SAND2015-5012PE

Orthotropic Plasticity in LAMÉ

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Exceptional service in the national interest

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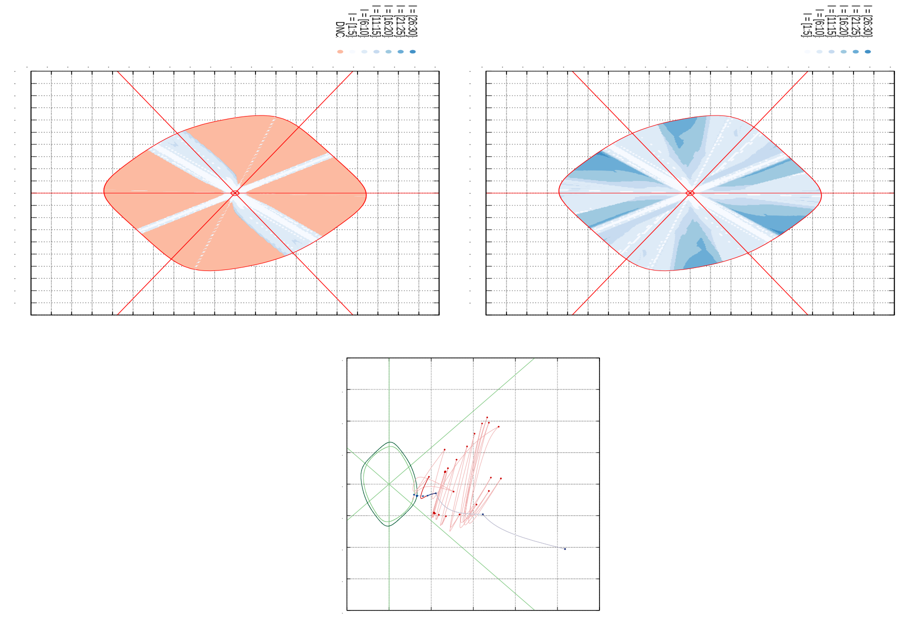

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Motivation

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- Traditional isotropic yield surfaces are not adequate for modeling some problems
- Anisotropy can be important factor
- Develop a plasticity model that is capable of modeling anisotropy
  - Yield surface
  - Hardening rule
  - Failure criterion

Yield Function

- The general form we will use for the yield function is as follows

$$f = \phi(\sigma_{ij}) - \bar{\sigma}(\kappa) = 0$$

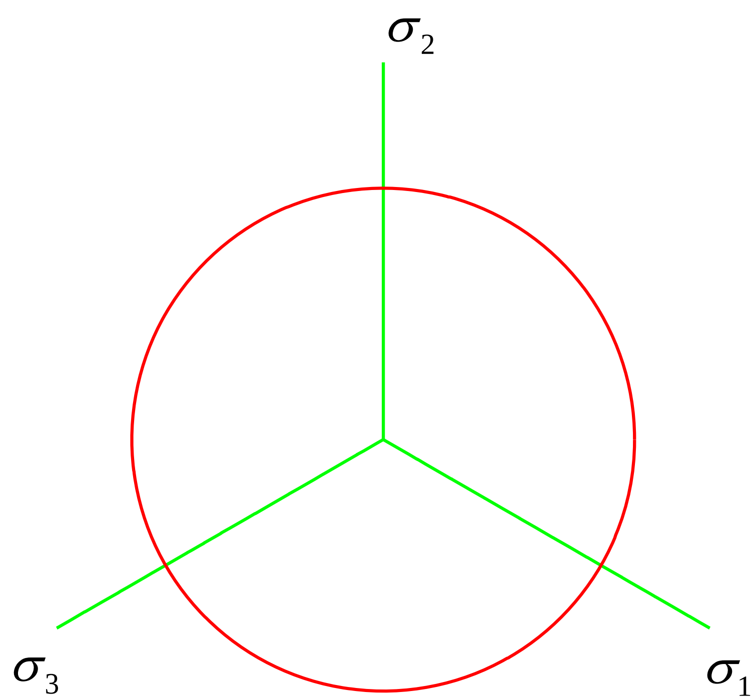
  - This defines a surface in stress space – the yield surface
  - This allows us to **model** the three-dimensional behavior of a material

- For a von Mises yield surface

$$\phi(\sigma_{ij}) = \sqrt{\frac{3}{2} s_{ij} s_{ij}} \quad ; \quad s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$$

Yield Function

- von Mises yield surface
  - Cylinder in stress space
  - "Look" down the hydrostat
  - Each point on the yield surface represents an **infinite** number of stress states



Isotropic Plasticity Models

von Mises – 1 parameter

$$\phi = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}}$$

Hosford – 2 parameters

$$\phi = \left\{ \frac{1}{2} \left[|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a \right] \right\}^{1/a}$$

Orthotropic Plasticity Models

Hill – 7 parameters

$$\phi^2(\boldsymbol{\sigma}) = F\left(\hat{\sigma}_{22} - \hat{\sigma}_{33}\right)^2 + G\left(\hat{\sigma}_{33} - \hat{\sigma}_{11}\right)^2 + H\left(\hat{\sigma}_{11} - \hat{\sigma}_{22}\right)^2$$

$$+ 2L\hat{\sigma}_{23}^2 + 2M\hat{\sigma}_{31}^2 + 2N\hat{\sigma}_{12}^2$$

$$\phi = \sqrt{\frac{3}{2}\boldsymbol{\sigma} : \mathbf{P} : \boldsymbol{\sigma}}$$

depends on material orientation

Orthotropic Plasticity Models

Barlat – 20 parameters *

$$\mathbf{s}' = \mathbf{L}' : \boldsymbol{\sigma} \quad ; \quad \mathbf{s}'' = \mathbf{L}'' : \boldsymbol{\sigma}$$

$$\phi(\boldsymbol{\sigma}) = \left\{ \frac{1}{4} \left[|s_1' - s_1''|^a + |s_1' - s_2''|^a + |s_1' - s_3''|^a + |s_2' - s_1''|^a + |s_2' - s_2''|^a \right. \right.$$

$$\left. \left. + |s_2' - s_3''|^a + |s_3' - s_1''|^a + |s_3' - s_2''|^a + |s_3' - s_3''|^a \right] \right\}^{1/a}$$

* Barlat et. al., "Linear transformation based anisotropic yield functions", IJP, v. 21, 2005.

Orthotropic Plasticity Models

Barlat – 20 parameters

$$\mathbf{s}' = \mathbf{L}' : \boldsymbol{\sigma} \quad ; \quad \mathbf{s}'' = \mathbf{L}'' : \boldsymbol{\sigma}$$

$$\mathbf{L}' = \mathbf{C}' : \mathbb{I}'$$

$$\mathbf{L}'' = \mathbf{C}'' : \mathbb{I}'$$

deviatoric projection

$$\mathbf{C}' = \begin{bmatrix} 0 & -c'_{12} & -c'_{13} & 0 & 0 & 0 \\ -c'_{21} & 0 & -c'_{23} & 0 & 0 & 0 \\ -c'_{31} & -c'_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_{66} \end{bmatrix}$$

$$\mathbf{C}'' = \begin{bmatrix} 0 & -c''_{12} & -c''_{13} & 0 & 0 & 0 \\ -c''_{21} & 0 & -c''_{23} & 0 & 0 & 0 \\ -c''_{31} & -c''_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c''_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c''_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c''_{66} \end{bmatrix}$$

Integration Algorithms

rate form of the model

$$\dot{\boldsymbol{\sigma}} = \mathbb{C} : \dot{\boldsymbol{\varepsilon}}^e$$

additive decomposition of strain rate

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$$

associated flow

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\gamma} \frac{\partial \phi}{\partial \boldsymbol{\sigma}}$$

$$\dot{\boldsymbol{\sigma}} = \mathbb{C} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\gamma} \frac{\partial \phi}{\partial \boldsymbol{\sigma}} \right)$$

Return Mapping Algorithms

yield function

$$f = \phi\left(\boldsymbol{\sigma}\right) - H\left(\Delta\gamma\right)$$

plastic strain residual

$$\mathbf{R} = -\Delta\boldsymbol{\varepsilon}^p + \Delta\gamma\frac{\partial\phi}{\partial\boldsymbol{\sigma}}$$

plastic strain increment

$$\Delta\boldsymbol{\varepsilon}^p = \mathbb{C}^{-1} : \left(\boldsymbol{\sigma} - \boldsymbol{\sigma}^{tr}\right)$$

equations we want to solve

$$f = 0$$

$$\mathbf{R} = \mathbf{0}$$

unknowns

$$\Delta\gamma \quad , \quad \boldsymbol{\sigma}$$

Iterative Algorithm

create iterative solution for unknowns

$$\Delta\gamma^{(k+1)} = \Delta\gamma^{(k)} + \Delta\left(\Delta\gamma\right)$$

$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} + \Delta\boldsymbol{\sigma}$$

two algorithm to solve for increment in unknowns

- Newton
- line search based on Newton

Newton Algorithm

$$\Delta\left(\Delta\gamma\right) = \frac{f^{(k)} - \mathbf{R}^{(k)} : \mathscr{L}^{(k)} : \dfrac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}}}{\dfrac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}} : \mathscr{L}^{(k)} : \dfrac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}} + H'_{(k)}}$$

$$\Delta\boldsymbol{\sigma} = -\mathscr{L}^{(k)} : \left(\mathbf{R}^{(k)} + \Delta\left(\Delta\gamma\right)\frac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}}\right)$$

slope of hardening curve

$$H' = \frac{dH}{d\Delta\gamma}$$

elastoplastic tangent

$$\mathscr{L}^{-1} = \mathbb{C}^{-1} + \Delta\gamma\,\frac{\partial^2\phi}{\partial\boldsymbol{\sigma}\partial\boldsymbol{\sigma}}$$

von Mises

$$\Delta\left(\Delta\gamma\right) = \frac{f^{(k)}}{3\mu + H'_{(k)}} \qquad \Delta\boldsymbol{\sigma} = -\Delta\left(\Delta\gamma\right)\frac{3}{2\phi}\mathscr{L}^{(k)} : \mathbf{s}$$

Line Search Algorithm

$$\Delta\gamma^{(k+1)} = \Delta\gamma^{(k)} + \alpha\, \Delta\left(\Delta\gamma\right)$$

search direction

$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} + \alpha\, \Delta\boldsymbol{\sigma}$$

search parameter

$$\alpha \in [0, 1]$$

search direction comes from Newton algorithm

need to determine “best” value for α

Line Search Algorithm *

residual

$$\mathbf{r} = \left(\frac{f}{2\mu} \, , \; \mathbf{R} \right)$$

merit function based on residual

$$\psi = \frac{1}{2} \mathbf{r} \cdot \mathbf{r}$$

…as a function of α

$$\psi(\alpha) = \frac{1}{2} \mathbf{r}(\alpha) \cdot \mathbf{r}(\alpha)$$

we want

$$\psi(\alpha) < \psi(0)$$

if we get this, then the solution is improving

* Perez-Foguet and Armero, "On the formulation of closest point projection algorithms in elastoplasticity – part II: globally convergent schemes", IJNME, v. 53, 2002.

