

# Predicting System Response at Unmeasured Locations



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## Motivation for improving system dynamic response estimates

- Components need to be qualified to survive dynamic environments in the system.
- Often one or more system level field/flight tests are conducted with some accelerometers at some convenient locations.
- These locations are usually not the attachment locations of the components.
- Ideally, all six dof at each attachment would be measured to provide an accurate input motion to the component. This is never provided.
- Enveloping of the few measurements at the nearest locations is performed and defined as the component input environment – this is usually wrong since the measurements are not at attachment degrees of freedom.
- No one knows the uncertainty or conservatism of this component environment.
- This work with AWE attempts to take a step toward providing the ideal estimate of all responses necessary to truly define a measured field/flight environment.

## Approach to estimate unmeasured system responses from a few measurements

- Obtain a validated finite element model of the system and calculate the modes of response in the desired frequency band.
- Install enough accelerometers at appropriate locations to extract all the modal coefficients in each pre-defined frequency band with a modal filter defined from the finite element model and the measured dof.
- Use the finite element model to interpolate the motion to any desired dof on the structure. As long as the FE shapes span the space of the true motion, the estimate is good. The modal filter is based on SEREP by O'Callahan and Avitabile.

$$\begin{Bmatrix} x_{meas} \\ x_{unmeas} \end{Bmatrix} = \begin{bmatrix} \Phi_{meas} \\ \Phi_{unmeas} \end{bmatrix} \{q\}$$

$$\{q\} = \Phi_{meas}^+ \{x_{meas}\}$$

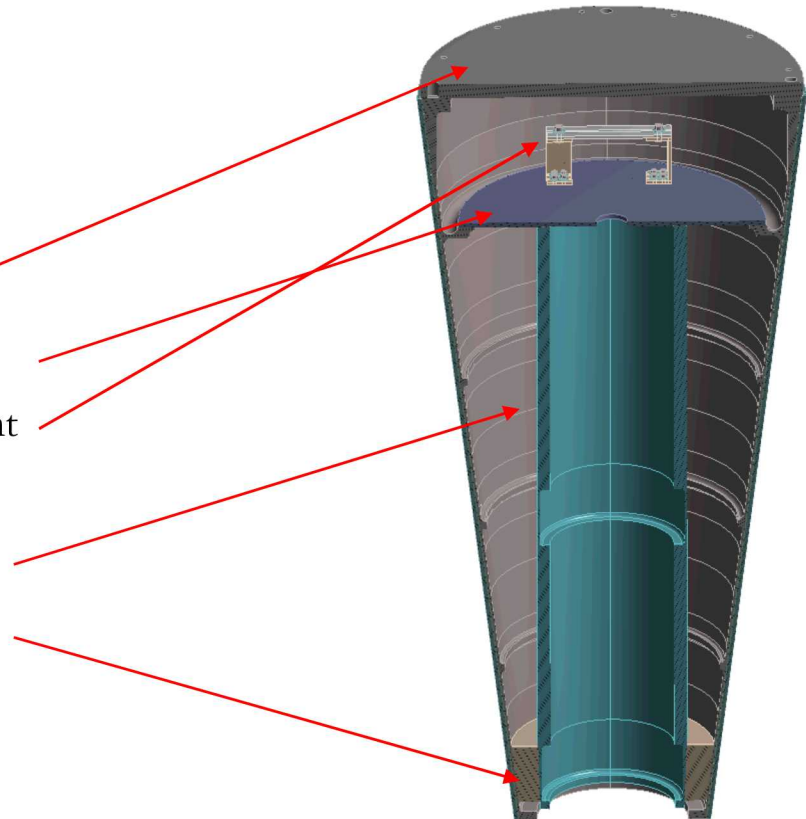
- The number of responses in  $x$  should be more than the number of modes to get a least squares estimate and some averaging of measurement errors.
- The desire was to extract the motion with around 30 gages. This required solving the problem in 5 different bands of the frequency domain to ensure the requirement of having more gages than modes. Bands had 13 to 28 modes.

# Proof of concept Hardware for estimating unmeasured responses MATV

- The project proposed to prove the estimation of unmeasured responses concept using research hardware provided by AWE known as the Modal Analysis Test Vehicle (MATV) which would be tested in a field random acoustic environment.

