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Verification of Experimental Component Mode Synthesis in the Sierra Analysis Framework

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Nomenclature

CMS	component mode synthesis
M	mass matrix
FE	finite element(s)
TS	transmission simulator

1) Abstract

Experimental component mode synthesis (CMS) seeks to measure the fundamental modes of vibration of a substructure and develop a structural dynamics model of an as-built structural component through modal testing. Experimental CMS has the potential to circumvent laborious and costly substructure model development and calibration in lieu of a structural dynamics model obtained directly from experimental measurements. Previous efforts of interfacing an experimental CMS model with a production finite element code proved cumbersome. Recently an improved "Craig-Mayes" approach casts an experimental CMS model in the familiar Craig-Bampton form. This form is easily understood by analysts and more readily interfaced with non-trivial, discrete finite element models. The approach/work-flow for interfacing an experimental Craig-Mayes CMS model with the Sierra analysis framework is discussed and the procedure is demonstrated on a verification problem.

Keywords – component mode synthesis, Craig-Bampton, substructure, Sierra, finite elements

2) Introduction

The concept of experimental component mode synthesis (CMS) seeks to measure the fundamental modes of vibration of a substructure and develop a structural dynamics model of a component or subsystem which may be inserted into an analytical model of a higher-level system. Strengths of experimental CMS allow for one to circumvent laborious and costly substructure

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model development and calibration in lieu of a structural dynamics model obtained directly from experimental measurements. Furthermore, experimental CMS allows for a better modelling capability of as-built structural components.

This work will interface an experimentally derived “Craig-Bampton” like substructure with a discrete finite element model within Sandia National Laboratories Sierra analysis framework [1,2]. A previously developed transmission simulation approach is employed to match interface locations with a discrete system level finite element model. Previous efforts will be discussed and strengths of the current approach in streamlining the use of experimentally derived substructures will be highlighted. This approach will be discussed and verification exercises will be presented.

3) Craig-Mayes Experimental Sub-structuring Method

The Craig-Mayes experimental dynamic sub-structuring method improves upon previous experimental sub-structuring methods by representing the substructure system matrices (mass, stiffness, and damping) in a Craig-Bampton [3] like form. This form contains structural matrices with generalized/modal and interface degrees of freedom. This approach uses a transmission simulator to model the interface of the sub-structure to the remainder of the system. The transmission simulator approach requires an accurate discrete finite element model of the transmission simulator/fixture to accurately recover the interface degrees of freedom in an experimental substructure. The Craig-Mayes experimental sub-structuring method and transmission simulator approach are discussed in References 4 and 5.

4) Interface of Experimental CMS Model to Sierra

Previous efforts of coupling experimental CMS models within the Sierra framework employed multi-point constraints and a non-Craig-Mayes CMS representation. This approach proved overly cumbersome for all but the simplest model configurations, and was prone to numerical conditioning issues. The Craig-Mayes format provides a readily realizable interface to a high fidelity structural dynamics model with an interface similar to CMS or “super element” model derived purely from analytical methods. The experimental CMS model is typically provided by experimentalists as a collection of Craig-Bampton like mass, stiffness, and damping matrices [4]. Note that the damping matrix is not required to define a baseline experimental CMS model, but is readily available from experimental measurements.

The Craig-Bampton like matrices are $p \times p$ in dimension, such that $p = m + n$. Here, m is the number of modes retained in the CMS reduction, and n is the number of interface degrees of freedom in the CMS model. The required form of these equations is shown in Equation 1. Note that the form of the mass matrix is shown, but identical forms are required for the stiffness and damping matrices. These matrices are symmetrical in nature. $\widehat{M}$ defines couplings between the generalized (modal) degrees of freedom in the CMS model (this matrix should be diagonal in nature), $\bar{M}$ defines couplings between the interface degrees of freedom, and $\widetilde{M}$ defines the couplings between generalized and interface degrees of freedom.

$$M_{CMS} = \begin{bmatrix} \widehat{M}_{m\times m} & \widetilde{M}_{m\times n} \\ \widetilde{M}^T_{m\times n} & \bar{M}_{n\times n} \end{bmatrix} \tag{1}$$

An $r \times 3$ coordinate array is also required that defines the coordinates of the r interface points. In addition to interface point coordinates an $n \times 1$ map array is required that specifies the “local” degrees of freedom of the interface degrees of freedom. The order of this array should be consistent with the ordering of the coordinate array. Details of the map array will be elaborated on in a subsequent section.

In summary, the following data is required to accompany an experimentally derived CMS model:

- Number of modes retained in the CMS reduction (m)
- Number of interface degrees of freedom (n)
- Interface point coordinate array
- CMS mass matrix
- CMS stiffness matrix
- Interface degree of freedom map array
- CMS damping matrix (optional)

This information can be provided to the MATLAB based “CMS Toolkit” to create a Sierra super element of the experimental CMS model. The CMS toolkit creates two files. First, an Exodus finite element mesh is created defining the geometry of the super element. This includes the coordinates of the interface nodes for the n-node super element. Next, the formulation of this super element (mass, stiffness, and damping) are characterized in a NetCDF binary file. The super element Exodus file is inserted into a discrete finite element model using a GJOIN [6] or a similar mesh joining utility. From here, the super element NetCDF file is referenced in a Sierra input deck and subsequent modal, vibration, or transient analysis accounts for the coupling between the discrete finite element model and the experimentally derived Craig-Mayes substructure. This workflow is depicted in Figure 1.