Line Search Algorithm

check

$$\psi(1) < \psi(0)$$

if so

$$\alpha = 1$$

we use Newton

if not

$$\hat{\psi} = \psi_0 + \alpha\psi_1 + \alpha^2\psi_2$$

$$\hat{\psi} = \left(1 - 2\alpha + \alpha^2\right)\psi(0) + \alpha^2\psi(1)$$

$$\alpha = \frac{\psi(0)}{\psi(1) + \psi(0)}$$

Line Search Algorithm

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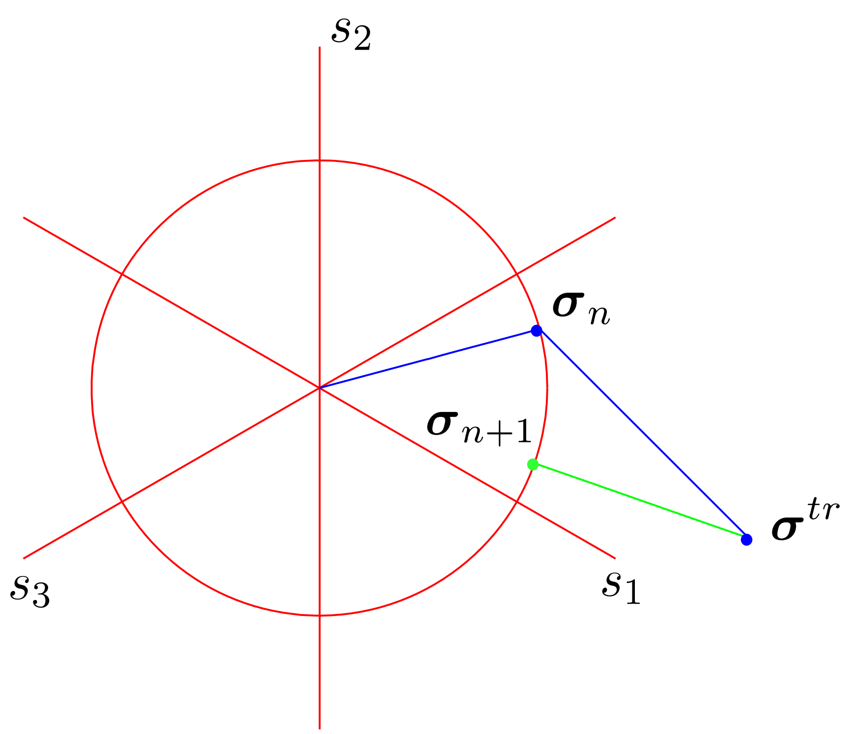
robustness

$$\frac{d\psi}{d\alpha}(0) = -2\psi(0)$$ (from Newton search direction)

$$\psi > 0 \quad \rightarrow \quad \frac{d\psi}{d\alpha}(0) < 0$$

there is always a neighborhood of $\psi(0)$
where the solution should be improving

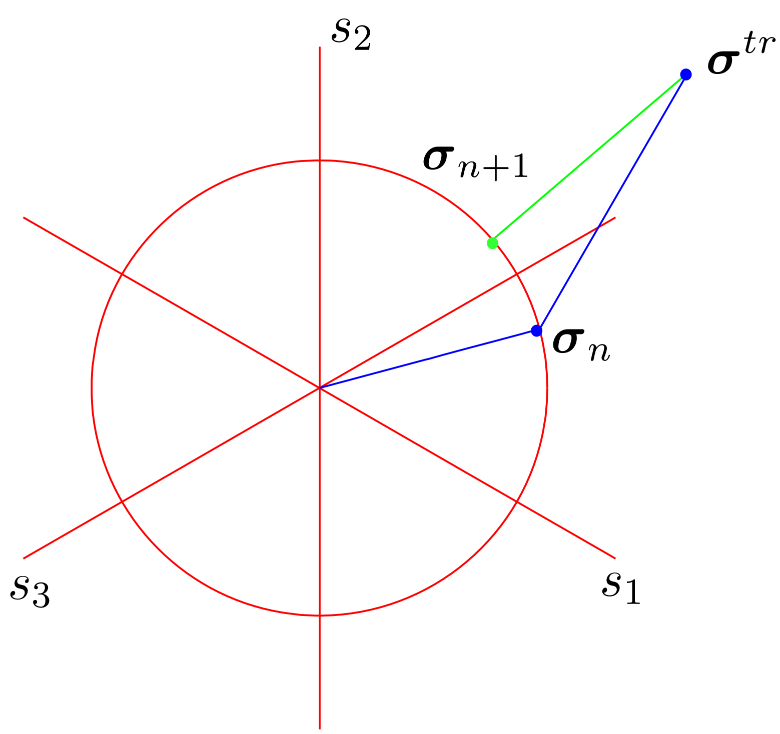
Trial Stress



$$\boldsymbol{\sigma}^{tr} = \boldsymbol{\sigma}_n + \mathbb{C} : \Delta\boldsymbol{\varepsilon}$$

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}^{tr} - \mathbb{C} : \Delta\boldsymbol{\varepsilon}^p$$

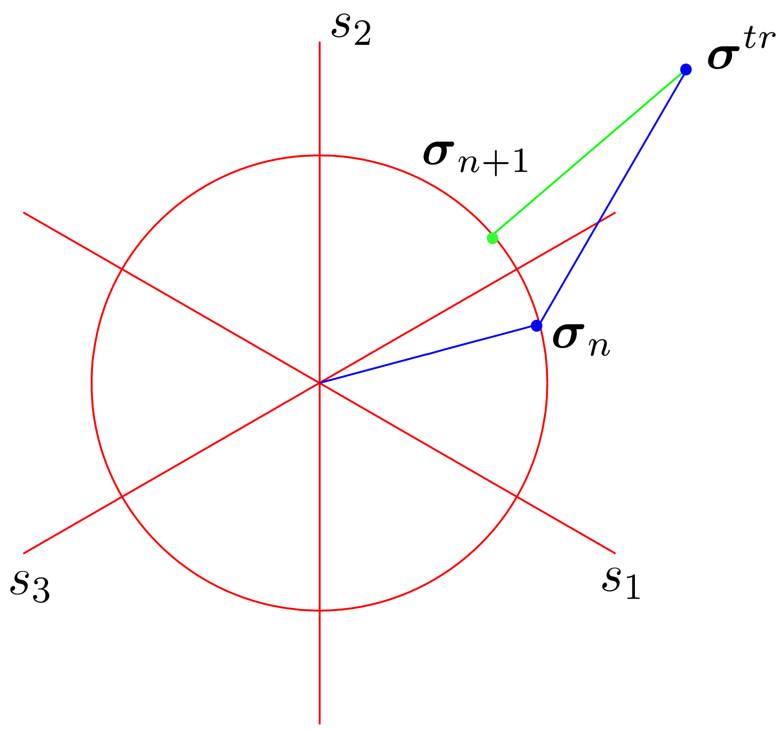
Trial Stress



$$\boldsymbol{\sigma}^{tr} = \boldsymbol{\sigma}_n + \mathbb{C} : \Delta\boldsymbol{\varepsilon}$$

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}^{tr} - \mathbb{C} : \Delta\boldsymbol{\varepsilon}^p$$

Trial Stress



$$\boldsymbol{\sigma}^{tr} = \boldsymbol{\sigma}_n + \mathbb{C} : \Delta\boldsymbol{\varepsilon}$$

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}^{tr} - \mathbb{C} : \Delta\boldsymbol{\varepsilon}^p$$

for the return mapping algorithm all that matters is the trial stress state and the yield surface

Scan of Trial Stress Space

principal stresses

$$\sigma_1 = s_1 + p$$
$$\sigma_2 = s_2 + p$$
$$\sigma_3 = s_3 + p$$

→

$$s_1 = c\cos\theta$$
$$s_2 = c\cos\left(\theta + \frac{2\pi}{3}\right)$$
$$s_3 = c\cos\left(\theta - \frac{2\pi}{3}\right)$$

what should we choose for c?

Scan of Trial Stress Space

set c = 1

$$\bar{s}_1 = \cos\theta$$
$$\bar{s}_2 = \cos\left(\theta + \frac{2\pi}{3}\right) \longrightarrow \bar{\phi} = \phi\left(\bar{\mathbf{s}}\right)$$
$$\bar{s}_3 = \cos\left(\theta - \frac{2\pi}{3}\right)$$

$$s_1 = \frac{\bar{\sigma}}{\bar{\phi}}\cos\theta$$
$$s_2 = \frac{\bar{\sigma}}{\bar{\phi}}\cos\left(\theta + \frac{2\pi}{3}\right) \longrightarrow \bar{\sigma} = \phi\left(\mathbf{s}\right)$$
$$s_3 = \frac{\bar{\sigma}}{\bar{\phi}}\cos\left(\theta - \frac{2\pi}{3}\right)$$

Scan of Trial Stress Space

algorithm

- pick $\bar{\sigma}$
- calculate $\bar{\phi} = \phi\left(\bar{\mathbf{s}}\right)$
- set $c = \bar{\sigma}/\bar{\phi}$
- sweep θ

$$s_1 = \frac{\bar{\sigma}}{\bar{\phi}} \cos\theta$$

$$s_2 = \frac{\bar{\sigma}}{\bar{\phi}} \cos\left(\theta + \frac{2\pi}{3}\right)$$

$$s_3 = \frac{\bar{\sigma}}{\bar{\phi}} \cos\left(\theta - \frac{2\pi}{3}\right)$$

$$\mathbf{s} = \sum_{i=1}^{3} s_i \hat{\mathbf{e}}_i \hat{\mathbf{e}}_i$$

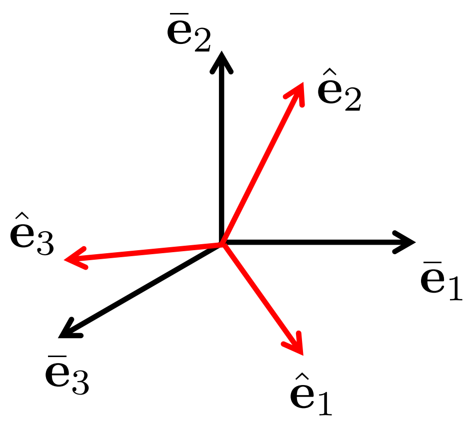
these stresses sweep out contours around the yield surface