- MATV Description

- A meter long
- 47 kg
- Composite wrapped on aluminum substrate cone
- Large end aluminum cover plate
- Aluminum internal flat component plate
- Bracket called the Removable Component (RC) bolted to the internal component plate
- Steel pipe bolted to the component plate
- Foam support between pipe and cone at small end



## 5 Field Acoustic Test for MATV

- A field acoustic test was run to 147 dB at the Institute of Sound and Vibration Research at Southampton University in a reverberant chamber with horn.
- Place in corner of chamber
- Horn
- MATV suspended by bungees
- 69 total accelerometer channels recorded

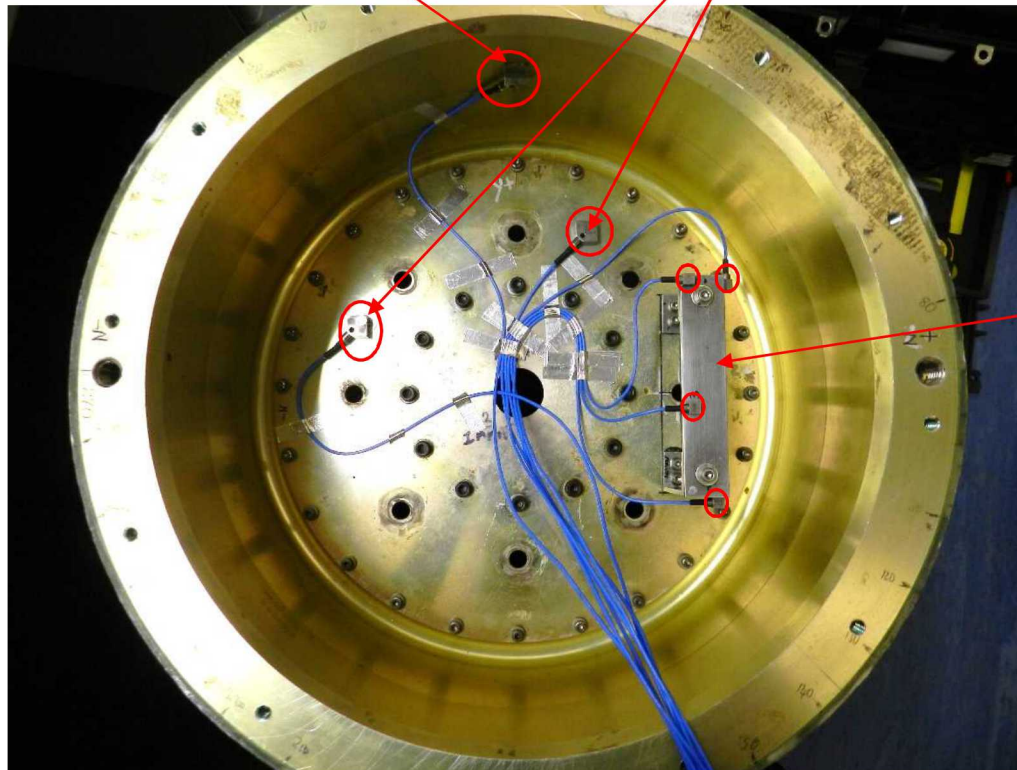


## 6 MATV Truth “Unmeasured” Accel Locations

- 14 truth accelerometer locations were chosen either on the RC or triaxial locations at typical mounting locations for a component. These were not used in the set to calculate the modal  $q$ 's, but were measured to compare the estimates generated from the FE model along with the  $q$ 's extracted from the 35 measurement accels.

1 Triax on Cone

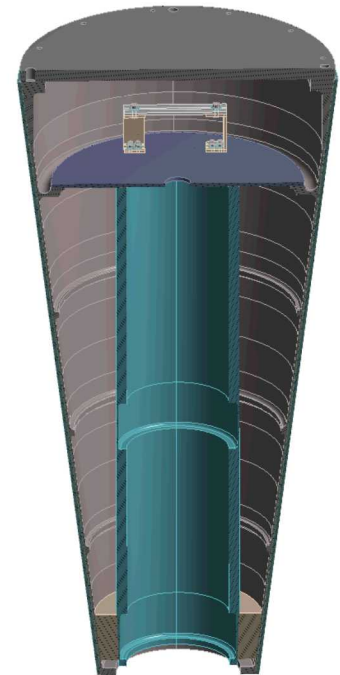
2 Triax on Component Plate



RC – 5 dof  
chosen on 4  
Triaxes

## 7 Finite Element Model Updating

- 78 modes were used in the 2000 Hz bandwidth of interest in the FE model.
- A three shaker modal test was performed up to 2000 Hz.
- All identified test ovaling modes were within 5 percent of the model frequencies.
- The foam contact at small end was adjusted to bring the first axial mode and the first torsion mode within 5 percent of the corresponding modal test modes.
- 22 modes were well correlated in frequency up to 1329 Hz torsion mode.
- Higher bending modes were not addressed due to resource constraints.



