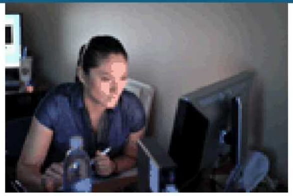
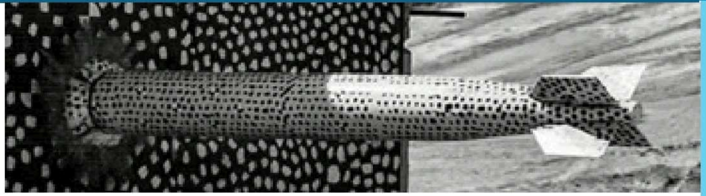
SAND2019-0223C

# Full-Field Strain Shape Estimation From 3D SLDV





SAND NUMBER
Unclassified Unlimited Release

Bryan Witt, Dan Rohe & Tyler Schoenherr
Sandia National Laboratories

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

# Overview

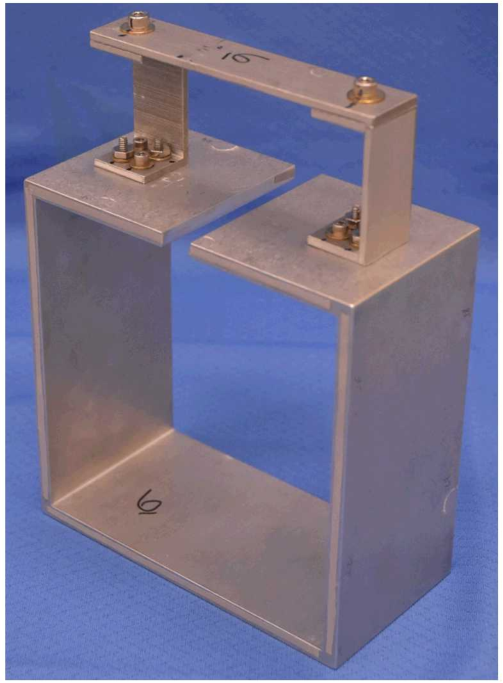
- Motivation
- Test Structure & Setup
- Process and Tools
- LDV Direct Methods
- LDV Transformation Methods
- Comparison of Methods
- Observations
- Future Work

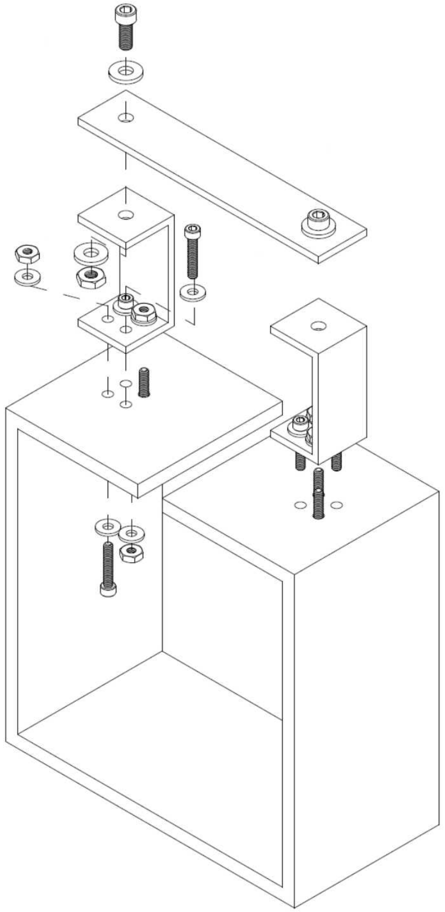
# Motivation

- Full-field strain response measurements are of great importance for:
  - Analytical model validation
  - Characterization of test article for which there is no finite element model (FEM)
  - Determination of environmental test boundary condition suitability
    - ***Does your test stress your part the same way it is stressed in a real system and environment?***
- Optical measurement methods have opened this possibility in recent history
  - Laser Doppler Vibrometry (LDV)
  - Digital Image Correlation (DIC)
- A mode-based model of full-field strain shapes would be advantageous
  - Can assess the contributions of individual modes to total strain field/damage
  - Calculate full-field strain response to a given environment without additional FEM analyses
- ***Desire to use the 3D Scanning LDV system to acquire full-field strain shapes***
  - ***Mode-based model for full-field strain response***

# Test Structure: Background

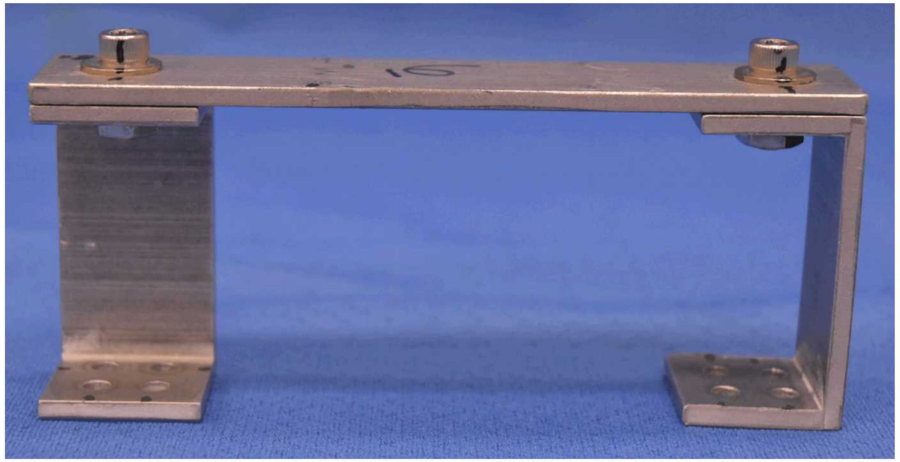
- Box Assembly with Removable Component (BARC)
  - Test bed structure currently being used to explore the effects of boundary conditions in environmental tests
  - As seen at IMAC, ESTECH, etc.

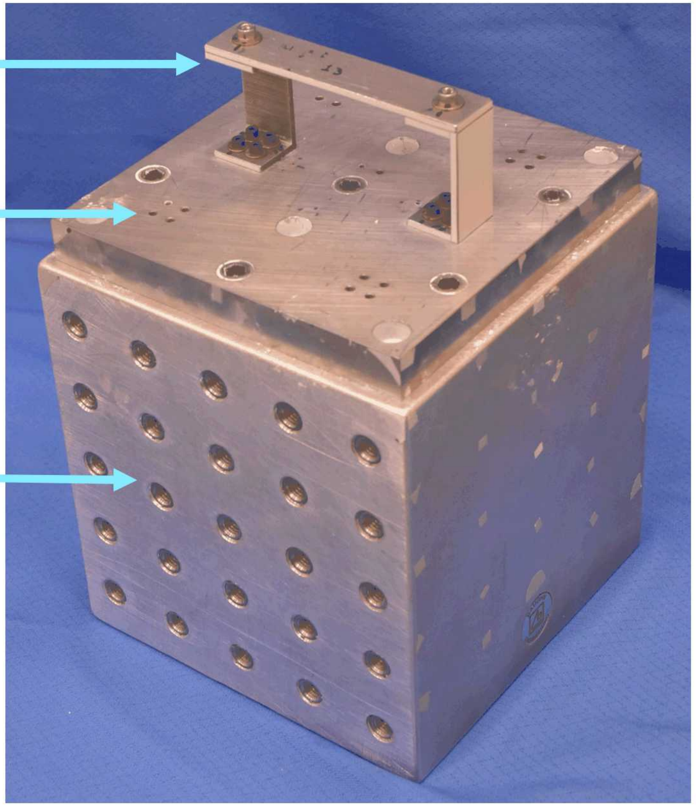




# Test Structure

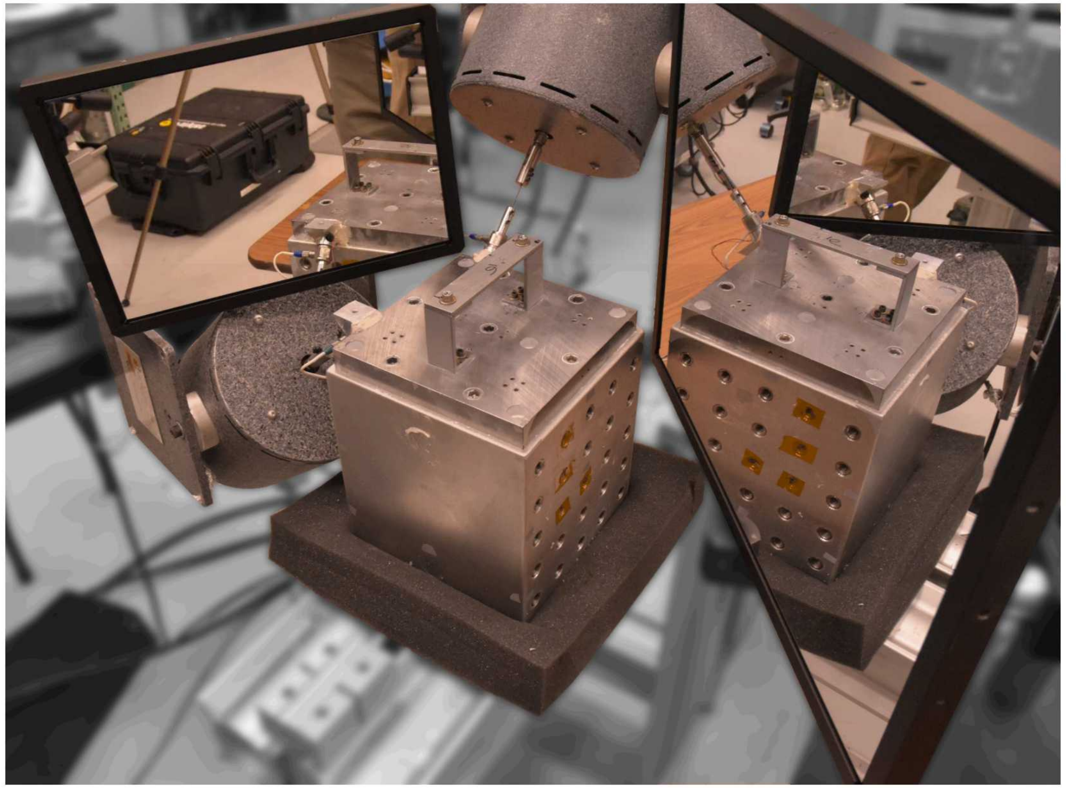
- Test article for this work comprises:
  - Removable Component (bench)
  - On vibration fixture plate
  - Attached to 7" vibration test cube
    - To stiffen the fixture plate





# LDV Test Setup

- Polytec PSV-500 Xtra (IR) 3D-SLDV system
- Two mirrors used to scan three sides of test article without changing configuration
- Test article supported on foam to approximate free-free boundary condition

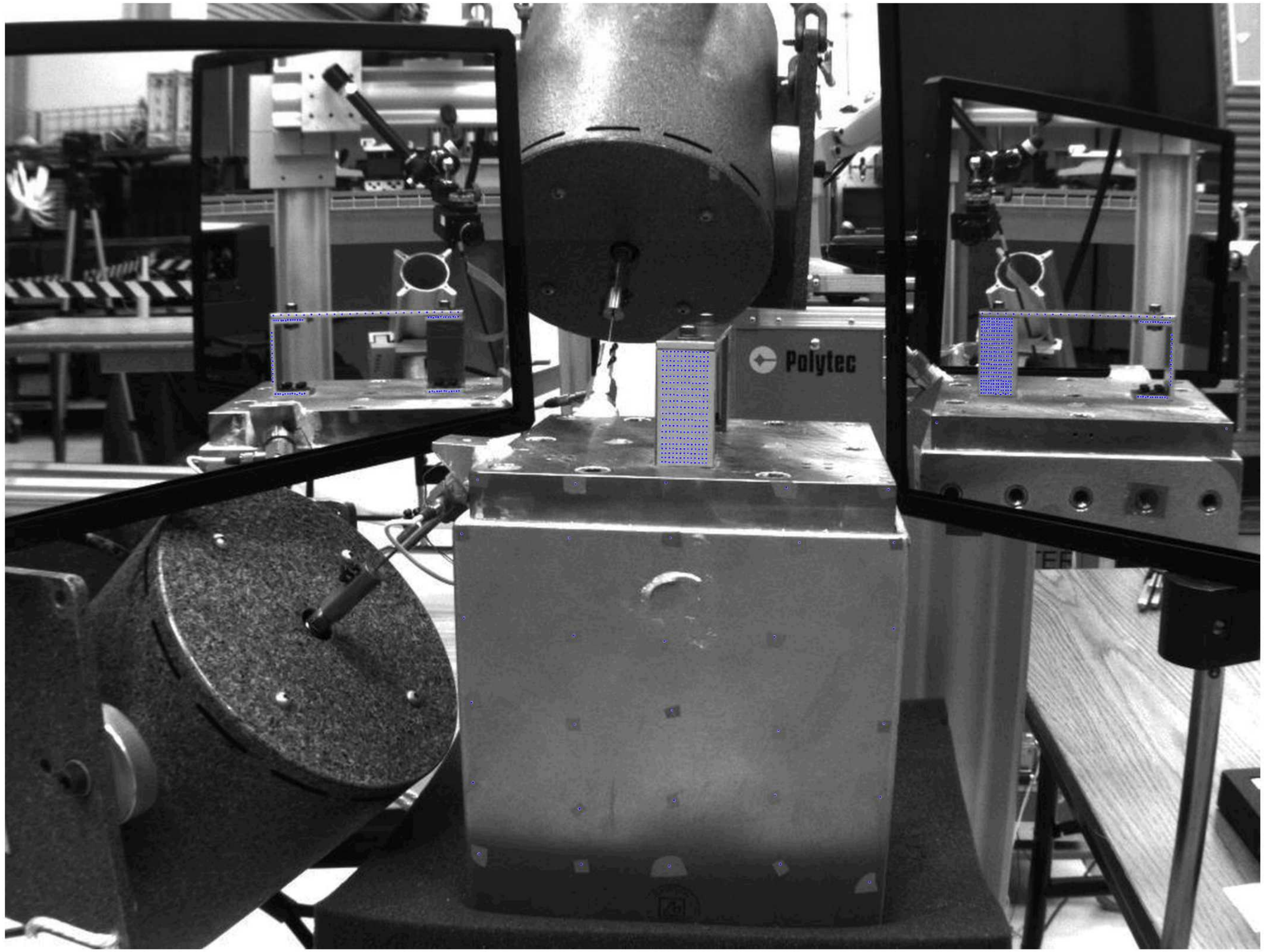


# LDV Test Setup

- Two types of tests were performed:
  - **Random Excitation**
    - Obtain **displacement mode shapes** and natural frequencies $(\boldsymbol{E}, \boldsymbol{\omega})$
  - **Sine-Dwell Excitation**
    - Obtain **operational deflection shapes** (ODS) at selected frequencies $(\boldsymbol{O}, \boldsymbol{f})$
    - Sine dwell at resonance frequencies determined from Random modal test

| Test | Vibrometer Settings | | | | | | |
|---|---|---|---|---|---|---|---|
| | Measurement Bandwidth (Hz) | Sample Rate (Hz) | Lines | Δf (Hz) | Averages | Signal Enhance | Speckle Tracking |
| Random | 1-6400 | 16000 | 6400 | 1.0 | 50 | Standard | Enabled |
| Sine Dwell | 1-6400 | 16000 | 6400 | 1.0 | 100 | Standard | Enabled |

| Test | Excitation | | |
|---|---|---|---|
| | Signal Type | Window | Excitation Bandwidth (Hz) |
| Random | Random | Hann | 10-6400 |
| Sine Dwell | Sine Dwell | None | Resonance Tones |



# Initial FEM Correlation

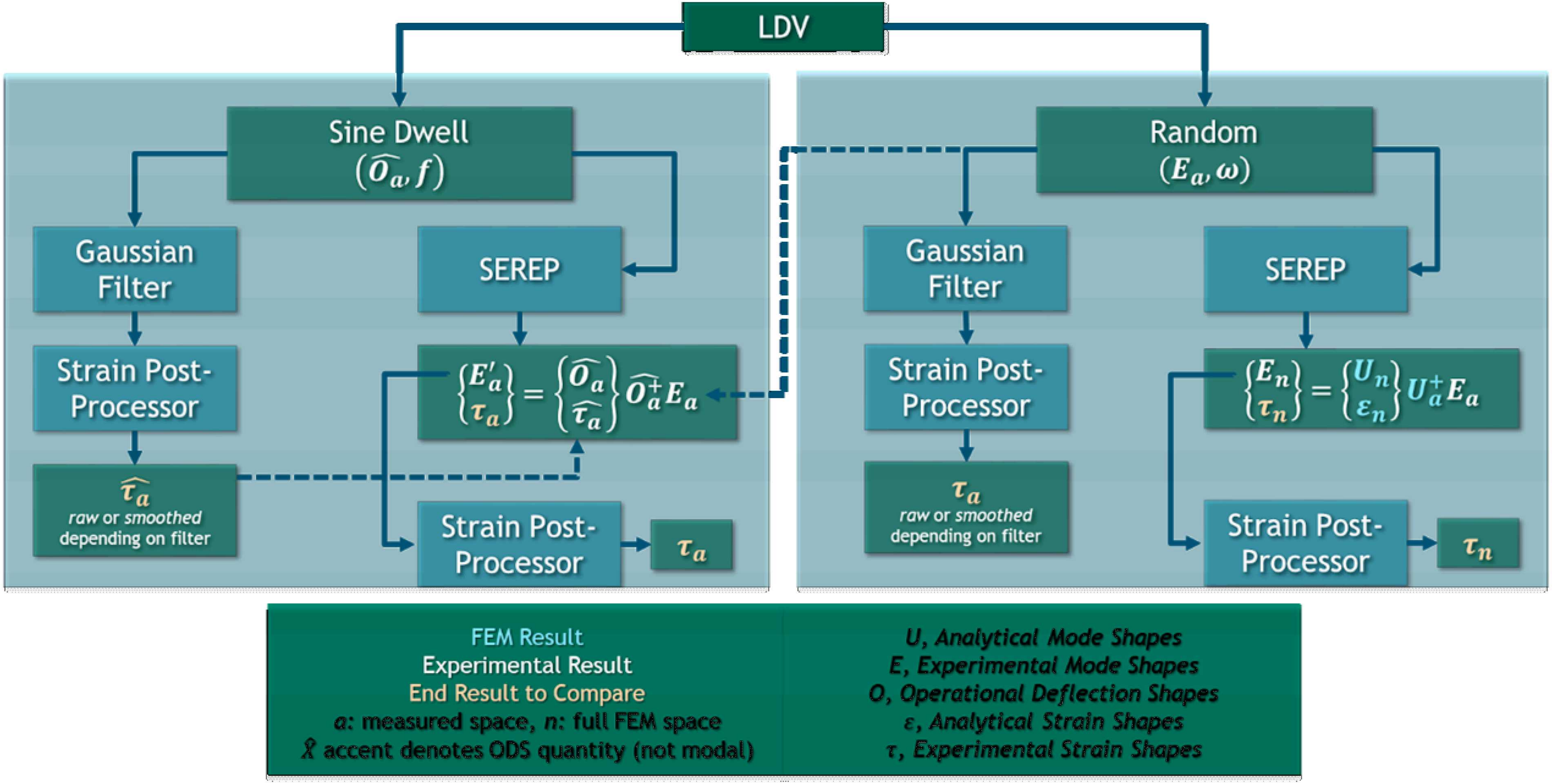
- Very good initial correlation to test data

| MAC | | Test Shapes | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| FEM Shapes | 1 | 0.02 | 0.18 | 0.08 | 0.00 | 0.02 | 0.08 | 0.02 | 0.03 | 0.05 | 0.05 |
| | 2 | 0.16 | 0.01 | 0.03 | 0.02 | 0.18 | 0.02 | 0.00 | 0.04 | 0.00 | 0.09 |
| | 3 | 0.07 | 0.09 | 0.27 | 0.00 | 0.02 | 0.10 | 0.01 | 0.00 | 0.03 | 0.03 |
| | 4 | 0.48 | 0.02 | 0.02 | 0.34 | 0.01 | 0.00 | 0.09 | 0.10 | 0.03 | 0.03 |
| | 5 | 0.00 | 0.18 | 0.58 | 0.01 | 0.01 | 0.20 | 0.00 | 0.01 | 0.13 | 0.03 |
| | 6 | 0.22 | 0.01 | 0.00 | 0.03 | 0.38 | 0.00 | 0.00 | 0.14 | 0.00 | 0.28 |
| | 7 | 0.99 | 0.02 | 0.00 | 0.14 | 0.01 | 0.00 | 0.05 | 0.00 | 0.00 | 0.04 |
| | 8 | 0.03 | 0.95 | 0.02 | 0.03 | 0.00 | 0.16 | 0.02 | 0.00 | 0.07 | 0.00 |
| | 9 | 0.00 | 0.17 | 0.95 | 0.04 | 0.01 | 0.10 | 0.00 | 0.02 | 0.09 | 0.01 |
| | 10 | 0.10 | 0.03 | 0.00 | 0.95 | 0.00 | 0.02 | 0.02 | 0.01 | 0.04 | 0.04 |
| | 11 | 0.01 | 0.01 | 0.01 | 0.07 | 0.93 | 0.01 | 0.00 | 0.03 | 0.01 | 0.11 |
| | 12 | 0.00 | 0.19 | 0.12 | 0.02 | 0.00 | 0.99 | 0.01 | 0.01 | 0.20 | 0.00 |
| | 13 | 0.05 | 0.03 | 0.00 | 0.01 | 0.01 | 0.00 | 0.98 | 0.01 | 0.00 | 0.00 |
| | 14 | 0.00 | 0.12 | 0.09 | 0.08 | 0.01 | 0.21 | 0.00 | 0.11 | 0.97 | 0.00 |
| | 15 | 0.01 | 0.12 | 0.08 | 0.07 | 0.00 | 0.22 | 0.00 | 0.02 | 0.93 | 0.02 |
| | 16 | 0.05 | 0.00 | 0.01 | 0.04 | 0.17 | 0.00 | 0.01 | 0.08 | 0.01 | 0.96 |
| | 17 | 0.04 | 0.15 | 0.04 | 0.04 | 0.00 | 0.22 | 0.06 | 0.01 | 0.20 | 0.00 |
| | 18 | 0.06 | 0.02 | 0.04 | 0.15 | 0.00 | 0.07 | 0.03 | 0.07 | 0.24 | 0.04 |