Scan of Trial Stress Space

one final note…

$$\mathbf{s} = \sum_{i=1}^{3} s_i \hat{\mathbf{e}}_i \hat{\mathbf{e}}_i$$

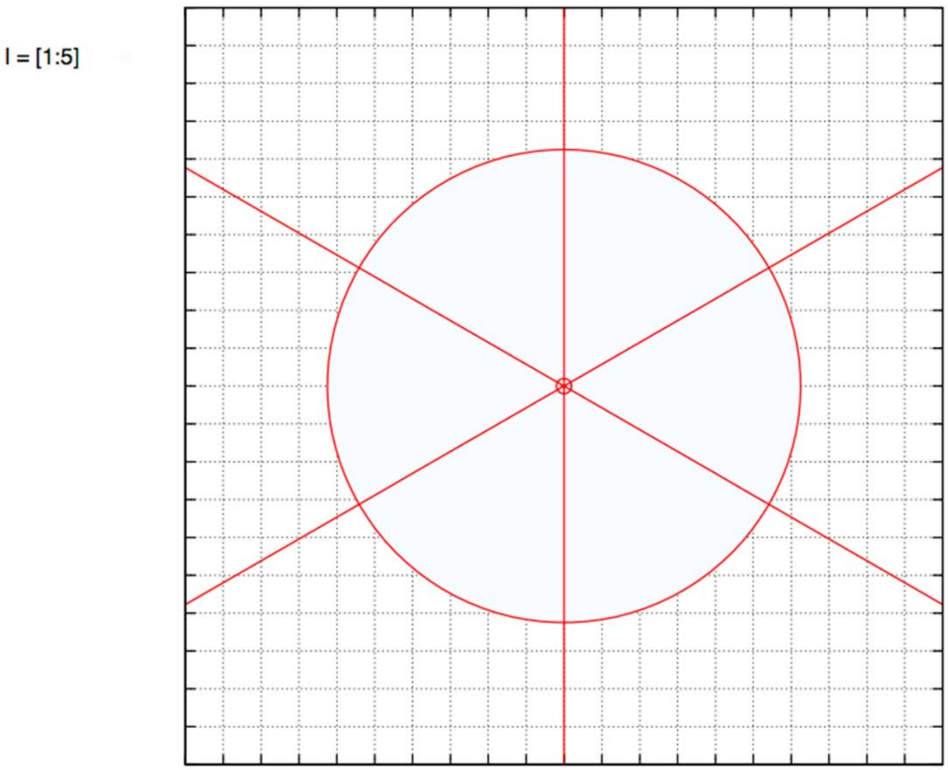
for isotropic models, the choice of eigenvectors does not matter, but for anisotropic models it does



The shape of the yield surface and the behavior of a return mapping algorithm depend on the direction of the principal stress axes (eigenvectors) relative to the material axes

von Mises

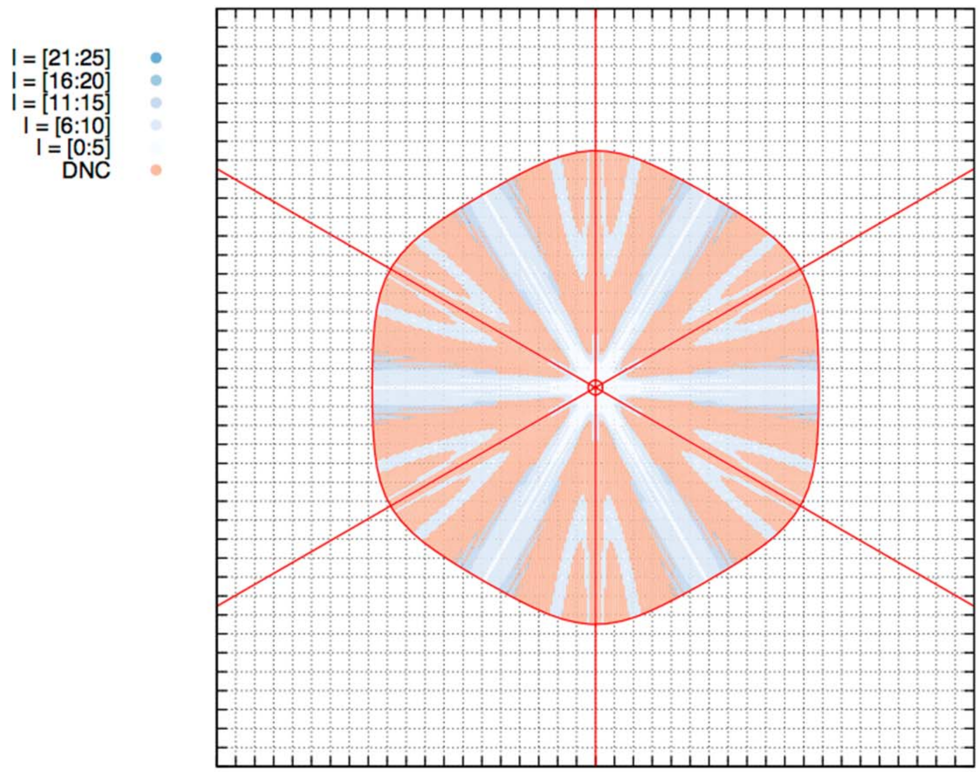
Newton



Newton algorithm for J_2 reduces to radial return

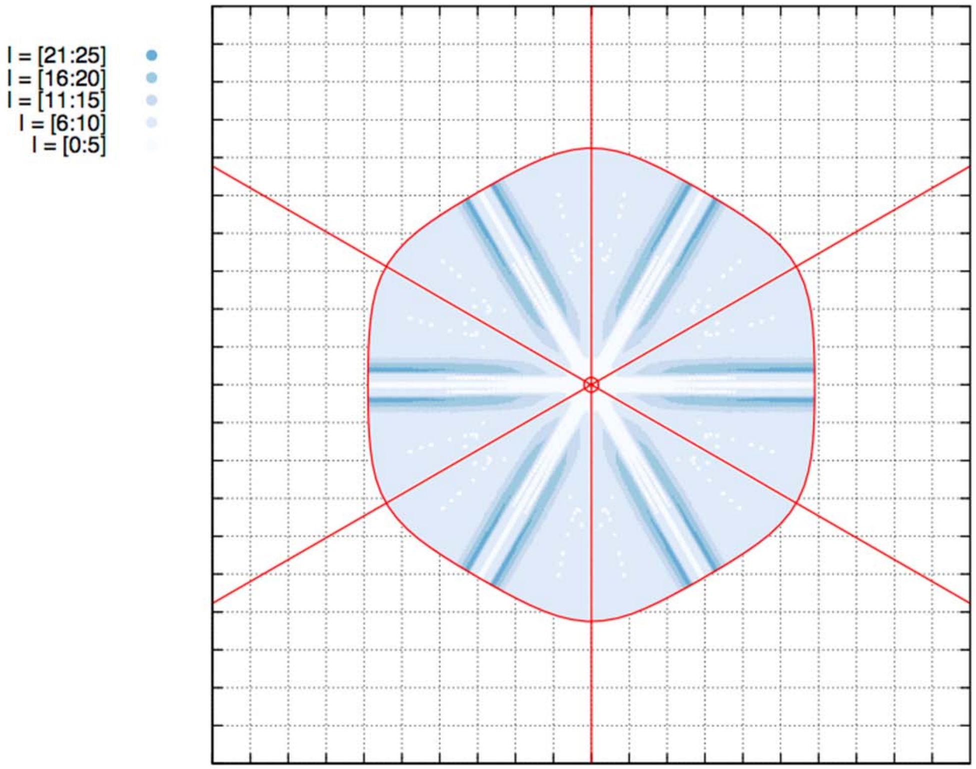
Hosford (a=8)

Newton



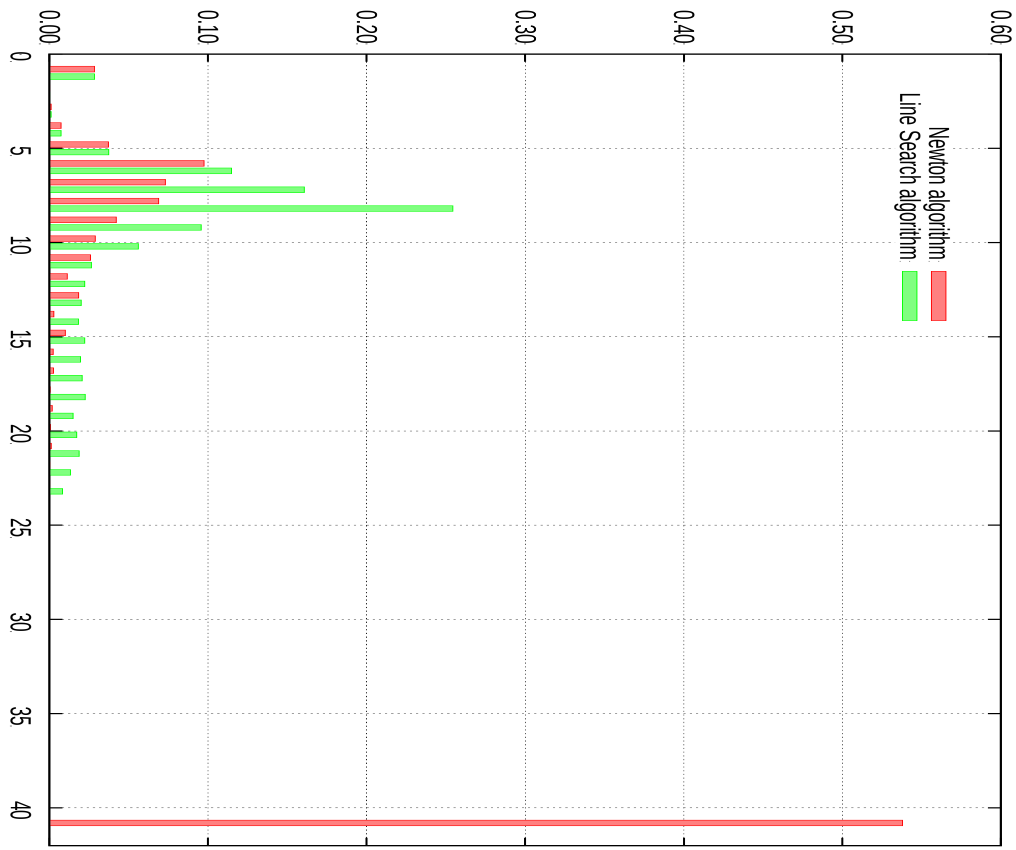
Hosford (a=8)

