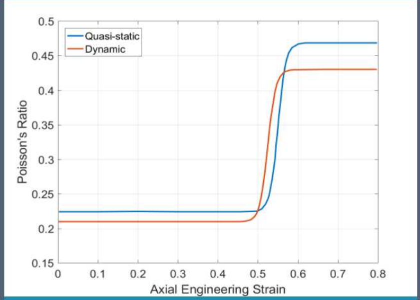
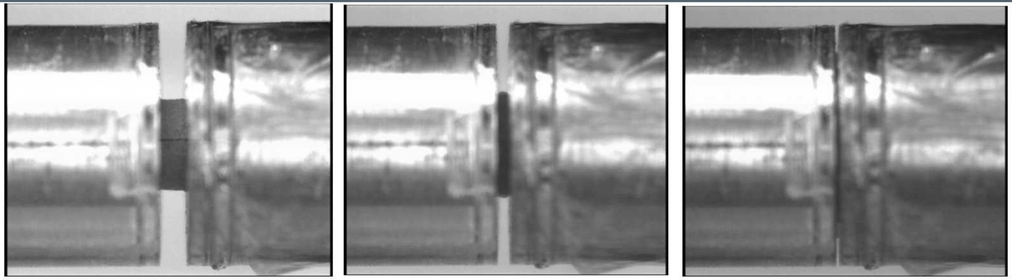
SAND2018-9627C

# Poisson's Ratio Induced Radial Inertia During Dynamic Compression of Hyperelastic Foams



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SAND2018-###

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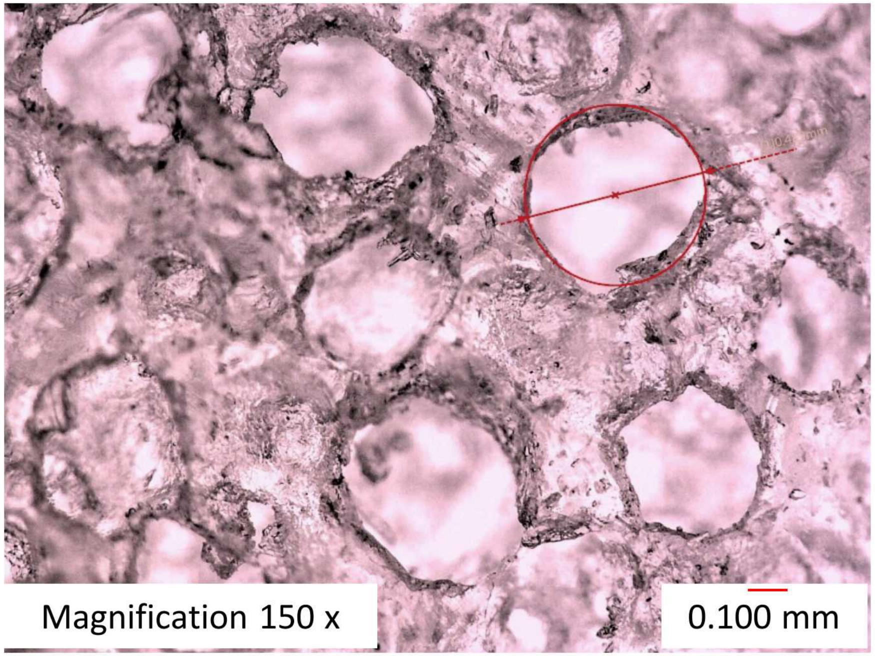
# Silicone Foam Background

- Shock Vibration Isolation and Reduction
  - Light weight
  - Excellent energy absorption
  - Full recovery after impact loading




Open-Cell Silicone foam

Density: 608 ± 21.85 kg/m$^3$

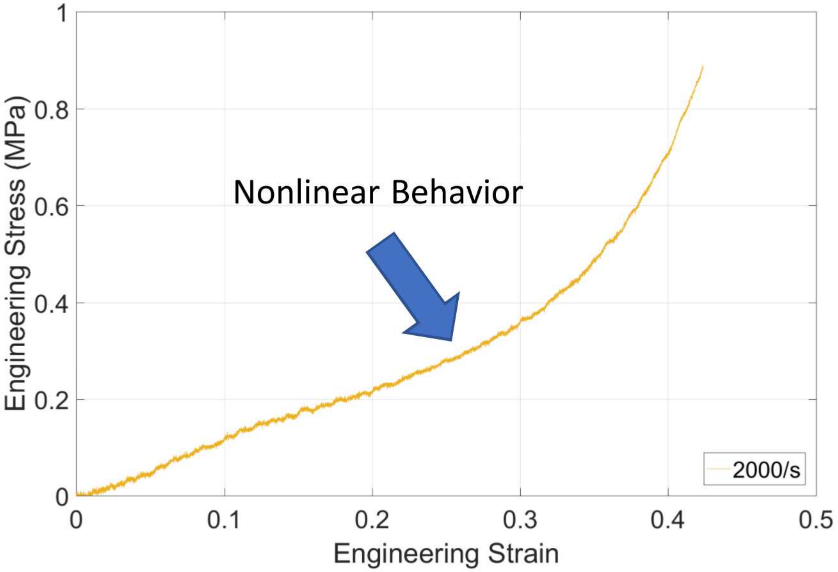
Average cell size: ~0.5 mm

Porosity:~50%

At first glance, it may seem OK to neglect Poisson's Ratio or assume a small number near zero due to apparent compressibility

# Silicone Foam

- Mechanical response of silicone foam may be altered by…
  - Strain rate
  - Stress-state
  - Temperature




Shock or vibration isolation pads can undergo

- High strain rate
- High densification state

Past densification, material may lose compressibility

How does Poisson's Ratio change with strain rate and densification?

