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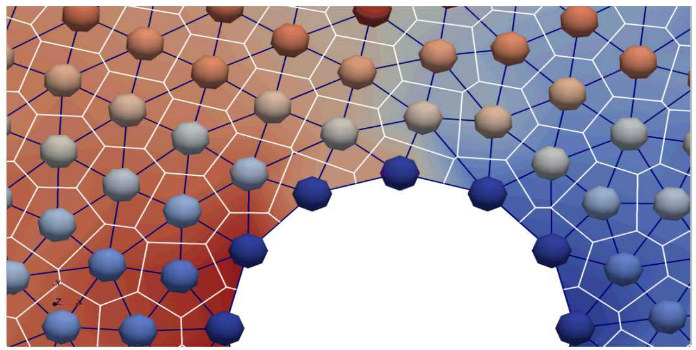

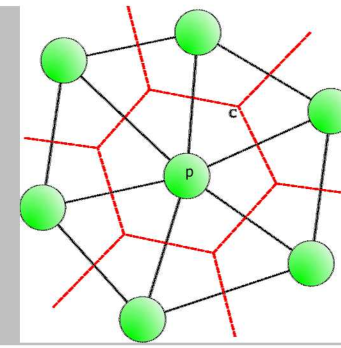


# New Hybrid Particle-Mesh Method for Incompressible Fluid Dynamics

Chris Chartrand
Blair Perot

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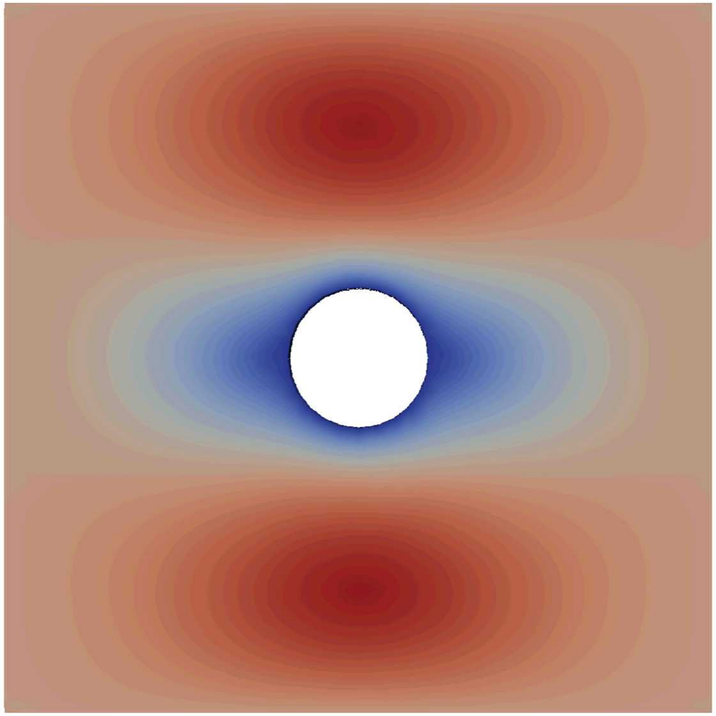
# Overview

- Classical methods vs Particle Methods
- Introduce two hybrid Particle-Mesh methods
  - Voronoi cell method
  - Staggered Method
- Comparison to Classical Results
- Conclusions and Future Work

# Classical Approach

- Velocity and Pressure are the primary unknowns
- Solve momentum equation to update velocity from forces
- Solve the pressure equation to enforce divergence free velocity field
  - Incompressibility enforced at an instance of time

$$\frac{\partial u_i}{\partial x_i} = 0$$



# Our Particle Approach

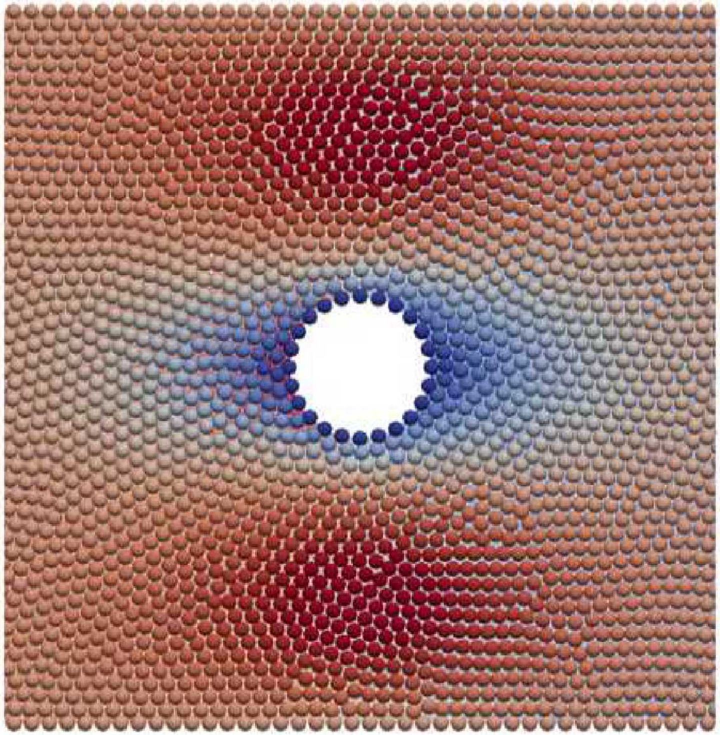
- Position and Pressure are the primary unknowns