Line Search



Hosford (a=8)

iterations



Visualization

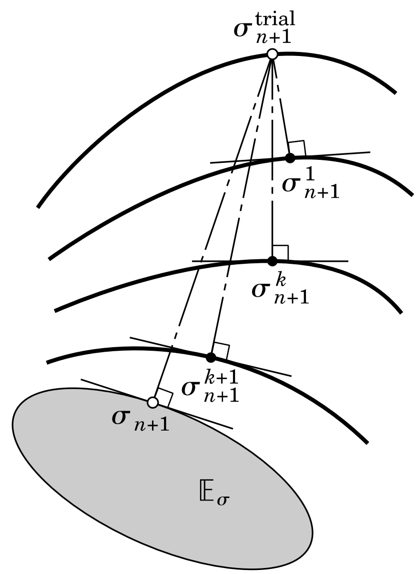
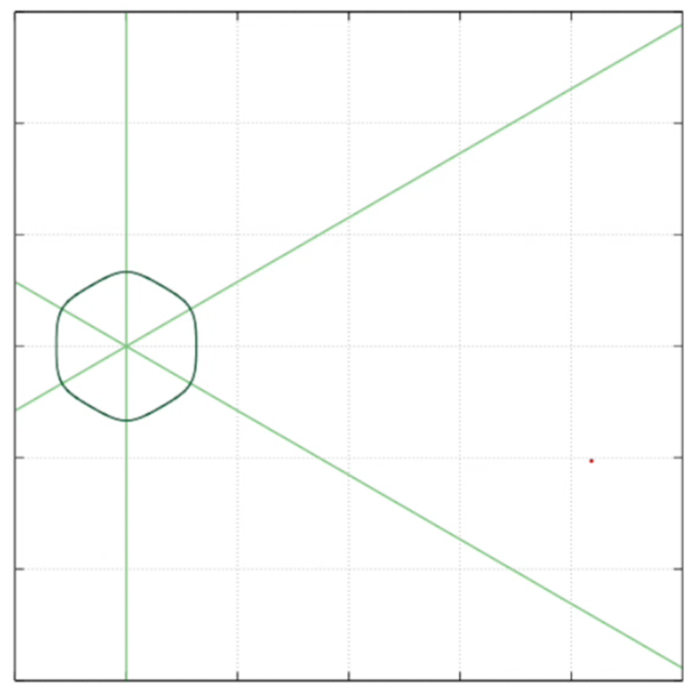


FIGURE 3-10. A geometric interpretation of the closest point projection algorithm in stress space. At each iterate $(\bullet)^{(k)}$, the constraint is linearized to find the intersection (cut) with $f = 0$. The next iterate $(\bullet)^{(k+1)}$, located on level set $f_{n+1}^{(k+1)} > 0$, is the closest point of that level set to the previous iterate $(\bullet)^{(k)}$ in the metric defined by the elasticities $\mathbf{C}$.

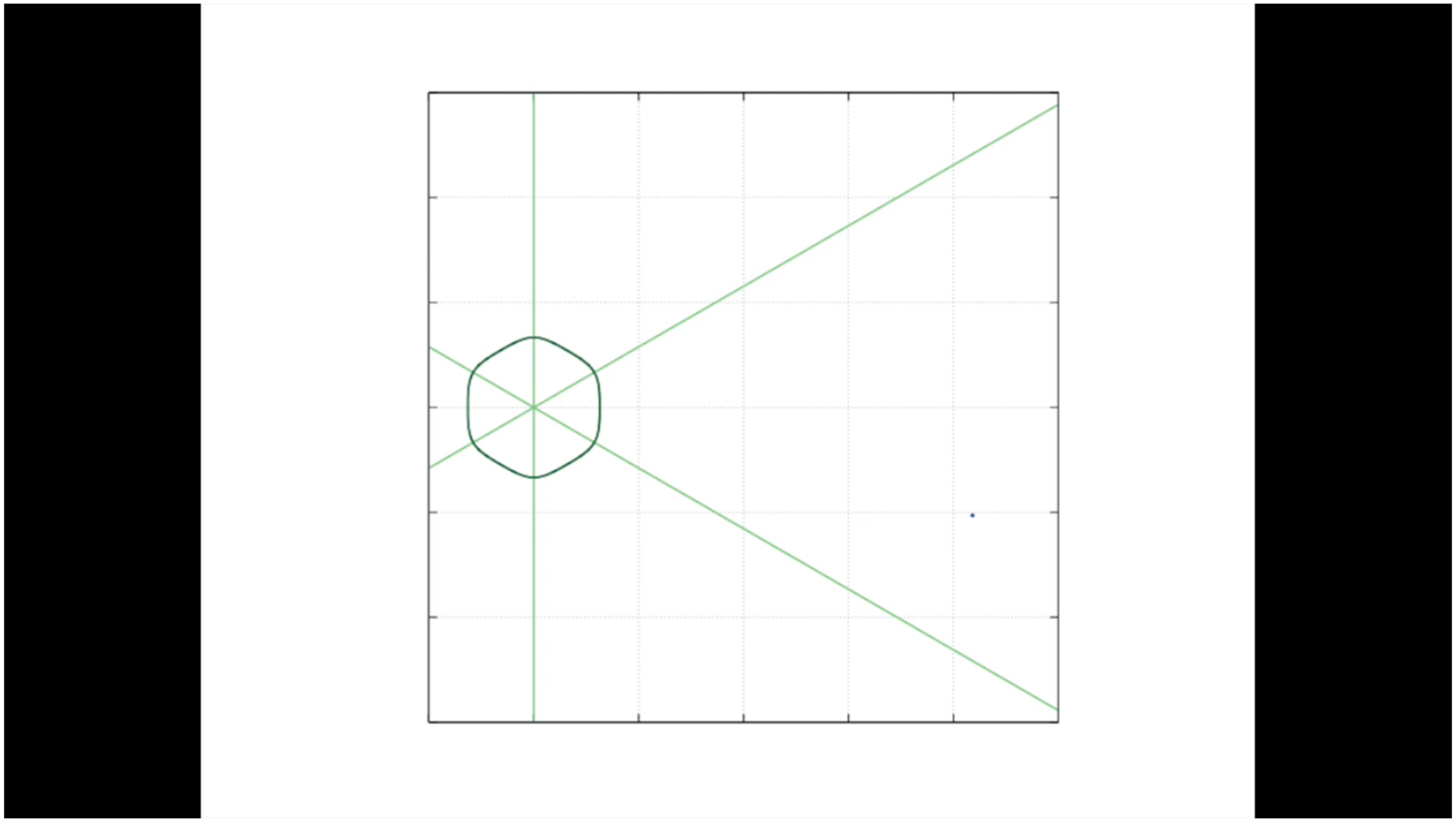
Hosford (a=8)

Newton algorithm

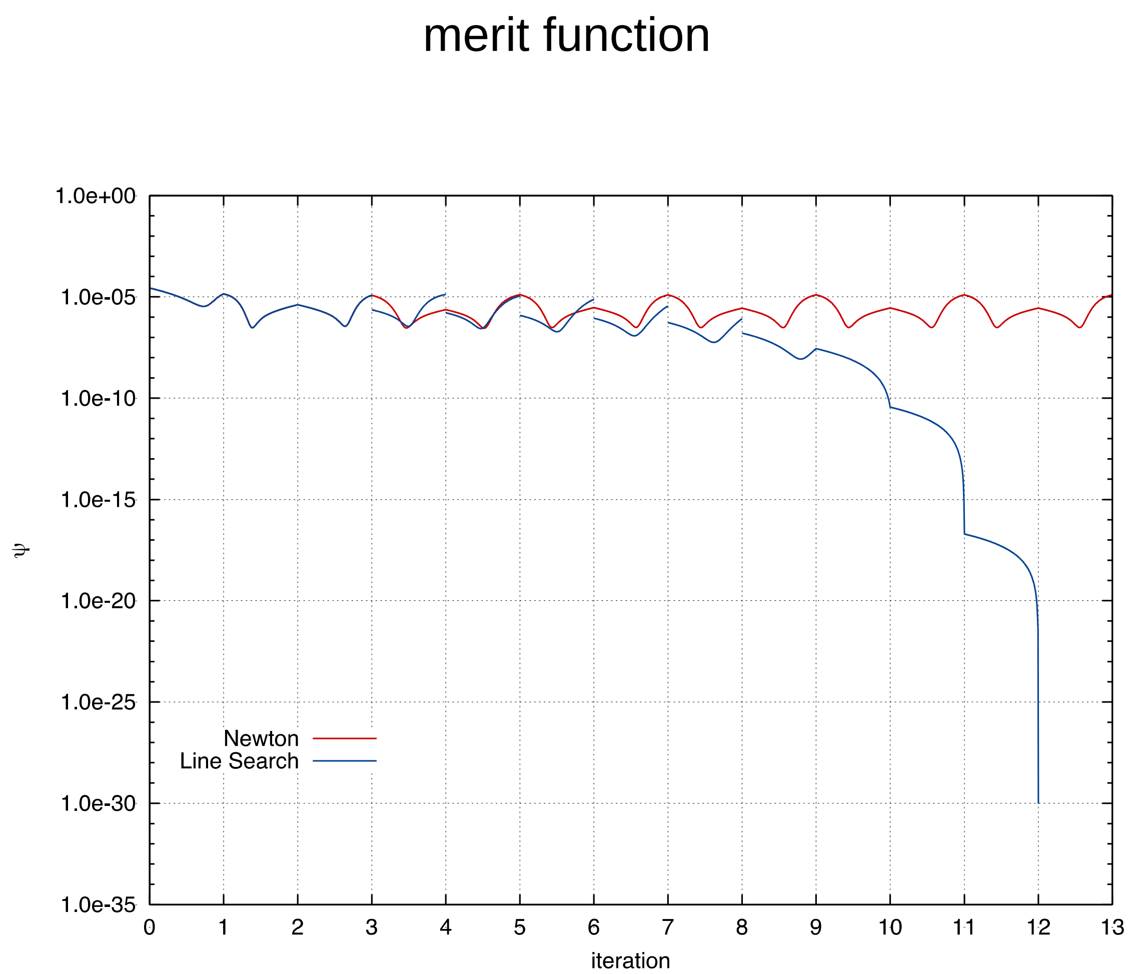


Hosford (a=8)

