

Frequency comb generation from spontaneous mode locking **in a semiconductor laser**

SAND2018-8811C

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Mode-locking without saturable absorber

Semiconductor laser theory

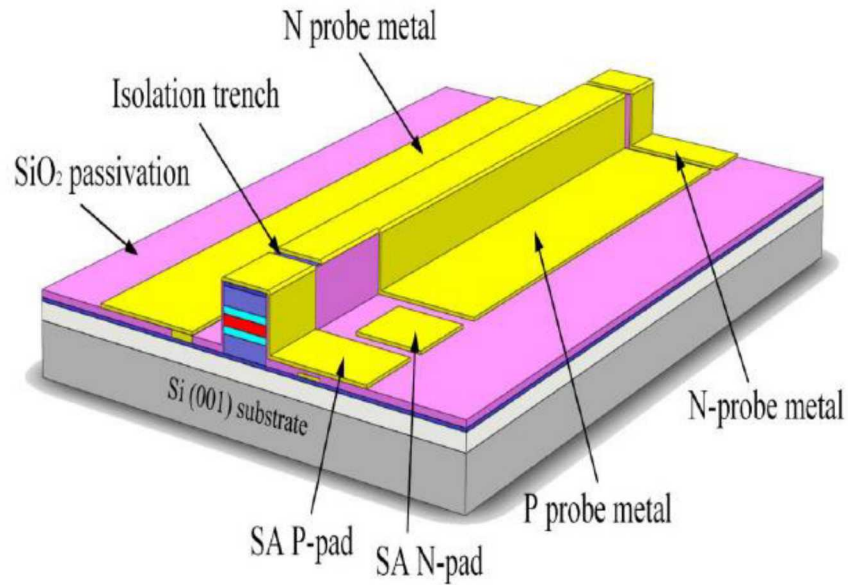
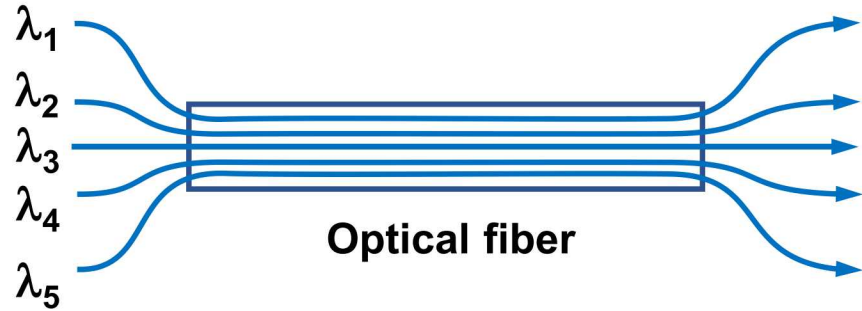
Loose ends

TAMU-Princeton-Baylor Summer School on Quantum Science and Engineering
Fest celebrating Lu Sham 80th birthday
Casper College, WY, July 22-28, 2018

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Mode-locking and frequency combs in a monolithic semiconductor laser diode

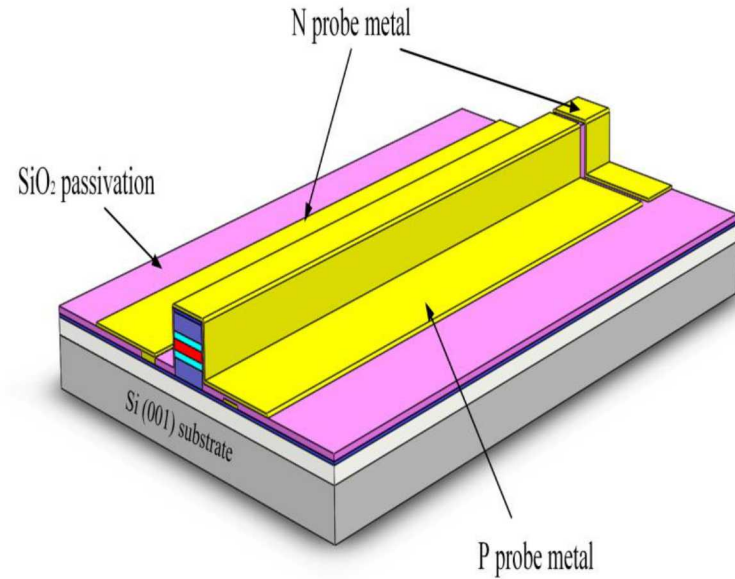
Application: Wavelength division multiplex (WDM) in optical communication