# Poisson's Ratio

- Poisson's Ratio is typically regarded as a material *constant*

$$\nu = -\frac{e_r}{e_x}$$

Good for metals in Elasticity

*Small strains*

Materials such as foams are undergo large deformation

$$\nu = -\frac{\ln \lambda_r}{\ln \lambda_x} = -\frac{\varepsilon_r}{\varepsilon_x}$$

True strains along axial and radial directions

*Secant Poisson's Ratio*

Applicable to Linear Response Only

In the case of nonlinear large deformation, ***tangent Poisson's ratio*** with true strains represents actual Poisson's ratio

$$\nu = -\frac{d\varepsilon_r}{d\varepsilon_x}$$

# Literature

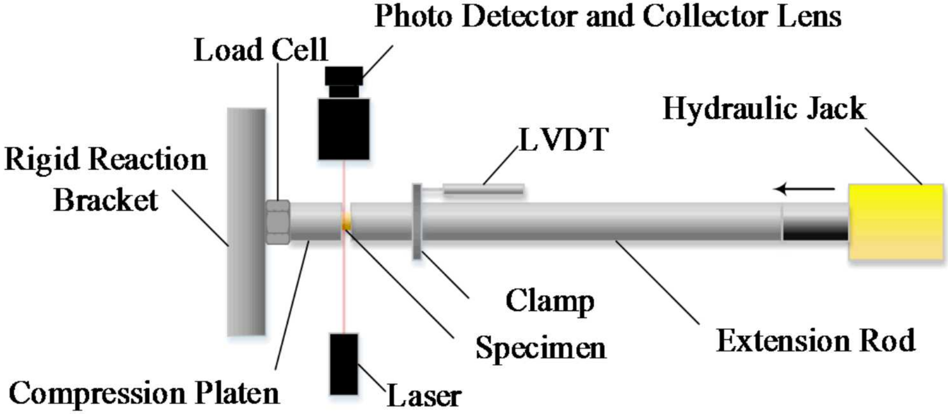
- Most available studies on Poisson’s ratio…

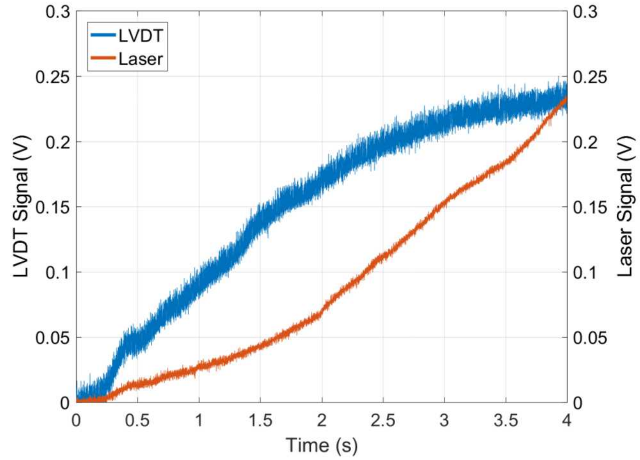
| | | |
|---|---|---|
| **Conducted at quasi-static strain rates** | → | **Response might not apply to high strain rates** |
| **Limited to small deformations** | → | **Material can undergo large deformation during use** |
| **Some measure large deformation, use secant ratio to calculate** | → | **Tangent method must be used for accurate results** |

**Our Approach**: Measure Poisson’s ratio at **high rate***, **large deformation**, and calculate using **tangent ratio** for best results

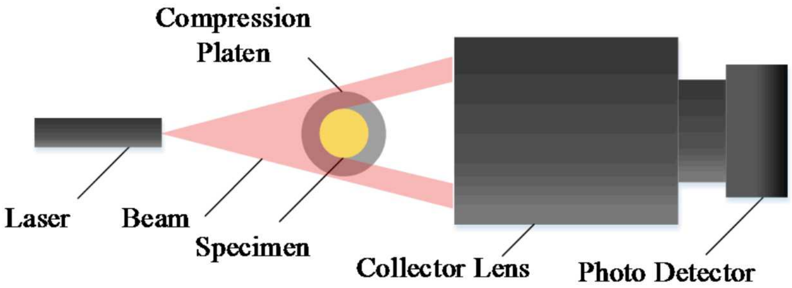
*quasi-static also included as a check

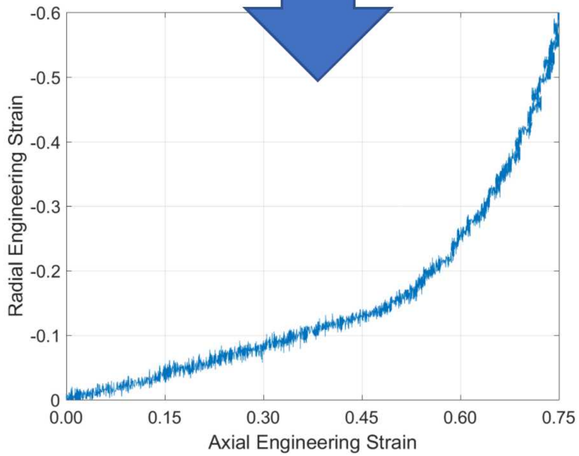
# Quasi-static experimental setup and calibration