Rigid Body Modes (FEM Shapes 1–6)

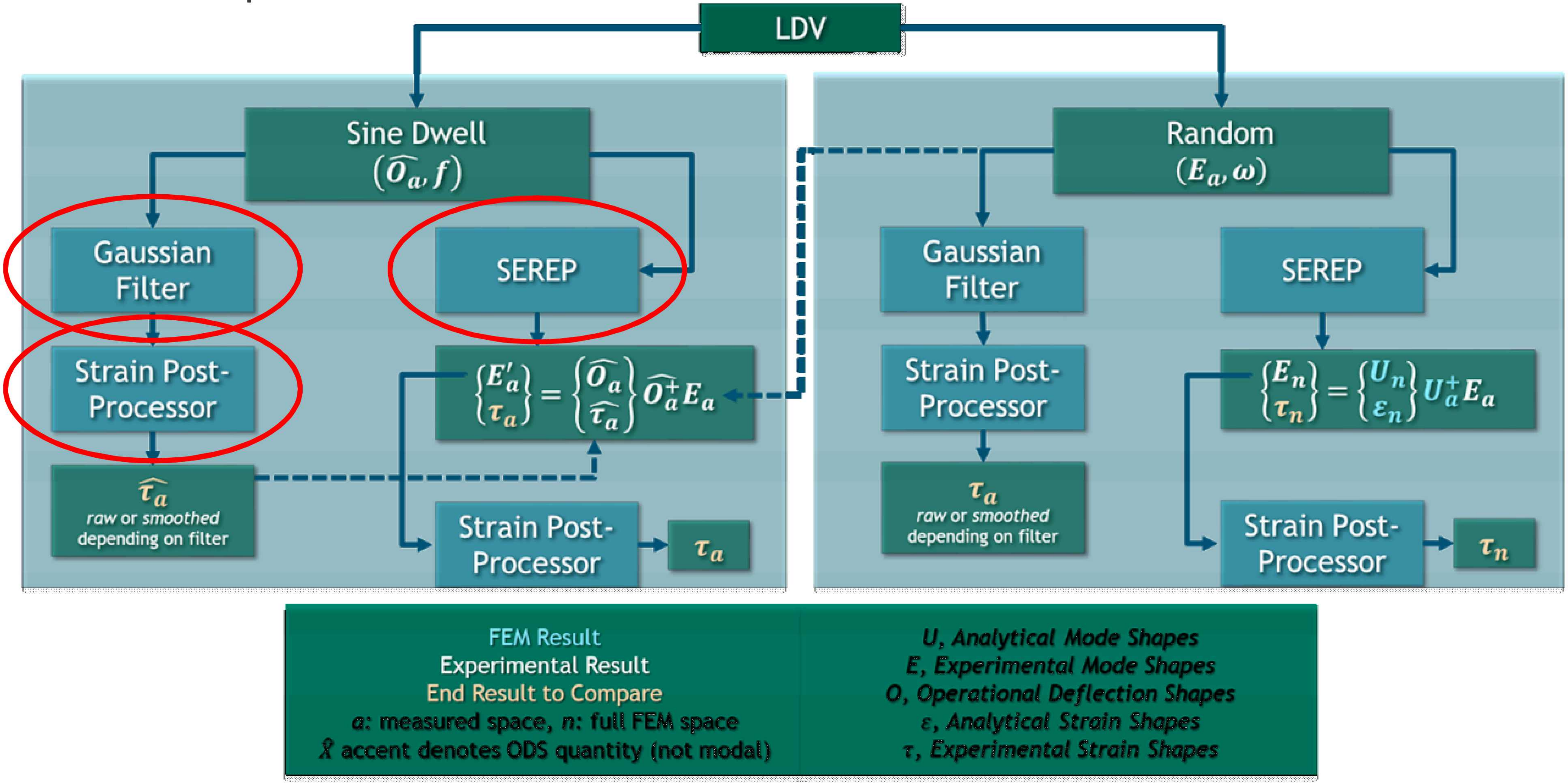
# Process and Tools

# Road Map



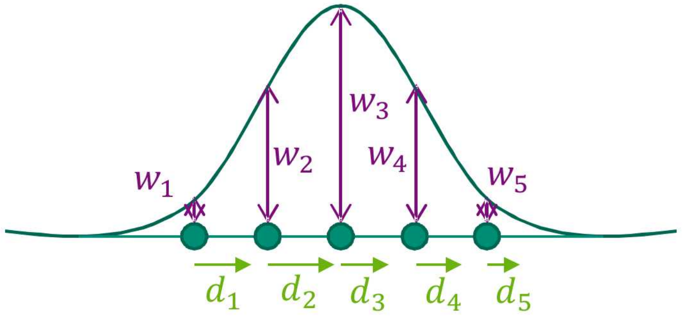
LDV
Sine Dwell
$(\widehat{O}_a, f)$
Gaussian Filter
Strain Post-Processor
$\widehat{\tau}_a$
raw or smoothed depending on filter
SEREP
$\begin{Bmatrix} E'_a \\ \tau_a \end{Bmatrix} = \begin{Bmatrix} \widehat{O}_a \\ \widehat{\tau}_a \end{Bmatrix} \widehat{O}_a^+ E_a$
Strain Post-Processor
$\tau_a$
Random
$(E_a, \omega)$
Gaussian Filter
Strain Post-Processor
$\tau_a$
raw or smoothed depending on filter
SEREP
$\begin{Bmatrix} E_n \\ \tau_n \end{Bmatrix} = \begin{Bmatrix} U_n \\ \varepsilon_n \end{Bmatrix} U_a^+ E_a$
Strain Post-Processor
$\tau_n$
FEM Result
Experimental Result
End Result to Compare
a: measured space, n: full FEM space
$\widehat{X}$ accent denotes ODS quantity (not modal)
U, Analytical Mode Shapes
E, Experimental Mode Shapes
O, Operational Deflection Shapes
ε, Analytical Strain Shapes
τ, Experimental Strain Shapes

# Road Map: Tools



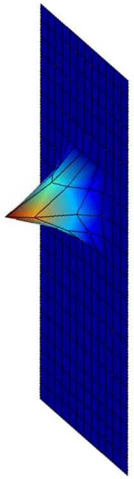
# Tools: Gaussian Spatial Filter

- Raw displacements from the scanning laser system tend to be noisy, and the derivative in the strain computation exacerbates this noise
- Low-pass filtering is used to smooth the data prior to calculating strain.
- A 2D gaussian function with a given size ($\sigma$) is used to perform a weighted average of the displacements around the point.

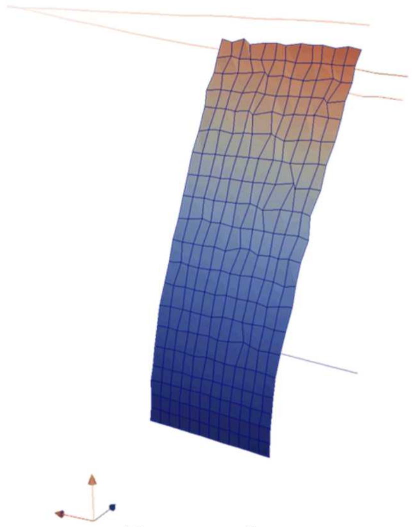



$$\overline{d_3} = \frac{w_1 d_1 + w_2 d_2 + w_3 d_3 + w_4 d_4 + w_5 d_5}{w_1 + w_2 + w_3 + w_4 + w_5} = \frac{\sum_i w_i d_i}{\sum_i w_i}$$

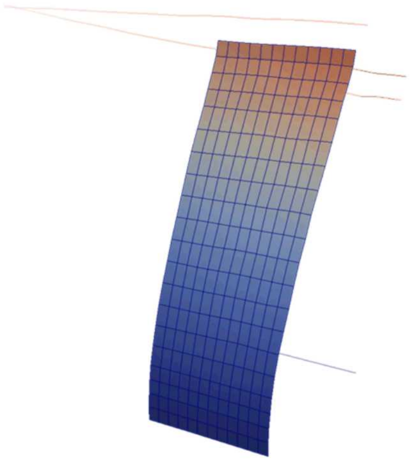
Filter Description



Filter weights in 3D



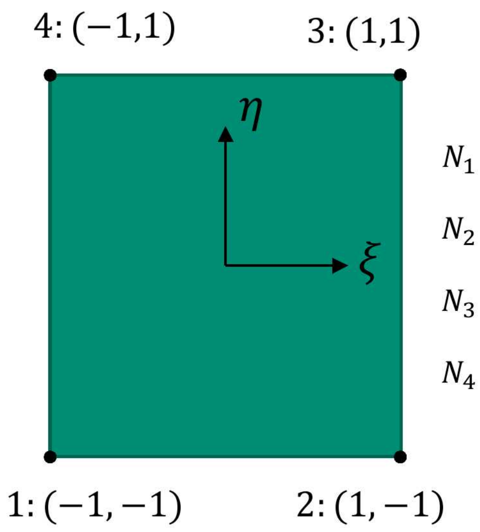
Raw shape



Filtered shape

# Tools: Strain Post-Processor

- Polytec Strain Post-processor operates on Band Data within scan files.
  - In order to directly compute strain from an arbitrary shape (mode shape, ODS, environment time step), it would need to be packaged into the scan file for the Polytec Strain Post-processor to operate on it.
- Rather than dealing with Polytec's software, a new Matlab-based Strain Post-Processor was written:
  - Extracts geometry and mesh connectivity from the scan file to create elements
  - Extracts nodal displacements from the scan file to deform the elements
  - Utilizes bilinear quadrilateral element (Q4) formulation to compute strain at various points on each element
  - We compute strain at the center $(x, y) = (0,0)$ of each element, interpolate to points common to FEM for comparison.




$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$
$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$
$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{xx} \\ \tau_{yy} \\ 2\tau_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \dots & \frac{\partial N_m}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & \dots & 0 & \frac{\partial N_m}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \dots & \frac{\partial N_m}{\partial y} & \frac{\partial N_m}{\partial x} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \mathbf{Bu}$$

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \boldsymbol{J}^{-1} \begin{bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{bmatrix}; \quad \boldsymbol{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

# Tools: SEREP

- System Equivalent Reduction/Expansion Process (SEREP) [1]
  - Method championed by Pete Avitabile at University of Mass. Lowell
  - Shape vectors are used as the transformation basis (opposed to M, K matrices)
  - Select which DOF and modes you want to retain
    - Can preserve arbitrary set of modes of interest
    - Insensitive to the number and location of DOF retained

- *n-Space* = full DOF set
- *a-Space* = reduced DOF set
- U = Analytical Shape (FEM)
- E = Experimental Shape

*Transformation Matrix* $\underset{[n,a]}{T} = \underset{[n,m]}{U_n}\,\underset{[m,a]}{U_a^+}$

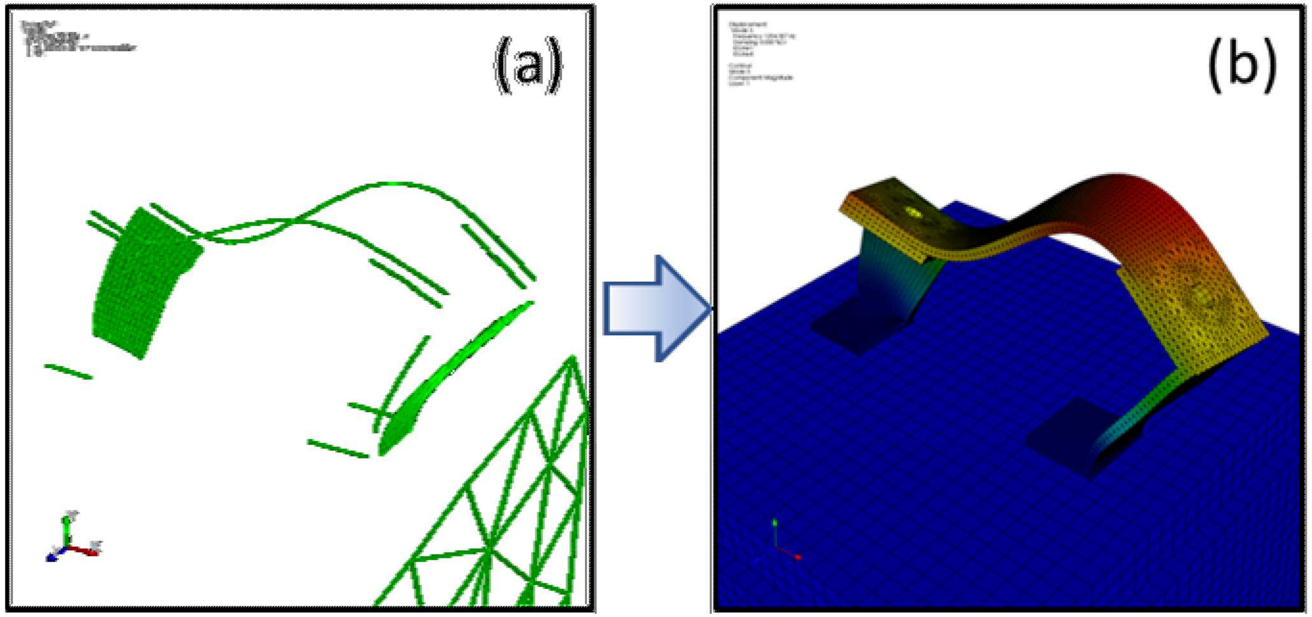
*Expanded Test Shapes* $E_n = TE_a$

It's simple!
But the devil is in the details…

[1] **System Equivalent Reduction Expansion Process (SEREP), J.O'Callahan, P.Avitabile, R.Riemer, Seventh International Modal Analysis Conference, Las Vegas, Nevada, February 1989**

# Tools: SEREP

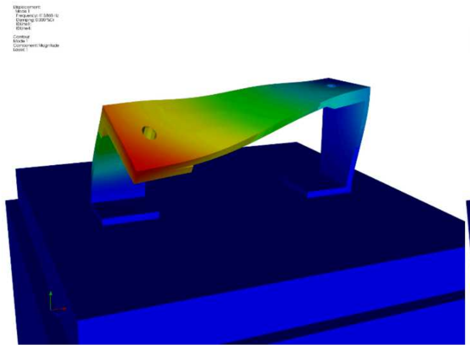
- Need to keep analytical rigid body modes
- Keeping too many DOF can lead to expansion problems
  - Including noisy DOF degrades the best fit
- Need to keep more DOF than the number of preserved modes (*a* > *m*)
- Analytical modes kept need to span the space of the test modes
  - Analytical modes dominated by components that were not measured in test will cause issues in expansion
  - FEM doesn't have to be perfect…but can't be totally off



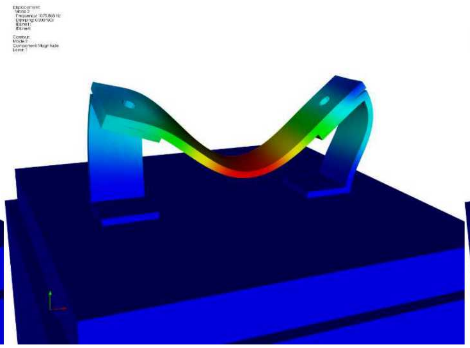
# Tools: SEREP

- Experimental test shapes expanded to full FEM space:

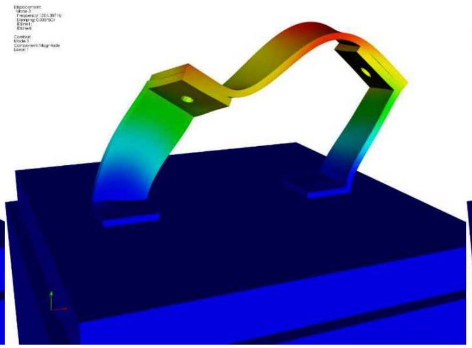
1: 414 Hz



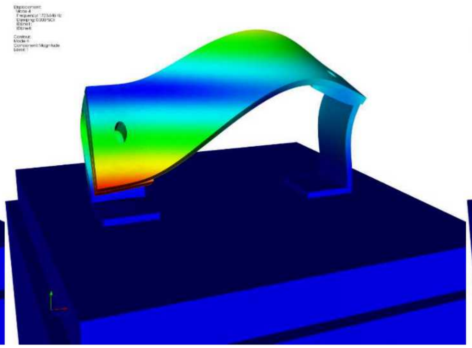
2: 1076 Hz



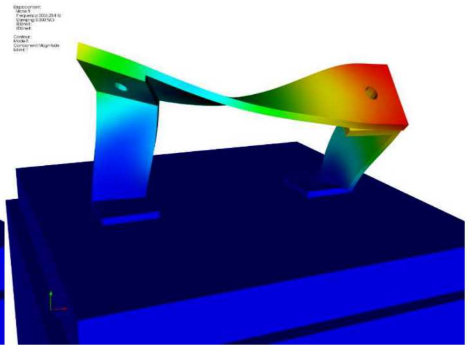
3: 1204 Hz



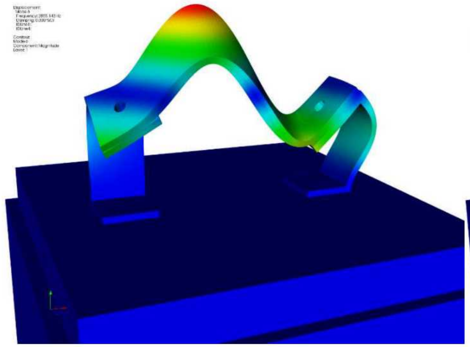
4: 1724 Hz



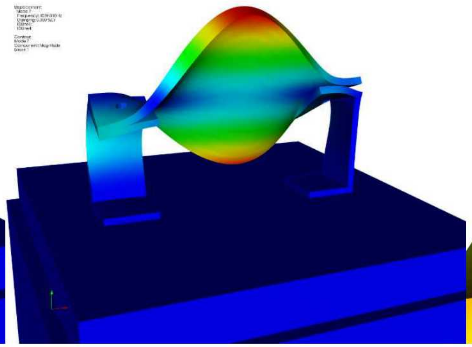
5: 2006 Hz



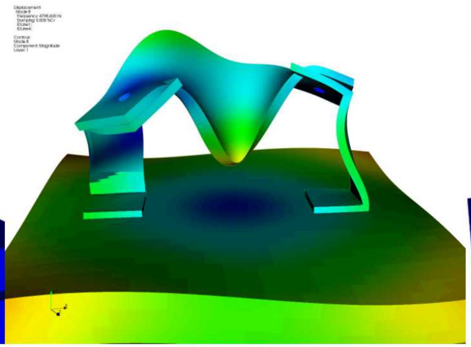
6: 2855 Hz



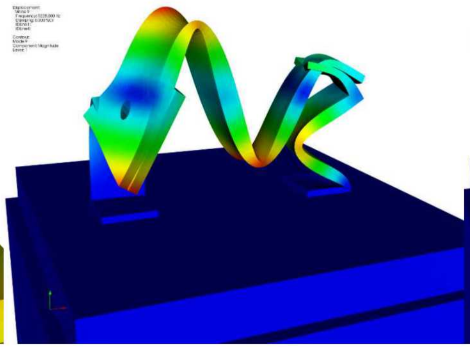
7: 4039 Hz



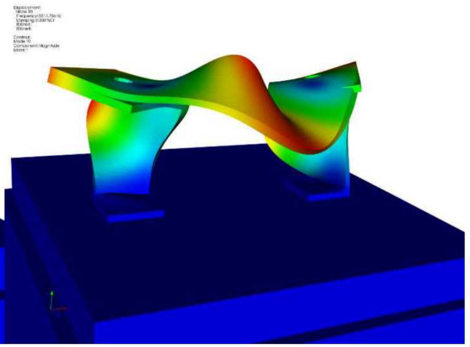
8: 4799 Hz



9: 5228 Hz



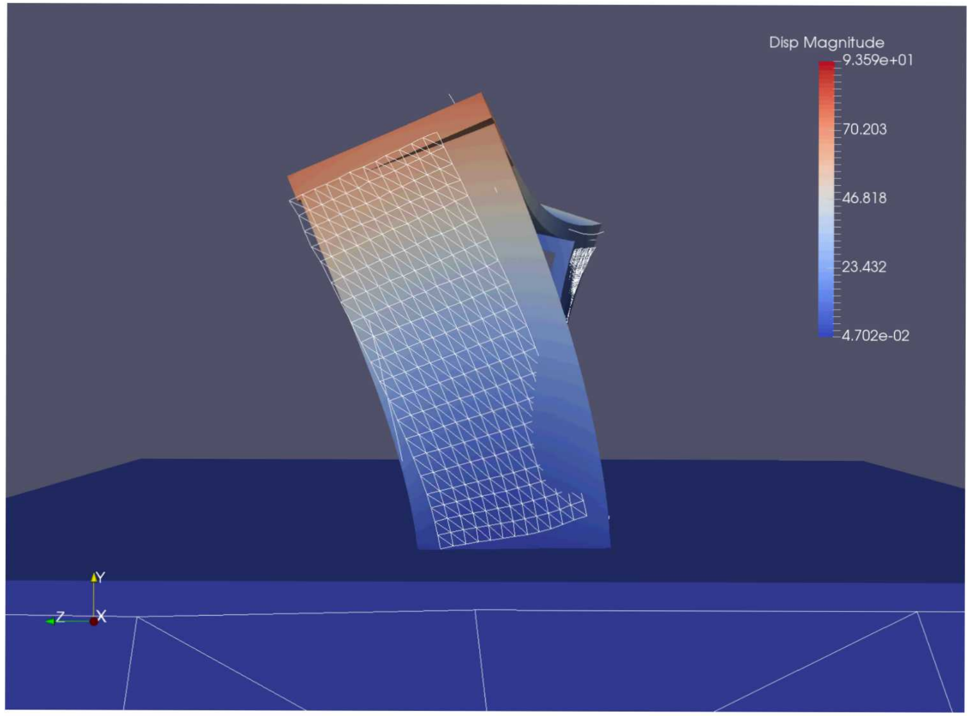
10: 5515 Hz



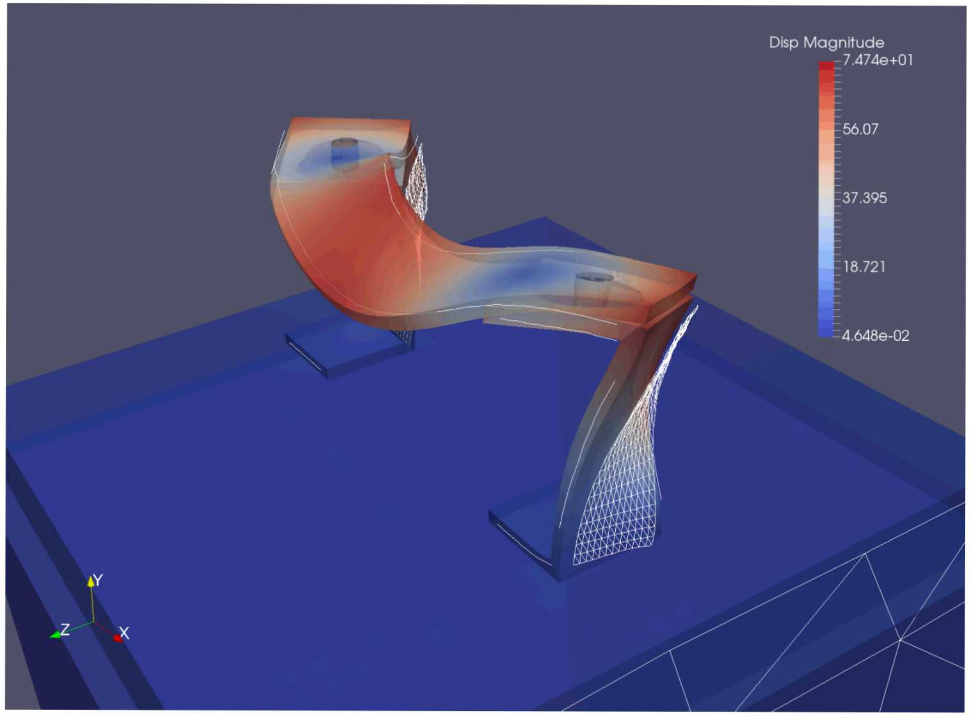
# Tools: SEREP

- Examples of test shape with less than perfect expansions…
  - Basis vectors (FEM shapes) don't quite span the space of certain experimental shapes we're trying to expand
  - Still very useful

Mode 5




Mode 10



# Tools: SEREP

- **Test shapes can be transformed/expanded using other modal quantities**
  - Direct to full *n*-space modal strain shapes, using FEM strains

$$\boldsymbol{\tau}_n = \boldsymbol{\varepsilon}_n \boldsymbol{U}_a^+ \boldsymbol{E}_a$$

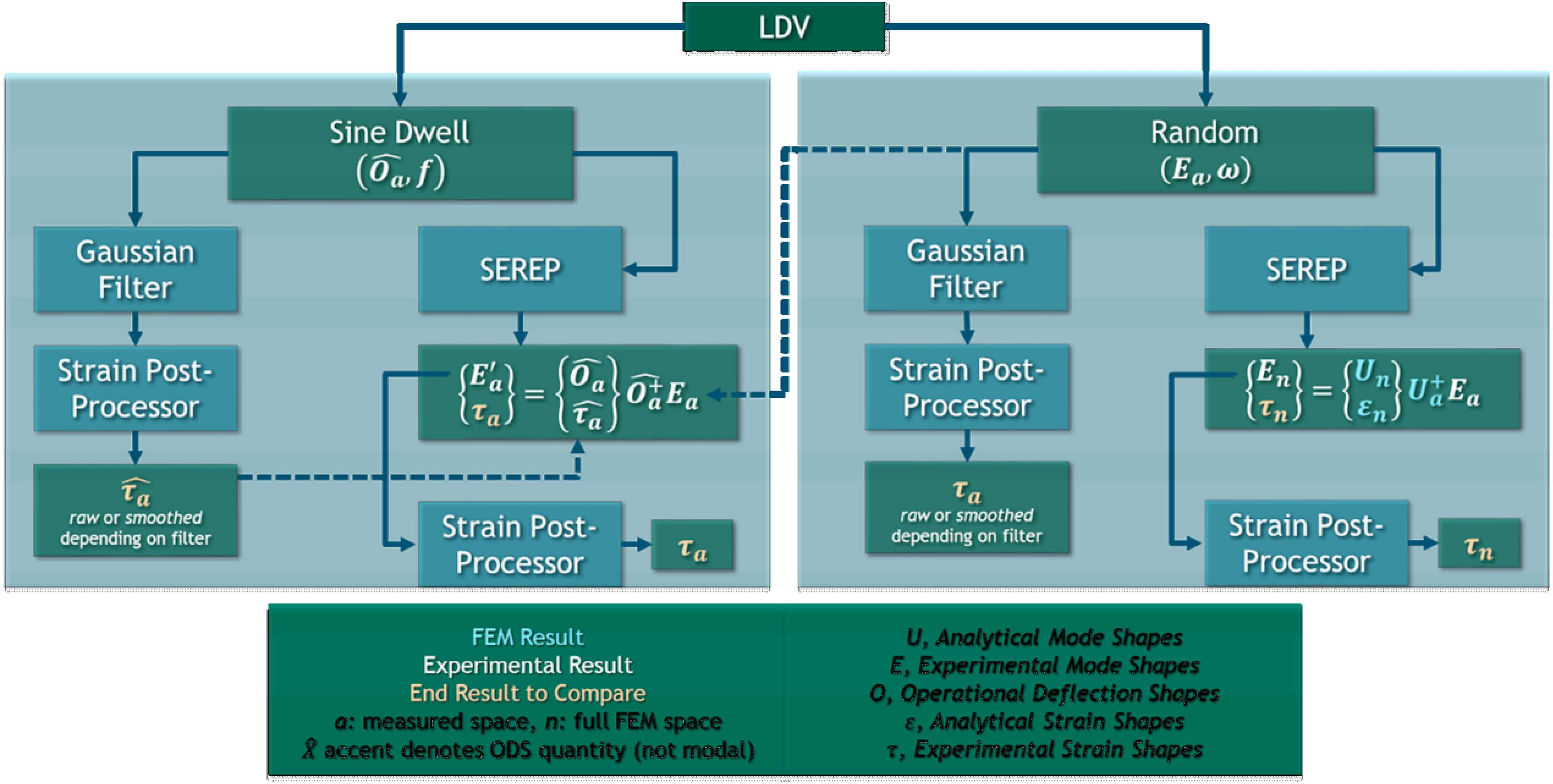
  - Direct to measurement *a*-space modal strain shapes, using ODS shapes and strains

$$\boldsymbol{\tau}_a = \widehat{\boldsymbol{\tau}_a} \widehat{\boldsymbol{O}_a}^+ \boldsymbol{E}_a$$

- **SEREP can also be used as a least squares error minimization**
  - Use to smooth noisy measured mode shapes using (potentially) cleaner ODS (higher signal-to-noise)

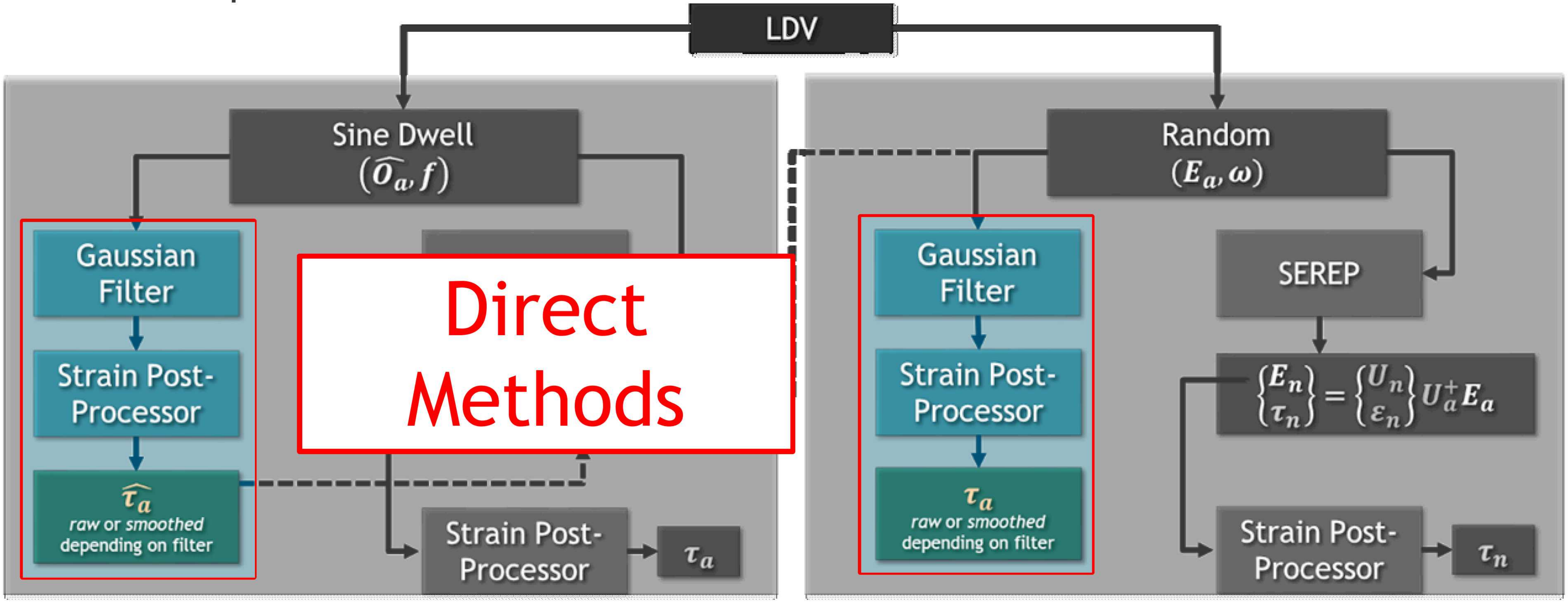
$$\boldsymbol{E}'_a = \widehat{\boldsymbol{O}_a} \widehat{\boldsymbol{O}_a}^+ \boldsymbol{E}_a$$

# Road Map



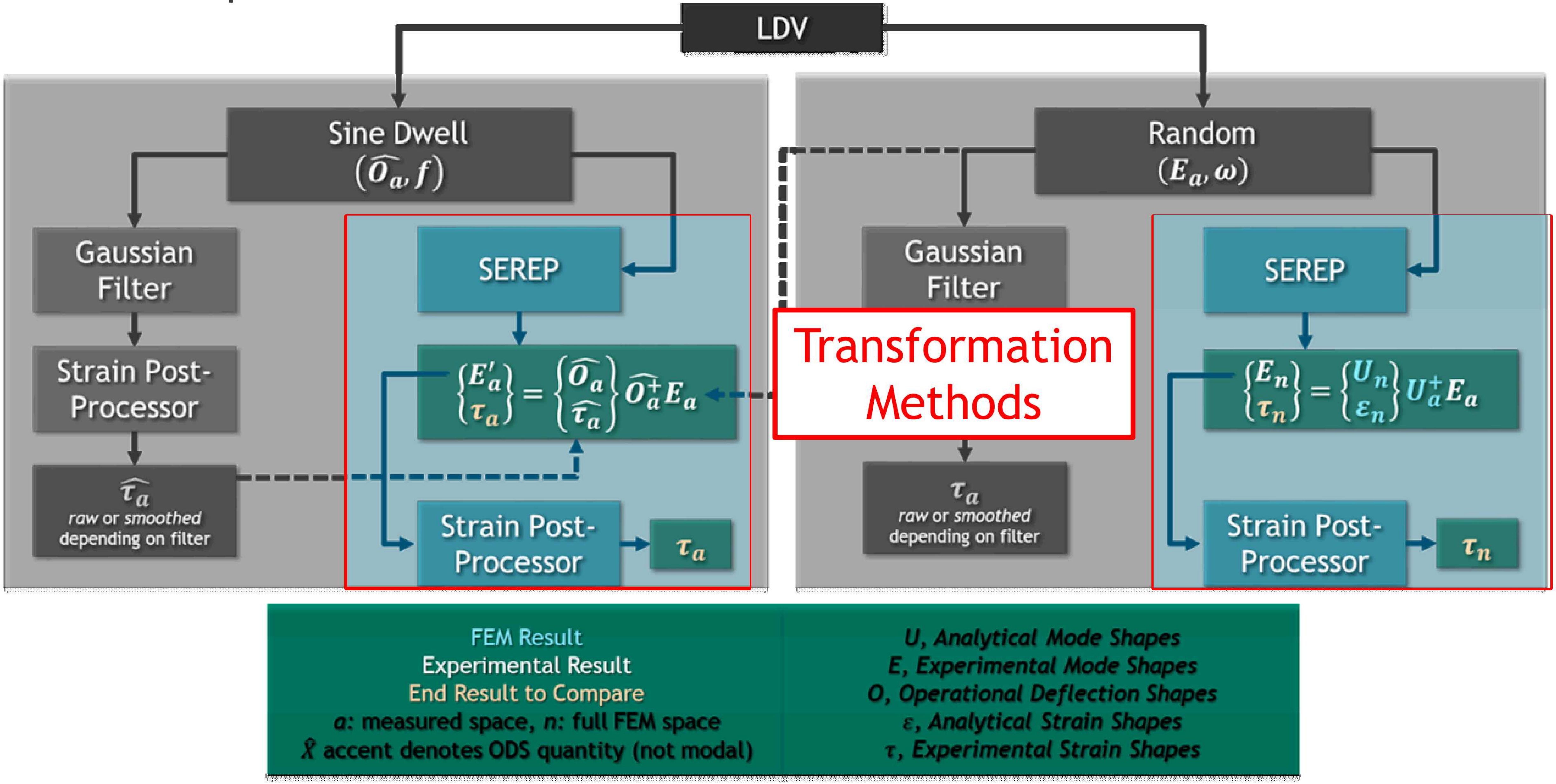
LDV
Sine Dwell
$(\widehat{O}_a, f)$
Gaussian Filter
SEREP
Strain Post-Processor
$\begin{Bmatrix} E'_a \\ \tau_a \end{Bmatrix} = \begin{Bmatrix} \widehat{O}_a \\ \widehat{\tau}_a \end{Bmatrix} \widehat{O}_a^+ E_a$
$\widehat{\tau}_a$
raw or smoothed depending on filter
Strain Post-Processor
$\tau_a$
Random
$(E_a, \omega)$
Gaussian Filter
SEREP
Strain Post-Processor
$\begin{Bmatrix} E_n \\ \tau_n \end{Bmatrix} = \begin{Bmatrix} U_n \\ \varepsilon_n \end{Bmatrix} U_a^+ E_a$
$\tau_a$
raw or smoothed depending on filter
Strain Post-Processor
$\tau_n$
FEM Result
Experimental Result
End Result to Compare
a: measured space, n: full FEM space
$\hat{X}$ accent denotes ODS quantity (not modal)
U, Analytical Mode Shapes
E, Experimental Mode Shapes
O, Operational Deflection Shapes
ε, Analytical Strain Shapes
τ, Experimental Strain Shapes

