at extreme conditions relevant to fusion and astrophysical plasmas

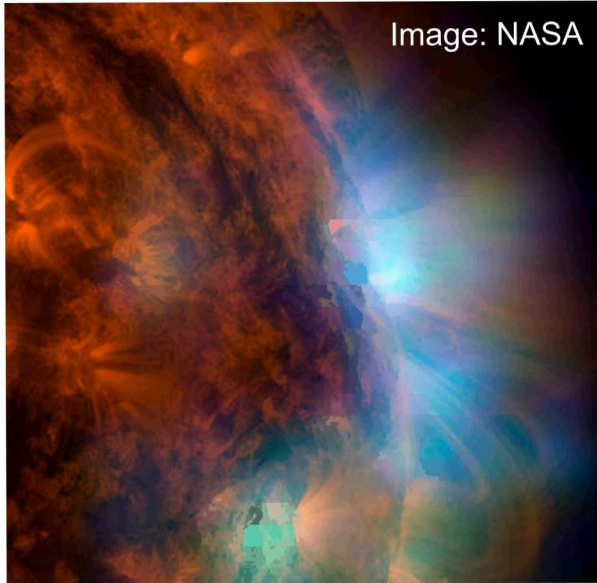


Image: NASA

10^{+9} meters
 10^{+17} seconds

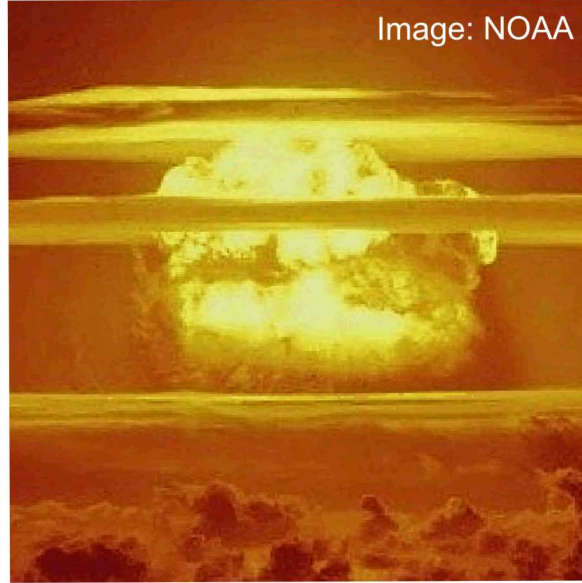


Image: NOAA

10^{+3} meters
 10^{+2} seconds

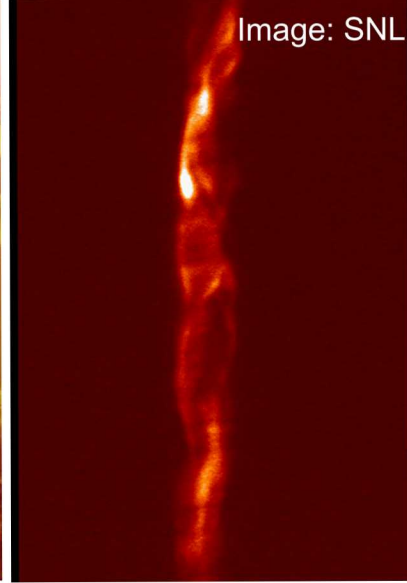
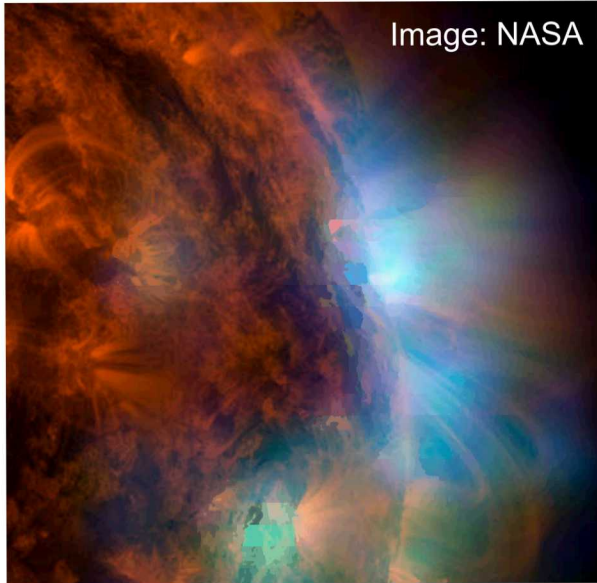


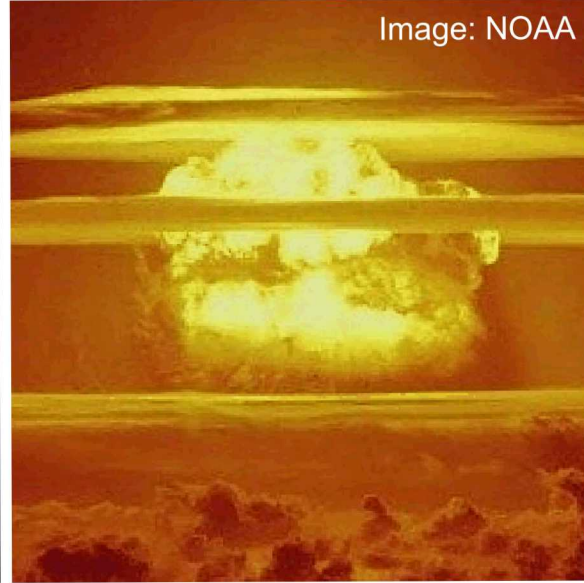
Image: SNL

10^{-4} meters
 10^{-9} seconds

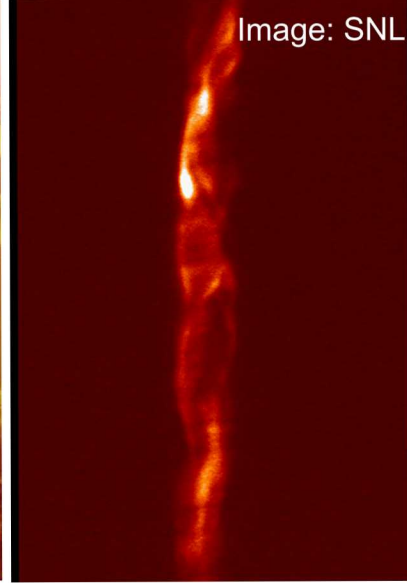
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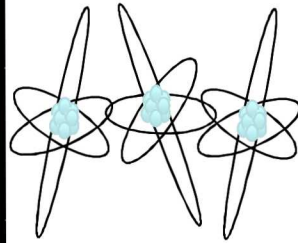
10^{+9} meters
 10^{+17} seconds



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 10^{+2} seconds

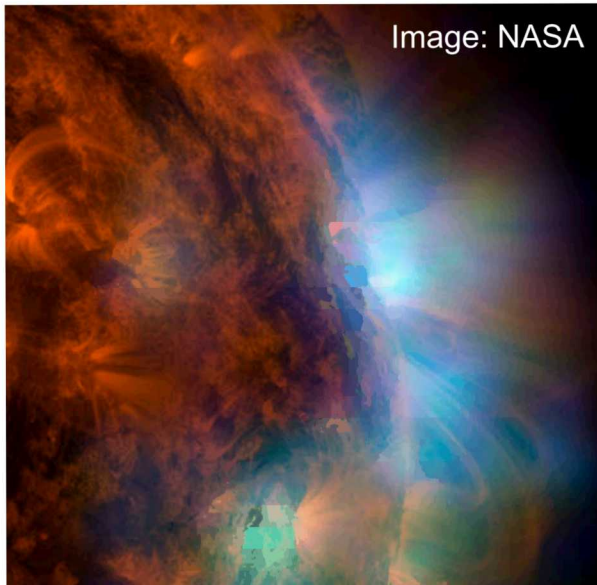


10^{-4} meters
 10^{-9} seconds

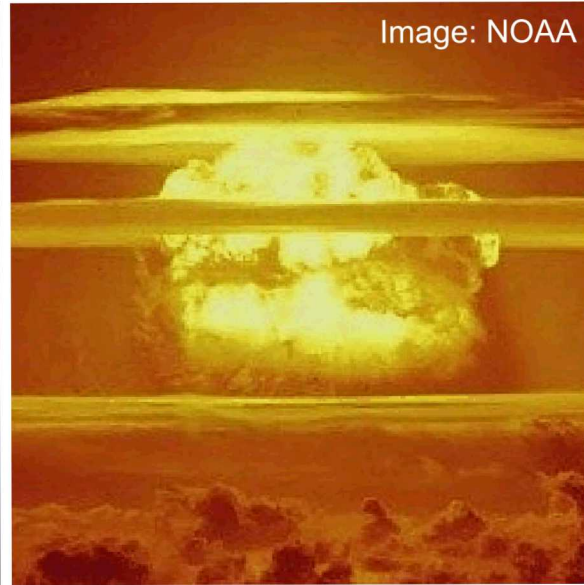


10^{-10} meters
 10^{-14} seconds

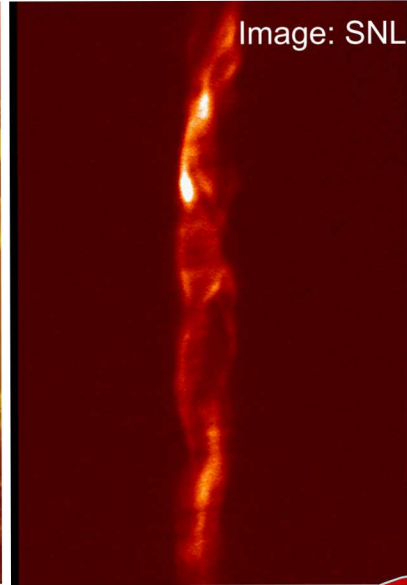
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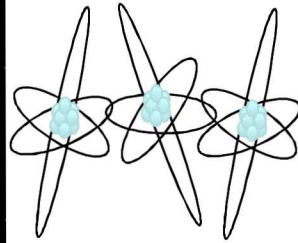
10^{+9} meters
 10^{+17} seconds



10^{+3} meters
 10^{+2} seconds



10^{-4} meters
 10^{-9} seconds

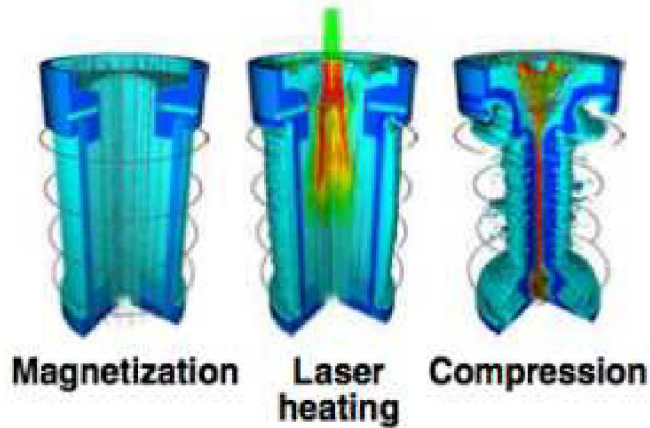


10^{-10} meters
 10^{-14} seconds

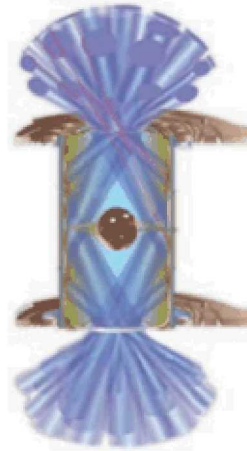
When we say “high-energy-density”, we don’t mean gasoline



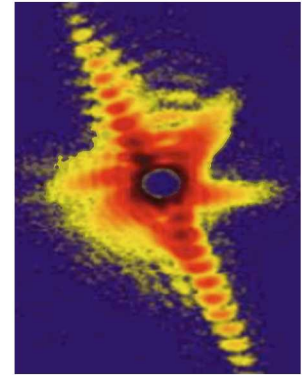
Context: How do we produce extreme conditions in the laboratory?



SNL's Z machine:
 $10 \text{ MJ} \rightarrow 10^{-9}\text{s}$, $100\text{-}1000 \text{ }\mu\text{m}$
 $0.3 - 3 \text{ keV}$, $0.01 - 1\text{g/cc}$
Fusion, Opacity, Rad. effects



LLNL's NIF:
 $2 \text{ MJ} \rightarrow 10^{-10}\text{s}$, $10\text{-}100 \text{ }\mu\text{m}$
 $0.3 - 3 \text{ keV}$, $0.01 - 1\text{g/cc}$
Fusion, Opacity, Rad. effects

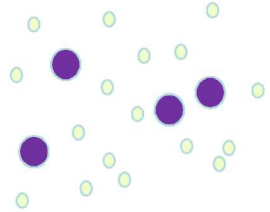


LCLS/ European XFEL:
 $2 \text{ mJ} \rightarrow 10^{-13}\text{s}$, $1\text{ }\mu\text{m}$
 10eV , 1g/cc
Fundamental material

We compress energy in space and time using pulsed power, lasers, or undulators

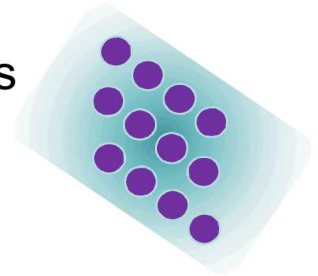
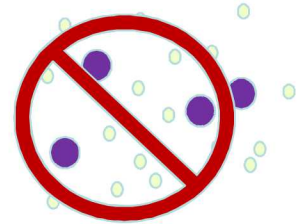
What's so difficult about modeling extreme conditions?