Active region

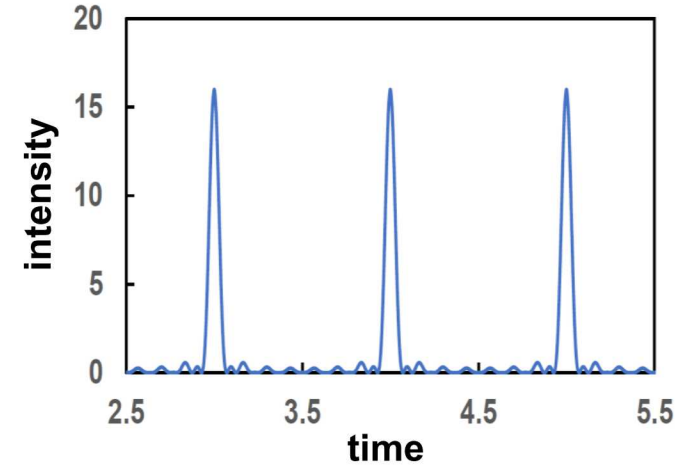


Frequency combs without saturable absorber

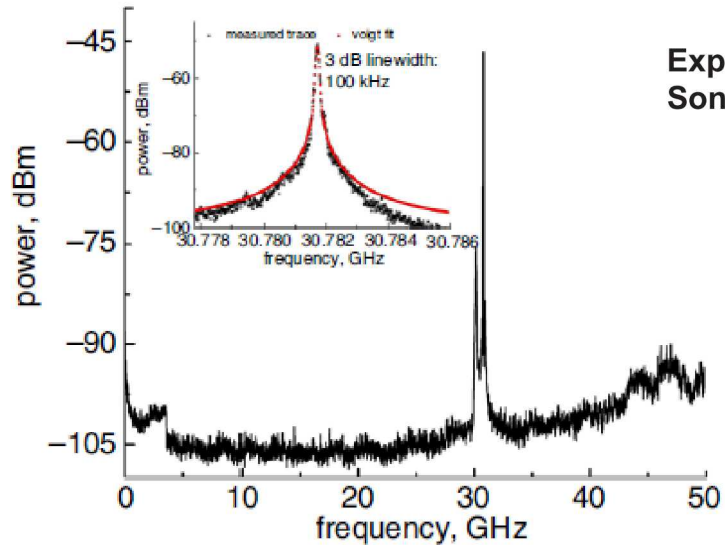
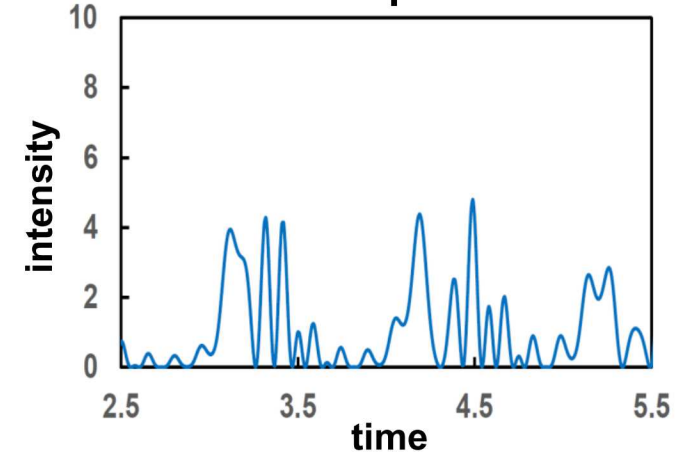


Why should there be spontaneous mode locking?

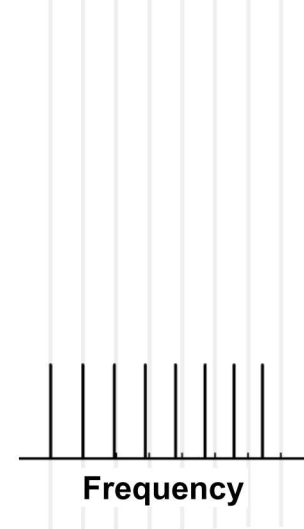
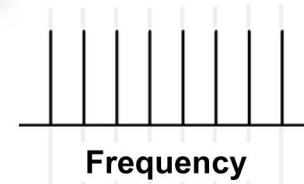
No dispersion