- Apply forces to particles to determine acceleration

$$f = ma$$

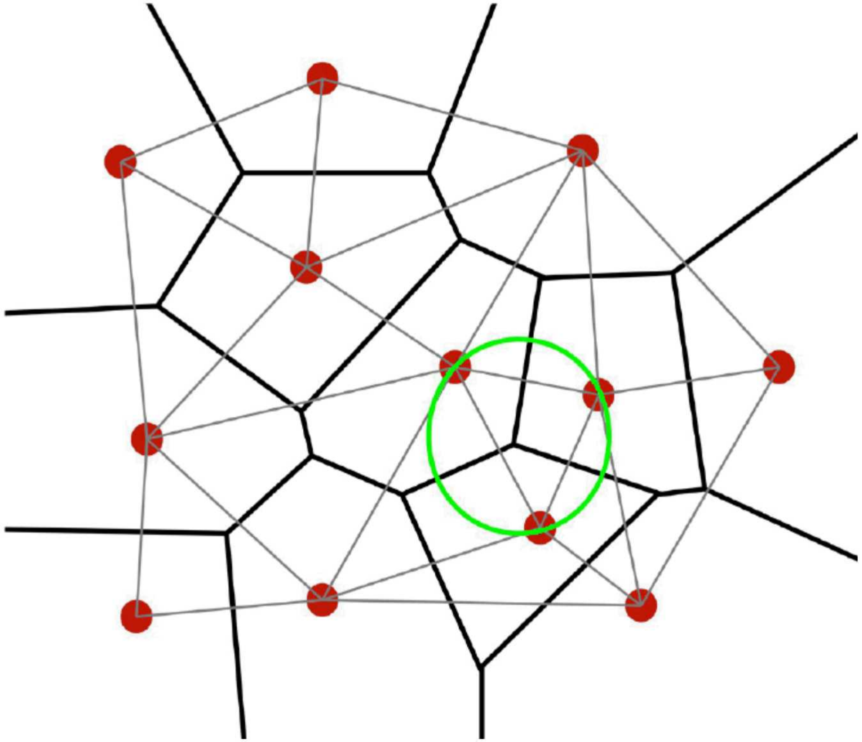
- Solve for the pressure required to preserve the volume of the particle
  - Incompressibility enforced over a space-time integral

$$\int\int_0^{\Delta t} \frac{\partial u_i}{\partial x_i} dt\, dV = 2\left(V_{\Delta t} - V_0\right) = 0$$



# Voronoi Particle Method

- Elegant method of dividing space into particle subsets
  - Convex polygon cells
  - Locally Orthogonal to its Dual
  - Ideal for "Flipping" algorithm

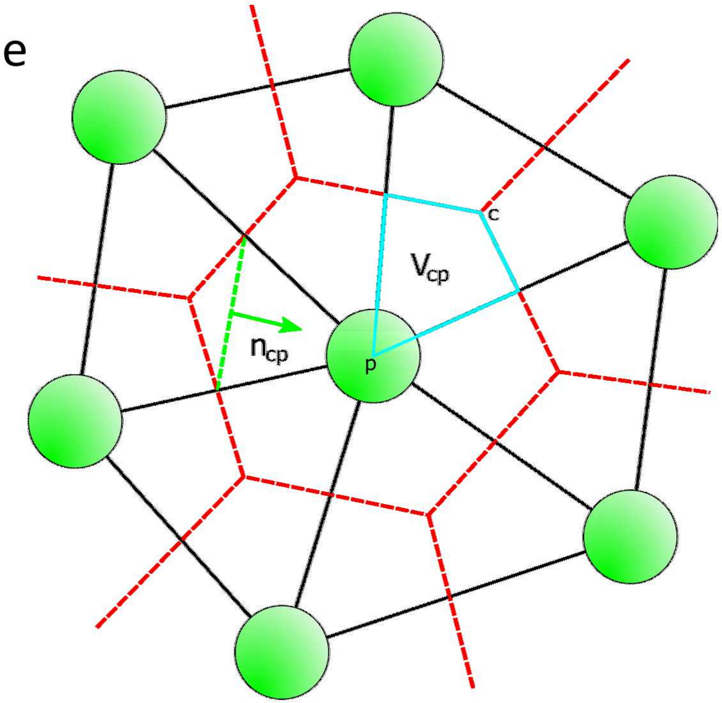


- Does not preserve volume as particle moves

- Particles not generally located at the CG of the cell

# Voronoi Particle Method

- Co-located
  - Pressure and velocity both at the particle location
  - Required interpolation to particle corne




# Vector Operators

- Gauss's Theorem

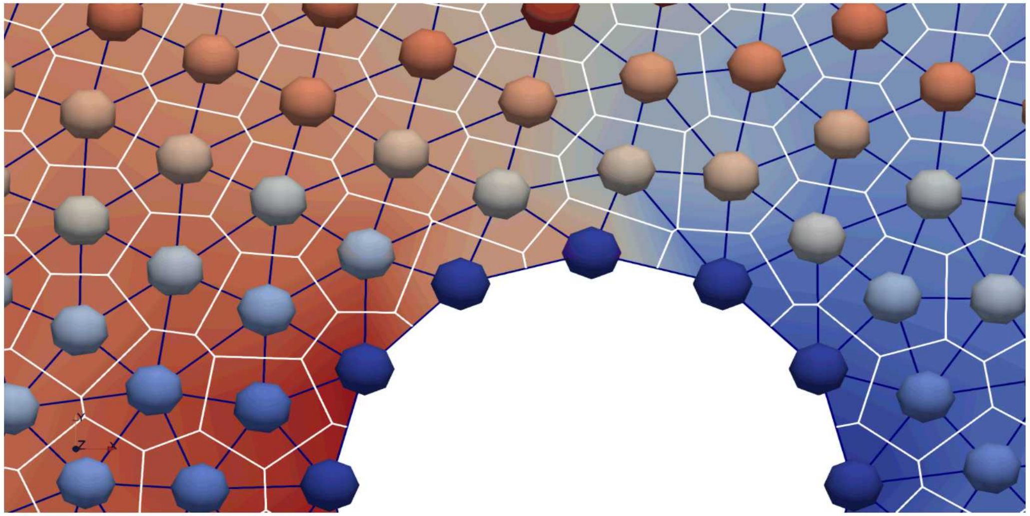
$$\int \nabla p \, dV_p = -\sum_c V_{pc} \sum_p n_{pc} p_p$$

$$\int \nabla \cdot \mathbf{u} \, dV_p = \sum_c n_{pc} \cdot \sum_p V_{pc} \mathbf{u}_p$$