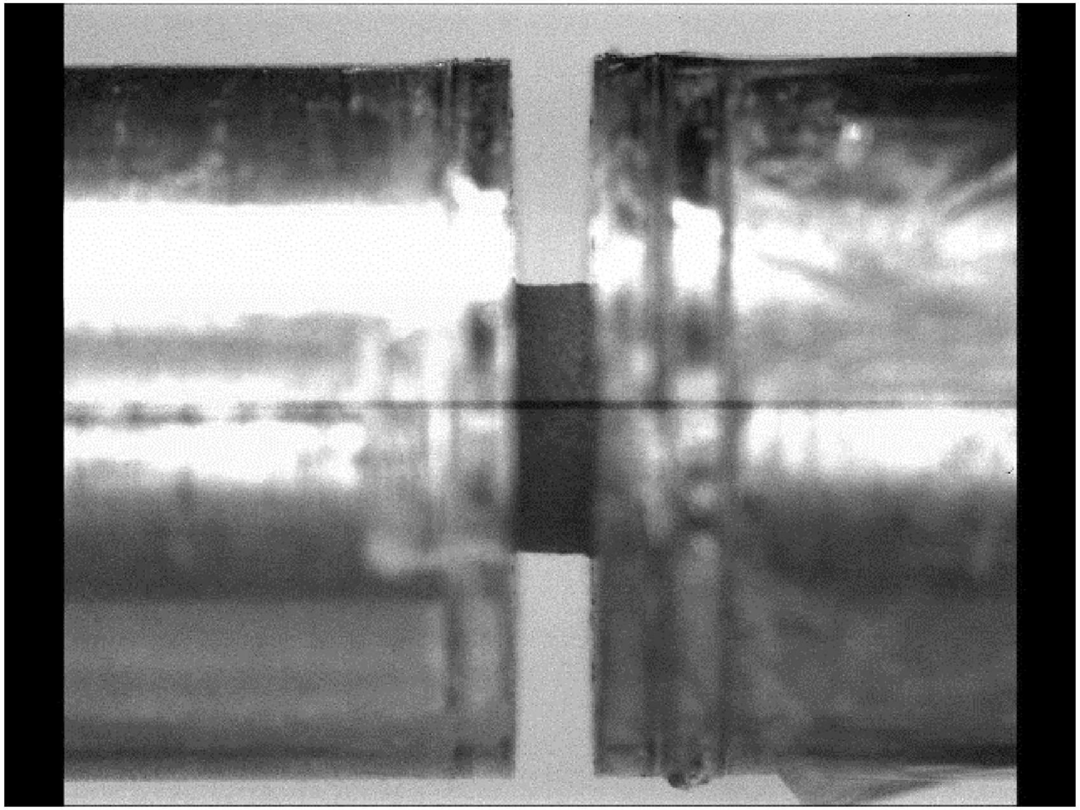




$$e_x = 1 - \frac{l_1}{l_0} \qquad e_r = 1 - \frac{d_1}{d_0}$$

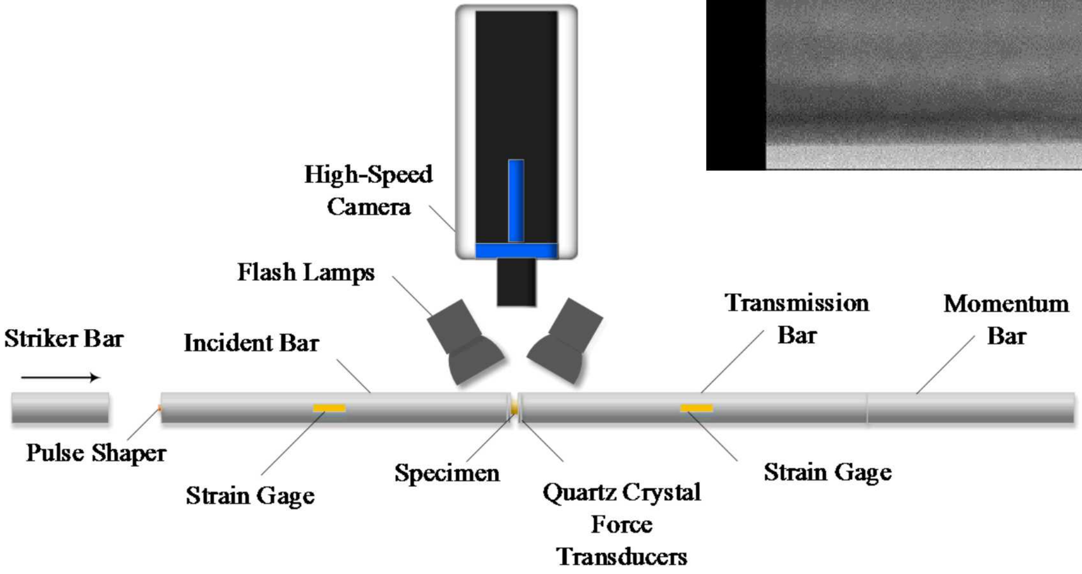


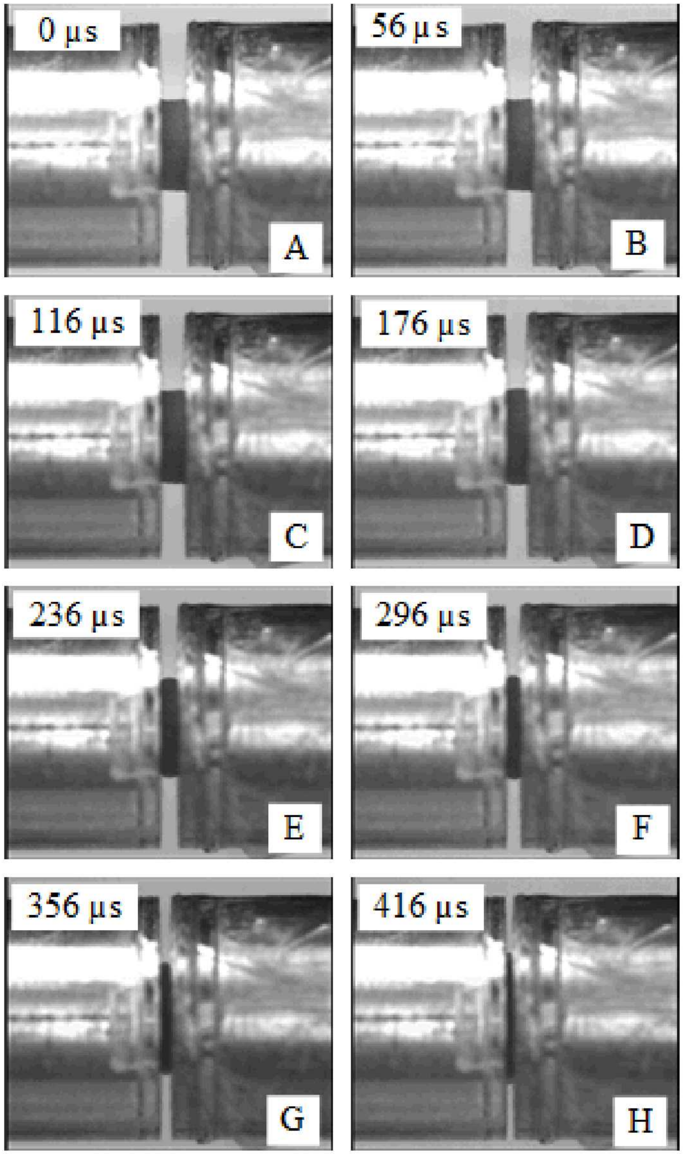
# High Rate Experiments

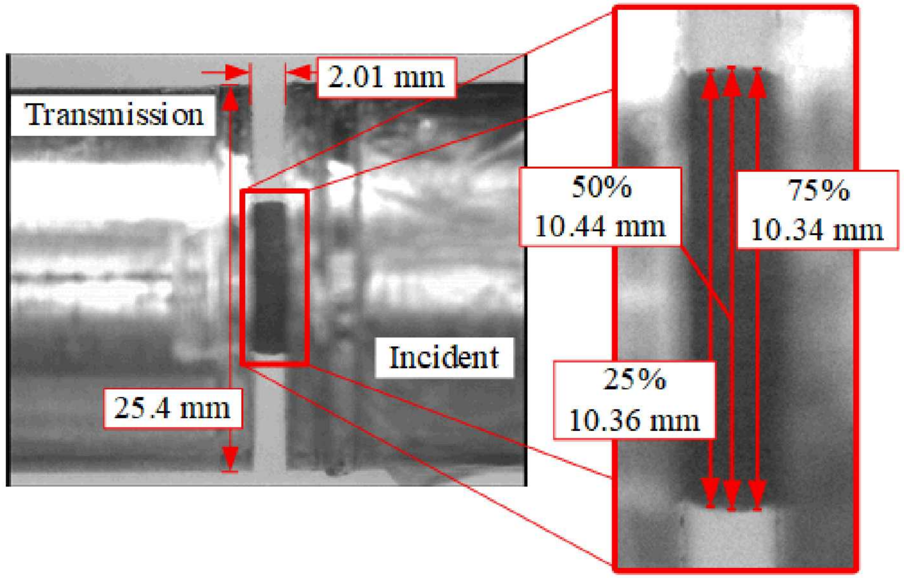


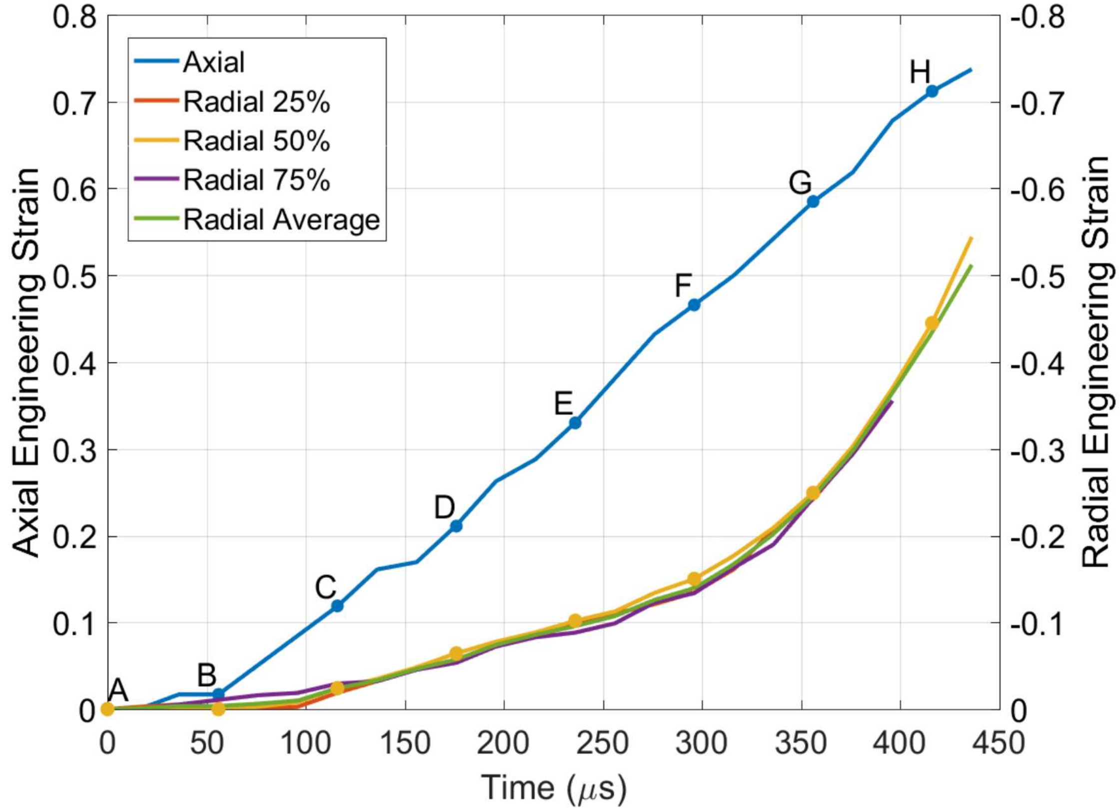
250 kFPS
2000 $s^{-1}$



# High Rate


0 µs
A
56 µs
B
116 µs
C
176 µs
D