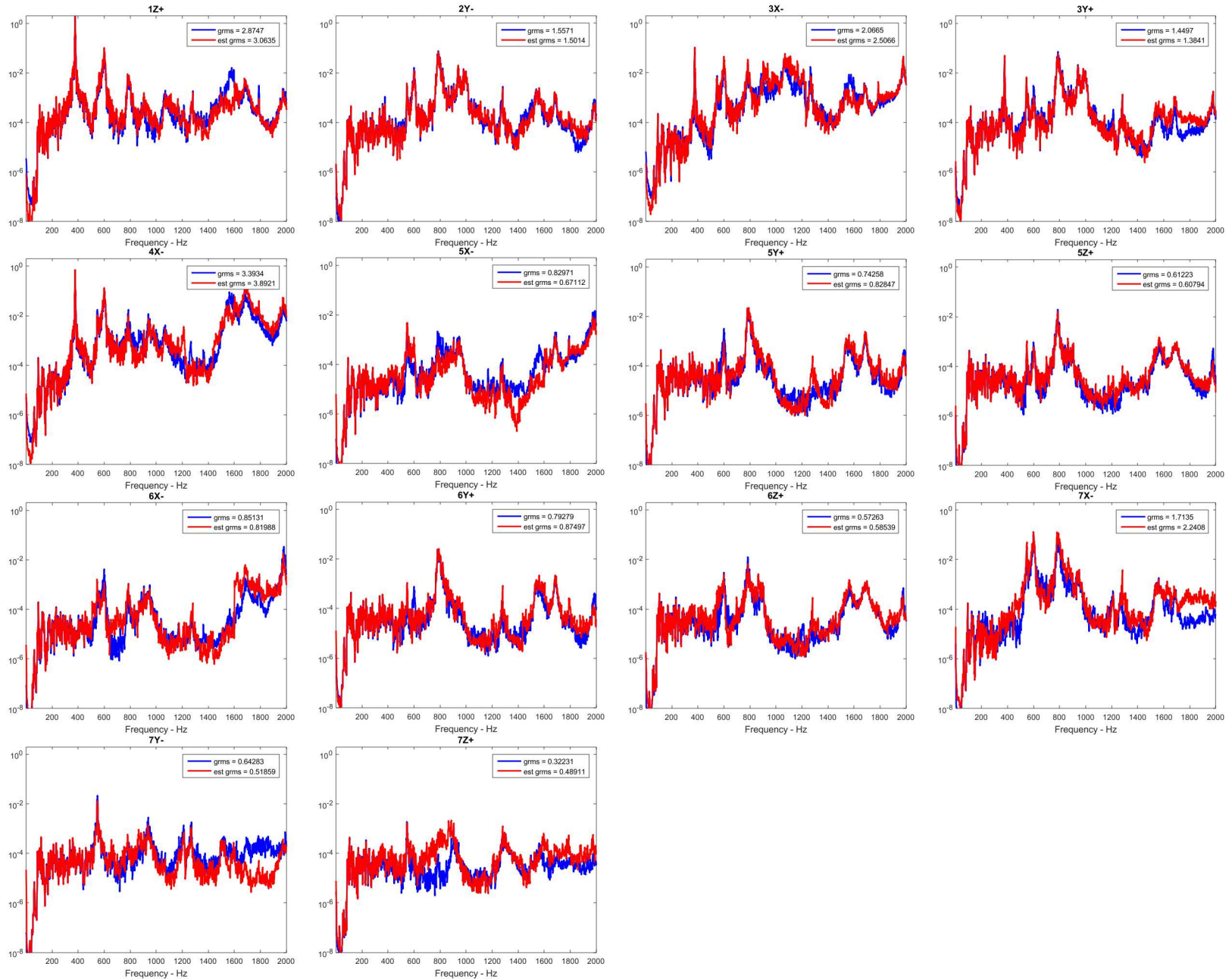
## 8 Selection of measurement gages

- 54 accelerometers were available from which to choose “measurement” gages
- 35 gages were selected using an algorithm that minimized the sum of the condition numbers of the FE mode shape matrix from each of 5 frequency bands.
- The estimate of the responses of the measurement gages were examined. It was noted that some responses had “dropouts”. It was found that dropouts in one band could be improved dramatically by adding in a few lower frequency “out of band” modes.
- Final condition numbers were 8, 55, 703, 12, 130 for  
18, 20, 13, 13, 28 modes in each band.
- We suspect this condition number based algorithm can be improved, but at least it is a theoretical way to select the measurement locations in advance and can be vetted by how well it reproduces the responses measured to extract the  $q$  values.

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# 14 Truth PSDs— Blue is Measured, Red is Algorithm Estimate



## Conclusions

- With good measurements, the process is effective if the FE mode shapes span the space of the actual motion. Results with the updated FE model were much better than before it was updated.
- By operating in multiple limited frequency bands, fewer measurements are required than total number of modes in the desired bandwidth associated with traditional modal filter theory. Here we utilized 35 measurements on a system with 78 modes.
- There is room for improvement in the condition number based measurement selection algorithm and the frequency band selection process. (Some are looking at other gage selection algorithms, e.g. Kammer's effective independence which can run much faster).
- This proof of concept adds fidelity to the idea that responses at unmeasured locations can be estimated with a limited set of measured responses.