Line Search Algorithm

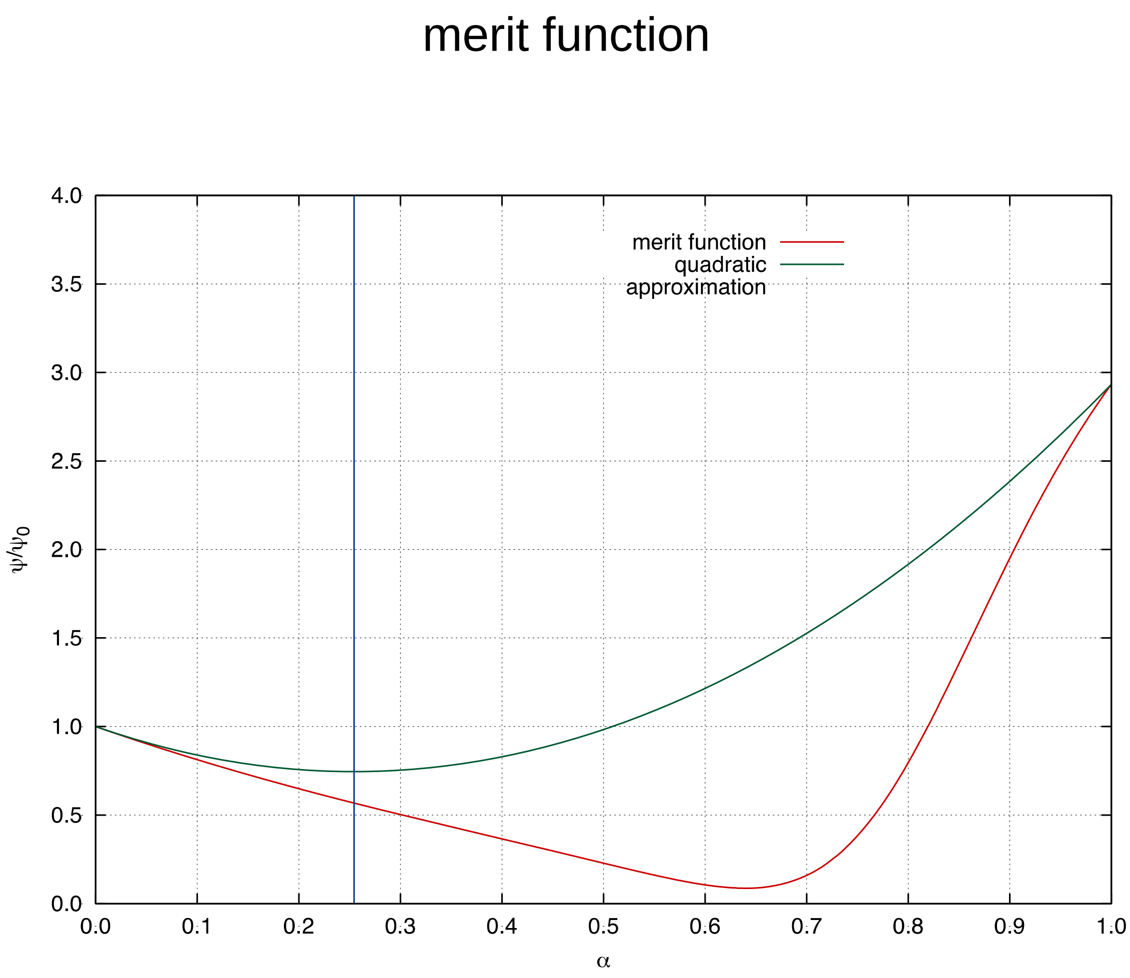


Hosford (a=8)



merit function
1.0e+00
1.0e-05
1.0e-10
1.0e-15
1.0e-20
1.0e-25
1.0e-30
1.0e-35
ψ
0
1
2
3
4
5
6
7
8
9
10
11
12
13
iteration
Newton
Line Search

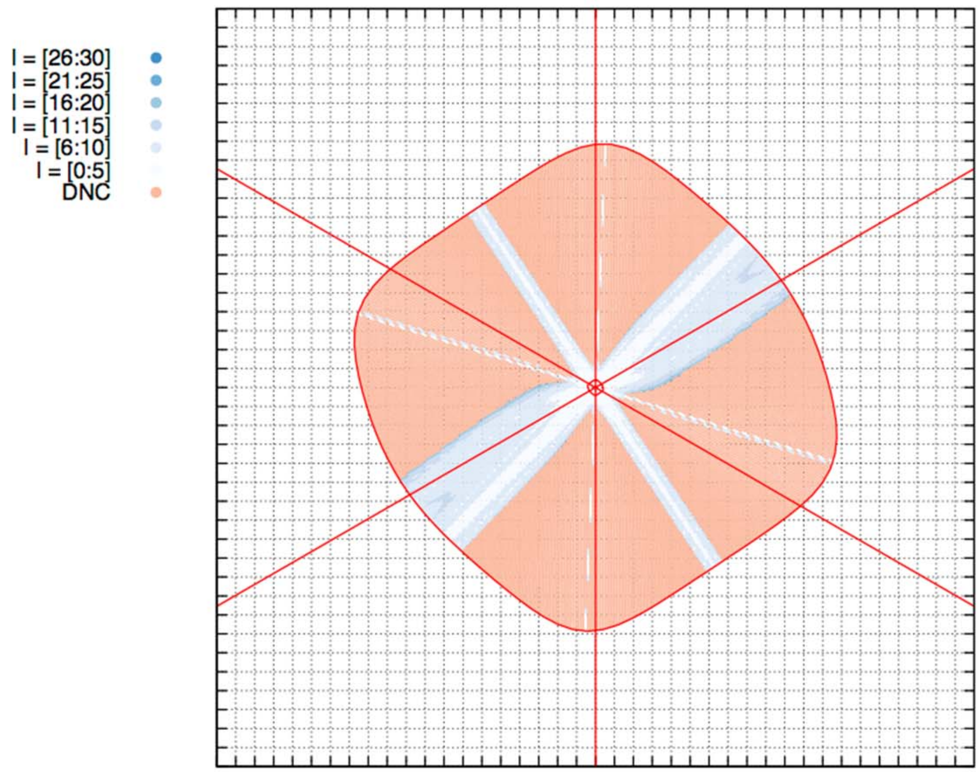
Hosford (a=8)



merit function
merit function
quadratic approximation
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
ψ/ψ0
α