236 µs
E
296 µs
F
356 µs
G
416 µs
H


2.01 mm
Transmission
Incident
25.4 mm
50%
10.44 mm
75%
10.34 mm
25%
10.36 mm

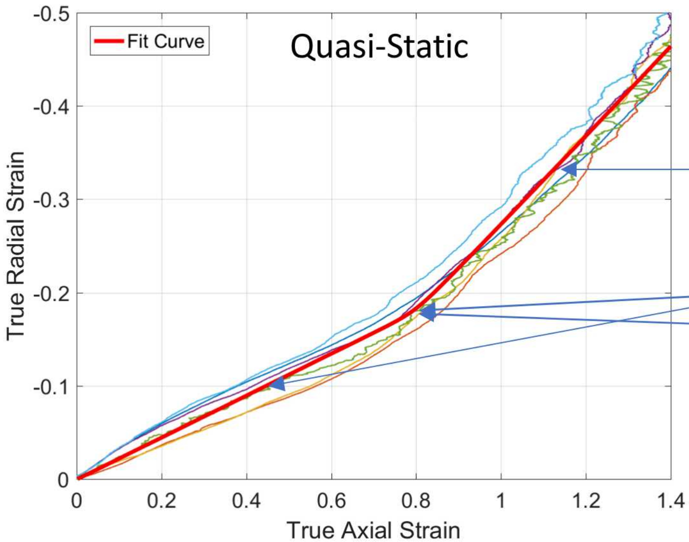

Axial
Radial 25%
Radial 50%
Radial 75%
Radial Average
A
B
C
D
E
F
G
H
Axial Engineering Strain
Radial Engineering Strain
Time (µs)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
50
100
150
200
250
300
350
400
450