- HED plasmas are (usually) not well described by classical plasma models
 - Partial ionization complicates simple ion + electron pictures
 - Degeneracy effects invalidate classical statistics
 - Density effects distort quantum orbitals
 - Ions can be strongly coupled



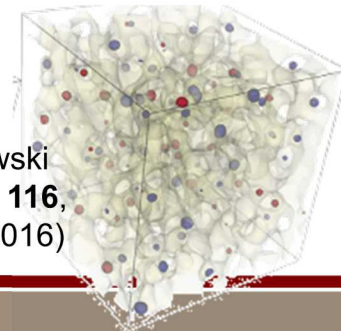
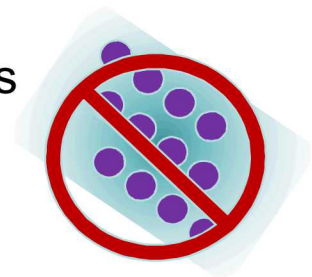
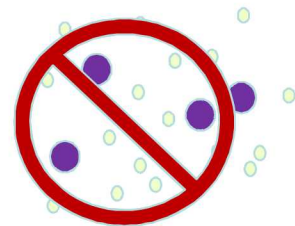
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- HED plasmas are (almost) never well described by solid-state models
 - Even modest temperatures can open enormous state space
 - Simplifications of ionic and electronic behavior are suspect



What's so difficult about modeling extreme conditions?

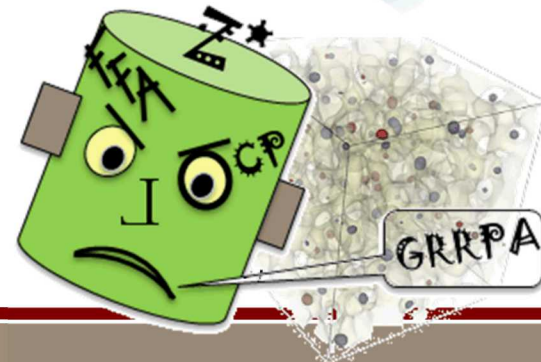
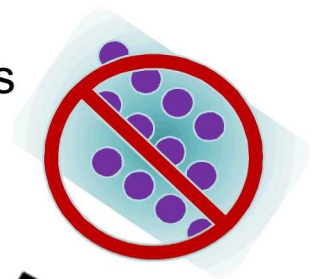
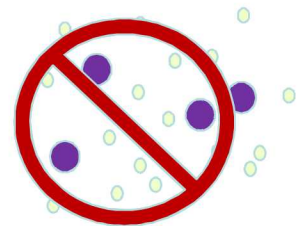
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- Rigorous and reliable models exist, but...
 - Quantum Molecular Dynamics (QMD)
 - Time-dependent Density Functional Theory (TD-DFT)
 - Computationally expensive and difficult to extend to high temperatures, low densities, and complex ions



A. Baczewski
et al, PRL **116**,
115004 (2016)

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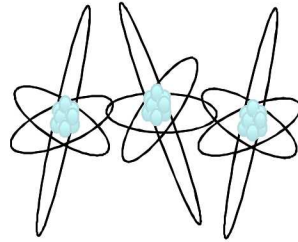


Central question: What happens to material when you squish it very hard and/or heat it quite a lot?

Experiments/Observables

Measurements from small, short-lived lab plasmas and large, distant astrophysical objects are inherently challenging

Observables (yields, images, spectra) can be difficult to interpret and may require both adequate material models and complex simulations



Simulations

“Magneto-radiation-hydrodynamic” simulations are used to design experiments and help interpret data from laboratory and astrophysical plasmas

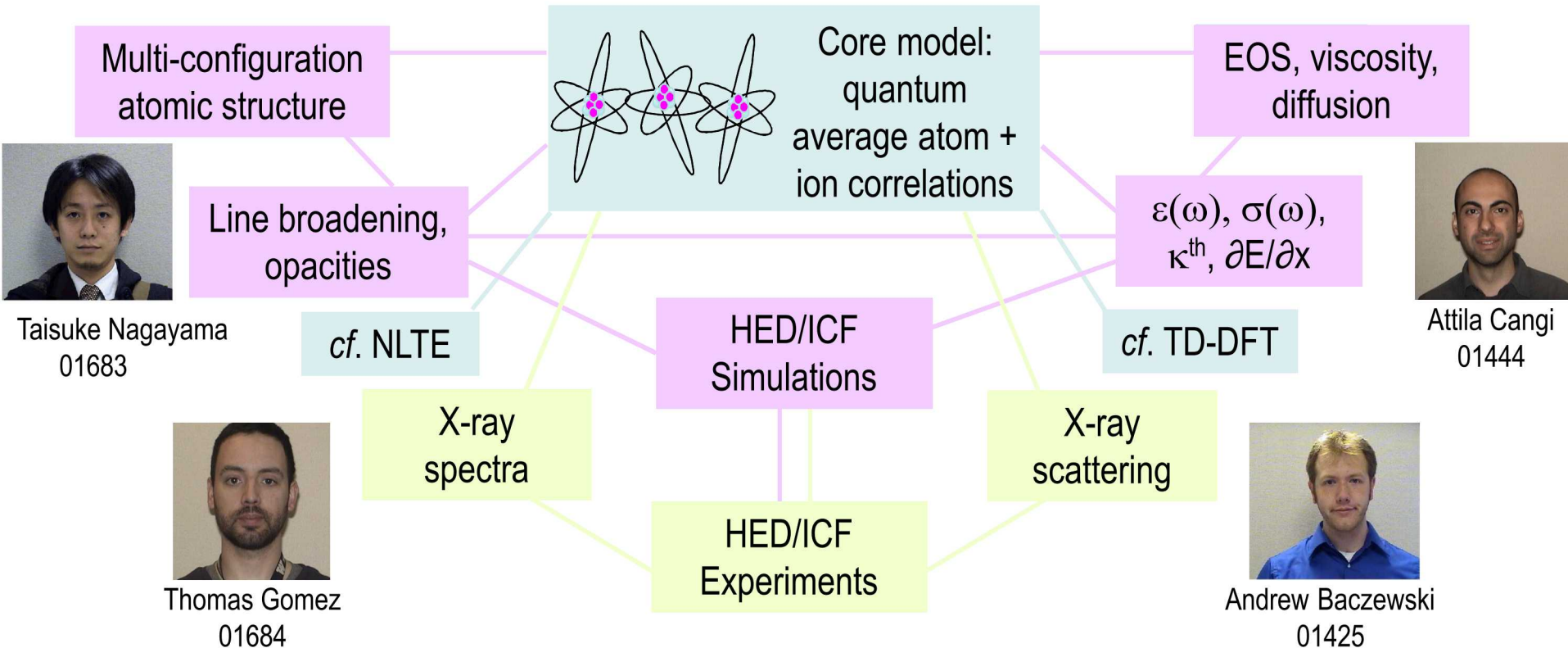
Reliable simulations themselves require extensive input from adequate material models (EOS, transport, opacity)

Additional questions:

How can we tell if our models are right?

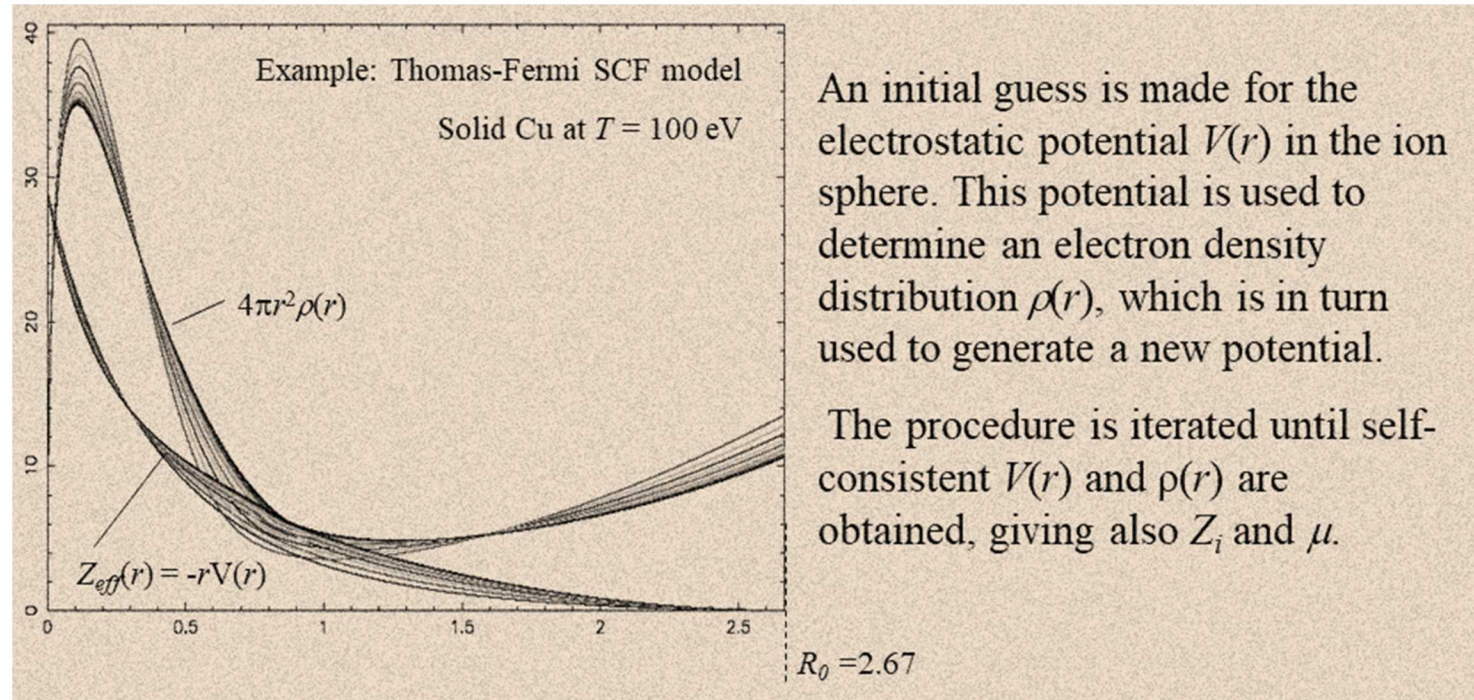
How important is model consistency?

Our central goal: Develop a unified, tractable, and consistent model for matter in extreme conditions



History: Thomas-Fermi fluid models developed in the 1920s still inform some present-day simulations

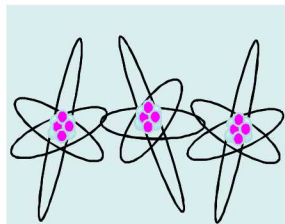
Fluid “Self-consistent field” models capture a lot of essential HED physics on the cheap!



But they neglect electronic shell structure and treat other ions as a uniform “jellium”

Our model builds on that history:

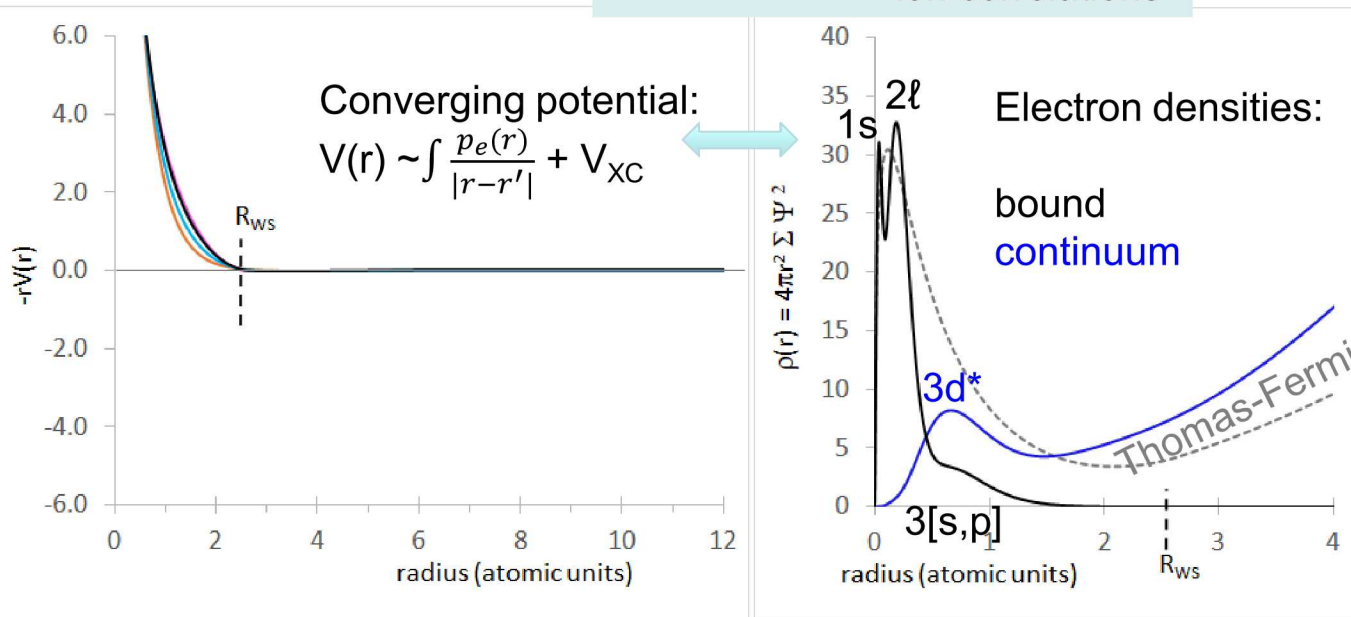
Fully quantum, semi-relativistic electrons for both the atom...