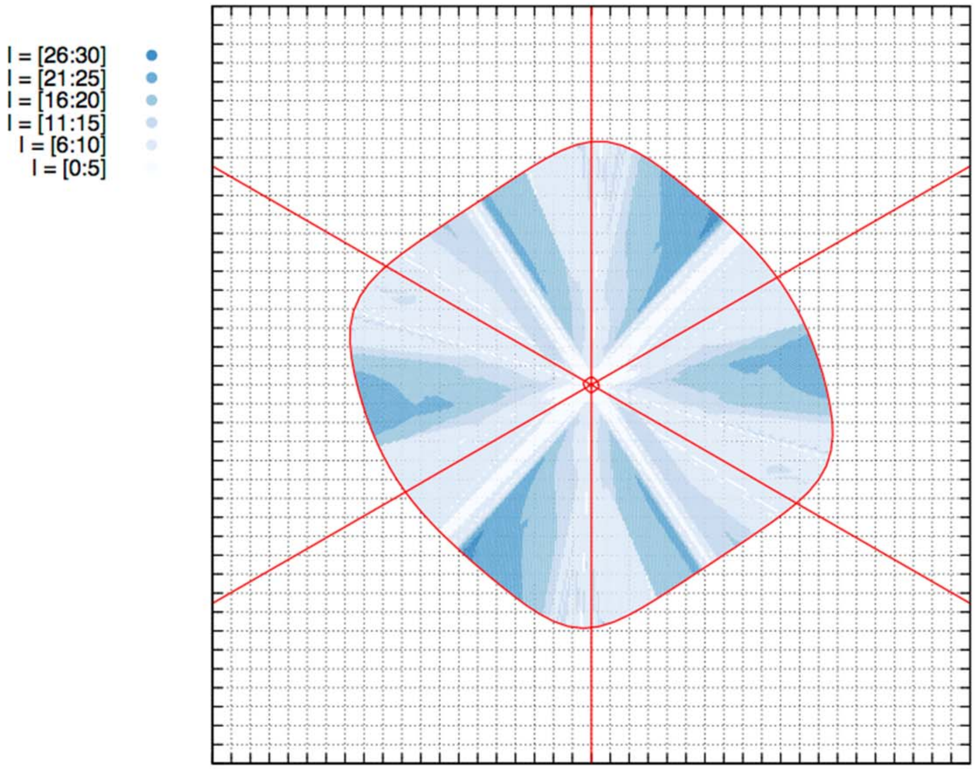
Barlat Model

Newton



Barlat Model

Line Search



Barlat Model

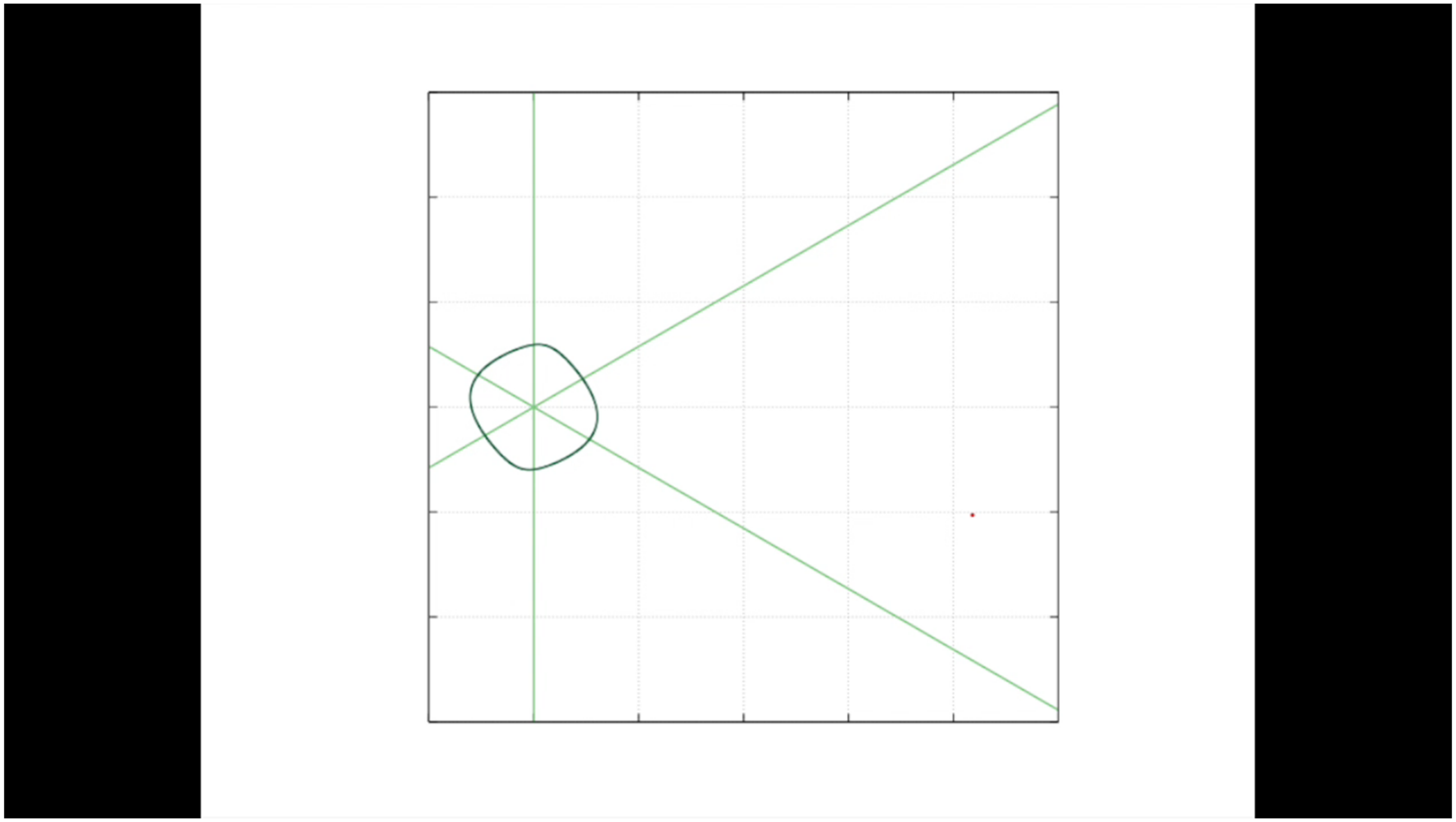
trial stress state for movies

$$\sigma_y = 200 \text{ MPa}$$

$$\boldsymbol{\sigma}^{tr} = \begin{bmatrix} 1000 & 20 & 300 \\ 20 & -300 & 150 \\ 300 & 150 & 100 \end{bmatrix} \text{ MPa}$$