# Road Map: Direct Methods

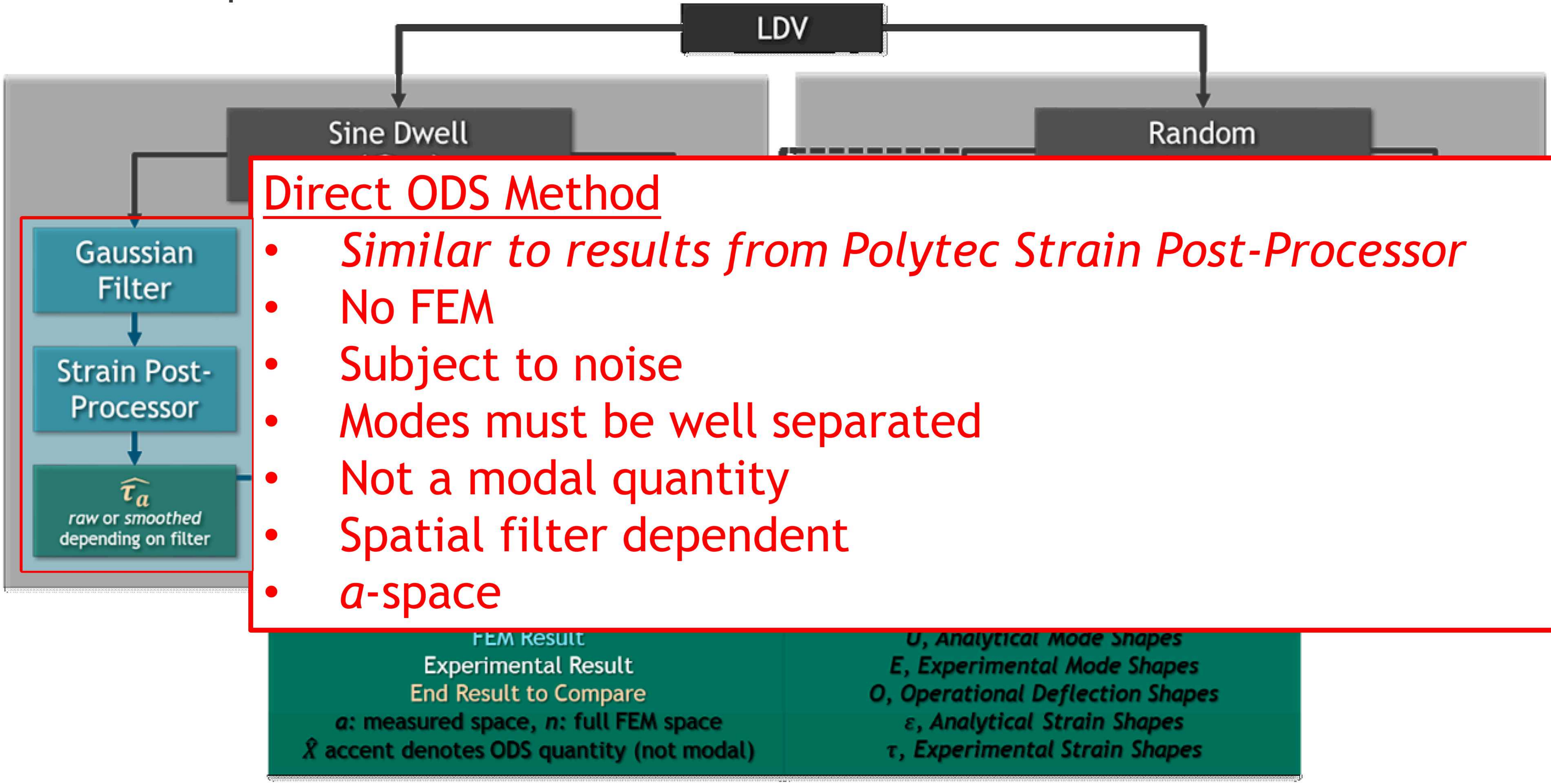



| | |
|---|---|
| FEM Result<br>Experimental Result<br>End Result to Compare<br>*a*: measured space, *n*: full FEM space<br>$\hat{X}$ accent denotes ODS quantity (not modal) | *U, Analytical Mode Shapes*<br>*E, Experimental Mode Shapes*<br>*O, Operational Deflection Shapes*<br>*ε, Analytical Strain Shapes*<br>*τ, Experimental Strain Shapes* |

# Road Map: Transformation Methods



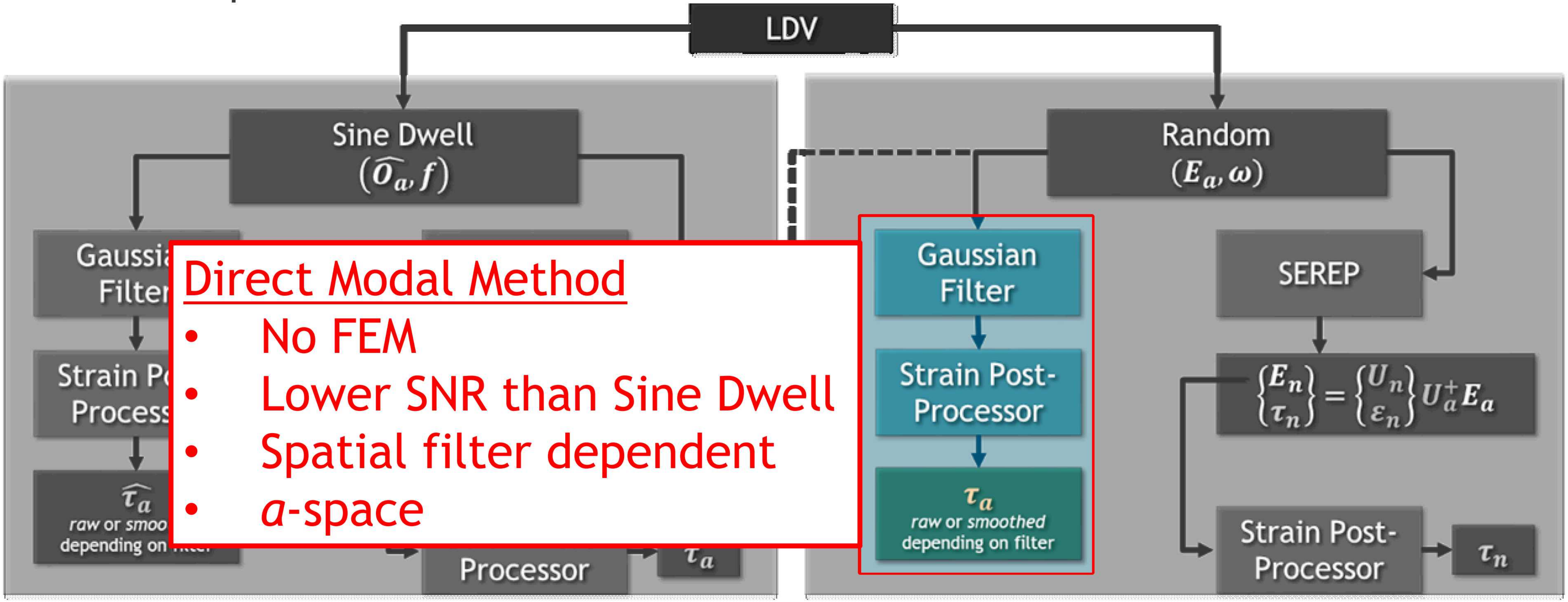
# Road Map: Direct Methods




Direct ODS Method

- *Similar to results from Polytec Strain Post-Processor*
- No FEM
- Subject to noise
- Modes must be well separated
- Not a modal quantity
- Spatial filter dependent
- *a*-space

# Road Map: Direct Methods

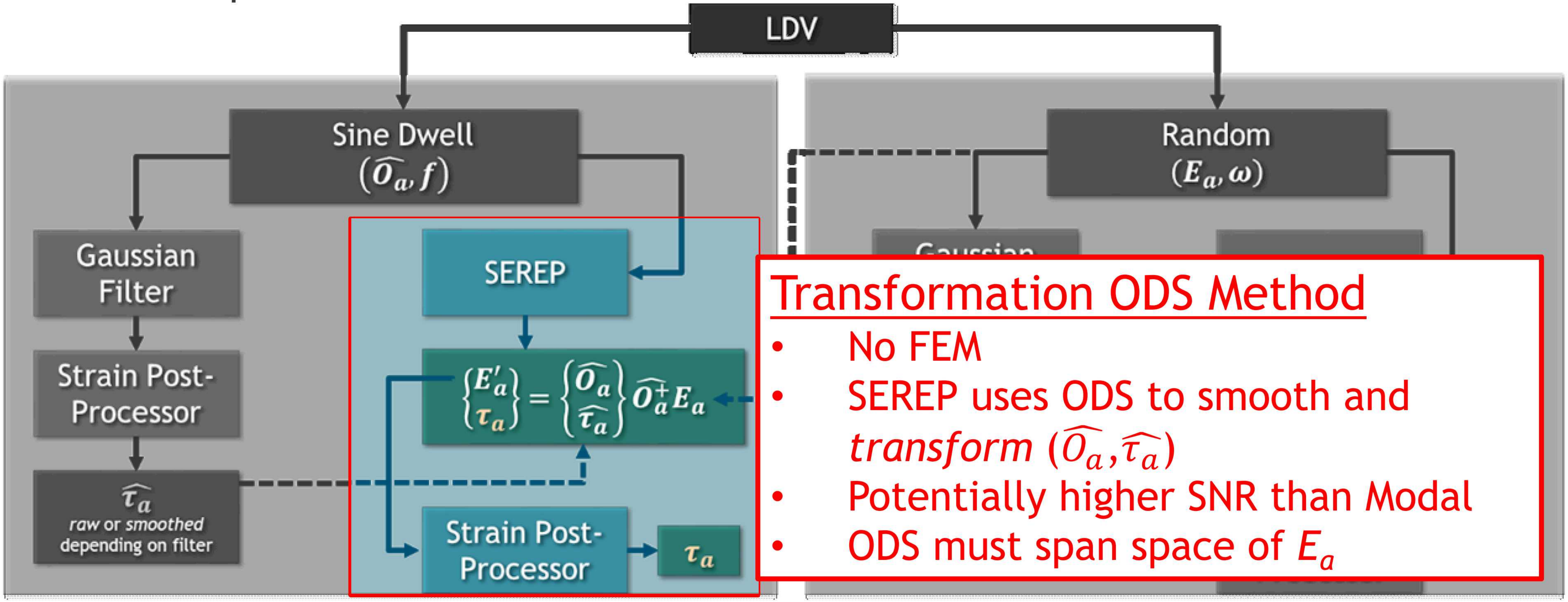



Direct Modal Method
- No FEM
- Lower SNR than Sine Dwell
- Spatial filter dependent
- *a*-space

FEM Result
Experimental Result
End Result to Compare
*a*: measured space, *n*: full FEM space
$\hat{X}$ accent denotes ODS quantity (not modal)

*U, Analytical Mode Shapes*
*E, Experimental Mode Shapes*
*O, Operational Deflection Shapes*
*ε, Analytical Strain Shapes*
*τ, Experimental Strain Shapes*

# Road Map: Transformation Methods

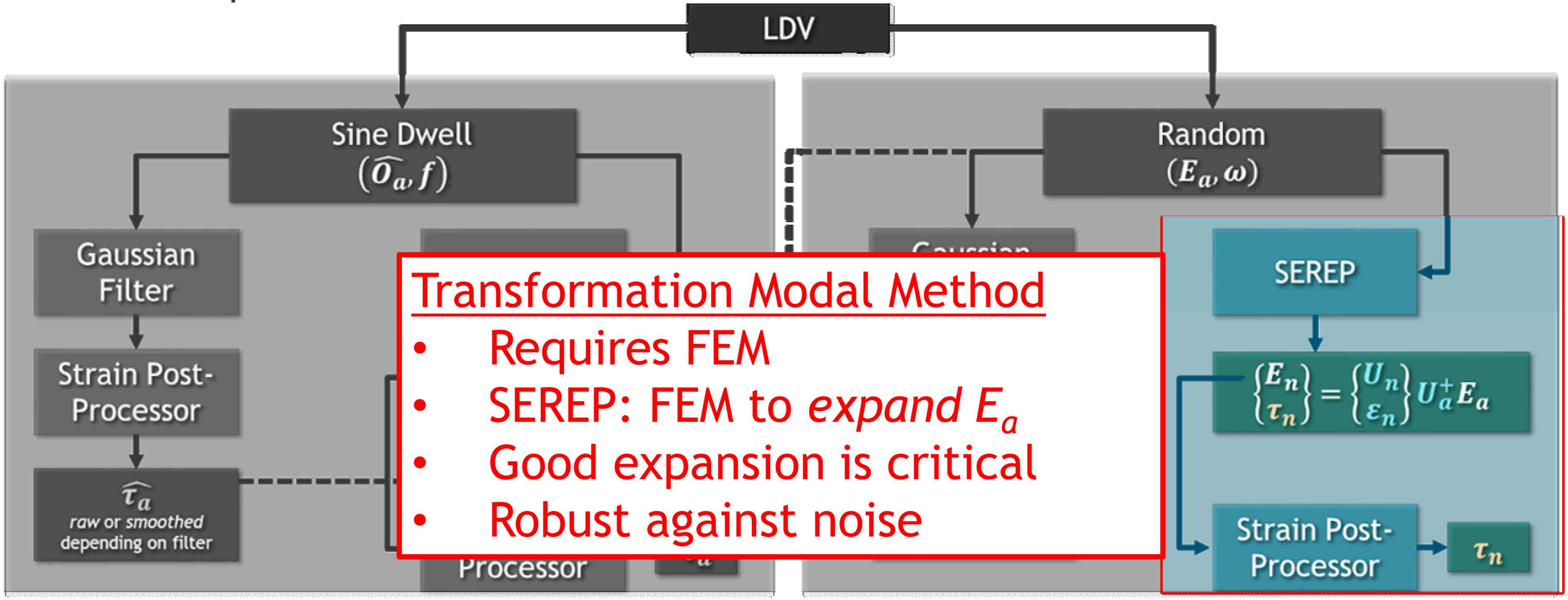



## Transformation ODS Method

- No FEM
- SEREP uses ODS to smooth and *transform* $(\widehat{O_a}, \widehat{\tau_a})$
- Potentially higher SNR than Modal
- ODS must span space of $E_a$

| | |
|---|---|
| FEM Result | *U, Analytical Mode Shapes* |
| Experimental Result | *E, Experimental Mode Shapes* |
| End Result to Compare | *O, Operational Deflection Shapes* |
| *a*: measured space, *n*: full FEM space | *ε, Analytical Strain Shapes* |
| $\hat{X}$ accent denotes ODS quantity (not modal) | *τ, Experimental Strain Shapes* |

# Road Map: Transformation Methods




## Transformation Modal Method

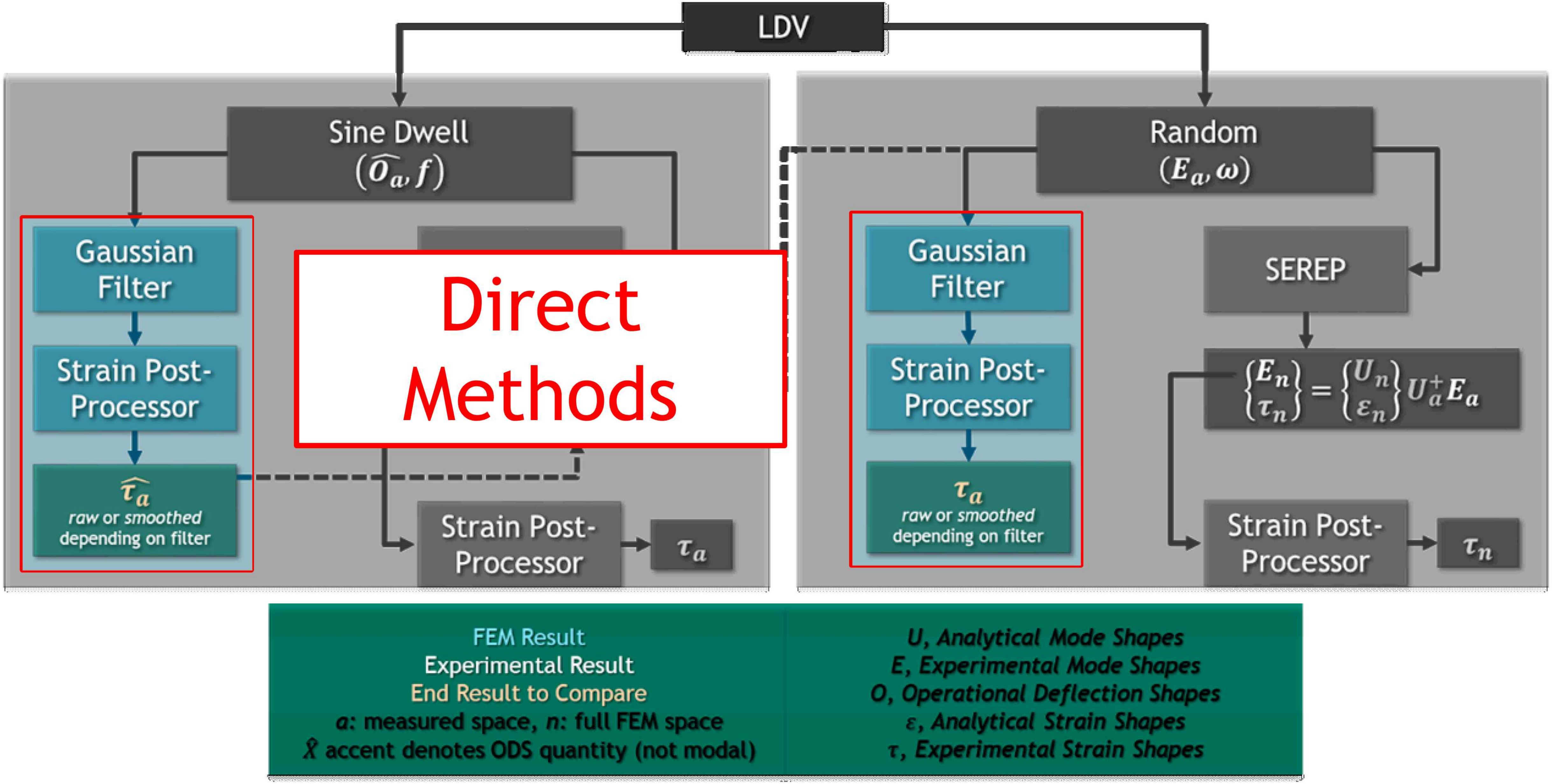
- Requires FEM
- SEREP: FEM to *expand* $E_a$
- Good expansion is critical
- Robust against noise

| | |
|---|---|
| FEM Result | $U$, *Analytical Mode Shapes* |
| Experimental Result | $E$, *Experimental Mode Shapes* |
| End Result to Compare | $O$, *Operational Deflection Shapes* |
| *a*: measured space, *n*: full FEM space | $\varepsilon$, *Analytical Strain Shapes* |
| $\hat{X}$ accent denotes ODS quantity (not modal) | $\tau$, *Experimental Strain Shapes* |

# LDV Direct Methods

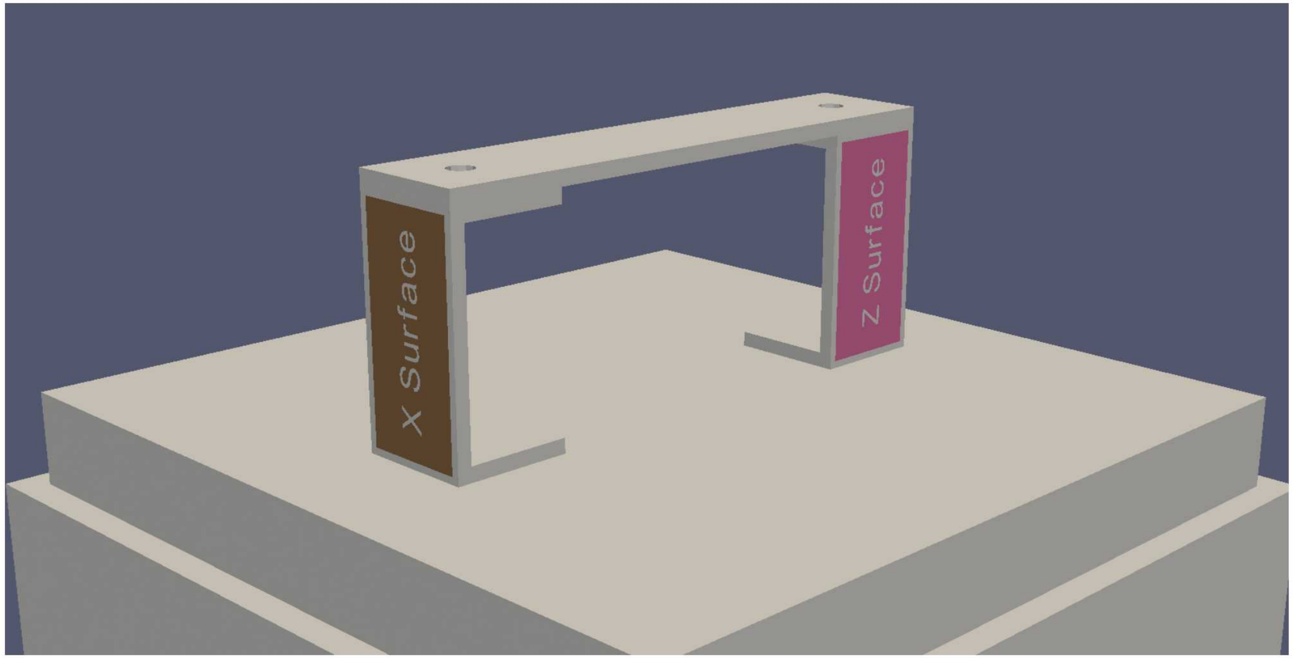
FEM? We don't need no stinking FEM...