Core model:
quantum
average atom +
ion correlations

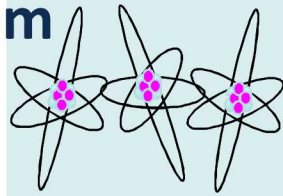
Bound and free atomic
wavefunctions are calculated in
a self-consistent potential

Solid-density iron, 10 eV



Our model builds on that history:

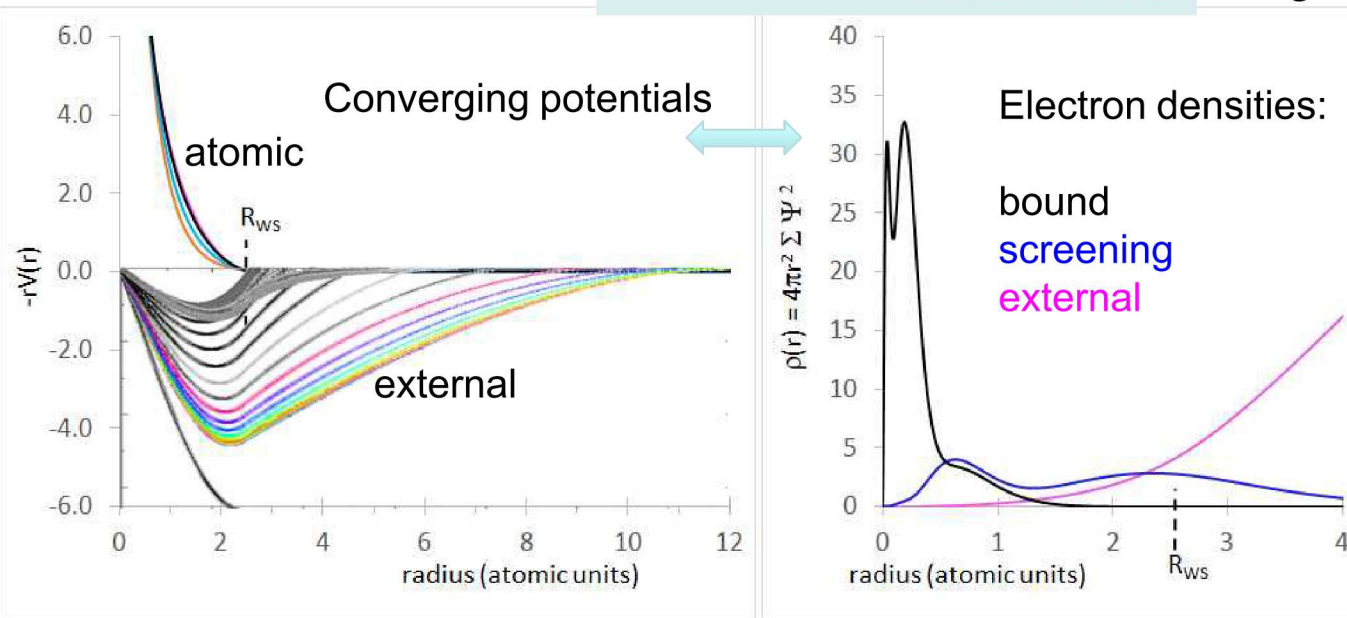
Fully quantum, semi-relativistic electrons for both the atom and the external system



Core model:
quantum
average atom +
ion correlations

The external system defines a neutral pseudo-atom (NPA) that extends beyond the Wigner-Seitz cell

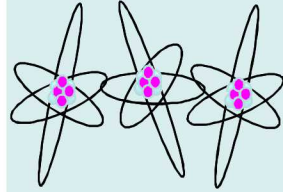
Solid-density iron, 10 eV



$$\rho^{\text{NPA}} = \rho^{\text{atomic}} - \rho^{\text{external}}$$

$$\rho^{\text{screening}} = \rho^{\text{NPA}} - \rho^{\text{ion}}$$

...which extends the model's self-consistency to ions:



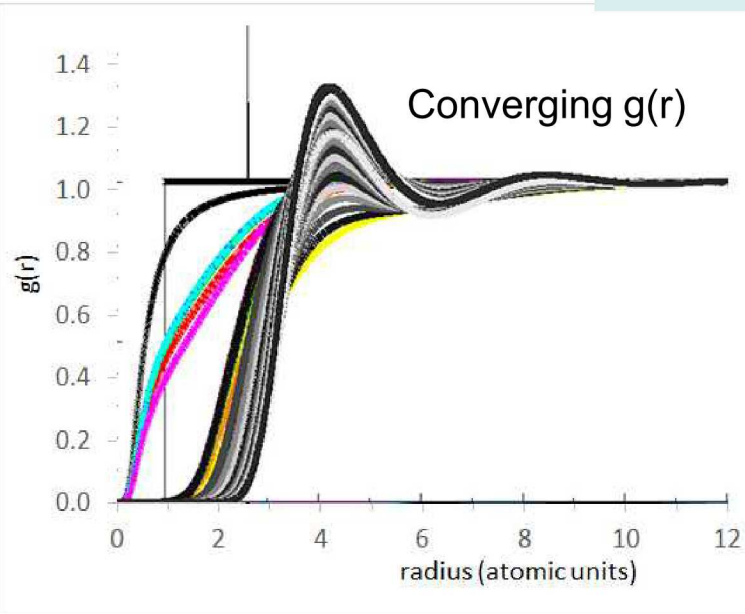
Core model:
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The screening electron density
determines the ion-ion
interaction potential:

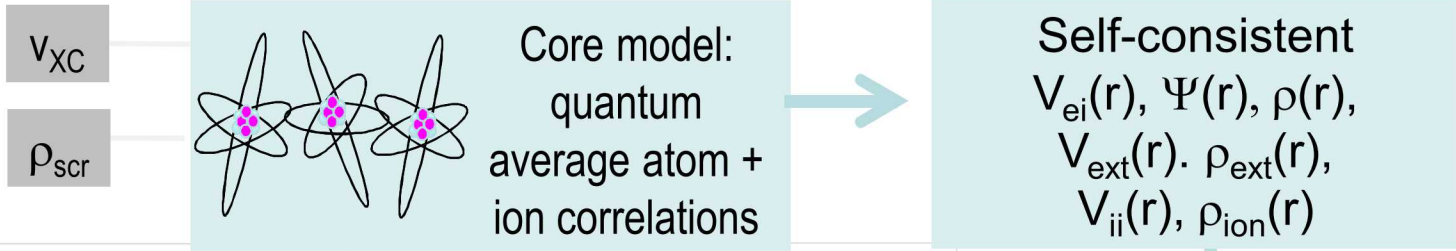
$$\beta V(k) = \frac{4\pi\beta}{k^2} \bar{Z}^2 - n_e^{\text{scr}}(k) C_{\text{le}}(k)$$

This potential constrains the
ion distribution through the
quantum Ornstein-Zernicke
equations

Solid-density iron, 10 eV

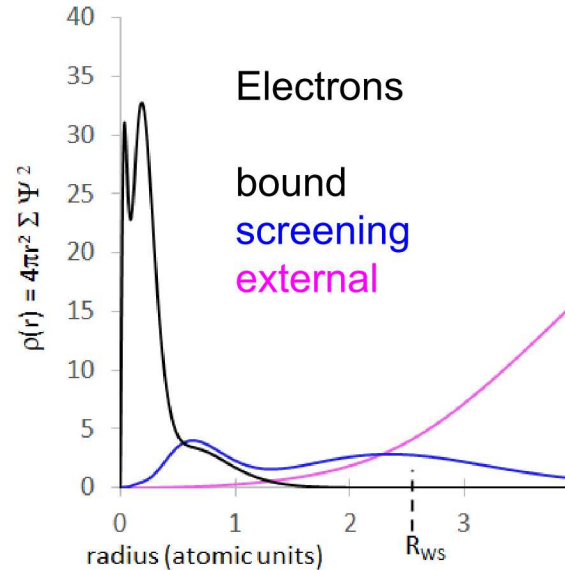
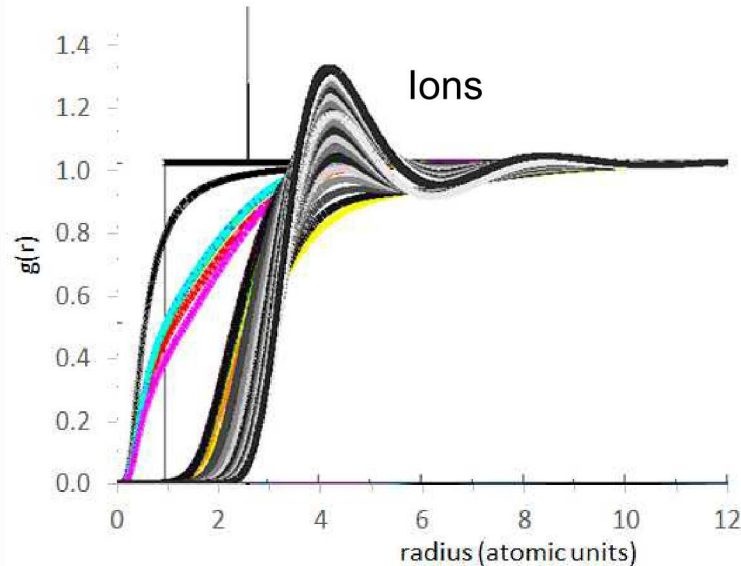


This is our fully self-consistent core model, which is sensitive to a limited number of constraints

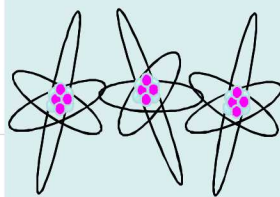


Observable and
constitutive
properties

- Spherical
- semi-relativistic
- + orthonormal $\Psi(r)$
- + runtime ~ 3 minutes

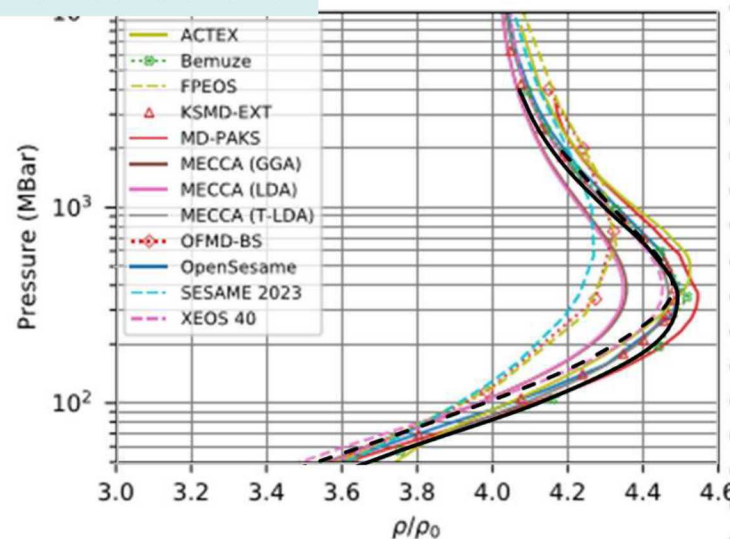
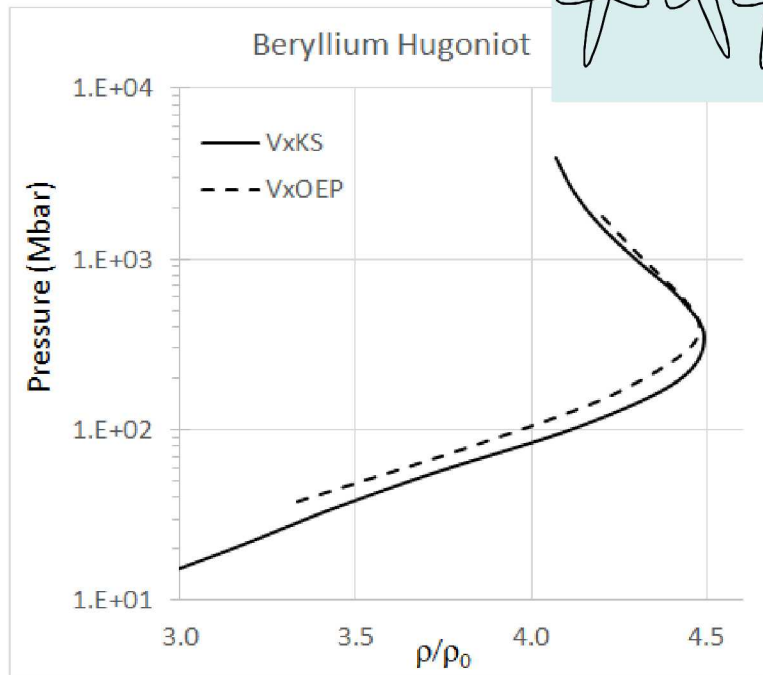


These constraints impact Equations of State and all other model-derived properties while preserving self-consistency



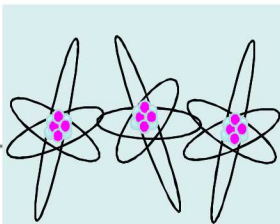
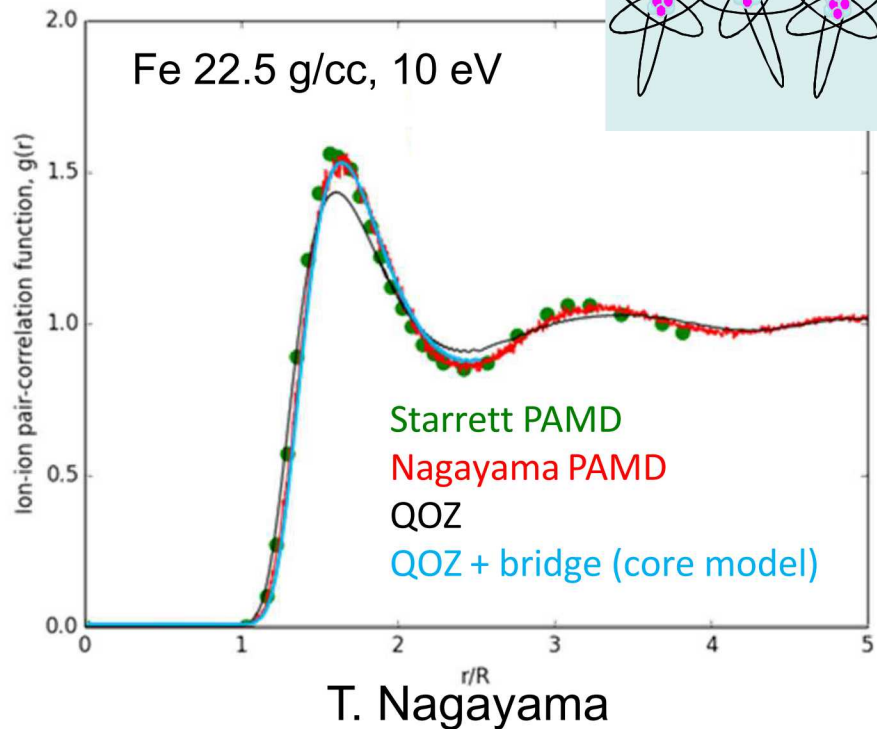
Core model:
quantum
average atom +
ion correlations