# Voronoi Method:

- Update velocity from forces (viscosity, gravity, but not pressure)

$$\hat{u}_p^n = u_p^n + \Delta t \frac{f_p^v}{\rho V_p}$$



- Update particle position

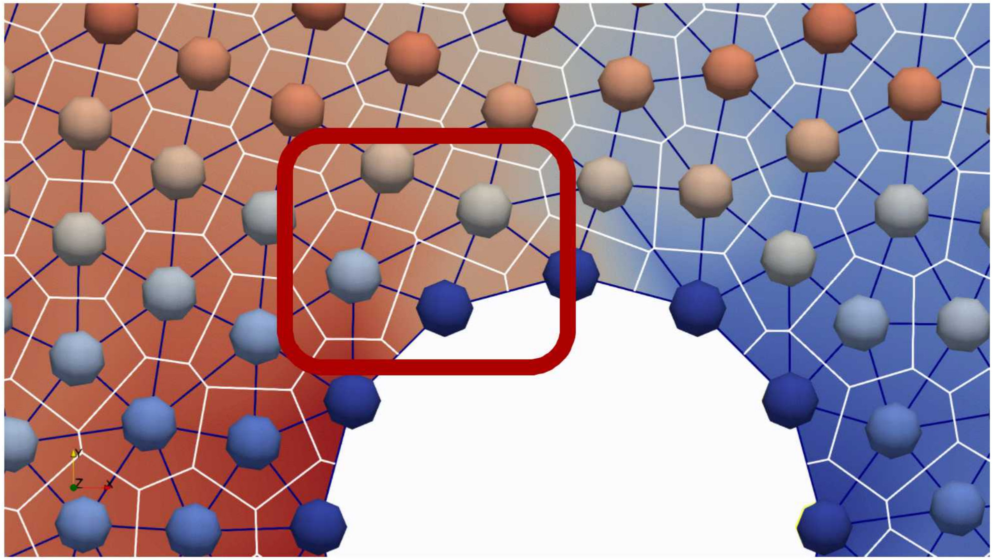
$$\hat{x}_p^n = x_p^n + \hat{u}_p^n \Delta t$$

- Redraw Voronoi partitions
  - Not necessarily volume conserving
  - Particles not necessarily at cell CG

Still need to enforce incompressibility constraint

# Voronoi Method:

- "Flip" if necessary
  - Prevents "twisted" grids
  - Defines new particle neighbors



- Solve for pressure change (Enforce incompressibility)

$$\sum_{p}\mathbf{n}_{cp}^{n+1}\cdot\frac{(\Delta t)^2}{V_c^{n+\frac{1}{2}}}\sum_{c}\mathbf{n}_{cp}^{n+1}\delta p_p=\mathbf{r}_p$$

Laplacian of Pressure Change

$$\mathbf{r}_p=\sum_{c}\mathbf{n}_{cp}^{n+1}\cdot\hat{\mathbf{x}}_c^{n+1}-2V_p^0$$

Volume Residual

$$p_i^{n+1}=p_i^n+\alpha\,\delta p_i^n$$

Pressure update

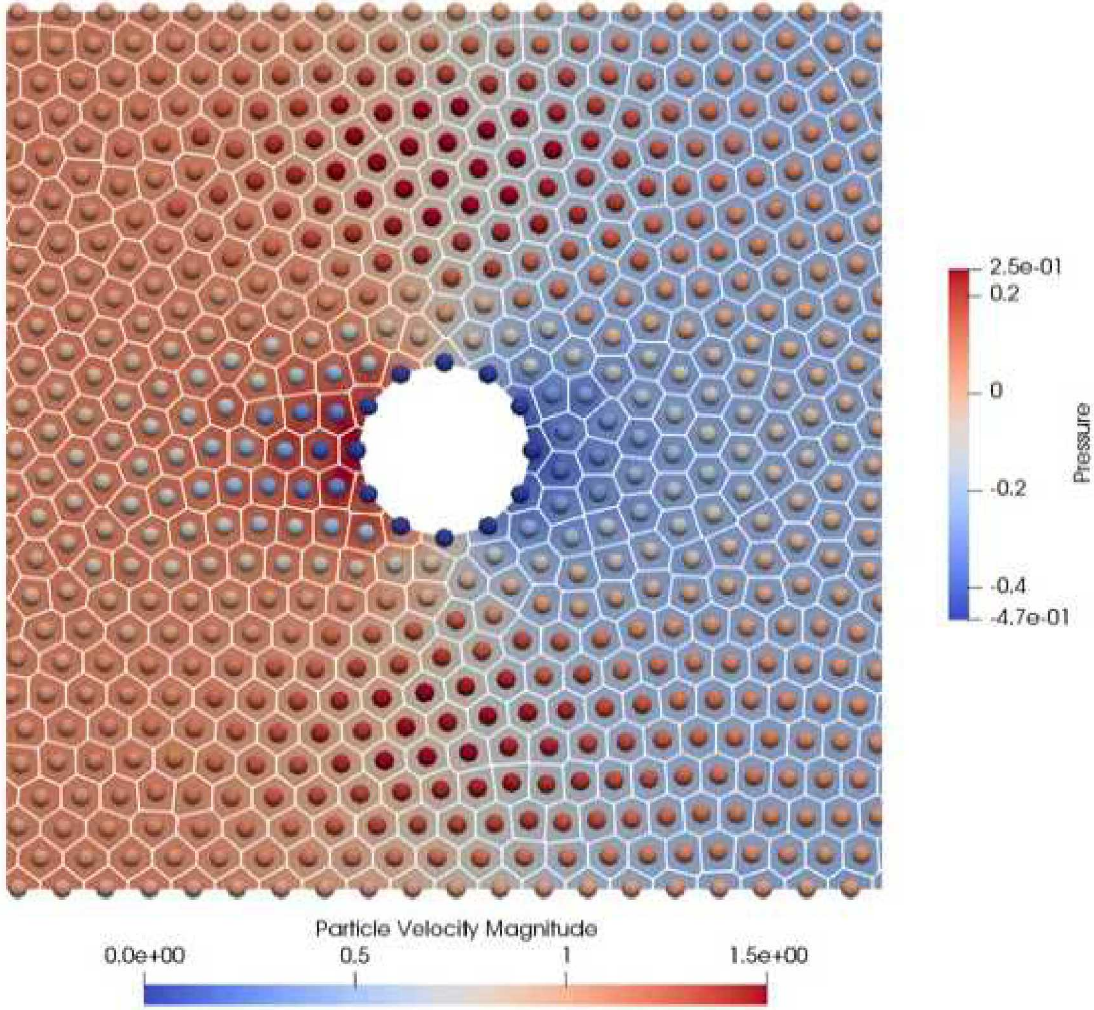
- Move particle corners (correct volume) due to pressure