# Direct Method Results

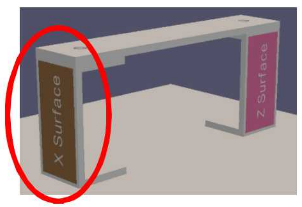


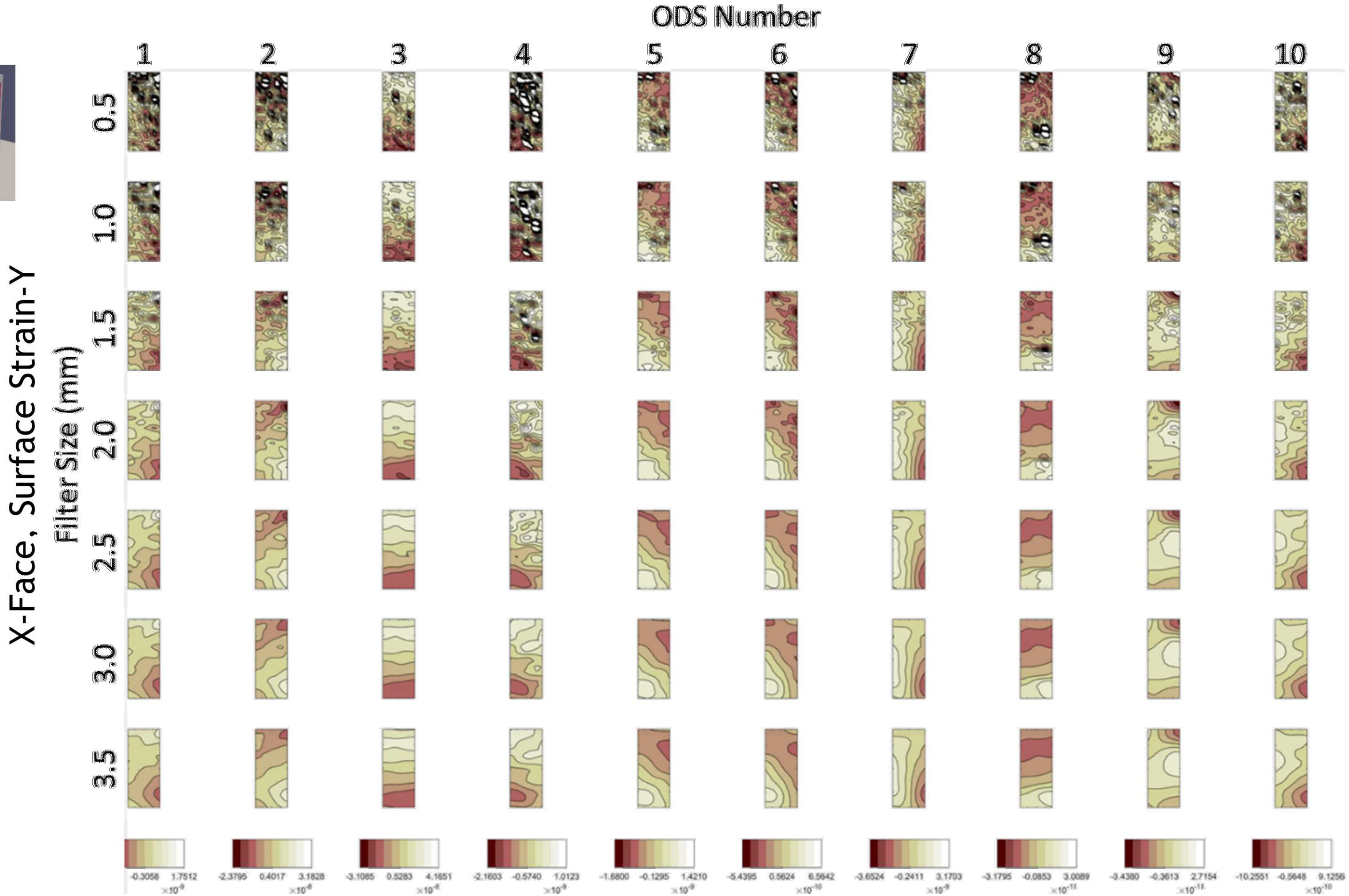
# Direct Method Results

- Gaussian filters of 0.5-3.5 mm were evaluated
  - 0.5 mm is essentially "unsmoothed"
  - Element size: 1.6 x 2.6 mm (horizontal, vertical)
- Comparison plots were made for the Surface Strains on the two C-channel faces
  - X-Face: (yy, yz, zz)
  - Z-Face: (xx, xy, yy)

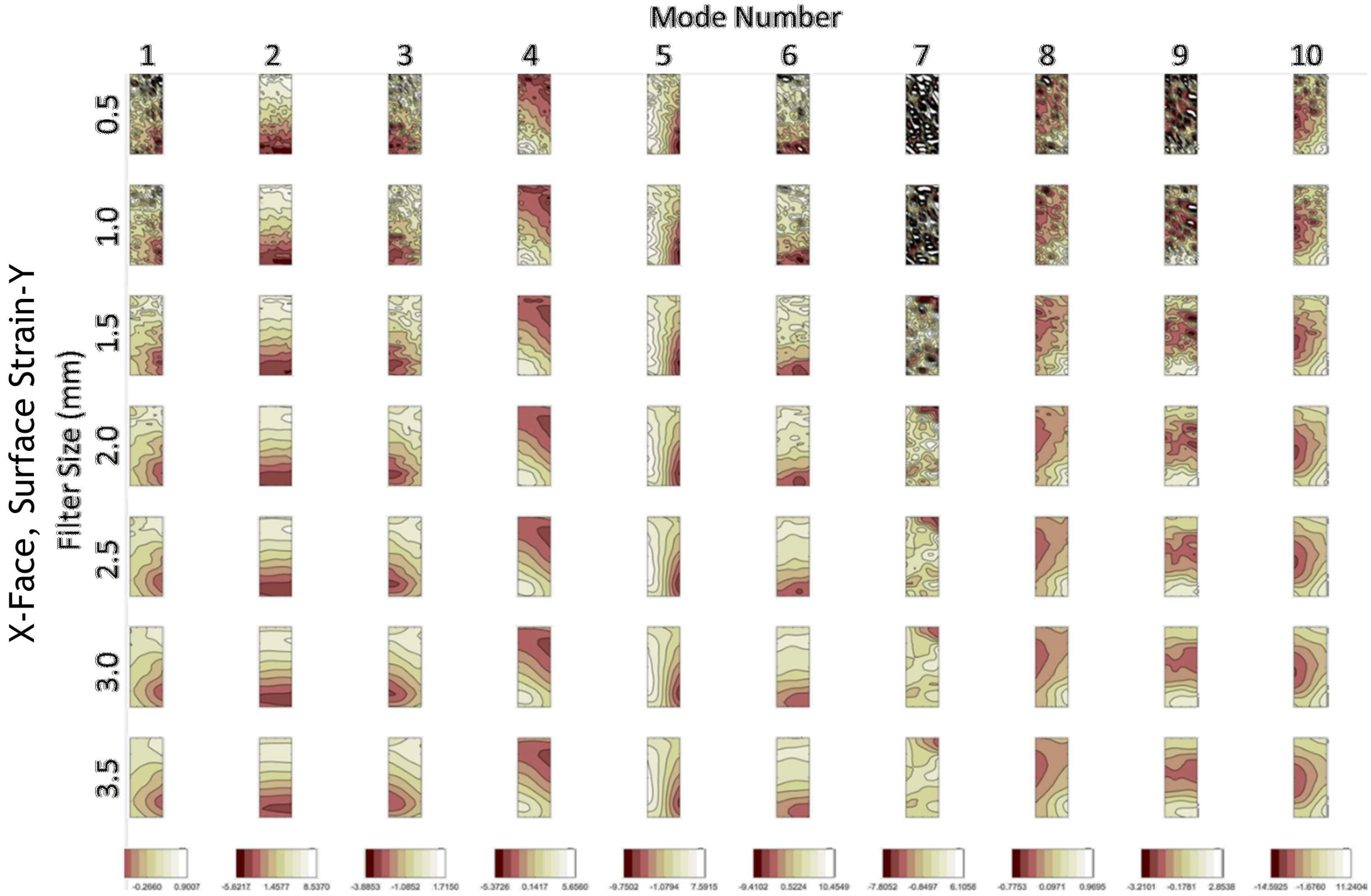


# Direct Method Results: ODS - Raw & Filtered



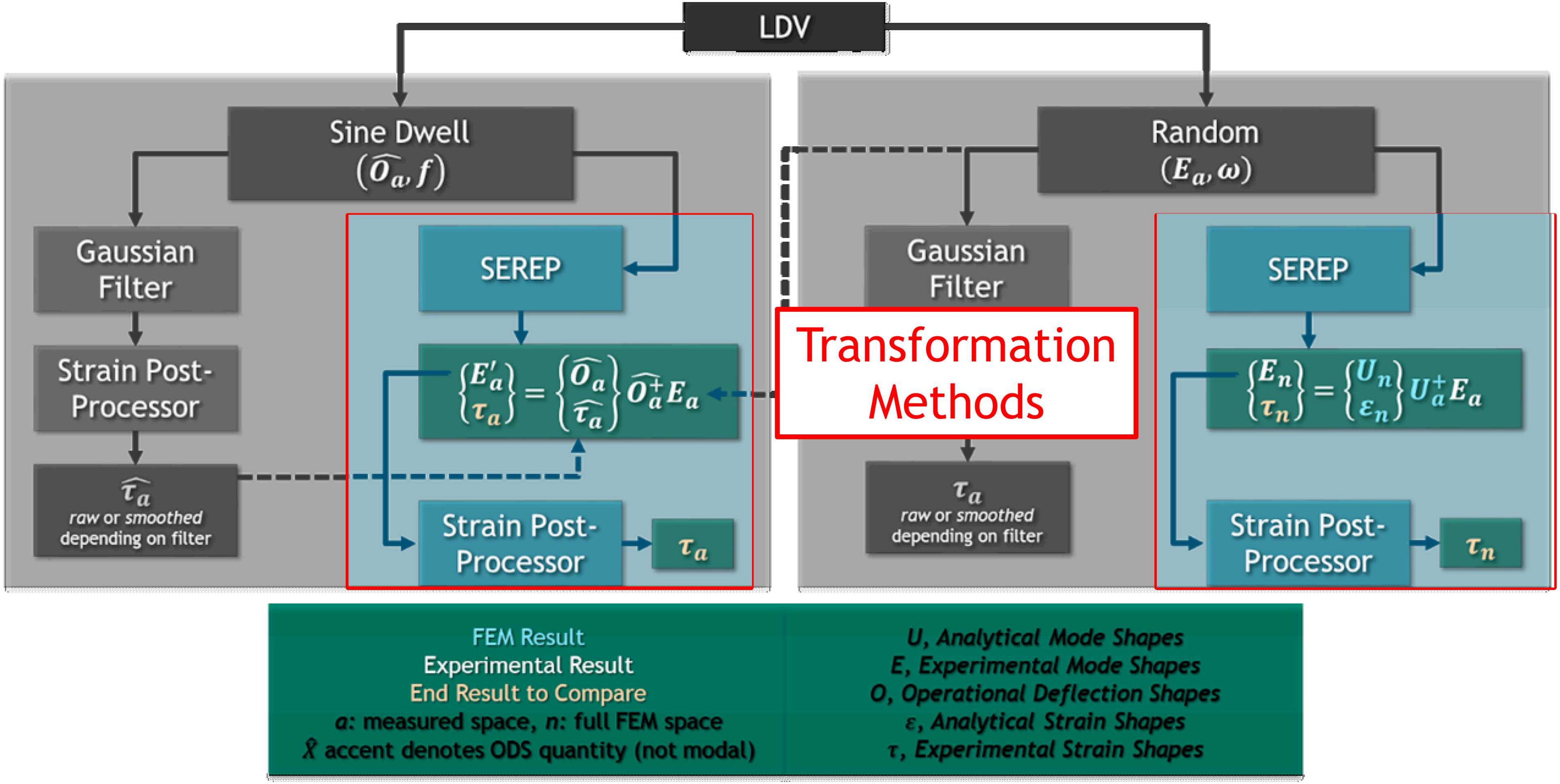
# Direct Method Results: Modal - Raw & Filtered



# LDV Transformation Methods

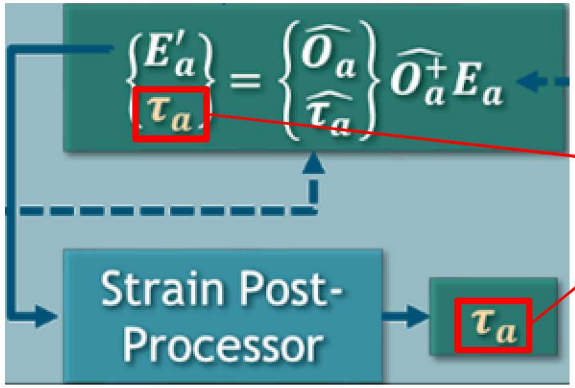
Maybe we need a FEM...

# Transformation Method Results



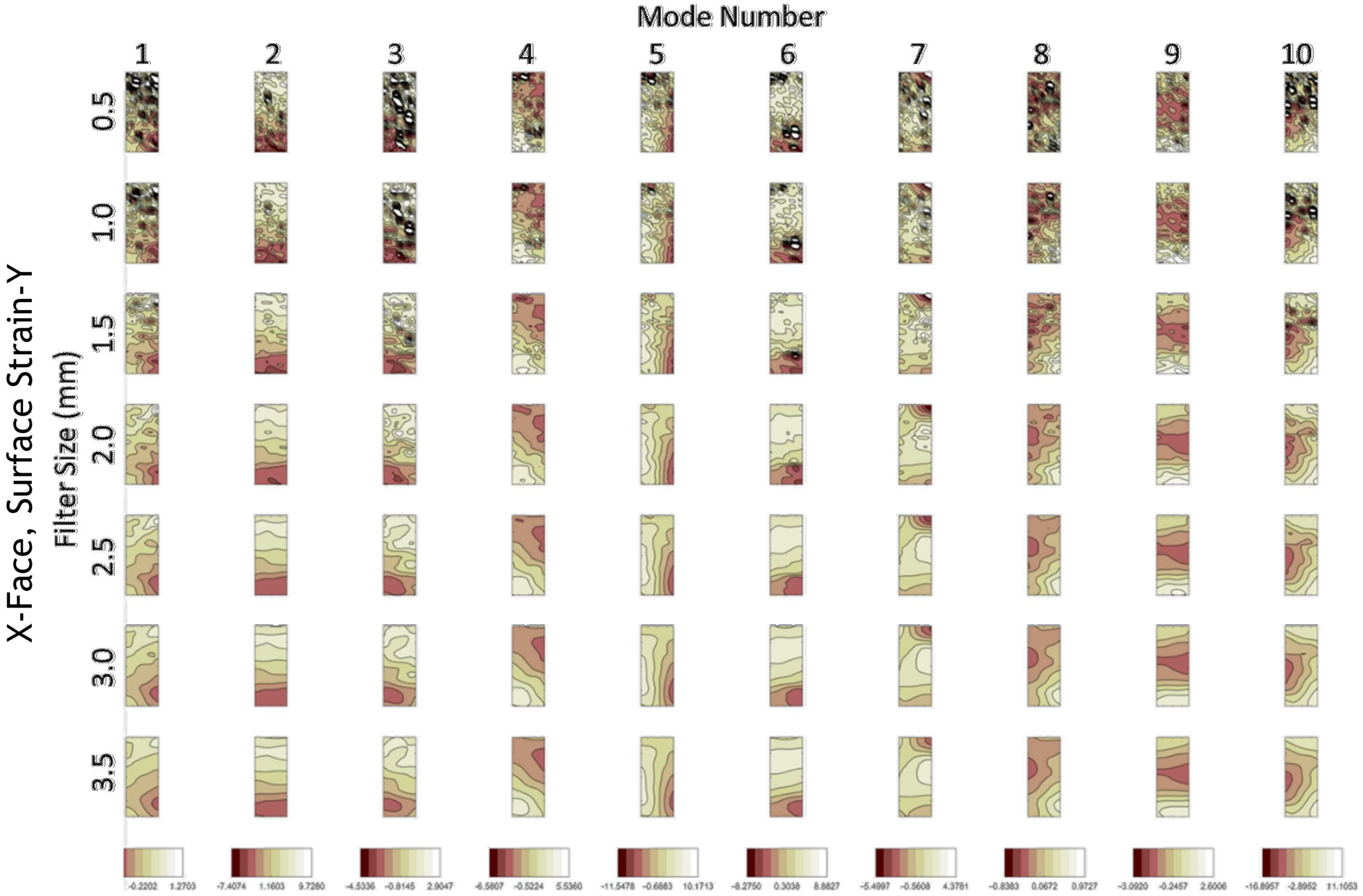
# Transformation Method Results: ODS

- Sine Dwell Test:
  - Extracted 10 ODS
  - 2202 DOF
- Kept all measured DOF (*a-set*)
- Retained all 10 ODS
- Two variations:
  - Use ODS as basis vectors for SEREP smoothing for $E_a$
  - Use ODS and ODS-derived strain shapes as basis vectors for SEREP transformation to $\tau_a$