EOS, viscosity,
diffusion



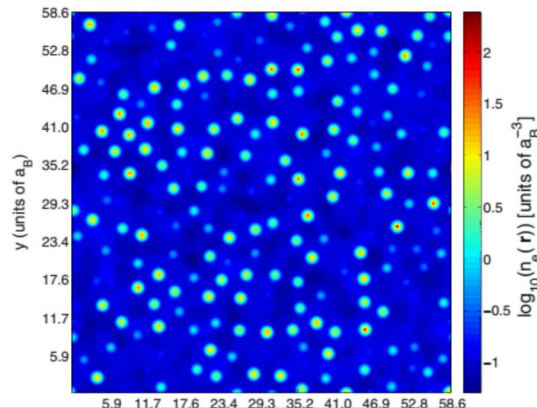
J. Gaffney, S. Hu *et al.*
LLE EOS code comparison

Screening from the core model drives PAMD simulations, which inform EOS & provide ionic transport coefficients



Core model:
quantum
average atom +
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EOS, viscosity,
diffusion

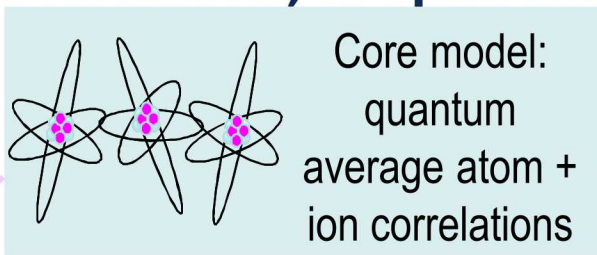


PHYSICAL REVIEW E **91**, 013104 (2015)

Pseudoatom molecular dynamics

C. E. Starrett,^{*} J. Daligault, and D. Saumon

The core-model wave functions produce optical properties: conductivities, dielectric functions, & opacities



Core model:
quantum
average atom +
ion correlations

Line broadening,
opacities $\alpha(\omega)$

$\sigma(\omega), \epsilon(\omega),$
 $\kappa^{\text{th}}, \partial E / \partial \chi$

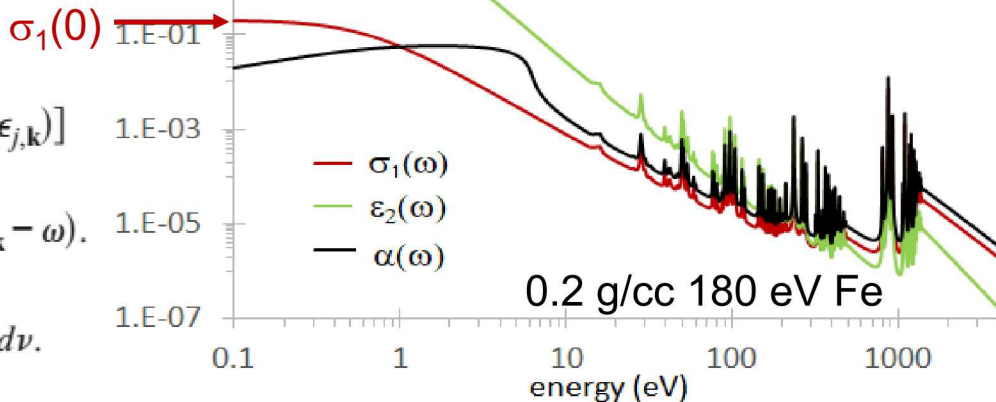
X-ray
spectra

X-ray
scattering

$$\sigma_1(\mathbf{k}, \omega) =$$

$$\frac{2\pi}{3\omega\Omega} \sum_{j=1}^{n_b} \sum_{i=1}^{n_b} \sum_{\alpha=1}^3 [F(\epsilon_{i,k}) - F(\epsilon_{j,k})] \\ \times \langle \Psi_{j,k} | \nabla_{\alpha} | \Psi_{i,k} \rangle^2 \delta(\epsilon_{j,k} - \epsilon_{i,k} - \omega).$$

$$\sigma_2(\omega) = -\frac{2}{\pi} \text{P} \int \frac{\sigma_1(\nu)\omega}{(\nu^2 - \omega^2)} d\nu.$$



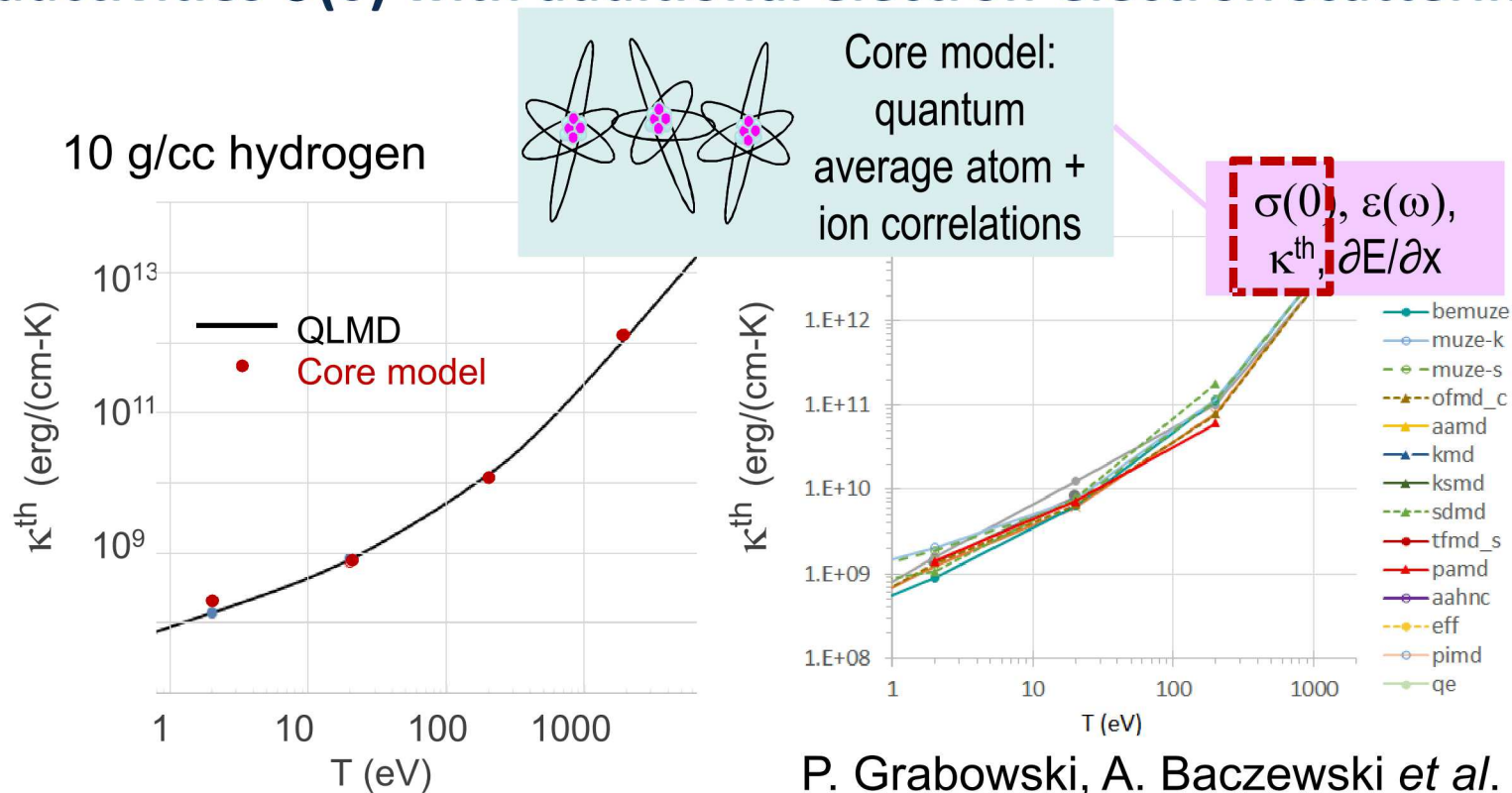
$$\epsilon_1(\omega) = 1 - \frac{4\pi}{\omega} \sigma_2(\omega),$$

$$\epsilon_2(\omega) = \frac{4\pi}{\omega} \sigma_1(\omega),$$

$$\alpha(\omega) = \frac{4\pi}{n(\omega)c} \sigma_1(\omega),$$

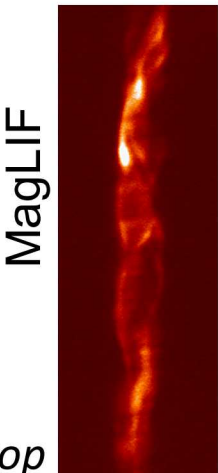
$$n(\omega) = \sqrt{\frac{1}{2} [|\epsilon(\omega)| + \epsilon_1(\omega)]}.$$

Thermal conductivities are derived from electrical conductivities $\sigma(0)$ with additional electron-electron scattering



M. Desjarlais

P. Grabowski, A. Baczewski *et al.*
SNL transport code comparison workshop

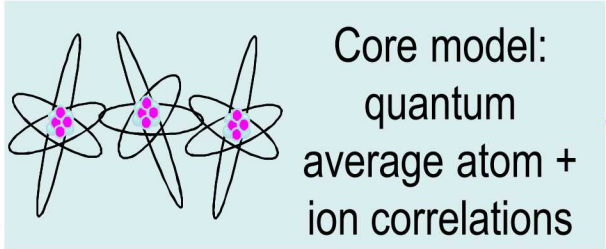


MagLIF

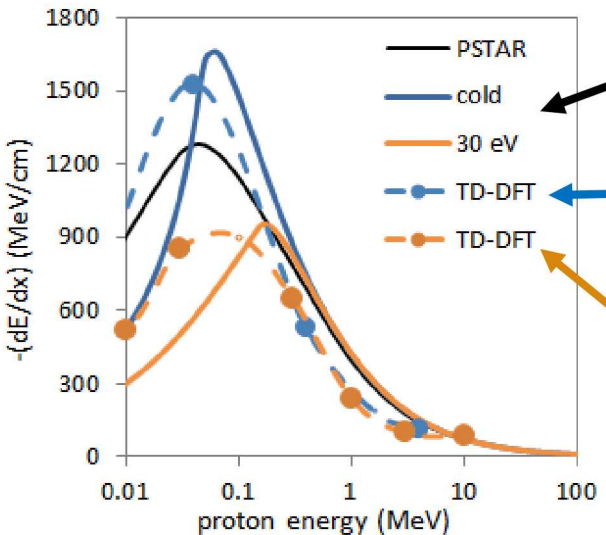
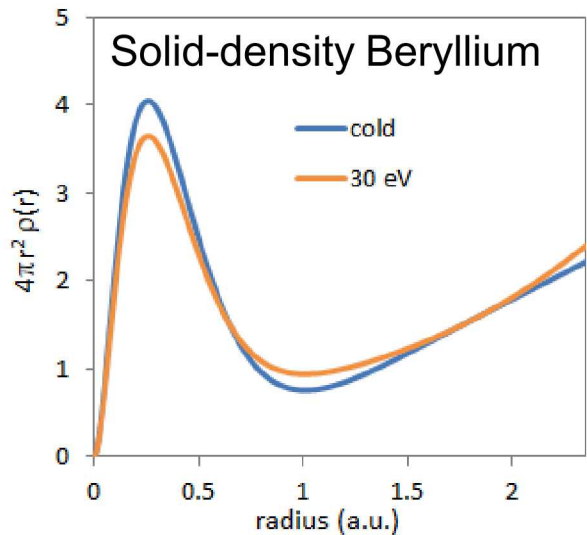
The dielectric function and electron density inform stopping powers (dE/dx) relevant to self-heating in fusion plasmas

$$L(\rho, V) = \frac{i}{\pi \omega_0^2} \int_0^\infty \frac{dk}{k} \int_{-kV}^{kV} \omega dw \left(\frac{1}{\epsilon(k, \omega)} - 1 \right)$$

$$\left(\frac{dE}{dx} \right) = - \frac{4\pi}{m} \left(\frac{Ze^2}{V} \right)^2 \int_0^\infty \rho(r) L(\rho, V) 4\pi r^2 dr$$



$$\sigma(\omega), \epsilon(\omega), \kappa^{\text{th}}, \partial E / \partial x$$

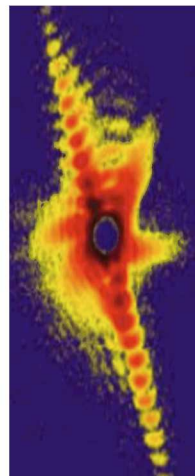


Core model:
~3 minutes on desktop

cold TD-DFT
(5 x 32 cores, 1 day)

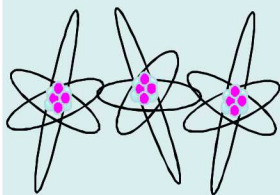
32 eV TD-DFT
(3000 x 16 cores, 1 day)