With dispersion

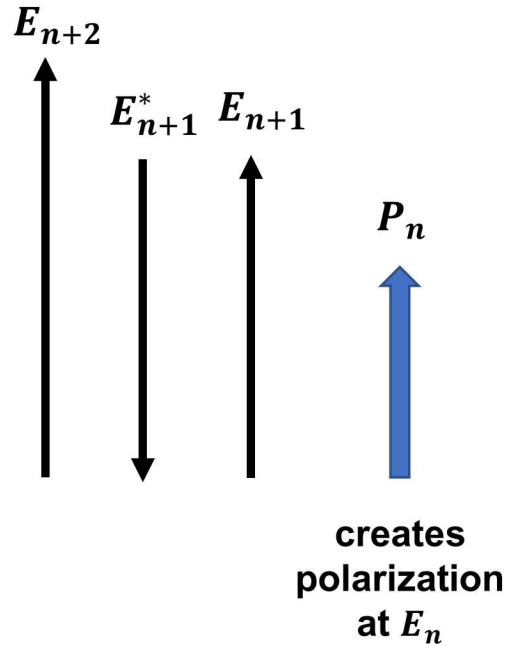


Experimental results: courtesy of Songtao Liu, John Bower's group, UCSB



Source of spontaneous mode locking

4-wave mixing

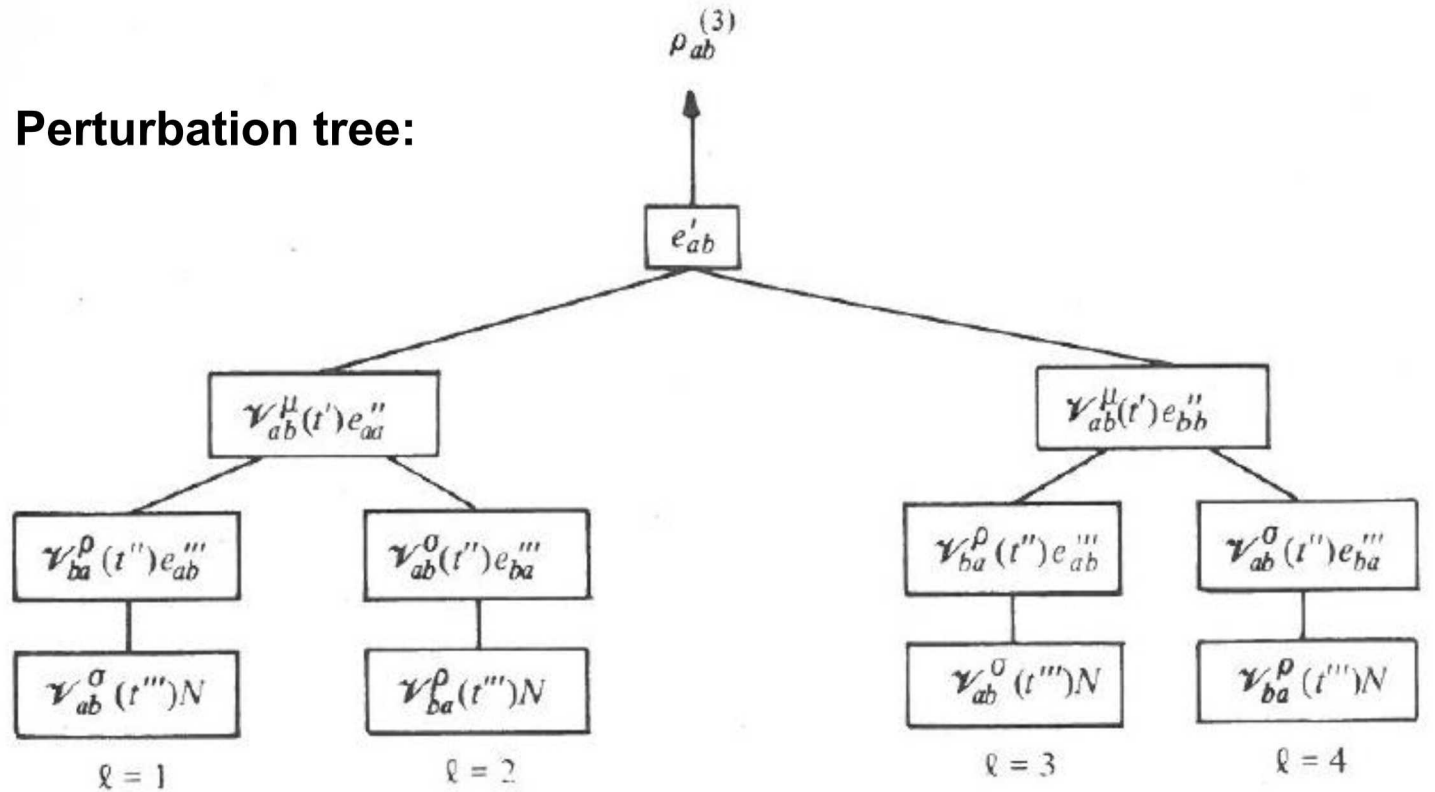


Sargent, Scully & Lamb:

$$\tilde{\rho} = \tilde{\rho}^{(1)} + \tilde{\rho}^{(3)} + \dots$$

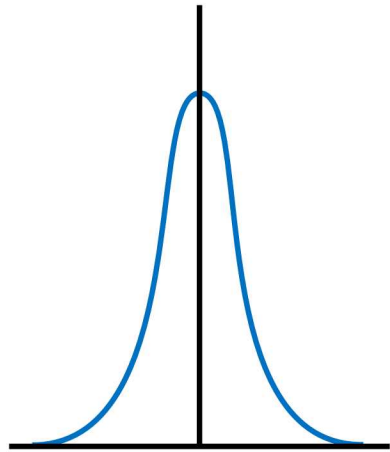
Gain
Mode pulling
↓
↑
Saturation
Mode pushing
Combination tones

Perturbation tree:



Homogeneous or Inhomogeneous broadening?

Semiconductor



or

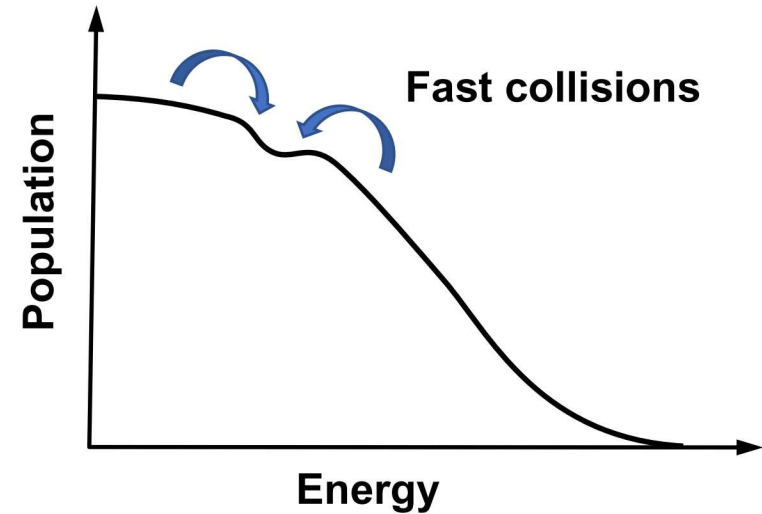
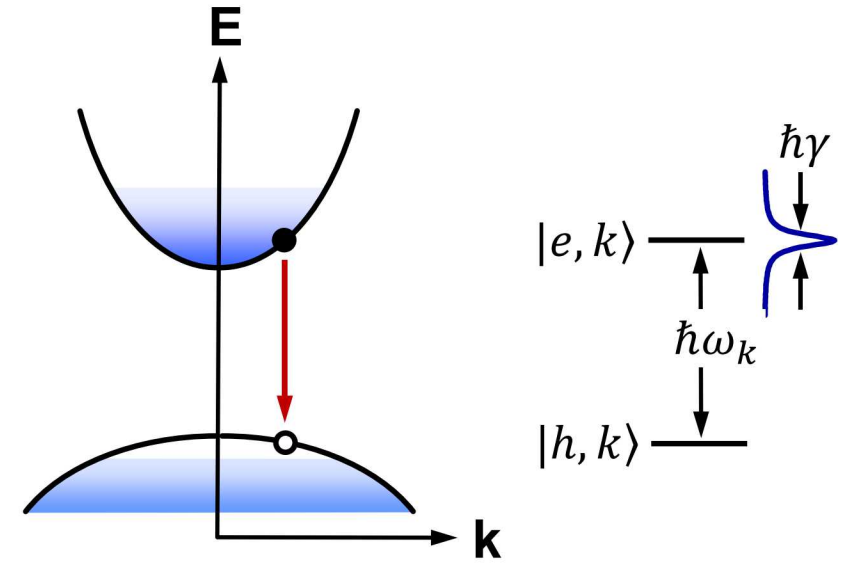
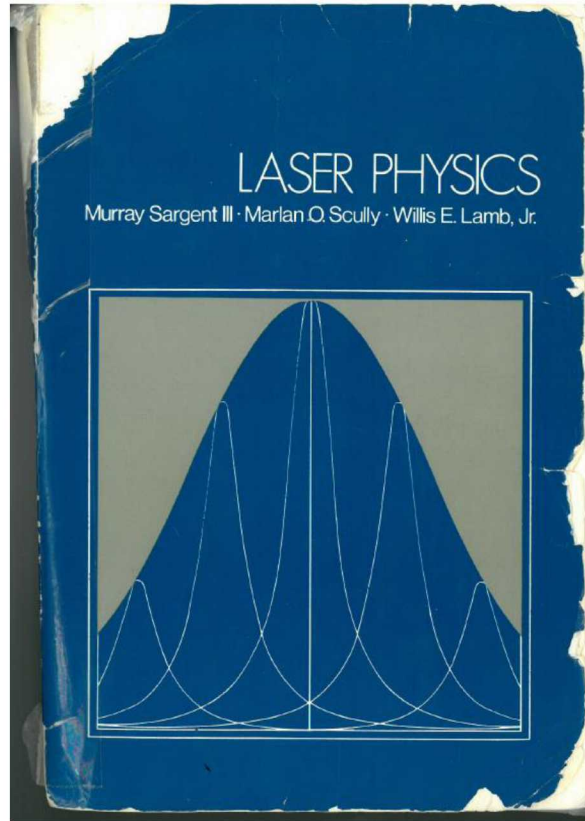