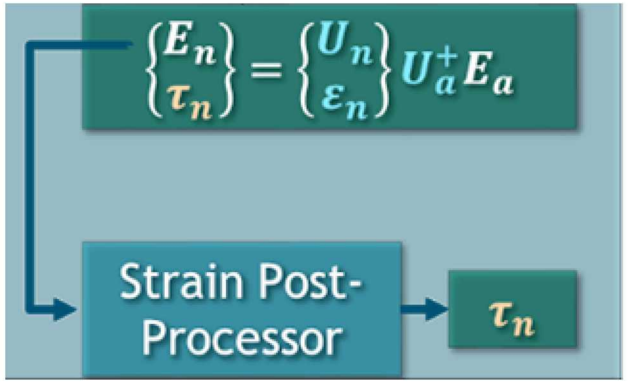

Both variations can be shown to be mathematically identical

# Transformation Method Results: ODS

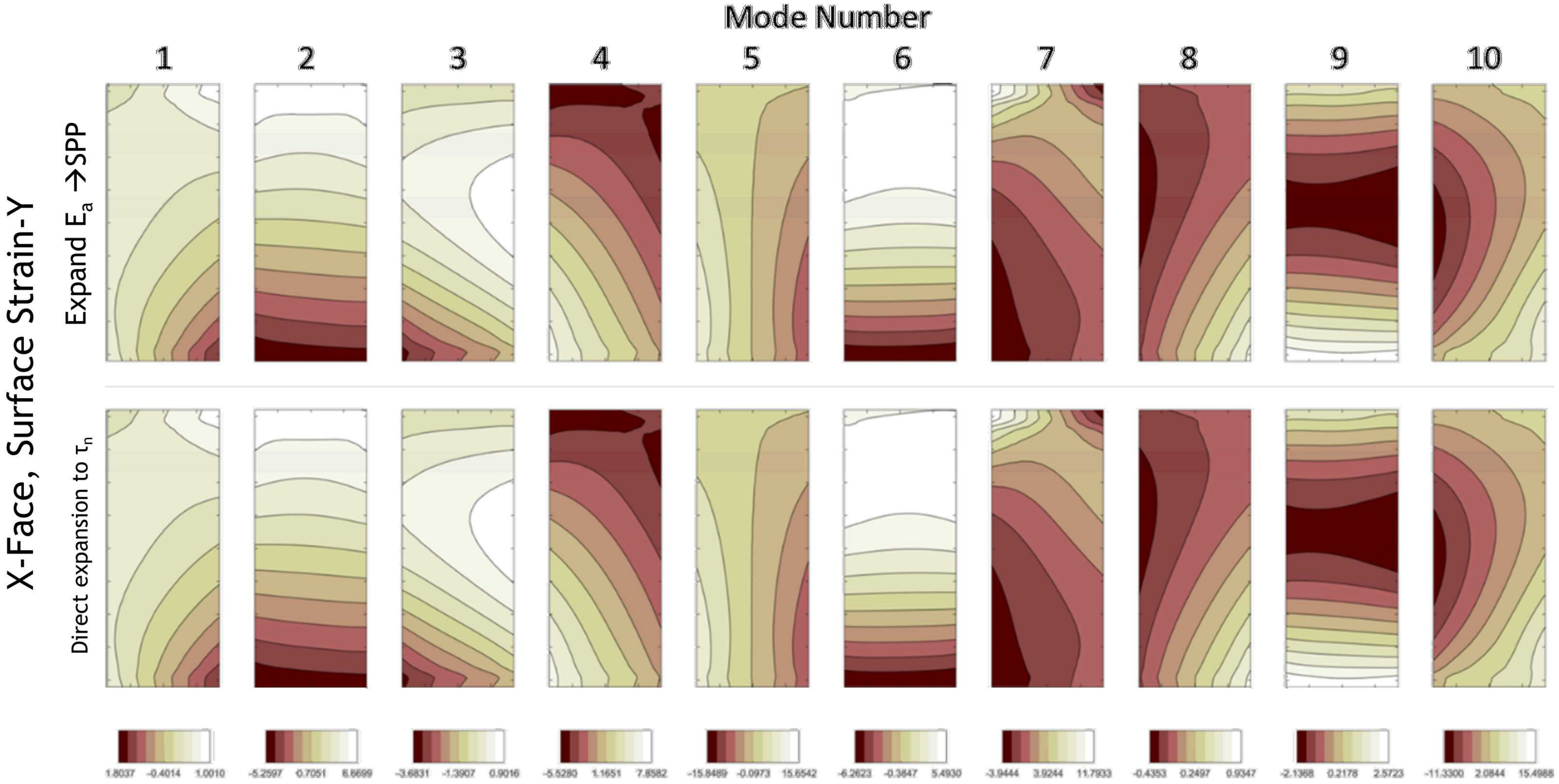


# Transformation Method Results: Modal

- FEM:
  - Delivered with 300 modes
  - 127,155 DOF
- Modal Test:
  - Identified 10 modes
  - 2,202 DOF
- Take full *n-Space* FEM shapes ($U_n$), reduce to *a-Space* set of DOF ($U_a$)
  - Kept 2x(Number of Test Shapes) = 20 DOF for reduction (*a-set*)
  - Retained first 18 analytical modes of the FEM
  - Important to include analytical rigid body modes
  - Important to exclude analytical modes of things you didn't measure (cube)

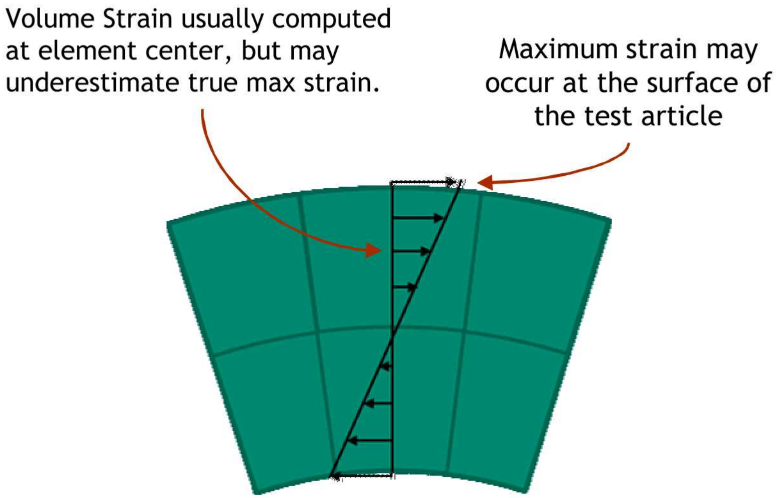


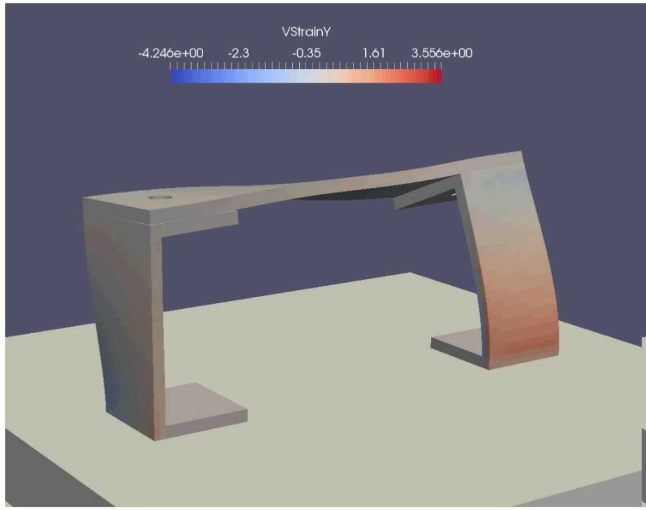
# Transformation Method Results: Modal



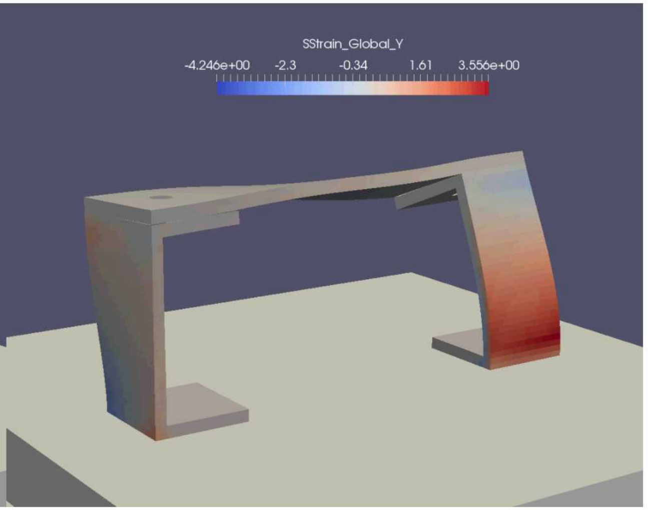
# Sidebar: Surface vs. Volume Strain

- The Scanning Laser Vibrometer measures strains at the surface of the test article.
- Surface strains may vary significantly from volume strains reported by the FEM depending on the mesh resolution.
- In the case of BARC, with only two elements through the thickness, the maximum surface strains were approximately twice the maximum volume strains.



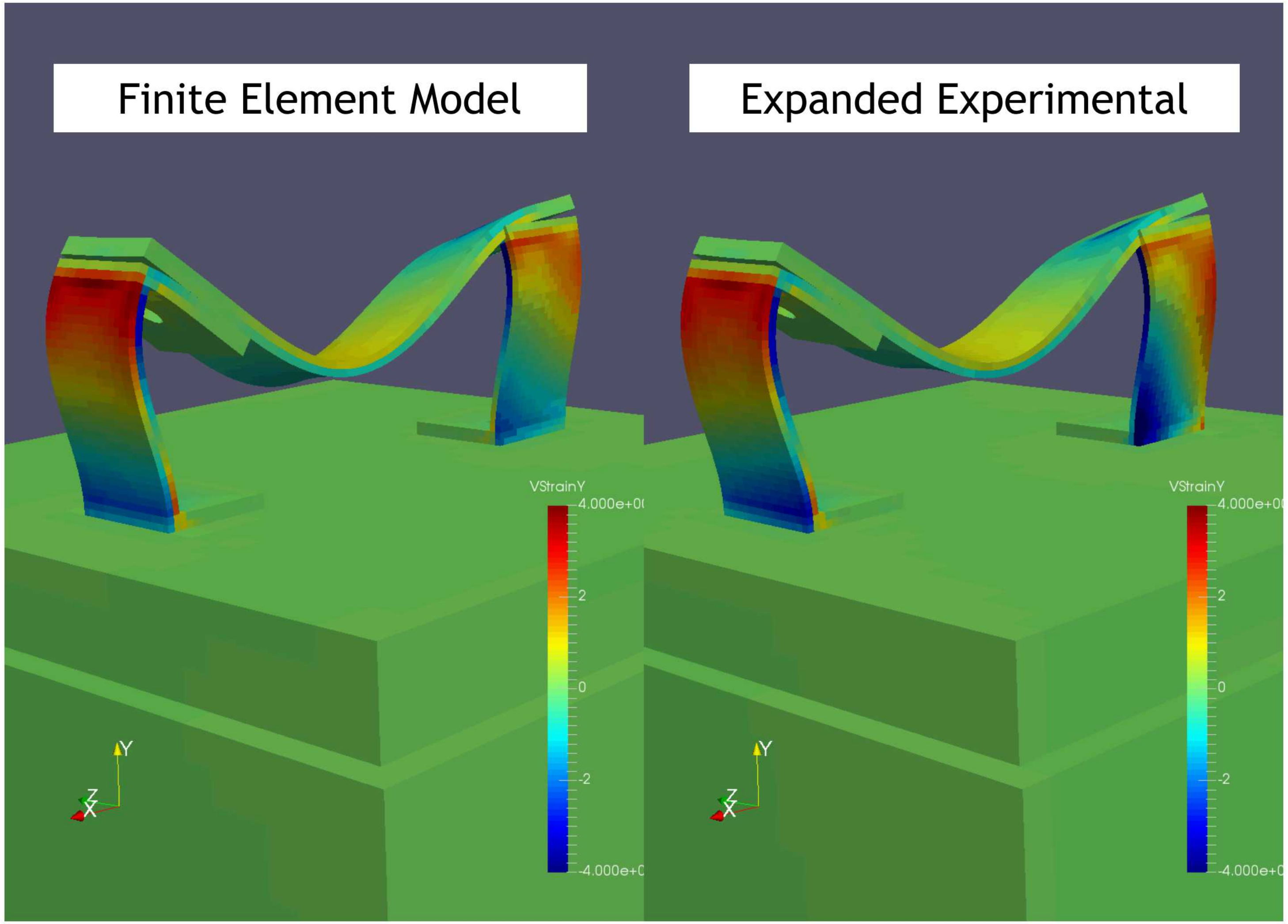

Volume Strain




Surface Strain

# LDV SEREP Methods: Modal Expansion

- Expansion works well for volume strain as well
  - Anticipated result
  - Different tool to have




VStrain-Y gradient plotted over 1st bending mode displacement

# Comparison of Methods

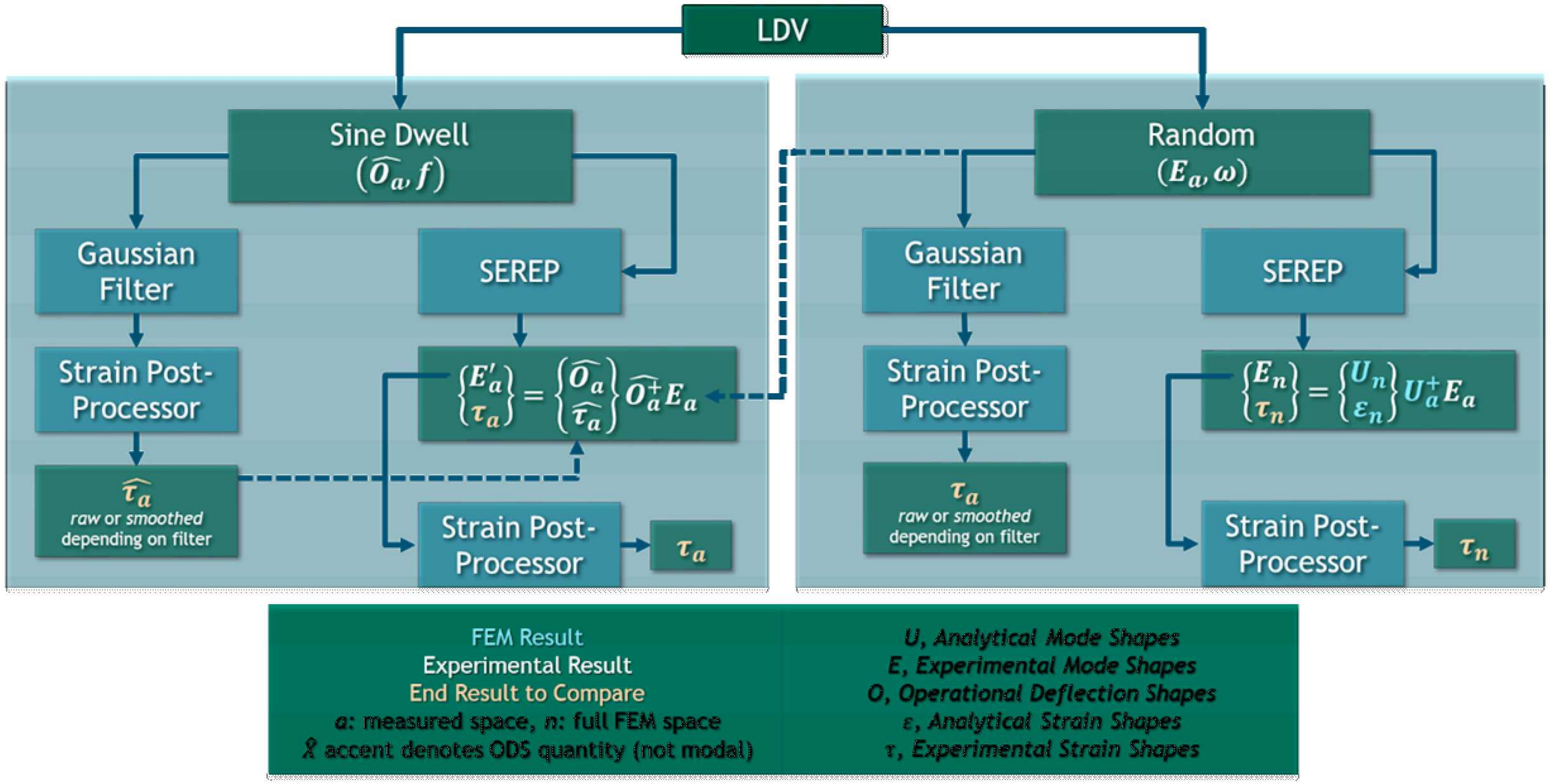
We probably need a FEM…

# Comparison of Methods

- **Direct**
  - ODS – Raw
  - ODS – Filtered
  - Modal – Raw
  - Modal - Filtered
- **Transformation**
  - ODS ($E_a \rightarrow$ SPP)
  - ODS ($\hat{\tau}_s \rightarrow \tau_s$)

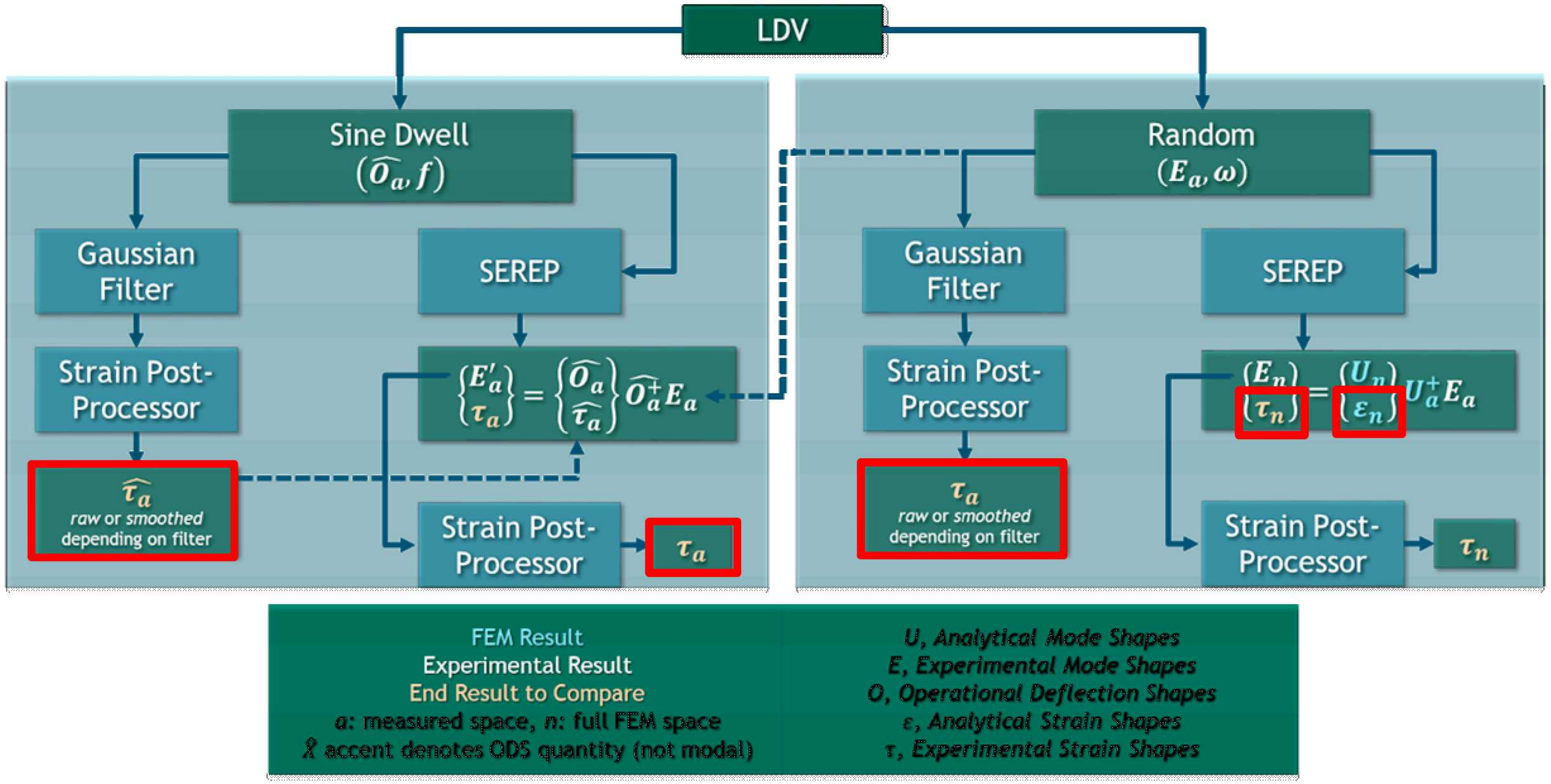
    Identical
  - Modal ($E_a \rightarrow E_n \rightarrow$ SPP)
  - Modal ($E_a \rightarrow \tau_s$)