A. Cangi, A. Baczewski



The dielectric function is also related to X-ray scattering, which is used as a diagnostic for laboratory plasmas

$$S(\omega) \propto \text{Im} \left[\frac{1}{\epsilon(\omega)} \right]$$

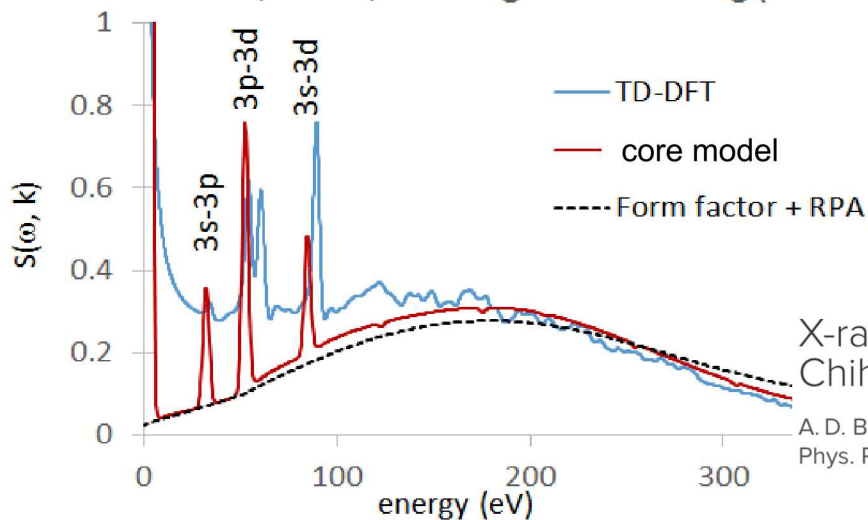


Core model:
quantum
average atom +
ion correlations

$$\sigma(\omega), \epsilon(\omega), \kappa^{\text{th}}, \partial E / \partial x$$

X-ray
scattering

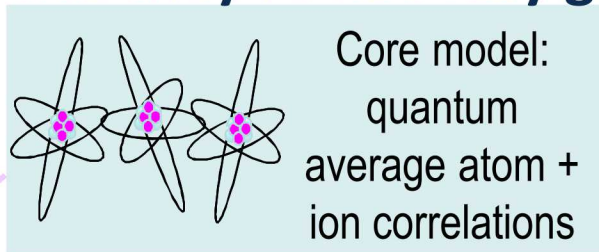
solid Fe, 20 eV, 130 degree scattering ($k = 3.4$)



X-ray Thomson Scattering in Warm Dense Matter without the Chihara Decomposition

A. D. Baczewski, L. Shulenburger, M. P. Desjarlais, S. B. Hansen, and R. J. Magyar
Phys. Rev. Lett. **116**, 115004 – Published 18 March 2016

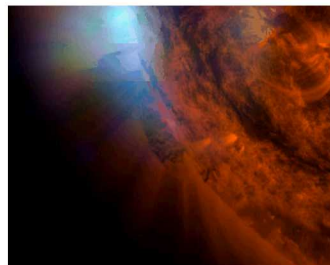
The optical properties that inform transport and scattering also provide opacities – but they're not very good



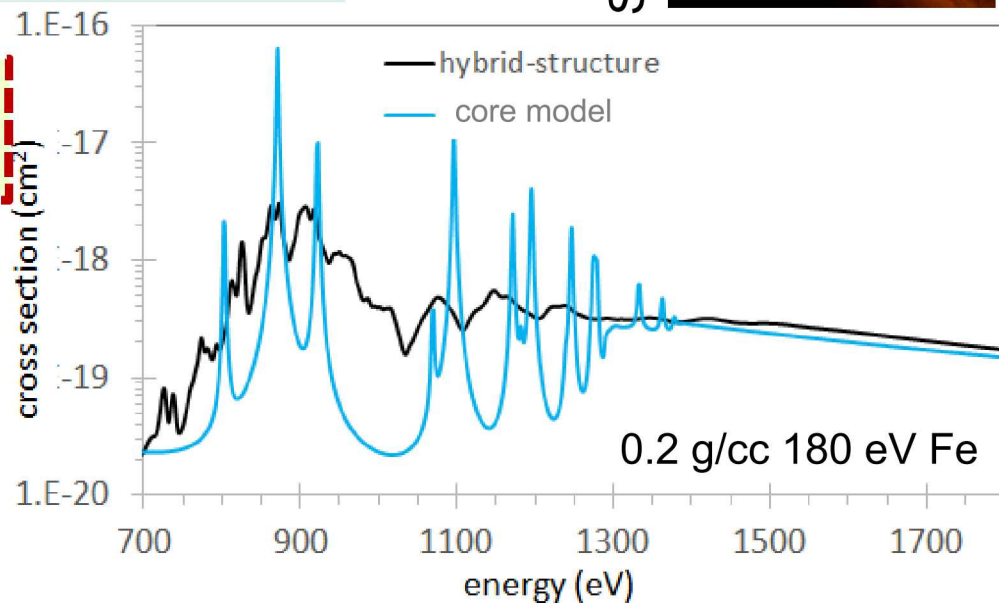
Line broadening,
opacities $\alpha(\omega)$

X-ray
spectra

Solar opacity*

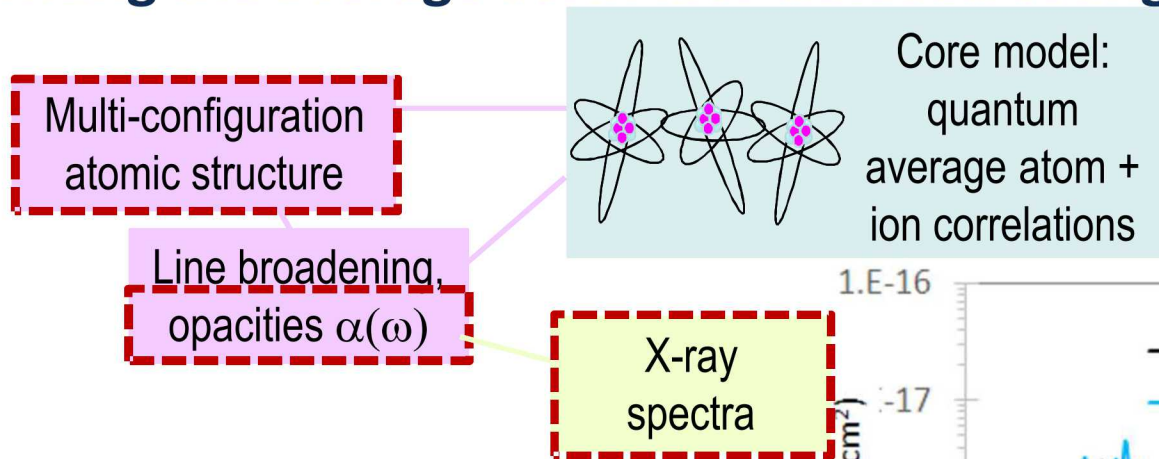


Average-atom opacities are strictly complete – which is not trivial for more complex models* – but they do not capture line splitting due to multiple configurations

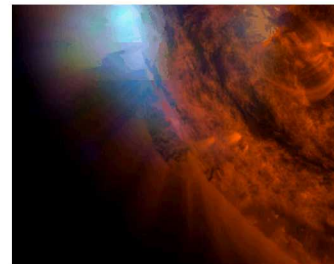


*Iglesias and Hansen, *Ap. J* **835**, 284 (2017); **Bailey et al, *Nature* **517**, 56 (2015)

The opacities can be significantly improved by splitting the average atom into detailed configurations



Solar opacity



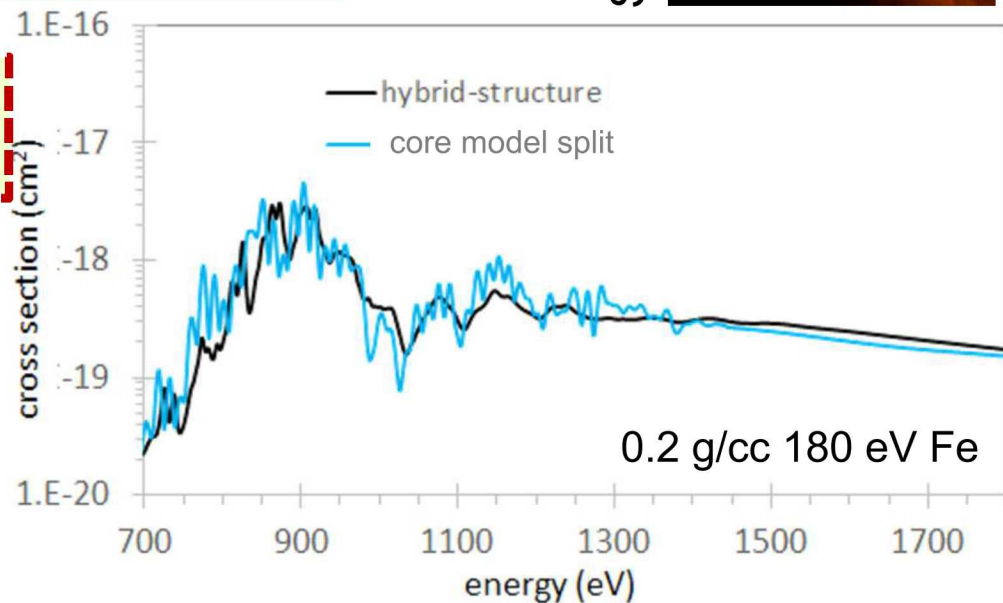
$$E_i = E_{c_{onfig}} + E_{c_{orr}} + E_{p_{las}} + E_{S_0}$$

$E_{c_{onfig}}$ from Slater coefficients/ Ψ

$E_{p_{las}}$ from ion distribution & $\Psi_{c_{onfig}}$

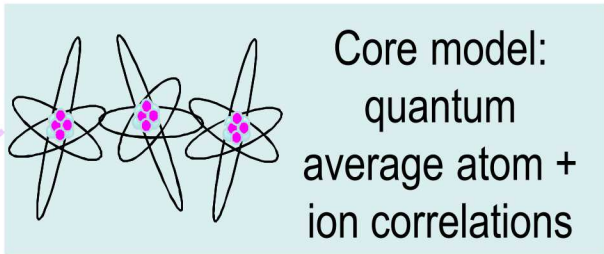
$$X_i = g_i w_i$$

$$\exp(\beta(m(\bar{\mu} - \bar{E}_p) - E_i))$$

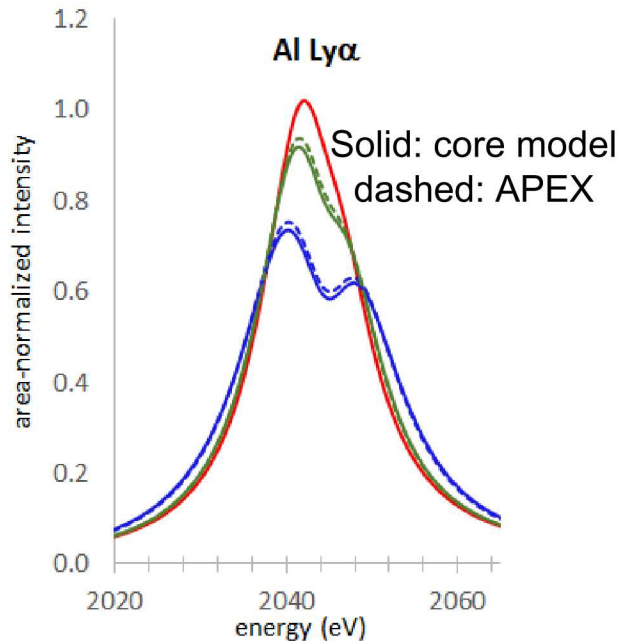
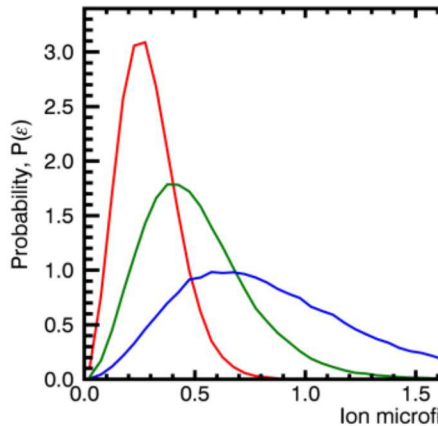
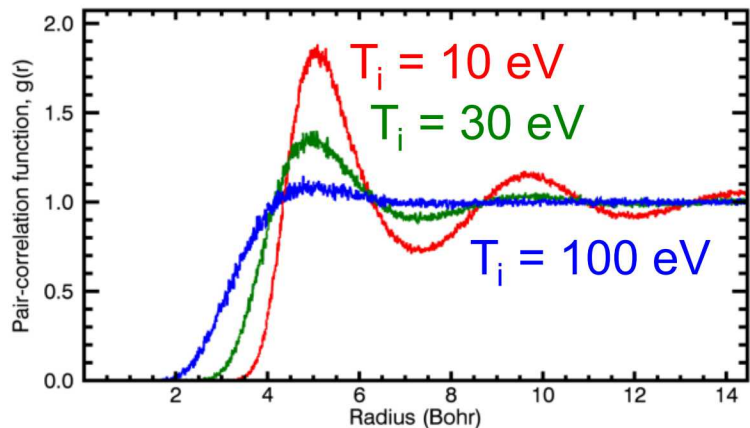


Opacities will be further refined using self-consistent line broadening from core-model PAMD & collision models

Line broadening, opacities



Solid-density Al, $T_e = 250$ eV



T. Gomez

T. Nagayama

We are working to build a self-consistent model of material at extreme conditions with:

- Constitutive properties adequate for use in simulations
 - Enforced consistency can improve sensitivity studies & increase constraints
- Observable predictions adequate for comparison with experiment
 - Enforced consistency means that if part of this model is wrong, the whole thing is wrong – and its wrongness should be detectable

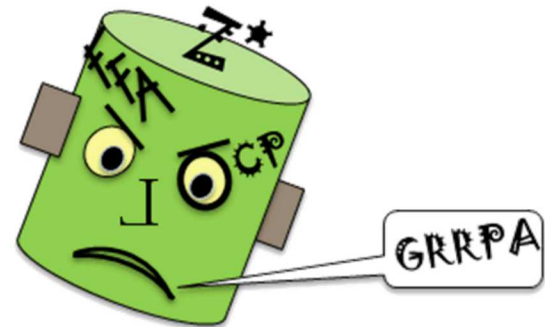
For complex systems, internally consistent models that can be falsified by comparison to detailed data have more epistemic value than tunable models made to fit integrated data