Table 9-1 or Table 10-2?

Semiconductor laser theory

Laser field

$$E(z, t) = \frac{1}{2} \sum_n \mathcal{E}_n(t) e^{-i[\nu_n t + \phi_n(t)]} u_n(z) + c. c.$$

Amplitude and frequency determining equations

$$\dot{\mathcal{E}}_n(t) = \left[\left(g_n(N) - \frac{\nu_n}{2Q} \right) - \sum_{m=1}^{n_{max}} \theta_{n,m}(N) \mathcal{E}_m^2(t) \right] \mathcal{E}_n(t) - \sum_{n' \neq n} \sum_{m \neq n} \sum_{m' \neq n} \text{Im} [\theta_{nn'mm'}(N) e^{i\psi_{nn'mm'}}] \mathcal{E}_{n'}(t) \mathcal{E}_m(t) \mathcal{E}_{m'}(t)$$

$$\nu_n + \dot{\phi}_n(t) = \Omega_n + \sigma_n(N) - \sum_{m=1}^{n_{max}} \tau_{n,m}(N) \mathcal{E}_m^2(t) - \sum_{n' \neq n} \sum_{m \neq n} \sum_{m' \neq n} \text{Re} [\theta_{nn'mm'} e^{i\psi_{nn'mm'}}] \mathcal{E}_{n'}(t) \mathcal{E}_m(t) \mathcal{E}_{m'}(t) \mathcal{E}_n^{-1}(t)$$

Combination tones



Solve simultaneously

Total carrier density

$$\dot{N}(t) = \frac{\eta J}{ed} - \gamma_{nr} N(t) - \sum_n \frac{\epsilon_0 c}{\hbar \nu_n} g_n \mathcal{E}_n^2(t)$$

Relative phase angle

$$\psi_{nn'mm'} = (\nu_n - \nu_{n'} + \nu_m - \nu_{m'})t + \phi_n(t) - \phi_{n'}(t) + \phi_m(t) - \phi_{m'}(t)$$

Hi Weng!

Well currently Kamie and I are in Bogota, but are returning to Seattle this evening. Colombia is a beautiful country with lots of mountains, sea, jungle, art, and spirit. People here are crazy about soccer, coffee, flowers.

Interesting that you're working on mode locking again. One of my favorite equations is

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{d\theta}{a + b \sin\theta} = \frac{1}{\sqrt{a^2 - b^2}}$$

which you can insert into an Office math zone (in Word, PowerPoint, OneNote,...) by typing `\integral<space>`. And one of my favorite subjects over the years has been four-wave mixing.

Kamie and I do visit Santa Fe to see Nicole & Co. We'll see her and Avalon this week in Seattle. It would be great to see you and catch up on everything!

Thanks,
Murray

Starting attempt: approximations

Multimode laser field

$$E(z, t) = \frac{1}{2} \sum_n \mathcal{E}_n(t) e^{-i[v_n t + \phi_n(t)]} u_n(z) + c. c.$$

Amplitude and frequency determining equations

~~$$\dot{\mathcal{E}}_n(t) = \left[\left(g_n(N) - \frac{\nu_n}{2Q} \right) - \sum_{m=1}^{n_{max}} \theta_{n,m}(N) \mathcal{E}_m^2(t) \right] \mathcal{E}_n(t) - \sum_{n' \neq n} \sum_{m \neq n} \sum_{m' \neq n} \text{Im} [\theta_{nn'mm'}(N) e^{i\psi_{nn'mm'}}] \mathcal{E}_{n'}(t) \mathcal{E}_m(t) \mathcal{E}_{m'}(t)$$~~

$$\nu_n + \dot{\phi}_n(t) = \Omega_n + \sigma_n(N) - \sum_{m=1}^{n_{max}} \tau_{n,m}(N) \mathcal{E}_m^2(t) - \sum_{n' \neq n} \sum_{m \neq n} \sum_{m' \neq n} \text{Re} [\theta_{nn'mm'} e^{i\psi_{nn'mm'}}] \mathcal{E}_{n'}(t) \mathcal{E}_m(t) \mathcal{E}_{m'}(t) \mathcal{E}_n^{-1}(t)$$

Total carrier density

$$\dot{N}(t) = \frac{\eta J}{ed} - \gamma_{nr} N(t) - \sum_n \frac{\epsilon_0 c}{\hbar \nu_n} g_n \mathcal{E}_n^2(t)$$



$$I_n = \frac{1}{N_{mode}} \frac{\eta \hbar \nu J_{th}}{ed G_{th}} \left[\frac{J}{J_{th}} - 1 \right]$$