    Identical
- **FEM**
  - Original analytical model



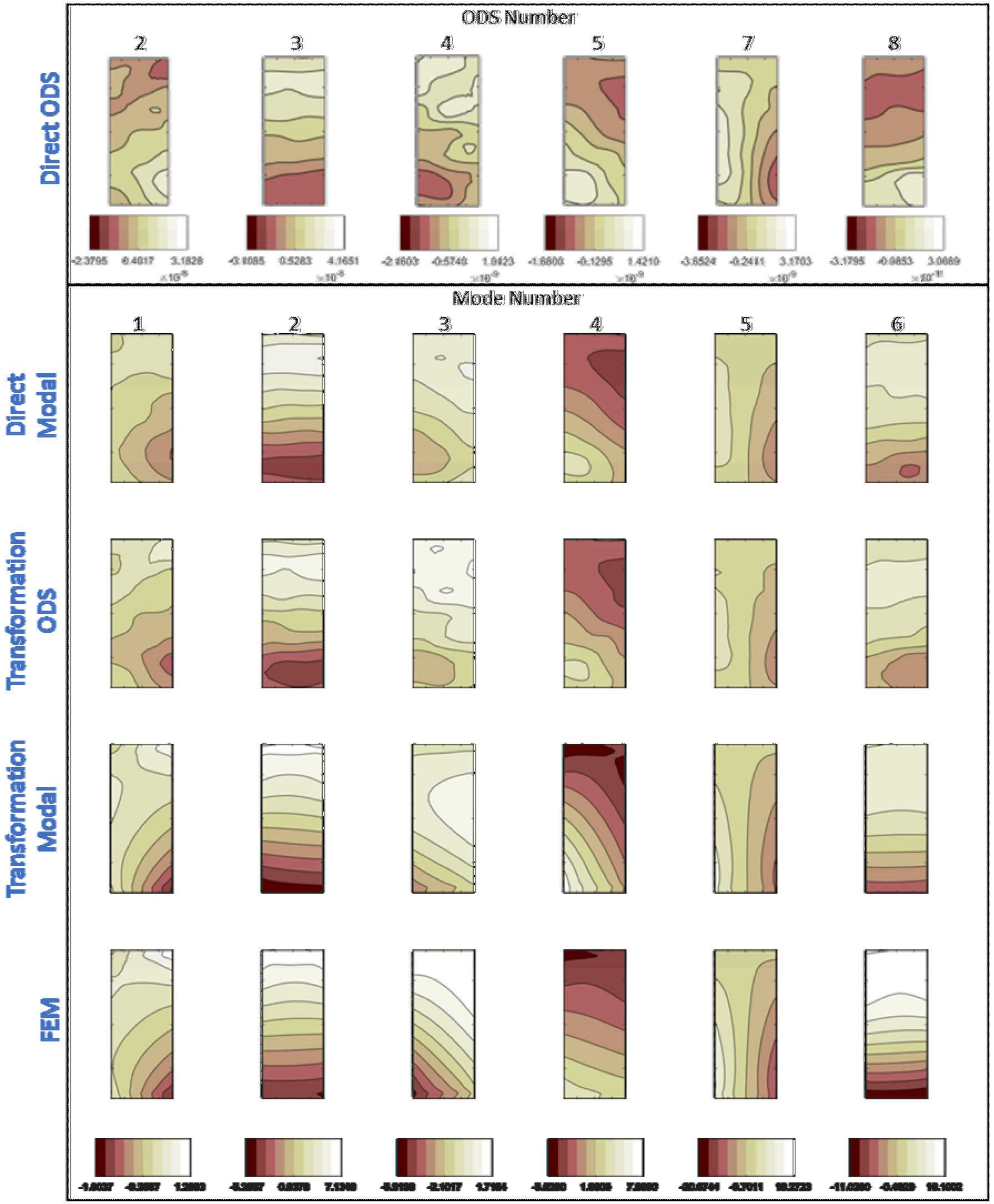
# Comparison of Methods

- **Direct**
  - ~~ODS – Raw~~
  - **ODS – Filtered**
  - ~~Modal – Raw~~
  - **Modal - Filtered**
- **Transformation**
  - **ODS ($E_a$→SPP)**
  - ~~ODS ($\hat{\tau}_s$→ $\tau_s$)~~
  - ~~Modal ($E_a$→$E_n$→SPP)~~
  - **Modal ($E_a$→ $\tau_s$)**
- **FEM**
  - **Original analytical model**



# Comparison of Methods

- Direct ODS have different displacement range than modal
  - Strains are scaled differently than modal quantities
  - Sign is opposite on ODS [2, 4, 6]
- Direct Modal works reasonably well, no FEM is needed
- Transform ODS puts you into modal domain and helps order and clean the shapes
- Transform Modal works very well overall, requires a FEM

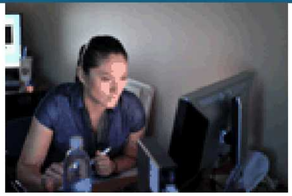


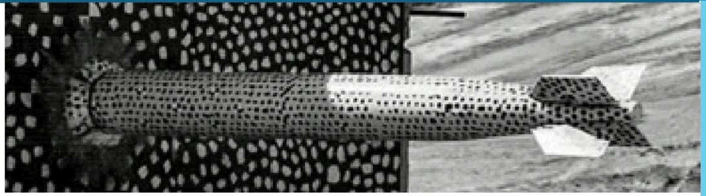

X-Face, Surface Strain-Y

# Comparison of Methods

- If you ***<u>have a decent FEM</u>***, would recommend trying ***<u>Transformation Modal Method</u>***
  - Best overall performance
  - Smooth modal model for strain response
- If you ***<u>don't have a FEM</u>***, would recommend ***<u>Direct Modal</u>***
  - For more linear structures with well-spaced modes, Transformation ODS method may work better (possibly better SNR)
  - For structures with closely spaced modes, Direct Modal will give a better basis for a strain-response modal model
- Neither of the above approaches fit well within the current Polytec strain post-processor
- Things that are ***<u>not</u>*** recommended:
  - Using Direct LDV measurements (ODS or Mode Shapes) without appropriate filtering
  - Assuming you don't need a FEM
  - Assuming your FEM is correct
  - Assuming you don't need a modal test
  - Assuming your modal test is correct

# Questions?





# Backup Slides

# Observations

Glad we had a FEM…

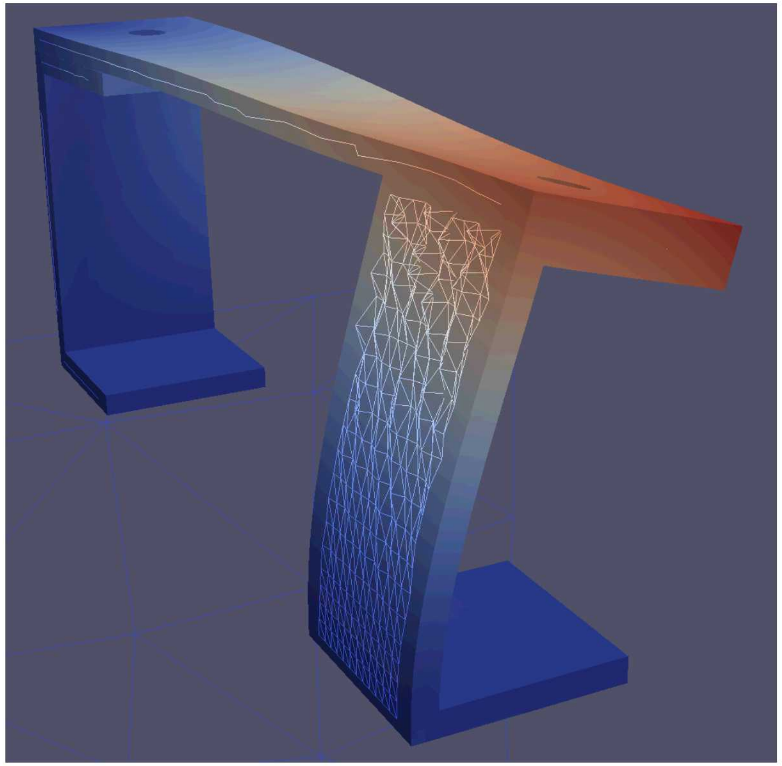
# Observations: Things that helped that may not always be true…

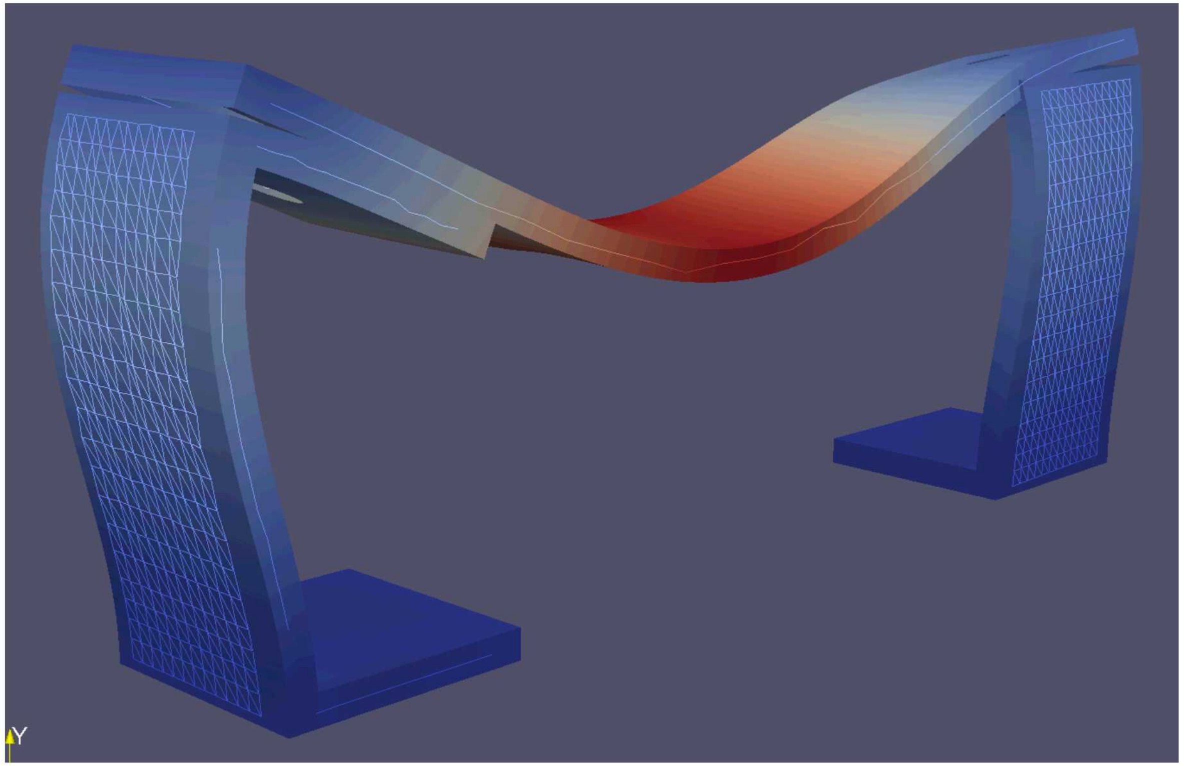
- Initial correlation of FEM was very good
  - Analytical shapes span space of test shapes
  - *Sensitivity of these methods to a poorly correlated FEM?*
- Test structure has shapes that are well separated in frequency
  - ODS are good approximations of the mode shapes →
  - *How well do ODS methods work with closely spaced modes?*

| MAC | | FEM | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ODS | 2 | 0.99 | 0.03 | 0.00 | 0.15 | 0.01 | 0.00 | 0.06 | 0.00 | 0.00 | 0.05 |
| | 3 | 0.02 | 0.99 | 0.11 | 0.02 | 0.01 | 0.17 | 0.03 | 0.00 | 0.08 | 0.00 |
| | 4 | 0.00 | 0.09 | 0.95 | 0.01 | 0.03 | 0.09 | 0.00 | 0.00 | 0.04 | 0.01 |
| | 5 | 0.11 | 0.02 | 0.01 | 0.99 | 0.02 | 0.01 | 0.02 | 0.02 | 0.06 | 0.02 |
| | 7 | 0.00 | 0.00 | 0.00 | 0.01 | 0.95 | 0.00 | 0.00 | 0.04 | 0.01 | 0.17 |
| | 8 | 0.01 | 0.15 | 0.10 | 0.03 | 0.01 | 0.88 | 0.00 | 0.00 | 0.13 | 0.01 |
| | 9 | 0.05 | 0.04 | 0.00 | 0.01 | 0.00 | 0.00 | 0.99 | 0.02 | 0.00 | 0.00 |
| | 10 | 0.00 | 0.01 | 0.02 | 0.02 | 0.05 | 0.01 | 0.01 | 1.00 | 0.14 | 0.11 |
| | 11 | 0.00 | 0.10 | 0.06 | 0.07 | 0.01 | 0.19 | 0.00 | 0.08 | 0.98 | 0.01 |
| | 12 | 0.05 | 0.01 | 0.02 | 0.02 | 0.17 | 0.00 | 0.00 | 0.12 | 0.00 | 0.98 |

# Observations: Nonlinearity

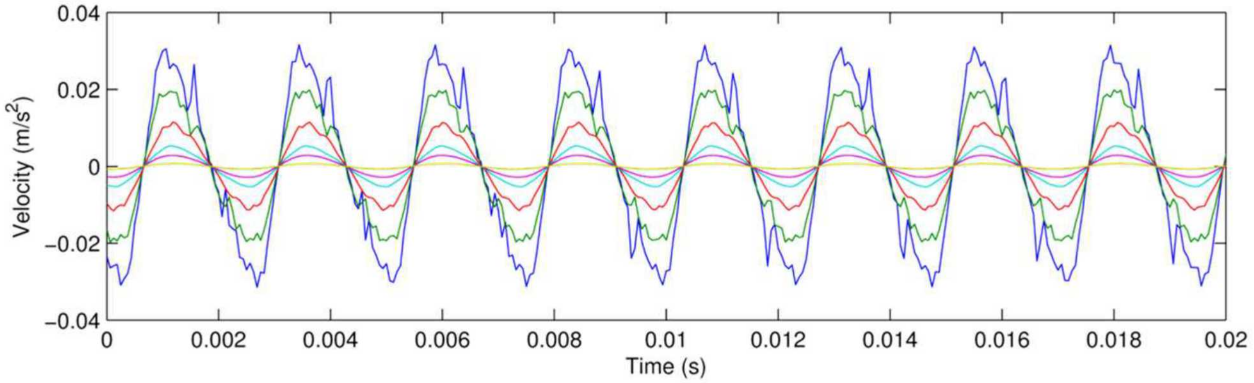
- There was significant noise in the first few shapes that could not be cleaned up by averaging
- This was thought to be due to nonlinearities in the test article
  - Perhaps due to the joints in the removable component
  - In many experimental and FEM shapes these joints can be seen separating

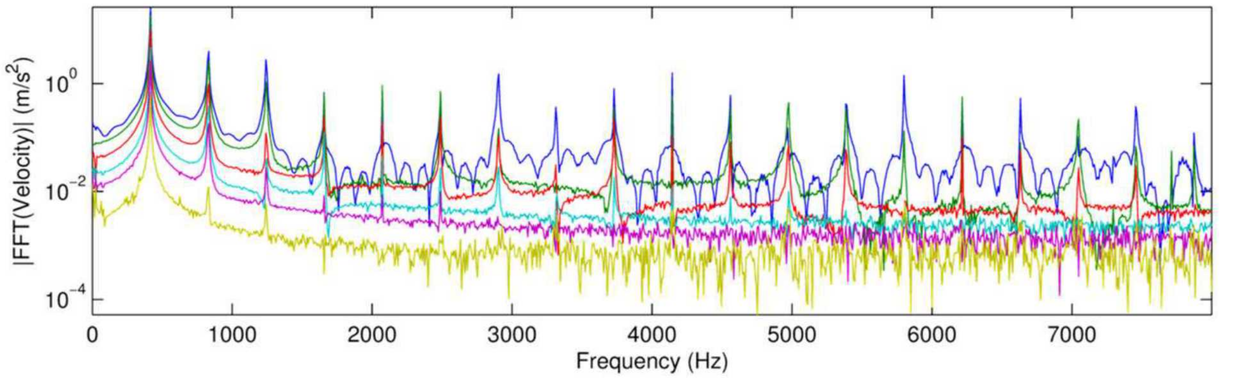




# Observations: Nonlinearity

- These nonlinearities are thought to be the reason that the sine dwell testing did not produce better strain results than computing the strain directly from the mode shapes
  - Playing a sinusoidal excitation force into the shaker resulted in non-sinusoidal responses with significant harmonics that got worse as the level increased.
  - The non-sinusoidal behavior was repeatable and therefore could not be corrected by averaging.
  - At some DOF, the noise floor of the laser was reached before the harmonics disappeared.
  - Strangely, points next to each other seemed to have significantly different responses, but they were completely repeatable.



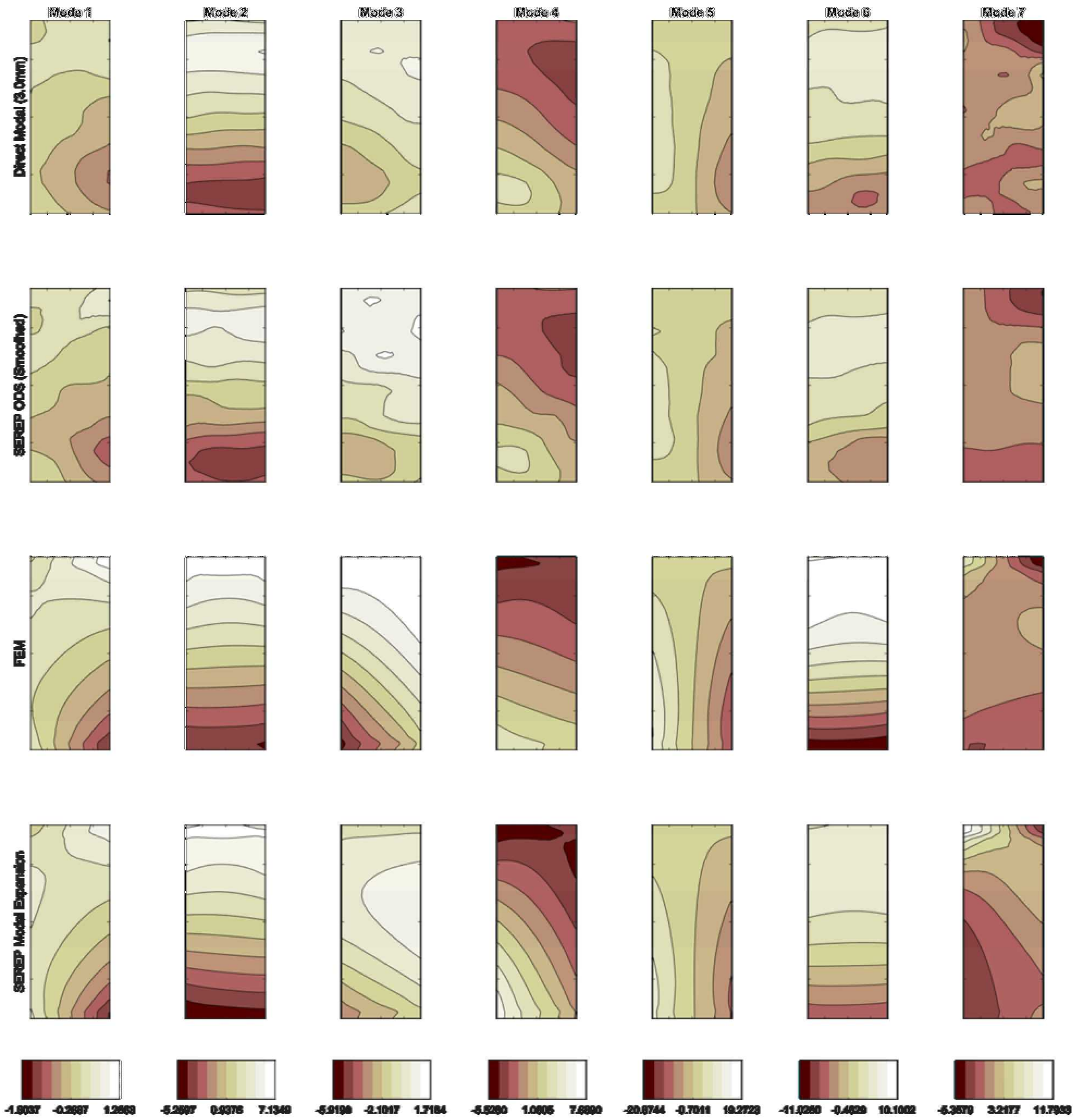



# Future Work

# Future Work

- Add 3D DIC strain measurements to this comparison
  - Same test article and configurations
- Use this work to establish a modal-based full-field strain model
  - Analytically apply an environment to modal model and calculate full-field strain response
  - Compare to FEM predictions for same environment
  - Experimentally apply environment to test article
    - Use modal filters to extract modal coefficients, multiply by strain shapes
    - Get strain response for each mode, total response is linear superposition
    - Compare to FEM strain response predictions
- Apply method to other structures!