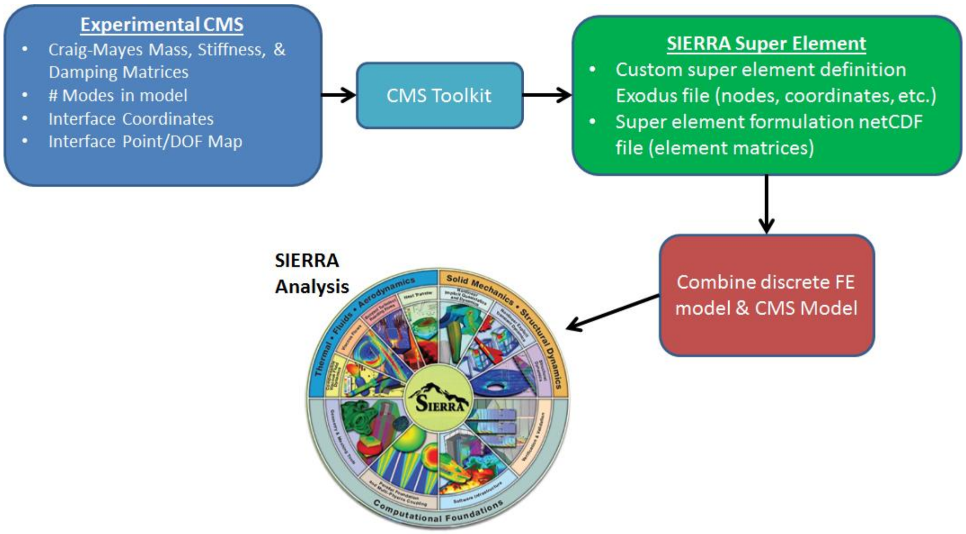

Fig. 1 Workflow for interfacing an experimentally derived CMS model in Sierra analysis

5) Demonstration

This section presents a demonstration of the aforementioned process for interfacing an experimentally derived Craig-Mayes substructure to a discrete finite element modal for Sierra-SD analysis. First the model/test configuration is described followed by results of the exercise.

5.1 Configuration

A 2-D simple beam configuration documented in Reference 4 was considered for a proof of concept analysis for interfacing an experimentally derived CMS substructure model with Sierra-SD analysis. The configuration is shown in Fig. 2. Two beams are connected together over a specified region of overlap. The left beam is to be modeled by finite elements, whereas the dynamics of the right beam are measured experimentally and an experimental CMS model is derived. This is done using a “transmission simulator” shown as “TS Beam” in Figure 2. Details of the transmission simulator are elaborated on in References 4 and 5. The transmission simulator essentially allows one to generate interface degree of freedom responses at discrete locations from those measured from a modal test. This is a very convenient means for interfacing an experimentally derived CMS model to discrete points of a finite element model.

Fig. 2 2D beam configuration [4]

Note that there are 5 nodes in the overlap between the left finite element beam and the right experimental CMS beam. Thus, the transmission simulator approach was used to derive an experimental CMS model with 5 interface points (coincident with the finite element nodes). Each interface point had 3 degrees of freedom (axial translation, bending translation, and rotation). Therefore, a total of 15 interface degrees of freedom exist in the model. Three modes were retained in the CMS reduction. This resulted in CMS mass, stiffness, and damping matrices that had dimension of 18 x 18.

5.2 Results

The Craig-Mayes substructure model of the beam was interfaced to the discrete “FE Beam” model in Sierra-SD described in Section 3. Results show good agreement between the “truth model” described in Reference 4 and the Sierra-SD implementation. Table 1 presents a comparison of modal frequencies. The first 5 modes have 1% error or less and the 6th to 9th modes have at most 1.4% error. Bending mode shape comparisons of a discrete finite element model of the entire system and those of the discrete “FE beam” coupled with the Craig-Mayes experimental beam are shown in Figures 3 and 4. Solid lines represent the finite element results of the complete system while markers represent the mode shape of the discrete left beam coupled with the Craig-Mayes right beam. Overall, good agreement is seen between the “truth” FEM mode shapes and those of the discrete finite element model of the left beam coupled to the Craig-Mayes substructure of the right beam. Some differences are apparent for the 4th to 7th bending mode shapes in the vicinity of the interface to the Craig-Mayes substructure. This may be due to some artifacts of the transmission simulator approach providing an increased stiffening effect at this location.