Relative phase angle

$$\psi_{nn'mm'} = (\nu_n - \nu_{n'} + \nu_m - \nu_{m'})t + \phi_n(t) - \phi_{n'}(t) + \phi_m(t) - \phi_{m'}(t)$$

Solve simultaneously

Starting attempt: approximations

Multimode laser field

$$E(z, t) = \frac{1}{2} \sum_n \mathcal{E}_n(t) e^{-i[v_n t + \phi_n(t)]} u_n(z) + c.c.$$

Amplitude and frequency determining equations

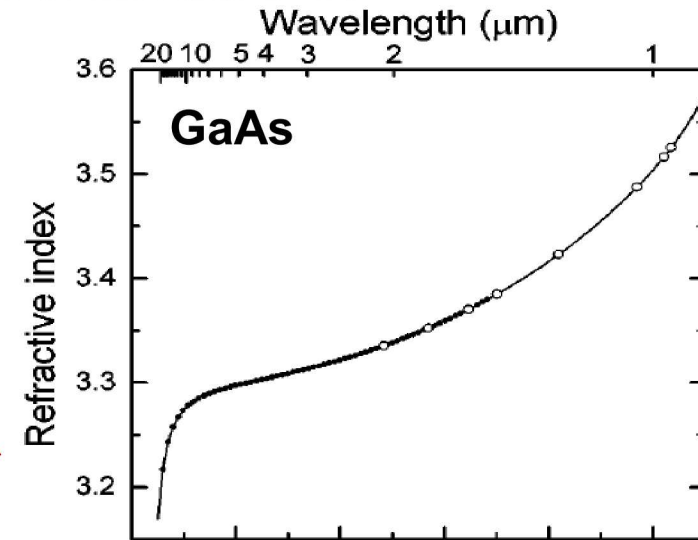
$$v_n + \dot{\phi}_n(t) = \Omega_n + \sigma_n(N) - \sum_{m=1}^{n_{max}} \tau_{n,m}(N) \mathcal{E}_m^2(t) - \sum_{n' \neq n} \sum_{m \neq n} \sum_{m' \neq n} \text{Re} [\theta_{nn'mm'} e^{i\psi_{nn'mm'}}] \mathcal{E}_{n'}(t) \mathcal{E}_m(t) \mathcal{E}_{m'}(t) \mathcal{E}_n^{-1}(t)$$

Total carrier density

$$I_n = \frac{1}{N_{mode}} \frac{\eta \hbar \nu J_{th}}{edG_{th}} \left[\frac{J}{J_{th}} - 1 \right]$$