Conclusions

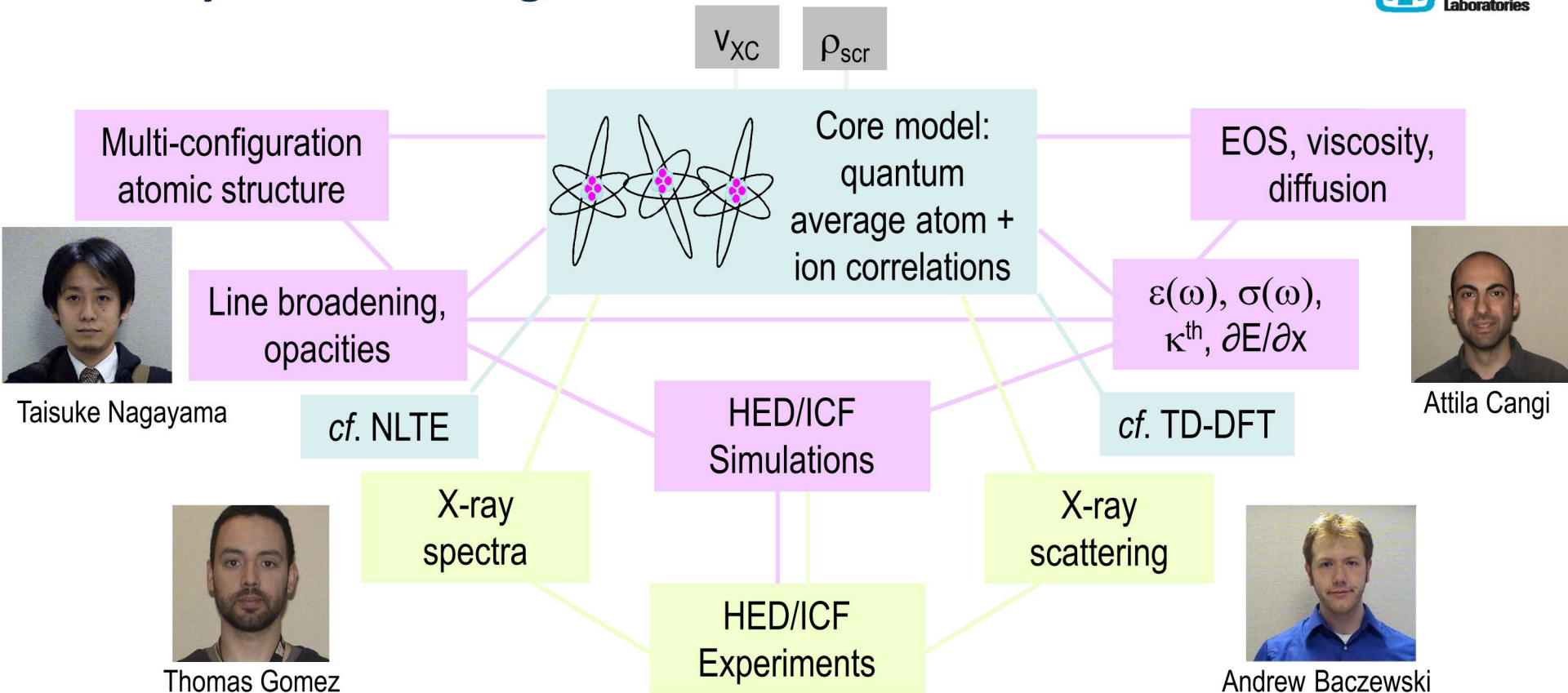
We are working to build a self-consistent model of material at extreme conditions with:

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For complex systems, internally consistent models that can be falsified by comparison to detailed data have more epistemic value than tunable models made to fit integrated data



Summary & acknowledgements



Thanks to:

Charles Starrett, David Kilcrease (LANL), Carlos Iglesias, Brian Wilson, Phil Sterne, Paul Grabowski (LLNL), Richard More

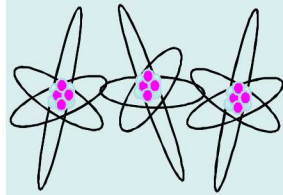
Primary funding: U.S. DOE, Office of Science Early Career Research Program, OFES FWP-14-017426

Thank you!

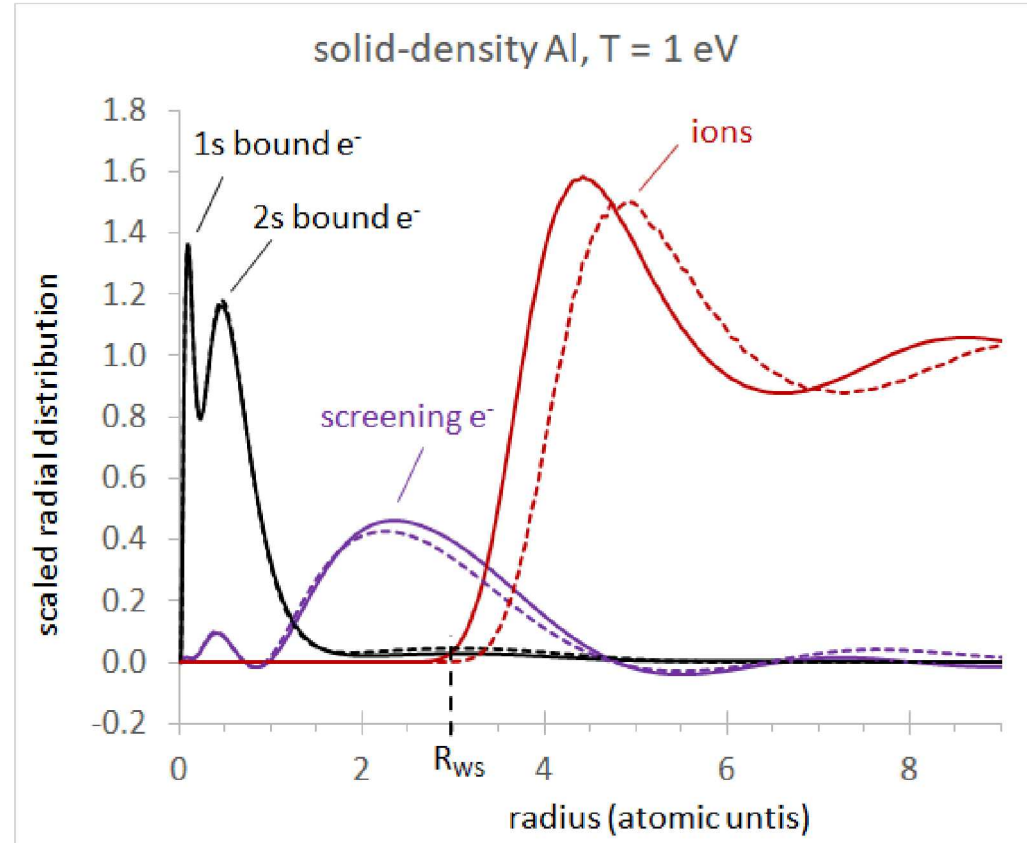
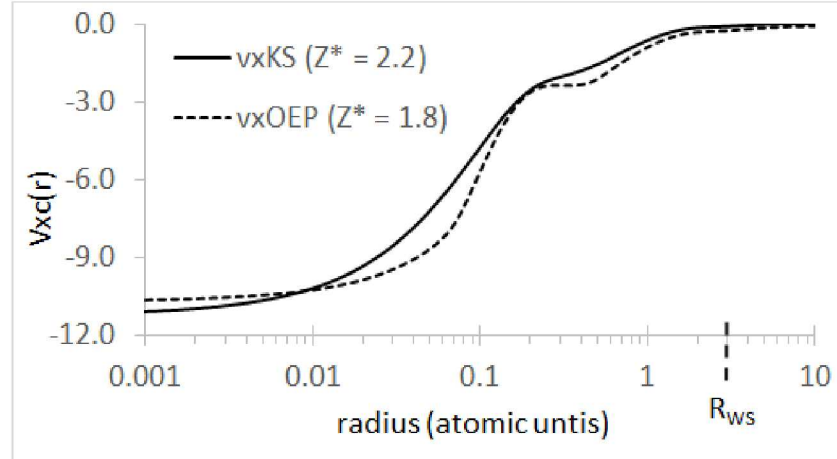
The core model is sensitive to the choice of exchange potential

V_{XC}

ρ_{scr}

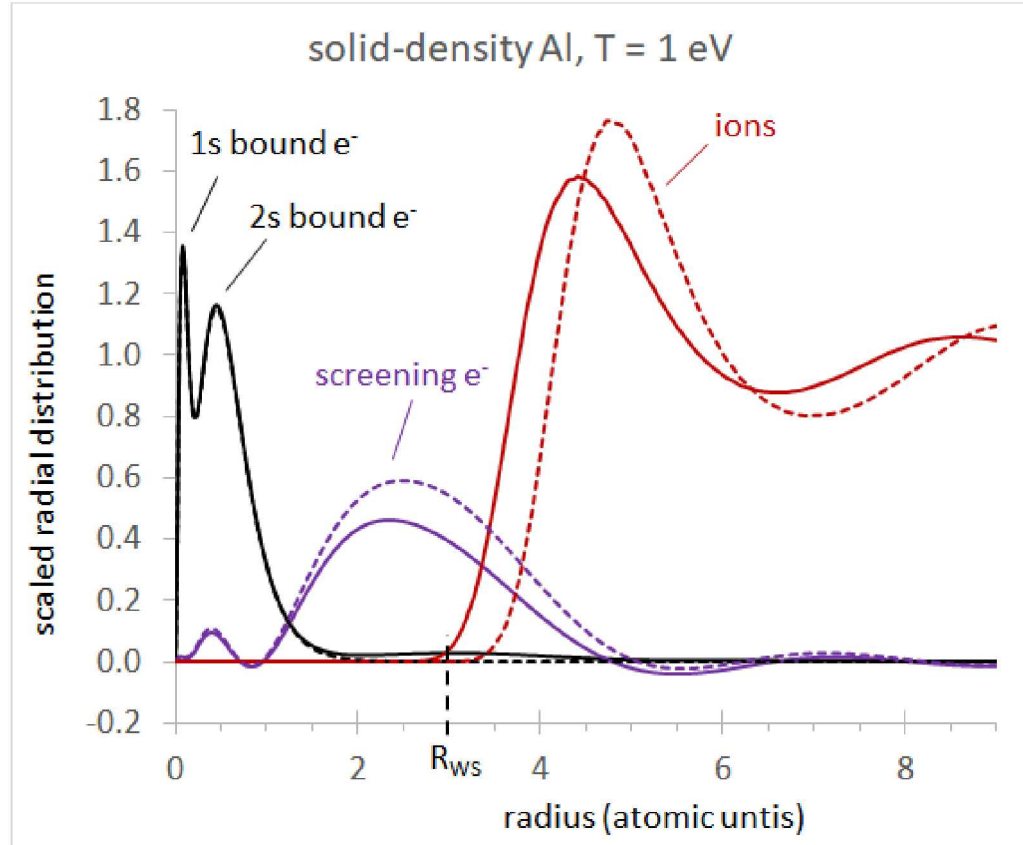
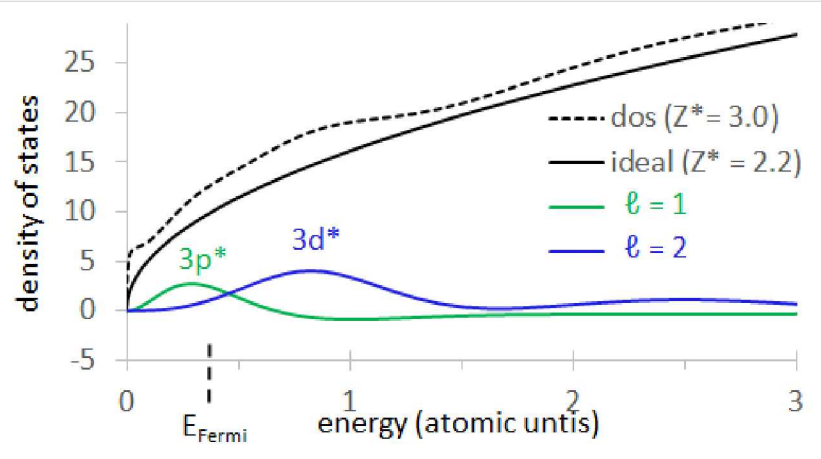
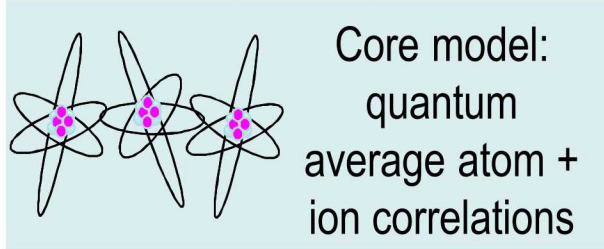


Core model:
quantum
average atom +
ion correlations



The core model is sensitive to whether the screening density includes the pressure-ionized “scars” of bound states

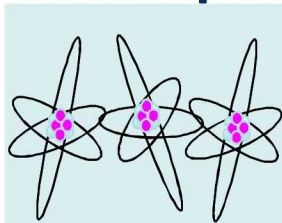
V_{XC} ρ_{scr}



Examining the full dielectric function indicates a missing piece in the standard Chihara decomposition

Chihara decomposition:

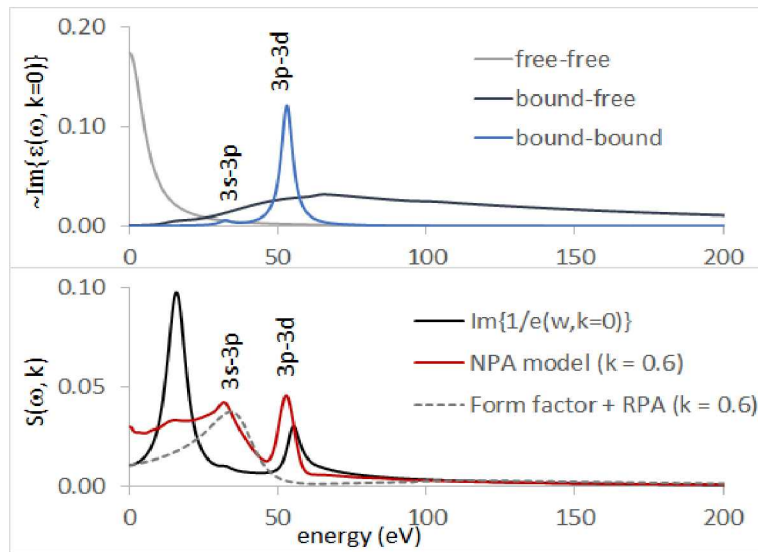
$$S(k, \omega) = \underbrace{|f_I(k) + q(k)|^2 S_{ii}(k, \omega)}_{\text{elastic}} + \underbrace{\bar{Z} S_{ee}(k, \omega)}_{\text{free-free}} + \underbrace{S_{bf}(k, \omega)}_{\text{bound-free}} + \underbrace{S_{bb}(k, \omega)}_{\text{bound-bound}}$$



Core model:
quantum
average atom +
ion correlations

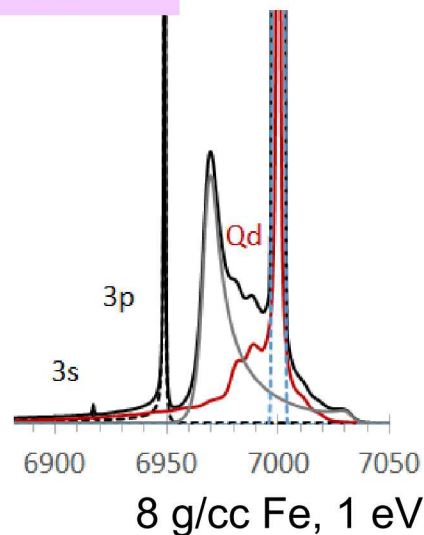
$$\sigma(\omega), \epsilon(\omega), \kappa^{\text{th}}, \partial E / \partial x$$

X-ray
scattering



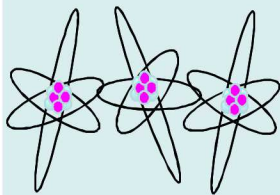
The bound-bound features in $\text{Im}\{1/\epsilon_{bb}(\omega)\}$ are reminiscent of sharp bound-free features previously noted by Johnson *et al.* and Souza *et al.**

5 g/cc Fe, 10 eV



*Johnson *et al.*, *PRE* **86**, 036410 (2012); Souza, Starrett *et al.*, *PRE* **89**, 023108 (2015)

Both the core model and TD-DFT capture bound-bound scattering features in $S(\omega, k)$

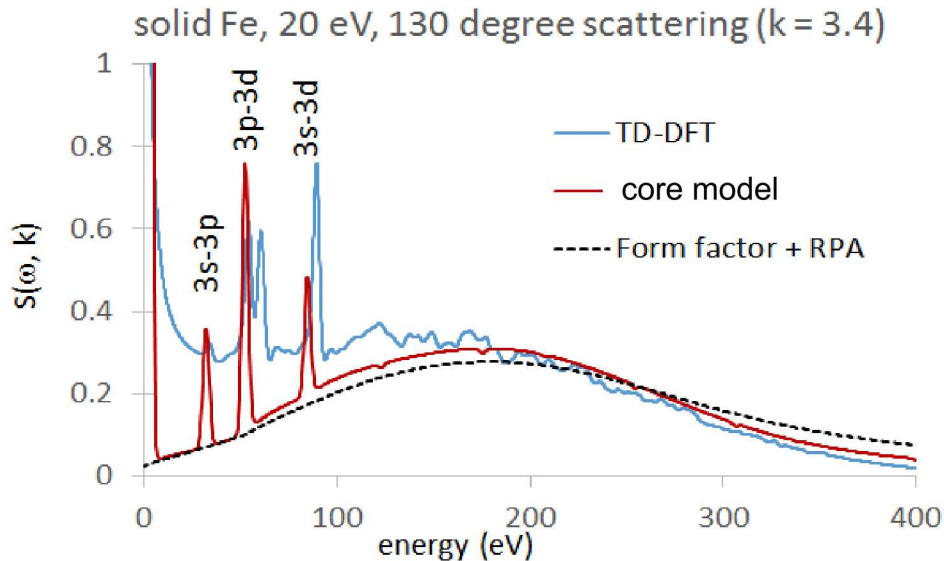


Core model:
quantum
average atom +
ion correlations

While $S(\omega, 0) \sim \text{Im}\{1/\epsilon(\omega, 0)\}$ roughly describes edges and line, a more general $S(\omega, k)$ can be obtained directly from core-model wavefunctions

X-ray
scattering

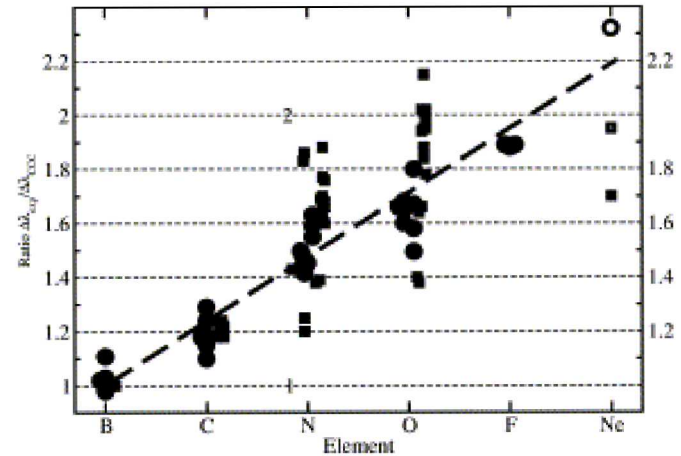
Experimental diagnostics benefit from consistent and complete atomic-scale models



A. Baczewski

A persistent puzzle: isolated line broadening for multi-electron atoms

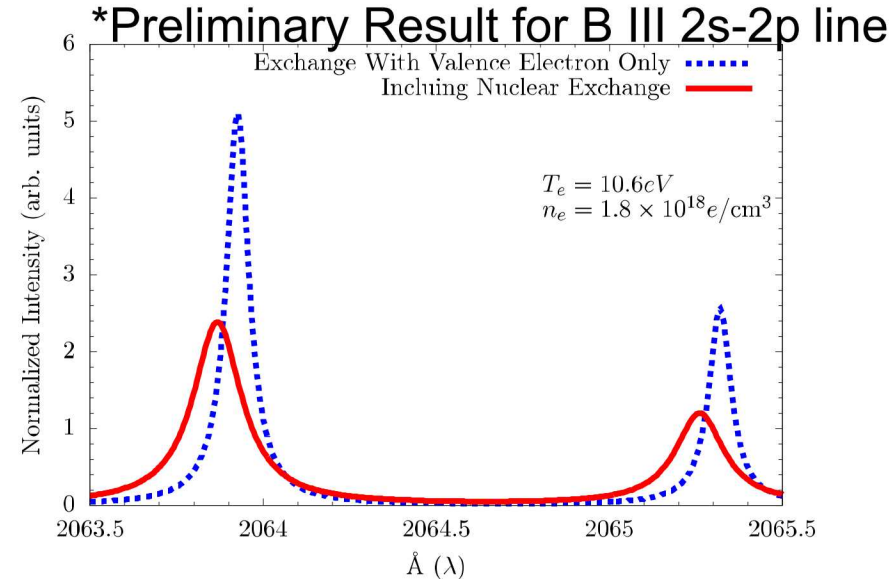
- Efforts to model simple multi-electron atoms (low-Z Li-like ions) show a disturbing disagreement with experiment
- These are not exotic conditions ($T_e \approx 15\text{eV}$; $n_e \approx 2 \times 10^{18} \text{ e/cm}^3$)
- This indicates that there might be some missing physics or something that the community is doing incorrectly
- In order to model more complex atoms, we must resolve this discrepancy with simple 3-electron atoms



Ralchenko et al.
JQSRT 81, 371 (2003)

Interaction Between Radiator and Plasma

- We need to use a complete Coulomb interaction (rather than the simpler dipole approximation)
- We also need to include the effects of the anti-symmetry between atomic and plasma electrons
- Approximations about wavefunctions can neglect some exchange interactions
- These neglected interactions have large consequences



More work is needed to complete this project, but we have identified some physics that requires careful attention because it has a significant effect on line shapes

These calculations (AA-LDA) described Zylstra's recent measurements of 14 MeV proton stopping reasonably well

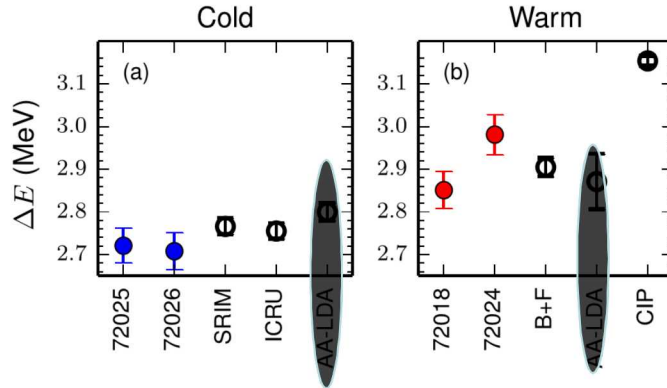


FIG. 4 (color online). Downshift (ΔE) for cold (a) and warm (b) shots compared to theory. The solid points are data (denoted by shot number), and theories are hollow points. The uncertainties in theoretical calculations are due to uncertainties in ρL and plasma conditions.

Mean ionization potential can be calculated simply by averaging $E_{\text{binding}} + E_{\text{Fermi}}$ for all electrons in the Average Atom calculation

The downshift of 14 MeV protons is determined by integrating calculated dE/dx over measured path length

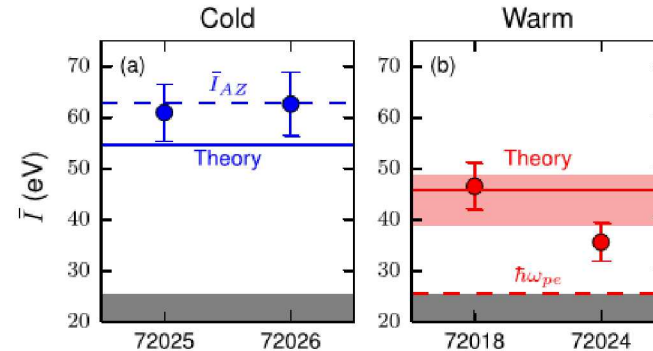
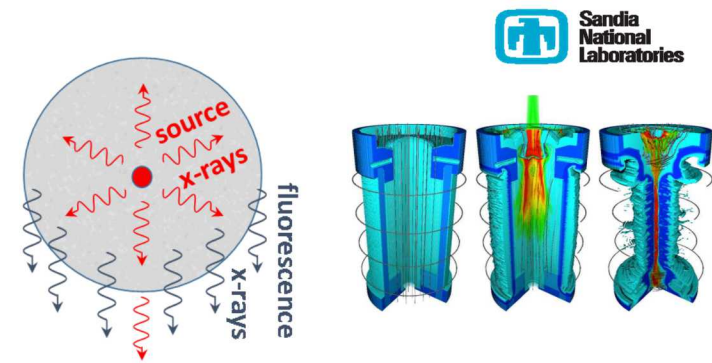
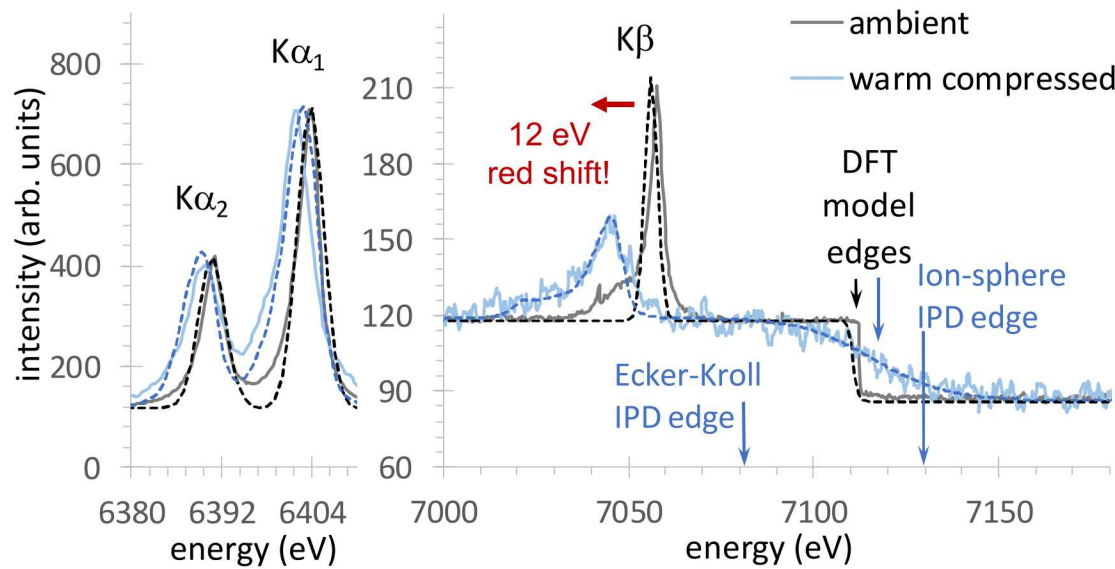


FIG. 5 (color online). Mean ionization potential ($\bar{I}$) inferred from the stopping-power data in the cold (a) and warm (b) cases compared to the Andersen-Ziegler empirical fits ($\bar{I}_{AZ}$), the ideal plasma case ($\hbar\omega_{pe}$), and electronic structure theory.