# Poisson's Ratio Calculation

$$\nu = -\frac{d\varepsilon_r}{d\varepsilon_x}$$

- Tangent Poisson's Ratio for large, nonlinear deformation

$$\varepsilon = -\ln\left(1-e\right)$$ True strain: Axial compression (+) and Radial Tension (-)

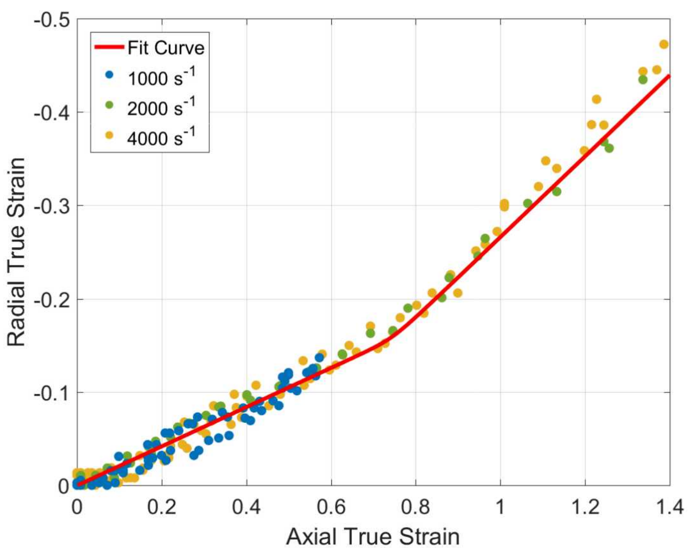



Boltzmann Sigmoidal Function

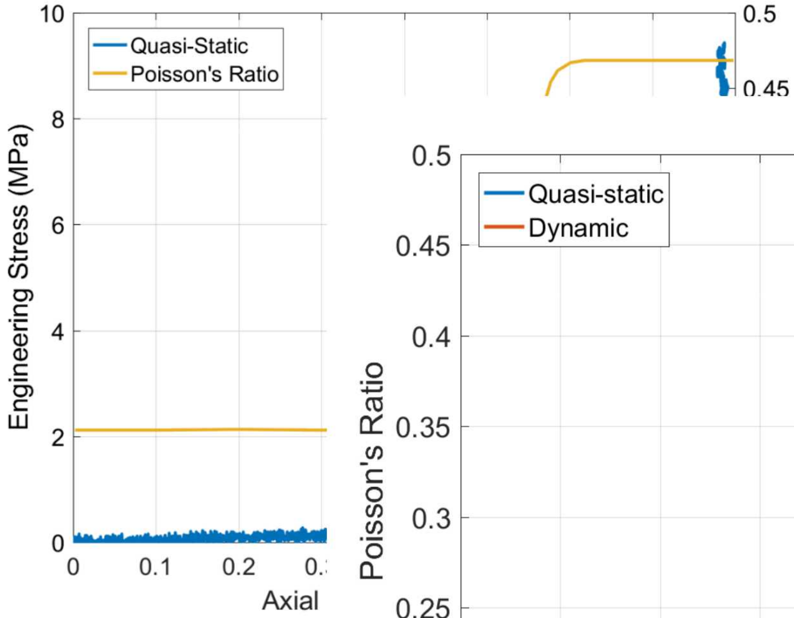
Bilinear

$$\nu\left(e_x\right) = \frac{\nu_1 - \nu_2}{1 + \exp\left(\frac{e_x - e_{x0}}{\delta}\right)} + \nu_2$$

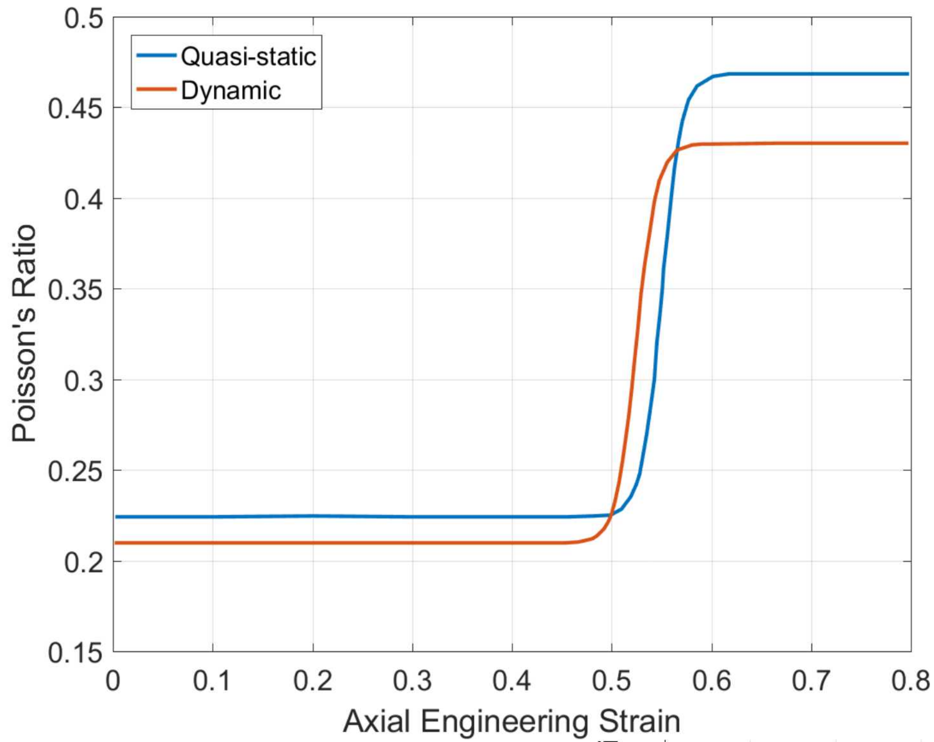
| Parameter | Quasi-static | Dynamic |
|---|---|---|
| $\nu_1$ | 0.22 | 0.21 |
| $\nu_2$ | 0.47 | 0.43 |
| $e_{x0}$ | 0.550 | 0.525 |
| $\delta$ | 0.01 | 0.01 |