Relative phase angle

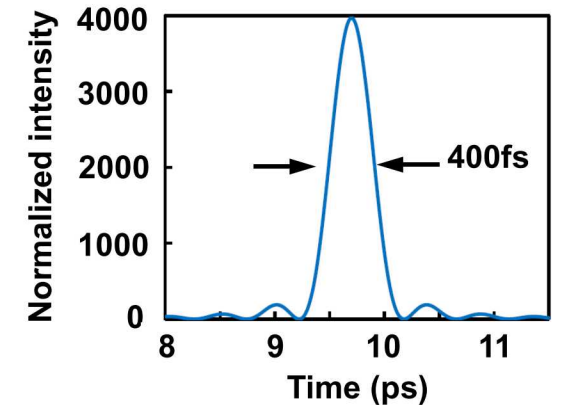
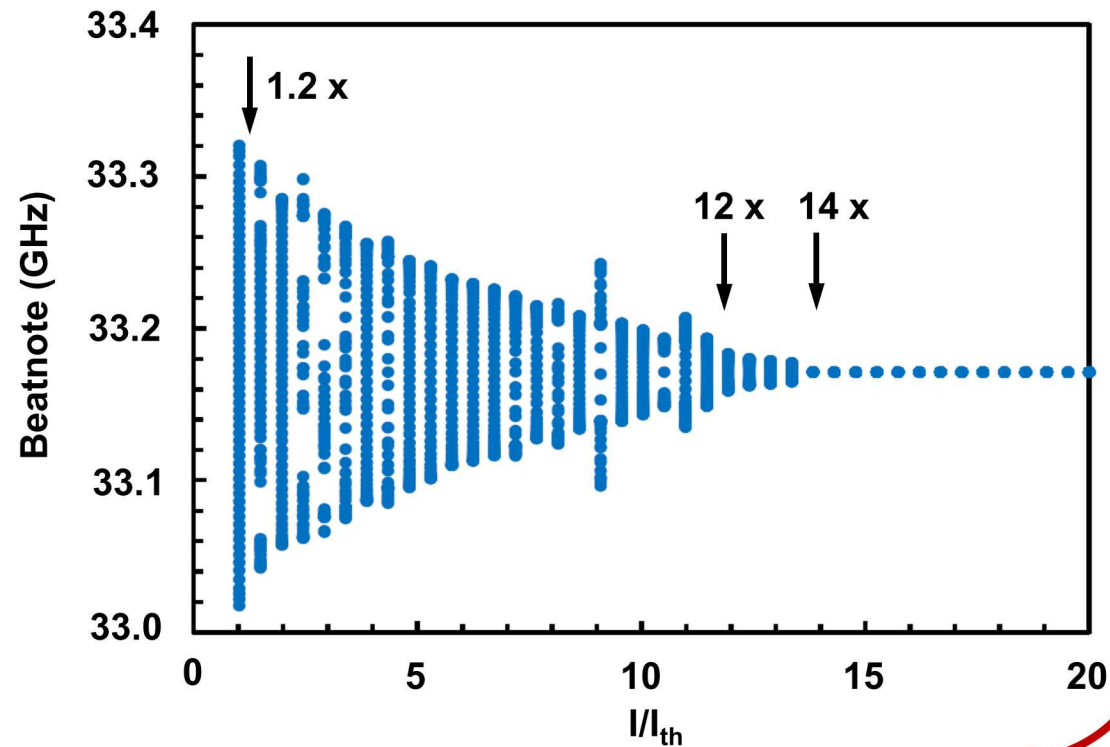
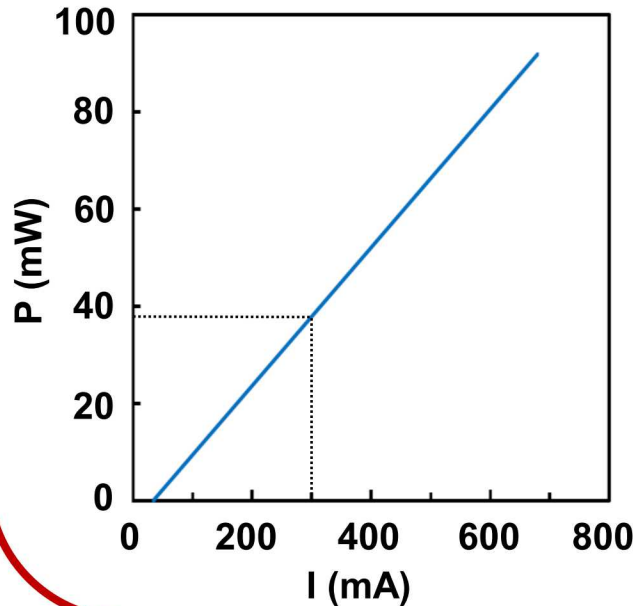
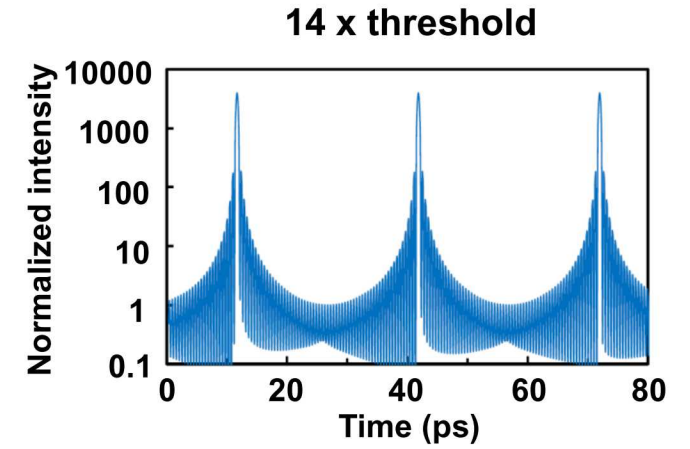
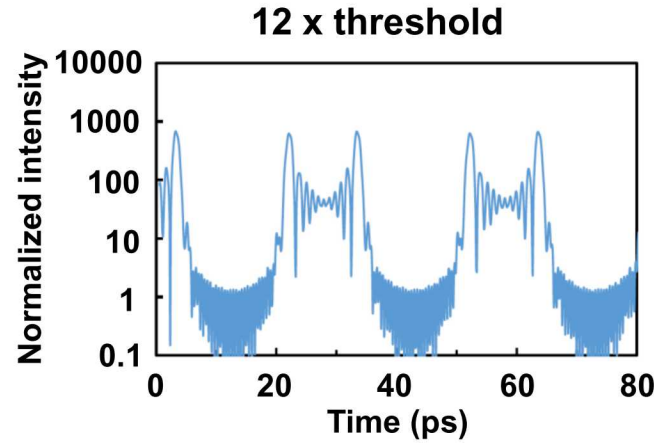
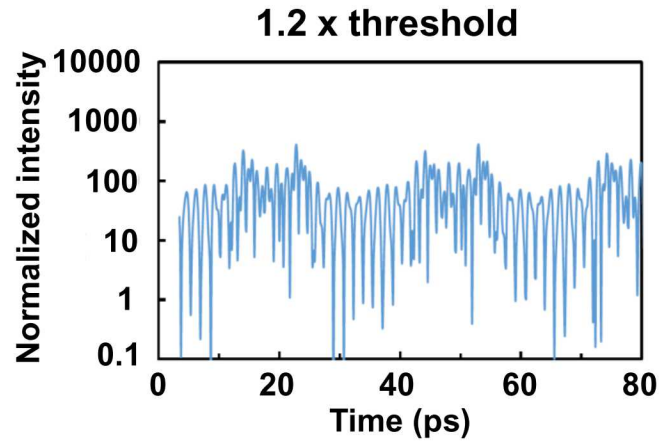
$$\psi_{nn'mm'} = (v_n - v_{n'} + v_m - v_{m'})t + \phi_n(t) - \phi_{n'}(t) + \phi_m(t) - \phi_{m'}(t)$$



Skauli, et al JAP 94, 6447 (2003)

Solve simultaneously

Progression to mode locking with increasing excitation

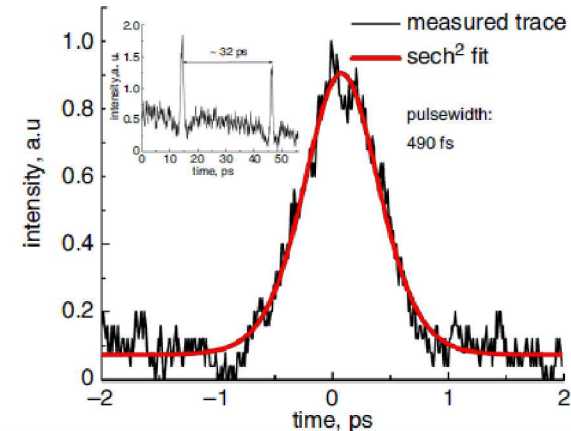
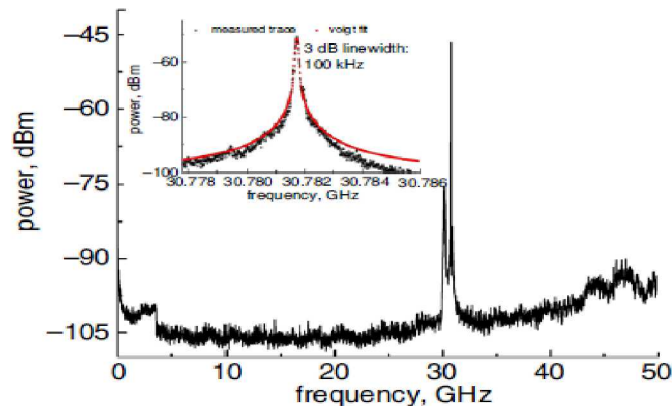
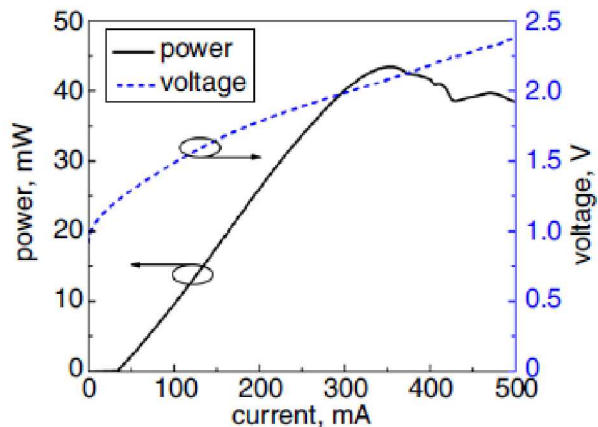