6) Conclusions

This paper has presented the motivation for using experimentally derived substructures within the Sierra analysis framework. The Craig-Mayes sub-structuring approach allows for a straightforward interface of an experimental CMS model with a discrete finite element model by using a representation similar to the Craig-Bampton CMS approach. This allows for the experimental CMS model to be treated virtually the same way as a numerically derived Craig-Bampton “super element”, although the experimental model may be prone to some numerical conditioning issues as a result of flaws in measurement data and mathematical operations being performed on that data. The work-flow of interfacing a Craig-Mayes model with the Sierra analysis framework was discussed and the process was demonstrated successfully on a proof-of-concept application.

Future work will consider more complicated substructures. This may include sub-structures generated from actual experimental data or substructures derived from “virtual” modal testing with the Sierra-SD analysis software. The concept of virtual modal testing allows for more idealized accelerometer data to be considered within the general process of an experimental sub-structuring method while allowing control of the imperfections in the test data through the introduction of measurement noise or other flaws.

Table 1 Comparison of Sierra-SD sub-structured modal frequency vs. truth frequency

Truth Frequency (Hz) [4]	Sierra-SD Sub-structured Frequency (Hz)	Error (%)
212.0	210.4	-0.7
574.6	568.6	-1.0
1121.0	1132.0	1.0
1867.3	1863.9	-0.2
2750.2	2767.6	0.6
3341.7	3383.9	1.3
3949.6	4003.2	1.4
5115.9	5105.0	-0.2
5965.5	5945.8	-0.3

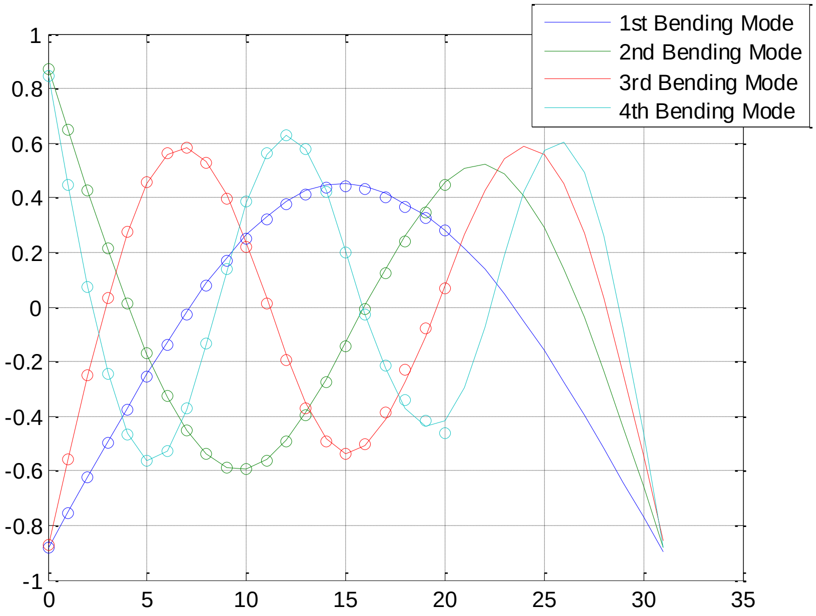

Fig. 3 Comparison of "truth" and Sierra-SD sub-structured lower bending mode shapes (solid line=truth model, marker = sub-structured model)

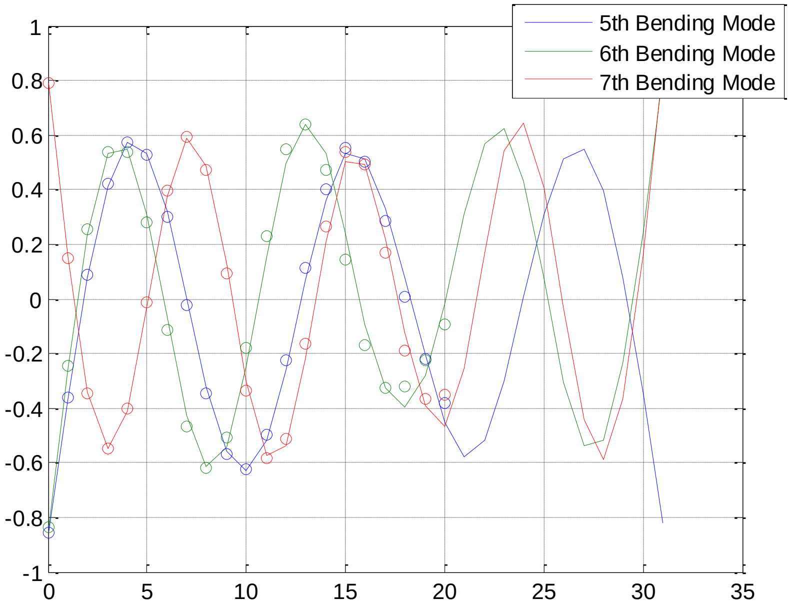

Fig. 4 Comparison of "truth" and Sierra-SD sub-structured higher bending mode shapes (solid line=truth model, marker = sub-structured model)

7) References

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