$$\mathbf{x}_c^{n+1}=\hat{\mathbf{x}}_c^n+\alpha\frac{(\Delta t)^2}{V_c^{n+\frac{1}{2}}}\sum_{c}\mathbf{n}_{cp}^{n+1}\delta p_p$$

- Particles to partition Centroid

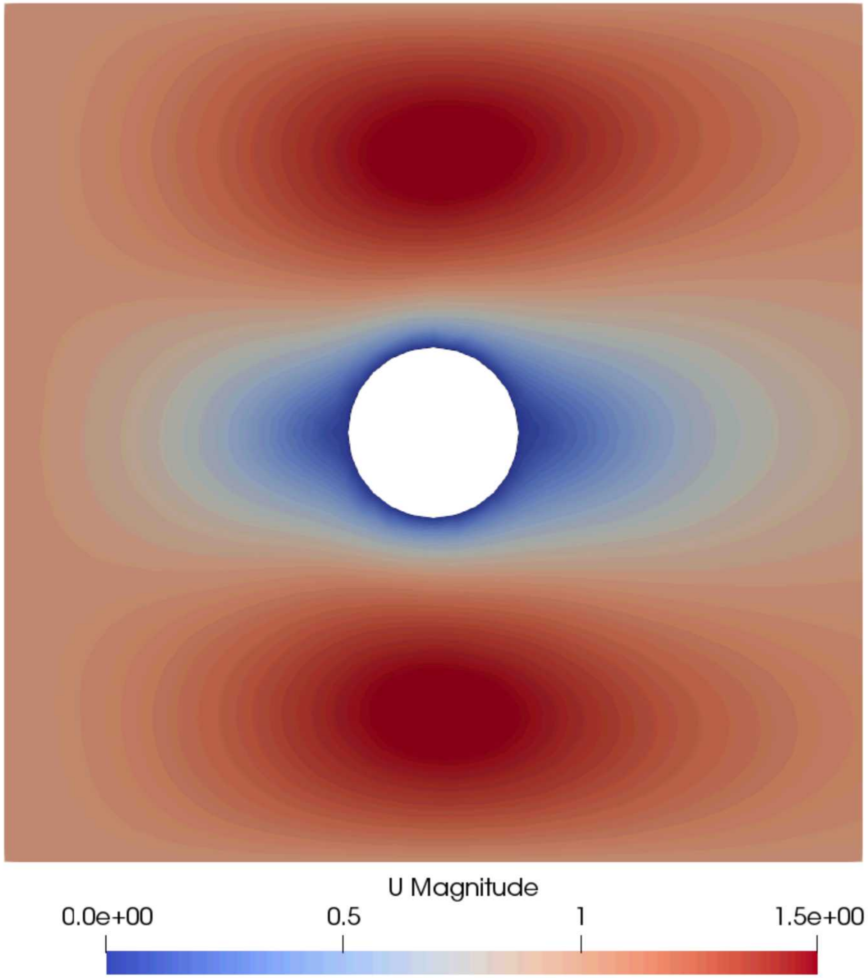