Device parameters used in model

Beam cross section = $6\mu\text{m} \times 1\mu\text{m}$
Active region thickness = 37.5nm
Cavity length = 1.330mm
 $R_1, R_2 = 0.99, 0.60$
Threshold current = 34mA
Injection efficiency = 0.6
Absorption = 10/cm

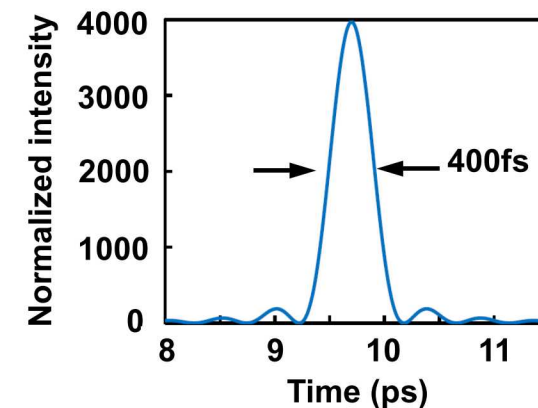
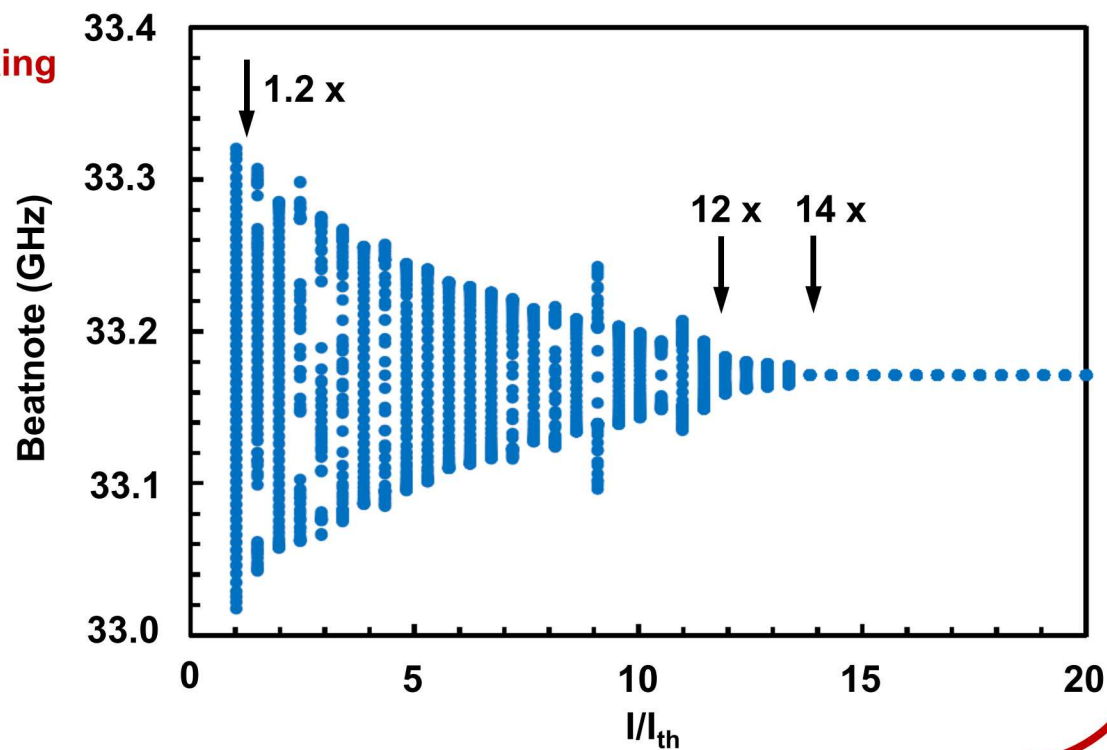
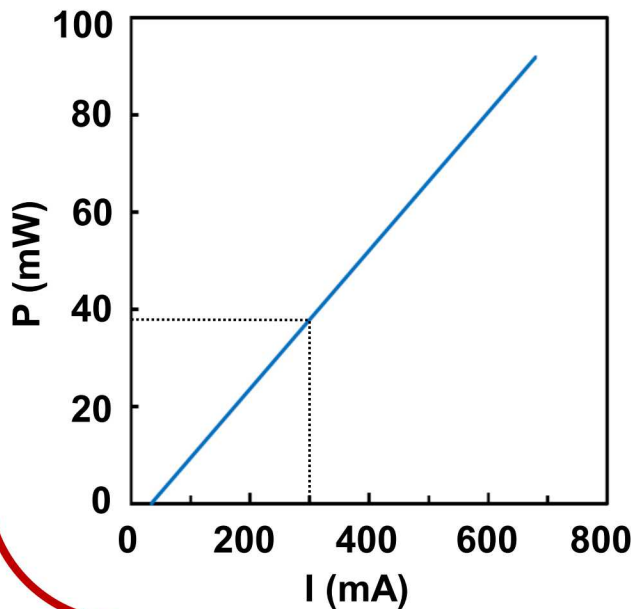
Progression to mode locking with increasing excitation