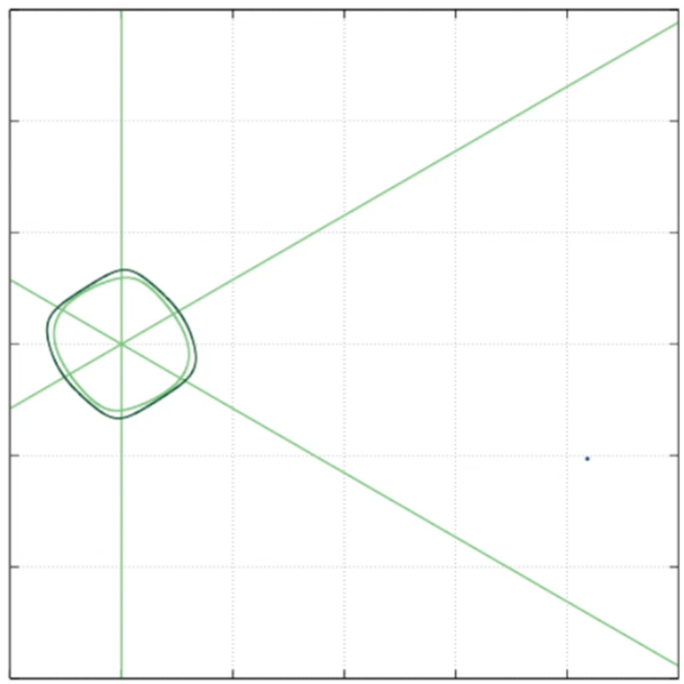
Barlat Model

Newton algorithm

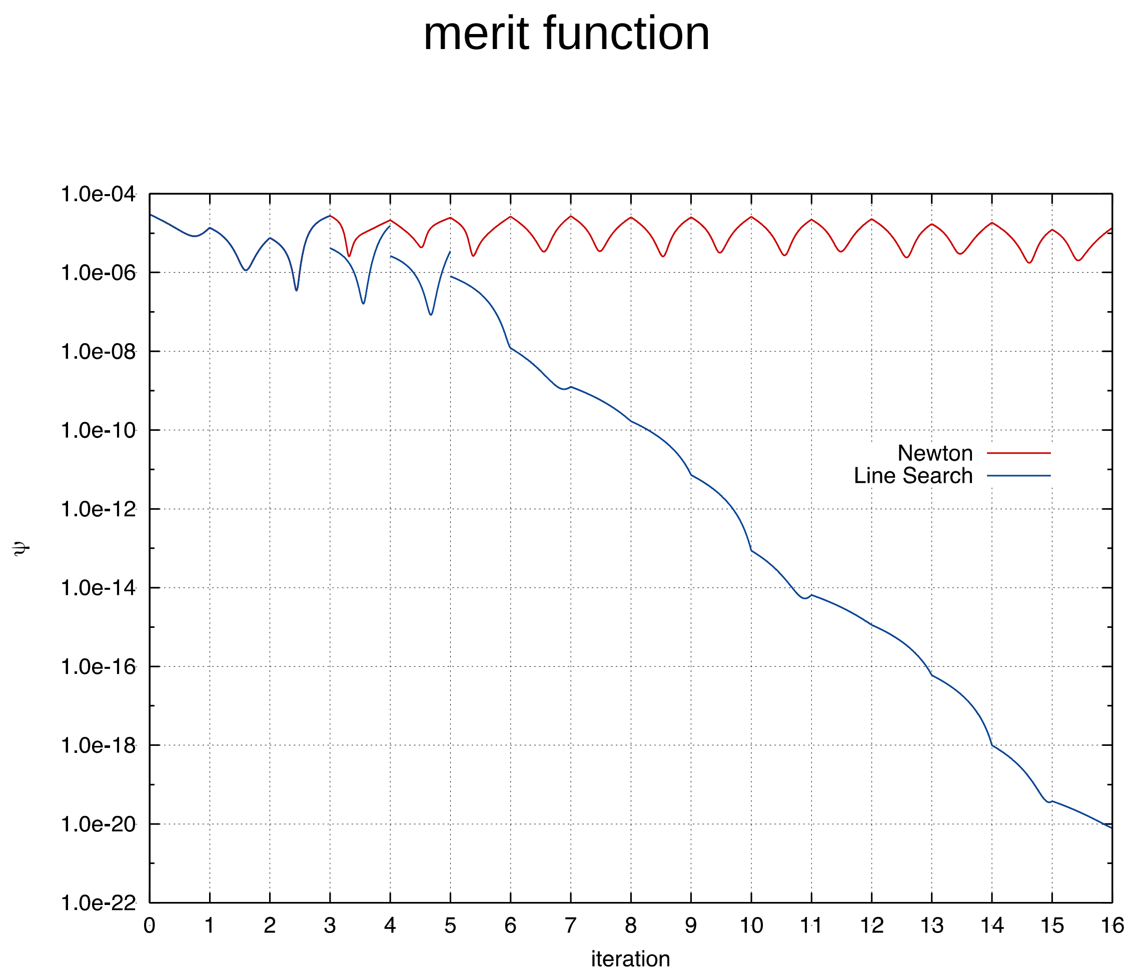


Barlat Model

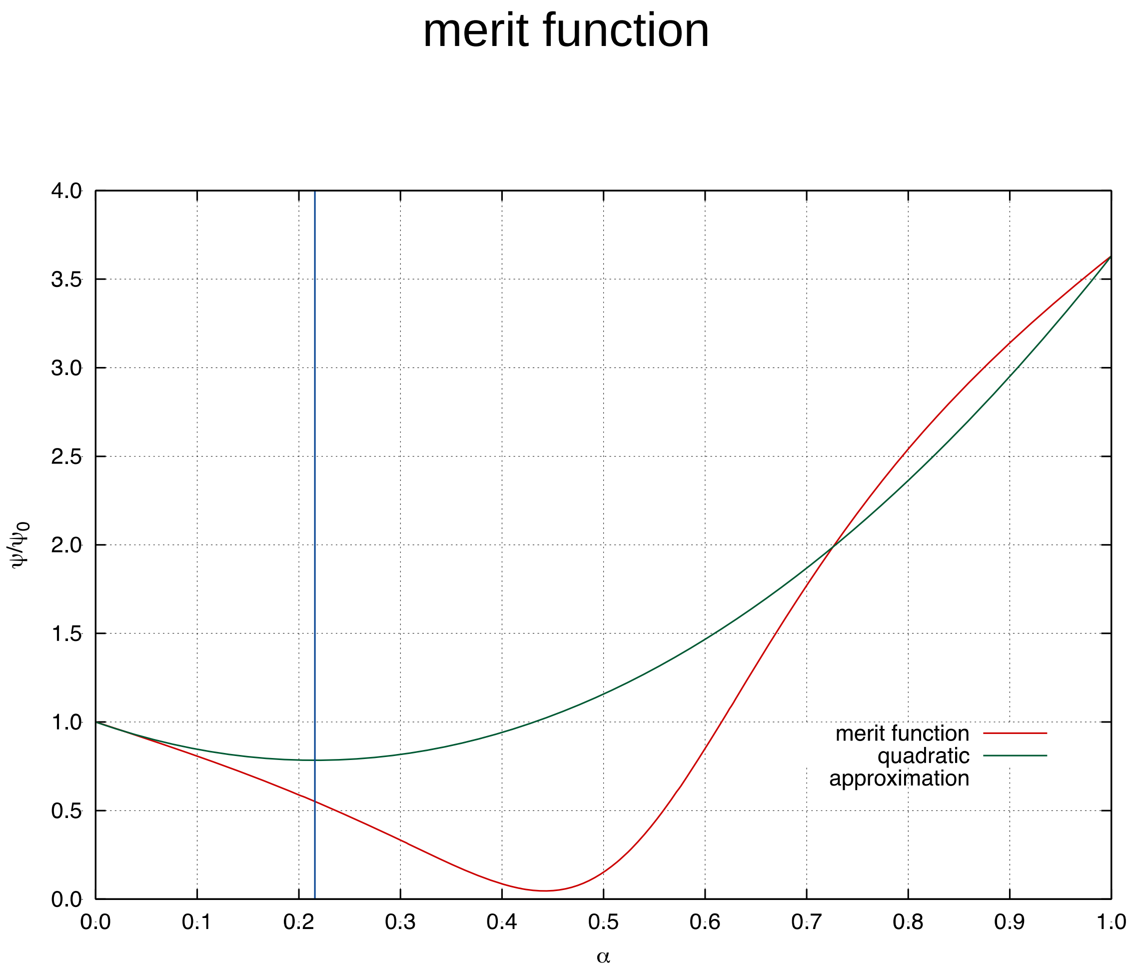
line search algorithm



Barlat Model



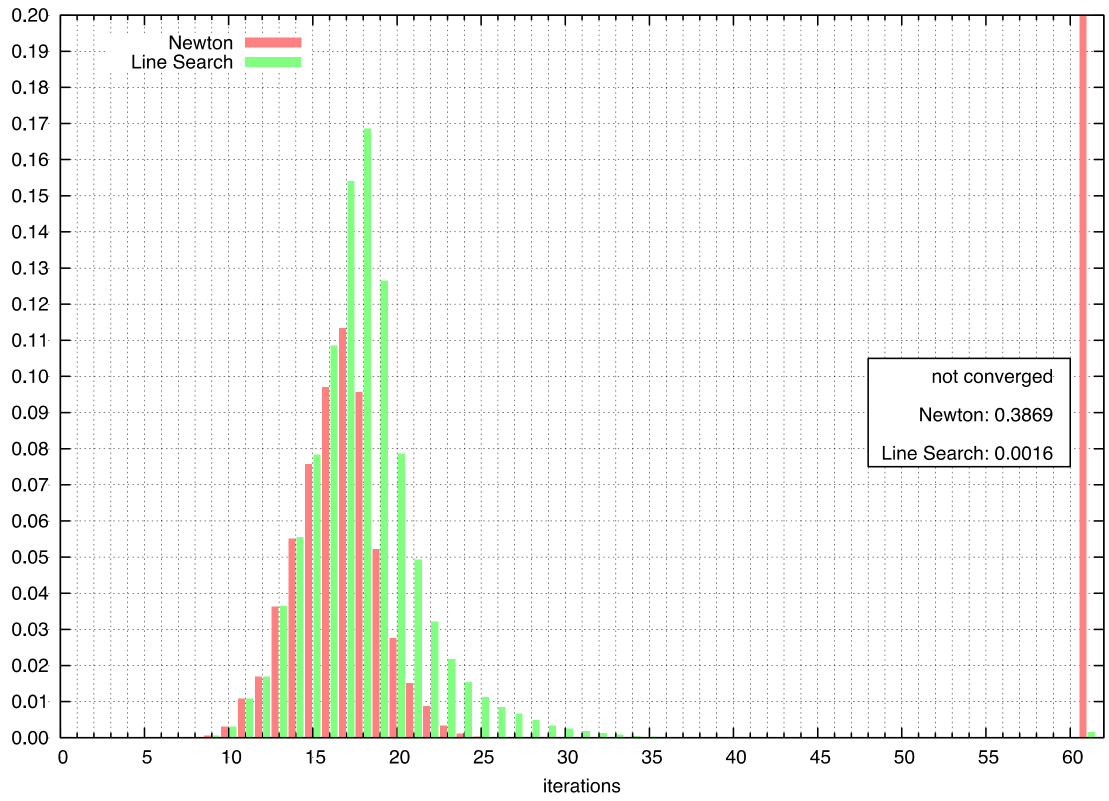
Barlat Model



Barlat Model

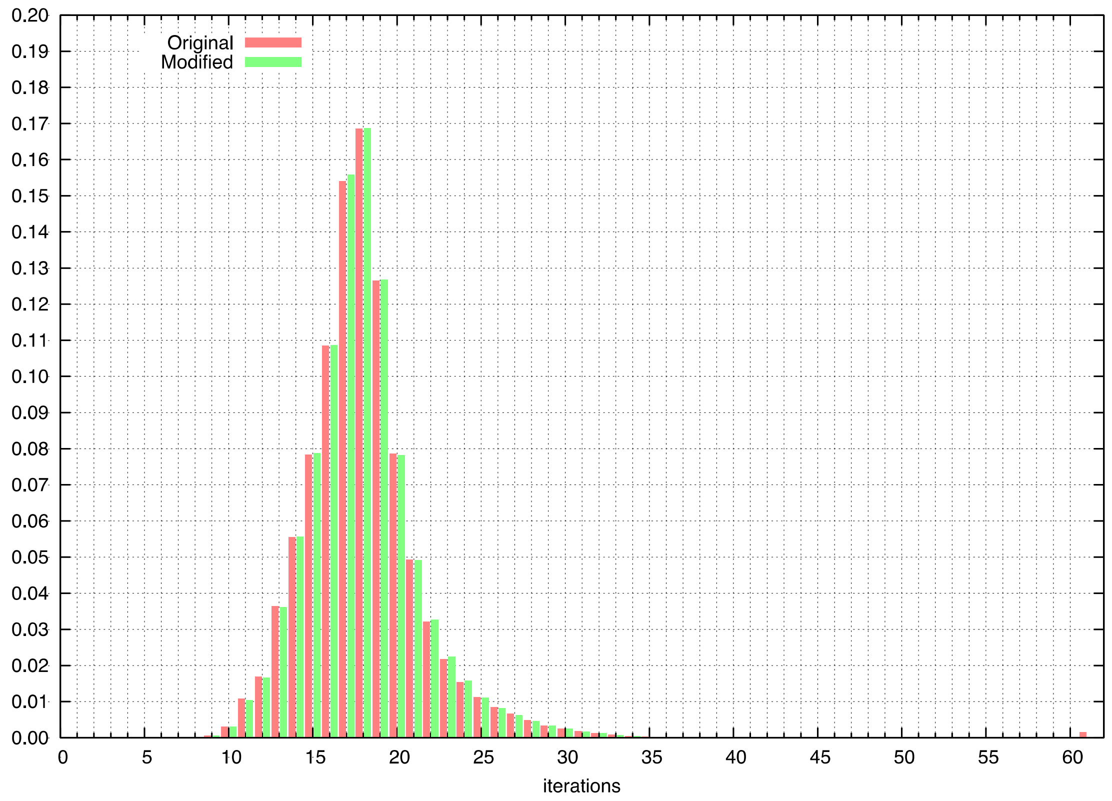
iterations

93,000,000 trial stress states



Barlat Model

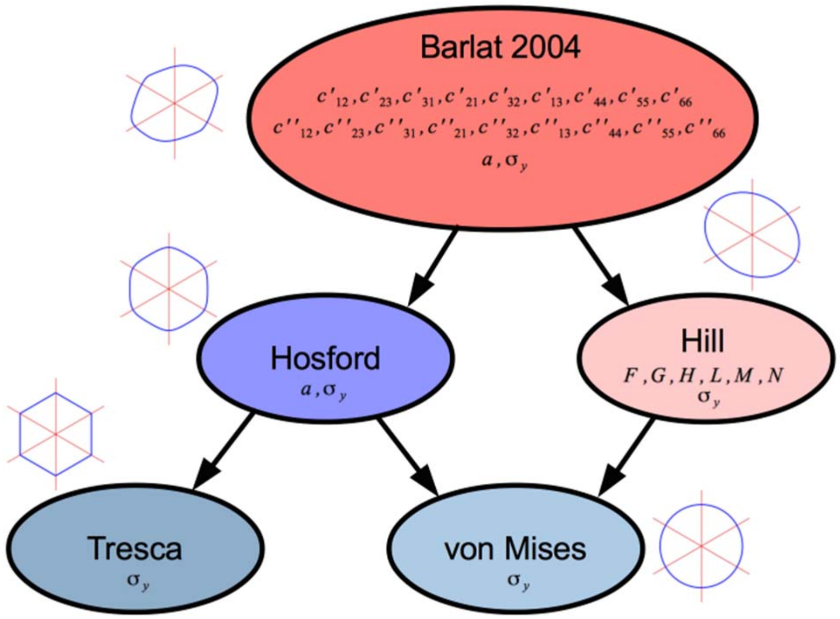
iterations



Questions

- How do we fit the plasticity models?
- What does the “hardening curve” mean?
- What do we do if the hardening observed experimentally doesn’t match the **form** of our model?
- How should we do failure?