# Results

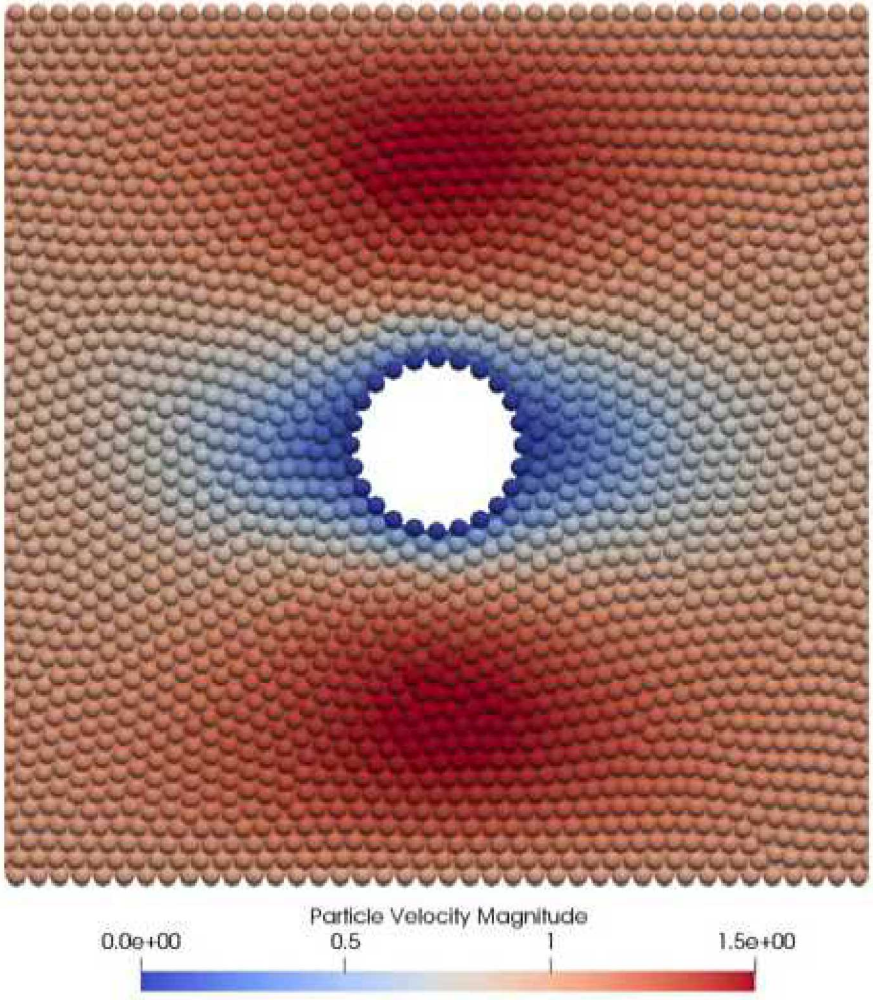
Flipping Algorithm



# Preliminary Results

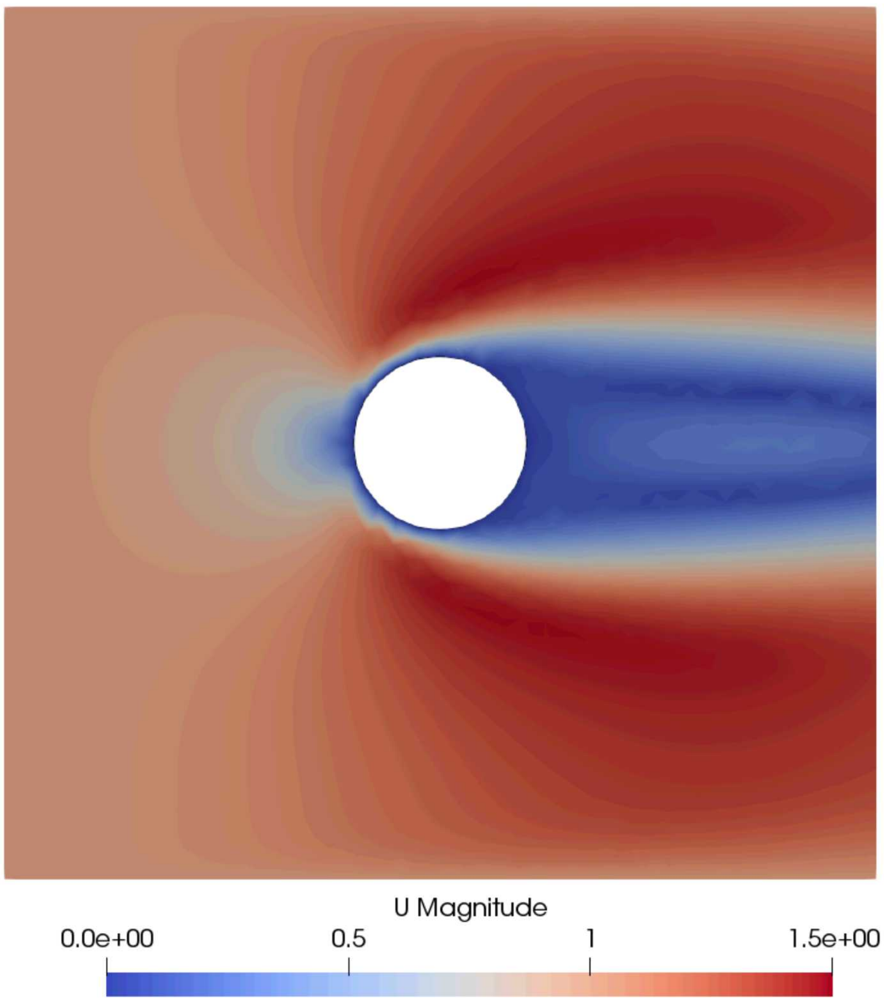
Velocity comparison to OpenFOAM, Re = 0.32