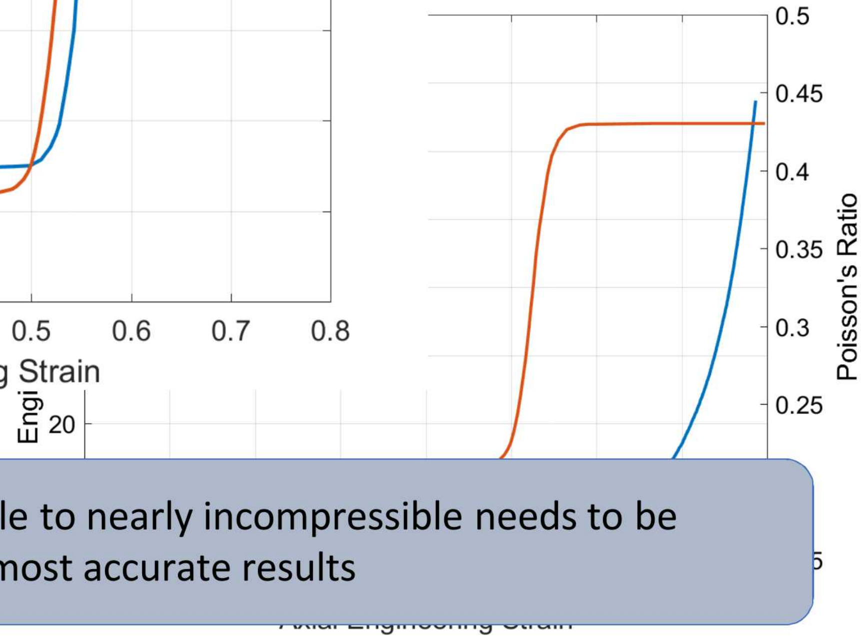


# QS and High Rate Poisson’s Ratio




- Change in ν coincided with
  r both rate

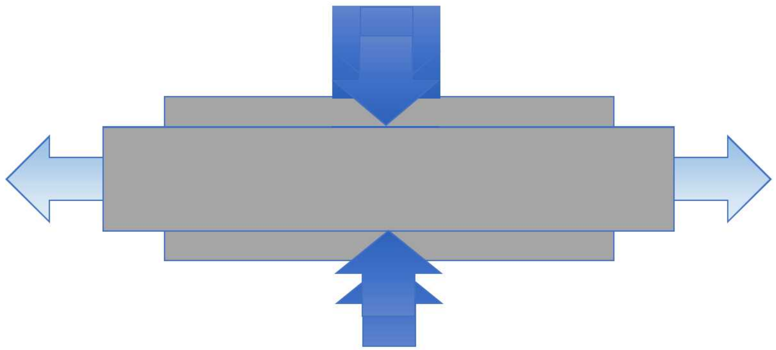



Change in Poisson’s ratio from compressible to nearly incompressible needs to be included in models for most accurate results

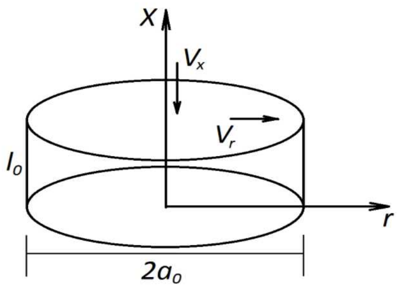
# Radial Inertia during Dynamic Compressive Loading

- Change in Poisson's Ratio affects the measured response of the material due to ***Radial Inertia***



Dharan and Hauser (1970): In a compression test with constant axial velocity:

- At a specific location, particle velocity along lateral (radial) direction increases with time (or increasing strain/decreasing specimen thickness)
- At a specific time, particle at outer diameter moves faster along lateral (radial) direction than the particle at inner diameter




$$V_r(t) = \frac{r}{2l} V_x(t)$$

Result: Radial Confinement

# Consequence of Radial Inertia

- Abnormal axial stress history → May overshadow material response (soft materials)

Abnormal spike in stress history due to radial inertia

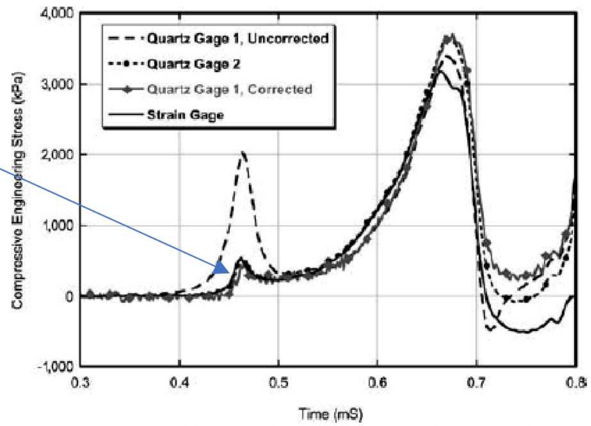



**Fig. 1** The abnormal initial spike-like features in the stress histories of a gelatin specimen

The bump is stress measured in axial direction due to radial-inertia-induced confinement in the sample

Could be incorrectly attributed to material response

*Song, B., Chen, W.W., Ge, Y., Weerasooriya, T., (2007) Radial inertia effects in Kolsky bar testing of extra-soft materials. Experimental Mechanics, 47:659-670.*