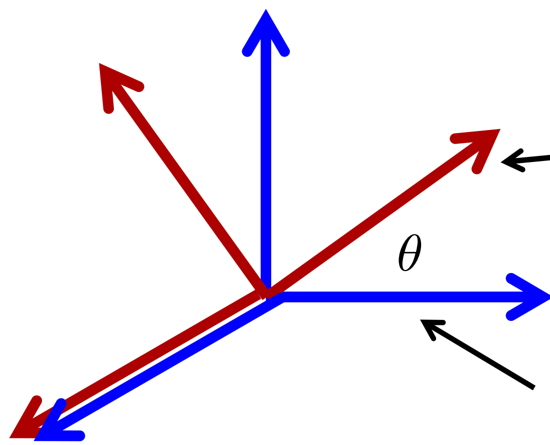
Model Hierarchy



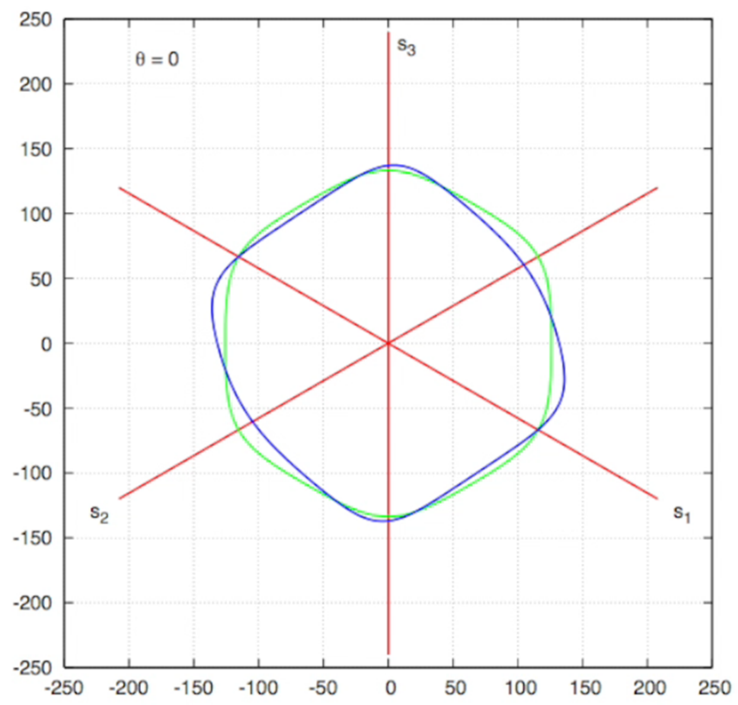
- UQ – can we use the Barlat model to same something **quantitative** about model form uncertainty?

Model Visualization

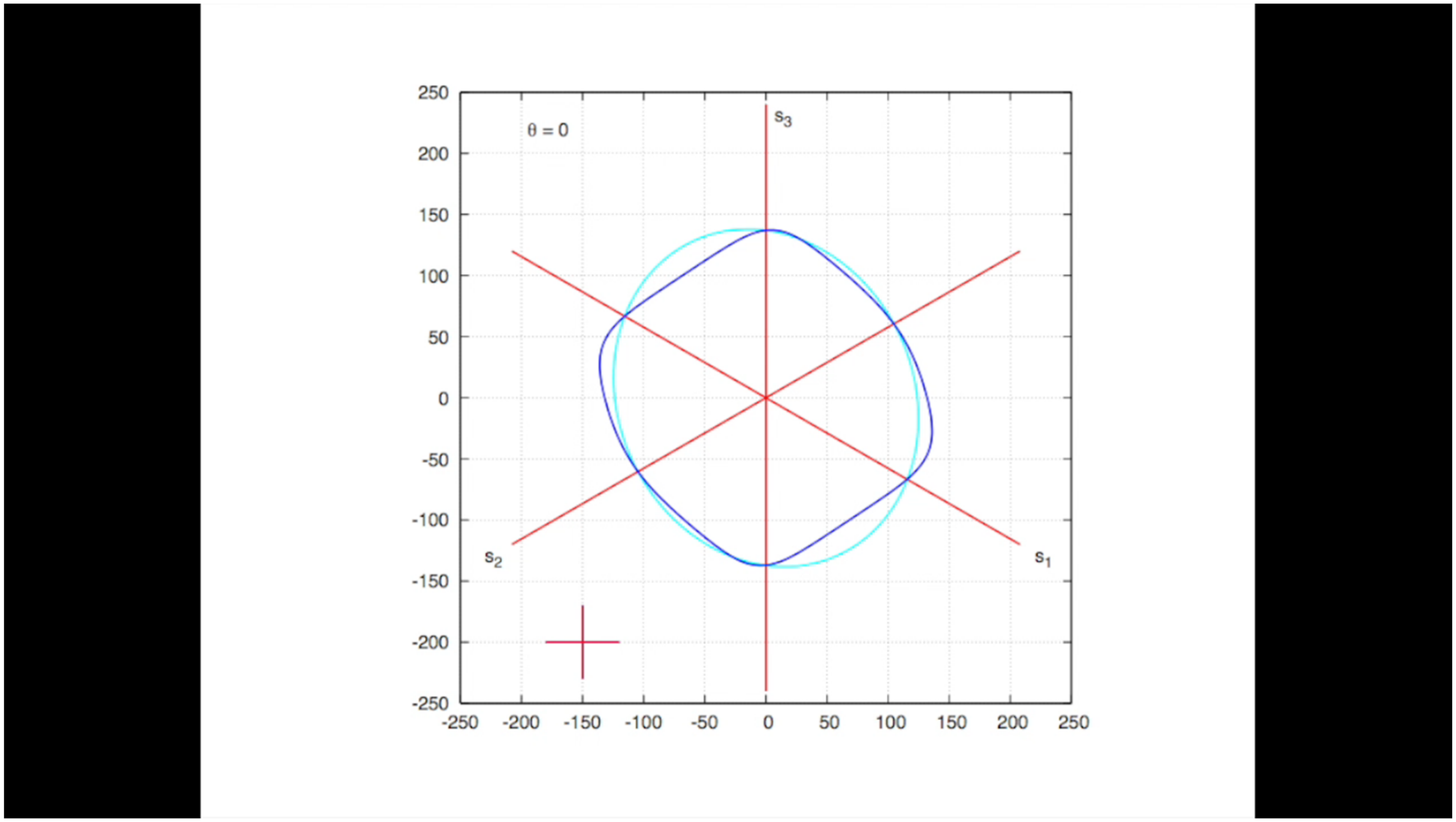
$$\boldsymbol{\sigma} = \sigma_{ij} \mathbf{e}_i \otimes \mathbf{e}_j$$
$$= \sum_{k=1}^{3} \sigma_k \hat{\mathbf{e}}_k \otimes \hat{\mathbf{e}}_k$$



Model Visualization



Model Visualization



Model Usage - Hosford

elastic properties

yield function exponent

hardening model

```
BEGIN PARAMETERS FOR MODEL HOSFORD_PLASTICITY
  YOUNGS MODULUS          = <real>
  POISSONS RATIO          = <real>
  A                       = <real>
  YIELD                   = <real>
  HARDENING MODEL         = <string>
  HARDENING MODULUS       = <real>
  HARDENING CONSTANT      = <real>
  HARDENING EXPONENT      = <real>
  HARDENING FUNCTION      = <string>
END PARAMETERS FOR MODEL HOSFORD_PLASTICITY
```

default value for **A** is 4, i.e. von Mises

Model Usage - Hill

elastic properties

yield function

hardening model

default values for **R**'s are 1

```
BEGIN PARAMETERS FOR MODEL HILL_PLASTICITY
  YOUNGS MODULUS        = <real>
  POISSONS RATIO        = <real>
  R11                   = <real>
  R22                   = <real>
  R33                   = <real>
  R12                   = <real>
  R23                   = <real>
  R31                   = <real>
  YIELD                 = <real>
  HARDENING MODEL       = <string>
  HARDENING MODULUS     = <real>
  HARDENING CONSTANT    = <real>
  HARDENING EXPONENT    = <real>
  HARDENING FUNCTION    = <string>

  ...

END PARAMETERS FOR MODEL HILL_PLASTICITY
```

Model Usage - Hill

orientation

```
BEGIN PARAMETERS FOR MODEL HILL_PLASTICITY

  ...

  COORDINATE SYSTEM                   = <string>
  DIRECTION FOR ROTATION              = <integer>
  ALPHA                               = <real>
  DIRECTION FOR SECOND ROTATION       = <integer>
  SECOND ALPHA                        = <real>
END PARAMETERS FOR MODEL HILL_PLASTICITY
```

Model Usage - Barlat

elastic properties

yield function

default value for **A** is 4

default values for **CP**'s are 1

```
BEGIN PARAMETERS FOR MODEL BARLAT_PLASTICITY
  YOUNGS MODULUS          = <real>
  POISSONS RATIO          = <real>
  A                       = <real>
  CP12                    = <real>
  CP13                    = <real>
  CP21                    = <real>
  CP23                    = <real>
  CP31                    = <real>
  CP32                    = <real>
  CP44                    = <real>
  CP55                    = <real>
  CP66                    = <real>

  ...

END PARAMETERS FOR MODEL BARLAT_PLASTICITY
```

Model Usage - Barlat

yield function

default values for **CPP**'s are 1

```
BEGIN PARAMETERS FOR MODEL BARLAT_PLASTICITY
  ...

  CPP12                    = <real>
  CPP13                    = <real>
  CPP21                    = <real>
  CPP23                    = <real>
  CPP31                    = <real>
  CPP32                    = <real>
  CPP44                    = <real>
  CPP55                    = <real>
  CPP66                    = <real>

  ...

END PARAMETERS FOR MODEL BARLAT_PLASTICITY
```

Model Usage - Barlat

hardening model

```
BEGIN PARAMETERS FOR MODEL BARLAT_PLASTICITY

  ...

  YIELD                     = <real>
  HARDENING MODEL           = <string>
  HARDENING MODULUS         = <real>
  HARDENING CONSTANT        = <real>
  HARDENING EXPONENT        = <real>
  HARDENING FUNCTION        = <string>

  ...

END PARAMETERS FOR MODEL BARLAT_PLASTICITY
```

Model Usage - Barlat

orientation

```
BEGIN PARAMETERS FOR MODEL BARLAT_PLASTICITY

  ...

  COORDINATE SYSTEM                 = <string>
  DIRECTION FOR ROTATION            = <integer>
  ALPHA                             = <real>
  DIRECTION FOR SECOND ROTATION = <integer>
  SECOND ALPHA                      = <real>
END PARAMETERS FOR MODEL BARLAT_PLASTICITY
```

Model Usage – Coordinate System

```
BEGIN COORDINATE SYSTEM <string>
  TYPE = <string>RECTANGULAR|CYLINDRICAL|SPHERICAL
  ORIGIN = <string>
  VECTOR = <string>
  POINT = <string>
  ORIGIN NODE = <string>
  VECTOR NODE = <string>
  POINT NODE = <string>
END COORDINATE SYSTEM <string>
```

```
DEFINE POINT <string> WITH COORDINATES <real> <real> <real>
```

- These commands are in the “sierra” scope

Conclusions

- new isotropic/anisotropic descriptions
- robust integration algorithm
  - capability can be used for other yield surfaces
- can we extend to viscoplastic models?
- can we get **quantitative** model form error and UQ?
- can we model anisotropic hardening and failure?