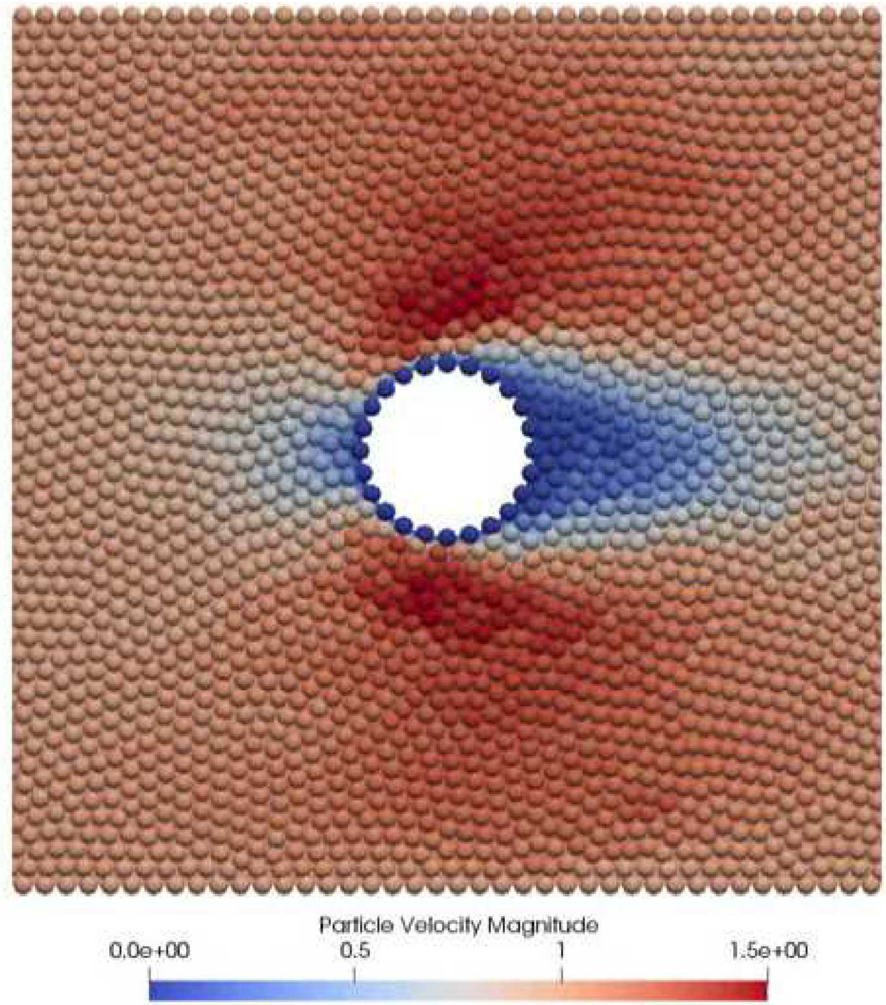


# Preliminary Results

Velocity comparison to OpenFOAM, Re = 32



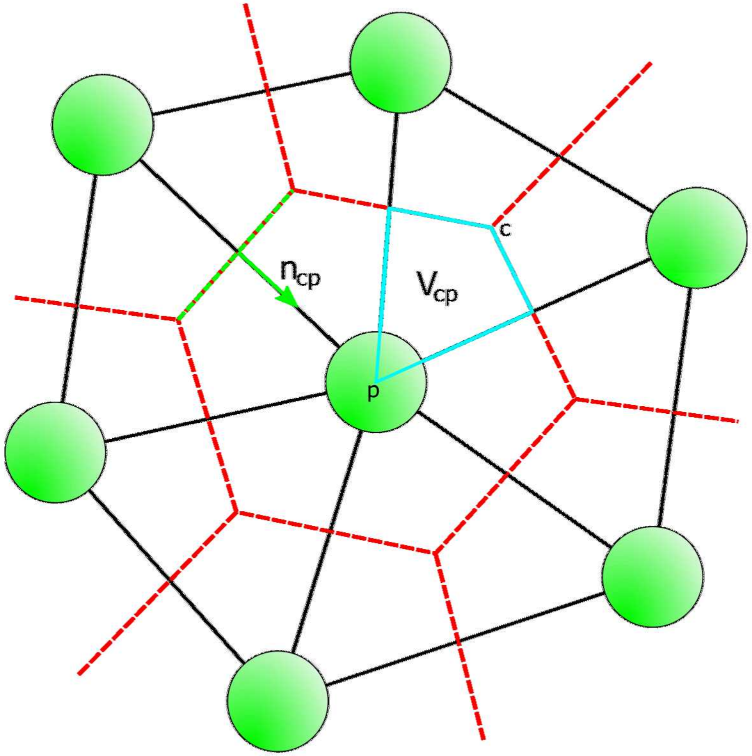



# Conclusions

- Voronoi Method
  - Volume conserving flips
  - Requires interpolation to corners for vector operations
  - Particle must constantly be shifted to Voronoi centroid ($1^{st}$ order)

# Staggered Particle Method

- Pressure and displacement on duals
  - Pressure at the particle location
  - Displacement on particle corners




# Vector Operators

- Gauss's Theorem

$$f_c = \left( \int \nabla p \, dV_p \right)_c = -\sum_p n_{pc} p_p$$

$$V_p = \left( \int \nabla \cdot \mathbf{x} \, dV_c \right)_p = \sum_c n_{pc} \cdot \mathbf{x}_c$$

# Method:

- Update Corner Momentum with forces

$$\hat{\mathbf{m}}_c^n = \sum_p V_{cp}^n \rho_p \mathbf{u}_p^n + \Delta t\, f_c$$

- Corner position prediction

$$\hat{\mathbf{x}}_c^n = \mathbf{x}_c^n + \frac{\Delta t}{\rho V_c^{n+1}} (\mathbf{m}_c^n)$$

- Update pressure

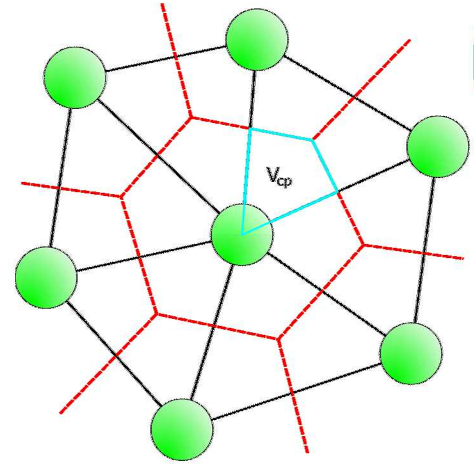
$$\sum_p \mathbf{n}_{cp}^{n+1} \cdot \frac{(\Delta t)^2}{\rho V_c^{n+\frac{1}{2}}} \sum_c \mathbf{n}_{cp}^{n+1} \delta p_p = \mathbf{r}_p \qquad \mathbf{r}_p = \sum_c \mathbf{n}_{cp}^{n+1} \cdot \hat{\mathbf{x}}_c^{n+1} - 2V_p^0$$

- Corner Location Pressure Correction

$$\mathbf{x}_c^{n+1} = \hat{\mathbf{x}}_c^n + \alpha \frac{(\Delta t)^2}{V_c^{n+\frac{1}{2}}} \sum_c \mathbf{n}_{cp}^{n+1} \delta p_p$$

- Move Particle locations to Centroid
- Final momentum (conservative weighting)

$$\mathbf{m}_p^{n+1} = \rho_p \sum_c V_{cp}^{n+1} \frac{\mathbf{x}_c^{n+1} - \mathbf{x}_c^n}{\Delta t}$$