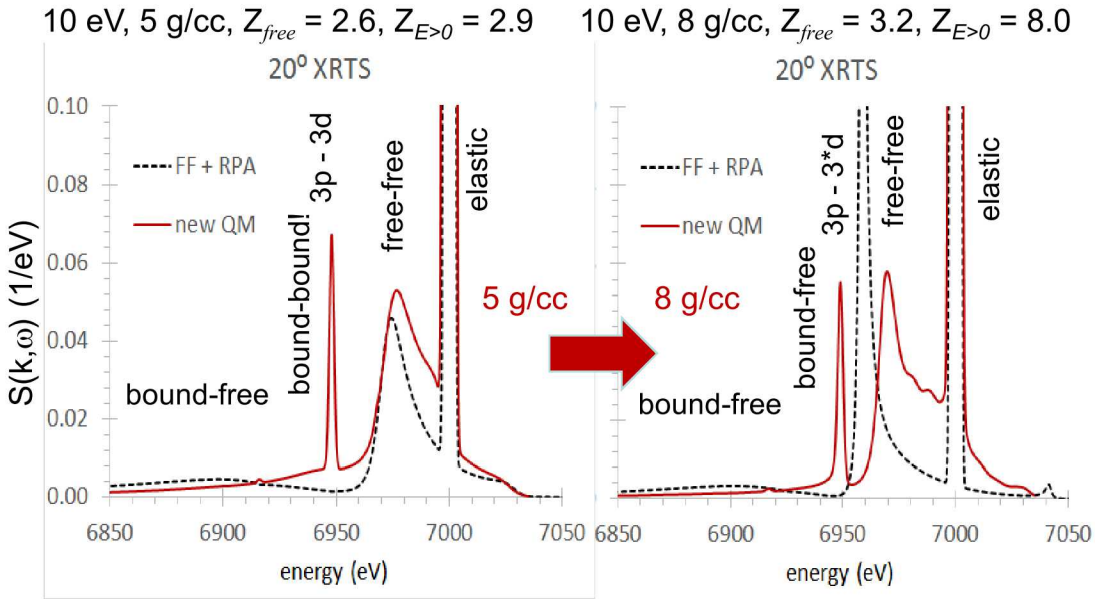
Observables in extreme WDM: absorption edges and fluorescence lines



Calculations (dashed lines) anchored to the K-edge of ambient data (solid gray) show good agreement with line and edge shifts and broadening due from a warm compressed MagLIF liner backlit by stagnation emission (solid blue) with $T \sim 10$ eV and $n_e \sim 2 \times 10^{24}$ e/cc.

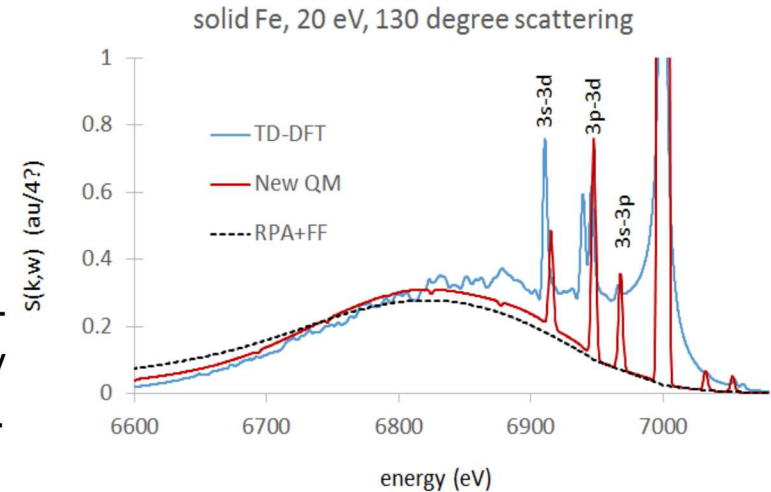
This agreement indicates that self-consistent DFT models describe electronic structure in extreme conditions with better fidelity than ad-hoc models of density effects.

Scattering calculations are also fully constrained



$$S(k, \omega) = \underbrace{|f_I(k) + q(k)|^2 S_{ii}(k, \omega)}_{\text{elastic}} + \underbrace{\bar{Z} S_{ee}(k, \omega)}_{\text{free-free}} + \underbrace{S_{bf}(k, \omega)}_{\text{bound-free}} + S_{bb}(k, \omega) \text{ (bound-bound)}$$

Most components of the scattering signal are calculated using fully self-consistent quantities^{1,2} (free-free uses RPA)



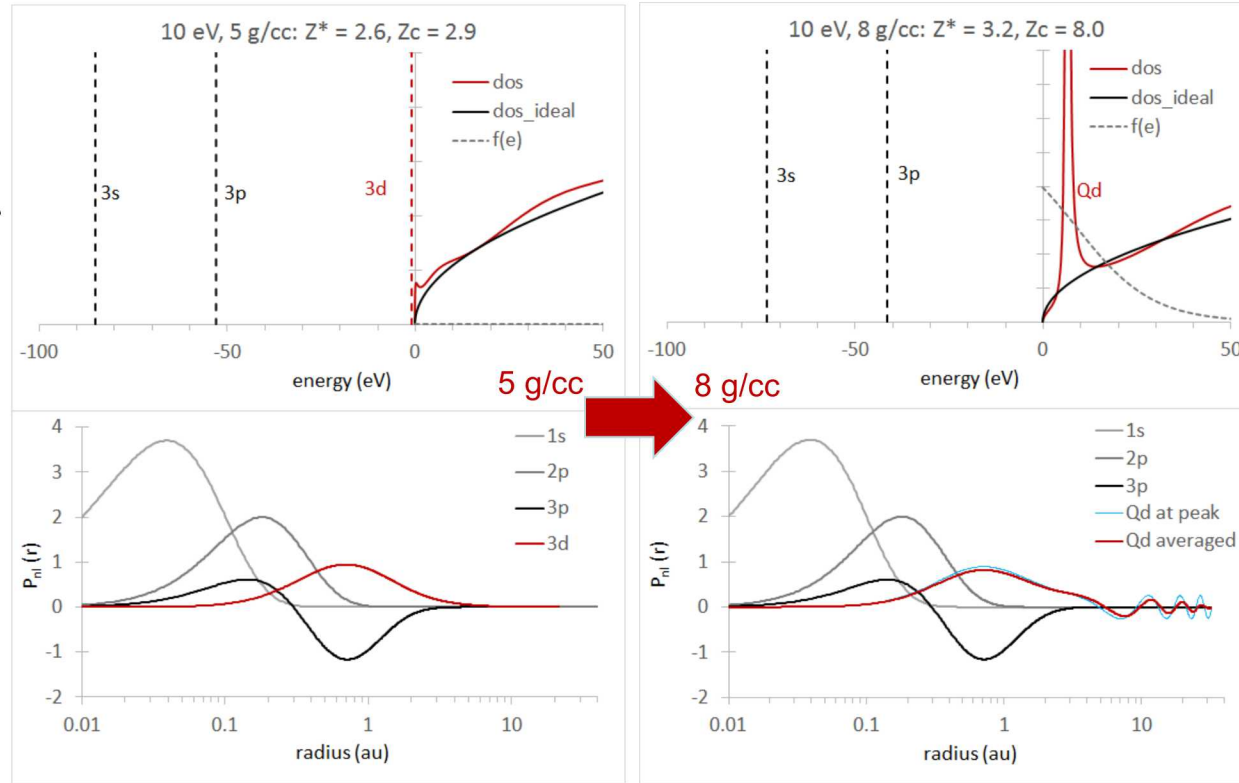
We find good agreement of the self-consistent average-atom model with time-dependent density functional theory
– A. Baczewski *et al.*, PRL 116, 115004 (2016).

Electrons: Quantum mechanical average atom

All-electron, fully quantum-mechanical* semi-relativistic self-consistent field solver with flexible exchange

density of states

wavefunctions



Near-solid iron at $T = 10$ eV

Key ansatz:

Quasi-bound states are averaged over resonance features in the DOS and treated *just like* bound states

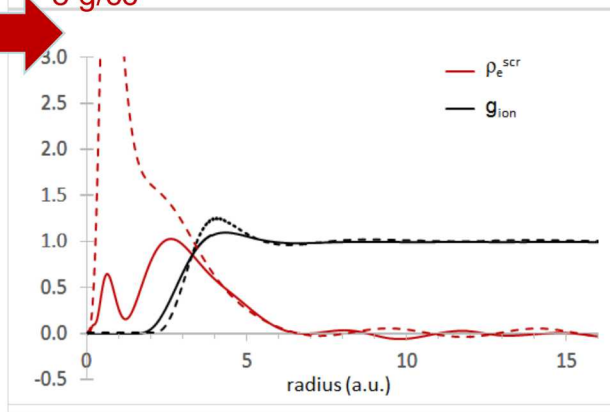
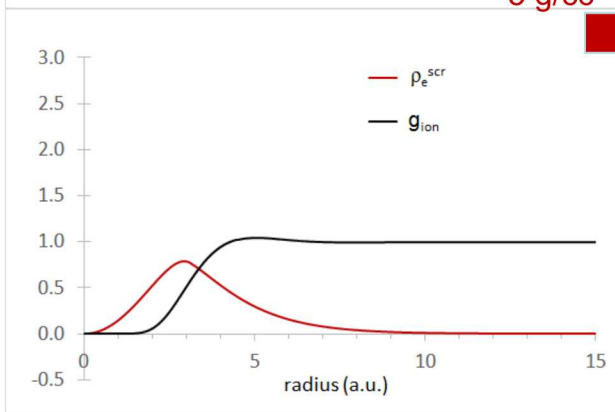
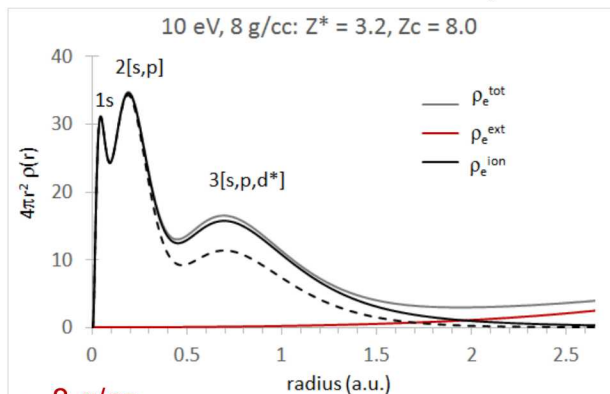
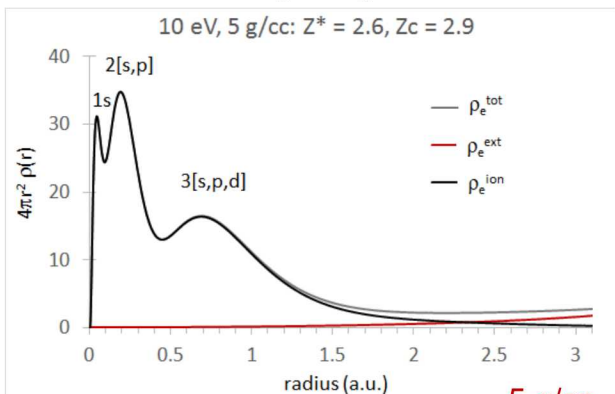
This ensures smooth variation of constitutive & observable properties under pressure ionization – and collapses multiple definitions of Z^* into a single value [cf. Murillo *et al.*, *PRE* **87**, 063113 (2013)]:

$$\begin{aligned}
 Z^* &= \int d\epsilon f(\mu, \epsilon) \text{DOS}_{\text{ideal}} & (Z_{\text{free}}) \\
 &= \int \rho_{\text{scr}} dr = \tilde{V}_{\text{ie}}(k=0) & (Z_{\text{scr}}) \\
 &= \int d\epsilon f(\mu, \epsilon) (1 - f(\mu, \epsilon)) \epsilon^{3/2} & (Z_{\text{Boltz}}) \\
 &= \frac{1}{3} (R_{\text{WS}}^3 / R_{\text{max}}^2) \rho(R_{\text{max}}) & (Z_{\text{asympt}}) \\
 &\approx \frac{1}{3} R_{\text{WS}} \rho(R_{\text{WS}}) & (Z_{\text{WS}}) \\
 &\neq \int d\epsilon f(\mu, \epsilon) \text{DOS} = Z_n - Z_b & (Z_{\epsilon > 0})
 \end{aligned}$$

*D. A. Liberman, *Phys. Rev. B* **20**, 4981 (1979), B.G. Wilson *et al*, *JQSRT* **99**, 658 (2006)

Ions: Quantum Ornstein-Zernike

Self-consistent V_{ij} & g_{ij} obtained by finding electron density with (ρ^{tot}) and without (ρ^{ext}) central charge*



$$\rho^{\text{PA}} = \rho^{\text{tot}} - \rho^{\text{ext}}$$

$$\rho^{\text{scr}} = \rho^{\text{PA}} - \rho^{\text{ion}}$$

Like Z^* , ρ^{ion} is not uniquely defined

New ansatz (solid): $\rho^{\text{ion}} = \rho^{\text{b}} + \rho^{\text{q-b}}$

Standard (dashed): $\rho^{\text{ion}} \approx \rho^{\text{b}}$

The new ansatz leads to smaller changes in screening under pressure ionization and softer $g(r)$

Combined with smooth changes in Z^* , this leads to smooth variations in model predictions for both constitutive properties and experimental observables

*C. Dharma-Wardana & F. Perrot, *Phys. Rev. E* **26**, 2096 (1982), C. Starrett & D. Saumon, *HEDP* **10**, 35 (2014)