X-Face, Surface Strain-Y



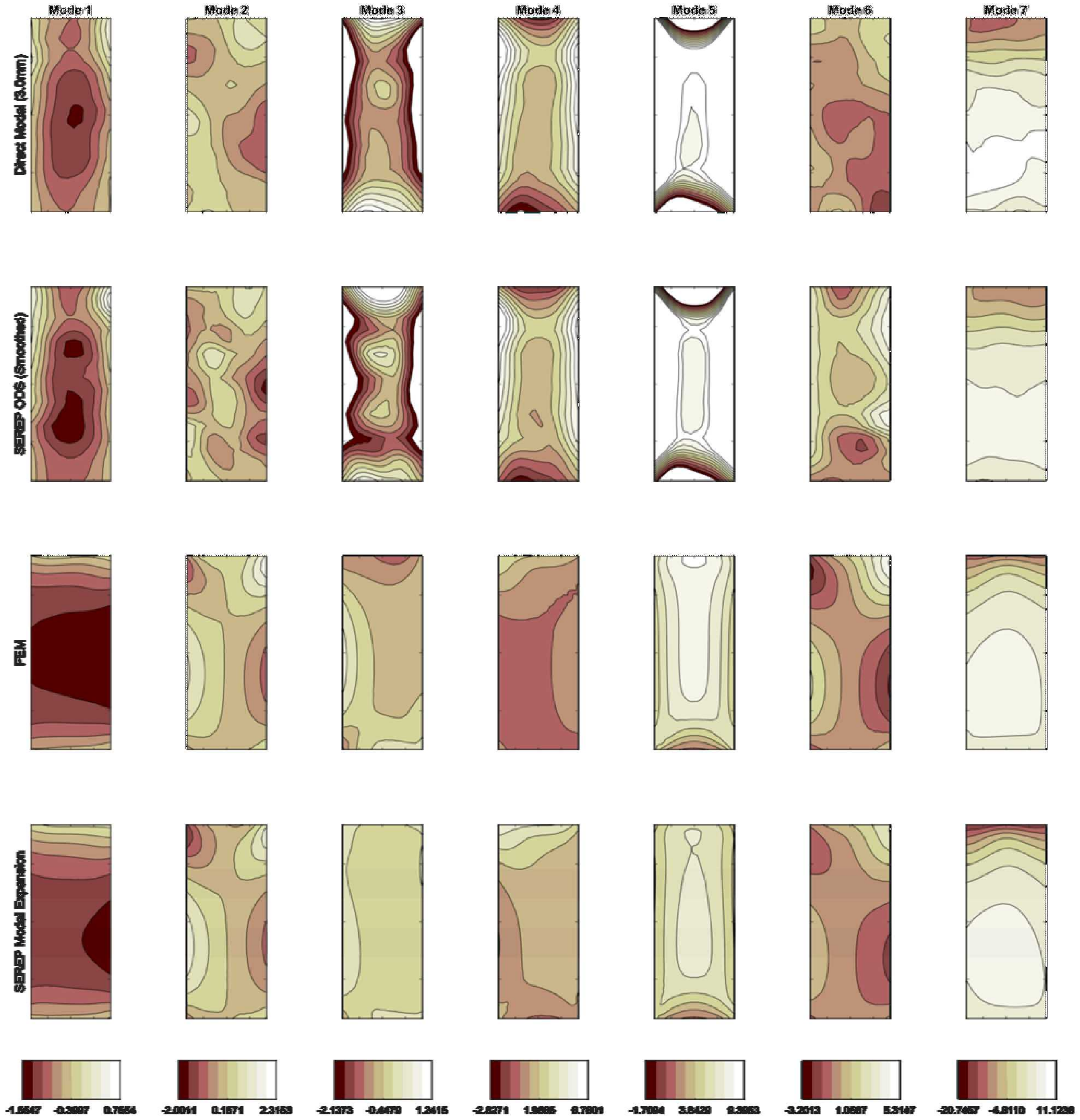

Direct Modal

Transformation ODS

FEM

Transformation Modal

X-Face, Surface Strain-YZ



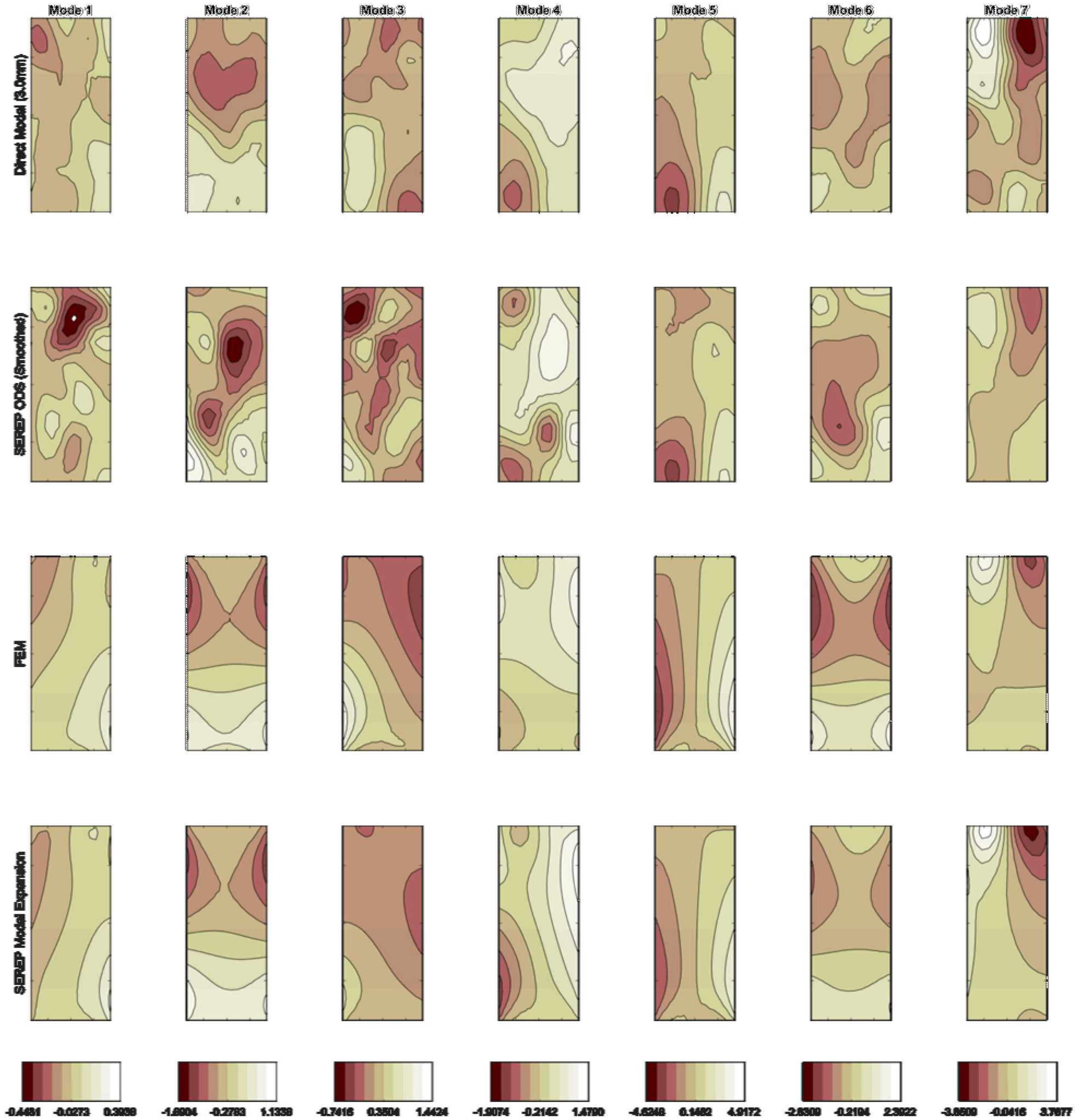

Direct Modal

Transformation ODS

FEM

Transformation Modal

X-Face, Surface Strain-Z



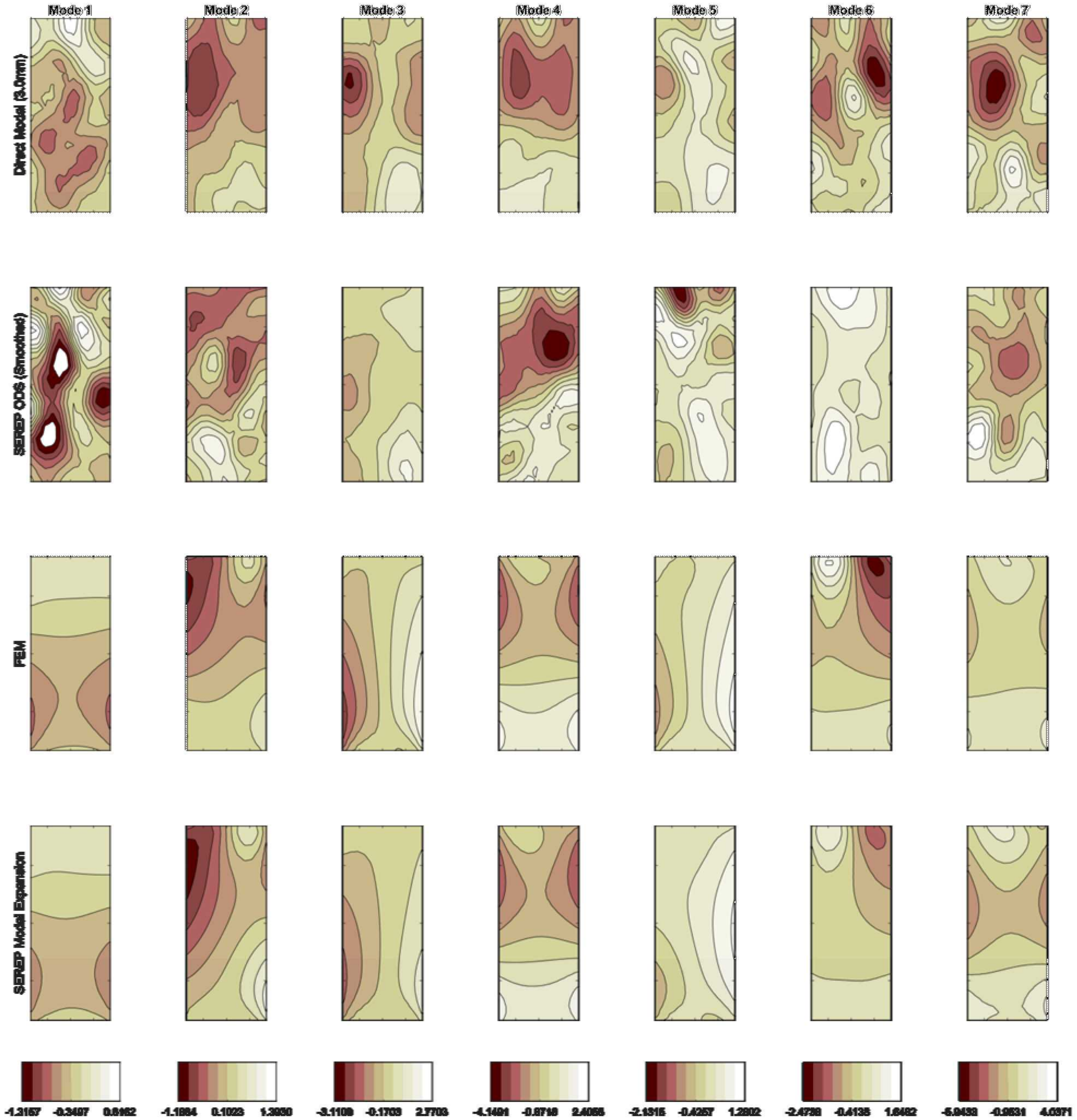

Direct Modal

Transformation ODS

FEM

Transformation Modal

Z-Face, Surface Strain-X

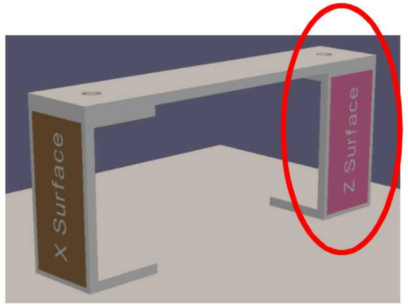



Direct Modal

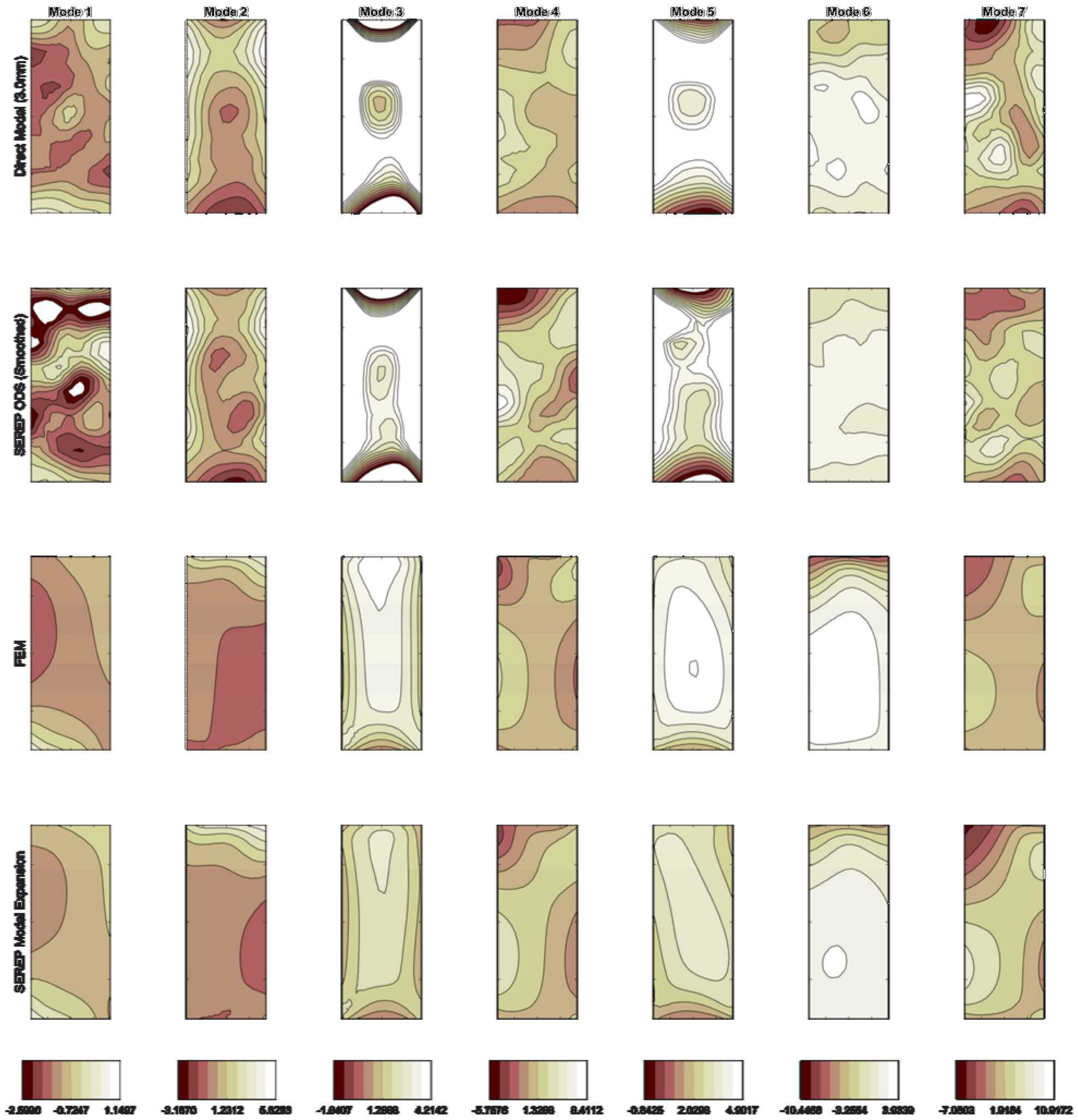
Transformation ODS

FEM

Transformation Modal



# Z-Face, Surface Strain-XY



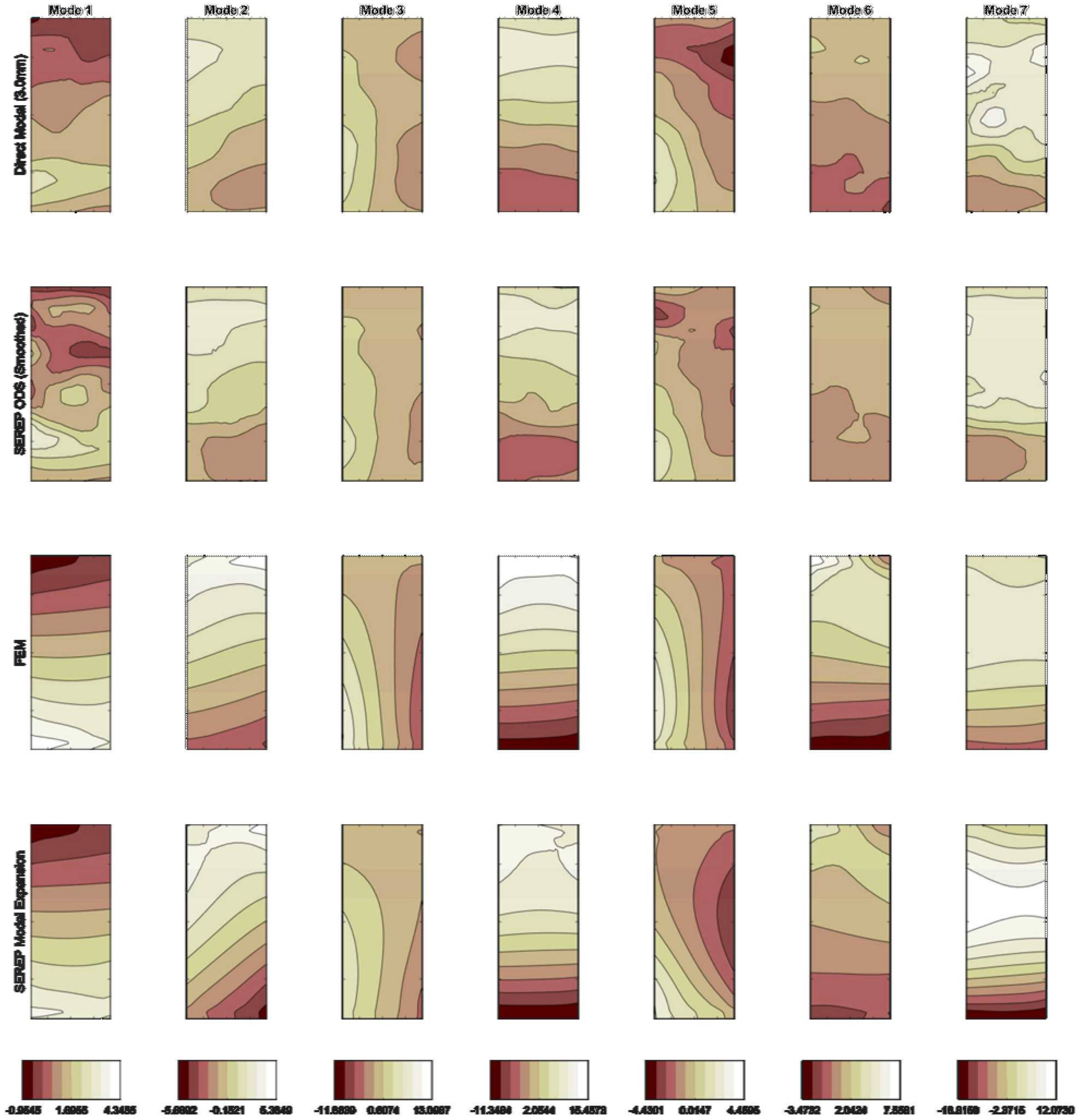

Direct Modal

Transformation ODS

FEM

Transformation Modal

Z-Face, Surface Strain-Y




Direct Modal

Transformation ODS

FEM

Transformation Modal