$V_{cp}$ Conservative Volume Weighting

# Preliminary Results

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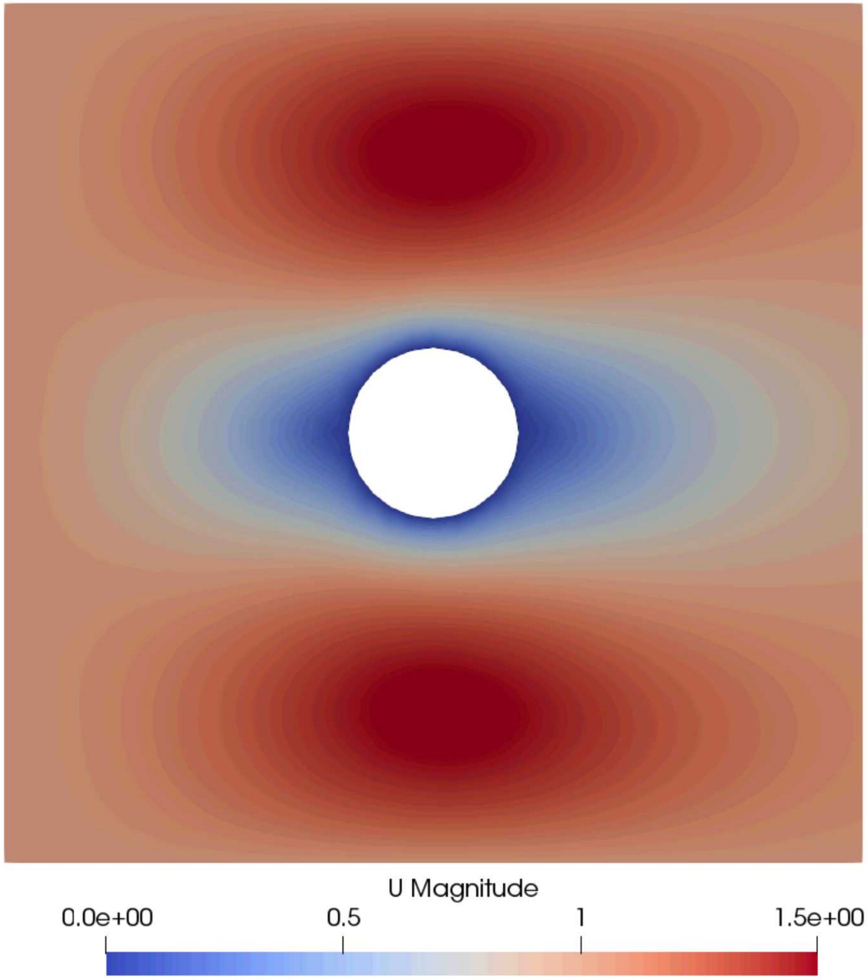