# Currently Existing Analysis

- Compressible Solid
- Small deformation

- Incompressible Solid
- Large deformation

➢ Kolsky (1949)

$$\sigma_z = \frac{\nu^2 a_0^2 \rho_0}{2} \cdot \ddot{e}$$

❖ Dharan and Hauser (1970)

$$\bar{\sigma}_z = \frac{\rho_0 a_0{}^2}{4l_0\left(1-e_x\right)^2}\left[\frac{3V_x^2}{2l_0\left(1-e_x\right)} + \frac{dV_x}{dt}\right]$$

$$\bar{\sigma}_z = \frac{1}{\pi a^2}\int_0^a \int_0^{2\pi} \sigma_z(r) r dr d\theta$$

➢ Forrestal et al. (2007)

$$\sigma_z = \frac{\nu^2\left(3-2\nu\right)}{4\left(1-\nu\right)}\left[a_0^2 - \frac{2r^2}{\left(3-2\nu\right)}\right]\rho_0 \cdot \ddot{e}$$

➢ Warren and Forrestal (2010)

$$\sigma_z = \frac{\rho}{4\left(1-e_x\right)^2}\left[\frac{3\dot{e}^2}{2\left(1-e_x\right)} + \ddot{e}_x\right]\left(a_0^2 - r^2\right)$$

Factors that affect radial inertia stress

Specimen Geometry:
- Specimen Diameter
- Specimen Thickness

Loading Conditions:
- Impact Velocity
- Strain Rate
- Strain

Specimen Properties:
- Density
- Poisson's Ratio

# Comprehensive Radial Inertia Analysis

- Re-deriving the previous analysis using **mass** and **momentum** conservation

***Mass:***

$$\frac{1}{\rho(t)}\cdot\frac{d\rho(t)}{dt}=-\left(\frac{V_x(t)}{l(t)}+2\frac{V_r(r,t)}{r(t)}\right)$$

***Momentum :***

$$\frac{\partial\sigma_r(r,t)}{\partial r}=-\rho(t)\left(\frac{\partial V_r(r,t)}{\partial t}+V_r(r,t)\frac{\partial V_r(r,t)}{\partial r}\right)$$

$$\frac{\partial\sigma_r(r,t)}{\partial r}=-\rho(t)\cdot\frac{r}{1-e_x(t)}\cdot\left[\nu(t)\ddot{e}_x(t)+\nu(t)\cdot(\nu(t)+1)\cdot\frac{\dot{e}_x^2(t)}{1-e_x(t)}+\frac{d\nu(e_x)}{de_x}\dot{e}_x^2(t)\right]$$

$$\sigma_r(r,t)=\rho(t)\cdot\frac{a^2(t)-r^2(t)}{2(1-e_x(t))}\cdot\left[\nu(t)\ddot{e}_x(t)+\nu(t)\cdot(\nu(t)+1)\frac{\dot{e}_x^2(t)}{1-e_x(t)}+\frac{d\nu(e_x)}{de_x}\dot{e}_x^2(t)\right]$$

$\nu(t)\to 0.5$, $\nu(t)\to 0.5$, $\nu(t)\to 0.5$, $\frac{d\nu(e_x)}{de_x}\to 0$

★ $$\sigma_r(r_0,t)=\rho_0\cdot\frac{a_0^2-r_0^2}{4(1-e_x(t))^2}\cdot\left[\ddot{e}_x(t)+\frac{3}{2}\cdot\frac{\dot{e}_x^2(t)}{1-e_x(t)}\right]$$

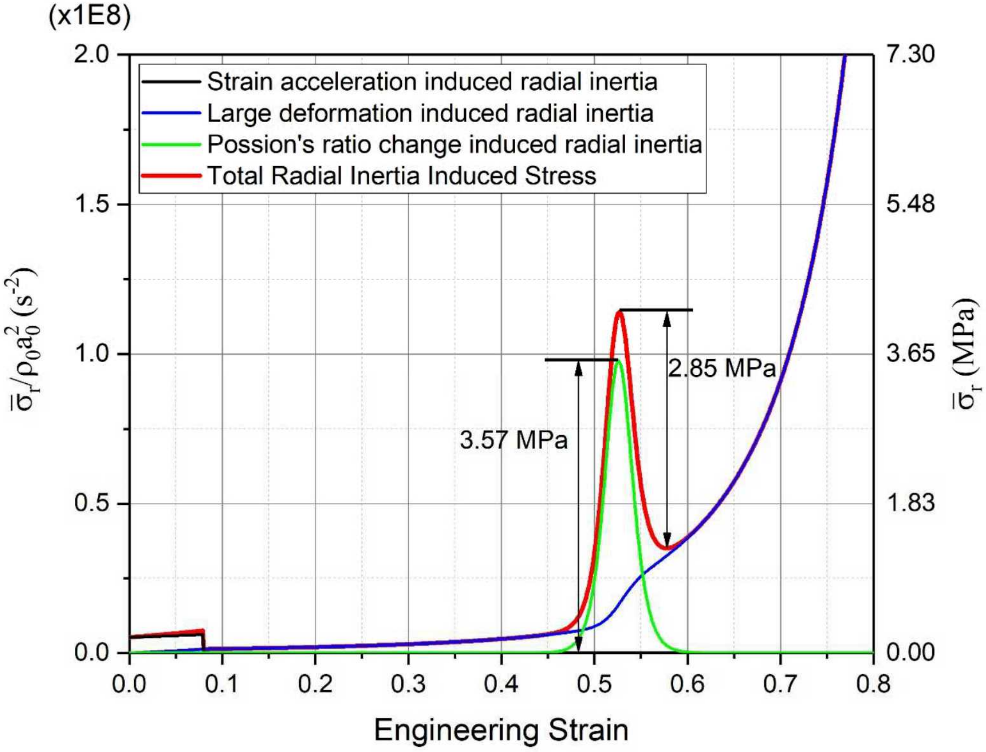
Recover equation from Warren and Forrestal (2010)