Expt



Reproduced experiment

- 1) L-I curve
- 2) Excitation at onset of mode locking
- 3) Mode locked pulse width



Device parameters used in model

Beam cross section = $6\mu\text{m} \times 1\mu\text{m}$
 Active region thickness = 37.5nm
 Cavity length = 1.330mm
 $R_1, R_2 = 0.99, 0.60$
 Threshold current = 34mA
 Injection efficiency = 0.6
 Absorption = 10/cm

Loose ends (needing fixing for device model)

Amplitude determining equations

$$\dot{\epsilon}_n(t) = \left[\left(g_n(N) - \frac{\nu_n}{2Q} \right) - \sum_{m=1}^{n_{max}} \theta_{n,m}(N) \epsilon_m^2(t) \right] \epsilon_n(t) - \sum_{n' \neq n} \sum_{m \neq n} \sum_{m' \neq n} \text{Im} [\theta_{nn'mm'}(N) e^{i\psi_{nn'mm'}}] \epsilon_{n'}(t) \epsilon_m(t) \epsilon_{m'}(t)$$

4-wave mixing coefficient from page 158, Sargent, Scully and Lamb

$$\theta_{n\mu\rho\sigma} = \frac{1}{4} i \left(\frac{\wp}{2\hbar} \right)^2 F_1 \gamma \left[1 + \frac{N_{2(\rho-\sigma)} + N_{2(\rho-\mu)}}{\bar{N}} \right] D(\omega - \nu_\mu - \nu_\rho + \nu_\sigma) [D_a(\nu_\rho - \nu_\sigma) + D_b(\nu_\rho - \nu_\sigma)] [D(\omega - \nu_\sigma) + D(\nu_\rho - \omega)]$$

Should really treat like in ZLAG problem

$$\vartheta_{n\mu\rho\sigma} = \sum_i f_i \sum_{a'} \sum_{b''} \sum_{a''} \sum_{b'} \frac{1}{L} \int_0^L ds \int_{-\infty}^{\infty} dv \int_0^{\infty} d\tau' \cdot \int_0^{\infty} d\tau'' \int_0^{\infty} d\tau''' \{ \dots \}.$$

TABLE III
SUMMARY OF THE COEFFICIENTS OF THE TERMS CONTAINING PHASE ANGLES. INTEGRAL TYPES 2 AND 3 VANISH IN THE DOPPLER LIMIT.

nppσ	d _μ	d _ρ	d _σ	Integral Type	d _n ⁺	p _n ⁺	Transitions	Phase Angle
1243	+	-	-	3	+	-	∨∨	+ψ
1342	-	-	+	2	+	-	∧∧	
2134	+	-	-	3	+	+	∨∨	-ψ
2431	-	-	+	2	+	+	∧∧	
3421	-	+	+	3	-	-	∨∨	-ψ
3124	+	+	-	2	-	-	∧∧	
4312	-	+	+	3	-	+	∨∨	+ψ
4213	+	+	-	2	-	+	∧∧	

nppσ	d _μ	d _ρ	d _σ	Integral Type	d _n	p _n	Transitions	θ _{nm}
1111	+	+	+	1	+	-	∨∨	11
2222	+	+	+	1	+	+	∨∨	22
3333	-	-	-	1	-	-	∨∨	33
4444	-	-	-	1	-	+	∨∨	44
1122	+	+	+	1	+	-	∧∧	12
1221	+	+	+	1	+	-	∧∧	12
2211	+	+	+	1	+	+	∧∧	21
2112	+	+	+	1	+	+	∧∧	21
3344	-	-	-	1	-	-	∧∧	34
3443	-	-	-	1	-	-	∧∧	34
4433	-	-	-	1	-	+	∧∧	43
4334	-	-	-	1	-	+	∧∧	43
1133	+	-	-	3	+	-	∧∧	13
1331	-	-	+	2	+	-	∧∧	13
3311	-	+	+	3	-	-	∧∧	31
3113	+	+	-	2	-	-	∧∧	31
2244	+	-	-	3	+	+	∧∧	24
2442	-	-	+	2	+	+	∧∧	24
4422	-	+	+	3	-	+	∧∧	42
4224	+	+	-	2	-	+	∧∧	42
2233	+	-	-	3	+	+	∧∧	23
2332	-	-	+	2	+	+	∧∧	23
3322	-	+	+	3	-	-	∧∧	32
3223	+	+	-	2	-	-	∧∧	32
1144	+	-	-	3	+	-	∧∧	14
1441	-	-	+	2	-	-	∧∧	14
4411	-	+	+	3	+	+	∧∧	41
4114	-	-	+	2	+	+	∧∧	41

Chow, Hambenne, Hutchings, Sanders, Sargent III, Scully, Multioscillator Laser Gyros, IEEE JQE-16 (1980)

Loose ends (needing fixing for device model)

Amplitude determining equations

$$\dot{\mathcal{E}}_n(t) = \left[\left(g_n(N) - \frac{\nu_n}{2Q} \right) - \sum_{m=1}^{n_{max}} \theta_{n,m}(N) \mathcal{E}_m^2(t) \right] \mathcal{E}_n(t) - \sum_{n' \neq n} \sum_{m \neq n} \sum_{m' \neq n} \text{Im} [\theta_{nn'mm'}(N) e^{i\psi_{nn'mm'}}] \mathcal{E}_{n'}(t) \mathcal{E}_m(t) \mathcal{E}_{m'}(t)$$

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Homogeneous broadening

$$\left. \frac{dp_k}{dt} \right|_{scat} = -\gamma_d p_k - \sum_{k'} \gamma_{k,k'}^{nd} p_{k'}$$