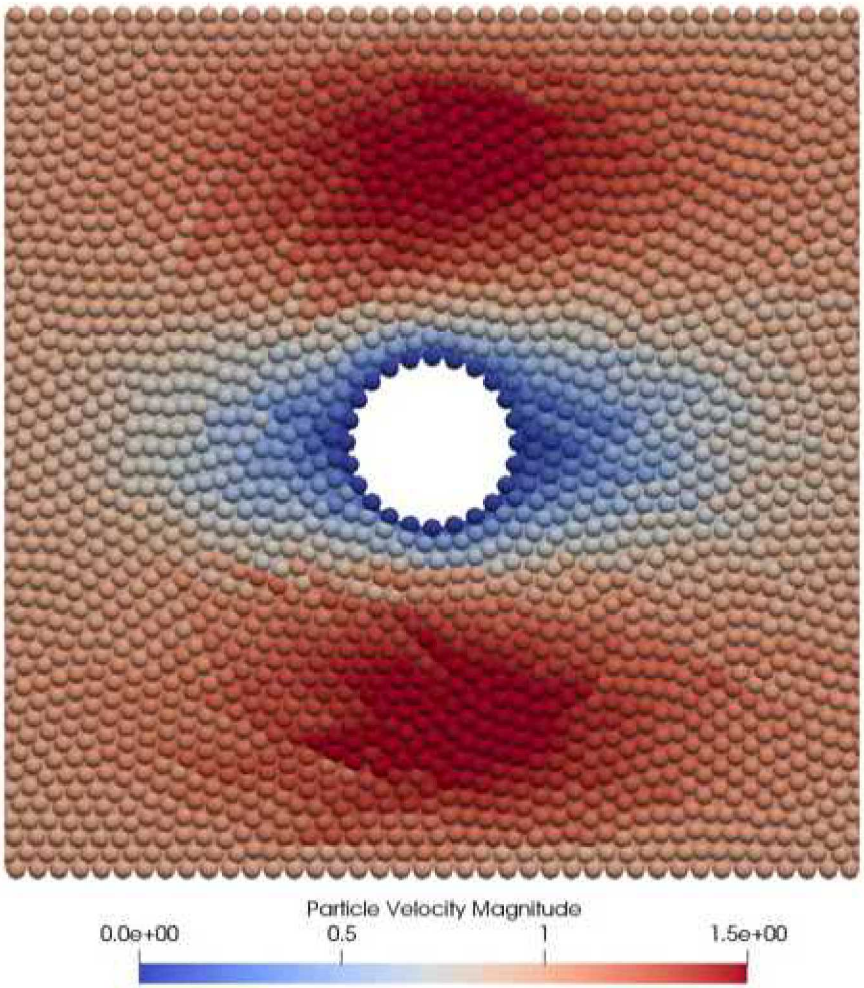

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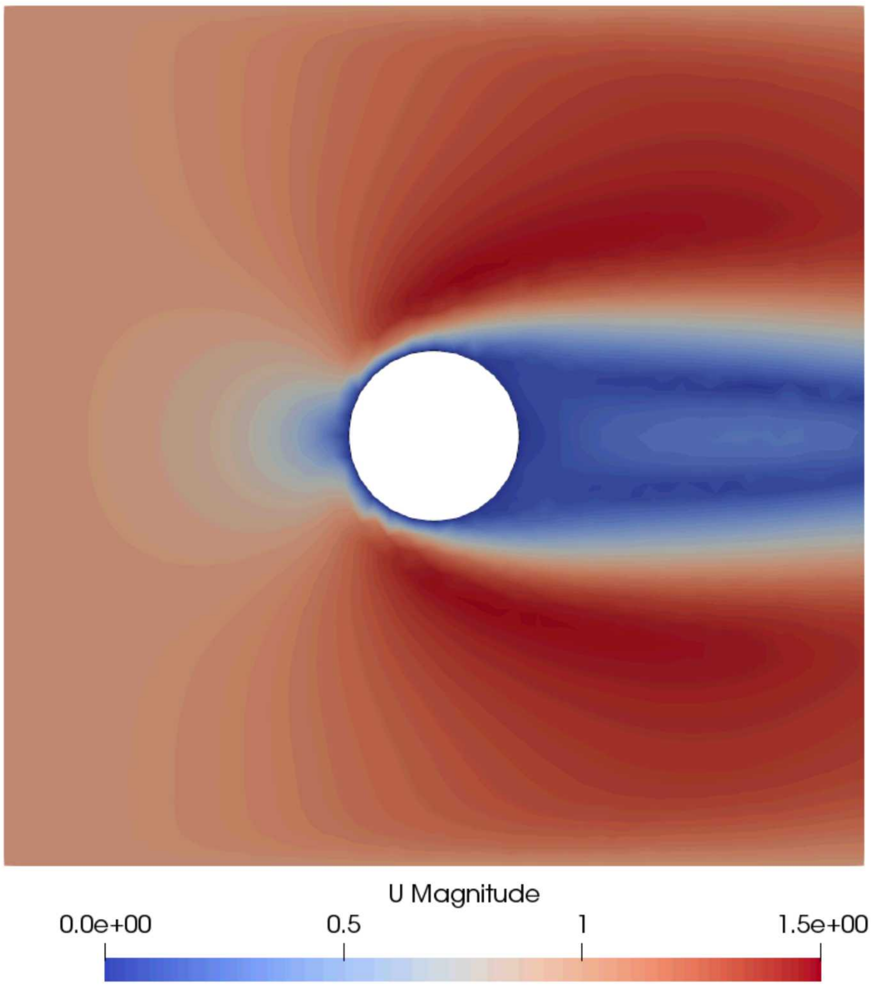


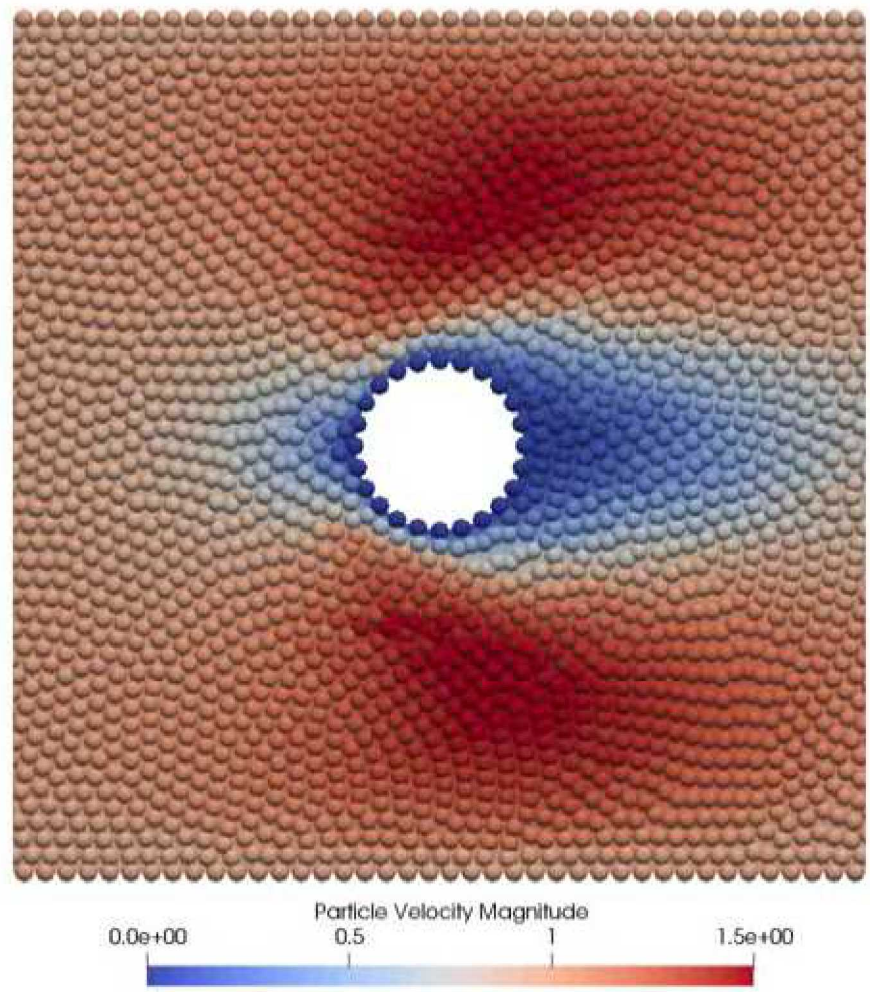

# Conclusions

- Staggered Method
  - Unknowns sit in appropriate locations for vector operations
  - Velocity at corners is consistent with incompressibility constraint
  - Particle centroid is a dependent variable of corner displacement

  - Flipping is less elegant and can cause fluctuations upon remapping the connectivity

Thank you.

Questions?