# Radial Inertia in Silicone Foam

Additional Measured Stress

$$\bar{\sigma}_r(t)=\frac{\rho_0\cdot a_0^2}{4\left(1-e_x(t)\right)^2}\cdot\left[\ddot{e}_x(t)\cdot\left(\nu_2+\frac{\nu_1-\nu_2}{1+\exp\left(\frac{e_x(t)-e_{x0}}{\delta}\right)}\right)+\left(\nu_2+\frac{\nu_1-\nu_2}{1+\exp\left(\frac{e_x(t)-e_{x0}}{\delta}\right)}\right)\cdot\left(1+\nu_2+\frac{\nu_1-\nu_2}{1+\exp\left(\frac{e_x(t)-e_{x0}}{\delta}\right)}\right)\cdot\frac{\dot{e}_x^2(t)}{1-e_x(t)}+\dot{e}_x^2(t)\cdot\frac{\nu_2-\nu_1}{\delta}\cdot\frac{\exp\left(\frac{e_x(t)-e_{x0}}{\delta}\right)}{\left[1+\exp\left(\frac{e_x(t)-e_{x0}}{\delta}\right)\right]^2}\right]$$

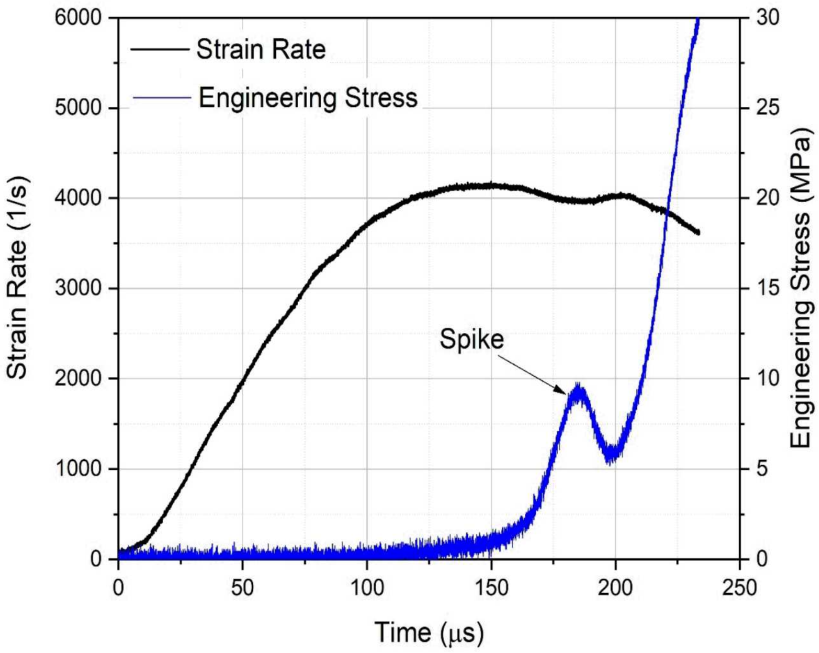
Extra Stress Including Poisson's Ratio Change Using Boltzmann Fit

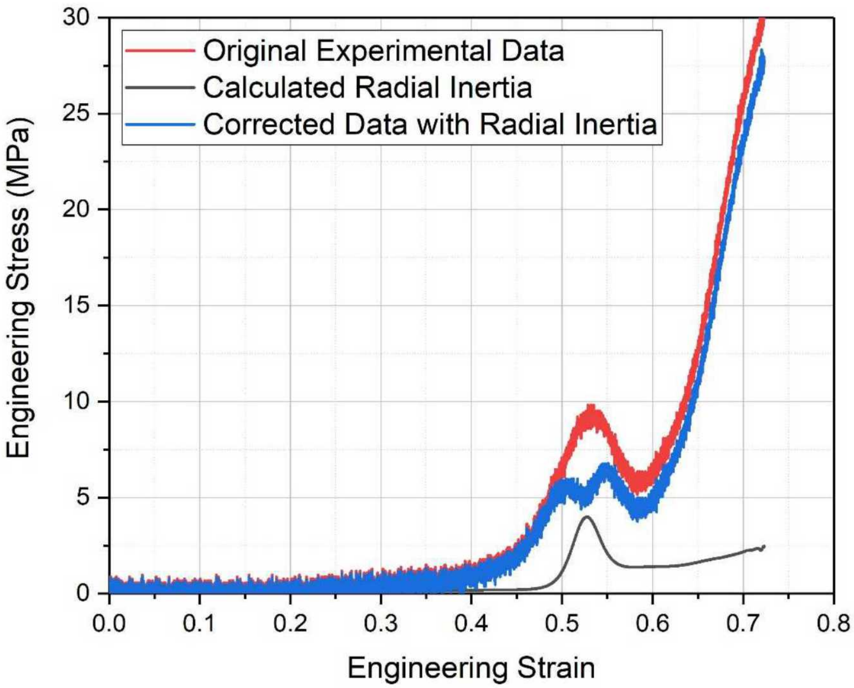



Contributors to Inertia Stress

- (Strain Acceleration)
- Large Deformation
- Poisson's Ratio Change

# Experimental Verification and Conclusion




Inertia stress induced by Poisson's ratio change is dominated during onset of densification (large strains)

Effect is worse at higher strain rates: almost no spike seen at 2000 $s^{-1}$, larger spike at 4000 $s^{-1}$ $(\dot{e}_x^2(t))$

This method can be used to correct inertia-induced stress spike seen in foam data

New radial inertia analysis is comprehensive especially accounting for Poisson's ratio change

# Conclusions

- Optical methods were used to measure deformation of silicone foam
- Radial-axial strains were collected and fit using a Boltzmann sigmoid function
- Tangent method to calculate Poisson’s ratio for large strain, nonlinear deformation
- Overall, foam transitioned from being compressible to nearly incompressible with densification
- Unique Poisson’s ratio behavior resulted in radial inertia during tests
- Comprehensive radial inertia analysis for SHPB experiment
- Specimen strain (large strains) and Poisson’s ratio change contributed to radial inertia in sample
- Numerical correction of stress-strain curve is possible but must be refined

# Questions

# Radial Laser Calibration